Chapter 5
The Effects of Changes in Risk on Risk Taking: A Survey

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Abstract We examine an important class of decision problems under uncertainty that entails the standard portfolio problem and the demand for coinsurance. The agent faces a controllable risk—his demand for a risky asset, for example—and a background risk. We determine how a change in the distribution in one of these two risks affects the optimal exposure to the controllable risk. Restrictions on first-order and second-order stochastic dominance orders are in general necessary to yield an unambiguous comparative statics property. We also review another line of research in which restrictions are made on preferences rather than on stochastic dominance orders.

Keywords Comparative statics under uncertainty • Increase in risk • Background risk • Portfolio decision • Insurance demand

5.1 Introduction

To start this survey, we present two problems that look very different at first glance. Consider an investor who has to allocate a given amount of money (w_0) between a safe asset paying a return (i) and a risky one paying a random return ($\tilde{x}$). If the mathematical expectation of $\tilde{x}$ exceeds i, it is optimal for an investor who obeys the axioms of expected utility to invest a strictly positive amount in the risky asset. Assume now that because of some good news, the prospects of the risky asset become "better" in the sense of improving the welfare of its holder. Intuition suggests that a rational investor should invest more in the risky asset because it has become relatively more attractive.

We now turn to the second problem. We consider the case of an insured whose wealth w_0 may be reduced by a random damage $\tilde{y}$. To protect himself against this damage he can buy insurance that is sold with a positive and proportional loading by an insurance company. The company and the insured have identical information about the initial risk $\tilde{y}$. It is well known that in this case, expected-utility maximizers should buy less than full insurance. Now assume that the insured receives a private information indicating that his risk deteriorates. Intuition suggests again that the insured should now demand more coverage to compensate for the deterioration in risk.

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The examples of portfolio and insurance decisions illustrate a more general problem that is the topics of this survey: how do changes in risk affect risk taking (e.g., portfolio) or risk avoidance (e.g., insurance) by a decision-maker? We basically show that unless specific restrictions are made on the change in risk and/or on the shape of the utility function, a risk-averse decision-maker may very well decide to increase his exposure to a risk whose distribution deteriorates.

While they have the same formal structure, the two examples just described share another important feature: the decision-maker faces only one risk and by his single decision about this risk, he optimally controls the total risk he will assume. An important part of this survey will be devoted to a more realistic case developed in the literature under the general heading of "background risk." In this problem two risks are involved: one is exogenous and is not subject to transformations by the decision-maker while the other one is endogenous and can be controlled in the way described in each of the two examples. The exogenous risk can be, for example, a risk related to labor income that is traditionally not insurable through standard insurance markets. The question raised in this new framework can be described as follows: how does the background risk affect the optimal decisions about the endogenous one? Is it true that, e.g., a deterioration in the background risk will always reduce risk taking vis-a-vis the other risk?

Before turning to this question, we present our basic model in Sect. 5.2 and we state some first results about it. Section 5.3 is devoted to a presentation of the standard stochastic orders. In Sect. 5.4, we survey results about the impact of a change in the distribution of the endogenous/controllable risk. As indicated earlier, the role and impact of background risk are examined in Sect. 5.5. Some extensions and a concluding remark are provided, respectively, in Sects. 5.6 and 5.7.

5.2 A Simple Model

The two problems presented in the introduction can be written in the following compact manner[1]:

$$\max_{\alpha} \; Eu(w_0 + \alpha\tilde{x} + \tilde{\epsilon}), \tag{5.1}$$

where α is the decision variable, the value of which measures the extent of risk taking. The random variable $\tilde{\epsilon}$ stands for the background risk. The utility function u is assumed to be increasing and concave. By assumption $\tilde{\epsilon}$ is independent of $\tilde{x}$, the endogenous/controllable risk.[2]

Notice finally that for the problem to make sense the random variable $\tilde{x}$ must take negative and positive values; otherwise, the optimal α would be either $-\infty$ or $+\infty$. The absolute value of α expresses the exposure to risk $\tilde{x}$. Its optimal level—denoted α^*—has two properties that can be stated as follows:

- If the mathematical expectation of $\tilde{x}$ is strictly positive, so will be α^*. This property which was shown to be true in the absence of background risk remains valid in its presence (for a proof in an insurance context, see Doherty-Schlesinger 1983).
- In the absence of background risk an increase in risk aversion decreases α^* (see Pratt 1964). However, as shown by Kihlstrom et al. (1981), this relationship does not extend when an

[1]For more details, see Dionne et al. (1993) and more especially pages 315–317. See also Eeckhoudt-Gollier (1995) and more specifically page 183, Exercise 10.1. The reader who is interested in an insurance interpretation of some results in this survey may also refer to Alarie et al. (1992).

[2]Gollier and Schlee (2006) examine the more general problem with a correlated background risk. More recent developments around these topics are also mentioned at the end of Sect. 5.6. Notice also that many results reviewed in this chapter also hold when final wealth is a concave function of α and $\tilde{x}$.

independent background risk is added to initial wealth. This result illustrates the importance of background risk, the presence of which may invalidate results that hold true in its absence.

5.3 Detrimental Changes in Risk

Suppose that random variable $\tilde{x}$ undergoes an exogenous change in distribution. The initial cumulative distribution function is denoted F, whereas the final one is denoted G. Economists usually consider two specific subsets of changes in risk: first-order or second-order stochastic dominance (respectively, FSD and SSD). In order to define these stochastic dominance orders, one looks at the effect of a change in risk on a specific class of agents.

5.3.1 *First-Order Stochastic Dominance*

F dominates G in the sense of FSD if the expected utility under F is larger than under G for any increasing utility function:

$$\int u(x)\mathrm{d}F(x) \;\geq\; \int u(x)\mathrm{d}G(x) \;\; \forall u \text{ increasing.} \tag{5.2}$$

Observe that among the set of increasing functions, we have the standard "step" (or indicator) function, which takes value 0 if x is less than a given y; otherwise, it takes value 1. Thus, applying the above definition to this function yields the necessary condition $1 - F(y) \geq 1 - G(y)$ or $F(y) \leq G(y)$. Notice also that any increasing function can be obtained by a convex combination of step functions, i.e., the set of step functions is a basis of the set of increasing functions. Observe finally that the expectation operator is linear, i.e., if u_1 and u_2 satisfy condition (5.2), then $\lambda u_1 + (1-\lambda)u_2$ also satisfies (5.2). All this implies that requiring $F(y) \leq G(y)$ for all y is not only necessary but also sufficient to guarantee that (5.2) holds. In conclusion, F dominates G in the sense of FSD if and only if

$$F(x) \leq G(x) \;\; \forall x. \tag{5.3}$$

Among other properties,[3] it is worth remembering that after an FSD deterioration the mathematical expectation of a random variable necessarily decreases while the converse is not necessarily true.

5.3.2 *Second-Order Stochastic Dominance*

Whereas this notion was already known in the statistical literature for a long time,[4] it became popular in the economics and finance literature after the publication of Hadar and Russell's article (1969). Distribution F dominates distribution G in the sense of SSD if all risk-averse agents prefer F to G. This is less demanding than FSD, since SSD requires F to be preferred to G just for increasing and concave utility functions, not for all increasing functions.

[3]For an excellent survey on stochastic dominance, see H. Levy (1992).

[4]See Hardy et al. (1929).

Observe that the set of "min" functions— $u(x) = \min(x, y)$—are increasing and concave. Thus a necessary condition for SSD is obtained by requiring condition (5.2) to hold for such functions. It yields

$$\int^y x\mathrm{d}F(x) + y(1 - F(y)) \geq \int^y x\mathrm{d}G(x) + y(1 - G(y)),$$

or, integrating by parts,

$$\int^y F(x)\mathrm{d}x \leq \int^y G(x)\mathrm{d}x. \tag{5.4}$$

Notice that any increasing and concave function can be obtained by a convex combination of "min" functions. Thus, using the same argument as before, it is true that condition (5.4) is not only necessary but is also sufficient for F to dominate G in the sense of SSD.

If F dominates G in the sense of SSD and if F and G have the same mean, then G is said to be an increase in risk (IR). Rothschild and Stiglitz (1970) showed that any increase in risk can be obtained either by adding noise to the initial random variable or by a sequence of mean-preserving spreads (MPS) of probabilities. A noise is obtained by adding a zero-mean lottery to any outcome of the initial random variable. A MPS is obtained by taking some probability mass from the initial density and by transfering it to the tails in a way that preserves the mean.

Finally, notice that any SSD deterioration in risk can be obtained by the combination of an FSD deterioration combined with an increase in risk.

5.4 The Comparative Statics of Changes in the Controllable Risk

In this section, we assume that some information is obtained that allows agents to revise the distribution of $\tilde{x}$, but $\tilde{\epsilon}$ remains unaffected. The literature devoted to this topic was mostly developed under the assumption that there is no background risk. Most often, this is without loss of generality. Indeed, for every increasing and concave u, define the indirect utility function v as follows:

$$v(z) = Eu(z + \tilde{\epsilon}). \tag{5.5}$$

This allows us to rewrite the initial problem (5.1) as

$$\max_{\alpha} Ev(w_0 + \alpha\tilde{x}). \tag{5.6}$$

Observe now that $u^{[n]}$, i.e., the nth derivative of u, and $v^{[n]}$ have the same sign, for any integer n. In particular v is increasing and concave. As long as no restriction on the utility function other than those on the sign of some of its derivatives is imposed, (5.1) and (5.6) are qualitatively the same problems.

As mentioned above, stochastic orders have been defined on the basis of how changes in distribution affect the welfare of some well-defined set of agents in the economy. In this section, we examine the effect of a disliked change in the distribution of $\tilde{x}$ on the optimal exposure α^* to this risk. For a while many researchers naturally extended the results about the agent's welfare to his optimal degree of risk taking. It turns out, however, that such an extension may not be correct.

The first-order condition on α^* under distribution F is written as

$$\int xu'(w_0 + \alpha^* x)\mathrm{d}F(x) = 0. \tag{5.7}$$

Given the concavity of the objective function with respect to the decision variable, the change in risk from F to G reduces the optimal exposure to risk if

$$\int xu'(w_0 + \alpha^* x)\mathrm{d}G(x) \leq 0. \tag{5.8}$$

It happens that F dominating G in the sense of FSD or SSD is neither necessary nor sufficient for α^* to be reduced, i.e., for condition (5.8) to be satisfied whenever (5.7) is satisfied. It is striking that an FSD deterioration in risk $\tilde{x}$ or an increase in risk $\tilde{x}$ can induce some risk-averse agents to increase the size α^* of their exposure to it! As counterexamples, let us examine the standard utility function $u(z) = z^{1-\gamma}/(1-\gamma)$. Consider in particular the case of a constant relative risk aversion $\gamma = 3$, which is within the range of degrees of risk aversion observed in the real world. Finally, take $w_0 = 2$ and an initial distribution of $\tilde{x} = (-1, 0.1; +4, 0.9)$. In this case, one can compute $\alpha^* = 0.6305$.

Suppose now that $\tilde{x}$ undergoes an FSD deterioration with a new distribution $(-1, 0.1; +2, 0.9)$. Contrary to the intuition, the agent reacts by increasing his exposure to $\alpha^* = 0.7015$! Alternatively, suppose that $\tilde{x}$ undergoes an increase in risk to the new distribution $(-1, 0.1; +3, 0.45, +5, 0.45)$. Again, it is a puzzle that the agent reacts to this increase in risk by increasing his exposure to $\alpha^* = 0.6328$.

From examples such as these, researchers tried to restrict the model in order to exclude the possibility of such puzzles. Two directions of research have been followed. One can either restrict preference functionals, or one can restrict the set of changes in risk. We hereafter examine these two lines of research separately.

5.4.1 *Restrictions on the Utility Function*

This line of research has been explored by Rothschild and Stiglitz (1971), Fishburn and Porter (1976), Cheng et al. (1987), and Hadar and Seo (1990). All their findings rely on the following observation. Define the function $\phi(x; w_0) = xu'(w_0 + \alpha^* x)$, where α^* is the optimal exposure under F. We hereafter normalize it to unity. Combining conditions (5.7) and (5.8), the change in risk reduces the optimal exposure α^* if

$$\int \phi(x; w_0)\mathrm{d}F(x) \geq \int \phi(x; w_0)\mathrm{d}G(x). \tag{5.9}$$

5.4.1.1 Conditions for FSD Shifts

Suppose first that F dominates G in the sense of FSD. Which condition is required on ϕ to guarantee that (5.9) holds? Comparing this condition to condition (5.2) directly provides the answer to this question: ϕ must be an increasing function. Because

$$\frac{\partial \phi}{\partial x}(x; w_0) = u'(w_0 + x) + xu''(w_0 + x),$$

ϕ is increasing if

$$A^{\mathrm{r}}(w_0 + x) - w_0 A(w_0 + x) \leq 1 \ \ \forall x, \tag{5.10}$$

where $A(z) = -u''(z)/u'(z)$ and $A^{\mathrm{r}}(z) = zA(z)$ are, respectively, the absolute and the relative degree of risk aversion measured at z. In conclusion, an FSD deterioration in $\tilde{x}$ always reduces the optimal exposure to it if relative risk aversion is uniformly less than unity. If condition (5.10) is not satisfied for some x, it is always possible to build a counterexample, as we have done above.

5.4.1.2 Conditions for Increases in Risk

The same argument can be used for increases in risk, which require ϕ to be concave in x. After some computations, we get that the second derivative of ϕ with respect to x is negative if and only if

$$P^{\mathrm{r}}(w_0 + x) - w_0 P(w_0 + x) \leq 2 \ \ \forall x, \tag{5.11}$$

where $P(z) = -u'''(z)/u''(z)$ and $P^{\mathrm{r}}(z) = zP(z)$ are, respectively, the absolute and the relative degree of prudence measured at z. In conclusion, an increase in risk $\tilde{x}$ always reduces the optimal exposure to it if relative prudence is positive and less than 2. Notice that we built the counterexample above on the basis of $P^{\mathrm{r}}(z) = \gamma + 1 = 4$.

5.4.2 Restrictions on the Change in Risk

5.4.2.1 First-Order Stochastically Dominated Shifts

In this section, we present some restrictions on FSD in order to guarantee that all risk-averse agents reduce their exposure after the shift in distribution.

A first step in this direction was made in a slightly different context by Milgrom (1981) and later on by Landsberger and Meilijson (1990) and Ormistion and Schlee (1993). We say that F dominates G in the sense of the monotone likelihood ratio order (MLR) if, crudely said, $\psi(x) = G'(x)/F'(x)$ is decreasing. It is easy to verify that MLR is a particular case of FSD. If F dominates G in the sense of MLR, we obtain that

$$\begin{aligned} \int xu'(w_0 + x)\mathrm{d}G(x) &= \int xu'(w_0 + x)\psi(x)\mathrm{d}F(x) \\ &\leq \psi(0) \int xu'(w_0 + x)\mathrm{d}F(x) = 0. \end{aligned} \tag{5.12}$$

The inequality is due to the fact that $x\psi(x)$ is always less than $x\psi(0)$. The last equality is the first-order condition on $\alpha^* = 1$ under F. In consequence, a MLR deterioration in risk reduces the optimal exposure to it for all risk-averse agents.

Since the FSD condition is already rather restrictive, the MLR property is even more so. Hence it is worth trying to extend the result we have just stated. First, observe that one can replace the monotonicity of ψ by a weaker single-crossing condition: $\psi(x)$ must single-cross the horizontal line at $\psi(0)$ from above. This is indeed the only thing that has been used in the proof (5.12). This single-crossing condition is much weaker than MLR.

Second, Eeckhoudt and Gollier (1995) considered the ratio of the cumulative distributions, that is $\frac{G(x)}{F(x)}$, and coined the term "monotone probability ratio" (MPR) when this expression is nondecreasing in x. As one can guess:

$$\text{MLR} \Rightarrow \text{MPR} \Rightarrow \text{FSD}.$$

MPR is weaker than MLR but is still a subset of FSD. It can be shown that the same comparative statics property holds under MPR. Hence the MPR condition is clearly an improvement on the MLR one.

5.4.2.2 Increases in Risk

Eeckhoudt and Hansen (1980) obtained a restriction on an increase in risk that yields the desired comparative statics property. They defined the notion of a "squeeze" of a density. This notion has been extended by Meyer and Ormiston (1985) who defined a strong increase in risk (SIR). A SIR is obtained when some probability weight is taken from the initial density of $\tilde{x}$ and sent either at its boundaries or outside the initial support. Meyer and Ormiston showed that all risk-averse agents reduce their exposure to a risk that undergoes a SIR.

In two subsequent articles, Black and Bulkley (1989) and Dionne et al. (1993) weakened the notion of a SIR. Contrary to a SIR, these restrictions allow for transferring probability masses inside the initial support of the distribution of $\tilde{x}$. However, to maintain the desired comparative statics result, they had to make assumptions about the behavior of the likelihood ratio between the initial and the final densities. Another sufficient condition for an increase in risk to have an unambiguous effect on α^* is the notion of a simple increase in risk, introduced by Dionne and Gollier (1992). A simple increase in risk is an IR such that F single-crosses G at $x = 0$.

To conclude this quick review, let us mention that much of this research resulted from A. Sandmo's discussion (1971) of the impact of the "stretching" of a random variable. A stretching of $\tilde{x}$ results from its linear transformation into $\tilde{y}$ with $\tilde{y} = t\tilde{x} + (1-t)E(\tilde{x})$ and $t > 1$. This transformation is mean preserving since $E\tilde{y} = E(\tilde{x})$. This intuitive notion was later on generalized by Meyer and Ormiston (1989) under the terminology of the "deterministic transformation" of a random variable. However to obtain intuitive comparative statics results with such transformation the assumption of decreasing absolute risk aversion (DARA) is required.

All the contributions dealing with special cases of either FSD or IR that we have surveyed so far share a common trend: one starts with rather restrictive sufficient conditions to yield the desired comparative statics result and then one progressively relaxes them. The endpoint of these successive improvements is given by a set of necessary and sufficient conditions that we now present.

5.4.2.3 The Necessary and Sufficient Condition

Gollier (1995, 1997) proposed a reversal in the agenda of research. Rather than trying to restrict the existing stochastic orders in order to obtain an unambiguous comparative statics property, one should solve the following problem: what is the stochastic order such that all risk-averse agents reduce their exposure to the risk that undergoes such a change in distribution? He coined the term "central dominance" (CR) for it.

Rothschild and Stiglitz (1971) already tried to solve this question, but their solution was wrong. Their argument went as follows: under which condition can we guarantee that

$$\int xu'(w_0 + x)\mathrm{d}G(x) \leq \int xu'(w_0 + x)\mathrm{d}F(x) \tag{5.13}$$

for all increasing and concave utility functions? Using the basis approach developed earlier in this chapter, the condition is that (replace u by any "min" function)

$$\int^{y} x\mathrm{d}G(x) \leq \int^{y} x\mathrm{d}F(x)$$

for all y. Contrary to the claim of Rothschild and Stiglitz (1971), this condition is sufficient, but not necessary for CR. Indeed, condition (5.13) is sufficient but not necessary for the comparative statics property. The correct necessary and sufficient condition is that the LHS of (5.13) be negative whenever the RHS is zero. Basing the analysis on this observation, Gollier (1995) obtained a correct characterization of CR, which is

$$\exists m \in R: \ \forall y: \int^{y} x\mathrm{d}G(x) \leq m \int^{y} x\mathrm{d}F(x). \tag{5.14}$$

All sufficient conditions mentioned above are particular cases of CR. Interestingly enough, strong and simple increases in risk satisfy condition (5.14) with $m = 1$, which was the condition proposed by Rothschild and Stiglitz (1971). But conditions like MLR and MPR and the weakenings of SIR by Black and Bukley (1989) and others satisfy the condition with $m \neq 1$. Observe also, whereas we already know that SSD is not sufficient for CR (see the numerical counter examples), it also appears that SSD is not necessary. That is, it can be the case that all risk-averse agents reduce their α^* after a change which is *not* a SSD.

5.5 The Comparative Statics of Background Risk

In the previous section, we explained why the presence of a background risk is unimportant to determine the *sign* of the impact of a change in the distribution of the controllable risk. However, the background risk has an impact on the optimal *value* of the exposure to $\tilde{x}$.

In this section, we do the comparative statics analysis that is symmetric to the one performed in the previous section. We take the distribution of $\tilde{x}$ as given and we perturbate the distribution of background risk $\tilde{\epsilon}$. Up to now, the literature focused mostly on the effect of *introducing* a background risk in the analysis. One compares the solution to program (5.6) to the solution of

$$\max_{\alpha} \ Eu(w_0 + \alpha\tilde{x}).$$

Remember that, as shown by Pratt (1964), the necessary and sufficient condition for an unambiguous comparison, independent of w_0 and the distribution of $\tilde{x}$, is that v be more risk averse than u. In this case, the introduction of a background risk reduces the optimal exposure to $\tilde{x}$. Thus, the problem simplifies to determining whether

$$-\frac{Eu''(z+\tilde{\epsilon})}{Eu'(z+\tilde{\epsilon})} \geq -\frac{u''(z)}{u'(z)} \tag{5.15}$$

for all z. If $\tilde{\epsilon}$ is degenerated at a negative value, this condition is just DARA. But it is logical to concentrate the analysis on the introduction of a *pure* background risk, viz., $E\tilde{\epsilon} = 0$.

The intuition that the introduction of a pure background risk should reduce the optimal exposure to other independent risks corresponds to the common wisdom that independent risks are substitutes. This intuition requires additional restrictions to the model, as shown by the following counterexample. Take $u(z) = \min(z, 50 + 0.5z)$, $w_0 = 101$, and $\tilde{x} = (-1, 0.5; +1.9, 0.5)$. Without background risk, one can compute $\alpha^* = 1$. But if pure background risk $\tilde{\epsilon} = (-20, 0.5; +20, 0.5)$ is added to wealth w_0, the agent increases his optimal exposure to $\alpha^* = 10.53$!

Several authors tried to find conditions on u that implies that a pure background risk reduces α^*. If $\tilde{\epsilon}$ is small, one can use second-order Taylor expansions of the numerator and denominator of the LHS of (5.15) to check that

$$-\frac{Eu''(z+\tilde{\epsilon})}{Eu'(z+\tilde{\epsilon})} \cong A(z) + 0.5\sigma_{\tilde{\epsilon}}^2\left[A''(z) - 2A'(z)A(z)\right]. \tag{5.16}$$

Thus, a necessary and sufficient condition for any pure small background risk to reduce the optimal exposure to other risks is

$$A''(z) \geq 2A'(z)A(z) \ \ \forall z. \tag{5.17}$$

Absolute risk aversion may not be too concave. But what is necessary and sufficient for small risk is just necessary if one wants the comparative statics property to hold for any risk. Gollier and Scarmure (1994) proved that a sufficient condition is that absolute risk aversion be decreasing and convex. The proof of this result is immediate. Indeed, let us define $h(t) = u'(z+t)/Eu'(z+\tilde{\epsilon})$. It yields

$$\begin{aligned}
-\frac{Eu''(z+\tilde{\epsilon})}{Eu'(z+\tilde{\epsilon})} &= Eh(\tilde{\epsilon})A(z+\tilde{\epsilon}) \\
&= EA(z+\tilde{\epsilon}) \ + \ E(h(\tilde{\epsilon})-1)A(z+\tilde{\epsilon}) \\
&\geq A(z+E\tilde{\epsilon}) \ + \ \operatorname{cov}(h(\tilde{\epsilon}), A(z+\tilde{\epsilon})) \\
&\geq A(z).
\end{aligned} \tag{5.18}$$

The first inequality is a direct application of Jensen's inequality, and $A'' > 0$. The second inequality comes from the fact that h and A are two decreasing functions of ϵ. This concludes the proof.

The convexity of absolute risk aversion is compatible with its positivity and its decrease. It is also an intuitive assumption as it means that the risk premium to any (small) risk decreases with wealth in a decreasing way. Observe that the familiar utility functions with constant relative risk aversion γ are such that $A(z) = \gamma/z$, so $A' < 0$ and $A'' > 0$. Thus, there is no ambiguity of the effect of background risk for this set of utility functions.

Eeckhoudt and Kimball (1992) and Kimball (1993) obtained an alternative sufficient condition that they called "standard risk aversion." Risk aversion is standard if absolute risk aversion A and absolute prudence P are both decreasing in wealth. Decreasing prudence means that the effect on savings of a risk on future incomes is decreasing with wealth.

Gollier and Pratt (1996) obtained the necessary and sufficient condition for a background risk with a non-positive mean to increase the aversion to other independent risks. They coined the term (background) "risk vulnerability." They used a technique of proof that has been systematized in Gollier and Kimball (1997) to solve other problems dealing with multiple risks.

Up to now, we examined the effect of introducing a background risk. Eeckhoudt, Gollier, and Schlesinger (1996) considered the more general problem of the effect of increasing the background risk, in the sense of a FSD or IR shift in distribution. In the case of an increase in background risk, they showed that the restrictions to impose on u to obtain an unambiguous effect on $\alpha*$ are much more demanding than risk vulnerability. Meyer and Meyer (1998) relaxed these conditions on u at the cost of restricting the changes in risk. For example, standard risk aversion is sufficient when limiting the analysis to the effect of a strong increase in background risk.

5.6 Extensions

Let us go back to the problem analyzed in Sect. 5.4. Indeed, the effect of a change in the distribution of $\tilde{x}$ and the effect of introducing a pure background risk are not without any link. Suppose that there is no background risk, but rather that the increase in risk in $\tilde{x}$ takes the form of adding an *independent* pure white noise $\tilde{\epsilon}$ to it. The derivative of the objective function with the new risk $\tilde{x}+\tilde{\epsilon}$ evaluated at the initial optimal exposure (normalized to 1) is written as

$$\begin{aligned} E(\tilde{x}+\tilde{\epsilon})u'(w_0+\tilde{x}+\tilde{\epsilon}) &= E\tilde{x}u'(w_0+\tilde{x}+\tilde{\epsilon}) + E\tilde{\epsilon}u'(w_0+\tilde{x}+\tilde{\epsilon}) \\ &= E\tilde{x}v'(w_0+\tilde{x}) + E\tilde{\epsilon}u'(w_0+\tilde{x}+\tilde{\epsilon}) \\ &\leq E\tilde{\epsilon}u'(w_0+\tilde{x}+\tilde{\epsilon}) \\ &\leq 0. \end{aligned} \tag{5.19}$$

The first inequality is obtained by using the fact that $\alpha^* = 1$ under the initial risk $\tilde{x}$, together with the fact that v is more concave than u under risk vulnerability. The second inequality is a direct consequence of the fact that $E\tilde{\epsilon} = 0$. We conclude that risk-vulnerable agents reduce their exposure to a risk that has been increased in the sense of adding a zero-mean independent white noise to it. This result is in Gollier and Schlesinger (1996).

Other developments of this field of research have been made to extend the basic model (5.1) to more than one source of endogenous risk. Landsberger and Meilijson (1990), Meyer and Ormiston (1985) and Dionne and Gollier (1996) considered the two-risky-asset problem, which is written as

$$\max_{\alpha} \; Eu(w_0+\alpha\tilde{x}_1+(1-\alpha)\tilde{x}_2).$$

These authors determined whether imposing MLR, SIR, or other restrictions on the change in the conditional distribution of $\tilde{x}_1$ generates the same conclusion in this more general context. Notice that rewriting final wealth as $w_0+\alpha(\tilde{x}_1-\tilde{x}_2)+\tilde{x}_2$ suggests that this problem is similar to the initial one, with a controllable risk $(\tilde{x}_1-\tilde{x}_2)$ and a "background" risk $\tilde{x}_2$. But the two risks are here correlated.

Another line of research is related to the management of multiple endogenous risks, a problem which can be formulated as follows:

$$\max_{\alpha_1,\ldots,\alpha_n} \; Eu\left(w_0+\sum_{i=1}^{n}\alpha_i\tilde{x}_i\right).$$

Eeckhoudt et al. (1994) examined the case where the $\tilde{x}_i$ are i.i.d., in which case all α_i^* are the same. They addressed the question of how α^* is affected by an increase in n. As an application, we have the optimal strategy of an agent who has to insure a fleet of vehicles. Gollier et al. (1997) showed that an increase in n reduces α^* if relative risk aversion is constant and less than unity.

While the extension presented so far was made in the 1990s to deal essentially with endogenous risks, it is worth mentioning that after 2000 there was a renewal of interest for the impact of background risks on the management of controllable ones. The first article in this direction was published by Arrondel–Calvo (2003). These authors considered the case of correlated small risks and obtained a first interesting result: an additive and negatively correlated background risk may increase the demand for the endogenous risky asset because the decision-maker will wish to benefit from the hedging effect induced by the increased holding of such an asset.

This line of research was much developed a few years later in two almost simultaneous articles by Tsetlin–Winkler (2005) and Franke et al. (2006) who extended the Arrondel–Calvo's contributions in two directions. They considered "large" risks and they also analyzed the (realistic) case of multiplicative background risks. Many examples of such a situation can be found in both articles and they give rise in Franke et al. (2006) to the notion of "multiplicative risk vulnerability" (see their (5.3) and its discussion) which complements that of additive risk vulnerability discussed in Sect. 5.5.

Finally attention was recently paid to the case where the background risk is not expressed in the same units as the endogenous one. Building upon a previous articles by Rey (2003), Li (2011) analyzed the behavior of an investor jointly facing an endogenous financial risk and an exogenous nonfinancial one that are not independent. Interestingly the analysis is developed using different notions of dependence, beyond the traditional one of correlation.

5.7 Conclusion

Stochastic dominance orders have been defined to determine the effect of a change in risk on the welfare of some category of economic agents. It is now apparent that these concepts are not well suited to perform comparative statics analyses. As an example, an increase in risk à la Rothschild–Stiglitz on the return of a risky asset may induce some risk-averse agents to increase their demand for it. Also, an increase in background risk à la Rothschild–Stiglitz may induce some risk-averse agents to raise their demand for another independent risk. In this chapter, we summarize the main findings that allow to solve these paradoxes. We tried to convince the reader that most restrictions to preferences or to stochastic orders make sense even if some are rather technical.

We examined a simple model with a single source of endogenous risk, plus a background risk. We separately considered the case of a change in the distribution of the endogenous risk and the case of a change in background risk. The current trends in this field are for the analysis of multiple risk taking situations, in which these two analyses are often combined to produce new results. Much progress must be still done on our understanding of the interaction between risks, but we now have the relevant tools and concepts to perform this work efficiently.

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