

# Feature Transformation from Configuration of Open-Loop Mechanisms into Linkages with a Case Study

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## 1 Introduction

This chapter proposes a method for feature synthesis of mechanisms and manipulators from user specifications based on a hybrid approach employing both neural network and optimization techniques. The mechanism design modeling problem with the lack of solution convergence observed in optimization is addressed by using a neural network method to generate reliable initial solutions. This chapter also discusses a module by which the validation of prescribed precision configuration points is evaluated. An excavator arm mechanism is used as a case study to test and validate the method. The necessary training data for the neural network is generated primarily through the use of forward kinematics equations, while the proposed method is analyzed using dimensional data collected from existing products.

Existing products are frequently modeled as a type of assembly features [9]. They can be redesigned and customized to meet specific operational needs and increase efficiency. Such customizable and yet conceptually proven products are commonly used to perform atypical tasks under space constraints, such as specialized manipulators. These products can be developed as a cluster of instances of a generic product because of their inherent common engineering principles. The generic product model are modeled in the form of assembly features. In most cases, the design objective can be achieved by adopting a different set of configuration parameter values based on a generic product model of the existing design features using the same design procedures developed during the initial design. Such well-defined assembly features, whose parameters can be assigned

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with different values, enable the product configuring mechanism to achieve increasing versatility and to address customization needs. However, specifying an effective and valid design feature data set for those existing feature models is difficult given the expected complicated and interdependent constraints [8].

For certain machinery equipment, such as excavators, the final spatial access envelope diagrams of the overall assemblies, which are referred to here as product *specification features*, are the basis of customer evaluation of the dimensional specifications. This kind of specification feature can be appreciated as a subtype of the assembly design feature as defined by Ma et al. [9]. Unlike those well-defined modular mechanical products, such as the mold bases used in the plastic molding industry [9], the conceptual design of manipulator products usually starts with a set of target specification features, i.e., the envelope diagrams or prescribed paths and motions identified by end users that need to be achieved by the overall mechanism.

There is another type of feature that has to be defined in this chapter: the design *configuration feature*, which is a product-level assembly feature [9] that represents the design intent with characteristic dimensions, geometry, patterns, and other associative relations to interfacing components. In the context of typical manipulator mechanism design, the configuration features are realized by materializing the *component design features* [10] involved. The process of design realization involves a feature dimensional synthesis phase in which engineers analyze relations among component design features to compose a workable and satisfactory product configuration that is governed by the aforementioned configuration feature. Of course, the final product design has to be completed by focusing on determining individual component feature dimensions and their related constraints, as well as material application patterns, in order to meet the configuration feature requirements after the mechanism is assembled. This chapter covers the design transformation process from the specification features to the configuration features. The remaining further design processes will be covered in the next chapter.

With reference to the context of manipulator design, a single pose (position and orientation) of the end effectors of the manipulator, or a valid instance of the specification feature, is defined by customers with their application demands; to translate the specification features into a set of configuration design features, the transformation process needs to use those known values of the manipulator's linkage dimensions together with the joint parameters to go through trials of inverse kinematic fitting. Typically, the specified poses (or instances of specification features) can only be verified and eventually confirmed by using kinematics procedures with the assumed (or known) linkage dimensions in the manipulator, or in other words, the related component design features.

However, the challenge arises when the customer requires and defines a set of expected configuration poses: what is the method to transform such input into those materialized linkage configuration features? In such a case, instead of solving a forward kinematics problem (direct configuration feature development) from the known components, the nature of the (which is common manipulator design) problems require solving kinematics problems in reverse based on a set of given specification feature instances. In other words, with the predefined specification

feature instances defined in the form of access envelopes, the configuration definition of the mechanism needs to be inversely calculated.

Multiple configuration solutions to such inverse kinematics problems usually exist. Obviously, with a set of required feature behavior instances (poses) to be targeted in the form of an access envelope space or, even more critically, an access path, the calculation for the mechanism dimensions becomes very complicated due to the combined constraints of kinematics and the existence of multiple solutions. Finding these solutions is already a challenge, but evaluating or validating them is even more difficult. For a single pose problem, the existing methods are manageable with reasonable effort. However, a multi-pose problem, which requires that the calculated linkage dimensions and joint variables fully satisfy a set of configurations or path parameters, makes the inverse kinematics approach difficult to implement.

Thus far most researchers have tried to solve the inverse kinematics and optimization problems by using a computational workspace searching method, but the optimization results are not reliable due to the fact that there are multiple solutions. To obtain the necessary convergence toward the expected range of a solution, researchers need initial suggestions as the input of the searching procedures. This initial solution requirement creates considerable challenges when trying to automate the conceptual design process and implement it using computer programs. There are dense correlations among the configuration feature parameters; arbitrary values cannot be assumed in their places when solving the system equations. Failure to use an appropriate starting parameter vector may produce mathematically accurate but physically impossible solutions.

The two remaining tasks are formulating a set of parametric geometric relationships for the specification feature of a manipulator, which has to be associated with typical linkage configuration feature parameters, and searching a workable solution, which needs an optimization technique.

This chapter proposes a method by which feature dimensional synthesis for manipulator mechanisms is performed based on the end user's specification parameter input. The implementation of this method requires a vector of initial suggestions of linkage configuration parameters that has to be close to the expected solution. A smart neural network procedure is used to generate the feasible initial suggestions of the linkage parameters. A case study of an excavator arm mechanism is carried out and the results are promising. The algorithm has been implemented in MATLAB, a numerical analysis software tool produced by MathWorks.

## **2 Background of Relevant Research**

Optimizing mechanism dimensions is a well-known design problem in the field of robotics and machinery, and a broadly studied research area. However, due to the multiple members of a mechanism and the dependencies among them that are constrained by kinematics, the problem is complicated. For example, Wu et al.

[13] formulated the kinematic constraints of closed chain mechanism as a mapping from Cartesian space to a higher dimensional projective space called image space by representing planar displacements with planar quaternions. The researchers pointed out that the use of this method enables one to reduce the problem of dimensional synthesis by determining algebraic parameters that define the image spaces. Computational simplification was achieved by transforming kinematic equations into geometric constraints. Dealing with the geometric parameters of the constraint manifold instead of the mechanism parameters provides ease and flexibility due to the decoupled nature of the relationships.

Optimization techniques are usually applied to solve the mechanism linkage dimensions). For example, a procedure of synthesizing the linkage dimensions of a four-bar spherical linkage mechanism was proposed by Alizade and Kilit [1]. The procedure used a polynomial approximation to transform 5 nonlinear equations into 15 linear equations and solve five design parameters. The objective of this study was to determine the dimensions of a spherical four-bar linkage mechanism by linearizing a set of nonlinear equations. The requirement for the mechanism was that it will be able to trace five precision points in space. The minimum deviation area (MDA) was proposed as a constraint criterion to select the most appropriate solution. The result of this investigation was tested by plotting the path of the mechanism against the prescribed precision points using AutoCAD 2000.

Jensen and Hansen [6] have suggested a method by which dimensional synthesis for both planar and spatial mechanisms are accomplished by taking the problem of non-assembly into consideration. The method makes use of a gradient-based optimization algorithm. Analytic calculation of sensitivities is performed by direct differentiation. The problem was mathematically formulated as a standard optimization problem with inequality to take the non-assembly nature of the problem into account. The Newton–Raphson method, due to its rapid convergence property, is used in the minimization of the kinematic constraints. Saddle's point and steepest descent methods were used to verify the direction of convergence and stability of the minimization method, respectively.

Kinematic synthesis of redundant serial manipulators has become the focus of research for Singla et al. [11]. They used an augmented Lagrangian optimization technique to determine optimum dimensions for a redundant serial manipulator. The algorithm was used for its robustness in identifying feasible solution ranges effectively. The formulation of the problem was based on the minimization of the positional error subject to the constraints of avoiding manipulator collisions with either external obstacles or its own links.

The workspace boundary definition can be more complicated. Laribi et al. [7] discussed an optimization technique used for determining the linkage dimensions) of a DELTA parallel robot for a prescribed workspace. The technique uses a genetic algorithm to minimize an objective function developed by writing expressions for the end effector location, based on a concept called the power of the point. The dimensions of the robots were calculated by minimizing a volume created by three intersecting surfaces that contain the prescribed cubic workspace. A penalty function screened out infeasible and select feasible solutions from the

available solution domain. Zhao et al. [15] used a similar approach, but the prescribed workspace was represented by a cylinder contained inside the minimum workspace volume and enclosed by the manipulator movement boundary surfaces. An optimization-based dimensional synthesis procedure was suggested to determine dimensional parameters for the design of a 2-UPS-PU parallel manipulator. The researchers used a cylindrical coordinate system when formulating the kinematic relationships, including the forward and inverse kinematics of the manipulator together with the Jacobian matrix for force and velocity analysis.

However, it has been identified that the multiple numbers of possible solutions is the primary disadvantage of analytical solutions methods. Gao et al. [4] reported that for their six degree of freedom (DOF) parallel manipulator, the traditional optimization techniques in the areas of dimensional synthesis lack the badly needed convergence property in their solutions when it is used for handling a larger number of geometric variables and complex objective functions. Non-traditional optimization methods need to be explored in order to address the problems of convergence uncertainties and limitations, on a maximum number of precision point problems solved using optimization and analytical techniques.

Gao et al. [4] also used generic algorithms and artificial neural networks (ANNs) as tools to deal with the optimization of the manipulator's stiffness and dexterity based on kinematic analysis procedures. Levenberg–Marquardt and standard back propagation algorithms were used in the neural network to approximate stiffness and dexterity analytical solutions. Because of the large numbers of variables included in the analysis, they have used two different approaches for the optimizations: Single Objective Optimizations (SOOs) and Multiple Objective Optimizations (MOOs) multiple objective optimization (MOO). With the first approach, the two objectives, stiffness and dexterity, were investigated separately; with the second approach, they were investigated together to understand their combined effect. Both approaches proved to be compatible.

It is worth pointing out, as Vasiliu and Yannou [12] did in their work, that “the absence of continuity between different morphologies prohibited and discouraged the use of interpolation techniques” in such a problem. Vasiliu and Yannou also proposed the use of ANNs. The ANN designed to be used for the synthesis application takes in the prescribed path and motion as an input and gives out the linkage parameters as an output. Erkaya and Uzmay [2] aimed to overcome problems arising from joint clearances in a four-bar mechanism. They used neural networks to characterize the clearances and the mechanism, and genetic algorithms to optimize them with the path and transmission angle errors used as part of the objective function. The clearances were represented by high stiffness and weightless links to make them suitable to be studied under rigid motion considerations but without affecting the overall inertial property of the mechanism. ANN procedures were also used by Hasan et al. [5] to study the relationship between the joint variables and the position and orientation of the end effector of a six DOF robot. The study was motivated by the fact that the use of ANN does not require an explicit knowledge of the physics behind the mechanism. The network was trained by the use of real-time data collected by sensors mounted on the robot. Designed

with an input layer of six neurons for three Cartesian location coordinates and three linear velocity components, the network was used to establish a mapping pattern between the input and output. The project mainly focused on finding the kinematic Jacobian solutions.

The advantage of using ANN is that it does not require any details of the mathematical and engineering knowledge involved [5]; it is thus suited to a wide range of similar applications. It was suggested that as long as there is sufficient data for training purposes, the ANN can be used to predict the Jacobian kinematics of other configurations without the need to learn and understand the explicit robot philosophies. Modifications and changes in existing robot structures can always be addressed by training the ANN with a new set of data reflecting the new modifications.

The problems and shortcoming associated with using ANN are also discussed in Hasan et al. [5]. The first challenge discussed is the difficulty of selecting the appropriate network architecture, activation functions, and bias weights. The other problem discussed is the difficulty and impracticality of collecting large amounts of data for the purpose of training the neural network.

As an alternative to using the ANN approach, some researchers are more interested in simulation and spatial configuration performance analysis of manipulators. Their work is motivated by the need to understand the manipulators' performance under certain environmental constraints. Frimpong and Li [3], for example, modeled and simulated a hydraulic shovel to investigate its kinematics and spatial configurations when the shovel is deployed in a constrained mining environment. Denavit-Hartenberg homogeneous coordinate transformation techniques were used to represent the relative orientations and configurations of adjacent links as well as the overall assembly. Forward kinematics of the machine was investigated as a five-linkage manipulator. After formulating the kinematics equations the manipulator was modeled in 3D and was simulated using the MSC ADAMS simulation software for selected time steps.

Therefore, as suggested by Vasiliu and Yannou [12], the requirement of a large number of data for training ANN can be addressed by simulation of the paths for a number of given sets of linkage parameters. The ANN can be trained using the simulated data in reverse, i.e., that for the given sets of mechanism parameters, the information of the access paths were determined. The other important point discussed in Vasiliu and Yannou's work is that neural networks perform well only within the data range they were trained with. Normalization of parameters during the utilization phase of the network is needed to bring the input values to the known range of the training set.

The constraints imposed for manufacturing the products usually dictates the capacities and efficiencies of the machineries. The general design and modification requirements can sometimes be achieved by merely redesigning an existing mechanism out of a different set of existing product sales materials.

Remote operability of hydraulic excavators, initiated due to operational safety and hazard issues, has recently become the focus of some researchers. The task of controlling the motion of excavator arm mechanisms has been attempted by

various remote control mechanisms. The method developed by Yoon and Manurung [14] is based on mapping the angular joint displacements of the human arm joints to that of the excavator arm joints. Their work is motivated by the need to include intuitivism into the control system.

### **3 The Proposed Hybrid Approach**

#### ***3.1 Overall Concept Description***

Most optimization techniques usually require a very good initial solution to be defined in order to produce sound solutions. One of the objectives of this chapter is to introduce a feature-based system by which a set of initial solutions that are reasonably close to the actual solution can be generated. Optimization techniques, when applied to the problems of dimensional synthesis of prescribed precision points, commonly encounter the difficulty of giving reasonable and practical results. There are two reasons for this: first is the proximity of the goal solution to the predefined initial solution; second is the compatibility or feasibility of the prescribed precision points. This is to say that prescription of unrealistic and ambitious specifications most likely produce, if the search converges to a solution at all, mathematically sound but physically inapplicable solutions.

The hybrid method proposed in this paper can be summarized by the flowchart as shown in Fig. 1. It is the objective of this chapter to introduce a new method in which a well-trained artificial neural network (ANN) tool is used to generate a set of high-quality initial solution suggestions for mechanism parameters based on user specifications, while optimization techniques are used to finally synthesize the necessary dimensions. The hybrid method attempts to jointly employ optimization and neural network procedures to synthesize the linkage design mechanisms' feature dimensions and further map them to the real manipulators. User specifications are also qualified with the checking of their priorities and ranges acceptable as the prescribed input values. The individual modules and procedures are explained in detail in the following subsection.

#### ***3.2 Synthesis and Validation Procedure***

The proposed method can be divided into the following stages: (1) ANN training; (2) input parameter validation; (3) system testing; (4) initial solution generation; (5) mechanism parameter synthesis; (6) result verification; and (7) random system check. To make full use of the neural network's advantage, its inner transformation matrices must first be trained to reflect the intricate nature of input and output relations.

### 3.2.1 Artificial Neural Network Training

Essentially, the purpose of training the ANN is to build a database that will be used to generate the feasible suggestions of the initial mechanism parameters according to new configuration specifications. The first step is to collect the training data. Ideally, such training data can be obtained from existing similar product information catalogues, usually in the form of product families, because the relevant data from that channel is proven workable with both input and output sets. As shown in Fig. 1, the proposed method makes use of such data as indicated by the top job block. Unfortunately, although these real product data sets are quite useful for training the ANN, the number of available data sets is never sufficient. To find a solution for the shortage of training data, forward mechanism simulation can be utilized to create as many input/output data sets as required [5]. Note that the generation of such simulation data is necessary because the available data is usually insufficient to serve the training purpose and the extra effort of collecting additional real product data is prohibitively costly.

In the case of the data generation process, the specification feature parameters which define the total workspace of the mechanism assembly will be generated from the given set of linkage configuration feature dimensions using forward kinematic equations. This is a mapping process in which the mechanism design feature parameters (linkage dimensions) are mapped to the specification feature terms, i.e., the envelope configuration parameters of the workspace or the working path in the case of a planar mechanism.

When training the ANN, both the existing real product feature data sets and the generated data sets will be used in reverse: the existing specification feature parameters are used as the input data for the training while the mechanism configuration feature parameters are used as the target output data. Note that most of the training data sets can be generated from the “artificial” forward mechanism behavior algorithm as used by Laribi et al. [7], which had provided a satisfactory outcome. In addition, real product data sets are collected from the market, and play a more important role in incorporating the industrial optimization factors into the ANN module. Those overall industrial design factors are embedded implicitly in real products on top of engineering mathematical solutions.

Since the ANN is expected to be effectively used only for those parameters lying within the ranges of its training data [7], to make the training data more generically useful, unification of the input vector as well as the output vector during the training cycles should be assured. Similarly, during the application of the trained neural network, the input and output for the new dimensional parameters have to be scaled or normalized to make sure they lie within the training ranges.



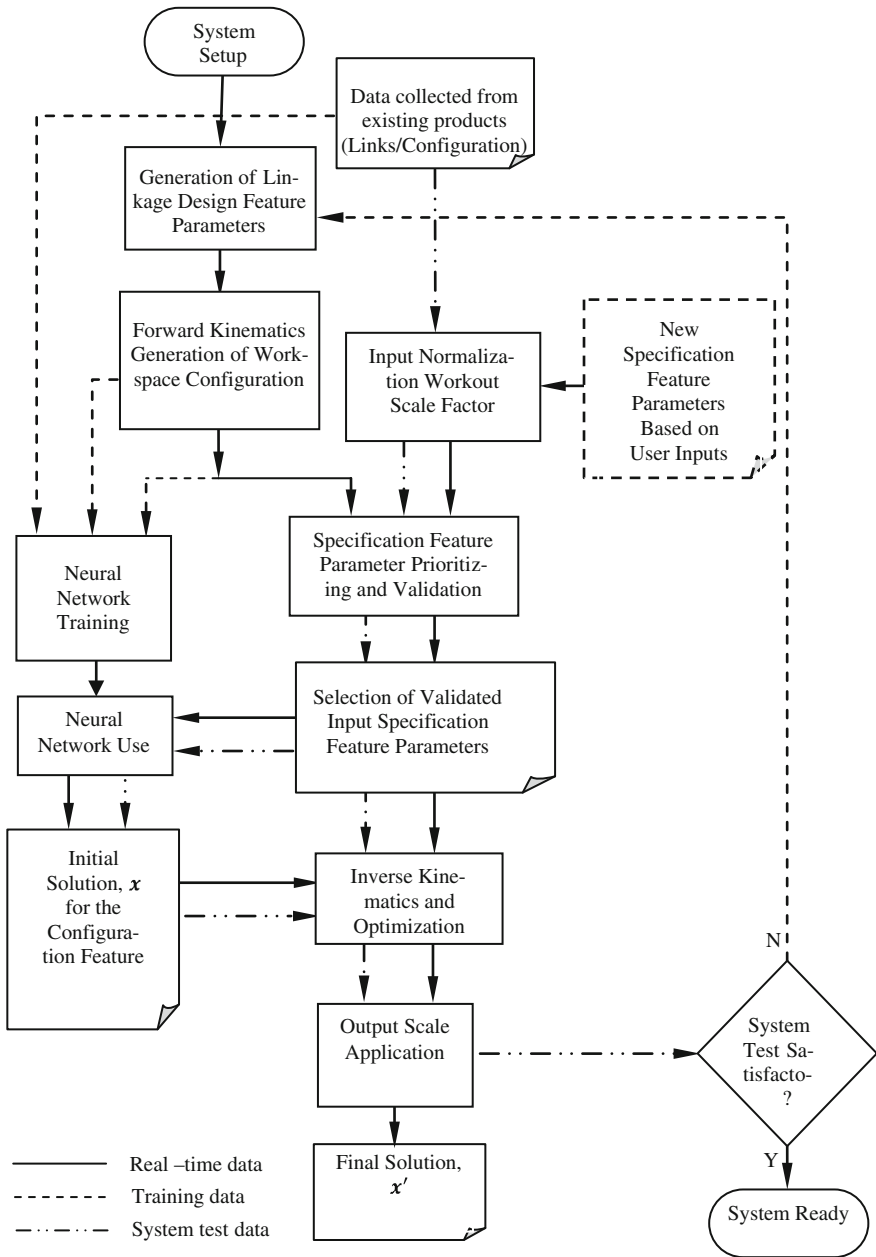


Fig. 1 Dimensional synthesis procedure

### 3.2.2 Input Specification Feature Parameter Validation

In addition to the training of the ANN, to search for a feasible mechanism parameter solution from a given set of configuration parameters it is necessary that the configuration parameter values be compatible with each other and their practical coexistence be feasible. If this condition is not met, the results of the analysis may be inapplicable. Figure 2 shows the procedure adopted to validate input configuration parameters. It is worth noting that the term *validation* is used here only to describe the applicability of a prescribed parameter set to a particular machine or manipulator configuration. The validation is performed by determining whether the configuration's given multiple input parameters, after being scaled or normalized, lie within the relative ranges established by the collected and generated data. The ranges derived from collected data are based on the results of statistical analysis of all the real product models available. The ranges derived from ANN-generated data are to be discussed in Sect. 4.3, which addresses the implementation algorithm.

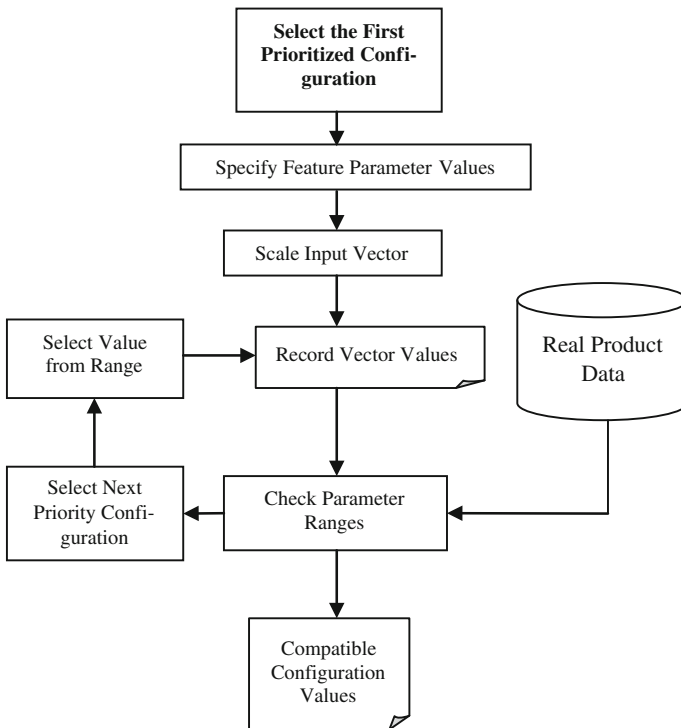


Fig. 2 Configuration prioritizing and selection

### 3.2.3 System Testing

To validate the overall procedure, real product specification feature data sets are to be used again for testing purposes, as shown by the step “Selection of Validated Input specification Feature Parameters” in Fig. 1. To test the system’s reliability, which is different from the ANN training process, the real product configuration parameters are fed into a trained ANN module to generate initial suggestions of the linkage configuration feature dimensions. Then, together with envelope specification feature parameters, the initial linkage configuration feature dimensions are used as the seeding vector to search for the goal vector of the targeted mechanism configuration dimensions. Then the output goal vector is compared with the real product mechanism dimension vectors. Theoretically, the system output deviations should be well within the specified tolerance of the system’s accuracy requirements. Note that the real product data sets are only a relatively small portion of the overall ANN training data sets. If the system does not meet the accuracy expectations, then more training data sets are required from both channels (as discussed previously).

## 3.3 *Application of the Smart Design Feature Transformation*

### 3.3.1 Initial Inverse Kinematic Solution Generation for Application

After the ANN has been successfully configured, it should be ready for application. At the beginning of the application design stage, the validated specification feature parameters are passed to the ANN module to generate initial design solutions. The initial solutions will then be used by the appropriate optimization procedure to refine and derive the goal solutions.

### 3.3.2 Mechanism Configuration Feature Dimension Synthesis

The dimension synthesis, which is specific to the nature of the mechanism in question, is carried out through optimization algorithms. The case that follows in Sect. 4 is a study of an excavator arm linkage system, and includes details of the algorithms. Ultimately, the resulting mechanism configuration feature parameter solutions in this research must be scaled back to the original ratio before being further processed.

### **3.4 Results Validation**

The optimization results are to be validated before they are adopted in the design and displayed in an appropriate CAD context. This straightforward procedure is to apply forward mechanism simulation and check the envelope space or path details against the specifications. If the results are not satisfactory, a troubleshooting procedure must be carried out. As discussed in the following case study, the reported research results have so far been satisfactory with a limited number of tests; the troubleshooting method was therefore not further explored.

### **3.5 Random System Validation Check**

To measure and validate the performance of the system, randomly selected configuration parameter data sets from the existing products' database can be selected and the corresponding mechanism configuration feature parameters generated using the proposed method. The results can be cross-checked against the actual dimensions and the efficiency of the method will be determined.

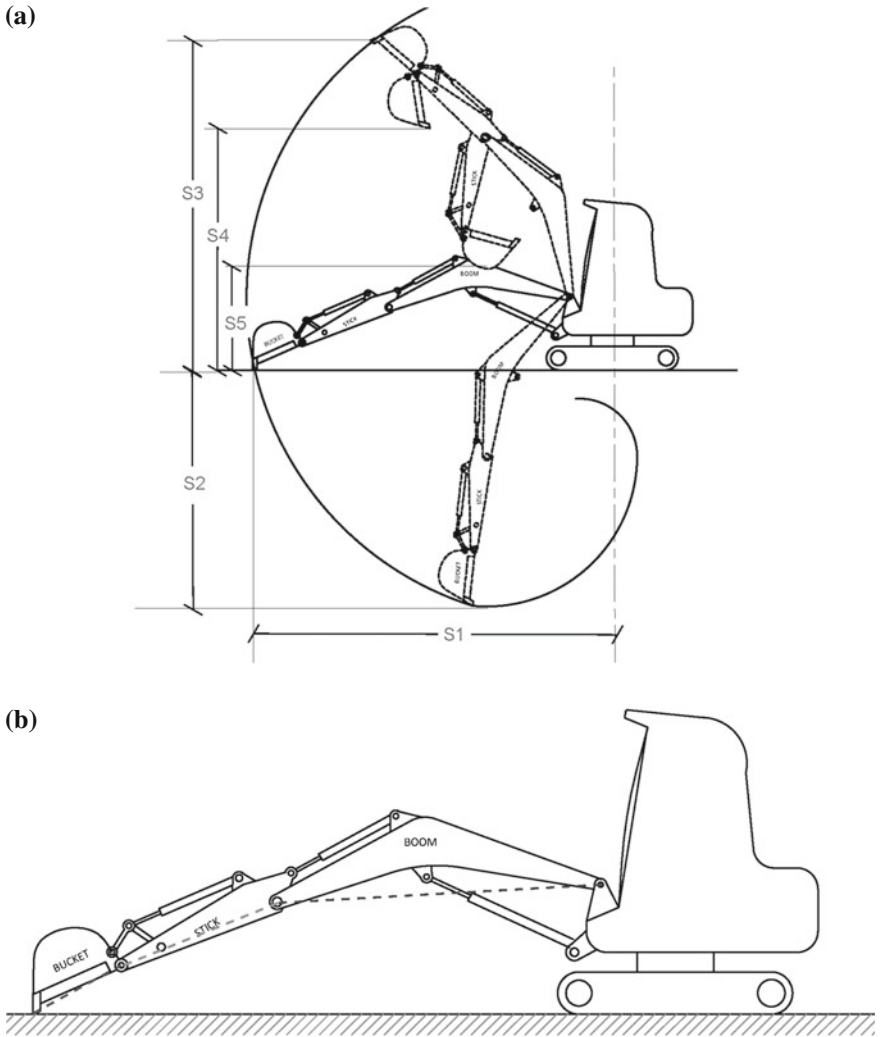
## **4 Case Study**

### **4.1 Excavator Case Representation**

In the conceptual process of designing an excavator, translating the access specification parameters (prescribed points or an envelope path) into linear dimensions of the arm mechanism represents the first stage. To do this, the boom, stick, and buckets of the planar mechanism are represented by linear linkages, and other elements, such as hydraulic cylinders and bucket transition four-bar linkages, are left out of consideration at this stage (see Fig. 3). These three links, connected in boom-stick-bucket sequence, are positioned and oriented in different poses such that their final configurations pass through the input specifications. Figures 3a and 3b show the traditional catalogue specification dimensions ( $S_1, S_2, \dots, S_5$ ), listed in Table 1, and the representation of the mechanism by linear elements ( $l_1, l_2, l_3, \beta$ ), respectively.

Hence, the design process involves determining a set of individual linkage dimensions) for the excavator arm mechanism (listed in Table 2) so that when they are connected to each other and conform to the overall mechanism, they will satisfy the working-range requirements.

Unlike forward kinematic problems in which the location and other properties of the end effector are to be calculated based on different joint variables and linkage dimensional inputs, this problem involves determining the joint variables



**Fig. 3** An example excavator arm configuration. **a** Typical commercial work-range specifications. **b** Linear arm elements

**Table 1** Hydraulic excavator workspace configuration parameters

S1	Maximum reach at ground level
S2	Maximum digging depth
S3	Maximum cutting height
S4	Maximum loading height
S5	Minimum loading height
S6	Maximum depth cut at level bottom
S7	Maximum vertical wall digging depth

**Table 2** Mechanism linkage dimensions

$l_1$	Hinge to hinge boom length
$l_2$	Stick length
$l_3$	Hinge to tip bucket length
$\beta$	Boom deflection angle

and linkage dimensions) for a given set of end effector configurations (bucket in this case). In forward kinematics or direct configuration analysis, the task is usually to determine the final configuration of the mechanism based on a given set of joint variables and linkage dimensions); this is a relatively simple and straightforward process, since the analysis usually leads to a unique solution. The inverse process in question, on the other hand, is relatively complex due to the availability of multiple solutions.

## 4.2 Data Generation for Neural Network Training

The main purpose of this task is to generate configuration and linkage parameter data sets to be used for training the proposed ANN. The ANN will be used in later stages to narrow down and select a physically viable set of linkage parameters to be used as initial solutions. This is entirely a forward kinematic procedure in which each final vector of configuration parameters,  $S$ , is determined from a given set of linkage dimensions) and joint variables,  $L$ .

Here  $S = (S_1, S_2, \dots, S_5)$ ;  $L = (l_1, l_2, l_3, \beta)$ .

The following subsections describe the mathematical model used for working out the envelope path configuration parameters ( $S_1, S_2, \dots, S_5$ ) from the mechanism linkage parameters,  $(l_1, l_2, l_3, \beta)$ .

### 4.2.1 Maximum Reach-Out at Ground Level ( $S_1$ )

The position of the bucket tip is calculated using forward kinematic methods. The individual rotational and linear transformation matrices are formulated using the Denavit-Hartenberg convention.

By applying the Law of Cosine to Fig. 4, the following mathematical relationship is formulated:

$$c^2 = (l_2 + l_3)^2 + l_1^2 - 2l_1(l_2 + l_3) \cos(180 - \beta) \quad (1)$$

$$c^2 = (l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta \quad (2)$$

$$c = \sqrt{(l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta} \quad (3)$$

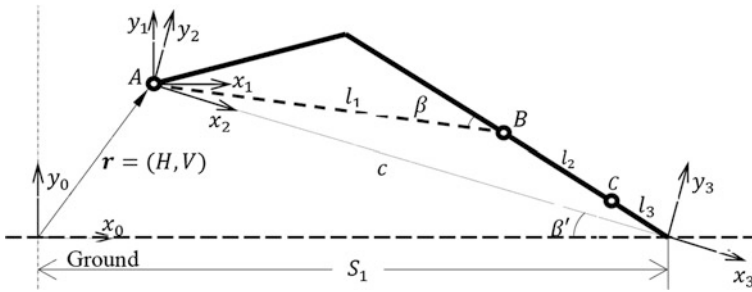


Fig. 4 Maximum out-reach at ground level

$$\sin \beta' = \frac{V}{c} = \frac{V}{\sqrt{(l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta}} \tag{4}$$

$$\beta' = \sin^{-1} \left( \frac{V}{c} \right) = \sin^{-1} \left( \frac{V}{\sqrt{(l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta}} \right) \tag{5}$$

$$(S_1 - H)^2 = (l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta - V^2 \tag{6}$$

$$(l_2 + l_3)^2 + l_1^2 + 2l_1(l_2 + l_3) \cos \beta - V^2 - (S_1 - H)^2 = 0 \tag{7}$$

The sequence of frame of reference translation from the origin to a frame located at the tip of the bucket is represented by the homogeneous transformation

$$A = T_{x'H} T_{y'V} R_{z'-\beta} R_{z'-\beta'} \tag{8}$$

where

$T_{x'H}$  Linear displacement in the positive  $x$  direction with  $H$  value

$T_{y'V}$  Linear displacement in the positive  $y$  direction with  $V$  value

$R_{z'-\beta}$  Rotation about the  $z$  axis by angular value of  $-\beta'$

$T_{x'c}$  Linear displacement in the positive  $x$  direction by a value of  $c$ .

The rotation sequences of Eq. 8, when represented by the corresponding matrices, take the following form:

$$A_{S1} = \begin{bmatrix} 1 & 0 & 0 & H \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & V \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\beta') & -\sin(-\beta') & 0 & 0 \\ \sin(-\beta') & \cos(-\beta') & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The resulting homogenous transformation matrix is then given by

$$A_{S1} = \begin{bmatrix} \cos \beta' & \sin \beta' & 0 & H + c \cos \beta' \\ -\sin \beta' & \cos \beta' & 0 & V - c \sin \beta' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The value of the *maximum out-reach at ground level* is then extracted from the above homogenous transformation matrix. The expression in cell (1, 4) is the value of the  $x$  coordinate of the bucket tip from the origin of the fixed reference frame, which in this case is the same as the value of the maximum reach-out at ground level,  $S_1$ .

$$S_1 = |A_{S1}(1, 4)| = |H + c \cos \beta'| \quad (11)$$

#### 4.2.2 Maximum Digging Depth ( $S_2$ )

The maximum digging depth requires the definition of angle  $\alpha_2$ , measured from the vertical to indicate the lower limit of the boom angular displacement around the base hinge. For a given value of this limiting angle, the maximum digging depth is expressed mathematically using the Denavit-Hartenberg convention:

Again, by using Law of Cosine,

$$l_1^2 = b^2 + b^2 - 2b^2 \cos(180 - 2\beta) \quad (12)$$

where  $b$  is the length of each side of the boom. For the purpose of simplification, they are assumed to be of equal length in this development.

$$l_1 = 2b \cos \beta \quad (13)$$

Referring to Fig. 5,

$$S_2 = l_1 \cos \alpha_2 + l_2 + l_3 - V \quad (14)$$

$$S_2 = l_1 \cos \alpha_2 + l_2 + l_3 - V \quad (15)$$

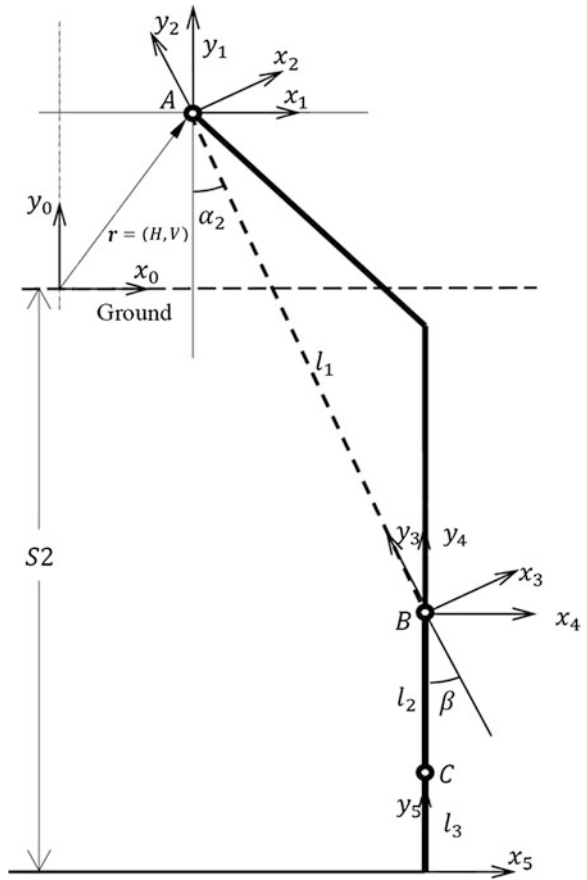
$$l_1 \cos \alpha_2 + l_2 + l_3 - V - S_2 = 0 \quad (16)$$

The homogeneous transformation sequence in this case is given by

$$A_{S2} = T_{x'H} T_{y'V} R_{z'(\alpha_2)} T_{y'-l_1} R_{z'-\beta} T_{y'-b} \quad (17)$$



**Fig. 5** Maximum digging depth



$$A_{S2} = \begin{bmatrix} 1 & 0 & 0 & H \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & V \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha_2) & -\sin(\alpha_2) & 0 & 0 \\ \sin(\alpha_2) & \cos(\alpha_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \dots \begin{bmatrix} \cos(-\beta) & -\sin(\beta) & 0 & 0 \\ \sin(-\beta) & \cos(-\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(18)

The resulting homogeneous transformation matrix takes the form of

$$A_{S2} = \begin{bmatrix} \cos(\alpha_2 - \beta) & -\sin(\alpha_2 - \beta) & 0 & H + l1 \sin(\alpha_2) + b \sin(\alpha_2 - \beta) \\ \sin(\alpha_2 - \beta) & \cos(\alpha_2 - \beta) & 0 & V - l1 \cos(\alpha_2) - b \cos(\alpha_2 - \beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

The cell in this matrix representing the maximum digging depth is the y displacement in cell (2, 1).

$$S_2 = |A_{S2}(2, 1)| = |V - l1 \cos(\alpha_2) - b \cos(\alpha_2 - \beta)| \quad (20)$$

### 4.2.3 Maximum Cutting Height ( $S_3$ )

For a given value of the upper angular limit of the boom rotation,  $\alpha_1$ , the procedure for the maximum cutting height expression formulation follows a procedure similar to the maximum digging depth calculation.

Referring to Fig. 6, the following relationship is developed for the maximum cutting height configuration:

$$H_2 = l_1 \cos \theta \quad (21)$$

where  $\theta$  in this case is given by

$$\theta = (\alpha_1 - \beta) \quad (22)$$

$$H_2 = l_1 \cos(\alpha_1 - \beta) \quad (23)$$

$$H_3 = l_2 \cos(\theta - \beta) \quad (24)$$

$$H_3 = l_2 \cos(\alpha_1 - 2\beta) \quad (25)$$

$$H_4 = l_3 \cos(\theta - \beta + \alpha_{bu}) \quad (26)$$

$$S_3 = V + H_2 + H_3 + H_4 \quad (27)$$

$$S_3 = V + l_1 \cos(\alpha_1 - \beta) + l_2 \cos(\alpha_1 - 2\beta) + l_3 \cos(\alpha_1 - 2\beta + \alpha_{bu}) \quad (28)$$

$$l_1 \cos(\alpha_1 - \beta) + l_2 \cos(\alpha_1 - 2\beta) + l_3 \cos(\alpha_1 - 2\beta + \alpha_{bu}) + V - S_3 = 0 \quad (29)$$

The homogenous coordinate transformation sequence for this configuration is given by

$$A_{S3} = T_{x'H} T_{y'V} R_{z'(\alpha_1 - \beta)} T_{y'l1} R_{z' - \beta} T_{y'b} \quad (30)$$

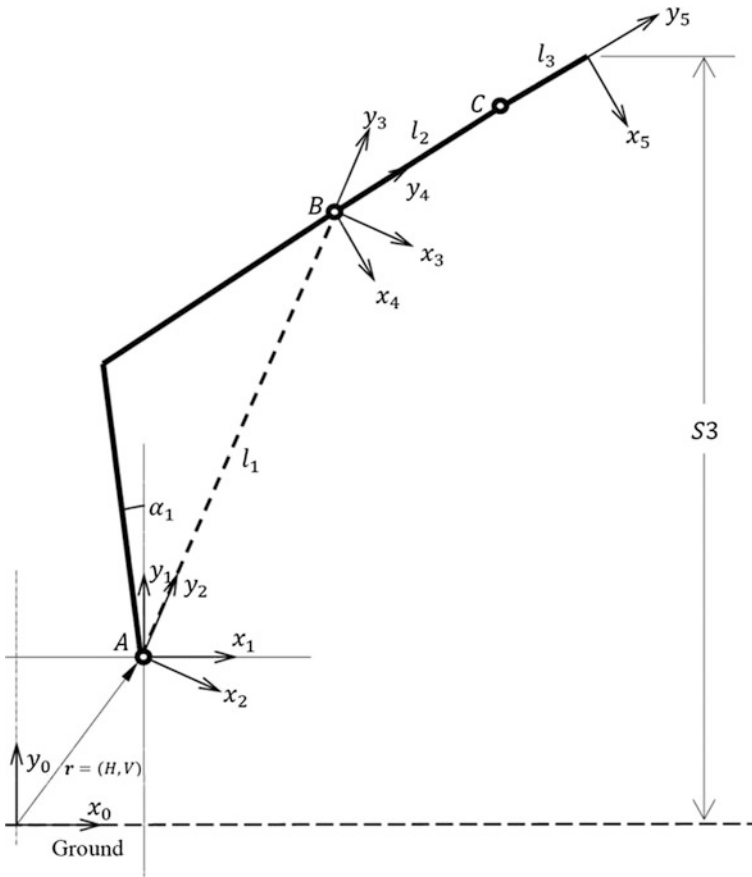


Fig. 6 Maximum cutting height

$$A_{S3} = \begin{bmatrix} 1 & 0 & 0 & H \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & V \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha_1 - \beta) & -\sin(\alpha_1 - \beta) & 0 & 0 \\ \sin(\alpha_1 - \beta) & \cos(\alpha_1 - \beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\beta) & -\sin(-\beta) & 0 & 0 \\ \sin(-\beta) & \cos(-\beta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

$$A_{S3} = \begin{bmatrix} \cos(\alpha_1 - 2\beta) & -\sin(\alpha_1 - 2\beta) & 0 & H - b \sin(\alpha_1 - 2\beta) - l1 \sin(\alpha_1 - \beta) \\ \sin(\alpha_1 - 2\beta) & \cos(\alpha_1 - 2\beta) & 0 & V + b \cos(2\beta - \alpha_1) + l1 \cos(\alpha_1 - \beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (32)$$

The y displacement component of this matrix represents the maximum cutting height.

$$S_3 = |A_{S3}(2, 4)| = |V + b \cos(2\beta - \alpha_1) + l1 \cos(\alpha_1 - \beta)| \quad (33)$$

#### 4.2.4 Maximum Loading Height ( $S_4$ )

The vertical position assumed by  $l_3$  in Fig. 7 is represented by slightly modifying the expression developed for maximum cutting height and ignoring the orientation angle of the last frame of reference, as follows:

$$S_4 = V + H_2 + H_3 - l_3 \quad (34)$$

$$l_1 \cos(\alpha_1 - \beta) + l_2 \cos(\alpha_1 - 2\beta) - l_3 + V - S_4 = 0 \quad (35)$$

The expression for maximum cutting height is modified with minor changes to make it fit this configuration. The last linear coordinate translation in this case is limited to  $l_2$  instead of  $(b = l_2 + l_3)$ . The bucket length  $l_3$  is further deducted from the y displacement component of the matrix.

The final result is given by the following matrix:

$$A_{S3} = \begin{bmatrix} \cos(\alpha_1 - 2\beta) & -\sin(\alpha_1 - 2\beta) & 0 & H - l_2 \sin(\alpha_1 - 2\beta) - l1 \sin(\alpha_1 - \beta) \\ \sin(\alpha_1 - 2\beta) & \cos(\alpha_1 - 2\beta) & 0 & V - l_3 + l_2 \cos(2\beta - \alpha_1) + l1 \cos(\alpha_1 - \beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (36)$$

$$S_4 = |A_{S3}(2, 4)| = |V - l_3 + l_2 \cos(2\beta - \alpha_1) + l_1 \cos(\alpha_1 - \beta)| \quad (37)$$

#### 4.2.5 Minimum Loading Height ( $S_5$ )

Following a similar procedure gives an expression for the homogeneous transformation matrix of the minimum cutting height configuration, as represented by Fig. 8.

$$S_5 = V + H_2 - l_2 - l_3 \quad (38)$$

$$S_5 = V + l_1 \cos(\alpha_2 - \beta) - l_2 - l_3 \quad (39)$$

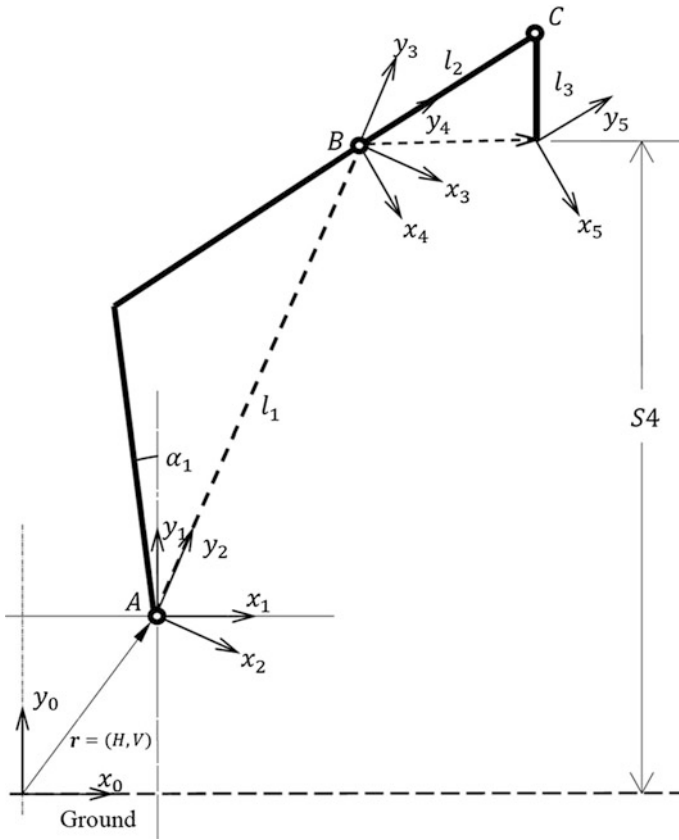


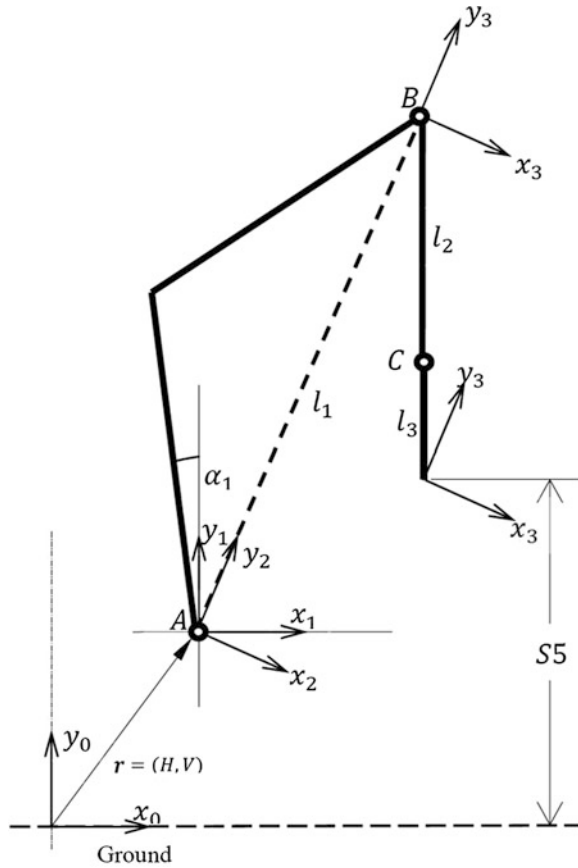
Fig. 7 Maximum loading height

$$V + l_1 \cos(\alpha_1 - \beta) - l_2 - l_3 - S_5 = 0 \tag{40}$$

$$A_{S3} = \begin{bmatrix} \cos(\alpha_1 - 2\beta) & -\sin(\alpha_1 - 2\beta) & 0 & H - l_1 \sin(\alpha_1 - \beta) \\ \sin(\alpha_1 - 2\beta) & \cos(\alpha_1 - 2\beta) & 0 & V + l_1 \cos(\alpha_1 - \beta) - l_2 - l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{41}$$

$$S_5 = |A_{S3}(2, 4)| = |V + l_1 \cos(\alpha_1 - \beta) - l_2 - l_3| \tag{42}$$

**Fig. 8** Minimum loading height



### 4.3 Generation of Training Data

The required training data is generated by mapping the configuration parameter for a set of mechanism dimension parameters. MATLAB is used to implement this task.

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ \beta \end{bmatrix} \rightarrow \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} \tag{43}$$

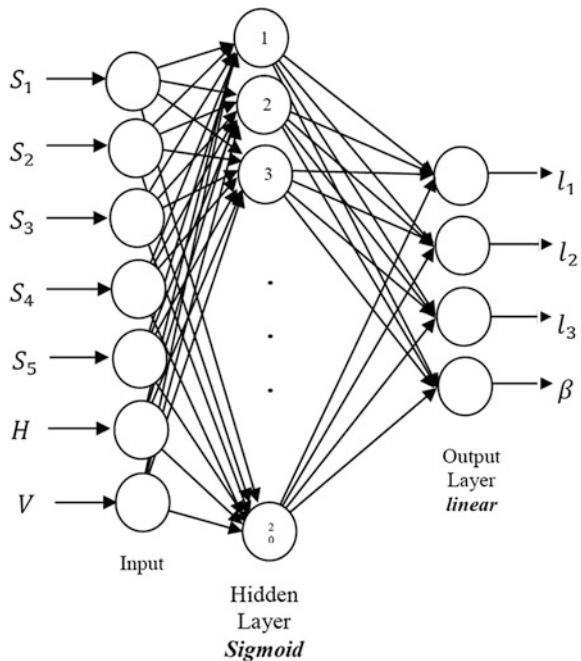
### 4.4 Neural Network Training

Since the ANN is needed to fulfill the purpose of preliminary inverse kinematic analysis, the output data generated from the forward simulation,  $S$ , will be used as the input data for its training, while the linkage parameters vector  $L$ , is the target data. Since the values of the configuration parameters depend also on the overall dimensions of the vehicle on which they are mounted, constant values for the  $xx$  and  $yy$  coordinates of the base hinge,  $H$  and  $V$ , are used in the analysis.

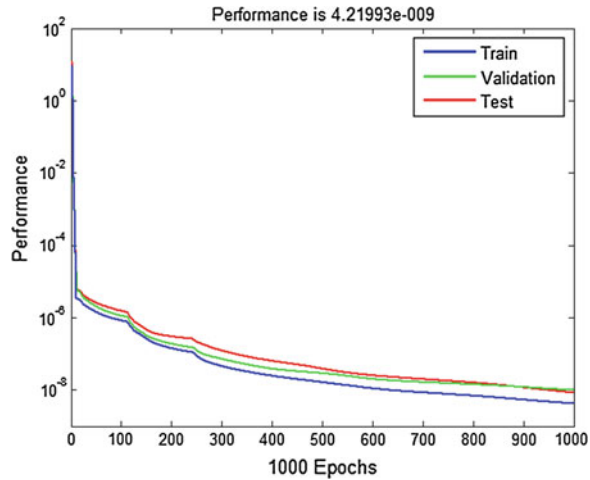
Accordingly, as shown in Fig. 9, a two-layer feed forward ANN is designed to map seven input configuration parameters to four target parameters. The ANN has one hidden layer with twenty neurons and one output layer with four neurons. The network is trained using the *Levenberg–Marquardt* back propagation algorithm. Sigmoid activation functions are used for the first layer and linear one-to-one activation functions for the output layer. The neural network is implemented using the neural network toolbox of the MATLAB programming language.

Given any one of the configuration parameters,  $S_1, S_2, \dots, S_5$ , the developed method identifies possible ranges of the other four configuration parameters based on the data generated in the previous section. Since the data is generated by simulating specific ranges of the linkage dimensions), this method scales input configuration parameters to make sure they lie within the available data range. Selected output ranges by this method are scaled back to the original before being displayed for the user.

Fig. 9 Architecture of the neural network



**Fig. 10** Performance of the neural network



The method implemented using a MATLAB program called *f\_Parameter\_Sorter* provides an option for the user to select a configuration parameter with which to begin and the sequence of upcoming selections. This option allows the flexibility to prioritize the operational configurations as needed. Once the first item is entered for the first choice of the configuration parameter, four different compatible configuration parameter ranges will be suggested for the others.

This process will be repeated on the remaining four parameters by selecting which configuration parameter to prioritize and picking its value from the range provided. The result of this second operation modifies the ranges of compatible values of the remaining three parameters. This process is repeated until all configuration parameters are assigned valid values. Figure 10 shows the convergence performance of the ANN training cycles, while Fig. 11 shows the standard ANN algorithm regression chart.

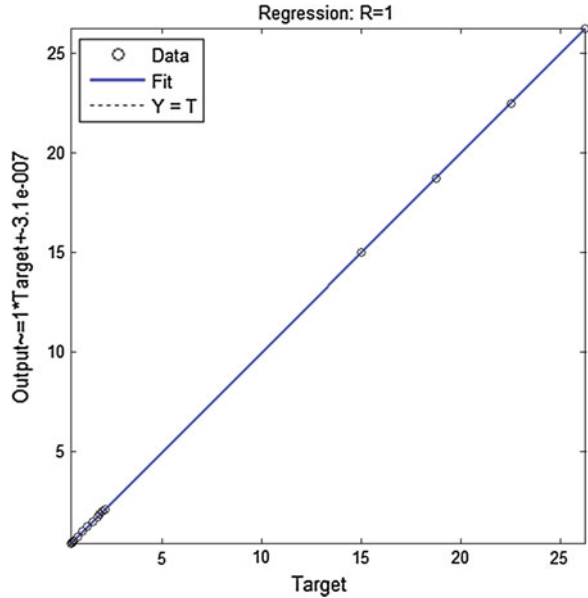
#### 4.5 Solving for Linkage Configuration Feature Parameters

Equations 11, 20, 33, 37, and 42 relate the specification values  $S_1, S_2, S_3, S_4,$  and  $S_5$  to the geometric dimensions of the excavator arm mechanism  $l_1, l_2, l_3,$  and  $\beta$ . Given the values of the other constants, these nonlinear equations can be solved using optimization techniques to determine the optimum linear and angular dimensions of the arm mechanism.

Since buckets are available as standard parts, the calculation of this algorithm focuses on determining the lengths of the boom and the stick together with the boom deflection angle, i.e.,  $l_1, l_2,$  and  $\beta$ . The selection of the bucket is made based on the initial solution suggested by the ANN. To determine the above three unknown variables, a combination of three of the above nonlinear equations is



Fig. 11 Regression result



solved using a MATLAB function, *fsolve()*, which employs the power of the *trust-region-reflective* algorithm

$$F(\mathbf{X}, \mathbf{S}) = \mathbf{0} \tag{44}$$

where  $\mathbf{X}$  and  $\mathbf{S}$  are vectors of unknown mechanism dimension variables and input configuration specification parameters.

$$\mathbf{X} = \begin{bmatrix} l_1 \\ l_2 \\ \beta \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} \tag{45}$$

Considering the maximum reach-out at ground level, maximum cutting height, and maximum loading height, the vector of equations will be formulated as follows:

$$\begin{bmatrix} l_1^2 + (l_2 + l_3)^2 + 2l_1(l_2 + l_3) \cos \beta - V^2 - (S_1 - H)^2 \\ l_1 \cos(\alpha_2 - \beta) + l_2 \cos(\alpha_2 - 2\beta) + l_3 \cos(\theta - \beta + \alpha_{bu}) + V - S_3 \\ l_1 \cos(\alpha_2 - \beta) + l_2 \cos(\alpha_2 - 2\beta) - l_3 + V - S_4 \end{bmatrix} = \mathbf{0} \tag{46}$$

The *trust-region-reflective* algorithm used to find the solution requires an initial solution to be defined as a starting point. The accuracy of the output for this particular problem greatly depends on the closeness of the initial solution to the actual solution. This is the stage where the suggested initial solution by the neural

network is used. It is also expected that at this stage the viability of the initial input parameters,  $S_1, S_2, \dots, S_5$ , is confirmed by the use of valid ranges developed according to the procedure discussed previously.

#### 4.6 Case Study Analysis Results and Discussion

In all, ten existing excavator product configuration data sets were collected; their contents are given in Table 3. A total of 1,296 forward simulation data sets were generated and they were used to train the ANN module developed with MATLAB. To test the system performance, the ten product configuration envelope path parameters were then fed into the ANN, and the output of the ANN, i.e., the initial suggestions for the downstream optimization module, was presented in Table 4 (left half). For the sake of comparison, the solutions generated after the optimization process are also listed in Table 4 (right half).

Clearly, the ANN module has served the purpose of providing useful initial suggestions that enabled the optimization module to find feasible solutions for the given mechanism. Furthermore, Table 5 shows the comparison results between the solutions and the original real product data obtained for the ten existing configurations. The average errors for linear dimensions are pretty close, i.e., within 10 %, but the angular  $\beta$  shows a bigger difference from the original dimension: about 24 %. The deviations of these errors are relatively small. Therefore, we can conclude that the proposed method is feasible and the results show a good agreement with the testing input data set. The method can be further improved by fine-tuning the optimization algorithms and the boundary conditions as well as by using more realistic product data sets for ANN training.

**Table 3** System testing data collected from the existing products (units: cm/degree)

Product	Configuration					Mechanism dimensions				Vehicle	
	S1	S2	S3	S4	S5	l1	l2	l3	$\beta$	H	V
1	359	183	344	226	107	174.1	88.2	51.9	24.5	63	75
2	413	252	384	271	109	205.9	102	61.1	25	68	86
3	412	260	359	246	111	201.2	99.4	67.9	28	74	93
4	435	228	422	283	106	203.3	105.2	64.9	30	78	90
5	409	248	385	267	125	201.3	99.5	61.5	25	66	84
6	372	208	371	257	110	171.4	89.1	58.2	24	77	82
7	352	196	331	235	92	159	86.3	49.6	22	77	71
8	345	203	338	238	99	165.1	88.4	49	20	64	73
9	332	184	335	238	104	163.4	83.8	47.5	24	55	71
10	415	254	368	272	110	204.9	102	63.1	25.76	68	81

**Table 4** The initial and final solutions generated from the system

Product	ANN initial solution (m)				Optimization final solution (m)			
	$l_1$	$l_2$	$l_3$	$\beta$	$l_1$	$l_2$	$l_3$	$\beta$
1	1.747	0.688	0.634	17.28	2.09	0.93	0.634	34.97
2	1.9697	0.9599	0.6556	19.4858	2.0022	0.9547	0.6556	30.0233
3	2.0453	0.8433	0.7019	26.524	2.0282	0.8581	0.7019	35.2464
4	1.8104	0.7914	0.783	10.7072	1.6981	0.8382	0.783	18.6694
5	1.7703	0.7381	0.6662	15.6772	2.0433	0.8323	0.6662	30.1607
6	1.6806	0.805	0.6103	13.2475	1.9883	0.9117	0.6103	25.8929
7	1.5389	0.8259	0.5279	15.2362	2.0148	0.9988	0.5279	30.014
8	1.6275	0.9462	0.5138	12.3882	1.97	0.9848	0.5138	32.8084
9	1.6105	0.909	0.4687	14.3593	2.1144	0.9756	0.4687	32.3521
10	1.9589	1.0773	0.6004	24.8409	1.879	0.926	0.6004	31.9021

**Table 5** Accuracy statistics of the system results

Dimensions	Average error (%)	Unbiased standard deviation	Root mean square error (RMSE)
$l_1$	8.627	0.1569	0.1489
$l_2$	1.4641	0.1356	0.1286
$l_3$	7.1778	0.085	0.0806
$\beta$	23.858	0.2652	0.2516

## 5 Conclusion

In this chapter, a hybrid feature transformation method from specification feature instances to mechanism configuration features was presented. The method uses ANN and optimization tools to solve feature-based dimension synthesis problems. The ANN, in order to reflect the mapping relations between accessing the envelope path and the linkage lengths in an excavator arm case study, needs to be trained before becoming usable. A mix of real product data sets from those existing product families and the generated data sets from forward kinematic simulation calculation methods are used for ANN training purposes. The forward data generation method is used to solve the problem of a shortage in real product data, and to produce enough “artificial” training data. The results of the analysis show a satisfactory estimation of the initial solutions based on the ANN model. For a set of existing product configurations, after testing the system on the whole cycle and searching for the final solutions with the optimization module, it can be concluded that the method is feasible and the results are promising, although more research analysis and evaluation are required. While this research used an excavator arm mechanism for the case study, the proposed method is not a product-dependent approach. Potentially, this hybrid method can be used for many other mechanism design processes as well.

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