Chapter 64
Research of Sub-Pixel Image Registration Based on Local-Phase Correlation

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Abstract A fast and effective sub-pixel image registration algorithm based on image local-phase correlation is presented. Using phase assessment methods, initial shift is performed by calculations. And the maximum cross-correlation value is obtained according to fast Fourier transform (FFT) of the local power spectrum after upsampling on the local area of the image. The experimental results show that the algorithm exhibits high running speed and it can work on common computers; moreover, it can restrain noise well.

Keywords Local-phase correlation • Fast fourier transform • Sub-pixel image registration • Image processing

64.1 Introduction

Image registration is the process of overlaying images (two or more) of the same scene taken at different times, from different viewpoints and/or by different sensors [1]. Image registration is one of the most important steps in image processing, machine vision, and medical imaging. It is the basis of multi-source image analysis, and it has wide applications in the fields of remote sensing, motion estimation, computer vision, medical imaging, image fusion, and splicing as well as image enhancement and restoration [2]. In some cases, there is small portion of image needed to process or recognize. In dealing with remote sensing measurements, medical image recognition, image recognition, etc., the image is difficult

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to meet people's requirements as a result of the complex conditions of image noise and movement of the imaging object [3]. Therefore, in order to get more efficient, faster image registration as well as able to been used in the harsh environment, people have undertaken extensive researches and put forward a variety of image registration algorithms [4]. In general, its applications can be divided into three main groups according to the manner of the image acquisition.

Spatial correlation, the disadvantage of such method is that it cannot be used for the images of different resolutions and features. Transform-based registration method: it has been applied to the image of the same nature and not for heterogeneous image. Feature-based registration method: in such methods, we need to firstly find feature points which are appropriate for the given task and then the two images are registrated according to the feature points [5]. If we use the traditional method for data processing registration, such as fast Fourier transform (FFT), the operator computation would be very large, which not only increases the load on the computer memory, but also reduces the image registration accuracy; especially, it is very demanding on the computer memory. For example, in the process of 1,024 $\times$ 1,024 resolution image, if the registration accuracy of images requirements reaches 0.01 pixel, it needs to store and compute 102,400 $\times$ 102,400 resolution using inverse fast Fourier transform (IFFT). Therefore, it is difficult to implement in the general computer for high-pixel image registration [6].

In this paper, we present an efficient, fast, and accurate registration algorithm based on the local phase of the sub-pixel image. Using the whole image cross-power spectrum, the pixel location is determined, and the sub-pixel precise peak value of phase cross-correlation spectrum is fond using computing matrix Fourier of the sampling the image of the local neighborhood. By virtue of the peak value, we can registrate images precisely. This registration algorithm inherits the frequency characteristics of FFT. In the upsampling process, it omits information unrelated to the template region; thus, the storage requirements and computation are reducing sharply. Therefore, one can obtain higher registration accuracy with our algorithm using cross-correlation spectroscopy; we adopt amplitude normalization operation which is robust under noise and imaging environment.

64.2 Phase Correlation Image Registration Principle

In general, there are three transformation relationship between target image $f(x, y)$ and pending registration image $g(x, y)$: rotation, zooming, and panning [wenx]. After decoupling, three parameters can be estimated; one can obtain rotation and scaling parameters of the two images with resorting to Fourier analysis, and so on. If we convert spatial space of images into frequency domain, we will deduce shifted parameter estimation methods on phase correlation [7].

$$E^2 = \frac{\sum |g(x, y) - f(x, y)|^2}{\sum |f(x, y)|^2} \tag{64.1}$$

According to Ref. [1], we will find the minimum value of E^2:

$$E^2 = \min_{a,x_0 y_0} \frac{\sum_{x,y} |ag(x - x_0, y - y_0) - f(x, y)|^2}{\sum_{x,y} |f(x, y)|^2} = 1 - \frac{\max_{x_0,y_0} \left| r_{fg}(x_0, y_0) \right|^2}{\sum_{x,y} |f(x, y)|^2 \sum_{x,y} |g(x, y)|^2} \tag{64.2}$$

Let $M \times N$ be the size of the image; then, the value of $r_{fg}(x_0, y_0)$ may be shown to be

$$\begin{aligned} r_{fg}(x_0, y_0) &= \sum_{x,y} f(x, y) g^*(x - x_0, y - y_0) \\ &= \sum F(u, v) G^*(u, v) \exp\left[i2\pi (\frac{ux_0}{M} + \frac{vy_0}{N}) \right] \end{aligned} \tag{64.3}$$

It is known as cross-correlation power spectrum; here, $F(u, v)$ is the Fourier transform of $f(x, y)$, and $G^*(u, v)$ denotes the complex conjugate of the Fourier transform of $g(x, y)$. The corresponding translational coordinates are obtained by the maximum of $r_{fg}(x_0, y_0)$.

64.3 Sub-Pixel Registration

Generally, sub-pixel level image registration can be achieved based on calculating the sampling matrix data of the image space of FFT with zero-padding method. However, this method will cause a sharp increase in storage requirements and decrease in computational efficiency. Consequently, the application is greatly restricted. Here, a local-phase technology to achieve high-precision image registration is present. On the basis of initial peak value obtained from the generally phase correlation by FFT, we sample the cross-power spectrum within a small region of the peak value. And we use Fourier transform on the sample matrix above to achieve local-phase correlation. Meanwhile, we refine the peak position in the accuracy of the sampling to achieve sub-pixel registration. This method only omits the information which is nothing to do with the area we concern and reduce the demand for storage space; meanwhile, it improves the operation efficiency and it does not lose any valuable information.

In order to facilitate analysis, we only consider one-dimensional image $f(x, y)$. And $f(x)$ is transformed into $F(u)$ using FFT.

$$F(u) = \sum_{n=0}^{N-1} f(x) e^{-i2\pi xu} \tag{64.4}$$

Here, $n = 0,1,\ldots,N-1$. And the discrete Fourier transform of $f(x, y)$ is given as follows

$$F(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-i2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \tag{64.5}$$

For simplify, we define that

$$E_1 = e^{-\frac{i2\pi ux}{M}}$$
$$E_2 = e^{-\frac{i2\pi yv}{N}} \tag{64.6}$$

The dimension of $f(x, y)$ is $N_A \times M_A$. The dimension of E_1 is $N_B \times N_A$. The dimension of E_2 is $M_A \times N_B$; especially, the Eq. (64.5) become DFT when $M_A = N_B = N$ and $dx \cdot du = 1/N$.

Using discrete Fourier transform, frequency domain sampling step becomes $K/N << 1$ from 1; here, k is the local sampling of scale and a partial sampling is achieved.

During the process of sub-pixel level image registration, we may obtain cross-correlation power spectrum according to Eq. (64.3). Its inverse Fourier transform is the Dirichlet function. The coordinates of the maximum of Dirichlet function are located at $(\overline{x}_0, \overline{y}_0)$, the estimated parameters of the initial translation. Using sample matrix Fourier transform method, we sample on the neighborhood of (u_k, v_k), the neighborhood of the initial shifted parameters. Suppose multiples of the sampling are λ (λ is the sample scale factor). It can be set so as to achieve sub-pixel registration accuracy. Correspondingly, the registration accuracy is 1/λ pixels. Therefore, $N_B = m \times \lambda$. And we will get the local power spectrum.

Through the sampling process, we obtain local power spectrum of $N_B \times N_B$. Its maximum coordinates are offset $(\Delta x_0, \Delta y_0)$ in shifted parameters of the sub-pixel level. After amendment of the initial shifted parameters, we may get estimated parameters. The process is called the local-phase correlation.

The specific steps of the algorithm are as follows:

1. Enter an image as reference image.
2. Shift the reference image $(\Delta x_0, \Delta y_0)$ to get pending registration image $g(x, y)$: The object image and reference image come from the same scene. Thus, their position is relatively fixed.
3. Determine $\left(\Delta x_0', \Delta y_0'\right)$ between $f(x, y)$ and $g(x, y)$ by means of $r_{fg}(x_0, y_0)$:The main function of this step is to locate the pixel level registration point.
4. Give the value of sample factor λ: The size of the sampling factor determines the size of the basis points in the neighborhood which to be toke, the greater its value, higher accuracy while the operation speed is slower.
5. Output the registration image.
6. Calculate the error according to the registration image and reference image.
7. Output time complexity.

64.4 Performance Analysis

Storage and computing scale: According to traditional sampling methods, it have to store 100 M × 100 N points to reach 0.01 registration accuracy (M and N are registration image pixel sizes). While using our method, it only need to store 100 λ × 100 λ points; here, λ is between 1 and 2. As a result, image registration accuracy has been increased and the load on the computer has been decreased.

The complexity of algorithm: The advantage of Fourier transform in sample matrix is that there is no need to pad zero in the cross-power spectrum. We directly sample on a small neighborhood specified by the spectrum. When $\lambda << N$ (N is the smaller value between N and M), the Fourier transform complexity of local sampling matrix is $O(N^2\lambda)$, while that of algorithm is $O(N^2\lambda^2 \log_2 N)$. Thus, the local-phase correlation has improved greatly than the latter.

64.5 Experimental Results and Analysis

In figure lenna, we set phase number 2 and accuracy 0.01, corresponding to the sample scale factor $\lambda = 100$. In this case, we shift the image (5.65, 12.09), (15.658, 10.349), and (7.1658, 10.5345), respectively, in the meantime and plus $f = 0.06$ impulse salt noise to the object image. And then calculate the error between the registration image and the reference one. Images processed, including before and after experiment, are shown in Figs. 64.1, 64.2, and 64.3. The related parameters are shown in Table 64.1.

Fig. 64.1 Reference images (**a**), the object image (**b**), and after shifted (5.65, 12.09) and the registered image (**c**) with $\lambda = 100$

Fig. 64.2 Reference images (**a**), the object image (**b**), and after shifted (15.658, 10.345) and the registered image (**c**) with $\lambda = 100$

Fig. 64.3 Reference images (**a**) with salt noise, the object image (**b**), and after shifted (9.49543, 8.7465) and the registered image (**c**) with $\lambda = 100$

Table 64.1 Comparison results of the estimated parameters of sub-pixel image registration

Phase	λ	Shifted(unit: pixels) ($x0$, $y0$)	Error of registration and reference image (Δx, Δy)	Running time
2	100	(5.65,12.09)	(0,0)	0.5279
		(15.658,10.345)	(0.002,0.005)	0.5221
		(7.1658,10.5345)	(0.0042,0.0045)	0.56
	1,000	(5.65,12.09)	(0,0)	1.2
		(15.658,10.345)	(0,0)	1.21
		(7.1658,10.5345)	(0.0002,0.0005)	1.207
6	100	(5.65,12.09)	(0,0)	0.5551
		(15.658,10.345)	(0.002,0.005)	0.5443
		(7.1658,10.5345)	(0.0042,0.0045)	0.5837
	1,000	(5.65,12.09)	(0,0)	1.2298
		(15.658,10.345)	(0,0)	1.21
		(7.1658,10.5345)	(0.0002,0.0005)	1.24

From the result, we can find that whether the movement unit is 0.01 pixels, the accuracy is also achieving 0.01 pixels, and the error is zero after registration. If moving accuracy exceeds 0.01 pixels, the registration error increases relatively, but its value is also less than 0.01. The increase in phase has little impact on error of phase-based image, but running time is affected slightly (it decreases with increasing phase). Accuracy increases by 10 times, while the running time increases by more than 100 times. Therefore, it fully reflects the characteristics of the running speed, registration error remained unchanged after adding impulse noise which mean that the good robustness of the proposed algorithm.

The comparative values between algorithm and algorithm (with an accuracy of 0.001) referred in reference are shown in Table 64.2.

From Table 64.2, it can be concluded that the proposed algorithm has higher registration accuracy than the algorithm in Ref. [1]. Nonetheless, this algorithm in this paper has certain advantages. So this algorithm has high registration accuracy.

Table 64.2 values of the two algorithms

Shift value(x0, y0)	Algorithm in Ref. [6]	Algorithm in the paper (with an accuracy of 0.001)
(10.7866,20.6167)	(10.79,20.62)	(10.7787,20.617)
(0.4537,0.8961)	(0.45,0.89)	(0.454,0.8962)
(10.6457,10.4356)	(10.64,10.61)	(10.646,10.43599)

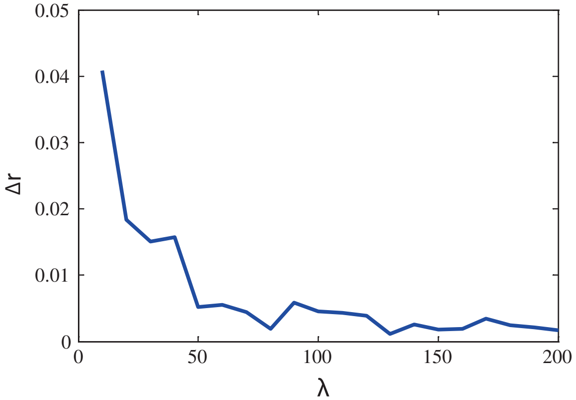

Fig. 64.4 Relationship between the sample scale factor λ and registration error Δr

When the level shifted of image (17.48574, 8.73837) pixels. The registration error is defined as follows (Δr):

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} \tag{64.8}$$

The relationship between sampling scale factor λ and the registration error Δr is shown in Fig. 64.4. It can be seen that the registration error Δr error of image registration decreases with the increasing of sampling scale factor λ. The size of the sampling scale factor is determined by the requirements of image registration. The higher the value of λ, the more the computer's storage space required, and the longer algorithm execution time. Therefore, the efficiency of the algorithm is greatly reduced. Thus, in order to avoid the increase in computer load, we just need to make image registration meet the requirements.

64.6 Conclusion

Image registration is the most popular areas in the image processing and analysis. It has wide applications in medicine, remote sensing, and machine areas. There are many techniques for image registration, but these methods take up space on computer and have high time complexity and long running time; especially, they has high requirements on the imaging environment. They cannot take effective registration on low-quality images caused by noise. Our algorithm is calculated by the smallest root mean square error theory. On the basis of traditional phase shift parameters, we propose a method of local-phase correction to get sub-pixel level image registration.

A large number of experiments show that the proposed algorithm has high registration accuracy, low complexity, and good robustness. Therefore, it can play a good role in practical applications.

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