Chapter 5
Friction

5.1 Introduction

Friction (from Latin fricare, to rub), is the term given to the resistance caused by the moving of the surfaces of bodies over each other. The resistance is due to the roughness of the surfaces. The first experiments on friction were made by Guillaume Amontons (1699) when he published his rediscovery of the laws of friction first presented by Leonardo da Vinci. Leonardo da Vinci (1452–1519) studied screws, gears, mechanisms, wear, bearings, friction, and lubrication. At Rochefort in 1781, Charles-Augustin de Coulomb verified the laws friction. The laws of dry friction are: 1. friction is directly proportional to the normal force between the surfaces of contact (Amontons 1st Law); 2. friction is independent of the apparent area of contact (Amontons 2nd Law); 3. friction is independent of the velocity with which the surfaces slide one on the other (Coulomb's Law). Arthur Jules Morin confirmed and extended Coulomb's work on friction (1830–1834). He build an experimental apparatus under the supervision of Jean-Victor Poncelet. He developed an apparatus to study the laws of falling bodies presented an accurate experimental proof of Galileo's result that distances travelled by a falling body increase as the square of the time.

If a body rests on an incline plane, the friction force exerted on it by the surface prevents it from sliding down the incline. The question is, what is the steepest incline on which the body can rest?

A body is placed on a horizontal surface. The body is pushed with a small horizontal force F. If the force F is sufficiently small, the body does not move.

Figure 5.1 shows the free-body diagram of the body, where the force W is the weight force of the body, and N is the normal force exerted by the surface on the body. The force F is the horizontal force, and F_f is the friction force exerted by the surface. Friction force arises in part from the interactions of the roughness, or asperities, of the contacting surfaces. The body is in equilibrium and $F_f = F$.

The force F is slowly increased. As long as the body remains in equilibrium, the friction force F_f must increase correspondingly, since it equals the force F. The body slips on the surface. The friction force, after reaching the maximum value, cannot

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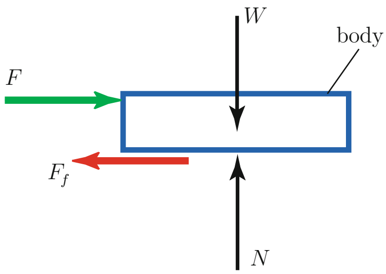


Fig. 5.1 Free-body diagram of the body

maintain the body in equilibrium. The force applied to keep the body moving on the surface is smaller than the force required to cause it to slip. Why more force is required to start the body sliding on a surface than to keep it sliding is explained in part by the necessity to break the asperities of the contacting surfaces before sliding can begin.

The theory of dry friction, or *Coulomb friction*, predicts:

- the maximum friction forces that can be exerted by dry, contacting surfaces that are stationary relative to each other;
- the friction forces exerted by the surfaces when they are in relative motion, or sliding.

5.2 Static Coefficient of Friction

The magnitude of the maximum friction force, F_f, that can be exerted between two plane dry surfaces in contact is

$$F_f = \mu_s N, \tag{5.1}$$

where μ_s is a constant, the *static coefficient of friction*, and N is the normal component of the contact force between the surfaces. The value of the static coefficient of friction, μ_s, depends on:

- the materials of the contacting surfaces;
- the conditions of the contacting surfaces namely smoothness and degree of contamination.

Typical values of μ_s for various materials are shown in Table 5.1.

Equation (5.1) gives the maximum friction force that the two surfaces can exert without causing it to slip. If the static coefficient of friction μ_s between the body and the surface is known, the largest value of F one can apply to the body without causing it to slip is $F = F_f = \mu_s N$. Equation (5.1) determines the magnitude of the maximum friction force but not its direction. The friction force resists the impending motion.

Table 5.1 Typical values of the static coefficient of friction

Materials	μ_s
Metal on metal	0.15–0.20
Metal on wood	0.20–0.60
Metal on masonry	0.30–0.70
Wood on wood	0.25–0.50
Masonry on masonry	0.60–0.70
Rubber on concrete	0.50–0.90

5.3 Kinetic Coefficient of Friction

The magnitude of the friction force between two plane dry contacting surfaces that are in motion relative to each other is

$$F_f = \mu_k N, \tag{5.2}$$

where μ_k is the *kinetic coefficient of friction* and N is the normal force between the surfaces. The value of the kinetic coefficient of friction is generally smaller than the value of the static coefficient of friction, μ_s.

To keep the body in Fig. 5.1 in uniform motion (sliding on the surface) the force exerted must be $F = F_f = \mu_k N$. The friction force resists the relative motion, when two surfaces are sliding relative to each other.

The body *RB* shown in Fig. 5.2a is moving on the fixed surface 0.

The direction of motion of *RB* is the positive axis x. The friction force on the body *RB* acts in the direction opposite to its motion, and the friction force on the fixed surface is in the opposite direction as shown in Fig. 5.2b.

5.4 Angle of Friction

The *angle of friction*, θ, is the angle between the friction force, $F_f = |\mathbf{F}_f|$, and the normal force to the surface $N = |\mathbf{N}|$, as shown in Fig. 5.3.

The magnitudes of the normal force and friction force, and θ are related by

$$F_f = R \sin\theta,$$
$$N = R \cos\theta,$$

where $R = |\mathbf{R}| = |\mathbf{N} + \mathbf{F}_f|$.

The value of the angle of friction when slip is impending is called the *static angle of friction*, θ_s,

$$\tan\theta_s = \mu_s.$$

Fig. 5.2 Directions of the friction forces

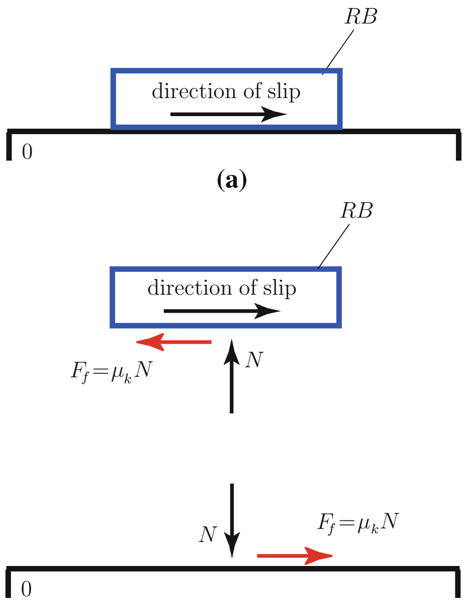
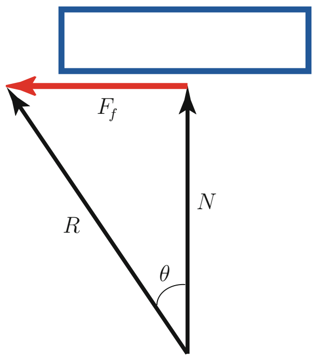


Fig. 5.3 Angle of friction, θ



The value of the angle of friction when the surfaces are sliding relative to each other is called the *kinetic angle of friction*, θ_k,

$$\tan \theta_k = \mu_k.$$

Example 5.1 The prism 1 of mass m_1 makes an angle α with the horizontal and can slide along the horizontal surface as shown Fig. 5.4. The slider 2 of mass m_2 is prevented from horizontal movement and can slide down on the inclined prism 1. The coefficients of static friction between the prism 1 and the slider 2, between the prism 1 and the horizontal surface 0, between the slider 2 and the vertical support 0

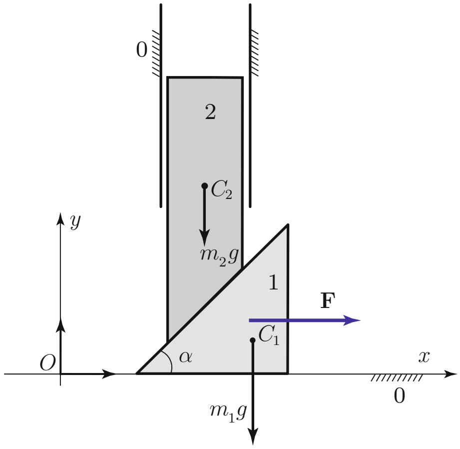


Fig. 5.4 Example 5.1

are equal to μ. The friction is sufficient to prevent the prism from moving without the application of any force.

Determine the greatest value of the horizontal force **F** that acts on the prism 1 without causing the motion of the system.

For the numerical example use $m_1 = 10$ kg, $m_2 = 5$ kg, $\alpha = 10°$, $\mu = 0.2$, and $g = 9.81$ m/s^2.

Solution A reference frame xy having the y-axis directed upward and the x-axis directed along the horizontal surface was considered, as shown Fig. 5.5.

Considering the mechanical system to be the slider-prism combination, the weight of the slider and the external force **F** will act on the prism making the prism to move. In addition to the weights and external force, the mechanical system will also be acted upon reaction and friction forces as shown in Fig. 5.5. The sum of all the forces acting on slider 2 can be expressed as:

$$\sum \mathbf{F}^{(2)} = \mathbf{F}_{02} + \mathbf{F}_{f02} + \mathbf{F}_{12} + \mathbf{F}_{f12} + \mathbf{G}_2,$$

where $\mathbf{F}_{02}$ is the of force of the ground (vertical support) on slider 2, $\mathbf{F}_{02} \perp y-$axis, $\mathbf{F}_{f02}$ is the friction force between the ground 0 and slider 2, $F_{f02} = \mu F_{02}$, $\mathbf{F}_{12}$ is the force of prism 1 on slider 2, $\mathbf{F}_{f12}$ is friction force between 1 and 2, $F_{f12} = \mu F_{12}$ and $\mathbf{G}_2$ is the weight of body 2. The frictional force is equal to the product of the static coefficient of friction with the normal force between the bodies in contact. As the direction of the frictional force must resist the tendency to slip. The MATLAB commands for the forces on slider 2 are:

```
F02_ = [-F02 0 0];
Ff02_ = [0 mu*F02 0];
F12_ = [-F12*sin(alpha) F12*cos(alpha) 0];
Ff12_ = [mu*F12*cos(alpha) mu*F12*sin(alpha) 0];
G2_ = [0 -m2*g 0];
```

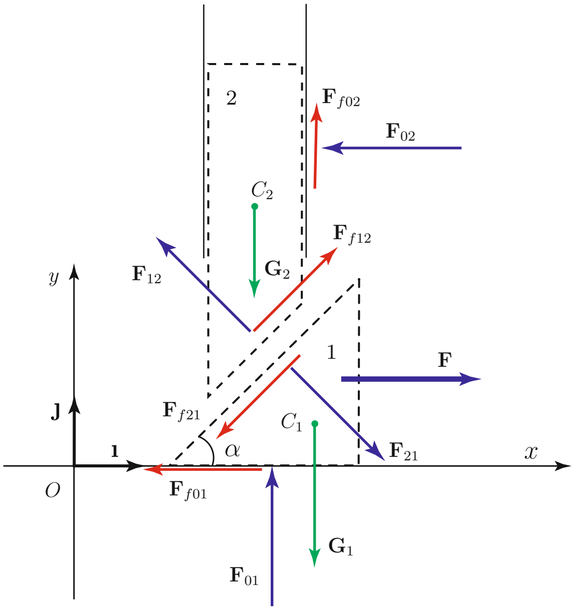


Fig. 5.5 Example 5.1 Free-Body diagrams

```
F2_  = F02_+Ff02_+F12_+Ff12_+G2_;
F2x = F2_(1);
F2y = F2_(2);
```

The two scalar equilibrium equations on $x-$axis and $y-$axis are:

```
Fx on 2:
F12*mu*cos(alpha) - F12*sin(alpha) - F02 = 0
Fy on 2:
F02*mu - g*m2 + F12*cos(alpha) + F12*mu*sin(alpha)=0
```

The joint forces reaction forces F_{02} and F_{12} are

```
F02 = (g*m2*(mu - tan(alpha)))/(mu^2 + 1)
F12 = (g*m2)/(cos(alpha)*(mu^2 + 1))
```

The sum of all the forces acting on the inclined prism 1 can be expressed as

$$\sum \mathbf{F}^{(1)} = \mathbf{F}_{01} + \mathbf{F}_{f01} + \mathbf{F}_{21} + \mathbf{F}_{f21} + \mathbf{G}_1 + \mathbf{F},$$

where $\mathbf{F}_{01}$ is the of force of the ground (vertical support) on wedge 1, $\mathbf{F}_{f01}$ is the friction force between the ground 0 and 1, $F_{f01} = \mu\, F_{01}$, $\mathbf{F}_{21} = -\mathbf{F}_{12}$ is the force of 2 on 1, $\mathbf{F}_{f21} = -\mathbf{F}_{f12}$ is friction force between 1 and 2, $\mathbf{G}_1$ is the weight of body 1 and $\mathbf{F}$ is the horizontal external force on 1. The MATLAB exressions for the forces on prism 1 are:

```
sol2 = solve(F2x,F2y,F02,F12);
F21_ = ...
-[-F12s*sin(alpha) F12s*cos(alpha) 0];
Ff21_ = ...
-[mu*F12s*cos(alpha) mu*F12s*sin(alpha) 0];
F01_ = [0 F01 0];
Ff01_ = -mu*[F01 0 0];
G1_ =  [0 -m1*g 0];
F_ = [F 0 0];
```

The equilibrium equations for the prism 1 are:

```
F1_ = F01_+Ff01_+F21_+Ff21_+G1_+F_;
F1x = F1_(1);
F1y = F1_(2);
```

or

```
Fx on 1: 0 =

               g m2 mu     g m2 sin(alpha)
  F - F01 mu - ------- + --------------------
                 2                     2
               mu  + 1   cos(alpha) (mu  + 1)
Fy on 1: 0 =

                g m2      g m2 mu sin(alpha)
  F01 - g m1 - ------- - --------------------
                 2                     2
               mu  + 1   cos(alpha) (mu  + 1)
```

The joint force reaction force F_{01} and the external force F are:

```
F01 =

         g m2 (mu tan(alpha) + 1)
  g m1 + ------------------------
                   2
                 mu  + 1
F =

                                  2 g m2 (mu - tan(alpha))
 g (m1 mu+m2 tan(alpha)) + ------------------------
                                            2
                                          mu  + 1
```

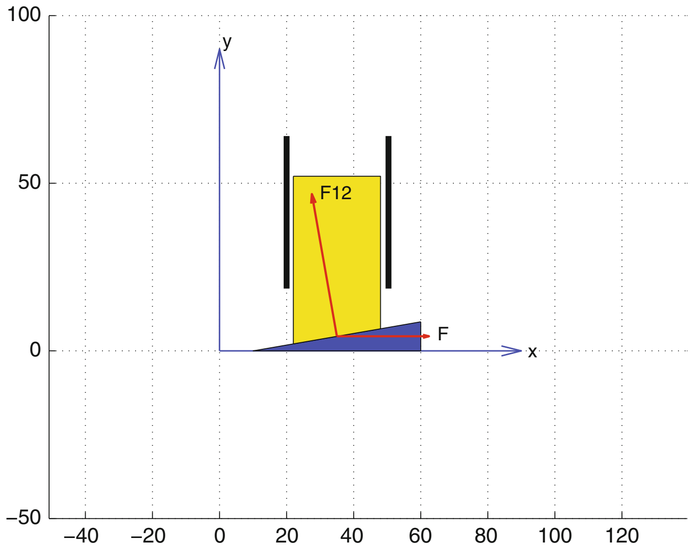


Fig. 5.6 Example 5.1 MATLAB representation of the mechanical system

The numerical results are for the joint forces are:

```
F02 =  1.117 (N), F01 = 146.927 (N), F12 = 47.891 (N)
```

and the greatest value of the horizontal force that acts on the prism 1 without causing the motion of the system is `F = 30.502 (N)`. The MATLAB representation of the mechanical system is shown in Fig. 5.6.

The MATLAB commands for the graphics are:

```
% system plot
aa = 100;
axis([-aa/2 aa -aa/2 aa])
grid on, hold on
axis equal

quiver(0,0,aa,0,...
'Color','b','LineWidth',1.0);
text(aa-8,0,'x','fontsize',12);
quiver(0,0,0,aa,...
'Color','b','LineWidth',1.0);
text(0,aa-8,'y','fontsize',12);

a=60; b=25;c=10;
alpha=10*pi/180;
x_F=c; y_F=0;
```

```
x_H=a; y_H=0;
x_E=x_H; y_E=(x_H-x_F)*sin(alpha);
x_O=0; y_O=0;
% slider vertices
vert = [x_F y_F 0; x_E y_E 0; x_H y_H 0;];
% slider faces
fac = [ 2 1 3];
% draw the slider
slider=patch...
('Faces',fac,'Vertices',vert,'FaceColor','b');

h=50;f=12;
x_A=x_F+f; y_A=(x_A-x_F)*sin(alpha);
x_B=x_A; y_B=y_A+h;
x_D=x_H-f; y_D=(x_D-x_F)*sin(alpha);
x_C=x_D; y_C=y_B;
% prism vertices
vert = ...
[x_A y_A 0; x_B y_B 0; x_D y_D 0; x_C y_C 0;];
% prism faces
fac = [ 2 1 3 4];
% draw the prism
prism=patch...
('Faces',fac,'Vertices',vert,'FaceColor','y');

% draw body left wall
s1x=x_A-f/6; s1y=x_A-f/6;
s2x=y_B+f; s2y=y_D+f;
s1 = [s1x s1y];
s2 = [s2x s2y];
line(s1,s2,'LineStyle','-',...
    'Color','k','LineWidth',4)

% draw body right wall
s3x=x_D+f/5; s3y=x_D+f/5;
s4x=y_B+f; s4y=y_D+f;
s3 = [s3x s3y];
s4 = [s4x s4y];
line(s3,s4,'LineStyle','-',...
    'Color','k','LineWidth',4)

quiver(x_A+(x_D-x_A)/2,y_A+(y_D-y_A)/2,...
F12n*cos(alpha+pi/2),F12n*sin(alpha+pi/2),...
'Color','r','LineWidth',1.5);
t0=text(abs(F12n*cos(alpha+pi/2))+20,...
```

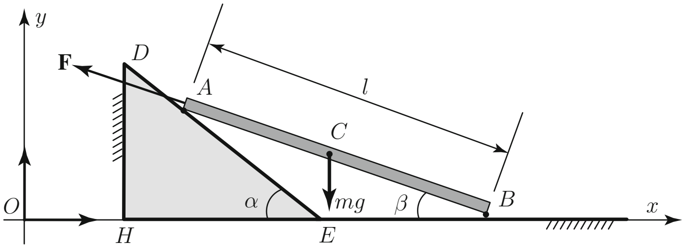


Fig. 5.7 Example 5.2

```
F12n*sin(alpha+pi/2), 0,' F12','fontsize',12);

quiver(x_A+(x_D-x_A)/2,y_A+(y_D-y_A)/2,Fn,0,...
'Color','r','LineWidth',1.5);
t0=text(abs(Fn)+33,5, 0,' F','fontsize',12);
```

Example 5.2 The rod *AB* of length l shown in Fig. 5.7 is supported at end B on a horizontal surface and at the other end (end A) by a inclined surface which makes an angle α with the horizontal. The coefficient of static friction between the rod and the inclined surface and the horizontal surface is μ. The weight of the rod is $G = mg$. The end A of the rod (supported by the inclined surface) is positioned in such a way that the angle between the rod and the horizontal supporting surface is equal to β. The end A of the rod can slid down on the inclined plane. A force F which has its directions parallel with the rod is applied at the left end A of the rod as shown in Fig. 5.7. Determine the greatest value of F without causing the motion of the rod. For the numerical example use: $l = 1$ m, $m = 10$ kg, $\mu = 0.2$, $\beta = \pi/6$ rad, $\alpha = \pi/4$ rad, and $g = 9.81$ m/s^2.

Solution The frictional force at the contact point of the end of the rod with the inclined plane is equal to the product of the static coefficient of friction with the normal force between that same end of the rod and the inclined surface. The normal force is always perpendicular at the contact point to the inclined plane, while the friction force is parallel to the inclined plane. As the direction of the frictional force must resist the tendency to slip, the frictional force must be acting “up” the inclined plane. At the other end of the rod resting on a horizontal surface. The force exerted by the horizontal surface must be vertical and the frictional force is equal to the product of the static coefficient of friction with the normal force as shown in Fig. 5.8.

The weight $\mathbf{G}$ is also a vertical force. The sum of all the forces acting on the left end A can be expressed as

$$\sum \mathbf{F} = \mathbf{N}_A + \mathbf{F}_{fA} + \mathbf{F} + \mathbf{N}_B + \mathbf{F}_{fB} + \mathbf{G} = \mathbf{0},$$

where $\mathbf{N}_A$ is the of force of the ground (prism) on the rod at A, $\mathbf{F}_{fA}$ is the friction force between the prism and the rod at A, $F_{fA} = \mu N_A$, $\mathbf{F}$ is the external force

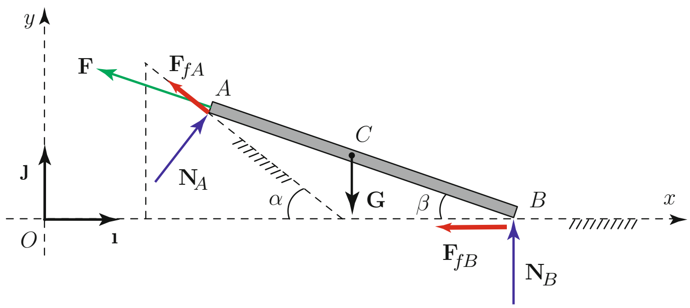


Fig. 5.8 Example 5.2 Free-Body diagram

along the rod at A, $\mathbf{N}_B$ is the of force of the horizontal surface (ground) on the rod at B, $\mathbf{N}_B \perp x-$axis, $\mathbf{F}_{fB}$ is the friction force between the ground and the rod at B, $F_{fB} = \mu N_B$, and $\mathbf{G}_2$ is the weight of the rod. The MATLAB commands for the forces on the rod are:

```
syms NA NB F
NA_=[NA*sin(alpha) NA*cos(alpha) 0];
FfA=mu*NA;
FfA_=[-FfA*cos(alpha) FfA*sin(alpha) 0];
F_=[-F*cos(beta) F*sin(beta) 0];
NB_=[0 NB 0];
FfB=mu*NB;
FfB_=[-FfB 0 0];
G_=[0 -m*g 0];
eqF_=NA_+FfA_+F_+NB_+FfB_+G_;
eqFx=eqF_(1);
eqFy=eqF_(2);
```

The sum of the moments of all forces about the left end A of the rod can be expressed as

$$\sum M_A = -\frac{l}{2}mg \cos\beta + l\, N_B \cos\beta = 0.$$

The scalar equilibrium equations in MATLAB are:

```
sum Fx:
NA*sin(alpha)-F*cos(beta)-NB*mu-NA*mu*cos(alpha)=0
sum Fy:
NB-g*m+NA*cos(alpha)+F*sin(beta)+NA*mu*sin(alpha)=0
moment about A:
NB*l*cos(beta)-(g*l*m*cos(beta))/2=0
```

The normal reactions are:

```
NA =

          g m (cos(beta) + mu sin(beta))
   --------------------------------------------
   2 (cos(alpha - beta) + mu sin(alpha - beta))

NA = 46.560 (N)

NB =

   g m
   ---
    2

NB = 49.050 (N)
```

and the greatest value of F without causing the motion of the rod is

```
 F =
                        2
  g m (sin(alpha) mu + 2 cos(alpha) mu - sin(alpha))
- ---------------------------------------------------
     2 (cos(alpha-beta) + mu sin(alpha-beta))

 F = 19.085 (N)
```

The MATLAB representation of the mechanical system is shown in Fig. 5.9.

5.5 Technical Applications of Friction: Screws

A screw thread is a uniform wedge-shaped section in the form of a helix on the external or internal surface of a cylinder (straight thread) or a cone (taper thread). The basic arrangement of a helical thread wound around a cylinder is illustrated in Fig. 5.10.

The terminology of an external screw threads is:

- *pitch* denoted by p is the distance, parallel to the screw axis, between corresponding points on adjacent thread forms having uniform spacing.
- *major diameter* denoted by d is the largest (outside) diameter of a screw thread.
- *minor diameter* denoted by d_r or d_1, is the smallest diameter of a screw thread.
- *pitch diameter* denoted by d_m or d_2 is the imaginary diameter for which the width of the threads and the grooves are equal.

The *lead* denoted by l is the distance the nut moves parallel to the screw axis when the nut is given one turn (distance a threaded section moves axially in one revolution).

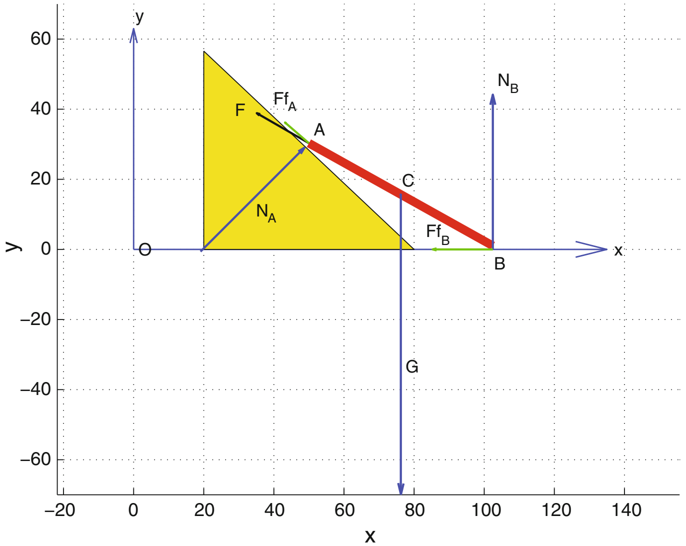
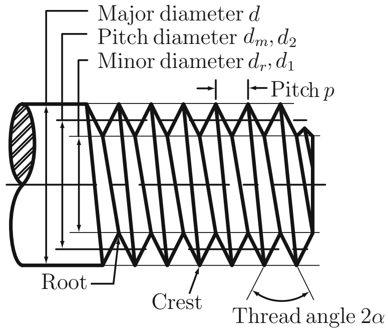


Fig. 5.9 Example 5.2 MATLAB representation of the mechanical system

Fig. 5.10 Screw thread



A screw with two or more threads cut beside each other is called a *multiple-threaded* screw. The lead is equal to twice the pitch for a double-threaded screw, and up to 3 times the pitch for a triple-threaded screw. The pitch p, lead l, and lead angle λ are represented in Fig. 5.11.

Figure 5.11a shows a single thread right-hand screw and Fig. 5.11b shows a double-threaded left-hand screw. If a thread traverses a path in a clockwise and receding direction when viewed axially, it is a *right-hand thread*. All threads are assumed to be right-hand, unless otherwise specified.

Metric threads are specified by the letter M preceding the nominal major diameter in millimeters and the pitch in millimeters per thread. For example: M 14 $\times$ 2, M is the SI thread designation, 14 mm is the outside (major) diameter, and the pitch is 2 mm per thread. Screw size in the Unified system is designated by the size number

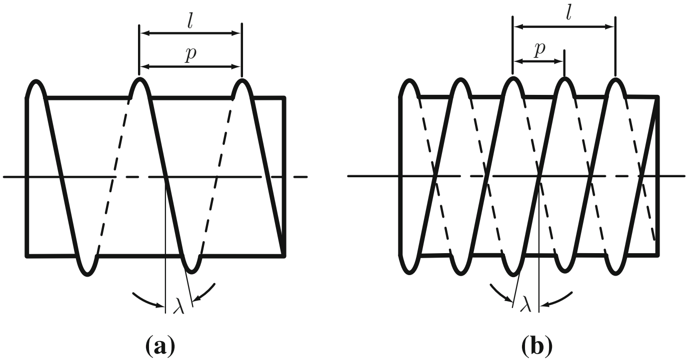
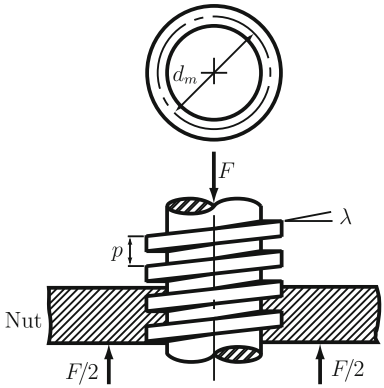


Fig. 5.11 Pitch p, lead l, and lead angle λ. **a** single thread-right hand, **b** double thread-left hand

Fig. 5.12 Power screw with a single thread



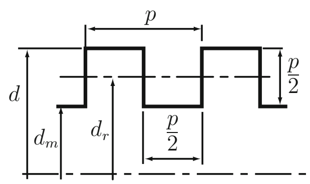
for major diameter (in.), the number of treads per in., and the thread form and series, like this: $\frac{5''}{8} - 18$, UNF $\frac{5''}{8}$ is the outside (major) diameter where the double tick marks mean inches, and 18 threads per in.

5.5.1 Power Screws

Power screws are used to convert rotary motion to linear motion of the meeting member along the screw axis. These screws are used to lift weights (screw-type jacks) or exert large forces (presses, tensile testing machines). The power screws can also be used to obtain precise positioning of the axial movement.

A square-threaded power screw with a single thread having the pitch diameter d_m, the pitch p, and the helix angle λ is considered in Fig. 5.12. A square thread profile is shown in Fig. 5.13.

Fig. 5.13 Square thread



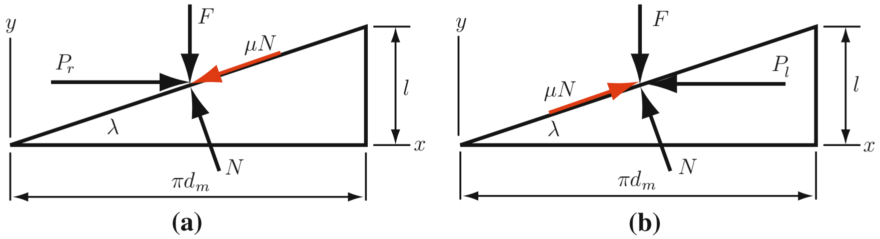


Fig. 5.14 **a** Force diagram for lifting the load and **b** force diagram for lowering the load

Consider that a single thread of the screw is unrolled for exactly one turn. The edge of the thread is the hypotenuse of a right triangle and the height is the lead. The base of the right triangle is the circumference of the pitch diameter circle (Fig. 5.14). The lead angle λ is the helix angle of the thread. The screw is loaded by an axial compressive force F (Figs. 5.12 and 5.14). The force diagram for lifting the load is shown in Fig. 5.14a, (the force P_r is positive). The force diagram for lowering the load is shown in Fig. 5.14b, (the force P_l is negative). The friction force is

$$F_f = \mu N,$$

where μ is the coefficient of dry friction and N is the normal force. The friction force is acting opposite to the motion. The equilibrium of forces for raising the load gives

$$\sum F_x = P_r - N \sin\lambda - \mu N \cos\lambda = 0, \tag{5.3}$$

$$\sum F_y = F + \mu N \sin\lambda - N \cos\lambda = 0. \tag{5.4}$$

Similarly, for lowering the load one may write the equations

$$\sum F_x = -P_l - N \sin\lambda + \mu N \cos\lambda = 0, \tag{5.5}$$

$$\sum F_y = F - \mu N \sin\lambda - N \cos\lambda = 0. \tag{5.6}$$

Eliminating N and solving for P_r

$$P_r = \frac{F (\sin\lambda + \mu \cos\lambda)}{\cos\lambda - \mu \sin\lambda}, \tag{5.7}$$

and for lowering the load

$$P_l = \frac{F\ (\mu\ \cos\lambda - \sin\lambda)}{\cos\lambda + \mu\ \sin\lambda}. \tag{5.8}$$

Using the relation

$$\tan\lambda = l/(\pi d_m),$$

and dividing the equations by $\cos\lambda$ one may obtain

$$P_r = \frac{F\ [(l\ \pi\ d_m) + \mu]}{1 - (\mu\ l\ \pi\ d_m)}, \tag{5.9}$$

$$P_l = \frac{F\ [\mu - (l\ \pi\ d_m)]}{1 + (\mu\ l\ \pi\ d_m)}. \tag{5.10}$$

The moment required to overcome the thread friction and to raise the load is

$$M_r = P_r \frac{d_m}{2} = \frac{F\ d_m}{2} \left(\frac{l + \pi\ \mu\ d_m}{\pi\ d_m - \mu\ l} \right). \tag{5.11}$$

The moment required to lower the load (and to overcome a part of the friction) is

$$M_l = \frac{F\ d_m}{2} \left(\frac{\pi\ \mu\ d_m - l}{\pi\ d_m + \mu\ l} \right). \tag{5.12}$$

When the lead, l, is large or the friction, μ, is low the load will lower itself. In this case the screw will spin without any external effort, and the moment M_l in Eq. (5.12) will be negative or zero. When the moment is positive, $M_l > 0$, the screw is said to be *self-locking*. The condition for self-locking is

$$\pi\ \mu\ d_m > l.$$

Dividing both sides of this inequality by $\pi\ d_m$, and using $l/(\pi\ d_m) = \tan\lambda$, yields

$$\mu > \tan\lambda. \tag{5.13}$$

The self-locking is obtained whenever the coefficient of friction is equal to or greater than the tangent of the thread lead angle.

The moment, M_0, required only to raise the load when the friction is zero, $\mu = 0$, is obtained from Eq. (5.11):

$$M_0 = \frac{F\ l}{2\ \pi}. \tag{5.14}$$

The screw efficiency e can be defined as

$$e = \frac{M_0}{M_r} = \frac{F\,l}{2\,\pi\,M_r}. \tag{5.15}$$

For square threads the normal thread load, F, is parallel to the axis of the screw. The preceding equations can be applied for square threads.

5.5.2 *Force Analysis for a Square-Threaded Screw*

Consider a square-threaded jack under the action of a axial load F and a moment M about the axis of the screw, Fig. 5.15. The screw has the mean radius r_m and the lead l. The force exerted by the frame thread on the screw thread is R. The angle θ made by R with the normal to the thread is the angle of friction (see Fig. 5.15)

$$\tan\theta = \mu = \frac{F_f}{N}.$$

The unwrapped thread of the screw shown in Fig. 5.15 is for lifting the load. The force equilibrium equation in the axial direction is

$$F = R\ \cos(\lambda + \theta),$$

where λ is the helix angle, $\tan\lambda = l/(2\,\pi\,r_m)$. The moment of R about the vertical axis of the screw is $R\,r_m\ \sin(\lambda+\theta)$. The moment equilibrium equation for the screw becomes

$$M = R\,r_m\ \sin(\lambda + \theta).$$

Combining the expression for F and M gives

$$M = M_r = F\,r_m\ \tan(\lambda + \theta). \tag{5.16}$$

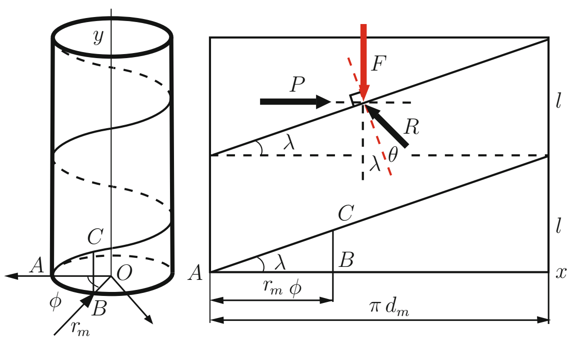


Fig. 5.15 Force diagram for a square-fhreaded screw

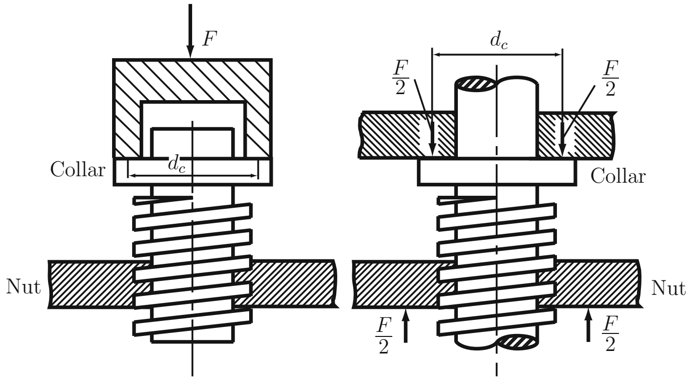


Fig. 5.16 Thrust collar

The force required to push the thread up is $P = M/r_m$. The moment required to lower the load by unwinding the screw is obtained in a similar manner:

$$M = M_l = F\, r_m\ \tan(\theta - \lambda). \tag{5.17}$$

If $\theta < \lambda$ the screw will unwind by itself.

In general, when the screw is loaded axially, a thrust bearing or thrust collar may be used between the rotating and stationary links to carry the axial component (Fig. 5.16). The load is concentrated at the mean collar diameter d_c. The moment required is

$$M_c = \frac{F\ \mu_c\ d_c}{2}, \tag{5.18}$$

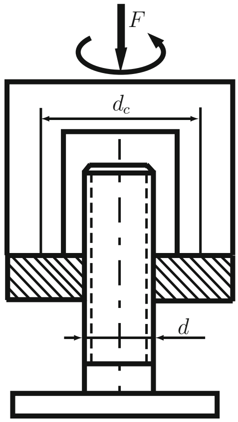
where μ_c is the coefficient of collar friction.

Example 5.3 A square-thread power screw, as shown in Fig. 5.17 has the major diameter $d = 38$ mm and the pitch $p = 6$ mm. The coefficient of friction of the thread is $\mu = 0.08$ and the coefficient of collar friction is $\mu_c = 0.1$. The mean collar diameter is $d_c = 45$ mm. The external load on the screw is $F = 9$ kN. Find: (a) the lead, the pitch (mean) diameter and the minor diameter; (b) the moment required to raise the load; (c) the moment required to lower the load; (d) the efficiency of the device.

Solution (a) From Fig. 5.13:
the minor diameter is $d_r = d - p$
the pitch (mean) diameter is $d_m = d - p/2$
the lead is$l = p$,
or:

```
l = p =  6.000 (mm)
dr = d - p = 32.000 (mm)
```

Fig. 5.17 Example 5.1




```
dm = d - p/2 = 35.000 (mm)
```

(b) The moment required to raise the load is

$$M_r = \frac{F d_m}{2}\left(\frac{l + \pi\mu d_m}{\pi d_m - \mu l}\right) + \frac{F \mu_c d_c}{2},$$

and:

```
Mr =0.5*F*dm*(pi*mu*dm+l)/(pi*dm-mu*l)...
    +0.5*F*dc*muc;
Mr = 41.537 (kN m)
```

(c) The moment required to lower the load is

$$M_l = \frac{F d_m}{2}\left(\frac{\pi\mu d_m - l}{\pi d_m + \mu l}\right) + \frac{F \mu_c d_c}{2},$$

and:

```
Ml =0.5*F*dm*(pi*mu*dm-l)/(pi*dm+mu*l)...
    +0.5*F*dc*muc;
Ml = 24.238 (kN m)
```

The sef-locking condition:

```
%  (pi*mu*dm - l) > 0
sf = (pi*mu*dm - l);
fprintf('sf = %6.3f \n',sf)
if sf > 0
fprintf('sf>0 => screw is sef-locking\n\n')
else
```

```
fprintf('sf<0 => screw is not sef-locking\n\n')
end
```

The screw is self-locking `sf = 2.796`.

(d) The overall efficiency is calculated with

$$e = \frac{F\,l}{2\pi M_r},$$

and:

```
e = F*l/(2*pi*Mr);
e =  0.207
```

5.6 Problems

5.1 Find the orientation angle θ of the force P for the smallest possible force P that can be applied so that the body shown in Fig. 5.18 is on the verge of moving. The body has weight the mass m and the coefficient of static friction at the surface is $\mu_s = 0.4$.

5.2 The car shown in Fig. 5.19 has the mass m, the center of mass at C, and travels along a track with a constant speed. Find the greatest slope θ of the track without causing the car to tip or to slip. The coefficient of static friction between the road and the car is μ. For the numerical example use: $m = 2000\,\text{kg}$, $\mu = 0.3$, $l = 1.75\,\text{m}$, $h = 0.5\,\text{m}$, and $g = 9.81\,\text{m/s}^2$.

5.3 A uniform bar of mass m and length l is placed on a rough wall at A and on a smooth floor at B at shown in Fig. 5.20. The coefficient of static friction between the bar and the wall is μ. The distance $OB = d$ is given. Find if the bar will remain in the initial position when it is released. For the numerical example use: $\mu = 0.4$, $l = 1\,\text{m}$, $d = OB = 0.3\,\text{m}$, and $g = 9.81\,\text{m/s}^2$.

5.4 The block 1, shown in Fig. 5.21, has the mass m and is placed on a rough wall. Find the minimum force F required to move the block of mass m. The coefficient of static friction is μ. The angle of the two wedges 2 and 3 is α. For the numerical example use: $m = 80\,\text{kg}$, $\mu = 0.4$, $\alpha = \pi/180$ rad, and $g = 9.81\,\text{m/s}^2$.

5.5 The wedge 3, shown in Fig. 5.22, has the mass $2\,m$. The wedges 2 has the mass m. Find the minimum force F required to move the wedge 3. The coefficient of

Fig. 5.18 Problem 5.1

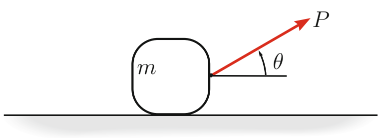


Fig. 5.19 Problem 5.2

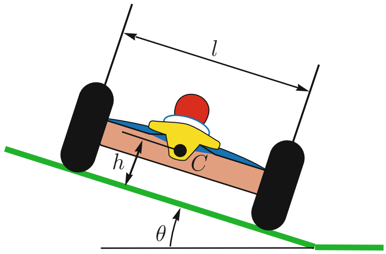


Fig. 5.20 Problem 5.3

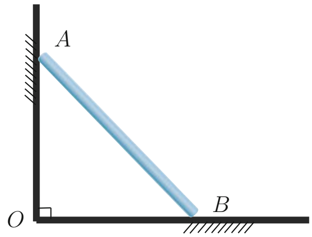


Fig. 5.21 Problem 5.4

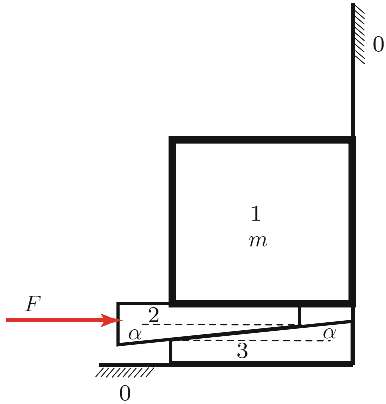
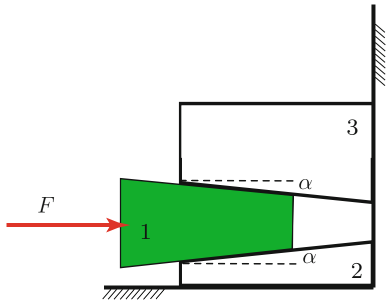


Fig. 5.22 Problem 5.5



static friction is μ. The angle of the wedges is α. For the numerical example use: $m = 10\,\text{kg}$, $\mu = 0.2$, $\alpha = 15°$ rad, and $g = 9.81\,\text{m/s}^2$.

5.6 A double square-thread power screw has a pitch (mean) diameter of 30 mm and a pitch of 6 mm. The coefficient of friction of the thread is 0.1 and the coefficient of collar friction is also 0.2. The mean collar diameter is 40 mm. The external load on the screw is 8 kN. Determine the moment required to lower the load and the overall efficiency.

5.7 A power screw has a double square thread with a mean diameter of 50 mm and a pitch of 8 mm. The coefficient of friction in the thread is 0.15. Determine if the screw is self-locking.

5.8 A triple-thread screw is used in a jack to raise a load of 3000 lb. The major diameter of the screw is 4 in. A plain thrust collar is used. The mean diameter of the collar is 5 in. The coefficient of friction of the thread is 0.08 and the coefficient of collar friction is 0.2. Determine: (a) the screw pitch, lead, thread depth, mean pitch diameter, and helix angle; (b) the starting moment for raising and for lowering the load; (c) the efficiency of the jack.

5.7 Programs

5.7.1 Program 5.1

```
% example 5.1
clear all; clc; close all

syms F02 F12 F01 F
syms m1 m2 g alpha mu

list ={m1,m2,g,alpha,mu};
listn={10,5,9.81,10*pi/180,0.2};

% slider 2
% force of ground (vertical support) on body 2
F02_ = [-F02 0 0];
% friction force between 0 and 2
Ff02_ = [0 mu*F02 0];
% force of prism 1 on body 2
F12_ = ...
[-F12*sin(alpha) F12*cos(alpha) 0];
% friction force between 1 and 2
Ff12_ = ...
[mu*F12*cos(alpha) mu*F12*sin(alpha) 0];
% weight of body 2
```

```
G2_ = [0 -m2*g 0];
% sum of forces on body 2
F2_ = F02_+Ff02_+F12_+Ff12_+G2_;
F2x = F2_(1);
F2y = F2_(2);

fprintf('Fx on 2:\n')
fprintf(' %s = 0 \n',char(F2x))
fprintf('Fy on 2:\n')
fprintf(' %s = 0 \n',char(F2y))

sol2 = solve(F2x,F2y,F02,F12);
F02s = simple(sol2.F02);
F12s = simple(sol2.F12);
fprintf('=>\n')
fprintf('F02 = %s\n',char(F02s))
fprintf('F12 = %s\n',char(F12s))
fprintf('\n')

% prism 1
% force of body 2 on prism 1
F21_ = ...
-[-F12s*sin(alpha) F12s*cos(alpha) 0];
% friction force between 2 and 1
Ff21_ = ...
-[mu*F12s*cos(alpha) mu*F12s*sin(alpha) 0];

% force of ground on prism 1
F01_ = [0 F01 0];
% friction force between 0 prism 1
Ff01_ = -mu*[F01 0 0];
% weight of prism 1
G1_ =  [0 -m1*g 0];
% external force of prism 1
F_ = [F 0 0];

% sum of forces on prism 1
F1_ = F01_+Ff01_+F21_+Ff21_+G1_+F_;
F1x = F1_(1);
F1y = F1_(2);

fprintf('Fx on 1: 0 = \n')
pretty(F1x)
fprintf('Fy on 1: 0 = \n')
pretty(F1y)
```

```
sol1 = solve(F1x,F1y,F01,F);
F01s = simple(sol1.F01);
Fs = simple(sol1.F);
fprintf('=>\n')
fprintf('F01 = \n')
pretty(F01s)
fprintf('F = \n')
pretty(Fs)

F02n = subs(F02s,list,listn);
F01n = subs(F01s,list,listn);
F12n = subs(F12s,list,listn);
Fn = subs(Fs,list,listn);

fprintf('\n\n')
fprintf('F02 = %6.3f (N) \n',F02n)
fprintf('F01 = %6.3f (N) \n',F01n)
fprintf('F12 = %6.3f (N) \n',F12n)
fprintf('F = %6.3f (N) \n',Fn)

% system plot
aa = 100;
axis([-aa/2 aa -aa/2 aa])
grid on, hold on
axis equal

quiver(0,0,aa,0,...
'Color','b','LineWidth',1.0);
text(aa-8,0,'x','fontsize',12);
quiver(0,0,0,aa,...
'Color','b','LineWidth',1.0);
text(0,aa-8,'y','fontsize',12);

a=60; b=25;c=10;
alpha=10*pi/180;
x_F=c; y_F=0;
x_H=a; y_H=0;
x_E=x_H; y_E=(x_H-x_F)*sin(alpha);
x_O=0; y_O=0;
% slider vertices
vert = [x_F y_F 0; x_E y_E 0; x_H y_H 0;];
% slider faces
fac = [ 2 1 3];
% draw the slider
```

```
slider=patch...
('Faces',fac,'Vertices',vert,'FaceColor','b');

h=50;f=12;
x_A=x_F+f; y_A=(x_A-x_F)*sin(alpha);
x_B=x_A; y_B=y_A+h;
x_D=x_H-f; y_D=(x_D-x_F)*sin(alpha);
x_C=x_D; y_C=y_B;
% prism vertices
vert = ...
[x_A y_A 0; x_B y_B 0; x_D y_D 0; x_C y_C 0;];
% prism faces
fac = [ 2 1 3 4];
% draw the prism
prism=patch...
('Faces',fac,'Vertices',vert,'FaceColor','y');

% draw body left wall
s1x=x_A-f/6; s1y=x_A-f/6;
s2x=y_B+f; s2y=y_D+f;
s1 = [s1x s1y];
s2 = [s2x s2y];
line(s1,s2,'LineStyle','-',...
    'Color','k','LineWidth',4)

% draw body right wall
s3x=x_D+f/5; s3y=x_D+f/5;
s4x=y_B+f; s4y=y_D+f;
s3 = [s3x s3y];
s4 = [s4x s4y];
line(s3,s4,'LineStyle','-',...
    'Color','k','LineWidth',4)

quiver(x_A+(x_D-x_A)/2,y_A+(y_D-y_A)/2,...
F12n*cos(alpha+pi/2),F12n*sin(alpha+pi/2),...
'Color','r','LineWidth',1.5);
t0=text(abs(F12n*cos(alpha+pi/2))+20,...
F12n*sin(alpha+pi/2), 0,' F12','fontsize',12);

quiver(x_A+(x_D-x_A)/2,y_A+(y_D-y_A)/2,Fn,0,...
'Color','r','LineWidth',1.5);
t0=text(abs(Fn)+33,5, 0,' F','fontsize',12);
% end of program
```

5.7.2 Program 5.2

```
% example 5.2
clear all; clc; close all

syms l m g mu alpha beta
syms NA NB F

NA_=[NA*sin(alpha) NA*cos(alpha) 0];

FfA=mu*NA;
FfA_=[-FfA*cos(alpha) FfA*sin(alpha) 0];

F_=[-F*cos(beta) F*sin(beta) 0];

NB_=[0 NB 0];

FfB=mu*NB;
FfB_=[-FfB 0 0];

G_=[0 -m*g 0];

eqF_=NA_+FfA_+F_+NB_+FfB_+G_;

eqFx=eqF_(1);
eqFy=eqF_(2);

eqMA=-l/2*m*g*cos(beta)+l*NB*cos(beta);

fprintf('sum Fx:\n')
fprintf(' %s = 0 \n',char(eqFx))
fprintf('sum Fy:\n')
fprintf(' %s = 0  \n',char(eqFy))
fprintf('moment about A:\n');
fprintf(' %s = 0 \n\n',char(eqMA))

sol=solve(eqFx,eqFy,eqMA,F,NA,NB);

NAs=simplify(sol.NA);
NBs=sol.NB;
Fs=simplify(sol.F);

list={l,m,g,mu,beta,alpha};
listn={1,10,9.81,0.2,pi/6,pi/4};
```

```
NAn=subs(NAs,list,listn);
NBn=subs(NBs,list,listn);
Fn=subs(Fs,list,listn);

fprintf('NA =\n')
pretty(NAs);
fprintf('\n')
fprintf('NA = %6.3f (N)\n\n',NAn)
fprintf('NB =\n')
pretty(NBs);
fprintf('\n')
fprintf('NB = %6.3f (N)\n\n',NBn)
fprintf('F =\n')
pretty(Fs);
fprintf('\n')
fprintf('F = %6.3f (N)\n\n',Fn)

% graphic
a = 70;
axis([-a/2 a -a a])
grid on, hold on
axis equal
xlabel('x'), ylabel('y')

quiver(0,0,2*a+10,0,...
'Color','b','LineWidth',1.0);
text(2*a,0,' x','fontsize',12);
quiver(0,0,0,a,...
'Color','b','LineWidth',1.0);
text(0,a,' y','fontsize',12);

a=80; b=35;c=20;
alpha=45*pi/180;
x_O=0; y_O=0;
x_F=c; y_F=0;
x_E=a; y_E=0;
x_D=c; y_D=(x_E)*sin(alpha);

t0=text(x_O, y_O, 0,' O','fontsize',12);
t1=text(x_F-1, y_F-4, 0,' F','fontsize',12);
t2=text(x_D-1, y_D+2, 0,' D','fontsize',12);
t3=text(x_E-1, y_E-4, 0,' E','fontsize',12);
% prism vertices
vert = [x_F y_F 0; x_D y_D 0; x_E y_E 0;];
```

```
% prism faces
fac = [ 2 1 3];
% draw the prism
prism=patch...
('Faces',fac,'Vertices',vert,'FaceColor','y');

offset=1;
beta=30*pi/180;
x_A=x_F+(x_E-x_F)/2; y_A=y_D/2+2*offset;
x_B=x_A+y_A/tan(beta); y_B=offset;
A = [x_A x_B];
B = [y_A y_B];
x_C=x_A+(x_B-x_A)/2;
y_C=(y_A)/2+offset/2;
C = [x_C y_C];

t4=text...
(x_B-1, y_B-5, 0,' B','fontsize',12);
t5=text...
(x_A, y_A+4, 0,' A','fontsize',12);
t6=text...
(x_C-1, y_C+4, 0,' C','fontsize',12);
% draw the rod
line(A,B,'LineStyle','-',...
    'Color','r','LineWidth',6)
scatter(x_C,y_C,3,5,'filled','b')

quiver(....
x_A-NAn*cos(alpha)+2*offset,...
y_A-NAn*sin(alpha)+2*offset,...
NAn*cos(alpha),NAn*sin(alpha),...
'Color','b','LineWidth',1.5);
t0=text(...
x_A-NAn*cos(alpha)/2,...
y_A-NAn*sin(alpha)/2-5*offset,...
0,' N_A','fontsize',12);

quiver(...
x_A,y_A,Fn*cos(beta+pi),...
Fn*sin(beta),...
'Color','k','LineWidth',1.5);
t0=text(...
x_A-Fn*cos(alpha)-9*offset,...
y_A+Fn*sin(alpha)-4*offset,...
0,' F','fontsize',12);
```

```
FfA=subs(FfA,list,listn);
FfA=subs(FfA,NAn,NA);
quiver(...
x_A,y_A,...
FfA*cos(alpha+pi)-offset,...
FfA*sin(alpha),...
'Color','g','LineWidth',1.5);
t0=text(...
x_A-Fn*cos(alpha)+2*offset,...
y_A+Fn*sin(alpha)-3.0*offset,...
0,' Ff_A','fontsize',12);

quiver(x_B,y_B-offset,0,NBn,...
'Color','b','LineWidth',1.5);
t0=text(...
x_B,y_B+NBn-4*offset,...
0,' N_B','fontsize',12);

quiver(x_B,y_B-offset,-Fn,0,...
'Color','g','LineWidth',1.5);
t0=text(...
x_B-Fn,y_B+2*offset, 0,...
'Ff_B','fontsize',12);

G=subs(G_,list,listn);
quiver(x_C,y_C,0,G(2),...
'Color','b','LineWidth',1.5);
t0=text(...
x_C,y_C+G(2)/2, 0,...
' G','fontsize',12);
% end of program
```

5.7.3 Program 5.3

```
% example 5.3
clear all; clc; close all

% major diameter d
d = 38; % mm
% screw pitch p
p = 6;  % mm
% coefficient of friction for thread
mu = 0.08;
```

```
% coefficient of collar friction
muc = 0.1;
% mean collar diameter dc
dc = 45; % mm
% external load F
F = 9;   % kN

% lead l
l = p;
% minor diameter dr
dr = d-p;
% (pitch) mean diameter dm
dm = d-p/2;

fprintf('l = p = %6.3f (mm)\n',l)
fprintf('dr = d - p = %6.3f (mm)\n',dr)
fprintf('dm = d - p/2 = %6.3f (mm)\n\n',dm)

% momemnt required to raise the load
Mr =0.5*F*dm*(pi*mu*dm+l)/(pi*dm-mu*l)...
    +0.5*F*dc*muc;
fprintf('Mr = %6.3f (kN m)\n\n',Mr)

% moment required to lower load
Ml =0.5*F*dm*(pi*mu*dm-l)/(pi*dm+mu*l)...
    +0.5*F*dc*muc;
fprintf('Ml = %6.3f (kN m)\n\n',Ml)

% sef-locking condition: (pi*mu*dm - l) > 0
sf = (pi*mu*dm - l);
fprintf('sf = %6.3f \n',sf)

if sf > 0
fprintf('sf>0 => screw is sef-locking\n\n')
else
fprintf('sf<0 => screw is not sef-locking\n\n')
end

% If  sf > 0 => the screw is sef-locking
% If  sf <= 0 => the screw is not sef-locking

% efficiency
e = F*l/(2*pi*Mr);
fprintf('e = %6.3f \n\n',e)
% end of program
```

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