

# Chapter 1

## Introduction

Suppose one has an amount of money  $M$  to be invested for a year at an annual interest rate  $I$  compounded continuously, i.e., total worth of the investment at the end of the year is the limit of what results from compounding  $2, 4, 8, 16, \dots$ , in each instance at equal time intervals. During the year, the net worth is continually growing, the rate of earning at each time is proportional to the current value. As the year progresses, the value of the account changes, but the *law* governing earning does not change. This is an example of an autonomous system. One-parameter semigroups in the problems to follow deal with such autonomous systems.

This book consists of a sequence of problems which develop a variety of aspects in the area of one-parameter semigroups of transformations. It is written in the ‘Discovery Style’ (Moore Method, Texas Method, Inquiry Based Learning, ...) in which definitions and problems, but not proofs, are given. The idea is to give an opportunity for the reader to discover important steps in the development of the subject. The hope is that this style will enable the reader to more quickly arrive at a point of independent research. Paul Halmos, who first informed me of plans for this series of problem books, was an advocate of problem based teaching, as opposed to lecture based teaching.

**Problem 1** *Express your opinion on how effective a lecture method might be in teaching someone to ride a bicycle.*

Someone who works through a substantial number of the problems to follow will gain an introduction to a variety of aspects of the subject of one-parameter semigroups of transformations. These problems are not represented as being encyclopedic, but rather seek to give a grounding in various issues connected with semigroup theory.

The problems vary widely in difficulty. Some will be considered easy, some quite difficult and a few pertain to open research questions. In a number of instances references are given in which further background may be found. If a reader finds a problem particularly difficult, they might still consider working

hard on it, delaying the pursuit of references for a time. It is hoped that such effort would help a reader to approach a reference from a position of strength.

The final chapter consists of notes on the other chapters. Along with supplying additional information about various problems, the notes give some glimpses of time scales for a research endeavor. They hint at some of the human drama of long-term research quests.

Semigroups, as in Definition 1 give a description of a broad class of semigroups. An underlying theme of this problem sequence is the question as to how semigroups relate to ‘time-dependent’ differential equations. For a broad class of linear problems, this question was well understood by the 1950s. The corresponding question for a similarly broad class of nonlinear problems started receiving attention in the later 1950s. How the nonlinear theory developed can be pieced together from problems and notes. By now a rather solid foundation for a theory of one-parameter nonlinear semigroups is in place but the the real work is just beginning.

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