Chapter 95
Synchronization of Uncertain Chaotic System by Nonlinear Sliding Mode Method

Aijun Zhou, Guang Ren, ChengYong Shao and Yong Liang

Abstract A kind of variable gain nonlinear sliding mode adaptive method is proposed to solve the synchronization problem of chaotic systems with unknown parameters and uncertain functions. The design of nonlinear sliding mode is very skillful. It is not only make sliding surface stable but not make the control easy to be solved. Since with a constant gain, the control system will be not sensitive enough to small signals or it will be unstable to big signals. So the variable gain method is adopted to improve the control accuracy. At last, detailed numerical simulation is done to testify the rightness and effectiveness of the proposed method.

Keywords Nonlinearity · Sliding mode · Chaos · Synchronization · Variable gain · Uncertainty

95.1 Introduction

It is very important to choose a proper gain for a real system. If the gain is not proper, the system will not behave well [1–10]. For example, if the gain is too big, the system will be unstable for the situation of big errors or big input signals. So the stable field will decrease from the state space points of views. But if the gain is

A. Zhou (✉) · G. Ren
Dalian Maritime University, 116026 Dalian, China
e-mail: zajwxy@163.com

C. Shao
Dalian Naval Academy, 116018 Dalian, China

Y. Liang
Naval Aeronautical and Astronautical University, 264001 Yantai, China

too small, the steady state error will be too big and the control accuracy is not enough. So to choose a proper gain not only the dynamic performance but also the robustness should be considered simultaneously.

In this paper, a kind of variable gain method is proposed to solve the above problem and it is integrated with a kind of nonlinear sliding mode synchronization method. The nonlinear integral sliding mode method can make the synchronization error to be small and make the system stable. But it is not easy to choose the proper gain for the nonlinear sliding mode method. So a kind a variable gain is introduced and the advantage is that it can adapt to the input signals automatically. Simulation result shows the effectiveness of the proposed method.

95.2 Problem Description

Take an universal chaos system for an example, the driven system can be described as follow

$$\dot{x} = f_x(x) + F_x(x)\theta_x + \Delta(x,t) \tag{95.1}$$

The response system can be written as

$$\dot{y} = f_y(y) + bu \tag{95.2}$$

The control objective for variable gain nonlinear sliding mode synchronization method is to design a synchronization law $u_i = u_i(k_i, S_i, \hat{d}_{ij})$, $k_i = f_{ki}(S_i, z_i)$ and $\dot{\hat{d}}_{ij} = H(S_i, z_i)$ such that the synchronization can be fulfilled then it has $f_{ki}(S_i, z_i)$, where $f_{ki}(S_i, z_i)$ is a variable nonlinear function and $H(S_i, z_i)$ is the turning law.

95.3 Assumptions

To make the below context more easy to understand, it is necessary to make two assumptions for the chaotic system.

Assumption 1: the response system has the some dimension as the driven system.

Assumption 2: there exists a positive constant d_{ij} such that the nonlinearities of the driven system satisfies

$$\left| f_{yi}(y_1, \ldots, y_4) - f_{xi}(x_1, \ldots, x_4) - \sum_{j=1}^{p_1} F_{xij}(x_1, \ldots, x_4)\theta_{x1j} - \sum_{j=1}^{q_1} \Delta_{xij}(x,t) \right| \tag{95.3}$$
$$\leq d_{i1} + d_{i2}|S_i|$$

where S_i is sliding mode and it is defined as follows.

Since the driven system is a chaotic system, and chaotic systems are bounded, so it is easy to be satisfied by many common chaotic systems.

95.4 Adaptive Nonlinear Sliding Mode with Variable Gain

Define a new error variable as $z_i = y_i - x_i$, for the above driven response system, the error system can written as

$$\dot{z}_i = f_{yi}(y_1, \ldots, y_4) - f_{xi}(x_1, \ldots, x_4) - \sum_{j=1}^{p_1} F_{xij}(x_1, \ldots, x_4)\theta_{x1j} - \sum_{j=1}^{q_1} \Delta_{xij}(x, t) + b_i u_i$$

(95.4)

Design the nonlinear sliding mode surface as

$$S_i = z_i \left[w_{i1} + w_{i2} \left\{ \iint z_i dt dt \right\}^2 \right] + \left(w_{i3} + w_{i4} \left(\int z_i dt \right)^2 \right) \int z_i dt$$ (95.5)

where $w_{ij} > 0$.

Solve the derivative of the above sliding mode surface it holds:

$$\dot{S}_i = \dot{z}_i \left[w_{i1} + w_{i2} \left\{ \iint z_i dt dt \right\}^2 \right] + z_i \left[2w_{i2} \int z_i dt \left\{ \iint z_i dt dt \right\} \right]$$
$$+ \left(w_{i3} + w_{i4} \left(\int z_i dt \right)^2 \right) z_i + 2w_{i4}z_i \int z_i dt \int z_i dt$$

(95.6)

Design the control as

$$u_i = b_i^{-1}(u_{ic} + u_{id})$$ (95.7)

Where

$$u_{ic} = -\frac{1}{\left[w_{i1} + w_{i2}\{\int z_i dt dt\}^2 \right]} \left\{ z_i \left[2w_{i2} \int z_i dt \left\{ \iint z_i dt dt \right\} \right] \right.$$
$$\left. + \left(w_{i3} + w_{i4} \left(\int z_i dt \right)^2 \right) z_i + 2w_{i4}z_i \int z_i dt \int z_i dt \right\}$$

(95.8)

And design u_{id} as:

$$u_{id1} = \frac{-k_{i1}S_i - \hat{d}_{i1} \, \mathrm{sgn}\,(S_i) - \hat{d}_{i2}S_i}{\left[w_{i1} + w_{i2}\{\int z_i dt dt\}^2 \right]}$$ (95.9)

It holds:

$$\dot{S}_i S_i \leq d_{i1}|S_i| + d_{i2}|S_i|^2 - k_{i1}|S_i|^2 - \hat{d}_{i1}|S_i| - \hat{d}_{i2}S_i^2$$ (95.10)

It can be simplified as:

$$\dot{S}_i S_i \leq \tilde{d}_{i1}|S_i| + \tilde{d}_{i2}|S_i|^2 - k_{i1}|S_i|^2 \qquad (95.11)$$

where

$$\tilde{d}_{ij} = d_{ij} - \hat{d}_{ij} \qquad (95.12)$$

Then it holds:

$$\dot{\tilde{d}}_{ij} = -\dot{\hat{d}}_{ij} \qquad (95.13)$$

Design the turning law as

$$\dot{\hat{d}}_{i1} = |S_i|, \quad \dot{\hat{d}}_{i2} = |S_i|^2 \qquad (95.14)$$

Choose the Lyapunov function as

$$V_i = \frac{1}{2}\left(S_i^2 + \tilde{d}_{i1}^2 + \tilde{d}_{i2}^2\right) \qquad (95.15)$$

It is easy to prove that

$$\dot{V}_i \leq 0 \qquad (95.16)$$

So the system is stable. Based on the above method, a kind of variable gain method is designed to improve the dynamic performance and reduce the system gain. Design

$$k_{i1} = 20\frac{|z_i| + \varepsilon_{i1}}{|z_i| + \varepsilon_{i2}} \qquad (95.17)$$

To make it easy to understand, take a example as $\varepsilon_{i1} = 0.2$, $\varepsilon_{i2} = 0.005$. If $|z_i| \geq 0.2$, it holds
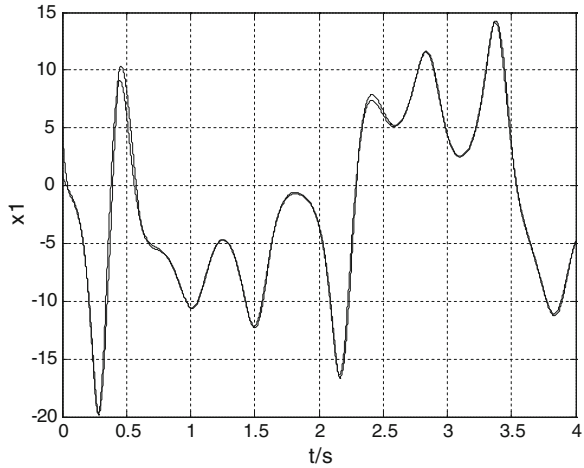
$$k_{i1} = 20\frac{|z_i| + \varepsilon_{i1}}{|z_i| + \varepsilon_{i2}} \rightarrow 20 \qquad (95.18)$$

If $|z_i| \rightarrow 0$, it holds

$$k_{i1} = 20\frac{|z_i| + \varepsilon_{i1}}{|z_i| + \varepsilon_{i2}} \rightarrow 800 \qquad (95.19)$$

It is obvious that the gain of the whole system will be very big when the input signal is small but the gain will decrease very quick if the input signal is very big.

Fig. 95.1 States x1 and y1

95.5 Numerical Simulation

Take a three dimension chaotic system as an example, the driven system can be written as

$$\dot{x}_1 = a(x_2 - x_1) + k_{lb}x_3 \cos x_2 \tag{95.20}$$

$$\dot{x}_2 = bx_1 + cx_2 - x_1x_3 + k_{lb}x_3 \cos x_2 \tag{95.21}$$

$$\dot{x}_3 = x_2^2 - hx_3 + k_{lb}(1 + \sin(x_2x_3))x_2 \tag{95.22}$$

where choose $a = 20, b = 14, c = 10.6, h = 2.8, k_{lb} = 0$ and the system has an attractor. a, b, c, h are unknown constants, k_{lb} items are unknown functions. The structure of response system is known, which can be expanded as:

$$\dot{y}_1 = a_y(y_2 - y_1) + u_1 \tag{95.23}$$

$$\dot{y}_2 = b_yy_1 - k_yy_1y_3 + u_2 \tag{95.24}$$

$$\dot{y}_3 = -c_yy_3 + h_yy_1^2 + u_3 \tag{95.25}$$

Choose parameters as $(a_y, b_y, c_y, k_y, h_y) = (10, 40, 2.5, 1, 4)$, and the initial state of response system can be set as $(y_1, y_2, y_3) = (1, -1, 2)$. The simulation result can be seen in below figures.

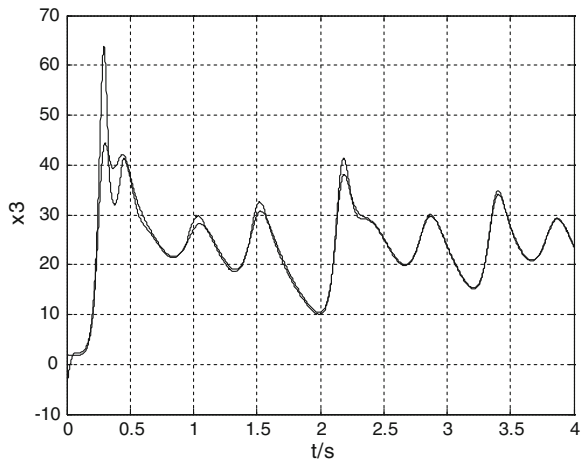Figure 95.1 shows the synchronization of response system state y1 and driven system state x1.

Figure 95.2 shows the synchronization of response system state y2 and driven system state x2.

Figure 95.3 shows the synchronization of response system state y3 and driven system state x3.

Fig. 95.2 States x2 and y2
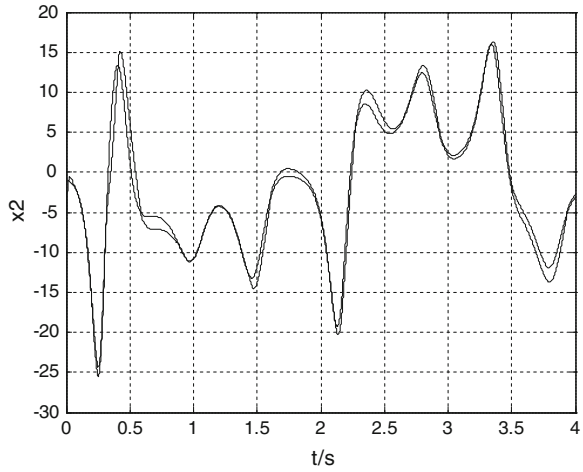
Fig. 95.3 States x3 and y3

With the proposed variable gain integral sliding mode synchronization method, it is obvious that the synchronization can be fulfilled but the synchronization error cannot be converged to zero that is mainly because of the disturbance of unknown nonlinear functions. With high gain controller design, the synchronization error can be decreased but it cannot be totally cancelled.

95.6 Conclusions

A kind of nonlinear integral sliding mode synchronization strategy is proposed with variable gain, which cannot only make the system to be sensitive to small signals but also can make the system to be stable with big input signals. Simulation

results shows that the proposed method is effective and synchronization can be fulfilled but the synchronization error cannot be converged to zero because of the existence of unknown nonlinear functions.

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