Chapter 1
Introduction

The representation theory of finite groups is a subject going back to the late eighteen hundreds. The pioneers in the subject were G. Frobenius, I. Schur, and W. Burnside. Modern approaches tend to make heavy use of module theory and the Wedderburn theory of semisimple algebras. But the original approach, which nowadays can be thought of as via discrete Fourier analysis, is much more easily accessible and can be presented, for instance, in an undergraduate course. The aim of this text is to exposit the essential ingredients of the representation theory of finite groups over the complex numbers assuming only knowledge of linear algebra and undergraduate group theory, and perhaps a minimal familiarity with ring theory.

The original purpose of representation theory was to serve as a powerful tool for obtaining information about finite groups via the methods of linear algebra, e.g., eigenvalues, inner product spaces, and diagonalization. The first major triumph of representation theory was Burnside's pq-theorem. This theorem states that a non-abelian group of order $p^a q^b$ with p, q prime cannot be simple, or equivalently, that every finite group of order $p^a q^b$ with p, q prime is solvable. It was not until much later [2, 14] that purely group theoretic proofs were found. Representation theory went on to play an indispensable role in the classification of finite simple groups.

However, representation theory is much more than just a means to study the structure of finite groups. It is also a fundamental tool with applications to many areas of mathematics and statistics, both pure and applied. For instance, sound compression is very much based on the fast Fourier transform for finite abelian groups. Fourier analysis on finite groups also plays an important role in probability and statistics, especially in the study of random walks on groups, such as card shuffling and diffusion processes [3, 7], and in the analysis of data [7, 8]; random walks are considered in the last chapter of the book. Applications of representation theory to graph theory, and in particular to the construction of expander graphs, can be found in [6]. Some applications along these lines, especially toward the computation of eigenvalues of Cayley graphs, are given in this text.

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