Experiencing 3d Simulated Space Through Different Perspectives

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Theoretical Background

The contribution of technology to the teaching and learning of geometry is perceived to be strongly linked with interactivity, multiple interlinked representations, including symbolic ones, dynamic manipulations, and dynamic visualizations (Laborde et al. 2006). However, relatively little research has been carried out on the way the above distinct characteristics of digital media can be exploited so as to engage students in meaningful 3d geometry investigations. Aiming to understand the way in which students' intuitions and ideas concerning spatial visualization and thinking (Presmeg 2006; Arcavi 2003) are challenged in 3d digital media, we developed a set of microworlds and a set of activities adopting a constructionist theoretical perspective (Kafai and Resnick 1996). A distinct feature of the microworlds was that they were "half-baked" (Kynigos 2007), i.e., incomplete or buggy digital artifacts that students had to investigate how they work and to change and fix them.

Our pedagogical aim was to engage the students in navigating a moving entity, the turtle, to construct graphical digital objects through Logo programming and the dynamic manipulation of procedure variable values in a 3d simulated space. Research seems to conclude that carefully designed Logo-based microworlds are an effective medium in offering rich mathematical experiences and encouraging the construction of meaning in 2d through the turtle metaphor (Clements and Sarama 1997; Kynigos 1992). Navigating the turtle requires the formation of essentially novel methods of spatial orientation, where the reference point is not the position of the user's body but the turtle's body, relative to which the entire system of orientation may change. In this framework, body-syntonicity is a critical concept in 2d Turtle Geometry (Papert 1980) that refers: (a) to navigating the turtle by coordinating one's body-posture,

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physically or imaginary, with the turtle-vehicle of motion and (b) to solving geometrical problems drawing upon ones embodied motional experiences.

Concurrently, students have to reconceptualize geometrical figures in terms of specific Logo commands, according to the distinct characteristics of Turtle Geometry (Papert 1980; Abelson and DiSessa 1981). Turtle geometry is based on a different geometrical system to those usually associated with the learning of geometry and it has been characterized as differential by Papert (1980) and as intrinsic by Abelson and DiSessa (1981) (see also Kynigos 1993). It is considered as differential since a given geometrical state of the turtle is fully defined by its relation to the turtle's immediately previous state. In a similar vein, it is characterized as intrinsic in the sense that there is no need to refer to places outside the turtle's immediate vicinity when deciding on an input to a procedure to change turtle's state. Recent extensions of Turtle Geometry in 3d space do not offer just a new perspective in the teaching and learning of geometry. New issues are raised related to the way the turtle metaphor is put to use and the way deeply rooted intuitions about experiencing space and locomotion can be exploited so as to make sense of geometric notions (Kynigos and Latsi 2007).

Recently there has been clear research interest on the perceptions students have in 3d virtual environments (Hauptman 2010) and the spatial dimensions of interactions though 3d avatars (Petrackou 2010). In mathematics education, these kinds of technological advances are investigated as far as their influences on students' learning are concerned (Hollebrands et al. 2008; Jones et al. 2010). However, to the best of our knowledge, there is little research on understandings formed by students using digital media, such as MachineLab Turtleworlds (MaLT), integrating symbolic Turtle Geometry with dynamic manipulation of the user's viewpoint of the 3d simulated space and the opportunities these media offer to revitalize the teaching of 3d geometry. The aim of our research was to investigate: (a) the way the students used the software's functionalities of changing viewpoints throughout the construction processes, (b) the interplay between the turtle metaphor and space visualization through various viewpoints, and (c) the interplay between the perception of figures considered in relation to different viewpoints and in relation to their geometric properties.

The Computational Environment

MaLT is a programmable environment for the creation and exploration of interactive virtual reality simulations developed within the ReMath project (ReMath 2005). MaLT was conceived as a constructionist microworld environment within MachineLab that extends the "Turtleworlds" turtle geometry to 3d geometrical space. Thus, an extension of Logo commands in 3d space is provided including the two conventional types of turtle turns (Reggini 1985): "uppitch/downpitch n degrees" (*up/dp n*), which pitches the turtle's nose up and down, and "leftroll/rightroll n degrees" (*lr/rr n*), which moves the turtle around its trunk/vertical axis. However, the distinct feature of MaLT is that the Logo-based Turtle Geometry is integrated with the dynamic

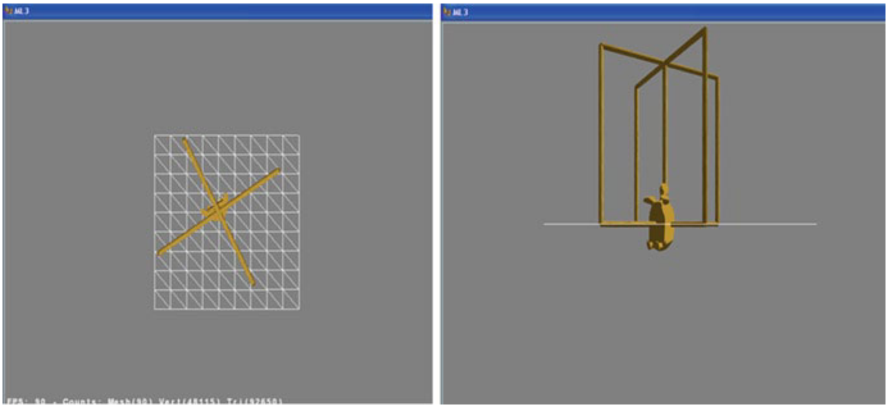


Fig. 1 The top–down and the side view of the simulated 3d space

Fig. 2 The 1d variation tool on the *left* and the active vector tool on the *right*

manipulation of interactive graphical representations – a functionality characteristic of Dynamic Geometry Environments. In particular, the dynamic manipulation tools available can be divided in two categories:

- Dynamic manipulation of the viewpoint of the 3d space: (a) by using toolbar's buttons where the user can pick among three default views (front, side, top–down, as shown in Fig. 1) and (b) by manipulating through mouse a specially designed vector tool, called the active vector, where the user can define either camera's direction or camera's position (see Fig. 2).
- Dynamic manipulation of graphical figures by means of sequentially changing the variable values of the programs they create them through the use of specially designed variation tools (see Fig. 2).

Methodology

Espousing an interpretive approach in educational research (Cohen et al. 2007) in the study reported here we followed a design-based research method (Van Den Akker et al. 2006), which entailed the "engineering" of tools and task, as well as the systematic study of both the process of learning and the means of supporting it (Gravemeijer and Cobb 2006). A critical component of design-based research is that the design is conceived not just to meet local needs but to advance a theoretical agenda, to uncover, explore and confirm theoretical relationships, and to create new theoretically expressed understandings about areas for which little is known. Thus, the analysis we have carried out does not comprise any kind of quantification of qualitative data, but rather refers to a nonmathematical process of interpretation, carried out for the purpose of discovering concepts and relationships in raw data and then organizing these into a theoretical explanatory scheme.

The research took place in the sixth grade of a public primary school in Greece. The class consisted of 23 pupils, who had totally 16 45 min teaching sessions with the experimenting teacher over 2 months. The pupils worked collaboratively in mixed-gender groups of two or three in the school's computer laboratory. The tasks were designed to bring in the foreground issues concerning the mathematical nature of 3d geometrical objects through their dynamic manipulation and transformation in mathematically meaningful ways. In particular, we divided the activity sequence in two phases and we developed for each one of them a strand of two tasks. In task 1, the pupils were asked to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft. In task 2 the pupils were asked to construct rectangles in at least two different planes of the graphical space of MaLT simulating the adjacent walls of a virtual room. In the second strand of activities, the pupils experimented with half-baked microwords. In particular, in task 3 the pupils were asked to use the 1d variation tool to control and experiment with the three variables that corresponded to different turtle turns in the half-baked microworld "Movedoor" (see Fig. 3), so as to create the simulation of a door opening and closing. The procedure was designed to have more than the variables needed. First the pupils had to decide what the role of each variable was and which values could be given to them. Then they had to build upon the half-baked microworld so as to develop a procedure that creates the simulation of a door opening and closing with the least possible variables.

In task 4, the pupils were asked to use the 1d variation tool to control the four variables corresponding to turtle turns in the "half-baked" microworld "Revolving door," so as to create the simulation of a revolving door (see Fig. 3). The procedure was designed to have more than the variables needed. First the pupils had to decide what the role of each variable was and which values could be given to them. Then, they had to build upon the half-baked microworld so as to develop a procedure that creates the simulation of a revolving door with the least possible variables. Finally, the pupils were asked to extend the procedure of the revolving door in order to create a simulation of the fan of a watermill. During the teaching sequence, the experimenting teacher intervened in the children's work by posing questions and encouraging them to clearly explain their ideas and strategies.

In order to describe the pupils' learning trajectories as they happened in real time, we adopted a participant observation methodology while the main corpus of data included video-recorded observational data, the experimenting teacher's observational notes as well as the sorting and archiving of the corpus of the pupils, work on and off computer. As far as the pupils' work on the computer is concerned we used a specially designed screen capture software – called Hypercam – which allowed us to record pupils' voices and at the same time to capture all their actions

```
to movedoor :a :b :c
uppitch(:a)
leftroll(:b)
repeat 2 [forward(7) right(:c) forward(4) right(:c)]
end
```

```
to revolving door :a :b :c :d
uppitch(:a)
leftroll(:b)
repeat 4 [repeat 2 [forward(7) right(:c) forward(4) rt(:c)] leftroll(:d)]
end
```

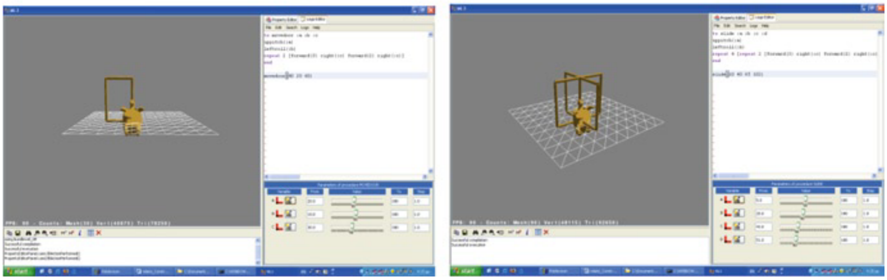


Fig. 3 The Logo code of the "Movedoor" and the "Revolving door" half-baked microworlds

on the screen. For the analysis we transcribed verbatim the audio recordings of all groups of pupils throughout the teaching sequence. Data were categorized in clusters of specific critical episodes that do not represent some quantifiable entity but are chosen to represent clearly the kind of activity that was going on in specific time in the classroom. The results presented here are based on the work of one group, consisted of one boy and one girl, and focusing on the way the viewpoint manipulation tools were used during the construction processes.

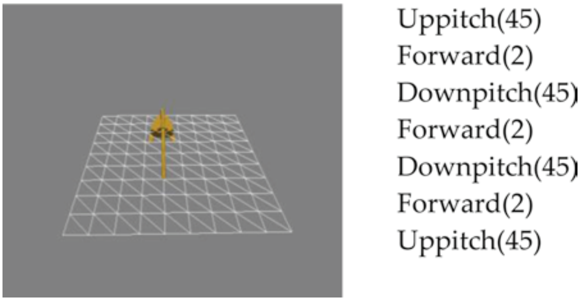
Construction Processes Through Different Perspectives

The analysis of our results has shown that the pupils' construction processes could be divided in two categories: construction processes through an intrinsic perspective and construction processes through an extrinsic perspective, depended on the point of focus and the way the simulated 3d space was experienced. This division rather reflects the two dominant perspectives people take on space (Tversky 2005); an external one when they observe space and they manipulate objects in it and an internal one when they explore an environment and when they navigate in it.

Construction Processes Through an Intrinsic Perspective

The results of the present research underline the importance of syntonizing one's body with the 3d turtle – vehicle of motion in the 3d simulated space. During the

Fig. 4 Simulating the take-off and landing of an aircraft along the *z* axis and the respective Logo code



construction processes of task 1 the pupils preferred "flying" the turtle along the *z* axis that gave the impression of depth, at a plane vertical to the display plane defined by the 2d computer screen. Moreover, they kept on working on the default front view (although slightly slanted through the use of the active vector manipulation tool) even though they did not have a clear representation of the turtle's journey (see Fig. 4).

It seems possible that the children preferred flying the turtle along the *z* axis (that gave the impression of depth) while viewing the simulated 3d space from the default front view since this way they could more easily coordinate the various frames of reference (Wickens et al. 2005) present. In order to drive the turtle in a body-syntonic way the pupils had to coordinate the following frames of reference: (a) the ego frame, defined in terms of the orientation of the trunk or location of the observer, (b) the display frame, defined in terms of the standard way of referring to things presented in the computer screen, where the right/left up/down directions are fixed, (c) the world frame, defined in terms of the fixed directions of "up" and "down," as a result of the gravitational effect, and (d) the vehicle frame of reference, defined in terms of the place and orientation of a moving entity, here the turtle.

Flying the turtle along the *z* axis, the orientation of the vehicle of motion, the turtle, coincided both with the orientation of the pupils' body in the lived-in 3d space and with the standard way of referring to the orientation of information on the computer screen as well as with the world frame of reference. The pupils' comments corroborate this result. When asked why they preferred this kind of flight they replied: "If we wanted to turn the turtle right or left, we could see from our hands. If we wanted to turn it right, let's say, we would think where our hand is and we would send it to the right."

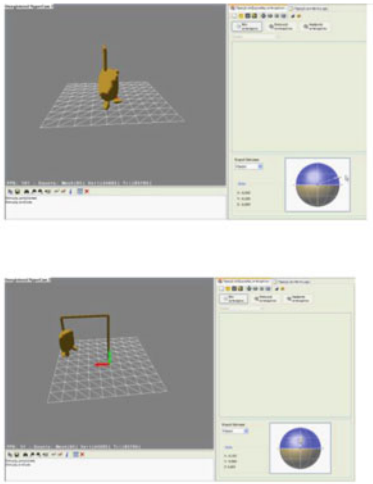
It is interesting that the children are focusing more on body-syntonicity while not being sidetracked by the visual effects even though only an inclined line – corresponding to the "taking off" of the turtle – was clearly visible on the computer screen. This result comes in contrast to the findings of other researches in the framework of 3d computational environments that have noted pupils' preference in working in a plane parallel to the computer's screen display plane (Kynigos and Latsi 2006, 2007). Working in a plane parallel to the display plane is considered closer to pupils' experiences with 2d figures in school textbooks or with 2d Logo and would eliminate the convention used in the representation of the 3d space. However, it seems that the kind of task and the metaphor used was of critical importance: the aim was

Episode 1:

S1: Fd, more Fd, more…we should go it forward 0.5. Does it have 0.5?

S2: Wait, wait the turtle is here.

S1: Yes, but it hasn't touch the ground. Has it? Wait I have to see it. (He changes the view from side to front through the active vector tool and continues forwarding the turtle.)



Up(180)
Dp(45)
Dp(45)
Fd(4)
Rt(90)
Rt(90)
Rt(90)
Rt(90)
Rt(90)
Fd(4)
Bk(1)
Bk(1)
Bk(1)
Fd(1)
Fd(1)
Fd(1)
Dp(45)
Rt(90)
Fd(1)
Fd(1)

Fig. 5 *First column*: Episode 1. *Second column*: Changing viewpoints. *Third column*: The respective Logo code, up to the point of the construction Episode 1 is referring to

S1: Let's see how many doors there are if the value is 720 (He plays with the 1d variation tool changing the values of the d variable). *Only one? This perspective is not convenient, I will change it* (He activates successively all the 3 default views and he opts for the top–down one).
S2 Yes, exactly like in the case of 360. It turns two rounds.

Fig. 6 Episode 2

not to construct just a slanted line or a geometrical figure but to simulate the take-off and the landing of the turtle – aircraft. In this framework, the use of the commands *uppitch/downpitch* as well as the motion of the turtle along the *z* axis, that gave the impression of depth, was rather more easily syntonized with everyday experiences and representations of flying aircrafts.

In the following tasks the pupils used extensively both the default 2d views and the active vector during their construction processes. It could be suggested that the various viewpoint manipulation tools were especially used: (a) when a bricolage construction strategy was adopted (episode 1, Fig. 5) and (b) when the pupils were experimenting with specific aspects of the half-baked microworlds (episode 2, Fig. 6). In episode 1 (see Fig. 5), the pupils are trying to construct "a wall" during task 2, giving commands to the turtle while using visual cues without having a clear strategy in mind. Their trial and error strategy is evident in the number of commands given to the turtle while they were trying to construct a parallelogram. It seems that every command is related only to the turtle's previous position and not to the whole

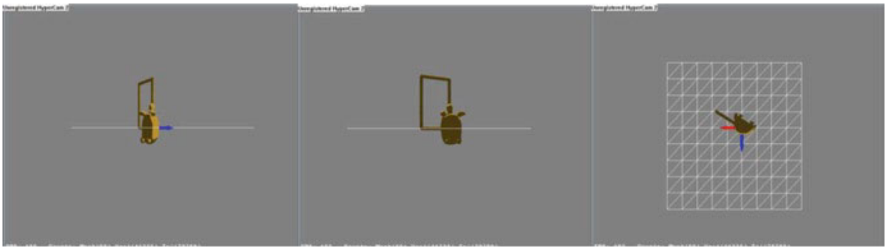


Fig. 7 The three default views of the revolving door half-baked microworld when the value of the *d* variable is 720

construction process and the figure's geometric properties. When it was not visually clear if they had constructed a closed figure, the pupils did not resort to the geometrical object's properties (e.g., that the opposite sides of the rectangular figure should have equal lengths) but to the viewpoint manipulation tools, so as to check if the figure was closed. Then they proceeded again forwarding the turtle little step by little step. The bricolage construction strategy followed by the pupils could not be attributed only to personal styles in programming (Turkle and Papert 1990) or to their Logo inexperience (Kafai 1995), but also to the intrinsic characteristics of Turtle Geometry. It seems that programming through the turtle metaphor promotes initially a step by step construction where emphasis is given on "guiding" the moving entity in relation to its immediately previous state rather than on the geometrical properties of the constructed objects. Following a step by step construction in 3d space, the pupils used multiple views of their construction so as to get the necessary information before giving the turtle the next command.

During task 4, the pupils initially experimented with the values of the variables of the half-baked microworld "Revolving door." They had extra difficulties in finding out the role of the: *d* variable, which determined the measure of turtle's turning and respective position in the 3d space before drawing each successive door of the revolving door model. It follows that the *d* variable determined also the position of the four rectangle doors in the 3d space as well as their position in relation to one another.

In episode 2 (Fig. 6), the pupils are conjecturing about the number of the visible rectangles (doors), if the value given to *d* is 720. However, they do not find the front default view convenient and after testing all the available default views (see Fig. 7), they choose to continue working with the top–down view active, where the number of the doors created by the turtle was more clearly visible. It should be also stressed that the preferred default view offered pupils a simplified 2d representation that possibly helped them focus on particular aspects of their construction: the turtle's rolling around its axis and the number of rectangle doors that in the top–down view were represented by line segments.

In sum, it could be argued that the pupils have initially preferred a body-syntonic way of navigating the turtle while opting for particular views that facilitated body-syntonicity, physically or imaginary. The emphasis on body-syntonicity with the turtle and generally the focus on the intrinsic characteristics of turtle geometry is rather depicted not only on the bricolage construction strategies followed but also on the preferred views that seem to have helped the pupils experience 3d space through an intrinsic perspective: get "immersed" to the 3d space through the turtle metaphor or explore it through multiple views according to challenges faced. However, as the pupils' construction strategies shifted to more analytic ones it seems that they ceased being so "immersed" in the 3d space, while starting paying more attention to the graphical results of the turtle's navigation. The multiple views that children used during their construction processes (e.g., episode 2, see Fig. 6) could not be interpreted only as a way of exploring the 3d space but also as a way of oscillating between focusing on turtle's navigation through an intrinsic perspective of the 3d space and focusing on the graphical results of its motion through an extrinsic perspective, a result that is treated in the next paragraph.

Construction Processes Through an Extrinsic Perspective

When using the turtle metaphor, pupils have to pass from the management of turtle's spatial movements to the construction of a graphic object (Fein et al. 1987), while making a distinction between the agent and the object, between the navigation of the turtle and the result of this navigation, the geometrical object. In parallel, pupils have to coordinate two different viewpoints: the viewpoint of the turtle which must be moved in an appropriate way so as to draw a figure and the viewpoint of an external observer who looks at the figural results of turtle's movement. The results of the present research suggest that as the activities unfolded, the pupils progressively adopted an extrinsic perspective of the 3d space, observing it as external viewers.

In the end of task 2 there was some free time available and the pupils spontaneously decided to try construct a closed figure building upon their experimentation during task 1. They were able to combine the flights they have previously constructed. Each take-off and landing of the turtle was used as the building block of a "peculiar" figure that came as result of four repeats of the initial turtle's journey, while turning the turtle 90° before each reexecution. It is also interesting – as it is evident in episode 3 (Fig. 8) – that the pupils adopted a more analytic programming strategy, visualizing the whole turtle's journey and explaining it to each other before entering commands to the microworld. Moreover, when they returned to the microworld they did not insert and execute the Logo commands one by one but they inserted and executed a group of Logo commands.

Another interesting point was that before starting their construction, the pupils adjusted their viewpoint through the active vector, so that there was a clear sense of perspective of the simulated 3d space (see Fig. 8). They then continued working on their construction keeping this viewpoint stable. However, this was not an occasional choice as the pupils followed the same strategy during the construction processes of the fan of the watermill during task 4: They adjusted their viewpoint so as to have again a sense of perspective (see Fig. 9) and they kept it stable throughout the whole construction process. When asked why they preferred this view, the pupils just

Episode 3:

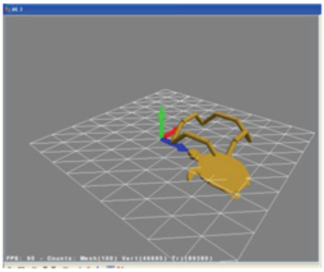
S1: Would you like to make a design?

S2: To make a circle?

S1: To go this way and then this way and again so.

S2: Let's make a triangle. First it goes this way and then it comes back. No, I have an idea, to insert 45 so as to go this way and then again 45 so as to go this way and then again 45 (They are showing on the screen and they are using their hands so as to simulate turtle's journey).

S1: Let's make a rhombus. So not right 45 but right 90 (So far they were talking to each other and now they return to the microworld inserting the commands).



Uppitch(45)
Forward(2)
Downpitch(45)
Forward(2)
Downpitch(45)
Forward(2)
Uppitch(45)
Right(90)

Fig. 8 *First column*: Episode 3; *Second column*: The closed figure constructed by the students; *Third column*: The respective logo commands that were executed 4 times

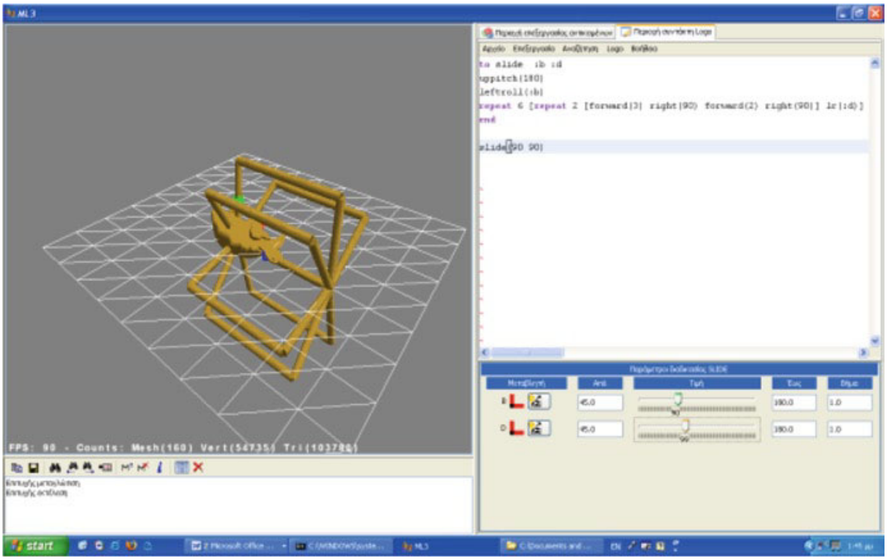


Fig. 9 The view preferred during task 4

replied: "It is more convenient because we can view the whole object." But the question that arises is: Why did the pupils keep on working with a fixed view during 3d constructions that seem to necessitate a high degree of spatial visualization and orientation? For instance would not it be easier or more body-syntonic to change viewpoints in order to decide turtle's turning before each reexecution of turtle's flight during the construction of the closed figure (episode 3, Fig. 8)? What were the reasons for this change as far as the use of the viewpoint manipulation tools is concerned as the activities unfolded? It seems that as the pupils got progressively more accustomed to the 3d turtle's motion and the software's representational infrastructure, they were not so much concerned about body-syntonicity and that it was more important for them to have a clear sense of the 3dness both of the simulated space and of the simulated objects.

Constructing the simulation of a 3d object while viewing the simulated space in perspective was probably more realistic and familiar. However, it could be also conjectured that the pupils preferred a fixed viewpoint during their constructions so as not to change position as observers and to have, thus, a stable point of reference which would probably be less cognitively demanding (Yakimanskaya 1991). A fixed 3d view rather gave the pupils a sense of space constancy, especially in cases that they adopted an analytic design strategy, as in episode 3, where they mentally visualized the whole turtle's journey before executing the relative commands so as to construct the figure. Thus, it could be argued that as the construction process became more complicated, the pupils preferred to view space from an extrinsic perspective, as external observers, focusing more on programming and geometric properties while taking into account the whole 3d space.

Conclusions

This study has tried to show that the way the available viewing angle manipulation tools were used was in a constant interplay both with the task at hand and with the construction strategies followed. When the focus was on turtle's navigation and orientation in 3d space, the body-syntonic metaphor (Papert 1980) came to the foreground while space was experienced through an intrinsic perspective (Tversky 2005): the user was immersed in space and was trying to view it from inside. In this case, the pupils used various viewpoints which helped them face specific challenges and focus on particular aspects of their construction. The intrinsic perspective and the use of multiple viewpoints seem also to be adopted in cases where a bricolage construction strategy was followed, when the pupils had not a clear idea about the actions that should be taken and when the construction was progressing command by command through trial and error. Thus, it seems that this intrinsic perspective of the simulated 3d space is rather associated – among the others – with an emphasis on the intrinsic characteristics of Turtle Geometry (Abelson and DiSessa 1981), where a given geometrical state of the turtle is fully defined by its relation only to the turtle's immediately previous state.

As the activities unfolded and as the pupils shifted focus from the management of turtle's spatial movements to the construction of a graphic object, they had started experiencing space through an extrinsic perspective, through the viewpoint of an external observer who looked at the figural results of the turtle's movement. In this case a fixed 3d view was less cognitive demanding and offered pupils both a realistic effect of familiar objects, and space and shape constancy. Moreover a holistic/external view of the 3d space was in accordance with analytic construction strategies, where the pupils were trying to visualize the turtle's journey taking into account the whole 3d space and constructed objects' geometrical properties before executing any commands on the computer. It goes without saying that there were not clear cut borders between the two perspectives and that there were a lot of instances that pupils oscillated between them according to their construction focus. This research was a tentative effort in appreciating an aspect of the large spectrum of the representational potential of a specific 3d microworld in the context of constructionist activities. Highly visual 3d Turtle Geometry microworlds, such as MaLT, seem to influence not only the kind of geometrical problems posed to students but most importantly the way students interact with the medium and the solution processes followed by them. However, a lot of further research is needed in order to investigate the way mathematical concepts can be integrated with spatial navigation and orientation in virtual environments, as well as in order to investigate the way these computational environments can be used in educational design.

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