# Chapter 2
# Techniques for Economic Modeling: Unlocking the Character of Data

**Abstract** In this chapter techniques for understanding economic data are described. These methods include measures such as the mean, variance, kurtosis, fractals, stationarity, frequency and time-frequency analysis techniques as well as their applications to the understanding economic data.

## 2.1 Introduction

The widespread availability of economic data has prompted researchers and practitioners to device ways of analyzing these data sets. Some of these data sets are estimated such as the inflation rate or they are priced by the market such as a trading share price. Data analysis is an area that has been around for a long time. Some of the techniques that have been used for estimation, like averages, are now so common that they are used as part of the normal daily conversation lexicology.

The conclusions that are drawn from these data and the statistical parameters that are derived are far reaching. For example, the conclusions that are drawn from the average changes of prices in an economy, also called inflation rate, are used to negotiate salary increases by unionized workers. The implication of a miscalculation of this very vital parameter is far reaching for the workers and the general economy of a country.

Recently, researchers and practitioners have developed decision making tools that are used to assist decision makers make their decisions. For example, a stock trader no longer has to necessarily visually look at complicated charts in order to make his or her decision. Instead, they can use a neural network that is able assist them in making a decision. Of course, this depends on the availability of data and an effective way of handling such data.

Normally before a piece of data is used, it is important to process it. For example, if we were to analyze a time history for the past 50 years of an economy of a particular country, sometimes it is not enough to just calculate the average or variance or any other statistical parameter of this time history. In such a situation,

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it might be more advisable to take the time data and convert it into the frequency domain because vital features would be revealed in the frequency domain rather than in the time domain.

This chapter gives a brief account of some of the techniques that are used to analyze data. We consider four domains, where we can analyze economic data and these are time, frequency, time-frequency and fractal domains. In the time domain or the so-called time series framework, we apply statistical concepts of mean, variance and kurtosis to analyze the data. In the frequency domain we use both the Fourier transform to analyze the data; in the time-frequency domain we apply wavelets to analyze the data while in the fractal domain we use the Hurst dimension to analyze the data. Another important factor is the characterization of the stationarity of data. Data is stationary if its characteristics are not changing as a function of time. In this regard, we apply the variance ratio test to characterize the stationarity of the data.

## 2.2 Time Domain Data

Data can be presented in the time domain which is what makes it known as time series data. In this domain, data is presented as a function of time. For example, we could present the quarterly GDP of the United States from 1 January 1947 to 1 April 2012 as shown in Fig. 2.1. From this figure, it can be observed that the GDP has been consistently growing during the specified time period. From this data, several statistical parameters can be derived and we pay attention to the average, variance and kurtosis.

The data in Fig. 2.1 can be expressed in terms of percentage change of GDP and this is shown in Fig. 2.2.

### *2.2.1 Average*

The average is the measure of the central tendency of data. For $N$ data points, the average $\bar{x}$ of series $x_1, x_2, \ldots, x_N$ can be calculated as follows (Hand 2008):

$$\bar{x} = \frac{1}{N}\sum_{i}^{N} x_i \tag{2.1}$$

Ni et al. (2013) studied the variable length moving average trading rules and its impact on a financial crisis period, while Pavlov and Hurn (2012) tested the profitability of moving-average rules as a portfolio selection strategy and found that, for a wide range of parameters, moving-average rules generate contrarian profits. Chiarella et al. (2012) applied moving averages successfully in a double auction

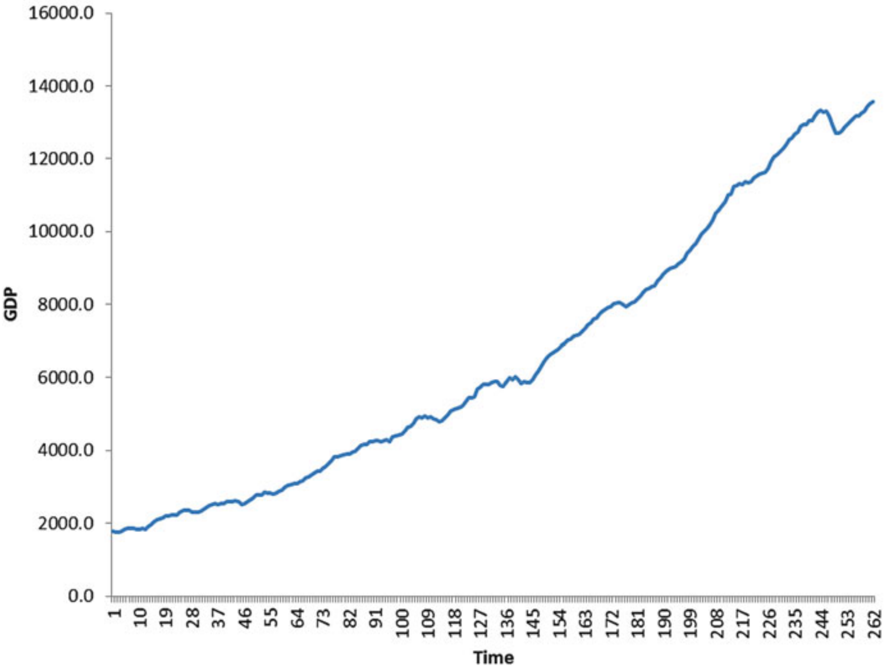


**Fig. 2.1** The quarterly GDP of the United States from 1947 to 2012

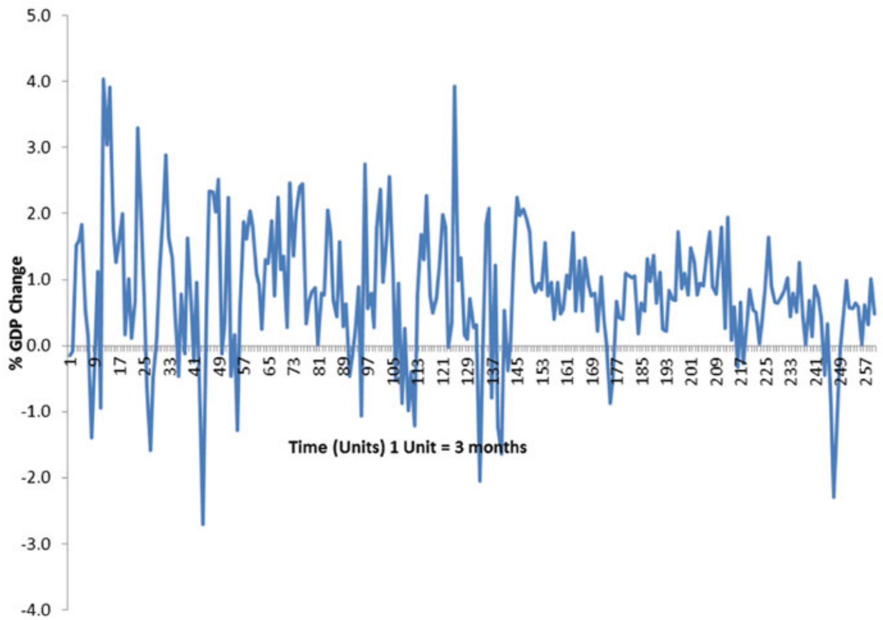


**Fig. 2.2** The percentage change of GDP of the United States from 1947 to 2012

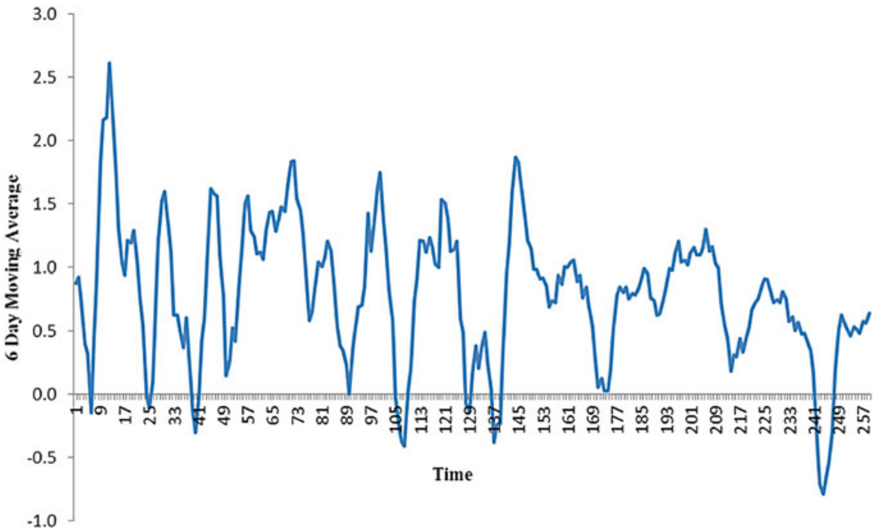


**Fig. 2.3** The 6 time unit moving average of the percentage change of GDP of the United States from 1947 to 2012

market. The data in Fig. 2.2 can be further processed by calculating the 6 time unit moving average of the percentage change of GDP and the results in Fig. 2.3 are obtained.

## 2.2.2 *Variance*

Variance is the measure of the spread of the data. It is calculated by finding the difference between the average value of the sum-of-squares and the square of the sum of averages, which can be written as follows (Hand 2008):

$$Varx = \frac{1}{N}\sum_{i}^{N}(x_i)^2 - \left(\frac{1}{N}\sum_{i}^{N}(x_i)\right)^2 \tag{2.2}$$

Variance has been used to estimate volatility of a stock market. Uematsu et al. (2012) estimated income variance in cross-sectional data whereas Chang et al. (2012) applied variance to estimate the rise and fall of S&P500 variance futures. Clatworthy et al. (2012) applied the variance decomposition analysis to analyze the relationship between accruals, cash flows and equity returns. The data in Fig. 2.2 can be transformed into the 6 time unit moving variance and this result is shown in Fig. 2.4.

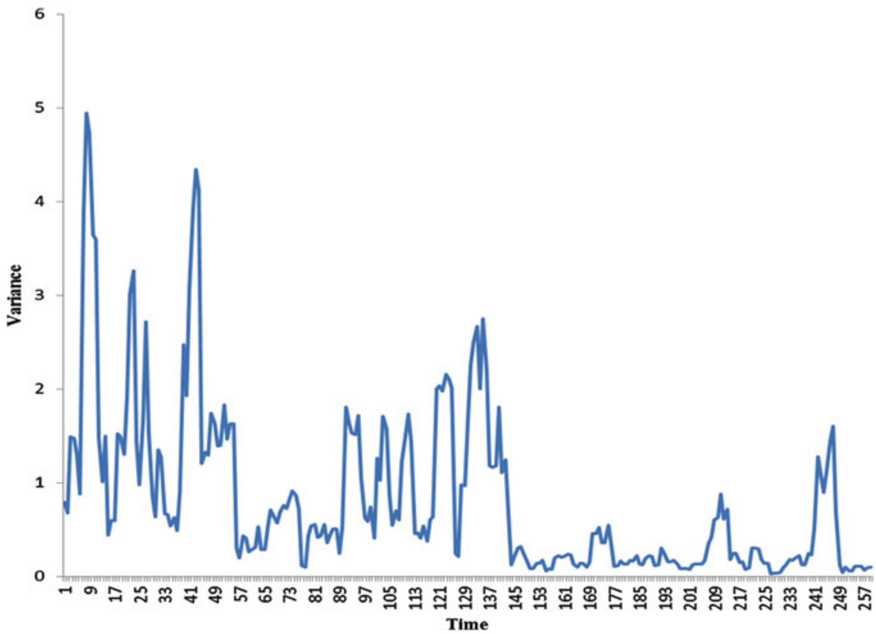


**Fig. 2.4** The 6 unit period moving variance of the percentage change of GDP of the United States from 1947 to 2012

From this figure, it can be observed in the earlier years the growth of the GDP was more volatile than in later years.

### *2.2.3 Kurtosis*

There is a need to deal with the occasional spiking of economic data and to achieve this task, Kurtosis is applied. Diavatopoulos et al. (2012) used Kurtosis changes to study the information content prior to earnings announcements for stock and option returns. They observed that changes in Kurtosis predict future stock returns. Some other applications of Kurtosis include in portfolio rankings (di Pierro and Mosevichz 2011), hedging (Angelini and Nicolosi 2010), and risk estimation (Dark 2010). The calculated Kurtosis value is typically normalized by the square of the second moment. A high value of Kurtosis indicates a sharp distribution peak and demonstrates that the signal is impulsive in character. Kurtosis can be written as follows (Hand 2008):

$$K = \frac{1}{N}\sum_{i=1}^{N}\frac{(x_i - \bar{x})^4}{\sigma^4} \tag{2.3}$$

where $\bar{x}$ is the mean and σ is the variance (Fig. 2.5).

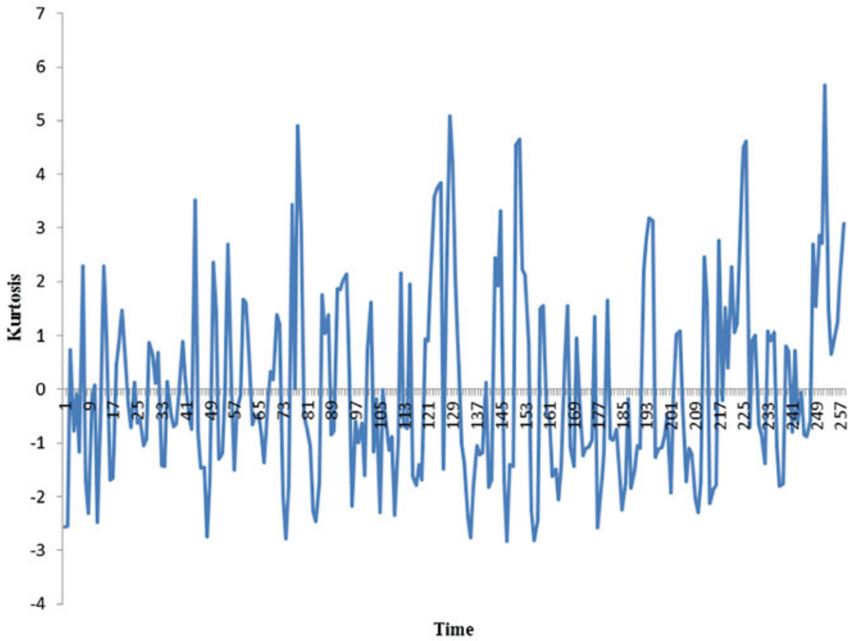


**Fig. 2.5** The 6 unit period moving kurtosis of the percentage change of GDP of the United States from 1947 to 2012

## 2.3 Frequency Doman

The roots of the frequency domain are in the Fourier series, which basically states that every periodic function can be estimated using a Fourier series. This Fourier series is in terms of sine and cosine. This in essence implies that each signal is represented in terms of a series of cycles with different amplitudes and frequencies. Within the economic modeling perspective this implies that we are representing an economic time series as a superposition of cycles. The function $f(x)$ can be estimated using the Fourier series and this is written as follows (Moon and Stirling 1999):

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{N} (a_n \cos(nx) + b_n \sin(nx)), N \geq 0 \tag{2.4}$$

where

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, n \geq 0 \tag{2.5}$$

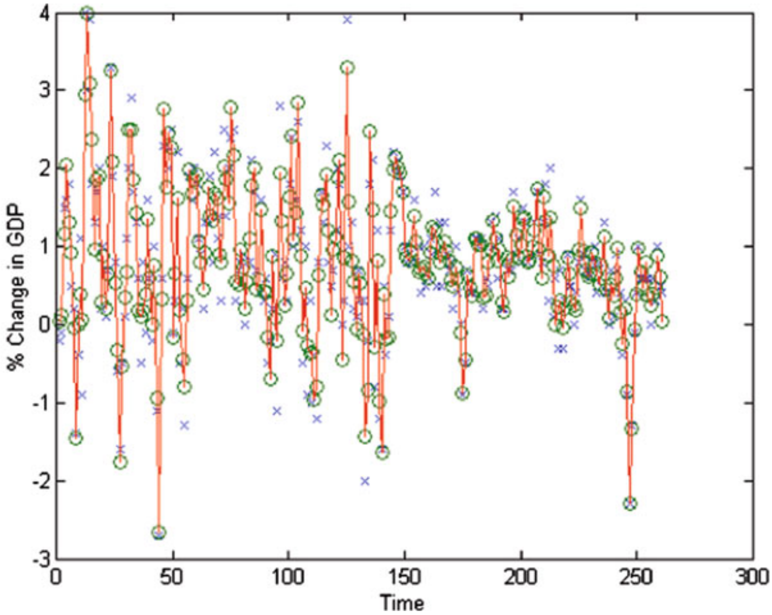


**Fig. 2.6** The Fourier series reconstruction of the percentage change of GDP of the United States from 1947 to 2012 (Key: x: 10 terms, o: 100 terms)

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, n \geq 1 \tag{2.6}$$

The estimation procedure outlined in Eqs. 2.4, 2.5, and 2.6 can be used to estimate Fig. 2.2 and this is shown in Fig. 2.6.

The representation of a signal using sine and cosine functions necessarily implies that a time domain signal can be transformed into the frequency domain. This can be achieved by applying the fast Fourier transform (FFT), which is essentially a computationally efficient technique for calculating the Fourier transform through exploiting the symmetrical relationship of the Fourier transform. If the FFT is applied to the function, $x(t)$, can be written as follows (Moon and Stirling 1999):

$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \tag{2.7}$$

where $\omega$ is the frequency and $t$ is the time. This relationship can be written as follows in the discrete form:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k\frac{n}{N}}, k = 0, \ldots, N-1 \tag{2.8}$$

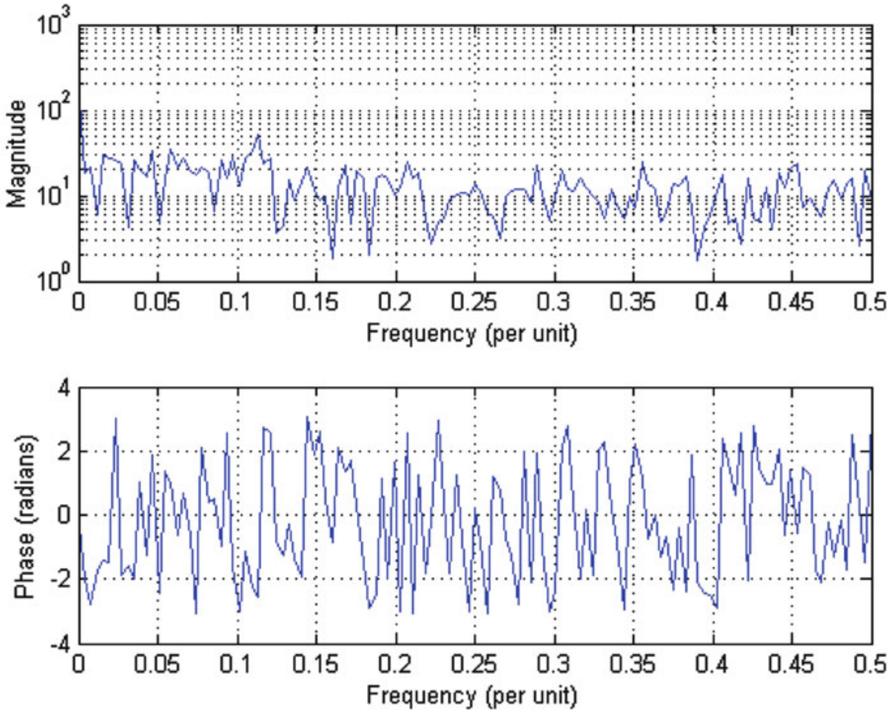


**Fig. 2.7** The magnitude and phase from the percentage change of GDP of the United States from 1947 to 2012

The Fourier transform of the signal in Fig. 2.2 is expressed in Fig. 2.7. In this figure, the peaks in the magnitude plot correspond to the cycles and since here we are dealing with an economic data it corresponds business cycle. The phase plot indicates the time lag associated with the magnitude. Grossmann and Orlov (2012) applied frequency domain methods to study exchange rate misalignments. They considered the deviations of Canadian, Japanese, and British spot exchange rates against the US dollar. The results showed that the Plaza Accord and the Euro introduction reduced the volatility of the exchange rate misalignments and extra returns for the Yen and the Pound along nearly all frequency components considered. Tiwari (2012) applied frequency domain methods to study causality between producers' price and consumers' price indices in Australia. The results obtained indicated that consumer price causes producer price at middle frequencies reflecting medium-run cycles, while producer price does not cause consumers price at any frequency. Shahbaz et al. (2012) applied the frequency domain methods to study causality between the Consumer Price Index (CPI) and the Wholesale Price Index (WPI). They observed one directional causal link from the CPI to WPI that varies across frequencies, that is, CPI Granger-causes WPI at all frequencies. Gradojevic (2012) applied frequency domain techniques to analyze foreign exchange order flows. The results indicated that causality depended on the customer type, frequency, and time period.

## 2.4 Time-Frequency Domain

Time-frequency techniques are methods where it is possible to see what is going on in both the time and frequency domains at the same instance. There is a number of time-frequency techniques and these include Short-time Fourier transform, Wavelet transform, Bilinear time-frequency distribution function (e.g. Wigner distribution function), modified Wigner distribution function, and Gabor–Wigner distribution function (Goupillaud et al. 1984; Delprat et al. 1992; Cohen 1995; Flandrin 1999; Papandreou-Suppappola 2002). In this chapter, we apply wavelet analysis to characterize an economic data in Fig. 2.2. There are many types of wavelets that can be used to analyze a signal and this chapter concentrates on the Morlet type (Chui 1992; Kingsbury 2001).

Zheng and Washington (2012) studied the choice of an optimal wavelet for detecting singularities in traffic and vehicular data. They found that choosing an appropriate wavelet mainly depends on the problem at hand and that the Mexican Hat wavelet offered an acceptable performance in detecting singularities in traffic and vehicular data. Ćmiel (2012) applied the wavelet shrinkage technique to estimate Poisson intensity of the Spektor-Lord-Willis problem. They found that the adaptive estimator gave the optimal rate of convergence over Besov balls to within logarithmic factors. Caraiani (2012) applied wavelets to study the properties of business cycles in Romania between 1991 and 2011 by analyzing the relationship between output and key macroeconomic variables in time and frequency. The results demonstrated that it is possible to separate the influence of definite events. Haven et al. (2012) applied wavelets to de-noise option prices. They demonstrated that the estimation of risk-neutral density functions and out-of-sample price forecasting is significantly improved after noise is removed using the wavelet method. Svensson and Krüger (2012) applied the wavelet method for analysing the mortality and economic fluctuations of Sweden between 1800, whereas Dajcman et al. (2012) successfully applied wavelets to analyze European stock market movement dynamics during financial crises and Hacker et al. (2012) successfully applied wavelets to study the relationship between exchange rates and interest.

In this chapter, we apply the Morlet wavelet which is a wavelet with a complex exponential multiplied by a Gaussian window. It has its roots in the Gabor transform, which applies concepts from quantum physics and Gaussian-windowed sinusoids for time-frequency decomposition and give the best trade-off between spatial and frequency resolution (Connor et al. 2012; Tao and Kwan 2012; Fu et al. 2013; Gu and Tao 2012; Agarwal and Maheshwari 2012). Goupillaud et al. (1984) adapted the Gabor transform to maintain the same wavelet shape over equal octave intervals, resulting in the continuous wavelet transform. The Morlet wavelet has been successfully applied before in a number of areas such as in analyzing earthquake ground motion (Shama 2012), simulating vehicle full-scale crash test (Karimi et al. 2012), seizure detection (Prince and Rani Hemamalini 2012), texture analysis for trabecular bone X-ray images (El Hassani et al. 2012), and fault diagnosis in rolling element bearing (Zhang and Tan 2012). The Morlet wavelet is described mathematically as follows (Goupillaud et al. 1984):

$$\Phi_\sigma(t) = c_\sigma \pi^{-\frac{1}{4}} e^{-\frac{1}{2}t^2} \left(e^{i\sigma t} - \chi_\sigma\right) \tag{2.9}$$

Here $\chi_\sigma = e^{-\frac{1}{2}\sigma^2}$ and is called the admissibility criterion while the normalization constant is:

$$c_\sigma = \left(1 + e^{-\sigma^2} - 2e^{-\frac{3}{4}\sigma^2}\right)^{-\frac{1}{2}} \tag{2.10}$$

The Fourier transform of the Morlet wavelet can be written as follows (Goupillaud et al. 1984):

$$\Phi_\sigma(\omega) = c_\sigma \pi^{-\frac{1}{4}} \left(e^{-\frac{1}{2}(\sigma-\omega)^2} - \chi_\sigma e^{-\frac{1}{2}\omega^2}\right) \tag{2.11}$$

The variable $\sigma$ permits the trade-off between time and frequency resolutions. The results obtained when the Morlet wavelets are used to analyze Fig. 2.2 are shown in Fig. 2.8.

## 2.5 Fractals

Fractal analysis is a method of defining complex shapes and numerous techniques for estimating fractal dimensions have been proposed (Marwala 2012). As described by Lunga (2007) and Lunga and Marwala (2006), fractal dimensions of an object are an indicator of the degree to which the object occupies space. Alternatively, a fractal dimension of a time series expresses how turbulent the time series is and also quantifies the extent to which the time series is scale-invariant (Lunga and Marwala 2006; Lunga 2007). The technique applied to approximate fractal dimensions using the Hurst exponent for a time series is known as the rescaled range (R/S) analysis and was proposed by Hurst (1951).

Lunga and Marwala (2006) successfully applied time series analysis applying fractal theory and online ensemble classifiers to model the stock market, while Nelwamondo et al. (2006a) applied fractals successfully for early classifications of bearing faults. Nelwamondo et al. (2006b) applied a multi-scale fractal dimension for speaker identification systems, while Nelwamondo et al. (2006c) applied fractals for improving speaker identification rates.

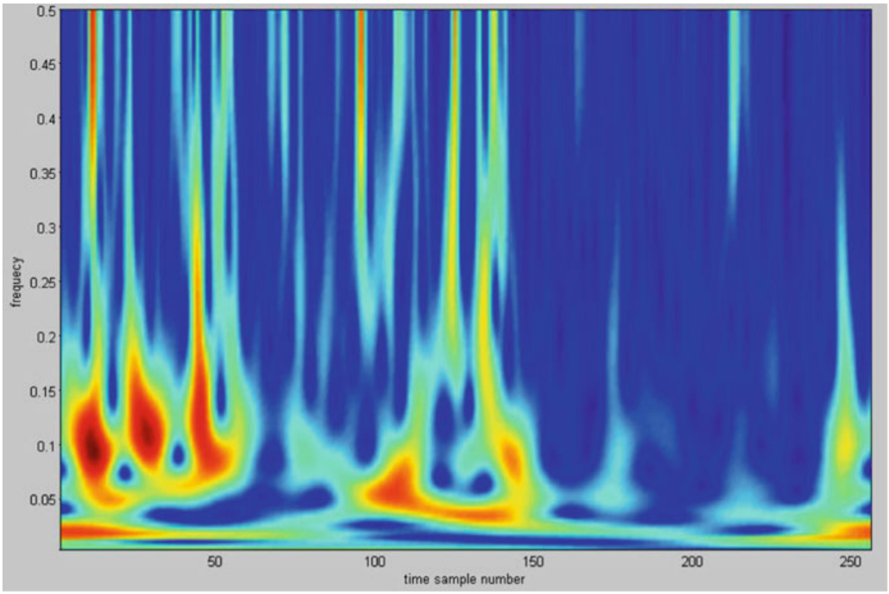


**Fig. 2.8** The wavelet transform of the percentage change of GDP of the United States from 1947 to 2012

Kristoufek (2012) studied the influence of the fractal markets hypothesis for liquidity and investment horizons on predictions of the dynamics of the financial markets during turbulences such as the Global Financial Crisis of late 2000s. They observed that fractal markets hypothesis predicted the observed characteristics sufficiently.

Krištoufek and Vošvrda (2012) used the fractal dimension, Hurst exponent, and entropy to quantify capital markets efficiency. The results obtained indicated that the efficient market was dominated by local inefficiencies and stock indices of the most developed countries were the most efficient capital markets.

Shin et al. (2012) applied fractals to organize distributed and decentralized manufacturing resources. The proposed technique was observed to reduce problem complexity through iterative decomposition of problems in resource management.

## *2.5.1 The Rescaled Range (R/S) Methodology*

As described by Lunga and Marwala (2006) as well as Lunga (2007), this section describes a method for approximating the quality of a time series signal to identify the intervals that are vital to classify a time signal. The rescaled range (R/S) analysis is a method that was proposed by Hurst (1951) to control reservoir on the Nile River dam project in 1907. The objective was to identify the optimal design of a reservoir from data of measured river discharges. A desirable reservoir does not run dry or overflow. Hurst proposed a statistical quantity, the Hurst exponent ($H$), a technique that can be applied to categorize time series signals into random and non-random series. By applying the R/S analysis, it is possible to identify the average non-periodic cycle and the measure of persistence in trends because of long memory effects (Skjeltorp 2000).

To implement R/S when we have a time series of length $M$ is to calculate the logarithm of a ratio with length $N = M - 1$ and this can be expressed mathematically as follows (Lunga 2007):

$$N_i = \log\left(\frac{M_{i+1}}{M_i}\right),\ i = 1, 2, 3, \ldots, (M - 1) \tag{2.12}$$

The average is estimated by splitting the time period into $T$ adjoining sub-periods of length $j$, in such a way that $T * j = N$, with each sub-period named $I_t$, with $t = 1{,}2 \ldots T$ and each element in $I_t$ named $N_{k,t}$ such that $k = 1{,}2, \ldots j$. This average can be written as follows (Lunga and Marwala 2006; Lunga 2007):

$$e_t = \frac{1}{j}\sum_{k=1}^{j} N_{k,t} \tag{2.13}$$

Therefore, $e_t$ is the average value of the $N_i$ enclosed in sub-period $I_t$ of length $j$. The time series of accrued data $X_{k,t}$ can be calculated from the mean for each sub-period $I_t$, as follows (Lunga 2007):

$$X_{t,t} = \sum_{i=1}^{k} (N_{k,t} - e_t)\, k = 1, 2, 3, \ldots, j \tag{2.14}$$

The range of the time series in relation to the mean within each sub-period can be mathematically written as follows (Lunga 2007):

$$R_{T_t} = \max(X_{k,t}) - \min(X_{k,t}),\, 1 < k < j \tag{2.15}$$

The standard deviation of each sub-period can be calculated as follows (Lunga 2007):

$$S_{I_t} = \sqrt{\frac{1}{j}\sum_{i=1}^{k} (N_{i,t} - e_t)^2} \tag{2.16}$$

The range of each sub-period $R_{Tt}$ can be rescaled by the respective standard deviation $S_{It}$ and is done for all the $T$ sub-intervals in the series and the following average R/S value is obtained (Lunga 2007):

$$e_t = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_{It}}{S_{It}}\right) \tag{2.17}$$

The computation from the Eqs. 2.12, 2.13, 2.14, 2.15, 2.16 and 2.17 can be recurred for different time ranges and this can be realized by sequentially increasing $j$ and reiterating the computation until all $j$ values are covered. After computing the R/S values for a large range of different time ranges $j$, $log(R/S)_j$ can be plotted versus $log(n)$. The Hurst exponent ($H$) can be estimated by conducting a least squares linear regression with $log(R/S)$ as the dependent variable and $log(n)$ as the independent variable and $H$ is the slope of the regression (Maragos and Potamianos 1999; Hurst 1951; Hurst et al. 1965). The fractal dimension and the Hurst exponent are related as follows (Lunga 2007; Wang et al. 2000):

$$D_f = 2 - H \tag{2.18}$$

There are other techniques for estimating the fractal dimensions and these include using the Box counting dimension (Falconer 1952; Nelwamondo et al. 2006a; Marwala 2012), the Hausdorff dimension (Falconer 1952) and Minkowski-Boulig and dimension (Schroeder 1991).

### 2.5.2 The Hurst Interpretation

In fractal theory, when $H \in (0.5; 1]$ then the time series is persistent and is described by long memory effects on all time scales (Lo 1991; Gammel 1998). Within the context of the stock market, this means that all prices are correlated with all future hourly price changes (Beran 1994). Persistence indicates that if the time series has been up or down in the previous time, then there is a probability that it will remain up and down, respectively, in the future time. The advantage of the tendency to reinforce behavior, or persistence, increases as $H$ tends to 1. The influence of the present on the future can be articulated as a correlation function $G$ as follows (Lunga 2007):

$$G = 2^{2H-1} - 1 \tag{2.19}$$

When $H = 0.5$ then $G = 0$ and the time series is uncorrelated. Nevertheless, when $H = 1$ then $G = 1$, representing ideal positive correlation. Alternatively, when $H \in [0; 0.5)$ then the time series signal is anti-persistent and this implies that every

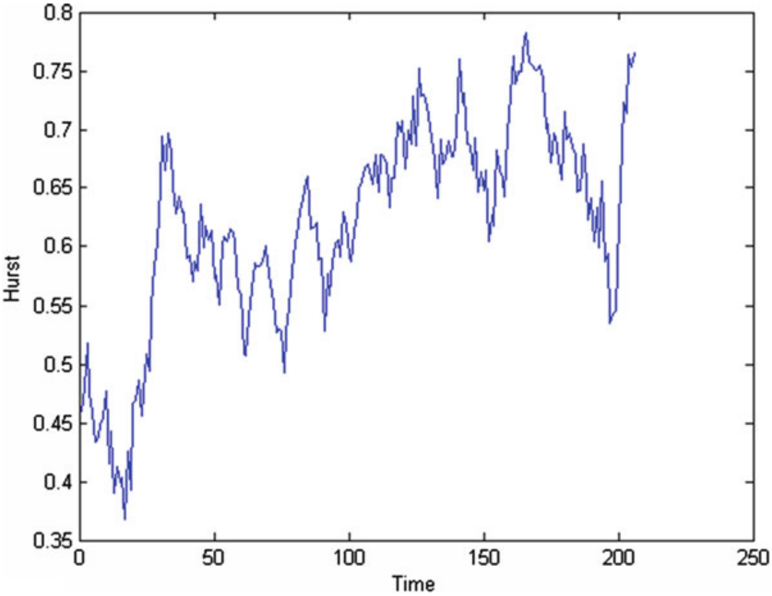


**Fig. 2.9** Hurst of the GDP of the United States from 1947 to 2012

time a time series has been up in the last period, it is more possibly going to be down in the next period. Therefore, an anti-persistent time series is spiky than a pure random walk.

The Hurst factor was used to analyse the data in Fig. 2.1 and the results in Fig. 2.9 were obtained. These results were obtained by calculating the $H$ over a period of 50 time units. Here, a unit corresponds to 3 months. The results in Fig. 2.9 indicate that the data during this period was largely persistent, which indicates that the signal has long memory effects. The results in Fig. 2.10 indicate that this data was anti-persistent and this means that every time the GDP change has been up in the previous period it is most likely going to be down in the next period.

## 2.6 Stationarity

A stationary process is a stochastic process whose joint probability distribution does not vary when shifted in space or time. Therefore, if certain parameters (for instance, the mean and variance) can be estimated, then they do not change over space or time (Priestley 1988). By the same logic, a non-stationary process is a process whose joint probability distribution varies when shifted in space or time. There are many techniques that have been proposed to evaluate whether a given signal is stationary or not.

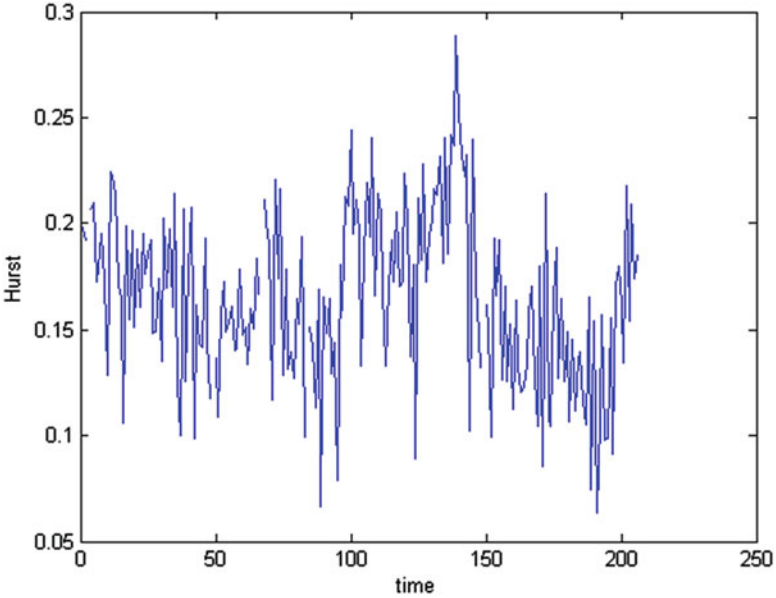


**Fig. 2.10** Hurst of the percentage change of the GDP of the United States from 1947 to 2012

Kiremire and Marwala (2008) proposed the stationarity index, which is a quantification of similarities of the auto correlation integral of a subdivision of a time series and the cross-correlation of that subdivision with others of the same time series. This quantification of similarity is a measure of the stationarity of the time series and, consequently, can be applied to detect and quantify non-stationarity.

The index was successfully applied in the analysis of electrocardiogram (ECG) and electroencephalogram (EEG) signals to detect the variations in the dynamics of the signals and the incidence of several events. The index showed sensitivity to variations in the dynamics shown in ECG signals that were the result of partial epileptic seizures. Zhou and Kutan (2011) applied non-linear unit root tests and recursive analysis to evaluate the relationship between the stationarity of real exchange rates and different currencies, different sample periods, and different countries. The results showed that a stationary real exchange rate is sensitive to sample period but not the currencies.

Caporale and Paxton (2013) investigated inflation stationarity in Brazil, Argentina, Chile, Mexico, and Bolivia from 1980 to 2004. They tested for structural interruptions in inflation, explained the interruptions using changes in monetary policies, and tested the relationship between structural interruptions and non-stationarity results.

Zhou (2013) successfully studied the nonlinearity and stationarity of inflation rates in the Euro-zone countries, whereas Fallahi (2012) successfully studied the stationarity of consumption-income ratios (CIR) and observed that the CSI was non-stationary in most of the countries.

Other studies on stationarity include energy consumption (Hasanov and Telatar 2011), inflation in Mexico (Caporale and Paxton 2011), interest rate in the European Union (Zhou 2011), commodity prices (Yang et al. 2012) and purchasing power parity of post-Bretton Woods exchange rate data for 20 industrialized countries (Amara 2011).

Suppose we have a stochastic process represented by $\{Y_t\}$ with $F_Y(y_{t_1+\zeta},\ldots, y_{t_n+\zeta})$ indicating a cumulative distribution function of the joint distribution $\{Y_t\}$ at times $\{t_1+\zeta,\ldots,t_n+\zeta\}$ then $\{Y_t\}$ is stationary if for all values of $n$, $\zeta$ and $\{t_1,\ldots,t_n\}$ then $F_Y(y_{t_1+\zeta},\ldots,y_{t_n+\zeta}) = F_Y(y_{t_1},\ldots,y_{t_n})$. There are many of techniques that have been proposed to quantify stationarity and these include Dickey-Fuller and Phillips-Perron Tests, Kwiatkowski-Phillips-Schmidt-Shin Test, as well as the Variance Ratio Test (Granger and Newbold 1974; Perron 1988; Kwiatkowski et al. 1992; Schwert 1989). In this chapter, we apply the Variance Ratio Test to determine the stationarity of a signal in Fig. 2.2. The variance ratio ($F$) can be written as follows mathematically (Lo and MacKinlay 1989):

$$F = \frac{Ve}{Vu} \tag{2.20}$$

Here, $E_v$ is the explained variance and $U_v$ is the unexplained variance and:

$$V_e = \frac{\sum_i n_i\left(\bar{X}_i - \bar{X}\right)^2}{K-1} \tag{2.21}$$

and

$$V_u = \frac{\sum_{ij}\left(X_{ij} - \bar{X}_i\right)^2}{N-K} \tag{2.22}$$

Here, $\bar{X}_i$ indicates the sample mean of the $i$th group, $n_i$ is the number of observations in the $i$th group, a $\bar{X}$ indicates the complete mean of the data, $X_{ij}$ is the $j$th observation in the $i$th out of $K$ groups and $N$ is the overall sample size. If the variability ratio is 1, then the data is following a random walk, if it is larger than 1, then it is trending and, therefore, non-stationary and if it is less than one, then it shows a mean reversal meaning than changes in one direction leads to probably changes in the opposite direction. The results obtained when we tested for stationarity using the variance ratio test are shown in Fig. 2.11 for the data in Figs. 2.1 and 2.12 for the data in Fig. 2.2.

These results were obtained by analyzing a moving window of 50 units with each unit corresponding to 3 months. These results indicate that the raw values of the GDP in Fig. 2.1 is non-stationary while the percentage change of GDP in Fig. 2.2 is stationary.

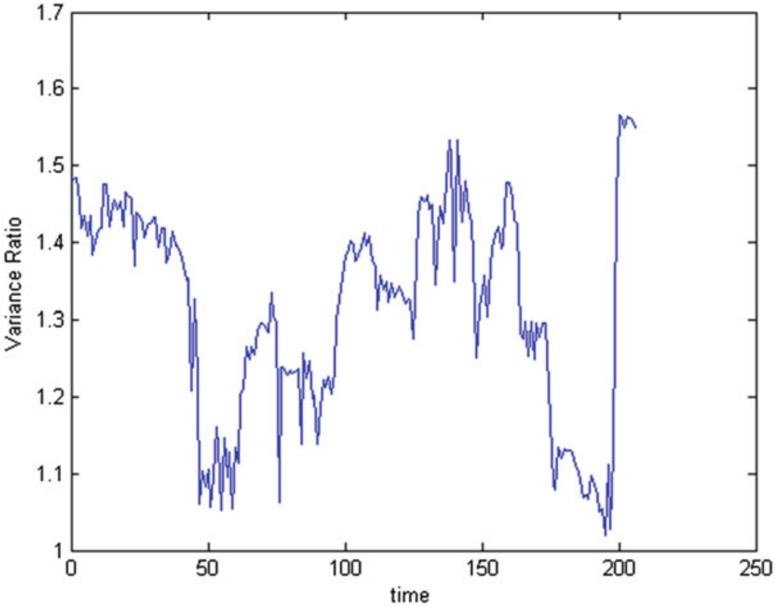


**Fig. 2.11** Variance ratio of the GDP of the United States from 1947 to 2012

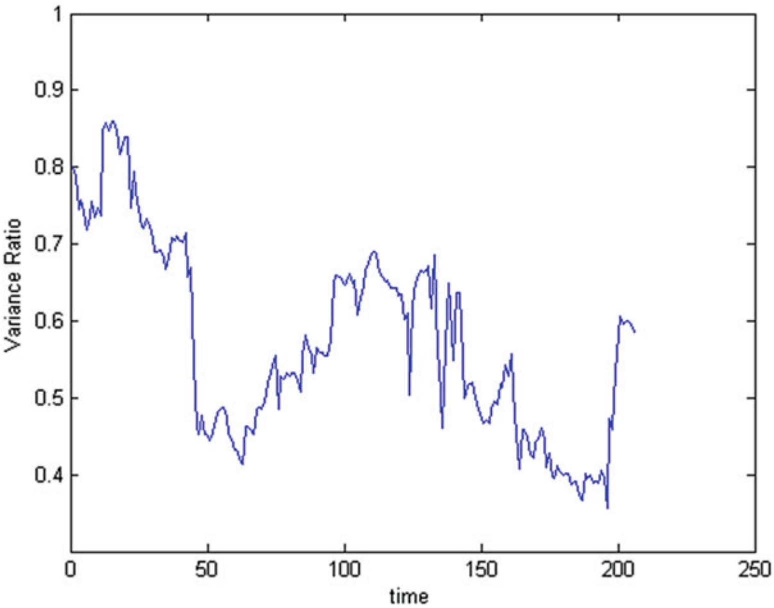


**Fig. 2.12** Variance ratio of the percentage change of the GDP of the United States from 1947 to 2012

## 2.7 Conclusions

In this chapter, techniques for economic data analysis were described and applied to analyze the GDP data. These methods were the mean, variance, Kurtosis, fractals, frequency, time-frequency analysis techniques and stationarity. The Fast Fourier transform method was used to decompose the GDP data from the time domain to the frequency domain. The Hurst parameter was applied to estimate the fractal dimension of the data. The variance ratio test was used to characterize stationarity.

## References

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