Chapter 4
A Joint Energy and Transmission Rights Auction on a Network with Nonlinear Constraints: Design, Pricing and Revenue Adequacy

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4.1 Introduction

The forward and real-time (spot) auction markets operated by independent system operators (ISOs) allow for trade in multiple wholesale electricity products, differentiated by time and location on the transmission network.[1] This chapter

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[1] In the United States, there are two types of independent system operators established under federal jurisdiction – Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs). RTOs have additional geographical requirements compared to the original ISOs, such as encompassing a larger multi-state region, as well as some functional differences, such as regional transmission planning. However, wholesale market design is not differentiated between the two types of organizations. Since ISO is a more generic term, we will use this term to refer to both types of organization in the remainder of the chapter. In the U.S., ISOs and RTOs include the California ISO, ERCOT (encompassing most of Texas, and not subject to federal jurisdiction), PJM RTO, the Midwest ISO (MISO), New York ISO, ISO New England, and the Southwest Power Pool (SPP). For a survey of the designs of some of these markets in the United States, see O'Neill et al. (2006). Each of the U.S. ISOs and RTOs also has a website with extensive documentation of market rules and procedures as well as data on market outcomes. We refer to some of these below.

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presents a general auction model that implements key features of the ISO markets, including definition of several market products, the rules for joint auctioning of the products in a sequence of forward and spot markets, the rules for financial settlement of those products, and the requirements to ensure revenue adequacy of the auctioneer. The model formulation is focused on a joint energy and transmission rights auction (JETRA; henceforth, the 'auction model' or 'auction'), along with a non-linear representation of the transmission network constraints. However, the formulation can be extended, in some cases with modification, to other market products. Our earlier paper (O'Neill et al. 2002) explored properties of this auction with linear transmission constraints.

At its inception, this auction model informed deliberations at the U.S. Federal Energy Regulatory Commission (FERC) in the early 2000s over a possible standard market design tariff for the wholesale power markets under its jurisdiction. A key objective at the time was to establish a framework for introducing a more complete set of financial transmission rights for the ISOs, including both point-to-point rights and "flowgate" rights, then considered to be mutually exclusive designs (see, e.g., Chao et al. 2000; Hogan 2000). Subsequently, political factors made it impossible for FERC to require implementation of a standardized wholesale market design.[2] Nevertheless, individual U.S. ISO market designs have since converged on certain products and pricing rules represented in our model formulation, such as point-to-point financial transmission rights and day-ahead and real-time markets with locational marginal pricing (LMP) of energy incorporating marginal congestion and loss charges. Other products discussed below have, however, not yet been introduced, such as forward locational energy sales integrated with the transmission rights auctions, and flowgate rights.

Despite this progress, the wholesale market design process has not been completed in the U.S., and there are almost continuous efforts at each ISO to introduce new products and pricing rules – some standardized across the ISOs, some not. This process advances market completeness by expanding the set of products and prices to a fuller range of the services provided by generation, non-generation,[3] and transmission assets, as is required for economic efficiency, especially under changing market and system conditions (such as integration of variable renewable generation). As some of these possible new market products, such as a reactive power product, require representation of non-linear transmission network constraints, whether for forward sales or real-time settlement purposes, our model continues to be applicable to the evolution of U.S. ISO market designs as well as regulatory reforms in other countries. At the same time, our illustrative extension to new products does not necessarily reflect an endorsement: as the history of market design in the U.S. has shown, for any specific ISO, the

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[2] The standard market design tariff was proposed by FERC in 2002, but failed to achieve sufficient political support in certain regions to be implemented in its original form.

[3] "Non-generation resources" is the term adopted by FERC to refer to demand response, storage and other non-generation resources that may provide market services.

determination of the products for inclusion should be left to the market participants based on their needs and preferences as well as the physical characteristics of the regional power system (as well as being subject to approval by FERC or state regulators).

In deciding whether or not to adopt a more elaborate market design, such as that proposed in this chapter, the market operator, stakeholders, and regulators have to balance several criteria. Two of them are emphasized in this chapter, and motivate our design: efficiency in the allocation and trading off of various market products (in our case here, forward energy and transmission rights), given what bidders say they are willing to pay for them; and revenue adequacy for the market operator. Others that are relevant include: incentive compatibility (the extent to which an auction design encourages bidders to reveal their true valuations and costs in their bids); complexity and cost of implementation relative to anticipated benefits to the market; transparency; and perceived fairness (definable in several ways).

4.1.1 General Features of the Forward and Spot Market Designs

We refer to forward markets[4] as any ISO market that clears prior to the ISO's physical dispatch, or real-time, market. As a general matter, offers and bids that clear forward markets are financially but not physically binding,[5] whereas those that clear the real-time auctions are treated as physical commitments that typically must follow the system operator's instructions or be subject to warnings or financial penalties.

In practice, ISOs hold forward markets on a variety of time-frames that reflect operating requirements and constraints, market needs or simply utility/regulatory conventions. The basic market sequence is characterized in Table 4.1. The types of market products shown are not offered uniformly in all ISOs (for example, only one ISO provides pre-day-ahead forward reserves); we provide further detail on product definition in the next section, but focus in this section on a general description of the market sequence and the features reflected in our auction formulation.

The number and timing of forward markets in JETRA is a market design decision that needs to reflect the conditions that pertain in the market and stakeholder preferences. The minimal requirement of the ISO is that it run a real-time market; it is possible to provide all forward products through formal or informal markets operated by other parties. However, non-ISO operated markets that do not clear using

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[4] We only consider ISO forward auction markets here, not any off-ISO bilateral power exchanges that can also operate in forward time-frames and in the same geographical territory. The existence of ISO auctions does not preclude operation of secondary non-ISO forward markets for transmission rights or bilateral energy transactions. In the U.S., ISO and non-ISO markets are generally regulated under a just and reasonable standard originating and under a fraud and abuse standard in the Federal Power Act.

[5] The exception to this rule is sales of forward capacity that create performance obligations in real-time.

Table 4.1 Characterization of existing forward and spot U.S. ISO markets

	Time-frame	Auction periodicity	Financial settlement interval	Types of market products
Spot markets (physical)	Real-time	Hourly	5–60 min	Energy (physical only)
Forward markets (financial)	Hour-ahead	Hourly	1 h	Energy, operating reserves
	Day-ahead	Daily (24 h)	1 h for energy Daily for 'make whole' payments	Energy, operating reserves, residual capacity
	Pre-day-ahead	Semi-annually or annually	Months, possibly differentiated by time of day	Operating reserves, financial transmission rights, capacity

a good representation of the network and the full dispatch run a significant risk of infeasible trades. This has happened in the CalPX in the early days of the California market and several European exchanges. For revenue adequacy, FTRs require assumptions about the network configuration, but if non-ISO markets only trade simple flowgates, then they can avoid the need to make such assumptions. However, the downside of only selling simple flowgates is that the rights holder is not guaranteed a perfect hedge for a bilateral power contract between two points. A distinct advantage of a central forward market operated by the ISO is that it is in the best position to incorporate network constraints along with the rest of the generation, load, and net imports. The other advantage is that the ISO can back FTR payments with congestion revenues, which an independent party cannot.

Pre-day-ahead markets. The pre-day-ahead ISO markets have conventionally been used to transact products denominated in time-periods of months or multiple months, such as financial transmission rights and capacity. Some ISOs have used such markets to procure forward operating reserves. In the auction design we propose in Sect. 4.3, the mathematical formulation explicitly represents only energy and financial transmission rights for pre-day-ahead auctions. A key generalization of the model has been to accommodate the joint auction of products that were previously advocated as mutually exclusive market designs, intended to support different visions of how forward market institutions should develop. Specifically, the auction model in (O'Neill et al. 2002) – and the analogous one presented here for the nonlinear case – synthesize and extend several prior auction models to allow for the simultaneous auction of flowgate, or flow-based, transmission rights and point-to-point transmission rights specified as options or obligations (Chao and Peck 1996; Harvey et al. 1997; Hogan 2000, 2002), in addition to real energy and possibly other products.[6]

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[6] The debate over the implementation of alternative transmission rights formulations is recounted in Hogan (2000, 2002) and O'Neill et al. (2002), among other sources, and will not be repeated here.

Pre-day-ahead energy transactions, whether in separate energy-only auctions[7] or in joint auctions with financial transmission rights, have not been introduced explicitly by any of the U.S. ISOs. However, conceptually, energy commitments could enter the pre-day-ahead auctions for transmission rights where the energy may be needed to make up losses on the transmission network, or to supply inertia or reactive power that are not explicitly modelled to create additional transmission capacity into transmission constrained areas. An example of the latter is the "San Francisco nomogram" constraint discussed in (O'Neill et al. 2002). Our introduction of energy into the forward transmission rights auction is a generalization of these applications.

Day-ahead and real-time markets. In day-ahead and real-time auction markets, products include real energy priced and settled using LMPs, and regulation and operating reserves settled at system-wide or zonal prices. With respect to day-ahead energy markets, ISOs typically use a two-phase day-ahead market clearing, in which first both physical[8] and financial (or virtual) bids are accepted and day-ahead prices determined, and second, a reliability unit commitment is conducted using bids associated with physical generation only and forecast load. Financial demand and supply bids[9] have some unique properties in that they are not associated with physical energy supply or demand, or physical transmission capacity. They can be used for financial hedging and are permitted, in part, in the forward markets to counter market power and to aid in producing better price convergence (on average) between and among the forward markets and the real-time market.

Simultaneously, transmission users are charged for marginal transmission costs (congestion and possibly losses) and congestion revenues are used to settle the financial transmission rights awarded in the pre-day-ahead auctions. Generally, these settlements take place using day-ahead market LMPs, unless the ISO only operates a real-time market, in which case they are settled against real-time LMPs. Point-to-point transmission rights are settled based on the differences in the LMP congestion components between their injection and withdrawal points, while if the ISO offered them, the flowgate rights would be settled using transmission shadow prices (called flowgate marginal prices in FERC 2002).[10] Settlement rules are defined precisely in the next section.

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[7] For example, some ISOs have evaluated additional energy auctions prior to the day-ahead auction, but not integrated with other products.

[8] That is, bids backed by physical assets. Selection in the day-ahead auction market does not require that the seller of the physical asset deliver in real-time; the seller still has the option to not perform and sell or buy back its position in real-time. The incentive to perform is thus primarily financial. In contrast, in real-time, failure to perform as instructed may result in administrative penalties.

[9] In this chapter we will use the term 'bid' at times to include either a bid or offer.

[10] While ISOs do not offer flowgate rights through auctions, there are a number of applications of flow-based capacity reservations that are used by the ISOs and affect energy prices in real-time. For example, currently, ISOs exchange flowgate capacity with their neighbors through Joint Operating Agreements to feasibly and optimally allocate loop flow.

Finally, the real-time markets begin at midnight of the operating day, clear every 5–10 min, and settle every 5–60 min.[11] In these markets, only physical bids are allowed, subject to performance requirements, and all financial positions are re-settled. Forward markets close in time to real-time, such as markets held one or more hours before the operating hour, are more "physical" in nature than financial, although the ISO has less time to recover from failure to perform than in the day-ahead market, where it has time to conduct reliability commitments and procure additional reserves.

In all these markets, various additional rules have been established to prevent market power and market manipulation by entities that also hold other property rights (including physical transmission scheduling), and appropriate creditworthiness rules are required for all cleared bids.

The actual timing of the sequence of ISO market clearing for the various market products is due to a mix of factors, including scheduling conventions inherited from predecessor utilities, regional system operators (e.g., power pools) and reliability organizations, market design decisions and computational constraints at the ISOs, and the interests of the market participants as new market designs were developed. Unfortunately, the timing of the sequence has tended to differ among ISOs, including contiguous ones, resulting in "seams" issues, some of which have been resolved over time through improved coordination (see, e.g., O'Neill et al. 2006).

4.1.2 Auctions with Non-linear Transmission Network Constraints

To formalize and generalize the design of these forward and real-time markets, the authors first introduced a multi-settlement, joint energy and transmission rights auction on a network characterized by an approximate linearized 'dc' load flow model (O'Neill et al. 2002, 2003) (for a derivation of the dc load flow approximation, see, e.g., Schweppe et al. 1988). In order to simplify auction clearing and financial settlements, linear network constraints are used in all U.S. ISO markets. For example, forward auctions for obligation and option point-to-point Financial Transmission Rights (FTRs) in PJM employ a dc load flow model.[12] As noted, some market operators create additional linear 'nomogram' constraints or 'cuts', often proxies for voltage limits, to ensure feasibility of the underlying physical system. According to our communications with software developers, the more general linear model in O'Neill et al. (2002) has been a basis of the development of the recently implemented transmission rights markets for the ISOs in ERCOT (Texas) and California in the U.S..

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[11] That is, some ISOs financially settle on a 5–10 min basis, while others settle on the basis of an hourly integrated price.

[12] www.pjm.com/markets/ftr

But other ISOs have implemented auctions with non-linear transmission constraints. In the New York ISO, the obligation point-to-point financial transmission rights are called Transmission Congestion Contracts (TCCs). In contrast to PJM, the auction is conducted using an approximate AC optimal power flow model that respects thermal, voltage and stability constraints within the New York control area.[13] There are other market products that would require consideration of non-linear constraints. The inclusion of reactive power in the auction market would also require the AC load flow model (FERC 2005; Hogan 1993; Kahn and Baldick 1994; O'Neill et al. 2008) or a linear or quadratic approximation to the model. Moreover, proposals for forward hedging of marginal losses through unbalanced point-to-point transmission rights would require auctions with a dc load flow model and quadratic losses (Harvey and Hogan 2002). This chapter thus generalizes the linear auction model in O'Neill et al. (2002) to the case with nonlinear constraints.

Whatever the final set of products, a key goal of the market design is to ensure the revenue adequacy of the auctions, which means that the ISO collects sufficient revenues to cover payment obligations. A theoretical result presented here is that for the auction with nonlinear transmission constraints that define a convex feasible region, the forward and spot auction sequence can be revenue adequate (the analogous proof for the linear case is shown by O'Neill et al. 2002). However, as with any transmission rights auction, additional rules are needed to account for revenue inadequacy due to changes in system topology. While we show the formal conditions for revenue adequacy, we do not explore in detail how market participants are affected financially when there is a shortfall. There are currently different rules for dealing with shortfalls. For example, in PJM, revenue inadequacy of FTRs is addressed by prorating the shortfall among the FTR holders. In NYISO, revenue inadequacy of TCC holders is covered by the transmission owners to provide incentives for efficient timing of transmission maintenance.

4.1.3 Additional Extensions of the General Auction Model

In each step of the sequence of auctions, our general model framework can be extended to include additional products,[14] pricing rules, settlements, or linkages with auctions for other wholesale market products. Some of these extensions are discussed in the subsequent sections, but we summarize several others here.

For example, some ISOs have established forward capacity (MW) auction markets to satisfy annual or multi-year local area and system-wide planning reserve margins (or resource adequacy requirements). These forward markets pay a locational clearing price for capacity, which in some designs is set by an administrative

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[13] www.nyiso.com

[14] Including those, such as generator start-up, that requires mixed integer programming formulations, as discussed in Sect. 4.4.

demand curve. The network models are also zonal rather than nodal, another difference with our model formulation as presented here. One linkage between the capacity auctions and the model of this chapter is that, as a general rule, offers that clear the capacity auction then have an offer obligation in the ISO day-ahead markets, making the capacity payment equivalent to the ISO buying a call option (on behalf of the load-serving entities that have the capacity obligation) on energy that pays the LMP when exercised. Hence, the model presented here can be viewed as a framework for final settlement of the energy call option associated with capacity rights.

Closer to actual operations, the sequential auction market design can be implemented with additional settlements between day-ahead and real-time energy markets in order to better accommodate variable energy generation by renewable sources, such as wind and solar generation, whose production forecast uncertainty decreases as the real-time market approaches. A sequence of auction markets, for example, occurring every six hours with rolling horizons, might allow for more efficient adjustments as the uncertainty decreases.

The remainder of the chapter is organized as follows. Section 4.2 offers a description of the types of energy and transmission right bids in the auction. Section 4.3 presents the mathematical statement of the auction model with nonlinear transmission constraints, and provides more mathematical detail on how transmission rights are specified for the auction. Section 4.4 discusses the settlement system and conditions for maintaining revenue adequacy. Section 4.5 provides an example based on a dc load flow with quadratic losses. Section 4.6 offers conclusions. An appendix presents the proof of revenue adequacy for a sequence of forward and real-time market auctions with 'expanding' transmission constraints that define a convex feasible region.

4.2 Auction Products

We now turn to the set of energy and transmission rights products modeled in the auction design, a subset of those discussed above. The types of electricity products that can be traded in the auction mechanism proposed in this chapter have been described by Baldick et al. (2005), Chao and Peck (1996), Chao et al. (2000), Harvey et al. (1997), and O'Neill et al. (2002, 2003, 2006). This section provides further qualitative description of these products, while the next section introduces our model's notation.

Energy. Several types of bids are typically allowed in energy and transmission auctions: supply offers, demand bids, financial bids, and transmission bids. Point-to-point transmission bids represent what a bilateral energy transaction is willing to pay for marginal congestion charges (and possibly losses) associated with its transmission schedule. If both the points are inside the ISO, the product is financial. Physical point-to-point bids are typically used on the boundaries of ISO systems where there is no fully arbitraged LMP on the "other side" of the boundary, which is often called a proxy bus or interface. The model presented here can accommodate each of these types of bids. For purposes of this discussion, some important aspects of energy auctions are not considered, such as the inclusion of unit commitment start-up and no-load costs, restrictions on bids to control the exercise of market power[15] and changes in network topology.[16]

Currently, energy offers (to sell) and bids (to buy) have only been allowed in the day-ahead and real-time markets. In a pre-day-ahead ISO auction market for energy and transmission, as discussed above, energy transactions could be used also to balance point-to-point transmission rights in a lossy system, or to increase transmission capacity for forward sale. These one-sided or unbalanced "rights" (actually, obligations) can be called "nodal revenue rights."

Simple Transmission Capacity Rights and Portfolio Combinations. As noted in the flowgate or flow-based rights literature (e.g., Chao and Peck 1996; Chao et al. 2000), there are two types of elementary transmission rights, which we call here the "simple rent collection right" and the "simple rent payment right." The simple rights are defined over single transmission elements, which include lines, transformers, other transmission elements or collections of transmission elements whose capacity is limited by exogenous thermal, stability, or contingency considerations. Such rights are often generically called "flowgate" rights (FERC 2002). For each element, the direction of the flows covered by the simple rights is defined separately and arbitrarily, in either a positive or negative direction. The simple rent collection right on a transmission element confers to the buyer the right to collect the rents that would occur when that element is congested, for the capacity specified in the right. Because the flow-based right is directional, the holder of a rent collection right only collects non-negative rents.

The simple rent payment right obliges the seller to pay any rents on a transmission element, for the capacity specified in the right. The rent payment right allows a market participant to create or consume financial capacity on a specific transmission element. Moreover, if the ISO did not itself allocate rights, but simply facilitated an auction of buyers and sellers (see Sect. 4.3), then all transmission owners could offer physical transmission rights. The simple rights can be aggregated into more complex rights through linear combinations or portfolios, for example, covering several transmission lines, nomograms, or constructing "point-to-point" rights on the basis of power flow distribution factors (O'Neill et al. 2002).

The combination of buying a rent collection rights on some transmission element and selling rent payment rights on other transmission element creates portfolio of

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[15] Bid restrictions for market power reasons can include a uniform, "safety net" bid cap for all generators, bid thresholds on generators that trigger market power mitigation, a requirement to bid approximate marginal costs, and other measures.

[16] Network topology changes can be either purposeful, to increase market surplus, or due to planned outages, such as maintenance, or to unplanned outages. Topology changes to increase market surplus, called optimal transmission switching, can ironically cause revenue inadequacy in the point-to-point transmission rights settlements. Corrective switching to stabilize or re-optimize the system can follow unplanned outages.

flowgate rights. For a set of simple rights that constructs a point-to-point right, holding this portfolio on each transmission element in the set is analogous in the linear dc JETRA model to the point-to-point obligation rights with a constant topology. In general, however, the individual rights and the portfolios are more likely to offer an imperfect rather than a perfect hedge against congestion charges associated with an energy transaction. Since an exact match between a particular point-to-point transaction and a portfolio of the rights would be difficult to create and maintain (although some authors propose that the ISO provide subsidies to maintain particular portfolios as complete hedges, for example, Chao et al. 2000). A transmission right that offers a perfect, or complete, congestion hedge is defined as one in which the congestion charges associated with real-time market transactions are equal to the congestion revenues obtained by the rights holder. An imperfect hedge is one in which the congestion charges are not equal to the revenues to transmission rights holders. For many holders, then, the flowgate right will be used to collect rents on heavily congested transmission elements rather than to hedge any particular power transaction.

Flowgate rights can be made available or withdrawn in the real-time market due to forced outages, the use of short-term ratings instead of steady-state ratings or unanticipated changes in weather. For example, changes in ambient temperature and wind speed can change the transmission line's carrying capacity.

Point-to-Point Transmission Rights. There are two types of point-to-point rights, the obligation right and the option right. An obligation right is more accurately described as a "contract" (Harvey et al. 1997), since it embodies an obligation to pay congestion revenues, but is now conventionally termed a financial transmission right. A point-to-point obligation transmission right is defined as the right to receive a payment or the obligation to pay the congestion charge rents that result from the physical flows associated with putting power into the system at a point of injection (POI) and taking power out of the system at a point of withdrawal (POW) (Harvey et al. 1997). Note that for a point-to-point obligation, flow in one direction adds an equivalent amount of "counterflow capacity" in the other direction. This can be generalized to multiple point-to-multiple point rights, which we will call network rights. These rights may simply aggregate point-to-point rights or may be "contingent" rights, when they hedge multiple possible POIs and POWs (discussed in O'Neill et al. 2002). The point-to-point obligation transmission right is equivalent to the forward transmission congestion contracts (TCCs) described in Harvey et al. (1997). The network rights were described in FERC's proposed capacity reservation tariff (FERC 1996).

The amount that is received (or paid, if negative) by the holder of the obligation right is the nodal price at the POW minus the nodal price at the POI multiplied by the quantity specified in the right. (A variant implemented at some ISOs pays only the difference in the congestion portion of the LMP price and not the loss component.) If the injections and withdrawals of power specified in the right are scheduled in the market in which the right is settled (and then executed in the real-time market, if different from the settlement market), then the right provides a complete congestion hedge, i.e., no additional payment for congestion will be necessary.

The point-to-point option transmission right is defined as the option to put power into the system at one or more POIs and take power out of the system at one or more POWs. The option TCCs discussed by Harvey et al. (1997) are similar to these point-to-point option rights in the linearized dc load flow model (O'Neill et al. 2002). It can be interpreted as the right to collect congestion rents if they exceed zero, without the obligation to pay that amount if negative. (I.e., they are options with a strike price of zero.) This option faces considerable computational challenges in an auction model with nonlinear transmission constraints, in that a separate load flow has to be calculated for each combination of possible exercised options (Hogan 2002). However, using a linearized dc load flow approximation model, the computation can be reduced sufficiently, thus facilitating the implementation of point-to-point options (alternatively, portfolios of flowgate rights could be used to approximate a point-to-point option right). In an auction with linear constraints, the point-to-point option is shown to be equivalent to setting aside capacity in each transmission constraint for positive increments of flow associated with the right but ignoring negative flows ("counterflow") in the opposite direction (e.g., O'Neill et al. 2002). This allows the auction to be run using a single set of power flow distribution factors (PTDFs), but no analogous reduction has been developed for the nonlinear case. Moreover, as we showed previously (O'Neill et al. 2002), the reduction in the linear case implies that an appropriately defined bundle of flowgate rights dominates the point-to-point option in the sense that there exists such a bundle whose cost is the same as the option right but which will pay off at least as much as an option right and, under some possible outcomes, it will pay strictly more. Although a point-to-point option has been included in some ISO markets, it has been excluded in others for various reasons. These include the fact that such rights would excessively diminish the available rights in locations where there are physical set-asides to honor prior physical transmission scheduling rights; a lack of stakeholder interest in such options as a hedging instrument; and to the software development costs and computational requirements of its implementation.

Point-to-point rights can be balanced or not balanced. A balanced right is one in which the quantity injected is equal to the quantity withdrawn. An unbalanced right does not have this requirement, so that an entity can approximate losses (average or marginal) by specifying a higher quantity injected than withdrawn.

Finally, as with the flowgate right, point-to-point rights can be bought from or sold into the auction.

4.3 The Auction with Nonlinear Constraints

4.3.1 Mathematical Statement

The types of energy bids and transmission rights described in Sect. 4.2 are represented in the mathematical statement of the auction model with non-linear constraints, JETRA-NL, below with more detail in Sect. 4.3.2. For ease of recognition, the notation used in the model borrows and extends from standard references, such as Chao and Peck (1996) and Harvey et al. (1997). All variables are assumed to be real power; however, the framework allows for the inclusion of reactive power (VARs). Units of the decision variables and right hand sides (RHS) of the constraints are in megawatts (MW or MWh/hour), while the objective function coefficients are in \$/MWh.

The JETRA-NL model is formally stated below. In brief, the formulation maximizes the net economic value (4.1) of accepted energy and transmission bids subject to definition of the net injection at each bus (4.2), inequality constraints upon injections and flows (4.3, 4.4, and 4.5) (whose capacity can be sold as rights), load flow constraints (4.6), upper bounds on transmission and energy rights (4.7, 4.8, 4.9, and 4.10), and nonnegativity restrictions.

$$\textit{JETRA-NL: max } v(t^F,\ t^P,\ g,\ x,\ y,\ f^+,\ f^-) = b^F t^F + b^P t^P + c^+ g^+ + c^- g^- \tag{4.1}$$

$$A^P t^P + A^+ g^+ + A^- g^- - y = 0 \qquad (\pi) \tag{4.2}$$

$$B^N t^F + K'(\mathrm{x},\ y,\ f) \le F^N \qquad (\mu^N) \tag{4.3}$$

$$B^+ t^F + f^+ \le\ F^+ \qquad (\mu^+) \tag{4.4}$$

$$B^- t^F + f^- \le F^- \qquad (\mu^-) \tag{4.5}$$

$$K''(x, y) - f^+ + f^- = 0 \qquad (\gamma) \tag{4.6}$$

$$t^F \le\ T^F \qquad (\psi^F) \tag{4.7}$$

$$t^P \le\ T^P \qquad (\psi^P) \tag{4.8}$$

$$g^+ \le\ G^+ \qquad (\rho^+) \tag{4.9}$$

$$g^- \le G^- \qquad (\rho^-) \tag{4.10}$$

$$t^F,\ t^P,\ g^+,\ g^-, f^+,\ f^- \ge\ 0$$

To avoid unnecessary notation, the bids are shown as having a lower bound of zero; more generally, quantity bids could have nonzero lower bounds. This generalization is a simple transformation in the linear parts of models. We assume a feasible solution exists; for instance, zero for all decision variables will be feasible if $K'(0,0,0) = 0$. The notation is defined as follows:

4.3.1.1 Index Sets

I is the set of nodes, $i = 1, \ldots, n^I$, in the system. F is the set of transmission (or flowgate) bids to buy or sell rights on individual transmission elements (e.g., a line, capacitor, transformer, or other transmission equipment) or a set of transmission elements, and is indexed by $k = 1, \ldots, n^F$. P is the set of transmission bids to buy or sell point-to-point rights, with index $k = 1, \ldots, n^P$. M^+ is the set of bids to sell (inject) energy, indexed by $m = 1, \ldots, n^{M+}$. M^- is the set of bids to buy (withdraw) energy, with index $m = 1, \ldots, n^{M-}$. H is the set of transmission elements in the system on which rights are purchased and sold, and the associated constraints are (4.4) and (4.5). It uses index $h = 1, \ldots, n^H$, where n^H defines the cardinality of H. H' is the set of additional interaction constraints that result from analysis of voltage, angle, stability, and contingency constraints, sometimes called nomogram or cut set constraints. On these constraints, rights can be purchased and sold. These constraints are indexed by $h = 1, \ldots, n^{H'}$, where $n^{H'}$ defines the cardinality of H'. The set H' is associated with the mapping K' in (4.3).

4.3.1.2 Variables

$f = f^+ - f^-$ is a vector-valued variable describing flows on the transmission elements. $f^+{}_h$ and $f^-{}_h$, $h \in H'$, representing the flow induced by x and y on transmission element h in the positive and negative direction respectively (defined arbitrarily).

$g = g^+ - g^-$ is a vector, where $g^+{}_m$, $m \in M^+$ represents the quantity of energy sold by the *mth* energy bid and $g^-{}_m$, $m \in M^-$ represents the quantity of energy purchased by the *mth* energy bid

t^F, $\{t^F{}_k,\ k \in F\}$, and t^P, $\{t^P{}_k,\ k \in P\}$, are vectors where $t^F{}_k$ represents the quantity of rights awarded to (bought by or sold to) the *kth* bid for flowgate (F) transmission type rights and $t^P{}_k$ represents the quantity of rights awarded to the *kth* bid for point-to-point (P) transmission type rights.

x is the set of variables that affect the topology and performance of the network, e.g., phase shifter settings, dc line settings, reactive power compensation and contingency set-asides on transmission elements for locational reserves. In today's practice, these variables are typically determined either exogenously or as a part of an iterative procedure, but the auction can accommodate bidding for these settings in the auction; see, e.g., O'Neill et al. (2002).

y is a vector, $\{y_i, i \in I\}$, where $y_i > 0$ is the amount of real power injected at node i, and $y_i < 0$ is the amount withdrawn at node i that is induced by the t^P, g^+ and g^- bids.

π, μ^N, μ^+, μ^-, γ, ψ^F, ψ^P, ρ^+, ρ^- are vectors of Lagrange multipliers associated with sets of primal constraints in the auction.

4.3.1.3 Parameters and Functions

b^F, $\{b^F{}_k,\ k \in F\}$, and b^P, $\{b^P{}_k,\ k \in P\}$, are vectors. $b^F{}_k$, $k \in F$ and $b^P{}_k$, $k \in P$ represents the \$/MWh value that the bidder associates with a transmission bid. Bids to buy are positive and bids to sell are negative.

$F^+, \{F^+{}_h,\ h \in H'\}, F^-, \{F^-{}_h, h \in H'\}$, and $F^N, \{F^N{}_h,\ h \in H''\}$, are transmission capacity constraints including thermal, stability or contingency limits associated with one or more transmission elements (e.g., several transmission elements grouped as a flowgate). Each individual constraint in the third category of capacity constraints (condition (4.3)) involve two or more flows simultaneously and so we refer them to interaction constraints. In practice, they are often called nomogram constraints.

B^+, B^- are matrices, $\{B^+{}_{hk},\ h \in H',\ k \in F\}$, $\{B^-{}_{hk},\ h \in H',\ k \in P\}$, where $B^+{}_{hk}$ represents the quantity in the positive direction on transmission element h that is requested in bid k and $B^-{}_{hk}$ represents the quantity in the negative direction on transmission element h that is requested in bid k.

B^N is a matrix, $\{B^N{}_{hk}, h \in H'', k \in F\}$, where $B^N{}_{hk}$ defines the quantity of the *hth* transmission network interaction constraint that the *kth* bid for a F right requires. An 'interaction' constraint is any constraint that is not simply a lower or upper bound on some variables (especially flows) or otherwise associated with a single transmission element. Examples include voltage and stability constraints. The set of network constraints H'' includes these constraints.

c^+, $\{c^+{}_m,\ m \in M\}$, and c^-, $\{c^-{}_m, m \in M^-\}$ are vectors where $c^+{}_m < 0$ represents the unit \$/MWh value to sell energy bid m and $c^-{}_m > 0$ represents the unit value to buy energy bid m.

A^+, $\{a^+{}_{im},\ i \in I,\ m \in M\}$, and A^-, $\{a^-{}_{im},\ i \in I,\ m \in M^-\}$, is a matrix where $a^+{}_{im} = 1$, if there is an injection of energy at node i associated with energy bid m; $a^-{}_{im} = -1$, if there a withdrawal at node i associated with energy bid m; and zero otherwise for simple trades. The formulation also permits energy portfolio bids where the matrix entries are not restricted to 0, 1 or -1.

$K'(x, y, f)$ is the mapping that defines additional inequality constraints upon flows resulting from off-line studies of contingencies, stability, voltage and angle constraints.

$K''(x, y)$ is the mapping from x and y to flows f. These are the basic load flow constraints, expressing flows as a function of injections. Consequently, $\partial K''(x,y)/\partial y$ can be viewed as a matrix of the power transfer distribution factors (PTDFs).

The set of optimal bids accepted by the auction is denoted as $\{t^{F*},\ t^{P*},\ g^{+*}, g^{-*}\}$ and the set of Lagrange multipliers that satisfy the Karush-Kuhn-Tucker (KKT) conditions for the auction is denoted $\{\pi^*,\ \mu^{N*},\ \mu^{+*},\ \mu^{-*},\ \gamma^*,\ \psi^{F*},\ \psi^{P*},\ \rho^{+*},\ \rho^{-*}\}$. If there are no losses, then the congestion rents (i.e., opportunity costs) resulting from flows are $\mu^N F^N + \mu^+ F^+ + \mu^- F^-$.

Constraint (4.2) includes the net injections from the energy part of the auction along with net injections implied by the point(s)-to-point(s) transmission auction; their sum yields the overall net injections at each node, y. Constraints (4.4 and 4.5) require that the flowgate F rights plus flows induced by y and x are subject to the bounds on each transmission element. Constraints (4.3) further require that the F rights and flows induced by y and x are subject to the interaction constraints on the system (i.e., represent a feasible physical dispatch with respect to those constraints). Constraint (4.6) calculates the flows induced by x and y. For instance, (4.2) and (4.6) together could be based on the linearized dc load flow analogues of Kirchhoff's current and voltage laws, respectively (as in the example at the end of this chapter). Constraints (4.7, 4.8, 4.9 and 4.10) enforce the upper bounds on each type of bid.

In general, the underlying physical constraints of a reliable AC system yield a nonconvex set. Let it be called C. Let $\underline{C}$ be the set that satisfies (4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9 and 4.10). $\underline{C}$ is often represented by an energy management system combined with judgment of experienced operators, various approximations and the results of contingency analyses. The set C includes relationships between power, reactive power, Kirchhoff's law, losses, voltage, phase angle regulators, dc lines and all specified contingencies. These constraints ensure the reliability/feasibility of the implied dispatch. Here we assume $\underline{C} \subset C$, that is, JETRA-NL is a restriction of the AC problem. In general, a full AC model would include a doubling of the size of y to include reactive power. More generally, we could define $g^{+}{}_{m} \in G^{+}{}_{m},\ g^{-}{}_{m} \in G^{-}{}_{m}$ could define additional constraints on generators and load such as ramp rate constraints or total energy limits over a series of hours (e.g., hydro energy constraints).

Several further generalizations are worth mentioning. First, the model could allow "all or nothing" or binary bids for rights. This can be accomplished by adding integer variables and replacing the upper bound constraints such as the following for g_m: If transmission switching was considered, it would also affect K'' (due to KVL); this complication is not considered in this chapter.

$$g_m - G_m z_m \leq 0$$

where z_m are 0/1 variables. Lower bounds could be similarly specified as follows:

$$g_m - \underline{G}_m z_m \geq 0$$

where the underlining denotes a lower bound.

Furthermore, the introduction of integer variables allows for unit commitment (i.e., dynamic optimization) of generation (e.g., Hobbs et al. 2001) and transmission switching (FERC 2005; O'Neill et al. 2005a), as well as for consideration by longer-term auction markets of entry by technologies with investment costs, as is characteristic of generation and transmission projects. Elsewhere, we have shown that efficient market-clearing prices in auction markets with non-convexities in technology and production exist using a two-part pricing scheme in which the integral activity (e.g., start-up) is offered a specific ("non-anonymous" or discriminatory) price while the associated commodity (e.g., energy) is cleared through a single or uniform market clearing ("anonymous" or non-discriminatory) price

(Elmaghraby et al. 2004; O'Neill et al. 2005b). Most ISOs have adopted such a two-part pricing regime (often called a revenue sufficiency or bid-cost recovery guarantee) for generator offers accepted in the day-ahead market and real-time market. The omission of these binary variables yields suboptimal solutions with lower market surplus and possibly an infeasible dispatch, but their inclusion threatens revenue adequacy and may induce changes in the settlement rules.

Finally, to this point, we have assumed that the ISO is defining and selling transmission rights. An initial allocation of rights can be done through an auction or by other methods. For example, in most U.S. ISO markets, the ISO first allocates transmission rights or the rights to a portion of transmission auction revenues. Next, the ISO conducts the transmission auction as if it owns the transmission rights under its control, but then returns auction revenues to transmission holders. In this approach, the capacity held by the ISO, F^+ and F^-, is the unallocated capacity. If $F^+ = F^- = 0$, the ISO offers no transmission rights and trading takes place among the rights holders.

4.3.2 Specifying the Bids for Energy and Transmission Rights

Because in some cases our notification diverges from familiar notation from prior transmission rights models (e.g., Chao and Peck 1996; Harvey et al. 1997), this section elaborates on the product definitions and characteristics introduced in Sect. 4.2, reviewing the mathematical formulation of the products as required by the auction model.

Energy. An simple energy bid (real or financial) to sell is defined by scalars, $G^+{}_m$ and $c^+{}_m$, and the vector $a^+{}_m$; $c^+{}_m$ (usually $c^+{}_m < 0$) is the cost (e.g., in $ per MWh) for a step m, and $G^+{}_m$ (e.g., in MWh) is the maximum quantity for sale in step m $(g^+{}_m \leq G^+{}_m)$. Adding the locational aspect, $a^+{}_m$ is a vector of 0 s and a single $a^+{}_{im} = 1$ defining the injection node i. Symmetrically, an energy bid (real or financial) to buy is defined by scalars, $G^-{}_m$, and $c^-{}_m$, and the vector $a^-{}_m$; $c^-{}_m$ specify the bid value (e.g., in $ per MWh) for step m up to $G^-{}_m$, the maximum quantity for the step $(g^-{}_m \leq G^-{}_m)$. Adding the locational aspect, $a^-{}_m$ is a vector of 0 s and a single $a^-{}_{im} = -1$ defining the withdrawal node i. For example, to define a simple bid to sell one unit of energy at node 6 in a network, $\mathrm{a}^+{}_{\mathrm{6m}} = 1$ and $\mathrm{a}^+{}_{\mathrm{im}} = 0$ for $i \neq 6$. If $\mathrm{a}^-{}_{\mathrm{6m}} = -1$, then it would be a bid to buy one unit of energy at node 6. An individual bid can be part of a step-wise function with each step a separate value of the index m.

Simple Transmission Capacity Rights and Portfolio Combinations. A bid for a transmission right of either the flowgate (F) or the point-to-point (P) type is defined by b and T. What differentiates the bids for F and P rights is that flowgate rights are directly associated with a transmission element and/or combination of transmission elements while point-to-point rights are associated with injections and withdrawals independent of the topology.

To sell the simple rent payment transmission right, ${b^F}_k < 0$ is the lowest amount a bidder is willing to accept to sell up to ${T^F}_k$ units. A bid, $k \in F$, for this right on transmission element j in the positive direction is defined by inserting ${B^+}_{hk} = -1$ in the flow constraint (4.4) for $h = j$ and 0 for $h \neq j$. Similarly, a bid on transmission element j in the negative direction is defined by ${B^-}_{hk} = -1$ in flow constraint (4.5) for $h = j$ and 0 for $h \neq j$.

To buy the simple rent collection transmission right, ${b^F}_k > 0$ is interpreted as the highest amount a bidder is willing to pay to buy up to ${T^F}_k$ units. A bid, $k \in F$, for this right on transmission element j in the positive direction is defined as ${B^+}_{hk} = 1$ for $h = j$ in (4.4) and 0 for $h \neq j$. Similarly, a bid on transmission element j in the negative direction is defined by ${B^-}_{hk} = 1$ in (4.5) for $h = j$ and 0 for $h \neq j$.

Those parameters (extending notation introduced by Chao and Peck 1996) indicate how much capacity on transmission element h is taken up by a unit of this type of right. In fact, a portfolio of flowgates k is defined by, ${B^+}_{hk}$, ${B^-}_{hk}$, ${B^N}_{hk}$, the proportions of each flowgate in the portfolio.

Point-to-Point Transmission Rights. As noted, the point-to-point transmission bids, $l \in P$, are defined over one or more POIs and one or more POWs at the n^I nodes in the system (more than one POI or POW defines a so-called network right). In the auction, the bidder would further have to specify whether the right is desired as an option or obligation; if options are allowed, this would result in different and more complicated computations (Hogan 2002). For the buyer of the P right, ${b^P}_l$ (usually ${b^P}_l > 0$) represents the highest amount bidder l is willing to pay to buy up to ${T^P}_l$ units. For sellers of the rights, ${b^P}_l$ (usually ${b^P}_l < 0$) is the lowest amount a bidder l is willing to accept to sell up to ${T^P}_l$ units. ${A^P}_l$ is a vector of net injection coefficients defining the net injection at each node i in each $l \in P$, with elements ${a^P}_{il}$. For a POI (conversely, POW), ${a^P}_{il} > 0$ (conversely, ${a^P}_{il} < 0$). Hence, for balanced rights in a lossless transmission system, $\sum_i {a^P}_{il} = 0$.

The portfolio of flowgate rights can be constructed that provides the same payoffs as a specified set of point-to-point rights if the topology is known and unchanging. However, if the network topology changes, then, in general, the flow patterns associated with a given point-to-point right will change. Generally, the point-to-point rights are independent of the topology, but flowgate rights depend specifically on the topology.

4.4 Forward and Dispatch Markets: Financial Settlement and Revenue Adequacy

ISO auction markets operate in a sequence of forward and real-time market auctions, with products such as transmission rights and generation capacity being traded pre-day-ahead, while energy and bid-based ancillary services are typically traded day-ahead and in real-time. As noted above, the exact timing and content of these product auctions are a matter of market design based on the history and characteristics of specific ISO markets. This section provides the general mathematical procedure for financial settlement and its link to revenue adequacy, focused on the two types of transmission rights and energy. A few brief simple examples are also given.

There are alternative sets of market rules that could be used for selling all or part of a set of transmission rights and/or forward energy commitments. Here, our formulation mathematically liquidates all rights in each auction. Carrying the rights to the next stage could be accomplished by bidding an equal specification to the current rights with a corresponding large bid value (although this rule could conflict with market power mitigation rules) or submitting a fixed bid, that is, a bid with an upper and lower bound equal to current holdings. Holdings are liquidated by simply not submitting a bid. By convention, in ISO markets, point-to-point transmission rights are formally settled in the day-ahead market, while financial energy trades through the ISO auctions can be transacted day-ahead but cashed out at the real-time physical dispatch prices. We do not require any financial bid to be cashed out until the real-time market. Energy sales and purchases are settled financially in each forward market.

The notation, s, is introduced to designate the sequence of energy and transmission auctions, where $s = S$, $S - 1$, ..., 1, 0, and the *sth* auction is defined as JETRA^s. JETRA^0 is the final, real-time dispatch auction. The optimal values for energy and transmission rights resulting from the *sth* auction are designated t^{Fs*}, t^{Ps*}, g^{+s*} and g^{-s*}. The optimal dual values will be similarly superscripted.

4.4.1 Multi-settlement System

Table 4.2 summarizes the multi-settlement system for the auction model using a uniform clearing price rule. The table shows the market design in which transmission rights contracts and nodal revenue rights contracts are settled finally in the real-time dispatch market (s = 0). In essence, for each auction s∈S, the ISO settles the rights contracts acquired in auction s + 1.

Row one of Table 4.2 shows that in each auction, s, transmission and energy rights contracts from auction $s + 1$ are settled (or liquidated) at the auction price times their contract holdings from the $s + 1$ auction (note again that incrementing by 1 is moving the auction backwards in time). Row two shows the contracts established in auction s will pay or are paid the auction price times the quantity of transmission rights and forward energy contracts that clear the market.

The real-time dispatch market, $s = 0$, settlements shown in rows three and four follow the same logic as the forward markets with respect to holders of transmission rights or forward energy contracts, who are paid the auction price times their holdings from the prior auction iteration, $s = 1$. Only physical injections and withdrawals are traded in auction 0, but the forward rights from $s = 1$ are settled.

Table 4.2 Calculation of settlement payments in auction s for rights allocated in auctions $s+1$ and s using uniform clearing price rule

	Flowgate rights (F)	Point-to-point rights (P)	Energy supply and demand (g)
JETRA^s ($s \geq 1$) (forward market): Payment to holders of contracts from previous auction $s+1$	$\mu^{N,s^*} B^{N,s+1} t^{F,s+1^*}$ (interaction constraints)	$\pi^{s^*} A^{P,s+1} t^{P,s+1^*}$	$\pi^{s^*} A^{+,s+1} g^{+,s+1^*}, \pi^{s^*} A^{-,s+1} g^{-,s+1^*}$
	$\mu^{+s^*} B^{+,s+1} t^{F,s+1}$ (flowgates in + direction)		
	$\mu^{-s^*} B^{-,s+1} t^{F,s+1}$ (flowgates in $-$ direction)		
JETRA^s ($s \geq 1$) (forward market): Payment by purchasers of contracts in auction s	$\mu^{N,s^*} B^{N,s} t^{F,s^*}$ (interaction constraints)	$\pi^{s^*} A^{P,s} t^{P,s^*}$	$\pi^{s^*} A^{+,s} g^{+,s^*}, \pi^{s^*} A^{-,s} g^{-,s^*}$
	$\mu^{+,s^*} B^{+,s} t^{F,s^*}$ (flowgates in + direction)		
	$\mu^{-s^*} B^{-,s} t^{F,s^*}$ (flowgates in $-$ direction)		
JETRA^0 (real-time market): Payment to holders of contracts from previous auction 1	$\mu^{N,0^*} B^{N,1} t^{F,1^*}$ (interaction constraints)	$\pi^{0^*} A^{P,1} t^{P,1^*}$	$\pi^{0^*} A^{+,1} g^{+,1^*}, \pi^{0^*} A^{-,1} g^{-1^*}$
	$\mu^{+0^*} B^{+,1} t^{F,0^*}$ (flowgates in + direction)		
	$\mu^{-0^*} B^{-,1} t^{F,1^*}$ (flowgates in $-$ direction)		
JETRA^0 (real-time market): Payment by purchasers of physical energy in auction 0	$\mu^{N,0^*} B^{N,0} t^{F,0}$ (interaction constraints)	$\pi^{0^*} A^{P,0} t^{P,0^*}$	$\pi^{0^*} A^{+,0} g^{+,0^*}, \pi^{0^*} A^{-,0} g^{-,0^*}$
	$\mu^{+0^*} B^{+,0} t^{F,0^*}$ (flowgates in + direction)		
	$\mu^{-0^*} B^{-,0} t^{F,0^*}$ (flowgates in $-$ direction)		

The implicit congestion charge associated with any pair of injections and withdrawals at different nodes is the difference in the auction LMPs at those two nodes. For instance, using our notation, if two awards for bids m and m' result in $g^*_m = g^*_{m'}$, where g^*_m is an injection at node 1 ($a_{1m} = 1$) and $g^*_{m'}$ is a withdrawal at node 2 ($a_{2m} = -1$), then the total congestion charge associated with these two transactions is $\pi^*_2 g^*_m - \pi^*_1 g^*_{m'} = (\pi^*_2 - \pi^*_1) g^*_m$.

A property of this settlement system that follows from convexity of the JETRA model and the optimality of its solution is that the prices in auction s are such that there remain no arbitrage opportunities among the rights awarded in that auction. As an example, a pair of energy rights, one involving injection of 1 MW at one node i and the other involving withdrawal at another node i' would result in exactly the same settlement as an equivalent point-to-point right from i' to i, so that no profitable arbitrage can be undertaken between those two types of rights. In a sense, the numerical process of finding an optimal solution can be viewed as consisting of searching for and taking advantage of all profitable arbitrage among the bids; if there remained profitable arbitrage opportunities at as solution, then the solution by definition could not be optimal.

Pre-day-ahead forward energy transactions, or nodal revenue rights, are not yet offered in ISO auctions. Hence their financial settlement deserves some further explanation. Settlement would take place, as with other transmission rights, in the day-ahead market (or in the real-time market if there is no day-ahead market). The holder of the injection right gets paid the nodal price for the energy it produces but is obligated to pay the nodal price to the ISO for energy represented in its nodal energy right, while the holder of the withdrawal right is obligated to pay the nodal price for the energy it actually consumes but is paid the nodal price for the energy quantity specified in its forward right. As with the two-sided, point-to-point right, executing the physical transaction specified in the right results in a net zero financial position in settlement. There are practical issues to implementing such a forward energy auction, most notably creditworthiness.

4.4.2 *Revenue Adequacy of the Auction Sequence*

Revenue adequacy could pertain to each pair of auctions in the sequence. Also, revenue adequacy could pertain to the entire sequence. If all pairs are revenue adequate, the full sequence is revenue adequate. A set of sufficient conditions for revenue adequacy is that the constraint sets are convex and the constraint set does not contract over the auction sequence. The proof is in the appendix. Even if the constraint set is not convex, if it is not contracting (i.e., if all feasible solutions in previous iterations remain feasible in subsequent iterations), then even if the prices do not result in revenue adequacy, each and every market participant can in theory be made better off by re-allocating the surplus. This is because the objective function (total surplus) can only improve if the feasible region is non-contracting.

This re-allocation may require a deviation from the uniform clearing price settlement, for example using two-part tariffs or fixed monetary transfers.

Beginning with each auction clearing, a requirement for revenue adequacy is that the auction result respects the set of transmission constraints. For point-to-point rights, this is commonly known as “simultaneous feasibility,” meaning that the power flow induced by the injections and withdrawals associated with the rights awarded is feasible (Harvey et al. 1997). Here “simultaneous feasibility” applies to all rights in each auction.

Turning next to the conditions on the auction sequence, we have assumed heretofore that each auction in the sequence, JETRA^s, is conducted with the same set of transmission constraints. However, an important feature of actual electricity markets is that in the forward markets for transmission rights, the transmission constraints modeled may be either more or less restrictive than the set operative in the real-time market.

The further ahead a forward market is of the physical dispatch auction $s = 0$, the greater the uncertainty about the network topology that will apply in the dispatch. This could justify a conservative transmission constraint set in the further forward auctions. For forward auctions closer in time to the dispatch, some uncertainty will be resolved and this will justify increased offerings by relaxing the constraints. For example, equipment may need to be derated if it is extremely hot, but temperature is not known until a time closer to the dispatch. The uncertainty can be captured in auction models through either multi-state or chance-constrained models, but these models are large and harder to solve and may require different settlement rules.

In general, the recursion of the auction markets is revenue adequate as long as the transmission capacity constraints form a nested, expanding sequence, a restriction which is stated more formally in the proof in the appendix. If K'' is linear and K' is convex, the constraint set is convex. For $s' > s$, if the constraint set defines a feasible region that is convex and non-contracting, that is, $F^{+,s'} \leq F^{+,s}$, $F^{-,s'} \leq F^{-,s}$ and $F^{N,s} \leq F^{N,s}$, then the auction sequence is revenue adequate. Non-contracting means that in each auction in the sequence, the transmission constraint set must be no more restrictive than the prior auction. This is an obvious requirement to prevent overselling of flowgate transmission rights.

A expanding constraint set can be thought of as the ISO holding back some of the rights until it is reasonably sure they will be available. Therefore, it is not unusual for the auction sequence to start with a conservative estimate of the availability of the network topology. Some ISOs have adopted simple rules to accommodate this requirement; for example, the California ISO sells forward transmission rights to only a small percentage of its transmission capacity (Bautista-Alderete 2010). Long-term point-to-point transmission rights are usually made available on conservative basis to account for the long-term uncertainty. Operational experience will be required to determine what quantity of alternative types of transmission rights can be made available in each forward market (annual, monthly, weekly, etc.).

As noted above, if the auction sequence is not revenue adequate in actual market operations, for example due to unplanned transmission outages affecting day-ahead and real-time market settlements, then each ISO has rules for how revenue shortfalls to rights holders are allocated.

4.5 Auction Example with Quadratic Losses

This section presents a numerical example of the auction model in a simplified network based upon a linearized dc load flow with quadratic losses (e.g., Hobbs et al. 2008; Schweppe et al. 1988). The only transmission elements considered are lines. Constraint (4.3) is omitted and (4.6) is modified to represent the dc analogues to Kirchhoff's Current and Voltage Laws:

$$\text{Current Law: } -y + D(f^{+} - f^{-}) + f^{-T}L^{-}f^{-} + f^{+T}L^{+}f^{+} \leq 0 \tag{4.11}$$

$$\text{Voltage Law: } R(f^{+} - f^{-}) = 0, \tag{4.12}$$

where the new notation is as follows:

D is a matrix that maps flow variables to the associated current law (enerrgy balance) constraints. The rows of the vector correspond to buses, and the columns correspond to lines of the network.

L^{+}, L^{-} are tensors of rank 3, where the only nonzero elements in L^{+} (L^{-}) are $l^{+}{}_{ikk}$ ($l^{-}{}_{ikk}$), representing the resistance loss coefficients (decrease in imports to bus i) due to a positive (negative) flow through transmission line k.

$R = \{r_{vk}\}$ are line reactances used in the voltage law analogues. Each element is the value of reactance for transmission line k that appears in voltage loop v. $r_{vk} = +R_k$ or $-R_k$ if line k occurs in loop v, depending on whether a positive flow $(f^{+} - f^{-})$ is in the same or opposite sense of flow around v. On the other hand, $r_{vk} = 0$ if link k does not occur in loop v. Consistent with the dc model, the number of independent loops v must be equal to $K - N + 1$, where K is the number of lines considered and N is the number of buses.

Note that (4.11) is a relaxation of the Kirchoff's Current Law (energy balance) equality constraint that results in a convex feasible region (Chao and Peck 1998). An example is given below to illustrate (4.11) and (4.12).

An important property, noted in Harvey and Hogan (2002), is that if $l_{ikk} > 0$, for some k, then in general no set of balanced P (point-to-point) rights will be feasible (revenue adequate) by themselves (except in the degenerate case of $t_P = 0$). This is because of losses. Revenue adequacy is thus possible only if sufficient energy rights are also sold (in particular, "rights" that oblige the rights holder to make payments to the ISO; i.e., rights g whose coefficients in Ag are positive). A combination of such energy and balanced point-to-point rights g and T_F can also be viewed as a set of imbalanced point-to-point rights.

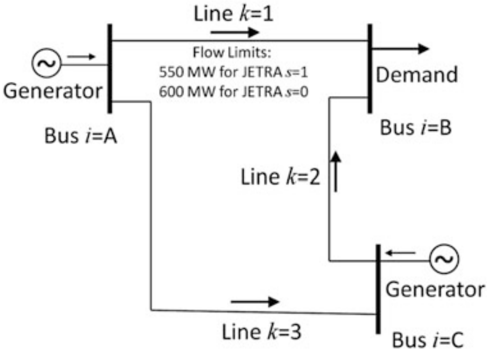

Fig. 4.1 Network for three bus JETRA-NL example

The numerical example takes place on the three node network in Fig. 4.1, in which the arrows show the direction of flow on lines $k = 1, 2, 3$ for injections at buses A and C (with a larger one at A) and a withdrawal at bus B. These directions also coincide with the positive directions of flows associated with those lines. All loss factors on all lines equal 0.0001 [MW/MW^2], and all lines have a physical flow limit of 600 MW (only the limit for $k = 1$ is shown because that is the only one that binds in the solutions below). All line reactances, $R_k = 1$. Then for this network, (4.11) and (4.12) become:

$$KCL_A : -y_A + (f^+{}_1 - f^-{}_1) + (f^+{}_3 - f^-{}_3) + 0.0001\left((f^-{}_1)^2 + (f^-{}_3)^2\right) \leq 0$$

$$KCL_B : -y_B - (f^+{}_1 - f^-{}_1) - (f^+{}_2 - f^-{}_2) + 0.0001\left((f^+{}_1)^2 + (f^+{}_2)^2\right) \leq 0$$

$$KCL_C : -y_C + (f^+{}_2 - f^-{}_2) - (f^+{}_3 - f^-{}_3) + 0.0001\left((f^-{}_2)^2 + (f^+{}_3)^2\right) \leq 0$$

$$KVL : (f^+{}_1 - f^-{}_1) - (f^+{}_2 - f^-{}_2) - (f^+{}_3 - f^-{}_3) = 0$$

Notice that if the only existing transmission or energy right is, say, a balanced t_P involving an injection of 1,000 MW at A ($y_A = +1{,}000$) and a withdrawal of 1,000 MW at B ($y_B = -1{,}000$), this would be infeasible. There are two reasons for this. First, such an injection-withdrawal pair would induce more than 600 MW of flow on line $k = 1$ (in the lossless case, 667 MW would flow). Second, because of line losses, there is no set of nonnegative flows $\{f^+{}_1, f^-{}_1, f^+{}_2, f^-{}_2, f^+{}_3, f^-{}_3\}$ that would simultaneously satisfy all four of the above constraints. Thus, there would either need to be some additional energy injected to make up for the loss, or the point-to-point right would need to be imbalanced, with more injected at A than withdrawn at B. The infeasibility of this right implies that the ISO might be revenue deficient if it settled that right at nodal prices from an optimal dispatches subject to the above constraints; this is indeed the case, as we see below

We illustrate a sequence of JETRA-NL with this network. In auction $s = 1$, due to ISO caution, only 550 MW of rights are released on each line k rather than the full 600 MW. As mentioned, this is the current policy of certain ISOs in order to lessen the likelihood of revenue inadequacy. We assume that in this auction there are the following bidders for transmission and energy rights:

- Bidder 1 is willing to pay up to \$60 per MWh per hour for up to 700 MW of point-to-point obligation transmission rights from node $i = \text{A}$ to $i = \text{B}$;
- Bidder 2 is willing to pay up to \$30 per MWh per hour for up to 300 MW of point-to-point rights from C to B;
- Bidder 3 bids is willing to pay up to \$80 per MW per hour for up to 100 MW of flowgate rights on line $k = 1$ in the direction from bus A to bus B; and
- Bidder 4 offers to sell up to 100 MW of forward energy rights at node B at a price of \$90/MWh.

The resulting formulation of JETRA-NL is as follows:

$$JETRA - NL, s = 1 : max\ 60t^{P}{}_{1} + 30t^{P}{}_{2} + 80\,t^{F}{}_{3} - 90g_{4}$$

$$t^{P}{}_{1} - y_{A} = 0$$

$$-t^{P}{}_{1} - t^{P}{}_{2} + g_{4} - y_{B} = 0$$

$$t^{P}{}_{2} - y_{C} = 0$$

$$t^{F}{}_{3} + (f^{+}{}_{1} - f^{-}{}_{1}) \leq 550$$

$$-(f^{+}{}_{1} - f^{-}{}_{1}) \leq 550$$

$$(f^{+}{}_{2} - f^{-}{}_{2}) \leq 550$$

$$-(f^{+}{}_{2} - f^{-}{}_{2}) \leq 550$$

$$(f^{+}{}_{3} - f^{-}{}_{3}) \leq 550$$

$$-(f^{+}{}_{3} - f^{-}{}_{3}) \leq 550$$

$$-y_{A} + (f^{+}{}_{1} - f^{-}{}_{1}) + (f^{+}{}_{3} - f^{-}{}_{3}) + 0.0001\left((f^{-}{}_{1})^{2} + (f^{-}{}_{3})^{2}\right) \leq 0$$

$$-y_{B} - (f^{+}{}_{1} - f^{-}{}_{1}) - (f^{+}{}_{2} - f^{-}{}_{2}) + 0.0001\left((f^{+}{}_{1})^{2} + (f^{+}{}_{2})^{2}\right) \leq 0$$

$$-y_{C} + (f^{+}{}_{2} - f^{-}{}_{2}) - (f^{+}{}_{3} - f^{-}{}_{3}) + 0.0001\left((f^{-}{}_{2})^{2} + (f^{+}{}_{3})^{2}\right) \leq 0$$

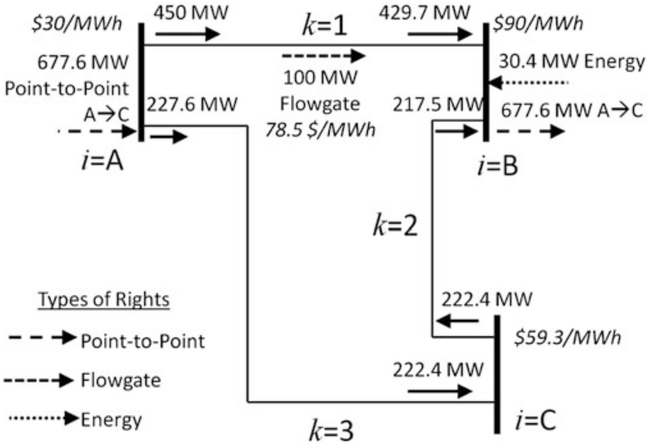

Fig. 4.2 Awarded financial transmission and energy rights and LMPs for dispatch round $s = 1$ of the JETRA-NL example

$$(f^+{}_1 - f^-{}_1) - (f^+{}_2 - f^-{}_2) - (f^+{}_3 - f^-{}_3) = 0$$

$$t^P{}_1 \le 700$$

$$t^P{}_2 \le 300$$

$$t^F{}_3 \le 100$$

$$g_4 \le 100$$

$$t^P{}_1,\ t^P{}_2,\ t^F{}_3,\ g_4 \ge 0$$

The ISO runs JETRA-NL for this auction, and makes the following awards of rights:

- 677.6 MW of point-to-point rights to Bidder 1, who pays the ISO $40,655 for these rights (equal to the nodal price difference between A and C times the awarded
- 0 MW of point-to-point rights to Bidder 2
- 100 MW of flowgate rights to Bidder 3, who pays $7856 for those rights (flowgate 1's shadow price times the award)
- 30.4 MW of energy rights from Bidder 4, who the ISO pays $2,734 (B's nodal price times the energy right sold)

Figure 4.2 shows this solution to the auction, along with the nodal and flowgate prices. The ISO's net receipts from the auction are $45,776, which is less than the

$45,922 objective function for the auction model.[17] This discrepancy arises because Bidder 3's upper bound is binding, meaning that she pays less for the rights than they are worth to her.

We now move to the next (and final) JETRA-NL iteration, $s = 0$, which is the physical dispatch. We assume that there are two power plants, neither with capacity limits. The plant at A offers to sell energy at $20/MWh (variable g_A), while C's plant offers at $50/MWh (variable g_C). There is a 1,000 MW load at B (variable g_B). The ISO makes available the full 600 MW of flow capacity in each line for this iteration. The resulting JETRA-NL is:

$$\mathit{JETRA\text{-}NL}, s = 0 : max - 20g_A - 50g_C$$

$$g_A - y_A = 0$$

$$-g_B - y_B = 0$$

$$g_C - y_C = 0$$

$$(f^+{}_1 - f^-{}_1) \leq 600$$

$$-(f^+{}_1 - f^-{}_1) \leq 600$$

$$(f^+{}_2 - f^-{}_2) \leq 600$$

$$-(f^+{}_2 - f^-{}_2) \leq 600$$

$$(f^+{}_3 - f^-{}_3) \leq 600$$

$$-(f^+{}_3 - f^-{}_3) \leq 600$$

$$-y_A + (f^+{}_1 - f^-{}_1) + (f^+{}_3 - f^-{}_3) + 0.0001\left((f^-{}_1)^2 + (f^-{}_3)^2\right) \leq 0$$

$$-y_B - (f^+{}_1 - f^-{}_1) - (f^+{}_2 - f^-{}_2) + 0.0001\left((f^+{}_1)^2 + (f^+{}_2)^2\right) \leq 0$$

$$-y_C + (f^+{}_2 - f^-{}_2) - (f^+{}_3 - f^-{}_3) + 0.0001\left((f^-{}_2)^2 + (f^+{}_3)^2\right) \leq 0$$

$$(f^+{}_1 - f^-{}_1) - (f^+{}_2 - f^-{}_2) - (f^+{}_3 - f^-{}_3) = 0$$

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[17] Round off errors result in slight discrepancies in results. For instance, $45,922 is the exact objective function value resulting from the exact decision variable values, while the values of the decision variables presented here, which are rounded off, yield $45,920 instead.

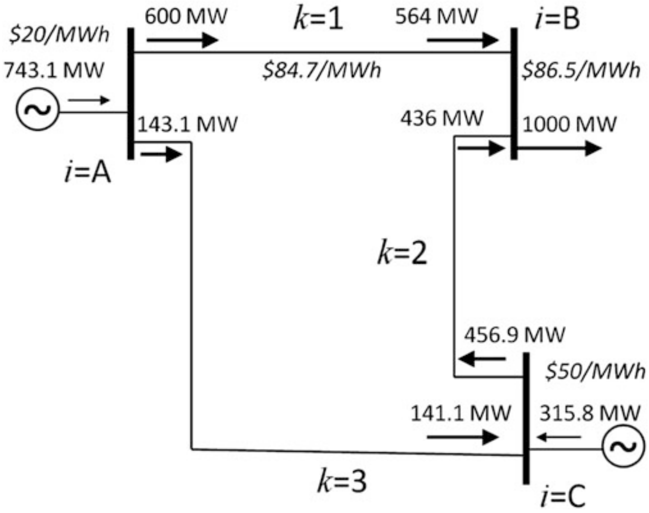

Fig. 4.3 Nodal injections and withdrawals and LMPs for dispatch round $s = 0$ of the JETRA-NL example

$$g_B = 1000$$

$$g_A, g_C \geq 0$$

The resulting dispatch is shown in Fig. 4.3, along with the nodal prices. The ISO pays a total of $30,652 to the two generators for their energy, while receiving $86,470 from the load at B. The resulting total surplus gained by the ISO is $55,818. If congestion portion of this surplus is calculated as the sum of the flowgate shadow prices times the flows, the congestion surplus is $50,797, while the loss surplus is the remaining $5020. The loss surplus arises because of the quadratic nature of losses, which means that marginal losses are roughly double the average loss. Consumers pay for marginal losses. In this example, the ISO essentially gets to keep the difference between marginal and average losses. In practice, the U.S. ISOs are required to refund excess revenues to market participants.

From its surplus, the ISO must pay the holders of financial transmission and energy rights awarded in the earlier JETRA-NL $s = 1$. The following awards are made to financial rights holders:

- Bidder 1, who holds 677.6 MW of point-to-point rights from A to B that were awarded in $s = 1$, is paid the nodal price difference ($86.5–$20) times those rights, or $45,039.
- Bidder 2 owns no rights, and so receives no payment
- Bidder 3 is paid 100 MW times the flowgate shadow price for k = 1, or $8466.
- Bidder 4 has to pay the ISO $2,627 for its 30.4 MW of energy injection rights at node B.

The net payments to financial rights holders by the ISO is \$50,879. Note that each bidder happens to make money on their financial rights. Bidders 1 and 3 get paid more in $s = 0$ for their transmission rights than they paid in $s = 1$, while Bidder 4 pays less to settle her energy right in $s = 0$ than she got paid in $s = 1$. Note that since Bidder 4 has no physical asset, her energy right is what is known in U.S. markets as a virtual energy right, in which energy is bought in one market, and then the same amount is sold back in the next, arbitraging the difference in prices. Bidder 4 is what is known as a virtual supplier, since she supplied power in the first auction $s = 1$. Because the energy price in $s = 1$ was greater than $s = 0$, she makes money on that energy transaction.

The fact that the financial rights holders made money on their rights has no implications for revenue neutrality of auction $s = 0$. In fact, the ISO's surplus in the final dispatch round $s = 0$ of JETRA-NL (\$55,818) exceeds its net payments to owners of financial rights awarded in $s = 1$ (\$50,879, as just noted)., This is necessarily the case because the dispatch model is convex (the feasible region defined by the load flow constraints (4.11 and 4.12) plus capacity constraints is convex, while the objective function is linear), and the transmission flows that would induced by the financial rights awarded in $s = 1$ are feasible in the dispatch model. In particular, note that the $s = 1$ flows in Fig. 4.2 are feasible if the transmission limits were the 600 MW values assumed in the $s = 0$ dispatch optimization.

As an example of financial rights that would not be revenue adequate, return again to the simple example mentioned before in which the only transmission or energy rights held after $s = 1$ are 1,000 MW of point-to-point rights from A to B. This set of rights would violate the load flow and capacity constraints of the network in Fig. 4.1. The settlement in that case, based on $s = 0$'s nodal prices, would be (\$86.5 − \$20) × 1,000 MW, or \$66,500; this would exceed the ISO's surplus of \$55,818 in $s = 0$, violating revenue adequacy.

4.6 Conclusion

The nonlinear auction model presented here provides a general framework for representing and implementing a more complete version of combined energy and transmission rights auctions that have been proposed and discussed in the United States. With all types of energy and transmission capacity bids allowed, the auction framework can be extended to most types of forward hedging. Frequent auctions increase liquidity by providing additional opportunities to trade while considering the network constraints that bilateral markets have difficulty factoring in. In addition, this framework could facilitate the efficient operation of off-ISO forward bilateral markets, which should benefit from more liquid transmission rights, such as the rights on commonly congested flowgates or possibly hub-to-hub rights. The proof of revenue adequacy that we previously provided for the auction with linear constraints (O'Neill et al. 2002) has been extended to the auction with convex constraints. However, the introduction of non-convex constraints invalidates the proof of revenue adequacy, assuming that a uniform pricing rule is used to determine the prices of rights and to settle them.

The practical obstacles to implementation of the model are computational requirements, implementation costs and transactions costs. For these reasons, while there is now broad consensus on many elements of market design, such as locational marginal prices for energy and financial transmission rights, market design proposals have allowed for phased implementation of different types of transmission rights and different auction products to allow for development of software and resolution of cost allocation issues.

The nonlinear auction model provides an analytic framework for exploration of additional market design features. For example, more frequent auctions add liquidity to the market. Future research to be conducted by the authors within this framework includes the modeling and pricing of locational reserves, pricing of reactive power (e.g., FERC 2005), property right awards for transmission expansion, pricing under optimal topologies, and unit commitment of transmission elements (e.g., O'Neill et al. 2005a).

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Appendix: Proof of Revenue Adequacy for the Auction Sequence

This appendix provides a set of sufficient conditions and a proof of revenue adequacy of the auction sequence. This proof extends the revenue adequacy proofs for transmission in Harvey et al. (1997) and O'Neill et al. (2002), both of which considered the case of linear transmission constraints, to an auction with both flowgate or flow-based and point-to-point rights together with nonlinear transmission constraints that define a convex feasible region. To simplify the presentation, the auction model is mapped into a more compact and general non-linear program (NLP) representing an auction in the following way:

As before, the rights bid for and awarded in the s-th auction in a sequence of auctions determine the distribution of revenues from the subsequent auction, $s-1$. Meanwhile, the prices obtained in the s-th auction determine how the rights awarded in the previous auction $s+1$ are financially settled, as well as how much winning bidders in auction s pay for the rights they win.

Define g^s as the vector of quantities awarded to P- and G-type bids (encompassing t^P, g^+ and g^- in the JETRA-NL model) with upper bound G^s in the s-th auction in the sequence. Define a general benefit function $c(g^s)$ (based on the bids by those seeking rights) for the bid award level, g^s. The vector y^s represents net injections in the s-th auction associated with rights g^s. $K^s(y)$ represents the flows induced by y^s as a result of the applicable load flow equations. Define t^s as the vector of F transmission rights (t^F in the JETRA-NL model) with upper bound T^s in the s-th auction. F^s is the vector of bounds in auction s for transmission elements and network flow constraints. Define π as the vector of dual values for the nodal energy balance constraint, which can be interpreted as the shadow or clearing prices for energy. Finally, define μ as the vector of dual values associated with transmission constraints, which can be interpreted as the shadow prices for transmission rights.

Using the resulting model NLP, the *sth* auction in the auction sequence $s+1, s, s-1, \ldots, 0$, termed NLP^s, is:

$$\text{NLP}^s : \max\ b^s t + c^s(g)$$

$$\begin{aligned} &Ag - y = 0 && (\pi) \\ &B^s t + K^s(y) \le F^s && (\mu) \\ &t \ \le\ T^s && (\psi) \\ &g \ \le\ G^s && (\rho) \end{aligned}$$

Note that all constraint and objective function parameters can depend on s.

The optimal solution to NLP^s is defined as $\{y^s, t^s, g^s\}$ and the corresponding optimal dual variables are $\{\pi^s, \mu^s, \psi^s, \rho^s\}$. To demonstrate revenue adequacy of the auction sequence, prices and payments must be defined for the bids for g and t that are accepted. Duals π^s are the market prices for g^s, and μ^s are the market prices for F^s, and are treated as row vectors in the below. The rights held as a result of the $s+1^{\text{st}}$ auction in the sequence are g^{s+1} and t^{s+1}. Financial settlements (payments by the auctioneer) in NLP^s for rights to its revenues, analogous to those defined above for the full auction model, are $\pi^s A g^{s+1}$ and $\mu^s B^{s+1} t^{s+1}$ for the two types of rights awarded in the previous auction NLP^{s+1}, where the superscript T is the transpose operator Meanwhile, the winning bidders for the two types of rights awarded by NLP^s pay $\pi^{s\,\mathrm{T}} A g^s$ and $\mu^{s\mathrm{T}} B^s t^s$, respectively.

The following theorem concerns the revenue adequacy of this sequence of auctions, and is a generalization of our earlier results for the linear JETRA (O'Neill et al. 2002):

Theorem 1 *If $B^s(g)$ is concave, $K^s(y)$ is convex, $K^s(y) \le K^{s+1}(y)$ for all y, and* $F^{s+1} \le F^s$, then each auction in the sequence of auctions $\{S-1, \ldots, s, \ldots, 1, 0\}$, is revenue adequate; that is:

$$\pi^{s\mathrm{T}}\left(A^s g^s - A^{s+1} g^{s+1}\right) + \mu^{s\mathrm{T}}\left(B^s t^s - B^{s+1} t^{s+1}\right) \ge 0.$$

Proof By convexity of K^s,

$$K^s(y^{s+1}) \ge K^s(y^s) + \nabla K^s(y^s)(y^{s+1} - y^s).$$

Rearranging, we obtain,

$$\nabla K^s(y^s)y^s \geq \nabla K^s(y^s)y^{s+1} + K^s(y^s) - K^s(y^{s+1}).$$

Premultiplying by the row vector of transmission capacity shadow prices $\mu^s \geq 0$,

$$\mu^s\nabla K^s(y^s)y^s \geq \mu^s\nabla K^s(y^s)y^{s+1} + \mu^s K^s(y^s) - \mu^s K^s(y^{s+1}) \tag{4.13}$$

From the KKTs to NLP^s,

$$\mu^s(B^s t^s + K^s(y^s)) = \mu^s F^s \tag{4.14}$$

Since $K^s(y) \leq K^{s+1}(y)$ and $F^s \geq F^{s+1}$ and because $(B^{s+1}t^{s+1}, y^{s+1})$ is a feasible solution to NLP^{s+1},

$$B^{s+1}t^{s+1} + K^s(y^{s+1}) \leq F^s \tag{4.15}$$

Multiplying both sides by $\mu^s \geq 0$,

$$\mu^s(B^{s+1}t^{s+1} + K^s(y^{s+1})) \leq \mu^s F^s \tag{4.16}$$

Combining (4.14) and (4.16) and multiplying both sides by -1 (which requires reversing the inequality),

$$-\mu^s(B^{s+1}t^{s+1} + K^s(y^{s+1})) \geq -\mu^s(B^s t^s + K^s(y^s)) \tag{4.17}$$

Adding (4.13) and (4.17), eliminating terms that cancel, and finally rearranging,

$$\mu^s\nabla K^s(y^s)y^s - \mu^s(B^{s+1}t^{s+1}) \geq \mu^s\nabla K^s(y^s)y^{s+1} - \mu^s(B^s t^s)$$

Substituting $\pi^s = \mu^s\nabla K^s(y^s)$ from the KKT condition for y^s for problem NLP^s and rearranging,

$$\pi^s(y^s - y^{s+1}) + \mu^s(B^s t^s - B^{s+1}t^{s+1}) \geq 0$$

Finally, in NLP^s, $A^s g^s = y^s$ while in NLP^{s+1}, $A^{s+1}g^{s+1} = y^{s+1}$; substitution of these constraints establishes the desired result:

$$\pi^s(A^s g^s - A^{s+1}g^{s+1}) + \mu^s(B^s t^s - B^{s+1}t^{s+1}) \geq 0.$$

Note that this result does not explicitly depend on the form of the objective function NLP^s. The objective can be linear or nonlinear, as long as it is concave so that the KKT conditions describe an optimal solution, then the use of the KKT conditions in the above proof remains valid.

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