Design of Reliability Test Plans: An Overview

E. A. Elsayed

Reliability prediction of new components, products, and systems is a difficult task due to the lack of well-designed test plans that yield "useful" information during the test and due to the stochastic nature of the normal operating conditions. The accuracy of the reliability prediction has a major effect on the warranty cost and repair and maintenance strategies. Therefore, it is important to design efficient test plans. In this chapter, we present an overview of reliability testing with emphasis on accelerated testing and address issues associated with the design of optimal test plans, stress application methods, and reliability prediction models. We further discuss the concept of equivalence of test plans and how it could be used for test time reduction. Finally, we present accelerated degradation modeling and the design of accelerated degradation test plans.

1 Introduction

The high rate of technological advances and innovations are spurring the continuous introduction of new products and services. Moreover, the intensity of the global competition for the development of new products in a short time has motivated the development of new methods such as robust design, just-in-time manufacturing, and design for manufacturing and assembly. More importantly, both producers and customers expect the product to perform the intended functions satisfactorily for extended periods of time. Hence, extended warranties and similar assurances of product reliability have become standard features of the product and serve as implied indicators of the product's reliability. Likewise, recalls of products and recent failures of systems, such as air traffic control systems and autos (sudden acceleration

E. A. Elsayed
Department of Industrial and Systems Engineering, Rutgers, The State University of New Jersey, 96 Frelinghuysen Road, Piscataway, NJ 08854-8018, USA
e-mail: elsayed@rci.rutgers.edu

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and brake failures) and products have emphasized the importance of testing. For example, a recent recall of a popular car is attributed, by the manufacturer, to lack of thoroughness in testing new cars and car parts under varying weather conditions, as demonstrated by the recently recalled gas-pedal mechanism that tended to stick more as humidity increased [40].

Careful reliability testing of systems, products, and components at the design stage is crucial to achieving the desired reliability at the field operating conditions. During the design stage of many products, especially those used in military, the elimination of design weaknesses inherent to intermediate prototypes of complex systems is conducted via the test, analyze, fix, and test (TAFT) process. This process is generally referred to as "reliability growth." Specifically, reliability growth is the improvement in the true but unknown initial reliability of a developmental item as a result of failure mode discovery, analysis, and effective correction. Corrective actions generally assume the form of fixes, adjustments, or modifications to problems found in the hardware, software, or human error aspects of a system [20]. Likewise, field test results are used in improving product design and consequently its reliability.

The above examples and requirements have magnified the need for providing more accurate estimates of reliability by performing testing of materials, components, and systems at different stages of product development.

There is a wide variety of reliability testing methodologies and objectives. They include testing to determine the potential failure mechanisms, reliability demonstration testing, reliability acceptance testing, reliability prediction testing using accelerated life testing (ALT), and others. This chapter focuses on ALT, reliability prediction models and the design of the ALT plans.

Testing under normal operating conditions requires a very long time especially for components and products with long expected lives, and it requires extensive number of test units, so it is usually costly and impractical to perform reliability testing under normal conditions.

In many cases, ALT might be the only viable approach to assess whether the product meets the expected long-term reliability requirements. ALT experiments can be conducted using three different approaches. The first is conducted by accelerating the "use" of the unit at normal operating conditions such as in cases of products that are used only a fraction of a time in a typical day which includes home appliances and auto tires. The second is conducted by subjecting a sample of units to stresses severer-than-normal operating conditions in order to accelerate the failure. The third is conducted by subjecting units that exhibit some type of degradation such as stiffness of springs, corrosions of metals, and wear out of mechanical components to accelerated stresses. The last approach is referred to as accelerated degradation testing (ADT).

The reliability data obtained from the experiments are then utilized to construct a reliability model for predicting the reliability of the product under normal operating conditions through a statistical and/or physics-based inference procedure. The accuracy of the inference procedure has a profound effect on the reliability estimates and the subsequent decisions regarding system configuration, warranties, and preventive maintenance schedules. Specifically, the reliability estimate depends on two factors, the ALT model and the experimental design of the ALT test plans. A "good" model can provide an appropriate fit to testing data and results in achieving accurate estimates at the normal conditions. Likewise, an optimal design of the test plans, which determines the stress loadings (constant-stress, ramp-stress, cyclic-stress, . . .), allocation of test units number stress level, optimum test duration, and other experimental variables, can indeed improve the accuracy of the reliability estimates. Indeed, without an optimum test plan, it is likely that a sequence of expensive and time-consuming tests results in inaccurate reliability estimates. This might also cause delays in product release, or the termination of the entire product as has been observed by the author.

We describe briefly the methods of stress application, types of stresses, and focus on the reliability prediction models that utilize the failure data at stress conditions to obtain reliability information at normal conditions. We begin by describing the three important methods including two of the most commonly used prediction models that relate the test results at stress conditions to failure rate at the normal operating conditions.

2 Reliability Prediction Models Using ALT Data

Many ALT models have been developed and successfully implemented in a variety of engineering applications. The important assumption for relating the accelerated failures to those at normal operating conditions is that the components/products operating at the normal conditions experience the same failure mechanisms as those at the accelerated conditions. Elsayed [14] classifies the existing ALT models into three categories: *statistics-based models*, *physics-statistics-based models*, and *physics-experimental-based models*, as shown in Fig. 1. In particular, the statistics-based models are generally used when the relationship between the applied stresses and the failure time of the product is difficult to determine based on physics or chemistry principles. In this case, accelerated failure times are used to determine the model parameters statistically after assuming either a linear or nonlinear life-stress relationship.

The statistics-based models can be further classified into parametric models and semiparametric/nonparametric models. The most commonly used failure time distributions in the parametric models are the exponential, Weibull, normal, lognormal, gamma, and extreme value distributions. The underlying assumption of these models is that the failure times of the products follow the same distributions at different stress levels. In reality, however, when the failure process involves complex and/or inconsistent failure time distributions, the parametric models may not interpret the data satisfactorily and the reliability prediction will be far from accurate. Consequently, semiparametric or nonparametric models appear to be attractive and more suitable for reliability estimation due to their "distribution-free" property. We briefly review the two most commonly used ALT models as they will be used in the design

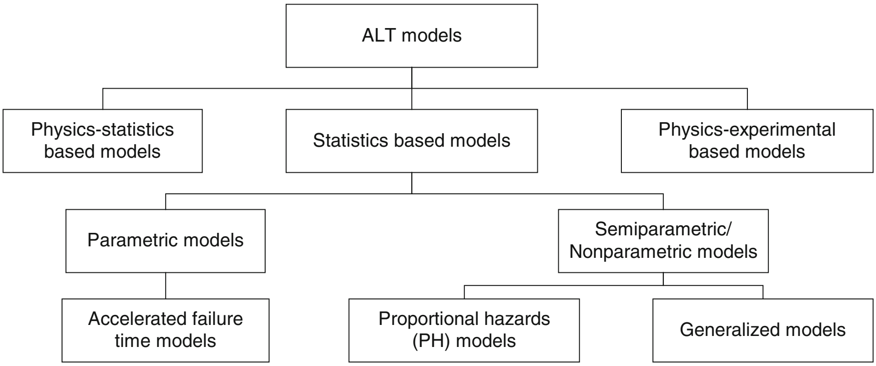

Fig. 1 Classification of ALT models [14]

of the ALT plans and describe a third model which relaxes the assumptions of the two models.

2.1 Proportional Hazards Model

Multiple regression models can be used to predict the time to failure (TTF) of a component under multiple covariates. A similar regression-based model that is widely used is the *proportional hazards* (PH) model introduced by Cox [8]. The PH model is generally expressed as:

$$\lambda(t; z) = \lambda_0(t)\exp(\boldsymbol{\beta} z)$$

where $z = (z_1, z_2, ...z_p)^T$ is a column vector of covariates (for ALT, it is the column vector of stresses and/or their interactions that components experience). $\boldsymbol{\beta} = (\beta_1, \beta_2, ...\beta_p)$ is a row vector of the unknown coefficients. $\lambda_0(t)$ is a baseline hazard rate function. Unlike standard regression models, the PH models assume that the applied stresses act multiplicatively, rather than additively, on the hazard rate—a much more realistic assumption in many cases [11, 16, 18]. The PH model is a class of models with the property that the hazard functions of two units at two different stress levels z_1 and z_2 are proportional to each other. In other words, the ratio of their hazard rates does not vary with time.

One of the advantages of the PH model is the ability to include time-dependent covariates. Let $z_i(t)$ be the covariate vector at time t for the ith individual unit under study, then the associated hazard rate function can be expressed as:

$$\lambda(t; z_i(t)) = \lambda_0(t)\exp(\boldsymbol{\beta} z_i(t))$$

where the hazard rate at time t depends only on the current stress level $z_i(t)$, and there is no effect caused by the previous stress history.

2.2 Accelerated Failure Time Models

Another widely used class of ALT models is *accelerated failure time* (AFT) models. For many products, there are well-established acceleration models that perform satisfactorily over the desired range of stresses. For instance, for temperature accelerated testing, the Arrhenius model has gained acceptance because of its many successful applications and general agreement of laboratory test results with long-term field performance. In an AFT model, it is assumed that for a unit under the applied stress vector z, the log-lifetime $Y = \log T$ has a distribution with a location parameter $\mu(z)$ depending on the stress vector z, and a constant scale parameter $\sigma > 0$ in the form of:

$$Y = \log T = \mu(z) + \sigma\varepsilon$$

where ε is a random variable whose distribution does not depend on z. The location parameter $\mu(z)$ follows some assumed life-stress relationship, e.g., $\mu(z_1, z_2) = \theta_0 + \theta_1 z_1 + \theta_2 z_2$, where z_1 and z_2 are some known functions of stresses. The popular Inverse Power law and Arrhenius model are special cases of this simple life-stress relationship. The AFT models assume that the covariates act multiplicatively on the failure time, or linearly on the log failure time, rather than multiplicatively on the hazard rate. The hazard function in the AFT model can be written in terms of the baseline hazard function $\lambda_0(\cdot)$ as:

$$\lambda(t; z) = \lambda_0(e^{\beta z} t)e^{\beta z}$$

The main assumption of the AFT models is that the TTFs are inversely proportional to the applied stresses, e.g., the TTF at high stress is shorter than the TTF at low stress. It also assumes that the failure time distributions are of the same type. In other words, if the failure time distribution at the higher stress is exponential then the distribution at the low stress is also exponential. Therefore, a general cumulative distribution function CDF for a two-parameters Weibull distribution under an applied stress vector z is

$$F(t; z) = 1 - \exp\left(-\left(\frac{t}{\theta(z)}\right)^{\beta}\right)$$

where β is the shape parameter and $\theta(z)$ is the scale parameter as a function of applied stresses which can be expressed as $\theta(z) = \theta_0 + \sum_{i=1}^{n} \theta_i z_i$, where θ_i is a coefficient of the covariate z_i. We illustrate the use of Weibull distribution for the true linear acceleration case in which the scale parameter at normal conditions θ_o is linearly related to the scale parameters at accelerated conditions θ_s using an acceleration factor A_F. The relationship between the failure time distributions at the accelerated and normal conditions can be derived as

$$F_s(t) = 1 - e^{-(\frac{t}{\theta_s})^{\beta_s}} \quad t \geq 0, \beta_s \geq 1, \theta_s > 0 \tag{1}$$

where β_s is the shape parameter of the Weibull distribution at stress conditions. The CDF at normal operating conditions is:

$$F_o(t) = F_s\left(\frac{t}{A_F}\right) = 1 - e^{-\left(\frac{t}{A_F\theta_s}\right)^{\beta_s}} = 1 - e^{-\left(\frac{t}{\theta_o}\right)^{\beta_o}} \tag{2}$$

As stated earlier, the underlying failure time distributions at both the accelerated stress and operating conditions have the same shape parameters, i.e., $\beta_s = \beta_o$, and $\theta_o = A_F\theta_s$. If the shape parameters at different stress levels are significantly different, then either the assumption of true linear acceleration is invalid or the Weibull distribution is inappropriate to use for analysis of such data.

Let $\beta_s = \beta_o = \beta \geq 1$. Then the probability density function at normal operating conditions is

$$f_o(t) = \frac{\beta}{A_F\theta_s}\left(\frac{t}{A_F\theta_s}\right)^{\beta-1} e^{-\left(\frac{t}{A_F\theta_s}\right)^{\beta}} \quad t \geq 0, \theta_s \geq 0 \tag{3}$$

The MTTF at normal operating conditions is

$$MTTF_o = \theta_o^{\frac{1}{\beta}}\Gamma\left(1 + \frac{1}{\beta}\right) \tag{4}$$

The failure rate at normal operating conditions is

$$\lambda_o(t) = \frac{\beta}{A_F\theta_s}\left(\frac{t}{A_F\theta_s}\right)^{\beta-1} = \frac{\lambda_s(t)}{A_F^{\beta}} \tag{5}$$

2.3 Extended Linear Hazard Regression Model

The PH and AFT models have very different assumptions (failure rate proportionality or failure time proportionality, respectively). The only model that satisfies both assumptions is the Weibull model. Assuming PH or AFT for a particular data set may lead to different results. Therefore, a simultaneous treatment of the two is of practical importance especially when the assumption regarding the PH or AFT is difficult to justify or does not hold. Ciampi and Etezadi-Amoli [6] propose the *extended hazard regression* (EHR) model which encompasses both the PH and AFT models as special cases. To further enhance the capability of modeling ALT, Elsayed et al. [18] propose a more generalized model - the *extended linear hazard regression* (ELHR) model by incorporating the time-varying coefficient effect into the EHR model. The ELHR model is expressed as:

$$\lambda(t; z) = \lambda_0(te^{(\beta_0+\beta_1 t)z})e^{(\alpha_0+\alpha_1 t)z} \tag{6}$$

The ELHR model encompasses all previous models—PH, AFT, and EHR as special cases. It incorporates the time-changing effects, proportional hazard effects, as well as time-varying coefficient effects into one model. The ELHR model outperforms the PH model and other extended models (e.g., Shyur et al. [35]) in that it can better interpret physical failure processes thus providing a better model fit to the corresponding failure time data. Furthermore, the ELHR model is essentially "distribution-free", and thus has a significant potential of dealing with complex failure processes. For example, by assuming the baseline hazard function $\lambda_0(\cdot)$ to be a quadratic function $\lambda_0(u) = \gamma_0 + \gamma_1 u + \gamma_2 u^2$, the model can be expressed as:

$$\lambda(t; z) = \gamma_0 e^{(\alpha_0+\alpha_1 t)z} + \gamma_1 t e^{(\theta_0+\theta_1 t)z} + \gamma_2 t^2 e^{(\omega_0+\omega_1 t)z} \tag{7}$$

where $\theta_0 = \alpha_0 + \beta_0$, $\theta_1 = \alpha_1 + \beta_1$, $\omega_0 = \alpha_0 + 2\beta_0$, $\omega_1 = \alpha_1 + 2\beta_1$. Then, the associated reliability is given by

$$\begin{aligned} R(t; z) &= \exp(-\Lambda(t; z)) \\ &= \exp\left(-\int_0^t \gamma_0 e^{(\alpha_0+\alpha_1 t)z} + \gamma_1 t e^{(\theta_0+\theta_1 t)z} + \gamma_2 t^2 e^{(\omega_0+\omega_1 t)z} du\right) \end{aligned}$$

where $\Lambda(t; z)$ is the cumulative hazard rate function. One of the drawbacks of the ELHR model is the number of parameters of the model. As the number increases it is likely that the accuracy of the estimated parameters decreases which might result in inaccurate reliability prediction at normal operating conditions. This drawback becomes more acute when the failure time data are small.

3 Accelerated Life Testing Plans

A detailed test plan is usually designed before conducting an accelerated life test. The plan requires determination of the type of stress, methods of applying stress, stress levels, the number of units to be tested at each stress level, and an applicable ALT model that relates the failure times at accelerated conditions to those at normal conditions. Of course, a clear objective of the test plan needs to be defined. We begin by the type of stresses followed by methods of stress loading.

3.1 Types of Stresses

In order to determine the type of stresses to be applied in ALT it is important to understand the potential failures of the components and the causes of such failures. This is usually based on engineering knowledge of the component's materials, function, and the stresses that induce such failures. A simplified design of experiments approach is usually conducted to study the effect of the type of stresses by using two levels of each stress (low and high). The high level of stress is the highest level that can be applied without causing a different failure mechanism other than that likely to occur at normal operating conditions. Therefore, a clear understanding of the physics of failure is necessary and testing such as highly accelerated life testing (HALT) is conducted to verify the failure mechanism and the magnitude of the highest stress. HALT subjects the test unit to vibration with random mode of frequency coupled with high temperature and shock in order to induce failures. The failure mechanism is investigated and the stress type and its maximum applied levels are determined accordingly.

In general, the type of applied stresses depends on the intended operating conditions of the product and the potential cause of failure.

We classify the types of stresses as:

1. Mechanical Stresses: *Fatigue* stress is the most commonly used accelerated test for mechanical components. Fatigue is the cause of failures of all rotating mechanical components. When the components are subject to elevated temperature, then *creep* testing (which combines both temperature and static or dynamic loads) should be applied. *Shock* and vibration testing is suitable for components or products subject to such conditions as in the case of bearings, shock absorbers, cell phones, tires, and circuit boards in airplanes and automobiles. Corrosion is another cause of failure of most ferrous material and is induced due to *humidity* and corrosive environment. Units that are subject to corrosion should then be tested using humidity and other corrosive environments as a stress. Wear out is another cause of moving mechanical parts. Depending on the actual use of the unit at normal operating conditions an accelerated test that mimics these conditions needs to be designed but with increased loads to cause significant wear out of the unit.
2. Electrical Stresses: These include power cycling, electric field, current density, and electromigration. Electric field is one of the most common electrical stresses as it induces failures in relatively short times as well as its effect is significantly higher than other types of stresses. Thermal fatigue which is induced by temperature cycling is another major cause of failure of electronic components.
3. Environmental Stresses: Temperature and thermal cycling are commonly used for most products. As stated earlier, it is important to use appropriate stress levels that do not induce different failure mechanisms than those at normal conditions. Humidity is as critical as temperature but its application usually requires a very long time before its effect is noticed. Other environmental stresses include ultraviolet light which affects the strength of elastomers, sulfur dioxide which causes corrosion in circuit boards, salt and fine particles and alpha rays which cause the failure of the read access memory (RAM) and similar components. Likewise, high levels of ionizing can cause electrons in outer orbits to be free which results in electronic noise and signal spikes in digital circuits. Therefore, radiation is an environmental stress that should be applied to the units subject to deployment in space and other similar environments.

3.2 Stress Loadings

Traditionally, ALT is conducted under constant stresses during the entire test duration. The test results are used to extrapolate the product life at normal conditions. In practice, constant-stress tests are easier to carry out but need more test units and a long time at low stress levels to yield sufficient degradation or failure data. However, in many cases the available number of test units and test duration are extremely limited. This has prompted the industry to consider different types of stress loading. Figure 2 shows examples of various stress loadings as well as their adjustable parameters. Some of these stress loadings have been widely utilized in ALT experiments. For instance, static-fatigue tests and cyclic-fatigue tests [23] have been frequently performed on optical fibers to study their reliability; dielectric-breakdown of thermal oxides [18] have been studied under elevated constant electrical fields and temperatures; the lifetime of ceramic components subject to slow crack growth due to stress corrosion have been investigated under cyclic stress by NASA [7]. These stress loadings are selected because of the ease and convenience of statistical analyses and familiarity of the existing analytical tools and industrial routines without following a systematic refinement procedure. Due to tight budgets and time constraints, there is an increasing need to determine the best stress loading in order to shorten the test duration and reduce the total cost while achieving an accurate reliability estimate. In the literature, most research has been focused on the design of optimum test plans when the stress loadings are given. However, until recently, fundamental research on the equivalency of these tests has not yet been investigated in reliability engineering literature. Without the understanding of such equivalency, it is difficult, if not impossible, for a test engineer to determine the best experimental settings before conducting actual ALT.

Furthermore, as is often the case, products are usually exposed to multiple stresses in actual use such as temperature, humidity, electric current, electric field, and various types of shocks and vibration. A typical example is automotive electronics located under the hood, where significant temperature fluctuation, vibration, corrosive gases, and dust contribute to various types of degradation leading to failures, such as cracks in solder joints, loss of connection of connectors, and sensor degradation. It is of interest to know with high confidence what the mileage of normal driving conditions is equivalent to each hour on test under accelerated conditions. Likewise, cellular phones are subject to different environmental conditions, shocks, and vibration. To study the reliability of such products, it is required to subject test units to multiple stresses simultaneously in ALT experiments. For constant-stress tests, it might not be difficult to extend the statistical methods for the design of optimum test plans for single stress to multiple stress scenarios. However, many practical and theoretical issues have to be dealt with when time-varying stresses such as step-stresses are considered. In a multi-stress multi-step test, when and in what order the levels of the stresses should be changed become challenging and unsolved problems. Figure 3 illustrates two example experimental settings out of thousands of choices as one can imagine in conducting a multi-stress multi-step ALT. In general, an arbitrary selection

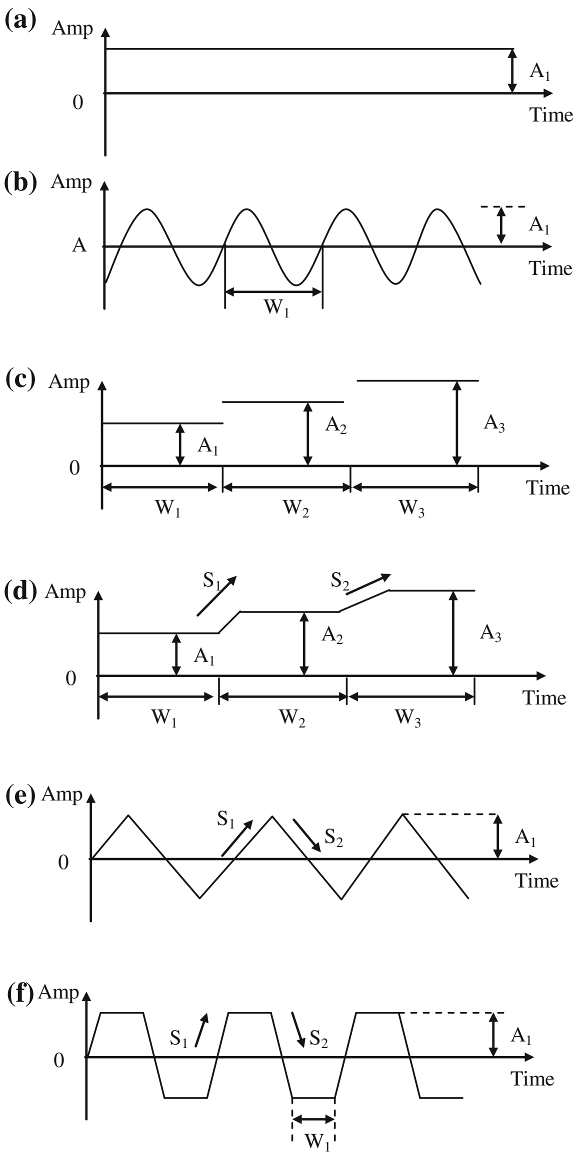

Fig. 2 Various loadings of a single type of stress; the vertical axis shows the amplitude of the applied stress. **a** Constant-stress. **b** Sinusoidal-cyclic-stress. **c** Step-stress. **d** Ramp-step-stress. **e** Triangular-cyclic-stress. **f** Ramp-soak-cyclic-stress

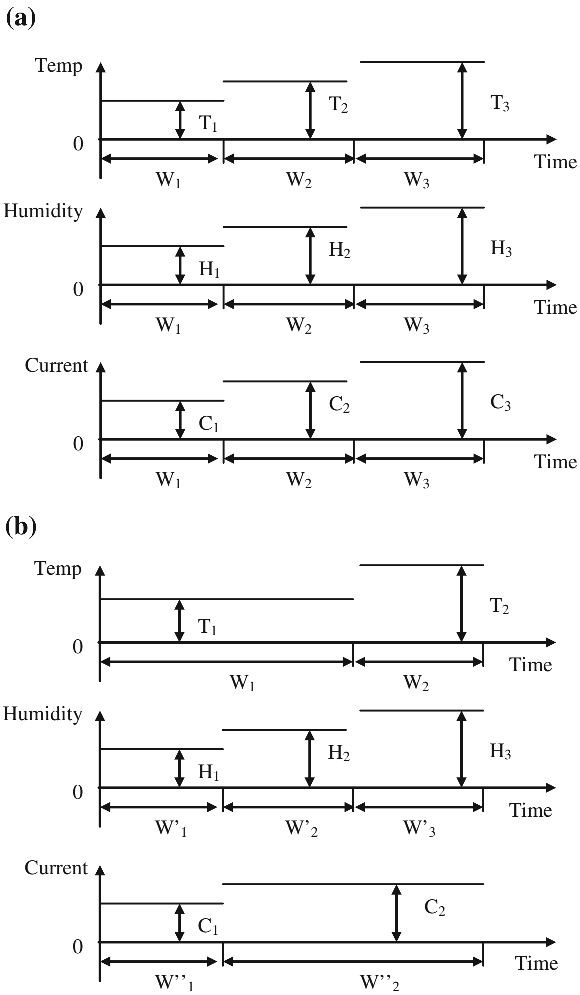

Fig. 3 Two example settings of an ALT involving temperature, humidity, and electric current. **a** Setting 1. **b** Setting 2

from combinations of multiple stress profiles may not result in accurate reliability estimates, especially when the effects of the stresses on the reliability of the product are highly correlated. Therefore, methods for tuning the high-dimensional decision variables under the constraints in time and cost need to be carefully researched and investigated.

3.3 Design of ALT Plans

An ALT plan requires the determination of the type of stress, method of applying stress, stress levels, the number of units to be tested at each stress level, and an applicable ALT model that relates the failure times at accelerated conditions to those at normal conditions.

When designing an *ALT*, we need to address the following issues: (a) Select the stress types to use in the experiment; (b) Determine the stress levels for each stress type selected; and (c) Determine the proportion of devices to be allocated to each stress level Elsayed and Jiao [17] and Elsayed [15]. In this chapter, we present an approach for designing test plans. We refer the reader to Meeker and Escobar [25], Escobar and Meeker [19] and Nelson [28–30] for other approaches for the design of ALT plans.

We consider the selection of the stress level z_i and the proportion of units p_i to allocate for each z_i such that the most accurate reliability estimate at use conditions z_D can be obtained. We consider two types of censoring: Type I censoring involves running each test unit for a prespecified time. The censoring times are fixed and the number of failures is random. Type II censoring involves simultaneously testing units until a prespecified number fails. The censoring time is random while the number of failures is fixed. We define the following notations:

ln	natural logarithm
ML	maximum likelihood
n	total number of test units
z_H, z_M, z_L	high, medium, low stress levels, respectively
z_D	specified design stress
p_1, p_2, p_3	proportion of test units allocated to z_L, z_M and z_L, respectively
T	prespecified period of time over which the reliability estimate is of interest at normal operating conditions
$R(t; z)$	reliability at time t, for given z
$f(t; z)$	*PDF* at time t, for given z
$F(t; z)$	*CDF* at time t, for given z
$\Lambda(t; z)$	cumulative hazard function at time t, for given z
$\lambda_0(t)$	unspecified baseline hazard function at time t

We assume the baseline hazard function $\lambda_0(t)$ to be linear with time:

$$\lambda_0(t) = \gamma_0 + \gamma_1 t$$

Substituting $\lambda_0(t)$ into the PH model described above, we obtain:

$$\lambda(t; z) = (\gamma_0 + \gamma_1 t) \exp(\boldsymbol{\beta} z)$$

We obtain the corresponding cumulative hazard function $\Lambda(t; z)$, and the variance of the hazard function as

$$\Lambda(t;z) = \left(\gamma_0 t + \frac{\gamma_1 t^2}{2}\right) e^{\beta \mathbf{z}}$$

$$Var[(\hat{\gamma}_0 + \hat{\gamma}_1 t)e^{\hat{\beta} Z_D}] = (Var[\hat{\gamma}_0] + Var[\hat{\gamma}_1]t^2)e^{2(\beta z + Var[\hat{\beta}]z^2)} + e^{2\beta z + Var[\hat{\beta}]z^2}(e^{Var[\hat{\beta}]z^2} - 1)(\gamma_0 + \gamma_1 t)^2$$

3.3.1 Formulation of the Test Plan

Under the constraints of available test units, test time, and specification of the minimum number of failures at each stress level, the objective of the problem is to optimally allocate stress levels and test units so that the asymptotic variance of the hazard rate estimate at normal conditions is minimized over a prespecified period of time *T*. If we consider three stress levels, then the optimal decision variables $(z_L^*, z_M^*, p_1^*, p_2^*, p_3^*)$ are obtained by solving the following optimization problem with a nonlinear objective function and both linear and nonlinear constraints [15].

$$\text{Min} \int_0^T Var[(\hat{\gamma}_0 + \hat{\gamma}_1 t)e^{\hat{\beta} z_D}]dt$$

Subject to

$$\underset{\sim}{\sum} = F^{-1}$$

$$0 < p_i < 1, \quad i = 1, 2, 3$$

$$\sum_{i=1}^{3} p_i = 1$$

$$z_D < z_L < z_M < z_H$$

$$np_i \Pr[t \leq \tau | z_i] \geq MNF, \quad i = 1, 2, 3$$

where, *MNF* is the minimum number of failures and $\underset{\sim}{\sum}$ is the inverse of the Fisher's information matrix.

Other objective functions can be formulated which result in different designs of the test plans. These functions include the D-Optimal design that provides efficient estimates of the parameters of the distribution. It allows relatively efficient determination of all quantiles of the population, but the estimates are distribution dependent.

3.3.2 Numerical Example

An accelerated life test is to be conducted at three temperature levels for MOS capacitors in order to estimate its life distribution at a design temperature of 50 °C. The test needs to be completed in 300 h. The total number of items to be placed under test is 200 units. To avoid the introduction of failure mechanisms other than those expected at the design temperature, it has been decided, through engineering judgment, that the testing temperature should not exceed 250 °C. The minimum number of failures for each of the three temperatures is specified as 25. Furthermore, the experiment should provide the most accurate reliability estimate over a 10-year period of time [15].

Consider three stress levels, then the formulation of the objective function and the test constraints follow the same formulation given in the above section. The plan derived that optimizes the objective function and meets the constraints is shown as follows:

$$z_L = 160\,^{\circ}\mathrm{C}, z_M = 190\,^{\circ}\mathrm{C}, z_H = 250\,^{\circ}\mathrm{C}$$

The corresponding allocations of units to each temperature level are:

$$p_1 = 0.5,\ p_2 = 0.4,\ p_3 = 0.1$$

3.3.3 Equivalent Accelerated Life Testing Plans

In design of ALT plans, estimate of one or more reliability characteristics, such as the model parameters, hazard rate, and the mean TTF at certain conditions are common. Accordingly, different optimization criteria might be considered. For instance, if the estimate of the model parameters is the main concern, D-optimality which maximizes the determinant of the Fisher information matrix is considered an appropriate criterion. When estimate of the time to quantile failure is of interest then the variance optimality that minimizes the asymptotic variance of time to quantile failure at normal operating conditions is commonly used. Meanwhile, different methods, e.g., maximum likelihood estimate (MLE) or Bayesian estimator can be used for estimation of the model parameters. However, each method has its inherent statistical properties and efficiencies. In light of this, we discuss equivalent test plans with respect to the same reliability characteristics and optimization criterion then determine equivalent test plans using the same inference procedure. In this chapter, we propose two possible definitions of equivalency as follows:

Definition 1 Two test plans are equivalent if the absolute difference of the objectives for reliability prediction is less than under the same set of constraints on the number of test units, expected number of failures, or total test time.

Definition 2 Two test plans are equivalent if they achieve the same objective for reliability prediction under the same constraints on the number of test units, expected number of failures, or total test time within a margin.

According to the above definitions, the equivalent test plans are not unique. Therefore, we recommend the following procedures for constructing equivalent plans [46].

The first step of the approach is to obtain an optimal baseline test plan. Since constant-stress test is the most commonly conducted ALT in industry and its statistical inference has been extensively investigated, we propose to use an optimal constant-stress plan as a baseline [45].

Suppose an optimal baseline test plan can be determined from the following general formulation:

$$\begin{aligned} &\text{Min} \quad f_B(x) \\ &\text{s.t.} \quad Lb \le x \le Ub \\ &C(x) \le 0, \;\; Ceq(x) = 0 \end{aligned} \tag{8}$$

where $f_B(x)$ is the objective function (e.g., the asymptotic variance of mean TTF) and x is its decision variable which can be expressed as either a vector or a scalar, Lb and Ub are the corresponding lower and upper bounds of x. $C(x) \le 0$ and $Ceq(x) = 0$ are the possible inequality and equality constraints, respectively.

The second step is to determine the equivalent test plan based on Definitions 1 or 2 using formulations (8) or (9), respectively. Formulation (9) is given as follows:

$$\begin{aligned} &\text{Min} \quad \Pi_i(y) \\ &\text{s.t.} \; |f_B(x) - f_E(y)| \le \delta \\ &\Pi_j(x) - \Pi_j(y) = 0 \\ &Lb' \le y \le Ub' \\ &C'(y) \le 0, \;\; Ceq'(y) = 0 \end{aligned} \tag{9}$$

where $f_B(x)$ and $f_E(y)$ are the base and equivalent objective functions on reliability prediction, respectively, and y is the decision variable of the equivalent test plan. $\Pi(\cdot)$ represents the constraint of the total number of test units, expected number of failures, or the test time. When $\Pi_j(y)$ is the total number of test units, $\Pi_i(y)$ can be the censoring time under Type-I censoring or expected number of failures under Type-II censoring and vice versa. The idea is to set the allowed difference between objective values as a constraint as well as seek other merits.

Similarly, based on Definition 2, the optimal equivalent test plan can be determined as,

$$\begin{aligned} &\text{Min } \Pi_i(y) \\ &\text{s.t. } f_B(x) - f_E(y) = 0 \\ &\left|\Pi_j(x) - \Pi_j(y)\right| \le \delta \\ &Lb' \le y \le Ub' \\ &C'(y) \le 0, \\ &Ceq'(y) = 0 \end{aligned} \tag{10}$$

An example that demonstrates these methods and develops equivalent step-stress and ramp-stress test plans and the baseline constant-stress test plan is given in Zhu and Elsayed [46].

4 Accelerated Degradation Testing (ADT)

In this section, we present the concept of degradation, degradation modeling, and the design of accelerated degradation test plans.

4.1 Degradation Models

There are many instances where few or no failures are observed even under accelerated conditions making reliability inference via failure-time analysis significantly inaccurate, if not impossible. However, if a product's performance indices related to failure mechanism experience degradation over time, degradation analysis may be a viable alternative to traditional failure-time analysis and ALT. Indeed, degradation data may provide more reliability information than would be available from traditional censored failure-time data.

In general, degradation testing can be conducted by observing the degradation of the units at normal operating conditions and use appropriate models to predict the reliability of such units. Alternatively, if the degradation rate is "small" then an ADT is conducted instead. Again, an appropriate prediction model is needed to relate degradation data at stress conditions to reliability estimate of the units at normal operating conditions.

Moreover, to save time and cost, ADT experiments are commonly conducted to provide immediate degradation data for predicting the reliability under normal operating conditions. However, in ADT analysis, an inaccurate prediction will result unless an appropriate degradation model and a carefully designed test plan are used.

An appropriate ADT model is the one that accurately accounts for the influences of the stresses (covariates) on the degradation process based on the product's physical properties and the associated probability distributions. Nelson [27] briefly surveys the degradation behavior of various products and materials subject to degradation, ADT models, and inference procedures. He also presents basic accelerated degradation models under constant stress. Meeker and Escobar [25] provide a review of degradation and describe the applications of ADT models. They propose mathematical models to analyze ADT data and suggest methods for estimating failure time distributions, distribution quantiles, and their confidence intervals. A part of the following is based on Liao [21].

Elsayed [14] provides a review of the degradation models and classifies ADT models into two types: physics-statistics-based models and statistics-based model. Furthermore, he classifies statistics-based model into two categories: parametric models and nonparametric/semiparametric models. This classification is summarized as follows.

1. Physics-statistics-based models
   Nelson [26] analyzes the degradation of an insulation material at different stress levels. He assumes that the temperature is the only acceleration factor that determines the degradation profile over time and presents a relationship among the absolute temperature, the median breakdown voltage, and time. He then estimates the lifetime distribution based on the performance degradation model. Based on Carey and Tortorella [4], Carey and Koenig [3] utilize ADT at higher temperature levels to infer the reliability of an integrated logic family, a component of a generation of submarine cables, at normal operating condition. They assume that the maximum propagation time delay (maximum degradation) and the absolute temperature are related by the Arrhenius law. The maximum likelihood estimator is then utilized to estimate the parameters of the Arrhenius relation, which is used for predicting the maximum degradation at normal operating conditions. Whitmore and Schenkelberg [41] model accelerated degradation process by a Brownian motion with a timescale transformation. The model incorporates the Arrhenius law for high stress testing. Inference methods for the model parameters based on ADT data are presented. Meeker et al. [24] use the Arrhenius law to describe the impact of temperature on the rate of a simple first-order chemical reaction and obtain a scale accelerated failure time model (SAFT). Approximate maximum likelihood estimation [33] is used to estimate model parameters. Confidence intervals for time-to-failure distribution are obtained by simulation-based methods. Chang [5] presents a generalized Eyring model to describe the dependence of performance aging on accelerated stresses in a power supply. The tests considered involve multiple measurements in a two-way design. The mean TTF of the power supply at the normal operating condition is estimated. Sometimes, the degradation indices (or rates) can be measured directly or by using surrogate indicators or by conducting destructive testing on the units.
2. Statistics-based models
   Statistics-based models consist of parametric models and nonparametric models. The parametric models assume that the degradation path of a unit follows a specific functional form with random parameters, or the degradation measure follows an assumed distribution with time-dependent parameters. Moreover, these

models assume that there is only a scaling transformation of the degradation paths or the degradation measure distributions at different stress levels but their forms remain unchanged. The nonparametric models relax the assumption about the form of the degradation paths or distribution of degradation and establish them in a nonparametric way. The models have greater flexibility in contrast to the parametric regression models, but they may not have explicit physical meaning.

a. Parametric models
   Based on the degradation paths, Crk [9] extends the methodology of the general degradation path approach to the development of the multivariate, multiple regression analysis of function parameters with respect to applied stresses.
   Tang and Chang [36] model nondestructive accelerated degradation data as a collection of stochastic processes for which the parameters depend on the stress levels. The model adopts the independent increment concept by assuming the incremental degradation within a time interval Δt is *i.i.d* random variable with mean $\mu_i \Delta t$ and variance $\sigma_i^2 \Delta t$. The constants μ_i and σ_i^2 are the parameters under the ith stress level, which are linked with applied stresses by a linear regression approach. The actual degradation path is the summation of these increments, whose first passage time to a threshold level D follows Birnbaum-Saunders distribution when $D \gg \mu_i \Delta t$. If the independent increment is s-normally distributed, then an inverse Gaussian distribution is used as it is a statistically more accurate model as discussed by Bhattacharyya and Fries [1] and Desmond [10].
   Among the approaches of degradation modeling by Brownian motion, Doksum and Hoyland [12] discuss ADT models for the variable-stress case and introduce a flexible class of models based on the concept of accumulated decay. The variable-stresses considered are simple-step-stress, multiple-step-stress, and progressive stress. The proposed model is a time-transformed Brownian motion with drift model, which assumes that certain deterministic stress level imposes the same scaling effect on drift and Brownian motion terms. Pieper et al. [32] propose a different model for the first passage time distribution under simple-step-stress condition. They also discuss an interesting extension that the time change point is random variable. However, the expression for the first passage probability density in this case cannot be obtained in an explicit form.
b. Nonparametric models
   Shiau and Lin [34] present a Nonparametric Regression Accelerated Life-stress (NPRALS) model for some groups of accelerated degradation curves (paths). They assume that various stress levels only influence the degradation rate but not the shape of the degradation curve. An algorithm is proposed to estimate the components of NPRALS such as the acceleration factor. By investigating the relationship between the acceleration factors and the stress levels, the mean TTF estimate of the product under the normal condition is obtained.

The nonparametric regression models bear the degradation-path-free property in contrast to the parametric models. They relax the specification of the form of the degradation path and perform much better than parametric models, if the assumed path function is far from true in the parametric modeling. However, nonparametric models require more data to obtain the same accuracy as that of the parametric models assuming that the parametric models are correct. In other words, the efficiency of nonparametric models is relatively low. Moreover, the time scaling assumption is important since it is required for predicting the form of degradation curve under normal operating conditions, but this assumption is rather weak. Moreover, to utilize the nonparametric regression model, the span of degradation curve under normal condition has to be covered by that of the accelerated degradation data after time scaling, and ADT must be conducted until test units fail.

Another nonparametric/semiparametric approach is to utilize the degradation hazard function. Eghbali [13] proposes an ADT model called proportional degradation hazards model (PDHM) assuming the logarithm of the degradation hazard is a linear function of the stress covariates $\underline{z}$, that is,

$$s(x; t, \underline{z}) = s_0(x; t) \exp(\underline{\beta}' \underline{z})$$

where $s_0(x; t) = g_0(x) q_0(t)$ can be expressed as two positive separable functions $g_0(x)$ and $q_0(t)$ of the degradation measure and the time, respectively. MLEs are utilized to obtain the model parameters. The model is applied to the ADT data of light emitting diode (LED) subject to accelerated temperature and current to predict reliability at normal operating conditions.

4.2 Design of ADT Plans

Design of ADT plans is similar to the design of ALT plans as both require the determination of the stress type, stress level, and allocation of test units to stresses. However, ADT plans require the identification of the degradation indicators, the frequency of measurements (sometimes the degradation can only be assessed via destructive testing). Of course, both ADT and ALT plans require the identification of the decision variables, constraints, and an optimization criterion such as the asymptotic variance of time to failure (TTF) estimate, variance of the reliability estimate, or variance of the estimated 100pth percentile of the lifetime distribution, etc. Although the optimization problem may be feasible, the obtained optimum test plan cannot correct the bias of a degradation model, therefore, a test plan is inappropriate if the degradation model is not accurate. We briefly discuss the common test plans.

4.2.1 Constant–Stress Degradation Test Plans

Boulanger and Escobar [2] present a method to determine the stress levels, sample size at each level, and observation times. However, their method is discussed under a predetermined termination time. Tseng and Yu [39] propose an intuitively appealing method for choosing the time to terminate a degradation test by analyzing the asymptotic convergence property of MTTF estimate but the termination rule is approximate since no constraint has been considered. Park and Yum [31] develop an optimal ADT plan under the assumptions of destructive testing and the simple constant rate relationship between the stress and the product performance. By solving a constrained nonlinear programming problem, the stress levels, the proportion of test units allocated to each stress level, and the inspection times are determined such that the asymptotic variance of the MLE of the MTTF at the normal operating conditions is minimized. Yu and Tseng [44] design an optimal degradation experiment under the constraint of the total experimental cost. They assume the degradation path can be transformed to a simple form. The optimal decision variables, sample size, inspection frequency, and termination time are determined by minimizing the variance of the estimated 100pth percentile of the lifetime distribution. As an application, Yu and Chiao [43] design an optimal degradation experiment for improving LED reliability. Wu and Chang [42] investigate the Nonlinear Mixed-effect Model and propose a step-by-step enumeration algorithm to determine the optimal sample size, inspection frequency, and termination time under the cost constraint. The variance of the estimator of percentile of the failure time distribution is minimized. They also study the sensitivity of the optimal plan to the changes of model parameters and cost. It shows that the optimal solution is slightly sensitive to the changes in the values of model parameters. Recently, Liao and Elsayed [22] propose the *Geometric Brownian Motion Degradation Rate* (GBMDR) model and inference procedure to estimate field reliability for a population and a specific individual unit.

4.2.2 Variable–Stress Degradation Test Plans

Since conducting a constant-stress ADT is costly due to the test duration, it may not be applicable for assessing the lifetime of a newly developed product because typically only a few test units are available. To overcome this difficulty, a variable-stress such as step-stress ADT experiment can be carried out. Tseng and Wen [38] provide an illustration of a statistical inference procedure for a step-stress ADT using a case study of LEDs. However, in the literature, variable-stress degradation test plans are rare. Tang et al. [37] investigates planning of an optimum step-stress ADT experiment where the test stress is increased in steps from a lower stress to a higher stress during the test. Based on the maximum likelihood theory, the asymptotic variance of TTF estimate at the normal operating conditions is then derived and used as a constraint instead of an objective function. The optimum testing plan which minimizes the testing cost gives the optimal sample size, number of inspections at each stress level, and number of total inspections. It is important to note that in such step-stress testing the sequence of load application has a significant impact on the reliability prediction at normal operating conditions, a fact that is rarely considered by researchers.

5 Summary

Reliability prediction of new components, products, and systems is a difficult task due to the lack of well-designed test plans that yield “useful” information during the test and due to the stochastic nature of normal operating conditions. The accuracy of the reliability prediction has a major effect on the warranty cost and repair and maintenance strategies. Therefore, it is important to design efficient test plans. In this chapter, we present an overview of reliability testing with emphasis on accelerated testing and address issues associated with the design of optimal test plans, stress application methods, and reliability prediction models. We further discuss the concept of equivalence of test plans and how it could be used for test time reduction. Finally, we present accelerated degradation modeling and the design of accelerated degradation test plans.

Dedication

This chapter is dedicated to my colleague and friend Dr. Shunji Osaki on his 70th Birthday for his contributions and leadership in the field of Reliability Engineering.

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