# Chapter 3
# Co-channel Interference

For cellular radio systems, the radio link performance is usually limited by interference rather than noise and, therefore, the probability of link outage due to CCI, $O_I$, is of primary concern. For the remainder of this chapter, the probability of outage refers to the probability of link outage due to CCI. The definition of the outage probability depends on the assumptions made about the radio receiver and propagation environment. One extreme occurs with fast moving mobile stations (MSs), that is, high Doppler conditions, where the radio receiver can average over the fast envelope variations using variety of coding and interleaving techniques. In this case, the transmission quality will be acceptable provided that the *average* received carrier-to-interference ratio, $\Lambda$, exceeds a critical receiver threshold $\Lambda_{\mathrm{th}}$. The receiver threshold $\Lambda_{\mathrm{th}}$ is determined by the performance of the radio link in the presence of envelope fading and CCI. Once $\Lambda_{\mathrm{th}}$ has been determined, the variations in $\Lambda$ due to path loss and shadowing will determine the outage probability. Another extreme occurs with stationary or very slowly moving MSs, that is, low Doppler conditions, where the radio receiver cannot average over the fast envelope variations because the required coding and interleaving depth is too large and will result in excessive transmission delay. Such delays may be acceptable for non-real-time services, but they are unacceptable for real-time services such as voice and streaming video. In this case, the transmission quality will be acceptable provided that the *instantaneous* received carrier-to-interference ratio, $\lambda$, exceeds another receiver threshold $\lambda_{\mathrm{th}}$.[1] The threshold $\lambda_{\mathrm{th}}$ is determined by the performance of the radio link in the presence of CCI under the condition that the link does not experience fading. Once $\lambda_{\mathrm{th}}$ has been determined, variations in $\lambda$ due to path loss, shadowing, *and envelope fading* will determine the outage probability. Sometimes the MSs will move with moderate velocities and the performance will lie somewhere between these two extreme cases.

The effect of CCI on the radio link performance depends on the ability of the radio receiver to reject CCI. Some of the more advanced receivers incorporate

[1] Note that $\Lambda_{\mathrm{th}}$ and $\lambda_{\mathrm{th}}$ are not the same.

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sophisticated signal processing techniques to reject or cancel the CCI, for example, single antenna interference cancelation techniques, or optimum combining with multiple antennas. In this case, the radio receiver is more tolerant to CCI and the receiver thresholds $\Lambda_{\text{th}}$ and $\lambda_{\text{th}}$ are generally reduced. This will reduce the outage probability or, conversely, improve the coverage probability.

Evaluating the outage probability for the log-normally shadowed signals that are typically found in land mobile radio systems requires the probability distribution of the total interference power that is accumulated from the sum of multiple log-normally shadowed interferers. Although there is no known exact expression for the probability distribution for the sum of log-normally random variables, several approximations have been derived by various authors in the literature. All of these approaches approximate the sum of log-normal random variables by another log-normal random variable. One such method that matches the first two moments of the approximation was developed by Fenton [98]. Sometimes Wilkinson is credited with this method, as in [236]. Here we called it the Fenton–Wilkinson method. Schwartz and Yeh developed another log-normal approximation that is based on the exact first two moments for the sum of two log-normal random variables [236]. The Schwartz and Yeh method generally provides a more accurate approximation than the Fenton–Wilkinson method but it is more difficult to use. Prasad and Arnbak [210] have corrected some errors in the equations found in Schwartz and Yeh's original paper, but the equations for Schwartz and Yeh's method in their paper also have errors. Another log-normal approximation is the cumulants matching approach suggested by Schleher [234]. With this approach, different log-normal approximations are applied over different ranges of the composite distribution. A good comparison of the methods of Fenton–Wilkinson, Schwartz-and-Yeh, Farley, and Schleher has been undertaken by Beaulieu, Abu-Dayya, and McLane [27].

The above log-normal approximations have been extensively applied to the calculation of the probability of outage in cellular systems. For example, Fenton's approach has been applied by Nagata and Akaiwa [190], Cox [69], Muammar and Gupta [185], and Daikoku and Ohdate [71]. Likewise, the Schwartz-and-Yeh approach has been applied by Yeh and Schwartz [296], Prasad and Arnbak [210], and Prasad, Kegel, and Arnbak [212].

Current literature also provides a thorough treatment of the probability of outage when the signals are affected by fading only, including the work of Yao and Sheikh [292], Muammar [184], and Prasad and Kegel [211]. Section 3.3 shows that the probability of outage is sensitive to the Rice factor of the desired signal, but it is insensitive to the number of interferers provided that the total interfering power remains constant. Calculations of the probability of outage for signals with composite log-normal shadowing and fading have considered the cases of Rayleigh fading by Linnartz [163], Nakagami fading by Ho and Stüber [131], and Ricean fading by Austin and Stüber [24]. Section 3.4 shows that shadowing has a more significant effect on the probability of outage than fading. Furthermore, the probability of outage is dominated by fading of the desired signal rather than fading of the interfering signals, for example, with Nakagami-$m$ fading, the probability of outage is sensitive to the shape factor $m$ of the desired signal but is insensitive to the shape factor of interfering signals. Finally, all of the above references assume a channel characterized by frequency nonselective (flat) fading. If the channel exhibits frequency selective fading, then the same general methodology can be used but the instantaneous carrier-to-interference ratio, $\lambda$, must be appropriately defined. The proper definition depends on the type of receiver that is used, for example, a maximum likelihood sequence estimation (MLSE) receiver or decision feedback equalizer.

Most of the literature dealing with the probability of outage assumes that the interfering co-channel signals add noncoherently. The probability of outage has also been evaluated by Prasad and Kegel [211, 213] for the case of coherent addition of Rayleigh faded co-channel interferers and a Ricean faded desired signal. The coherent co-channel interferers are assumed to arrive at the receiver antenna with the same carrier phase. However, as discussed by Prasad and Kegel [213] and Linnartz [163], it is more realistic to assume noncoherent addition of co-channel interferers in mobile radio environments. Coherently, addition of co-channel interferers generally leads to pessimistic predictions of the probability of outage.

The remainder of this chapter begins with approximations for the sum of multiple log-normally shadowed interferers in Sect. 3.1. The various approximations are compared in terms of their accuracy. Section 3.2 derives the probability of outage with log-normal/multiple log-normal (desired/interfering) signals. Section 3.3 considers the outage probability for Ricean/multiple Rayleigh signals without shadowing. Section 3.4 does the same for log-normal-Nakagami/multiple log-normal Nakagami signals.

## 3.1 Multiple Log-normal Interferers

Consider the sum of $N_{\mathrm{I}}$ log-normal random variables

$$I = \sum_{k=1}^{N_{\mathrm{I}}} \Omega_k = \sum_{k=1}^{N_{\mathrm{I}}} 10^{\Omega_{k(\mathrm{dBm})}/10}, \tag{3.1}$$

where the $\Omega_{k\,(\mathrm{dBm})}$ are Gaussian random variables with means $\mu_{\Omega_{k(\mathrm{dBm})}}$ and variances $\sigma^2_{\Omega_k}$, and the $\Omega_k = 10^{\Omega_{k(\mathrm{dBm})}/10}$ are log-normal random variables. Unfortunately, there is no known closed form expression for the probability density function (pdf) of the sum of multiple ($N_{\mathrm{I}} \geq 2$) log-normal random variables. One may think to apply the central limit theorem, provided that $N_{\mathrm{I}}$ is large enough, and approximate $I$ as a Gaussian random variable. However, since $I$ represents a power sum, it cannot assume negative values so that the resulting approximation is invalid. Moreover, the value of $N_{\mathrm{I}}$ will be small in the case of a few dominant co-channel interferers, so the central limit theorem will not apply anyway. There is a general consensus that the sum of independent log-normal random variables can be approximated by another log-normal random variable with appropriately chosen parameters. That is,

$$I = \sum_{k=1}^{N_{\mathrm{I}}} 10^{\Omega_{k(\mathrm{dBm})}/10} \approx 10^{Z_{(\mathrm{dBm})}/10} = \hat{I}, \tag{3.2}$$

where $Z_{(\mathrm{dBm})}$ is a Gaussian random variable with mean $\mu_{Z\,(\mathrm{dBm})}$ and variance $\sigma_Z^2$. The problem is to determine $\mu_{Z\,(\mathrm{dBm})}$ and $\sigma_Z^2$ in terms of the $\mu_{\Omega_k\,(\mathrm{dBm})}$ and $\sigma_{\Omega_k}^2$, $k = 1,\ldots,N_{\mathrm{I}}$. Several methods have been suggested in the literature to solve this problem including those by Fenton [98], Schwartz and Yeh [236], and Farley [236]. Each of these methods provides varying degrees of accuracy over specified ranges of the shadow standard deviation $\sigma_\Omega$, the sum $I$, and the number of interferers $N_{\mathrm{I}}$.

### 3.1.1 Fenton–Wilkinson Method

With the Fenton–Wilkinson method, the mean $\mu_{Z\,(\mathrm{dBm})}$ and variance $\sigma_Z^2$ of $Z_{(\mathrm{dBm})}$ are obtained by matching the first two moments of the sum $I$ with the first two moments of the approximation $\hat{I}$. To derive these moments, it is convenient to use natural logarithms. We write

$$\Omega_k = 10^{\Omega_{k\,(\mathrm{dBm})}/10} = \mathrm{e}^{\xi\Omega_{k\,(\mathrm{dBm})}} = \mathrm{e}^{\hat{\Omega}_k}, \tag{3.3}$$

where $\xi = \ln(10)/10 = 0.23026$ and $\hat{\Omega}_k = \xi\Omega_{k\,(\mathrm{dBm})}$. Note that $\mu_{\hat{\Omega}_k} = \xi\mu_{\Omega_k\,(\mathrm{dBm})}$ and $\sigma_{\hat{\Omega}_k}^2 = \xi^2\sigma_{\Omega_k}^2$. The $n$th moment of the log-normal random variable $\Omega_k$ can be obtained from the moment generating function of the Gaussian random variable $\hat{\Omega}_k$ as

$$\mathrm{E}[\Omega_k^n] = \mathrm{E}[\mathrm{e}^{n\hat{\Omega}_k}] = \mathrm{e}^{n\mu_{\hat{\Omega}_k}+(1/2)n^2\sigma_{\hat{\Omega}_k}^2}. \tag{3.4}$$

To find the required moments for the log-normal approximation, we can use (3.4) and equate the first two moments on both sides of the approximation

$$I = \sum_{k=1}^{N_{\mathrm{I}}} \mathrm{e}^{\hat{\Omega}_k} \approx \mathrm{e}^{\hat{Z}} = \hat{I}, \tag{3.5}$$

where $\hat{Z} = \xi Z_{(\mathrm{dBm})}$. For example, suppose that the $\hat{\Omega}_k, k = 1,\ldots,N_{\mathrm{I}}$ have means $\mu_{\hat{\Omega}_k}, k = 1,\ldots,N_{\mathrm{I}}$ and identical variances $\sigma_{\hat{\Omega}}^2$. Identical variances are often assumed for the sum of log-normal interferers because the standard deviation of log-normal shadowing is largely independent of the radio path length [151, 153]. Equating the means on both sides of (3.5) gives

$$\mu_I = \mathrm{E}[I] = \sum_{k=1}^{N_{\mathrm{I}}} \mathrm{E}[\mathrm{e}^{\hat{\Omega}_k}] = \mathrm{E}[\mathrm{e}^{\hat{Z}}] = \mathrm{E}[\hat{I}] = \mu_{\hat{I}} \tag{3.6}$$

and substituting (3.4) with $n = 1$ into (3.6) gives the result

$$\left(\sum_{k=1}^{N_{\mathrm{I}}} \mathrm{e}^{\mu_{\hat{\Omega}_k}}\right) \mathrm{e}^{(1/2)\sigma_{\hat{\Omega}}^2} = \mathrm{e}^{\mu_{\hat{Z}}+(1/2)\sigma_{\hat{Z}}^2}. \tag{3.7}$$

Likewise, we can equate the variances on both sides of (3.5), that is,

$$\sigma_I^2 = \mathrm{E}[I^2] - \mu_I^2 = \mathrm{E}[\hat{I}^2] - \mu_{\hat{I}}^2 = \sigma_{\hat{I}}^2. \tag{3.8}$$

Under the assumption that the $\hat{\Omega}_k, k = 1, \ldots N_\mathrm{I}$ are independent random variables when calculating the second moments in the above equation, this gives the result

$$\left( \sum_{k=1}^{N_\mathrm{I}} \mathrm{e}^{2\mu_{\hat{\Omega}_k}} \right) \mathrm{e}^{\sigma_{\hat{\Omega}}^2} (\mathrm{e}^{\sigma_{\hat{\Omega}}^2} - 1) = \mathrm{e}^{2\mu_{\hat{Z}}} \mathrm{e}^{\sigma_{\hat{Z}}^2} (\mathrm{e}^{\sigma_{\hat{Z}}^2} - 1). \tag{3.9}$$

By squaring each side of the equality in (3.7) and dividing each side of resulting equation by the respective sides of the equality in (3.9), we can solve for $\sigma_{\hat{Z}}^2$ in terms of the known values of $\mu_{\hat{\Omega}_k}, k = 1, \ldots, N_\mathrm{I}$ and $\sigma_{\hat{\Omega}}^2$. Later, $\mu_{\hat{Z}}$ can be obtained by substituting the obtained expression for $\sigma_{\hat{Z}}^2$ into (3.7). This procedure yields the following solution:

$$\sigma_{\hat{Z}}^2 = \ln \left( (\mathrm{e}^{\sigma_{\hat{\Omega}}^2} - 1) \frac{\sum_{k=1}^{N_\mathrm{I}} \mathrm{e}^{2\mu_{\hat{\Omega}_k}}}{\left( \sum_{k=1}^{N_\mathrm{I}} \mathrm{e}^{\mu_{\hat{\Omega}_k}} \right)^2} + 1 \right). \tag{3.10}$$

$$\mu_{\hat{Z}} = \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln \left( \sum_{k=1}^{N_\mathrm{I}} \mathrm{e}^{\mu_{\hat{\Omega}_k}} \right), \tag{3.11}$$

Finally, we convert back to base 10 logarithms by scaling, such that $\mu_{Z\ (\mathrm{dBm})} = \xi^{-1} \mu_{\hat{Z}}$ and $\sigma_Z^2 = \xi^{-2} \sigma_{\hat{Z}}^2$.

The accuracy of this log-normal approximation can be measured in terms of how accurately the first two moments of $I_{(\mathrm{dB})} = 10\log_{10} I$ are estimated, and how well the cumulative distribution function (cdf) of $I_{(\mathrm{dB})}$ is described by a Gaussian cdf. It has been reported in [236] that the Fenton–Wilkinson method breaks down in the accuracy of the values obtained for $\mu_{Z\ (\mathrm{dBm})}$ and $\sigma_Z^2$ when $\sigma_\Omega > 4\,\mathrm{dB}$. For cellular radio applications $\sigma_\Omega$ typically ranges from 6 to 12 dB and the Fenton–Wilkinson method has often been discredited in the literature on that basis. However, as pointed out in [27], the Fenton–Wilkinson method breaks down only if one considers the application of the Fenton–Wilkinson method for the prediction of the first two moments of $I_{(\mathrm{dB})}$. Moreover, in problems relating to the probability of CCI outage in cellular radio systems, we are usually interested in the tails of the complementary distribution function (cdfc) $F_I^c(x) = \mathrm{P}[I \geq x]$ and the cdf $F_I(x) = 1 - F_I^c(x) = \mathrm{P}[I < x]$. In this case, we are interested in the accuracy of the approximation

$$F_I(x) \approx \mathrm{P}[\mathrm{e}^{\hat{Z}} \geq x] = Q\left( \frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}} \right), \tag{3.12}$$

for large and small values of $x$. It will be shown later that the Fenton–Wilkinson method approximates the tails of the cdf and cdfc functions with good accuracy, a result that was reported in [27].

### *3.1.2 Schwartz and Yeh Method*

The Schwartz and Yeh method [236] calculates exact values for the first two moments of the sum of two independent log-normal random variables. Nesting and recursion techniques are then used to find exact values for the first two moments for the sum of $N_{\mathrm{I}}$ independent log-normal random variables. For example, suppose that $I = \Omega_1 + \Omega_2 + \Omega_3$. The exact first two moments of $\ln(\Omega_1 + \Omega_2)$ are first computed. We then define $Z_2 = \ln(\Omega_1 + \Omega_2)$ as a new Gaussian random variable, let $I = \mathrm{e}^{Z_2} + \Omega_3$, and again compute the exact first two moments of $\ln I$. Since the procedure is recursive, we only need to detail the Schwartz and Yeh method for the case when $N_{\mathrm{I}} = 2$, that is,

$$I = \mathrm{e}^{\hat{\Omega}_1} + \mathrm{e}^{\hat{\Omega}_2} \approx \mathrm{e}^{\hat{Z}} = \hat{I} \tag{3.13}$$

or

$$\hat{Z} \approx \ln\left(\mathrm{e}^{\hat{\Omega}_1} + \mathrm{e}^{\hat{\Omega}_2}\right), \tag{3.14}$$

where the Gaussian random variables $\hat{\Omega}_1$ and $\hat{\Omega}_2$ have means $\mu_{\hat{\Omega}_1}$ and $\mu_{\hat{\Omega}_2}$, and variances $\sigma^2_{\hat{\Omega}_1}$ and $\sigma^2_{\hat{\Omega}_2}$, respectively.

Define the Gaussian random variable $\hat{\Omega}_d \stackrel{\triangle}{=} \hat{\Omega}_2 - \hat{\Omega}_1$ so that

$$\mu_{\hat{\Omega}_d} = \mu_{\hat{\Omega}_2} - \mu_{\hat{\Omega}_1}, \tag{3.15}$$

$$\sigma^2_{\hat{\Omega}_d} = \sigma^2_{\hat{\Omega}_1} + \sigma^2_{\hat{\Omega}_2}. \tag{3.16}$$

Taking the expectation of both sides of (3.14) and assuming that the approximation holds with equality gives

$$\begin{aligned}
\mu_{\hat{Z}} &= \mathrm{E}\left[\ln\left(\mathrm{e}^{\hat{\Omega}_2} + \mathrm{e}^{\hat{\Omega}_1}\right)\right] \\
&= \mathrm{E}\left[\ln\left(\mathrm{e}^{\hat{\Omega}_1}\left(1 + \mathrm{e}^{\hat{\Omega}_2 - \hat{\Omega}_1}\right)\right)\right] \\
&= \mathrm{E}\left[\hat{\Omega}_1\right] + \mathrm{E}\left[\ln\left(1 + \mathrm{e}^{\hat{\Omega}_d}\right)\right].
\end{aligned} \tag{3.17}$$

The second term in (3.17) is

$$\mathrm{E}\left[\ln\left(1 + \mathrm{e}^{\hat{\Omega}_d}\right)\right] = \int_{-\infty}^{\infty} \left(\ln\left(1 + \mathrm{e}^x\right)\right) p_{\hat{\Omega}_d}(x)\mathrm{d}x. \tag{3.18}$$

We now use the power series expansion

$$\ln(1+x) = \sum_{k=1}^{\infty} c_k x^k, \qquad c_k = \frac{(-1)^{k+1}}{k}, \tag{3.19}$$

where $|x| < 1$. To ensure convergence of the power series and the resulting series of integrals, the integration in (3.18) is broken into ranges as follows:

$$\begin{aligned}\int_{-\infty}^{\infty} \left(\ln\left(1+\mathrm{e}^x\right)\right) p_{\hat{\Omega}_d}(x)\mathrm{d}x &= \int_{-\infty}^{0} \left(\ln\left(1+\mathrm{e}^x\right)\right) p_{\hat{\Omega}_d}(x)\mathrm{d}x \\ &\quad + \int_{0}^{\infty} \left(\ln\left(1+\mathrm{e}^{-x}\right)+x\right) p_{\hat{\Omega}_d}(x)\mathrm{d}x. \end{aligned} \tag{3.20}$$

The second integral in the above equation was obtained using the identity

$$\begin{aligned}\ln(1+\mathrm{e}^x) &= \ln\left((\mathrm{e}^{-x}+1)\mathrm{e}^x\right) \\ &= \ln(1+\mathrm{e}^{-x}) + \ln(\mathrm{e}^x) \\ &= \ln(1+\mathrm{e}^{-x}) + x. \end{aligned} \tag{3.21}$$

After a very long derivation that is detailed in [236],

$$\mu_{\hat{Z}} = \mu_{\hat{\Omega}_1} + G_1, \tag{3.22}$$

where

$$\begin{aligned} G_1 &= \mu_{\hat{\Omega}_d} \Phi\left(\frac{\mu_{\hat{\Omega}_d}}{\sigma_{\hat{\Omega}_d}}\right) + \frac{\sigma_{\hat{\Omega}_d}}{\sqrt{2\pi}} \mathrm{e}^{-\mu^2_{\hat{\Omega}_d}/2\sigma^2_{\hat{\Omega}_d}} \\ &\quad + \sum_{k=1}^{\infty} c_k \mathrm{e}^{k^2\sigma^2_{\hat{\Omega}_d}/2} \left(\mathrm{e}^{k\mu_{\hat{\Omega}_d}} \Phi\left(\frac{-\mu_{\hat{\Omega}_d} - k\sigma^2_{\hat{\Omega}_d}}{\sigma_{\hat{\Omega}_d}}\right) + T_1\right) \end{aligned} \tag{3.23}$$

with

$$T_1 = \mathrm{e}^{-k\mu_{\hat{\Omega}_d}} \Phi\left(\frac{\mu_{\hat{\Omega}_d} - k\sigma^2_{\hat{\Omega}_d}}{\sigma_{\hat{\Omega}_d}}\right) \tag{3.24}$$

and where $\Phi(x)$ is the cdfc of a standard normal random variable, defined as

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-y^2/2}\mathrm{d}y. \tag{3.25}$$

The variance of $\hat{Z}$ can be computed in a similar fashion, resulting in the expression [236]

$$\sigma^2_{\hat{Z}} = \sigma^2_{\hat{\Omega}_1} - G_1^2 - 2\sigma^2_{\hat{\Omega}_1} G_3 + G_2, \tag{3.26}$$

where

$$G_2 = \sum_{k=1}^{\infty} b_k T_2 + \left(1 - \Phi\left(-\frac{\mu_{\hat{\Omega}_d}}{\sigma_{\hat{\Omega}_d}}\right)\right)\left(\mu^2_{\hat{\Omega}_d} + \sigma^2_{\hat{\Omega}_d}\right) + \frac{\mu_{\hat{\Omega}_d}\sigma_{\hat{\Omega}_d}}{\sqrt{2\pi}} \mathrm{e}^{-\mu^2_{\hat{\Omega}_d}\Big/\left(2\sigma^2_{\hat{\Omega}_d}\right)}$$
$$+ \sum_{k=1}^{\infty} b_k \mathrm{e}^{-(k+1)\mu_{\hat{\Omega}_d} + (k+1)^2\sigma^2_{\hat{\Omega}_d}/2} \Phi\left(\frac{\mu_{\hat{\Omega}_d} - \sigma^2_{\hat{\Omega}_d}(k+1)}{\sigma_{\hat{\Omega}_d}}\right) \tag{3.27}$$
$$-2\sum_{k=1}^{\infty} c_k \mathrm{e}^{-k\mu_{\hat{\Omega}_d} + k^2\sigma^2_{\hat{\Omega}_d}/2}\left(\xi_{\hat{\Omega}_k}\Phi\left(-\frac{\xi_{\hat{\Omega}_k}}{\sigma_{\hat{\Omega}_d}}\right) - \frac{\sigma_{\hat{\Omega}_d}}{\sqrt{2\pi}}\mathrm{e}^{-\xi^2_{\hat{\Omega}_k}\Big/\left(2\sigma^2_{\hat{\Omega}_d}\right)}\right)$$

$$G_3 = \sum_{k=0}^{\infty} (-1)^k \mathrm{e}^{k^2\sigma^2_{\hat{\Omega}_d}/2} T_1 + \sum_{k=0}^{\infty} (-1)^k T_2 \tag{3.28}$$

with

$$T_2 = \mathrm{e}^{\mu_{\hat{\Omega}_d}(k+1) + (k+1)^2\sigma^2_{\hat{\Omega}_d}/2} \Phi\left(\frac{-\mu_{\hat{\Omega}_d} - (k+1)\sigma^2_{\hat{\Omega}_d}}{\sigma_{\hat{\Omega}_d}}\right) \tag{3.29}$$

and

$$b_k = \frac{2(-1)^{k+1}}{k+1}\sum_{n=1}^{k}\frac{1}{n}, \tag{3.30}$$

$$\xi_{\hat{\Omega}_k} = -\mu_{\hat{\Omega}_d} + k\sigma^2_{\hat{\Omega}_d}. \tag{3.31}$$

It has been reported in [236] that approximately 40 terms are required in the infinite summations for $G_1$, $G_2$ and $G_3$ to achieve four significant digits of accuracy in the moments of $\hat{Z}$. On the next step of the recursion, it is important that we let $\sigma^2_{\hat{\Omega}_1} = \sigma^2_{\hat{Z}}$ and $\mu_{\hat{\Omega}_1} = \mu_{\hat{Z}}$; otherwise, the recursive procedure will fail to converge.

### 3.1.3 Farley's Method

Consider $N_{\mathrm{I}}$-independent identically distributed (i.i.d.) normal random variables $\hat{\Omega}_k$ each with mean $\mu_{\hat{\Omega}}$ and variance $\sigma^2_{\hat{\Omega}}$. Farley approximated the cdfc of the sum

$$I = \sum_{k=1}^{N_{\mathrm{I}}} \Omega_k = \sum_{k=1}^{N_{\mathrm{I}}} \mathrm{e}^{\hat{\Omega}_k} \tag{3.32}$$

as [236]

$$\mathrm{P}[I > x] \approx 1 - \left(1 - Q\left(\frac{\ln x - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right)\right)^{N_{\mathrm{I}}}. \tag{3.33}$$

As shown in [27], Farley's approximation is actually a strict lower bound on the cdfc. To obtain this result, let

$$F_I^c(x) = \mathrm{P}\left[\Omega_1 + \Omega_2 + \ldots + \Omega_{N_\mathrm{I}} > x\right] \tag{3.34}$$

and define the two events

$$\begin{aligned} A &= \{\text{at least one } \Omega_i > x\} \\ B &= A^\mathrm{c}, \text{ the complement of event } A. \end{aligned} \tag{3.35}$$

Events $A$ and $B$ are mutually exclusive and partition the sample space. Therefore,

$$\begin{aligned} \mathrm{P}[I > x] &= \mathrm{P}[I > x \cap A] + \mathrm{P}[I > x \cap B] \\ &= \mathrm{P}[A] + \mathrm{P}[I > x \cap B]. \end{aligned} \tag{3.36}$$

The second term in (3.36) is positive for continuous pdfs such as the log-normal pdf. For example, the event

$$C = \left\{ \bigcap_{i=1}^{N_\mathrm{I}} x/N_\mathrm{I} < \Omega_i < x \right\} \tag{3.37}$$

is a subset of the event $B$. Under the assumption that the $\Omega_i$ are i.i.d., event $C$ occurs with nonzero probability because

$$\mathrm{P}[C] = \left( Q\left(\frac{\ln(x/N_\mathrm{I}) - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right) - Q\left(\frac{\ln x - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right)\right)^{N_\mathrm{I}} > 0. \tag{3.38}$$

Therefore, $\mathrm{P}[I > x] > \mathrm{P}[A]$. Since the $\Omega_i$ are i.i.d.,

$$\begin{aligned} \mathrm{P}[A] &= 1 - \prod_{i=1}^{N_\mathrm{I}} \mathrm{P}[\Omega_i \le x] \\ &= 1 - \left(1 - Q\left(\frac{\ln x - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right)\right)^{N_\mathrm{I}}. \end{aligned} \tag{3.39}$$

Finally, we have the lower bound on the cdfc

$$\mathrm{P}[I > x] > 1 - \left(1 - Q\left(\frac{\ln x - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right)\right)^{N_\mathrm{I}} \tag{3.40}$$

or, equivalently, the upper bound on the cdf

$$\mathrm{P}[I \le x] > \left(1 - Q\left(\frac{\ln x - \mu_{\hat{\Omega}}}{\sigma_{\hat{\Omega}}}\right)\right)^{N_\mathrm{I}}. \tag{3.41}$$

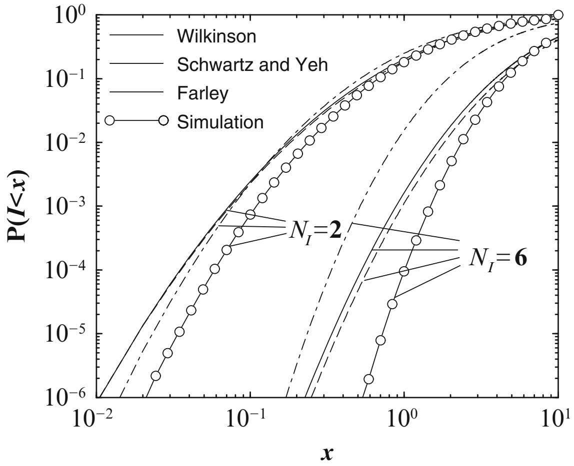



**Fig. 3.1** Comparison of the cdf for the sum of two and six log-normal random variables with various approximations; $\mu_{\Omega_k \text{ (dB)}} = 0\,\text{dB}$, $\sigma_\Omega = 6\,\text{dB}$

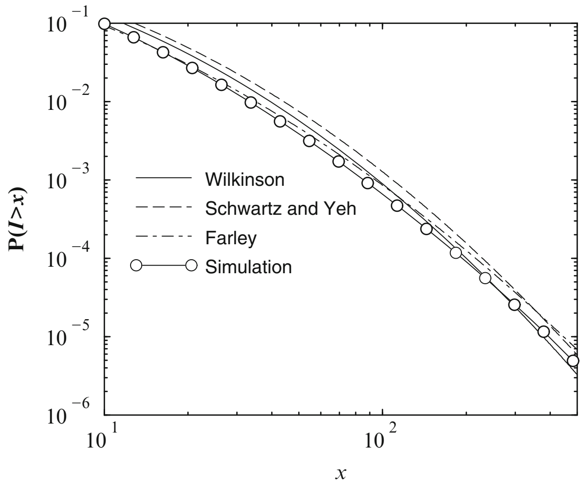



**Fig. 3.2** Comparison of the cdfc for the sum of two log-normal random variables with various approximations; $\mu_{\Omega_k \text{ (dB)}} = 0\,\text{dB}$, $\sigma_\Omega = 6\,\text{dB}$

### *3.1.4 Numerical Comparisons*

Figure 3.1 compares the cdf for the sum of $N_\text{I} = 2$ and $N_\text{I} = 6$ log-normal random variables, obtained with the various approximations. Likewise, Figs. 3.2–3.4 compare the cdfc obtained with the various approximations. Exact results are also shown that have been obtained by computer simulation. Observe that the cdfc is approximated quite well for all the methods, but the best approximation depends on the number of interferers, shadow standard deviation, and argument of the cdfc. The cdf is approximated less accurately, especially for $N_\text{I} = 6$ log-normal random variables.

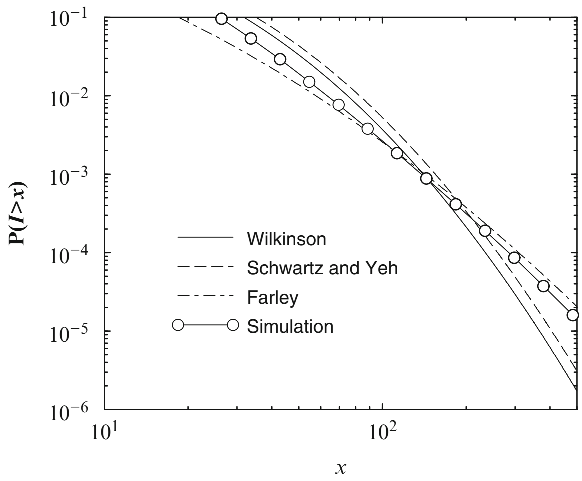



**Fig. 3.3** Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\mu_{\Omega_k\,(\mathrm{dB})} = 0\,\mathrm{dB}$, $\sigma_\Omega = 6\,\mathrm{dB}$

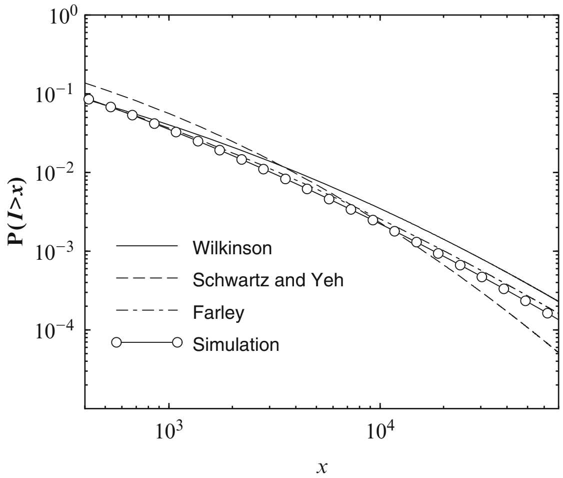



**Fig. 3.4** Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\mu_{\Omega_k\,(\mathrm{dB})} = 0\,\mathrm{dB}$, $\sigma_\Omega = 12\,\mathrm{dB}$

## 3.2 Log-Normal/Multiple Log-Normal Interferers

Consider the situation shown in Fig. 1.15, where a mobile station (MS) is at distance $d_0$ from the serving base station (BS) and at distances $d_k, k = 1, 2, \ldots, N_\mathrm{I}$ from the first tier of $N_\mathrm{I}$ co-channel BSs. Define the vector $\mathbf{d} = (d_0,\ d_1, \ldots,\ d_{N_\mathrm{I}})$ as the set of distances of a particular MS from the serving BS and surrounding BSs. The average received carrier-to-interference ratio as a function of the vector $\mathbf{d}$ is

$$\Lambda_{(\mathrm{dB})}(\mathbf{d}) = \Omega_{(\mathrm{dBm})}(d_0) - 10\log_{10}\sum_{k=1}^{N_\mathrm{I}} 10^{\Omega_{(\mathrm{dBm})}(d_k)/10}. \tag{3.42}$$

For the case of a single interferer ($N_{\mathrm{I}} = 1$), the sum on the right side of (3.42) has only one term. In this case, $\Lambda_{(\mathrm{dB})}(\mathbf{d})$ is Gaussian distributed with mean $\mu_{\Omega_{(\mathrm{dBm})}(d_0)} - \mu_{\Omega_{(\mathrm{dBm})}(d_1)}$ and variance $2\sigma_\Omega^2$. For the case of multiple log-normal interferers, the second term of (3.42) is approximated as a normal random variable $Z_{(\mathrm{dBm})}$ with mean $\mu_{Z\ (\mathrm{dBm})}$ and variance $\sigma_Z^2$ using the techniques discussed in Sect. 3.1. Then

$$\Lambda_{(\mathrm{dB})}(\mathbf{d}) = \Omega_{(\mathrm{dBm})}(d_0) - Z_{(\mathrm{dBm})}(d_1,\, d_2,\, \ldots,\, d_{N_{\mathrm{I}}}), \tag{3.43}$$

where we, again, show the dependency of the CCI on the set of distances. Note that $\Lambda_{(\mathrm{dB})}(\mathbf{d})$ has mean and variance

$$\mu_{\Lambda_{(\mathrm{dB})}(\mathbf{d})} = \mu_{\Omega_{(\mathrm{dBm})}(d_0)} - \mu_{Z\ (\mathrm{dBm})} \tag{3.44}$$

$$\sigma^2_{\Lambda(\mathbf{d})} = \sigma_\Omega^2 + \sigma_Z^2. \tag{3.45}$$

If there were only one possible choice of serving BS, then the probability of outage at a particular MS location is

$$O_I(\mathbf{d}) = Q\left(\frac{\mu_{\Omega_{(\mathrm{dBm})}(d_0)} - \mu_{Z\ (\mathrm{dBm})} - \Lambda_{\mathrm{th(dB)}}}{\sqrt{\sigma_\Omega^2 + \sigma_Z^2}}\right). \tag{3.46}$$

If handoffs are allowed, then the analysis is more complicated. In this case, the probability of outage will depend on the handoff algorithm that is used. In the simplest case, we can consider soft handoffs where the BS that provides the best link is always used. In this case, an outage occurs only when no BS can provide a link having a carrier-to-interference ratio that exceeds the receiver threshold $\Lambda_{\mathrm{th}}$. In this case, the probability of outage at a particular location is

$$O_I(\mathbf{d}) = \prod_{k=0}^{M} Q\left(\frac{\mu_{\Omega_{k\ (\mathrm{dBm})}(d_0)} - \mu_{Z_k\ (\mathrm{dBm})} - \Lambda_{\mathrm{th(dB)}}}{\sqrt{\sigma_\Omega^2 + (\sigma_{Z_k})^2}}\right), \tag{3.47}$$

where $M$ is the number of handoff candidates. The outage can then be calculated by averaging the probability of outage over the random location of a MS within the reference cell.

## 3.3 Rician/Multiple Rayleigh Interferers

Sometimes propagation conditions may exist such that the received signals experience envelope fading, but not shadowing. In this section, we calculate the outage probability for the case of envelope fading only. The case of combined shadowing and envelope fading is deferred until the next section. In the case of envelope fading only, the received desired signal may consist of a direct line of sight (LoS) component, or perhaps a specular component, accompanied by a diffuse component. The envelope of the received desired signal experiences Ricean fading. The co-channel interferers are often assumed to be Rayleigh faded, because a direct LoS condition is unlikely to exist between the co-channel interferers and target receiver due to their large physical separation. Let the instantaneous power in the desired signal and the $N_{\rm I}$ interfering signals be denoted by $s_0$ and $s_k$, $k = 1, \ldots, N_{\rm I}$, respectively. Note that $s_i = \alpha_i^2$, where $\alpha_i^2$ is the squared envelope. The carrier-to-interference ratio is defined as

$$\lambda \stackrel{\triangle}{=} \frac{s_0}{\sum_{k=1}^{N_{\rm I}} s_k}. \tag{3.48}$$

For a specified receiver threshold $\lambda_{\rm th}$, the outage probability is

$$O_I = {\rm P}[\lambda < \lambda_{\rm th}]. \tag{3.49}$$

The instantaneous received power of the desired signal, $s_0$, has the noncentral chi-square (Ricean fading) distribution in (2.59), while the instantaneous power of each interferer, $s_k$, has the exponential distribution (Rayleigh fading) in (2.52).

For the case of a single interferer, the outage probability reduces to the simple closed form expression [292]

$$O_I = \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_1} \exp\left\{ -\frac{KA_1}{\lambda_{\rm th} + A_1} \right\}, \tag{3.50}$$

where $K$ is the Rice factor of the desired signal, $A_1 = \Omega_0/(K+1)\Omega_1$, and $\Omega_k = {\rm E}[s_k]$. Note that $A_1$ can be interpreted as the ratio of the average desired signal power to the total interfering power. If the desired signal is Rayleigh faded, then the outage probability can be obtained by setting $K = 0$ in (3.50). For the case of multiple interferers, each with mean power $\Omega_k$, the outage probability has the closed form expression [292]

$$O_I = 1 - \sum_{k=1}^{N_{\rm I}} \left( 1 - \frac{\lambda_{\rm th}}{\lambda_{\rm th} + A_k} \exp\left\{ -\frac{KA_k}{\lambda_{\rm th} + A_k} \right\} \right) \prod_{\substack{j=1 \\ j \neq k}}^{N_{\rm I}} \frac{A_j}{A_j - A_k}, \tag{3.51}$$

where $A_k = \Omega_0/(K+1)\Omega_k$. This expression is valid only if $\Omega_i \neq \Omega_j$ when $i \neq j$, that is, the different interferers are received with distinct mean power levels. If some of the interfering signals are received with the same mean power, then an appropriate expression for the outage probability can be derived in straight forward manner. If all the interferers are received with the same mean power, then the total interference power $s_{\rm M} = \sum_{k=1}^{N_{\rm I}} s_k$ has the Gamma pdf

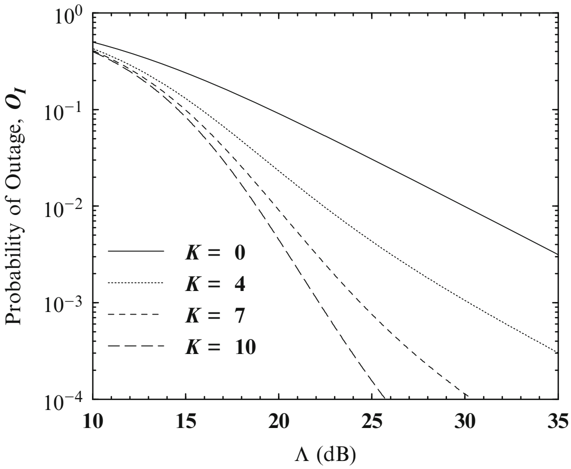



**Fig. 3.5** Probability of CCI outage with a single interferer. The desired signal is Ricean faded with various Rice factors, while the interfering signal is Rayleigh faded; $\lambda_{\rm th} = 10.0\,$dB

$$p_{s_{\rm M}}(x) = \frac{x^{N_{\rm I}-1}}{\Omega_1^{N_{\rm I}}(N_{\rm I}-1)!}\exp\left\{-\frac{x}{\Omega_1}\right\}. \tag{3.52}$$

The outage probability can be derived as [292]

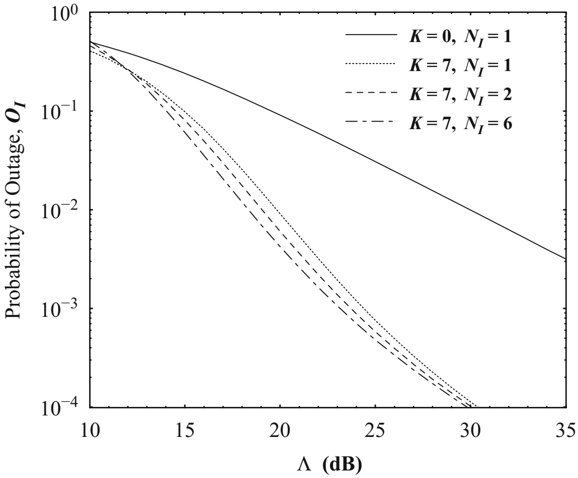
$$O_I = \frac{\lambda_{\rm th}}{\lambda_{\rm th}+A_1}\exp\left\{-\frac{KA_1}{\lambda_{\rm th}+A_1}\right\} \times \sum_{k=0}^{N_{\rm I}-1}\left(\frac{A_1}{\lambda_{\rm th}+A_1}\right)^k \sum_{m=0}^{k}\binom{k}{m}\frac{1}{m!}\left(\frac{K\lambda_{\rm th}}{\lambda_{\rm th}+A_1}\right)^m. \tag{3.53}$$

Again, if the desired signal is Rayleigh faded, then the probability of outage with multiple Rayleigh faded interferers can be obtained by setting $K = 0$ in either (3.51) or (3.53) as appropriate. In Fig. 3.5, the outage probability is plotted as a function of the average received carrier-to-average-interference ratio

$$\Lambda = \frac{\Omega_0}{N_{\rm I}\Omega_1} \tag{3.54}$$

for various Rice factors and a single interferer. Observe that the Rice factor of the desired signal has a significant effect on the outage probability. Figure 3.6 plots the outage probability for $K = 0$ and 7 and a varying number of interferers. Observe that the number of interferers does not affect the probability of outage as much as the Rice factor of the desired signal, provided that the total interfering power remains constant.

**Fig. 3.6** Probability of CCI outage with multiple interferers. The desired signal is Ricean faded with various Rice factors, while the interfering signals are Rayleigh faded and of equal power; $\lambda_{\mathrm{th}} = 10.0\,\mathrm{dB}$




## 3.4 Log-Normal Nakagami/Multiple Log-Normal Nakagami Interferers

The probability of outage has been evaluated in the literature for a single Nakagami interferer [287] and multiple Nakagami interferers [6, 293], in the absence of shadowing. Here we analytically formulate the probability of outage with multiple log-normal Nakagami interferers. For the case when the interfering signals have the same shadowing and fading statistics, we derive an exact mathematical expression for the probability of outage.

Let the instantaneous power in the desired signal and the $N_{\mathrm{I}}$ interfering signals be denoted by $s_0$ and $s_k,\ k = 1,\ldots,N_{\mathrm{I}}$, respectively. The instantaneous carrier-to-interference ratio is $\lambda = s_0 / \sum_{k=1}^{N_{\mathrm{I}}} s_k$. For a specified receiver threshold $\lambda_{\mathrm{th}}$, the probability of outage is

$$O_I = \mathrm{P}[\lambda < \lambda_{\mathrm{th}}] \equiv \mathrm{P}\left[ s_0 < \lambda_{\mathrm{th}} \sum_{k=1}^{N_{\mathrm{I}}} s_k \right]. \tag{3.55}$$

The $k$th interfering signal, $s_k,\ k = 1,\ldots,N_{\mathrm{I}}$, is affected by log-normal shadowing and Nakagami fading with the composite pdf c.f. (2.313)

$$\begin{aligned} p_{s_k}(x) = &\int_0^\infty \left(\frac{m_k}{w}\right)^{m_k} \frac{x^{m_k - 1}}{\Gamma(m_k)} \exp\left\{-\frac{m_k x}{w}\right\} \\ &\times \frac{1}{\sqrt{2\pi}\xi\sigma_\Omega w} \exp\left\{ -\frac{\left(10\log_{10}\{w\} - \mu_{\Omega_k\,(\mathrm{dBm})}\right)^2}{2\sigma_\Omega^2} \right\} \mathrm{d}w. \end{aligned} \tag{3.56}$$

Let $W = \sum_{k=1}^{N_\mathrm{I}} s_k$ be the total instantaneous received power from the $N_\mathrm{I}$ interfering signals such that $\lambda = s_0/W$, and define the auxiliary random variable $Y = W$. Then using a bivariate transformation of random variables, the joint pdf of $\lambda$ and $Y$ is $p_{\lambda,Y}(x,y) = yp_{s_0,W}(xy,y) = yp_{s_0}(xy)p_W(y)$ and

$$p_\lambda(x) = \int_0^\infty yp_{s_0}(xy)p_W(y)\mathrm{d}y, \tag{3.57}$$

where we used the fact that $s_0$ and $W$ are statistically independent random variables. It follows that the outage probability is

$$\begin{aligned} O_I &= \mathrm{P}[\lambda < \lambda_\mathrm{th}] \\ &= 1 - \int_{\lambda_\mathrm{th}}^\infty \int_0^\infty yp_{s_0}(xy)p_W(y)\mathrm{d}y\mathrm{d}x. \end{aligned} \tag{3.58}$$

Suppose for the time being that the desired signal is affected by Nakagami fading only, that is, there is no shadowing and $\Omega_0 = \mathrm{E}[s_0]$ is fixed. Then $s_0$ has the Gamma distribution in (2.63) and

$$\int_{\lambda_\mathrm{th}}^\infty p_{s_0}(xy)\mathrm{d}x = \sum_{h=0}^{m_0-1} \left(\frac{m_0}{\Omega_0}\right)^h \frac{y^{h-1}\lambda_\mathrm{th}^h}{h!} \exp\left\{-\frac{m_0\lambda_\mathrm{th}y}{\Omega_0}\right\}. \tag{3.59}$$

Hence, the conditional outage probability in (3.58) becomes

$$\mathrm{P}(\lambda < \lambda_\mathrm{th}|\Omega_0) = 1 - \sum_{h=0}^{m_0-1} \left(\frac{m_0\lambda_\mathrm{th}}{\Omega_0}\right)^h \frac{1}{h!} \int_0^\infty \exp\left\{-\frac{m_0\lambda_\mathrm{th}y}{\Omega_0}\right\} y^h p_W(y)\mathrm{d}y. \tag{3.60}$$

### 3.4.1 Statistically Identical Interferers

Here we assume statistically identical interferers so that $m_k = m_\mathrm{I}$ and $\mu_{\Omega_k\,(\mathrm{dBm})} = \mu_{\Omega_I\,(\mathrm{dBm})}$, $i = 1, \ldots, N_\mathrm{I}$. Following Linnartz [163], the integral in (3.60) can be obtained using Laplace transform techniques. The Laplace transform of the pdf $p_W(y)$ is

$$\mathscr{L}_W(s) = \int_0^\infty \mathrm{e}^{-sy} p_W(y)\mathrm{d}y. \tag{3.61}$$

The integral in (3.60) is then equal to the $h$th derivative of $\mathscr{L}_W(s)$ with respect to $s$ evaluated at the point $s = (m_0\lambda_\mathrm{th})/\Omega_0$. That is,

$$\begin{aligned} \int_0^\infty \mathrm{e}^{-sy} y^h p_W(y)\mathrm{d}y &= (-1)^h \mathscr{L}_W^{(h)}(s) \\ &= (-1)^h \frac{\mathrm{d}^h}{\mathrm{d}s^h} \left\{ \prod_{k=1}^{N_\mathrm{I}} \int_0^\infty \mathrm{e}^{-sy_k} p_{s_k}(y_k)\mathrm{d}y_k \right\}, \end{aligned} \tag{3.62}$$

where the last line follows under the assumption of statistically independent interferers. Using the composite distribution in (3.56) with $m_k = m_{\rm I}$ and $\mu_{\Omega_k\,(\rm dBm)} = \mu_{\Omega_I\,(\rm dBm)}$, $i = 1, \ldots, N_{\rm I}$, it can be shown that

$$\int_0^\infty \mathrm{e}^{-sy_k} p_{s_k}(y_k)\mathrm{d}y_k = \frac{m_{\rm I}^{m_{\rm I}}}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-x^2}}{\left(10^{(\mu_{\Omega_I\,(\rm dBm)}+\sqrt{2}\sigma_\Omega x)/10} s + m_{\rm I}\right)^{m_{\rm I}}} \mathrm{d}x. \tag{3.63}$$

Using this result and averaging over the log-normal shadowing distribution of the desired signal gives the final result

$$\begin{aligned} O_I = 1 - \int_0^\infty & \left( \sum_{h=0}^{m_0-1} \left(-\frac{m_0 \lambda_{\rm th}}{\Omega_0}\right)^h \frac{1}{h!} \right. \\ & \times \frac{\mathrm{d}^h}{\mathrm{d}s^h} \left\{ \left( \int_{-\infty}^{\infty} \frac{m_{\rm I}^{m_{\rm I}} \mathrm{e}^{-x^2}\,\mathrm{d}x}{\sqrt{\pi}\left(10^{(\mu_{\Omega_k\,(\rm dBm)}+\sqrt{2}\sigma_\Omega)x/10} s + m_{\rm I}\right)^{m_{\rm I}}} \right)^{N_{\rm I}} \right\} \Bigg|_{s=\frac{m_0\lambda_{\rm th}}{\Omega_0}} \Bigg) \\ & \times \frac{1}{\sqrt{2\pi}\xi\sigma_\Omega\Omega_0} \exp\left\{ -\frac{(10\log_{10}\{\Omega_0\} - \mu_{\Omega_0\,(\rm dBm)})^2}{2\sigma_\Omega^2} \right\} \mathrm{d}\Omega_0. \end{aligned} \tag{3.64}$$

Equation (3.64) is an exact expression for log-normal Nakagami fading channels. When $m_0 = m_{\rm I} = 1$, it reduces to the simpler expression obtained by Linnartz [163] for log-normal Rayleigh fading channels. In (3.64), let

$$F(s) = \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-x^2}}{\left(10^{(\mu_{\Omega_I\,(\rm dBm)}+\sqrt{2}\sigma_\Omega x)/10} s + m_{\rm I}\right)^{m_{\rm I}}} \mathrm{d}x \tag{3.65}$$

and use the identity [118]

$$\begin{aligned} G(s) &= \frac{\mathrm{d}^h}{\mathrm{d}s^h} (F(s))^{N_{\rm I}} \\ &= N_{\rm I} \binom{h - N_{\rm I}}{N_{\rm I}} \sum_{i=1}^{h} (-1)^i \binom{h}{i} \frac{(F(s))^{N_{\rm I}-i}}{N_{\rm I}-i} \frac{\mathrm{d}^h}{\mathrm{d}s^h} F(s). \end{aligned} \tag{3.66}$$

Observe that $G(s)$ is a function of the derivatives of $F(s)$ only, and

$$\frac{\mathrm{d}^h F(s)}{\mathrm{d}s^h} = \frac{\mathrm{d}^h}{\mathrm{d}s^h}\left\{\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-x^2}}{\left(10^{\left(\mu_{\Omega_I\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega x\right)/10} s + m_\mathrm{I}\right)^{m_\mathrm{I}}}\mathrm{d}x\right\}$$

$$= (-1)^h \frac{(m_\mathrm{I}+h-1)!}{(m_\mathrm{I}-1)!}\int_{-\infty}^{\infty} \frac{\left(10^{\left(\mu_{\Omega_I\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega x\right)/10}\right)^h \mathrm{e}^{-x^2}}{\left(10^{\left(\mu_{\Omega_I\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega x\right)/10} s + m_\mathrm{I}\right)^{m_\mathrm{I}+h}}\mathrm{d}x. \quad (3.67)$$

We can obtain $G(s)$ from (3.66) and (3.67), and substitute it into (3.64). Then using the following change in the variable of integration

$$x = \frac{10\log_{10}\{\Omega_0\} - \mu_{\Omega_0\,(\mathrm{dBm})}}{\sqrt{2}\sigma_\Omega} \quad (3.68)$$

the outage probability in (3.64) becomes

$$O_I = 1 - \sum_{h=0}^{m_0-1}\left(-m_0\lambda_\mathrm{th}10^{-\mu_{\Omega_0\,(\mathrm{dBm})}/10}\right)^h \frac{m_\mathrm{I}^{m_\mathrm{I}N_\mathrm{I}}}{\sqrt{\pi}^{N_\mathrm{I}+1}h!}$$
$$\times \int_{-\infty}^{\infty} 10^{-\sqrt{2}\sigma_\Omega xh/10}\mathrm{e}^{-x^2} G\left(m_0\lambda_\mathrm{th}10^{-\left(\mu_{\Omega_0\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega xh\right)/10}\right)\mathrm{d}x. \quad (3.69)$$

The integrals in (3.67) and (3.69) can be efficiently computed using Gauss–Hermite quadrature integration. Applying the Gauss–Hermite quadrature formula to (3.67) gives

$$\frac{\mathrm{d}^h F(s)}{\mathrm{d}s^h} = (-1)^h \frac{(m_\mathrm{I}+h-1)!}{(m_\mathrm{I}-1)!}\sum_{t=1}^{N_\mathrm{p}} H_{x_t} \frac{10^{\left(\mu_{\Omega_I\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega x_t\right)h/10}}{\left(10^{\left(\mu_{\Omega_I\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega x_t\right)/10} s + m_\mathrm{I}\right)^{m_\mathrm{I}+h}} \quad (3.70)$$

where $H_{x_t}$ are weight factors, $x_t$ are the zeros of the Hermite polynomial $H_\mathrm{p}(x)$, and $N_\mathrm{p}$ is the order of the Hermite polynomial. When obtaining numerical results, a Hermite polynomial of order 16 resulted in sufficient accuracy and the corresponding values for $H_{x_t}$ and $x_t$ are listed in Table 3.1. Likewise, for (3.69) we have

$$O_I = 1 - \sum_{h=0}^{m_0-1}\left(-m_0\lambda_\mathrm{th}10^{\mu_{\Omega_0\,(\mathrm{dBm})}/10}\right)^h \frac{m_\mathrm{I}^{m_\mathrm{I}N_\mathrm{I}}}{\sqrt{\pi}^{N_\mathrm{I}+1}h!}$$
$$\times \sum_{\ell=1}^{N_\mathrm{p}} H_{x_\ell} 10^{-\sqrt{2}\sigma_\Omega x_\ell h/10} G\left(m_0\lambda_\mathrm{th}10^{-\left(\mu_{\Omega_0\,(\mathrm{dBm})}+\sqrt{2}\sigma_\Omega xh\right)/10}\right)\mathrm{d}x. \quad (3.71)$$

**Table 3.1** Zeros and weight factors of 16 order Hermite polynomials [2]

| *Zeros* $x_i$ | *Weight factors* $H_{x_i}$ |
|---|---|
| $\pm 0.27348104613815$ | $5.079294790166 \times 10^{-1}$ |
| $\pm 0.82295144914466$ | $2.806474585285 \times 10^{-1}$ |
| $\pm 1.38025853919888$ | $8.381004139899 \times 10^{-2}$ |
| $\pm 1.95178799091625$ | $1.288031153551 \times 10^{-2}$ |
| $\pm 2.54620215784748$ | $9.322840086242 \times 10^{-4}$ |
| $\pm 3.17699916197996$ | $2.711860092538 \times 10^{-5}$ |
| $\pm 3.86944790486012$ | $2.320980844865 \times 10^{-7}$ |
| $\pm 4.68873893930582$ | $2.654807474011 \times 10^{-10}$ |

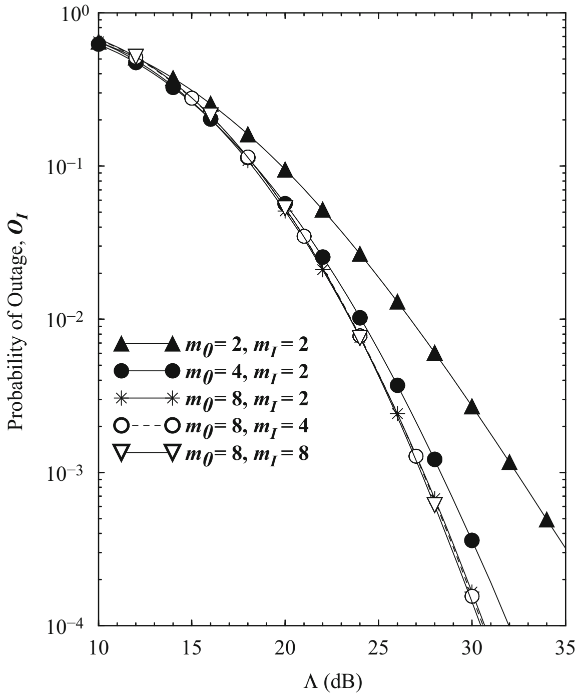



**Fig. 3.7** Probability of CCI outage when the desired and interfering signals are Nakagami faded. Results are shown for various fading distribution parameters; $\sigma_\Omega = 6\,\text{dB}$, $\lambda_{\text{th}} = 10.0\,\text{dB}$

Figure 3.7 shows the probability of outage as a function of the average-carrier-to-average-interference ratio

$$\Lambda = \frac{\mu_{\Omega_0}}{N_{\text{I}}\mu_{\Omega_I}}. \tag{3.72}$$

Results are plotted for $N_{\text{I}} = 6$ interfering signals and varying degrees of fading on the desired and interfering signals. Observe that the outage probability is insensitive to changes in the Nakagami shape factor, $m$, for interfering signals. This phenomenon

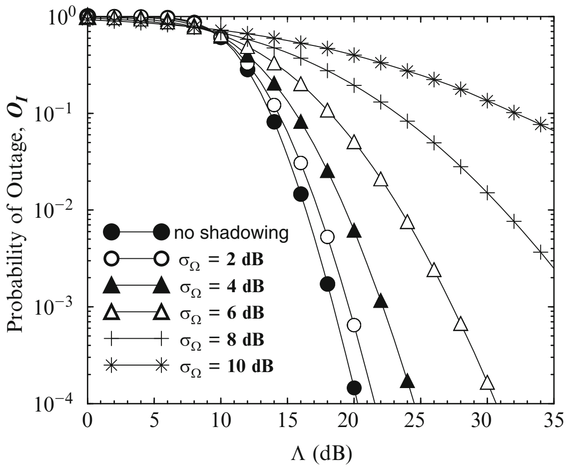



**Fig. 3.8** Probability of CCI outage when the desired and interfering signals are Nakagami faded. Results are shown for various shadow standard deviations; $m_0 = 8$, $m_{\rm I} = 2$, $\lambda_{\rm th} = 10.0\,{\rm dB}$

demonstrates that the probability of CCI outage is dominated by the fading of the desired signal rather than fading of the interfering signals. Figure 3.8 shows the outage probability for different values of the shadow standard deviation $\sigma_\Omega$. The shadow standard deviation has a significant effect on the outage probability.

### *3.4.2 Statistically Non-identical Co-channel Interferers*

If the interferers are statistically nonidentical, then (3.64) still applies with $m_{\rm I}$ replaced by $m_k$. Since the product in (3.64) does not reduce to taking the $n$th power, the numerical evaluation is difficult. This difficulty can be overcome using approximations. Sect. 2.6.2.1 showed that the composite distribution of the squared-envelope due to Nakagami fading and log-normal shadowing can be approximated by a log-normal distribution with the parameters in (2.314). Moreover, the sum of log-normal random variables can be approximate by still another log-normal random variable using either the Fenton–Wilkinson method in Sect. 3.1.1 or the Schwartz and Yeh method in Sect. 3.1.2. Hence, we can use (2.314) to find individual approximated log-normal distribution for each of the interfering signals, and then apply Schwartz and Yeh's method or the Fenton–Wilkinson method to find a pure log-normal distribution for the total interference power $s_{\rm I}$. This results in the density

$$p_{s_{\rm I}}(x) = \frac{1}{x\sigma_I \xi\sqrt{2\pi}} \exp\left\{ -\frac{\left(10\log_{10}\{x\} - \mu_{s_I\,({\rm dBm})}\right)^2}{2\sigma_I^2} \right\}. \tag{3.73}$$

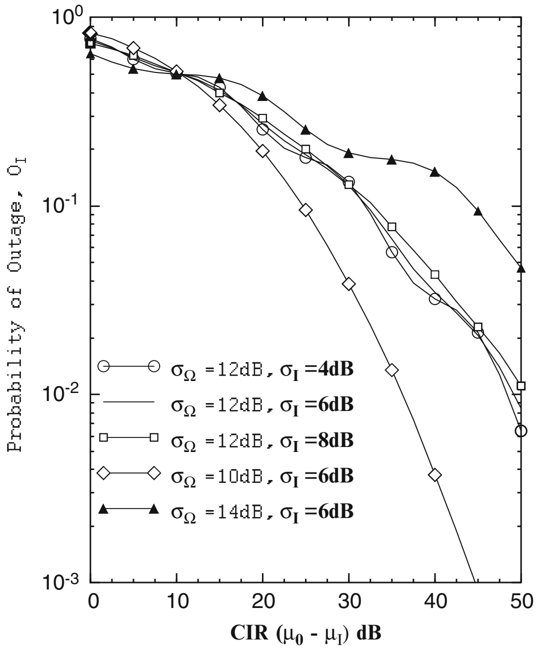



**Fig. 3.9** Probability of CCI outage for different dB spreads and statistically nonidentical interferers; $m_0 = 4$, $\lambda_{\text{th}} = 10.0\,\text{dB}$

To maintain accuracy, we still treat the desired signal as a composite Nakagami log-normal signal with the *pdf* in (3.56). After repeated integrations

$$O_I = 1 - \sum_{k=0}^{m_0-1} \frac{\left(m_0 \lambda_{\text{th}} 10^{\left(\mu_{s_I\,(\text{dBm})} - \mu_{\Omega_0\,(\text{dBm})}\right)/10}\right)^k}{k!\pi}$$
$$\times \int_{-\infty}^{\infty} \mathrm{e}^{-y^2} \mathrm{e}^{\sqrt{2}\sigma_I \xi k y} \int_{-\infty}^{\infty} \mathrm{e}^{-x^2} \mathrm{e}^{-\sqrt{2}\sigma_\Omega \xi k x}$$
$$\times \exp\left\{-m_0 \lambda_{\text{th}} \mathrm{e}^{\sqrt{2}\xi(\sigma_I y - \sigma_\Omega x) + \xi\left(\mu_{s_I\,(\text{dBm})} - \mu_{\Omega_0\,(\text{dBm})}\right)}\right\} \mathrm{d}x \mathrm{d}y. \tag{3.74}$$

We note that when the number of interferers increases, $\sigma_I$ decreases, while $\mu_I$ increases. For a fixed $\mu_d$, the CIR will be reduced when the number of interferers is increased. Once again (3.74) can be evaluated using double Gauss-Hermite quadrature integration. Figure 3.9 shows the probability of CCI for interferers with various statistics. Observe that the number of interferers and shadowing are the dominant factors in determining the probability of CCI outage.

## Problems

**3.1.** A receiver is affected by three log-normally shadowed co-channel signals having the power sum

$$I = \sum_{k=1}^{3} I_k,$$

where

$$I_{1(\mathrm{dB})} \sim \mathscr{N}(-10\ \mathrm{dBm}, \sigma_\Omega^2),$$

$$I_{2(\mathrm{dB})} \sim \mathscr{N}(-15\ \mathrm{dBm}, \sigma_\Omega^2),$$

$$I_{3(\mathrm{dB})} \sim \mathscr{N}(-20\ \mathrm{dBm}, \sigma_\Omega^2),$$

and where $\sigma_\Omega = 8$ dB, and $\mathscr{N}(\mu, \sigma_\Omega^2)$ refers a Gaussian random variable with mean $\mu$ and variance $\sigma_\Omega^2$. The sum $I$ is to be approximated as another log-normal random variable, $Z$, using the Fenton–Wilkinson method.

(a) Find the mean and variance of $Z_{(\mathrm{dB})}$.
(b) Suppose that the received carrier power $C_{(\mathrm{dB})}$ has the distribution

$$C_{(\mathrm{dB})} \sim N(0\ \mathrm{dBm}, \sigma_\Omega^2),$$

where $\sigma_\Omega = 8$ dB. Using your result from part a), what is the distribution of the carrier-to-interference ratio $\Lambda_{(\mathrm{dB})} \equiv (C/I)_{(\mathrm{dB})}$?

**3.2.** Consider a system where the local mean signal strength is described by (1.3), where $d_o = 1$ m, $\mu_{\Omega_p\ (\mathrm{dBm})}(d_o) = 0$ dBm, $\beta = 3.5$, and $\varepsilon_{(\mathrm{dB})}$ is a zero-mean Gaussian random variable with standard deviation $\sigma_\Omega = 8$ dB.

(a) Suppose that a mobile station is 1 km from its serving base-station and at distances 3 and 4 km from two co-channel base-stations. Using the Fenton–Wilkinson method, derive the probability density function of the received interference power $I_{(\mathrm{dB})}$.
(b) What is the probability density function of the received carrier-to-interference ratio $\Lambda_{(\mathrm{dB})}$?

**3.3.** The scenario in Fig. 3.10 depicts the worst case CCI situation for the first tier of co-channel interferers on the forward channel. Assume a reuse cluster size of seven cells, a cell radius of $R = 3$ km, a path loss exponent of $\beta = 3.5$, and a receiver carrier-to-interference threshold $\Lambda_{\mathrm{th}\ (\mathrm{dB})} = 10$ dB. Ignore the effect of handoffs and assume that the MS must stay connected to the BS in the center cell.

(a) Assuming that the local mean signal strength is described by (1.3) with $\mu_{\Omega_p\ (\mathrm{dBm})}(d_o) = -10$ dBm at $d_o = 1$ km, a shadow standard deviation $\sigma_\Omega = 8$ dB, calculate the probability of outage $O_I(\mathbf{d})$ in (3.46) using the Fenton–Wilkinson method.

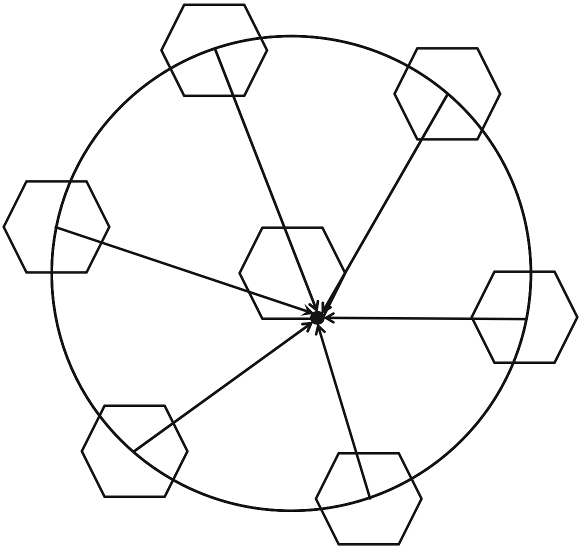


**Fig. 3.10** Worst case CCI situation on forward channel in Problem 3.3

(b) For $\sigma_\Omega = 4$ dB, what is required threshold $\Lambda_{\text{th}}$ such that the probability of outage is less than 1%?
(c) Repeat b) for $\sigma_\Omega = 12$ dB.

**3.4.** Consider the Fenton–Wilkinson method for approximating the sum of log-normal random variables. Consider the sum of $N$ log-normal random variables

$$I = \sum_{k=0}^{N} \mathrm{e}^{\hat{\Omega}_k},$$

where the $\hat{\Omega}_k$ are independent zero-mean Gaussian random variables with $\sigma_\Omega = 8$ dB. Plot the mean $\mu_{Z\ (\text{dBm})}$ and variance $\sigma_Z^2$ of the approximate Gaussian random variable $Z_{(\text{dB})}$ as a function of $N$ for $N = 2,3,4,\ldots,10$.

**3.5.** This problem uses Monte Carlo simulation techniques to verify the usefulness of the Schwartz and Yeh approximation and the Fenton–Wilkinson approximation for the sum of two log-normal random variables. Consider the sum of two log-normal random variables

$$I = \Omega_1 + \Omega_2,$$

where the corresponding Gaussian random variables $\Omega_{1\ (\text{dB})}$ and $\hat{\Omega}_{2\ (\text{dB})}$ are independent and identically distributed with zero mean and variance $\sigma_\Omega^2$. Using the Schwartz and Yeh method, plot the values of $\mu_{Z\ (\text{dB})}$ and $\sigma_Z^2$ as a function of the variance $\sigma_\Omega^2$. Repeat for the Fenton–Wilkinson method. Now obtain the same results using computer simulation and compare the analytical results. What are your conclusions?

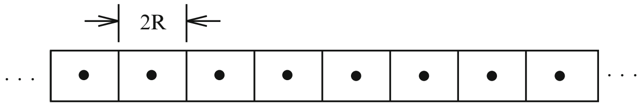



**Fig. 3.11** Highway microcell deployment for Problem 3.6

**3.6.** You are asked to design a highway microcell system as shown in Fig. 3.11. Each cell has length $2R$.

(a) A BS with an omnidirectional antenna is placed at the center of each cell. Ignoring shadowing and envelope fading, determine the minimum reuse factor needed so that the worst case carrier-to-interference ratio, $\Lambda$, is at least 17 dB. State whatever assumptions you make.
(b) Now suppose that directional antennas are used to divide each cell into two sectors with boundaries perpendicular to the highway. Repeat part (a).
(c) Consider again the sectored cell arrangement in part (b). If shadowing is present with a standard deviation of $\sigma_\Omega$ dB, what is the probability of CCI outage on a cell boundary? Assume soft handoffs between adjacent cells.

**3.7.** Derive (3.50).

**3.8.** Derive (3.51).

**3.9.** Derive (3.53).

**3.10.** Derive (3.74).

**3.11.** Consider a microcellular environment where a Ricean faded desired signal is affected by a single Rayleigh faded interferer. Neglect the effect of path loss and shadowing. Suppose that the transmission quality is deemed acceptable if both the instantaneous carrier-to-noise ratio and the instantaneous carrier-to-interference ratio exceed the thresholds, $\gamma_{th}$ and $\lambda_{th}$, respectively. Analogous to (3.53), derive an expression for the probability of outage.