Chapter 90
A Bi-level Multi-objective Optimization Model of Multiple Items for Stone Industry under Fuzzy Environment

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Abstract Traditionally, stone industry is produced essential materials for the construction industry but stone industry is always debated as a high emission industry for stone dust and waste water. This emission has an adverse impact on environment, humans, agriculture and ground water. This paper focuses on how to optimize the stone industry. The government is considered as the leader level which will make a strategy to plan the exploring amount of every stone plant and sustainable development in stone industry to create employment opportunity and economic growth. The stone plants are considered as the lower-level decision-makers which optimize their objective functions under the constraint of leader. The stone plants have individual objectives of maximizing the profit and produce different product according to the demand constraints under the limited exploring amount. Due to the lack of historical data, some emission coefficients are considered as fuzzy numbers according to experts advices. Therefore, a bi-level multi-objective optimization model with possibilistic and predetermined constraints under the fuzzy environment is developed to control the pollution and get sustainable development in stone industry. For some special fuzzy coefficients, the equivalent model is obtained. At the end, a practical case is proposed to show the efficiency of the proposed model.

Keywords Bi-level multi-objective programming · Possibilistic constraint · Stone industry · Fuzzy simulation

90.1 Introduction

Natural and artificial stone industry partly contains the stone quarrying, processing stone, recycling stone and so on. As a result of above processes the stone industry is

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produced different sorts of essential material product for the construction industry. Especially the building culture around worldwide is growing very rapidly. The principle rock types used as ornamental stone are marble and granite products. The stone industry supply and demand is increasing internationally. Since a large demand has been placed on building material in the building industry owing to increasing population this has caused a chronic shortage of building material. Accumulation of unmanaged waste especially in the developing country has resulted in an increasing environmental concern. Recycling of such waste as a building material appears to be a viable solution not only to such pollution problem, but also the economical design of building [7]. During the processes of quarrying stone, cutting stone and processing stone, stone industry is emitted huge amount of emission. That why stone industry is blamed as a high emission industry for the stone dust and waste water. During the stone cutting process, water is used for the purpose of cooling and collecting dust. The resulting water is a suspension of limestone powder. The annual amount of waste generated by this process includes 700,000 tons of slurry waste in addition to 1 million tons solid waste. The dumping of this waste in open area has created several environmental problems and negatively impacts agriculture humans and ground water [1]. In addition, destroying vegetation cover, regional topographic changes, soil erosion and disordering landscape are other negative environmental impacts [5]. Currently, waste water is treated in order to recycle the water for reuse, while the produced slurry is being dumped to open areas. The resulting solid waste is mainly limestone powder, which can be recycled in different forms of useful products like those related to construction materials [4]. Ammary [6] proves that stone cutting industry can be modified to convert them into zero discharging industry by recycling the waste water. The sludge can be used for producing bricks when it is in the slurry phase thus eliminating the need for water for producing bricks and the need for sludge disposal. Nasserdine et al [2] use the review of existing practice and jar test experiment to optimize the water recycling and treatment facilities in the stone cutting industry. Almeida et al [3] proposed and an overview of solutions to absorb the stone slurry and demonstrate the technical viability for producing white cement concrete with carbonate stone slurry in order to solve the problem of the waste generated by the natural stone industry. Furthermore, it may be said that marble and granite replacement rendered a good condensed matrix. The increased durability of concrete can be attributed to the glass content and chemical composition of the granite. The results of one study showed that the marble and granite waste aggregates can be used to improve the mechanical properties, workability and chemical resistance of the conventional concrete mixtures [8].

An important milestone in understanding the relationship between economic growth and the environment was laid during the second quinquennium of the 1980s, which recognized the complementarities that existed between them, with an emphasis on the need to mainstream environmental concerns into the planning process in order to ensure sustainable development [9, 10, 20], in their path-breaking work on the potential environmental impacts of the North American Free Trade Agreement (NAFTA), had extended this milestone by providing seminal evidence in support of an inverted U-shaped relationship between economic growth (measured by increases in per capita income) and some indicators of environmental quality. This relationship is the so-called environmental Kuznets curve (EKC).

Bi-level optimization has also been previously used for related applications in process systems engineering, like supply chain planning [11], design of reliable process networks [12] and collaborative design decision making for forearm crutch [13]. Oftentimes, for such problems, the leader and follower objectives are conflicting. At the same time uncertainties in their objectives and constraints exist. However, a satisfactory (near-optimal or satisficing) solution can be reached by providing tolerances in the objective functions and constraints, and by defining corresponding degrees of satisfaction through membership functions to indicate the preference of the decision-makers as is typical of decision-making in a fuzzy environment [14]. For example, Shih et al [15], Sinha [16] and Arora and Gupta [17] developed interactive fuzzy mathematical programming to obtain the best compromise solution, which simultaneously satisfies the upper- and lower-level objectives and constraints. All these techniques are based on the upper-level decision-maker specifying tolerances for his objective and variables, and then allowing the lower-level decision-makers to optimize their objective functions, provided that these tolerances are met. The followers then communicate their results to the leader, who modifies his goals and control variables if the original tolerances are not met. The process continues iteratively until a solution which satisfies the goals of both leader and follower is reached. For the application considered in this paper the decision hierarchy is illustrated with the government as the upper-level decision-maker having the objective of minimizing the emissions, maximize the social employment and economic growth and the stone plants as the lower-level decision makers having individual objectives of maximizing the profit and minimizing emissions. Since the emissions were not constantly monitored in the process of exploring the stone mine and producing the stone products, it results in the lack of the historical data about the emissions of the stone dust and waste water. We have to consider them as fuzzy numbers according to those experts' advice in the stone industry.

The other sections of this paper are organized as follows. In Sect. 90.2, a bi-level multi-objective problem is described. An example is also presented that will help readers to understand the problem background. A possibilistic bi-level multi-objective programming model is developed and it's equivalent model is obtained in Sect. 90.3. In Sect. 90.4, a case study is proposed to show the significance of the proposed bi-level multi-objective programming model with fuzzy coefficients. In the last Sect. 90.5, some conclusions are made.

90.2 Problem Statement

The stone industry is produced essential materials for the construction industry. The construction industry is growing very rapidly that why the stone industry supply and demand is increasing internationally. Stone plants are over exploiting to meet the increasing demand of stone materials. Traditionally, the stone industry partly contains the stone quarrying, stone processing, stone recycling and so on. During this processes the stone industry is emitted huge amount of stone dust and waste water due to quantitative relationship between emissions and the exploring and processing amount. This emission has adverse impact on environment, humans, agriculture and ground water. Here is needed a planing for the stone industry, local government and plants play the important role to perform the responsibilities, respectively.

Government is considered as a leader in this paper. As a leader level, the government has responsibility to insure the local environment from pollution. Natural and artificial stone industry, imitates large volume of stone waste that's why stone industry is always regarded as a high-emission industry for the stone dust and waste water. The greatest waste concerns in the stone industry are the stone dust and waste water. Both are significantly affected the environment and local system, badly ruin the vegetations and pollute the air and rivers. Government has duty to make a suitable plan to avoid over-exploitation and the pollution of environment. Therefore, planning a reasonable exploring limitation for every stone plant is very important for the government to ensure the local environment. Governement usually, want a sustainable development to overcome employment issue and also economic growth. In the following level, every stone plant has their predetermined level of profit. They are wanted to achieve their predetermined level of profit under the limited amount of exploring stone waste. Stone plants have responsibility to keep the environment unpolluted according to the plan of government. Stone plants should also increase their investment to attain the sustainable development according to the increasing demand and supply of stone products. Stone plants try their best to overcome the problem of unemployment according to their capacity under the policy of government and economic growth. So it is clear that the problem mentioned above is considered as a bi-level optimization problem. In this bi-level model government acts as a leader and stone plants are followers. It is assumed here, that there is perfect exchange of information among all the participants such that objective functions and constraints are known.

For the stone industry which contains several plants, these are the objectives of the government authority (upper-level decision-maker) to minimize the environmental pollution, and maximize the social employment and economic growth. This can be achieved by optimizing the design of exploring amount of the stone resources between the participating plants, which are assumed to cooperate among themselves and thus act as a lower-level decision-maker. Note that industrial symbiosis implicitly requires cooperative behavior of the participants [16, 19]. The government can influence the stone plants by imposing disincentives in the form of assigning different exploring amounts to plants according to their scale of production and clean technology. The plants operate independently of each other.

Since the emissions were not constantly monitored in the process of exploring the stone mine and producing the stone products, it results in the lack of the historical data about the emissions of the stone dust and waste water. We have to consider them as fuzzy numbers according to those experts advice in the stone industry. In this paper, the coefficient of stone dust and waste water amount cannot be estimated by the statistical method and hence they are regarded as fuzzy numbers.

90.3 The Optimal Model

The problem of optimizing the stone industry is formulated as a bi-level multi-objective optimization problem with fuzzy coefficients, in which the government is taken as the leader level decision maker and the stone plants are taken as the followers level decision maker.

90.3.1 Assumptions and Notations

(1) Assumptions

1. Government has considered a possibilistic level of pollution that environment can be tolerate.
2. Emission of a stone dust is directly proportional to the amount of stone mine that explores and the amount of stone which is used to process into different kinds of product.
3. Emission of waste water is directly proportional to the amount of stone which is used to produce different kinds of product.
4. Employment is directly related to the amount of production.
5. Variable cost is directly proportional to the amount of production.
6. Every plant has their predetermined level of profit.

(2) Notations

Indices

Φ : set of stone materials, i is an index, $\Phi = \{1,2,\cdots,m\}$;
Ψ : Set of stone plants, j is an index, $\Psi = \{1,2,\cdots,n\}$;
Ω : set of product k is an index, $\Omega = \{1,2,\cdots,w\}$.

Parameters

$\widetilde{Ed}_{ij}$: coefficient of emission of stone dust when plant j explores stone mines i;
$\widetilde{ed}_{ijk}$: coefficient of emission of stone dust when plant j produce k kind of product by using stone mines i;
$\widetilde{ew}_{ijk}$: coefficient of emission of waste water when plant j produced k kind of product by using stone mines i;
p_{ij} : coefficient of employment that stone plant j explores stone mine i;
P_{ijk} : coefficient of employment that plant j produce k sorts of product with stone mine i;
$\bar{f}ep$: predetermined employment level;
c_k : unit price of product k;
C_{ijk} : unit veritable cost that plant j produce product k;
h_{ij} : holding cost that plant j hold the remanent stone materials i;
θ_{ijk} : transfer rate that plant j produces product k by using stone material i;
Y_{ij} : the amount of stone mine i that explore stone plant j;
X_{ijk} : the amount of product k that produced stone plant j by stone mine i;

$\bar{f}_p$: predetermined level of profit of plant;
$\overline{PC}_j^U$: upper limitation of production cost of plant j;
$\overline{D}_k$: forecasted future demand of product k;
ED^u : upper limitation of total emission of stone dust in this region;
EW^u : upper limitation of total emission of waste water in this region;
$\overline{Y}_{EC}$: predetermined level of economic output;
IV_j^U : upper limitation of the inventory for stone plant j;
t_{ij} : coefficient of exploring cost that bear stone plant j to explore stone mine i;
t_{ijk} : coefficient of processing cost that bear stone plant j to produced product k by stone mine i;

Decision variables

Y_{ij} : amount of stone mine i that government allows the stone plants j to explore;
X_{ijk} : the amount of product k that produced stone plant j by stone mine i;
Z_{ijk} : binary variable; 1, the plant j produces the product k; 0, otherwise.

90.3.2 Model Formulation

The bi-level multi-objective optimization programming model under fuzzy environment of multiple items in stone industry can be mathematically formulated as follows.

(1) Government model (leader)

Government is leader in this model. As a leader level the government has debt instrument to ensure the local environment and sustainable development in stone industry, solve the employment issue and economic growth. The following objectives are counted by the government.

• To get the minimum emission including the stone dust and waste water when all the plants explores the stone mine and produce different sorts of stone products.

$$\min F_1 = \sum_{i\in\Phi}\sum_{j\in\Psi}\widetilde{Ed}_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\left(\widetilde{ed}_{ijk}X_{ijk}+\widetilde{ew}_{ijk}X_{ijk}\right),$$

where $\widetilde{Ed}_{ij}$ is a coefficient of emission of a stone dust when plant j explores stone mine i and Y_{ij} is an amount of stone mine i that explores plant j. The $\widetilde{ed}_{ijk}$ and $\widetilde{ew}_{ijk}$ are the coefficient of emission of stone dust and waste water respectively when plant j produces product k of amount X_{ijk} by using stone mine i.

Atypically, it is very hard to get the precise amount of minimum emission and decision makers only need the minimum objective under a certain possiblistic level. Therefore above objective is normally converted into a possiblistic constrained.

$$\begin{cases}\min \overline{F}_1\\ \text{s.t. } Pos\left\{\sum_{i\in\Phi}\sum_{j\in\Psi}\widetilde{Ed}_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\widetilde{ed}_{ijk}X_{ijk}+\widetilde{ew}_{ijk}X_{ijk})\leq\overline{F}_1\right\}\geq\delta_1^U,\end{cases} \tag{90.1}$$

where *Pos* is the possibility measure proposed by Dubois and Prade [18], and $\geq \delta_1^U$ is the possibilistic level which respects the possibility that decision makers get the minimum objective.

• To get the maximum employment the government has obtained the following objective function.

$$\max F_2 = \sum_{i\in\Phi}\sum_{j\in\Psi} p_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} P_{ijk}X_{ijk},$$

where p_{ij} is the employment that plant j explores the stone mine i and P_{ijk} is the employment that stone plant j produced product k by stone mine i.

To maximize employment is the second objective of government. Government is proposed predetermined amount of employment to maximize the employment. So that above objective is converted into constraint as follow.

$$\begin{cases} \max F_2 \\ \text{s.t.} \left\{ \sum_{i\in\Phi}\sum_{j\in\Psi} p_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} P_{ijk}X_{ijk} \right\} \geq \bar{f}_{ep} \end{cases} \tag{90.2}$$

• Government want to maximize their economic output than that of predetermined level.

$$\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \theta_{ijk}X_{ijk} \geq \overline{Y}_{EC}. \tag{90.3}$$

• The stone dust and waste water should be less than the predetermined levels in order to guarantee the air and water quality. We get two constraints under the possibilistic levels.

$$Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi} \widetilde{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \widetilde{ed}_{ijk}X_{ijk} \leq ED^u \right\} \geq \delta_2^U, \tag{90.4}$$

$$Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \widetilde{ew}_{ijk}X_{ijk} \leq EW^u \right\} \geq \delta_3^U. \tag{90.5}$$

(2) Plant Model (followers)

As a follower level, the stone plants have their own objectives to get the highest profit and sustainable development for increases production to meet the increasing forecasted demand of stone materials. The following objectives are considered by stone plants.

• Maximizing their profit is the first objective of plants, that is obtained follow.

$$\max H_1 = \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} c_k\theta_{ijk}X_{ijk} - \sum_{i\in\Phi}\sum_{j\in\Psi} f(Y_{ij}) - \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} f(X_{ijk})$$

$$-\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}C_{ijk}-h_{ij}\left(Y_{ij}-\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}X_{ijk}\right),$$

where $c_k\theta_{ijk}X_{ijk}$ is the total revenue of stone plant j, $f(Y_{ij})$ is the cost that plant j occurs to explore stone mine i, $f(X_{ijk})$ is the cost occurs when stone plant j produce product k by mine stone i, C_{ijk} is the constant cost of stone plant j and $h_{ij}(Y_{ij}-\sum X_{ijk})$ is the holding cost of the remnant of stone mine i that hold the stone plant j, where $f(X)_{ijk}$ is the production caste function as follows,

$$f(X_{ijk})=\begin{cases}t_{ij}Y_{ij}+t_{ijk}X_{ijk}+C_{ijk}Z_{ijk}, & \text{if } X_ijk\geq 0\\ 0, & \text{if } X_ijk=0.\end{cases} \tag{90.6}$$

Maximizing profit is the first objective of stone plant. To get the maximum profit stone plants have predetermined level of profit. Profit should not be less than the predetermined level of profit. This is right constraint of above objective. Which is stated as follow.

$$\begin{cases}\max\overline{H}_1\\ \text{s.t.}\sum_{j\in\Psi}\sum_{j\in\Psi}\sum_{k\in\Omega}c_k\theta_{ijk}X_{ijk}-\sum_{i\in\Phi}\sum_{j\in\Psi}f(Y_{ij})-\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}f(X_{ijk})\\ \quad-\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}C_{ijk}-h_{ij}\left(Y_{ij}-\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}X_{ijk}\right)\geq\bar{f}_p.\end{cases} \tag{90.7}$$

• Maximizing production is the second objective of stone plant, which is stated as follow.

$$\max H_2=\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\theta_{ijk}X_{ijk}.$$

In the postponement literature, most researchers assume that product demand in each period are random and are independent across time. Especially in developing country, product demand increase with the passage of time. As a result of these assumptions, the stone plants plane to forecast future demand of market and increase their production according to it. So the aggregate demand for time period t is right constraint of above objective. That is as follow,

$$\begin{cases}\max H_2\\ \text{s.t.}\sum_{i\in\Phi}\sum_{j\in\Psi}X_{ijk}\geq\overline{D}_k\end{cases} \tag{90.8}$$

• Production cost should not exceed the predefined level of cost.

$$\sum_{i\in\Phi}t_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{k\in\Omega}t_{ijk}X_{ijk}+\sum_{i\in\Phi}\sum_{k\in\Omega}C_{ijk}Z_{ijk}-h_{ij}\left(Y_{ij}+\sum_{i\in\Phi}\sum_{k\in\Omega}X_{ijk}\right)\leq\overline{PC}_j^U. \tag{90.9}$$

• Inventory should not exceed the maximum limitation.

$$Y_{ij} - \sum_{i\in\Phi}\sum_{k\in\Omega} X_{ijk} \le IV_j^U. \tag{90.10}$$

90.3.3 Global Model

As a complicated system, both the leader and the followers should simultaneously consider the objectives and constraints with each other and then make the decision. Therefore, from Equations (90.1) $\sim$ (90.10), the whole bi-level optimization model under fuzzy environment should be given as follows,

$$\begin{cases}
\min \overline{F}_1 \\
\max \overline{F}_2 \\
\text{s.t.} \begin{cases}
Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi} \widetilde{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} (\widetilde{ed}_{ijk}X_{ijk} + \widetilde{ew}_{ijk}X_{ijk}) \le \overline{F}_1 \right\} \ge \delta_1^U \\
\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \theta_{ijk}X_{ijk} \ge \overline{Y}_{EC} \\
Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi} \widetilde{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \widetilde{ed}_{ijk}X_{ijk} \le ED^u \right\} \ge \delta_2^U \\
Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} \widetilde{ew}_{ijk}X_{ijk} \le EW^u \right\} \ge \delta_3^U \\
\sum_{i\in\Phi}\sum_{j\in\Psi} p_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} P_{ijk}X_{ijk} \ge \bar{f}_{ep} \\
\max H_1 \\
\max H_2 \\
\text{s.t.} \begin{cases}
\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} c_k\theta_{ijk}X_{ijk} - \sum_{i\in\Phi}\sum_{j\in\Psi} f(Y_{ij}) - \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} f(X_{ijk}) \\
- \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} C_{ijk} - h_{ij}\left(Y_{ij} - \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} X_{ijk} \right) \ge \bar{f}_p \\
\sum_{i\in\Phi}\sum_{j\in\Psi} X_{ijk} \ge \overline{D}_k \\
\sum_{i\in\Phi} t_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{k\in\Omega} t_{ijk}X_{ijk} + \sum_{i\in\Phi}\sum_{k\in\Omega} C_{ijk}Z_{ijk} - h_{ij}\left(Y_{ij} + \sum_{i\in\Phi}\sum_{k\in\Omega} X_{ijk} \right) \\
\le \overline{PC}_j^U \\
Y_{ij} - \sum_{i\in\Phi}\sum_{k\in\Omega} X_{ijk} \le IV_j^U.
\end{cases}
\end{cases}
\end{cases} \tag{90.11}$$

90.3.4 Equivalent Model

As we all know that, it is hard for decision maker's to find optimal strategies for the multi-objective programming with fuzzy coefficients. In this paper we had possibilistic constraints to get the minimum objective, so without membership functions we cannot convert the model into crisp equivalent model. According to expert's experience in stone industry we are used L-R membership functions. $\widetilde{Ed}_{ij}$, $\widetilde{ed}_{ijk}$ and

$\widetilde{ew}_{ijk}$ ($i \in \Phi = 1,2,\cdots,m$; $j \in \Psi = 1,2,\cdots,n$; $k \in \Omega = 1,2,\cdots,w$) are fuzzy numbers with L-R membership functions in this paper.

Lemma 90.1. *Assume that* $\widetilde{Ed}_{ij}$, $\widetilde{ed}_{ijk}$ *and* $\widetilde{ew}_{ijk}$ ($i \in \Phi = 1,2,\cdots,m$; $j \in \Psi = 1,2,\cdots,n$; $k \in \Omega = 1,2,\cdots,w$) *are L-R fuzzy numbers with the following membership functions,*

$$u_{\widetilde{Ed}_{ij}}(t) = \begin{cases} L\left(\frac{Ed_{ij}-t}{\alpha_{ij}^{Ed}}\right), \ t < Ed_{ij}, \ \alpha_{ij}^{Ed} > 0 \\ R\left(\frac{t-Ed_{ij}}{\beta_{ij}^{Ed}}\right), \ t \geq Ed_{ij}, \ \beta_{ij}^{Ed} > 0, \end{cases} \tag{90.12}$$

$$u_{\widetilde{ed}_{ijk}}(t) = \begin{cases} L\left(\frac{ed_{ijk}-t}{\alpha_{ijk}^{ed}}\right), \ t < ed_{ijk}, \ \alpha_{ijk}^{ed} > 0 \\ R\left(\frac{t-ed_{ijk}}{\beta_{ijk}^{ed}}\right), \ t \geq ed_{ijk}, \ \beta_{ijk}^{ed} > 0, \end{cases} \tag{90.13}$$

$$u_{\widetilde{ew}_{ijk}}(t) = \begin{cases} L\left(\frac{ew_{ijk}-t}{\alpha_{ijk}^{ew}}\right), \ t < ew_{ijk}, \ \alpha_{ijk}^{ew} > 0 \\ R\left(\frac{t-ew_{ijk}}{\beta_{ijk}^{ew}}\right), \ t \geq ew_{ijk}, \ \beta_{ijk}^{ew} > 0. \end{cases} \tag{90.14}$$

where α_{ij}^{Ed}, β_{ij}^{Ed} *are positive numbers expressing the left and right spreads of* $\widetilde{Ed}$, α_{ijk}^{ed}, β_{ijk}^{ed} *are the positive numbers expressing the left and right spreads of* $\widetilde{ed}$, *and* α_{ijk}^{ew}, β_{ijk}^{ew} *are positive numbers expressing the left and right spreads of* $\widetilde{ew}$, ($i \in \Phi = 1,2,\cdots,m$; $j \in \Psi = 1,2,\cdots,n$; $k \in \Omega = 1,2,\cdots,w$). *Reference functions* $L,R : [0,1] \longrightarrow [0,1]$ *with* $L(1) = R(1) = 0$ *and* $L(0) = R(0) = 1$ *are non-increasing, continuous functions. Then we have*

$$Pos\left\{ \sum_{i\in\Phi}\sum_{j\in\Psi} \widetilde{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} (\widetilde{ed}_{ijk}X_{ijk} + \widetilde{ew}_{ijk}X_{ijk}) \leq \overline{F}_1 \right\} \geq \delta_1^U,$$

if and only if

$$\overline{F}_1 \geq \sum_{i\in\Phi}\sum_{j\in\Psi} Ed_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} (ed_{ijk} + ew_{ijk})X_{ijk}$$
$$-L^{-1}(\delta_1^U)\left(\sum_{i\in\Phi}\sum_{j\in\Psi} \alpha_{ij}^{Ed}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega} (\alpha_{ijk}^{ed} + \alpha_{ijk}^{ew})X_{ijk} \right).$$

Proof. Let $w \in [0,1]$ be any positive real number and

$$L\left(\frac{Ed_{ij}-x}{\alpha_{ij}^{Ed}}\right) = L\left(\frac{ed_{ij}-y}{\alpha_{ijk}^{ed}}\right) = L\left(\frac{ew_{ijk}-y}{\alpha_{ijk}^{ew}}\right) = w,$$

then from Equations (90.12), (90.13) and (90.14) we have

$$x = Ed_{ij} - \alpha_{ij}^{Ed}L^{-1}(w),$$

$$y = ed_{ijk} - \alpha^{ed}_{ijk} L^{-1}(w),$$
$$z = ew_{ijk} - \alpha^{ew}_{ijk} L^{-1}(w).$$

For any

$$Y_{ij},\ X_{ijk} \geq 0,\ (i \in \Phi = 1,2,\cdots,m;\ j \in \Psi = 1,2,\cdots,n;\ k \in \Omega = 1,2,\cdots,w),$$

it is easily follows that,

$$\begin{aligned} t = & \sum_{i\in\Phi}\sum_{j\in\Psi} xY_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(yX_{ijk} + zX_{ijk}) \\ = & \left[\sum_{i\in\Phi}\sum_{j\in\Psi} Ed_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(ed_{ijk} + ew_{ijk})X_{ijk}\right] \\ & - \left[\sum_{i\in\Phi}\sum_{j\in\Psi} \alpha^{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\alpha^{ed}_{ijk} + \alpha^{ew}_{ijk})X_{ijk}\right] L^{-1}(w). \end{aligned}$$

Therefore, we have

$$L\left(\frac{\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} Ed_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(ed_{ijk} + ew_{ijk})X_{ijk} - t}{\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} \alpha^{Ed}_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(\alpha^{ed}_{ijk} + \alpha^{ew}_{ijk})X_{ijk}}\right) = w. \tag{90.15}$$

It is also proved by the same method that,

$$R\left(\frac{t - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} Ed_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(ed_{ijk} + ew_{ijk})X_{ijk}}{\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} \beta^{Ed}_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(\beta^{ed}_{ijk} + \beta^{ew}_{ijk})X_{ijk}}\right) = w. \tag{90.16}$$

Hence, it is easily found that

$$\sum_{i\in\Phi}\sum_{j\in\Psi} \widetilde{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\widetilde{ed}_{ijk}X_{ijk} + \widetilde{ew}_{ijk})X_{ijk}$$

is also a L-R fuzzy number with the left spread

$$\sum_{i\in\Phi}\sum_{j\in\Psi} \alpha^{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\alpha^{ed}_{ijk} + \alpha^{ew}_{ijk})X_{ijk}$$

and right spread

$$\sum_{i\in\Phi}\sum_{j\in\Psi} \beta^{Ed}_{ij}Y_{ij} + \sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\beta^{ed}_{ijk} + \beta^{ew}_{ijk})X_{ijk}.$$

According to the definition of possibility measure proposed by Dubois and Prade [18], it can be obtained as follows,

$$Pos\left\{\sum_{i\in\Phi}\sum_{j\in\Psi}\widetilde{Ed}_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\widetilde{ed}_{ijk}X_{ijk}+\widetilde{ew}_{ijk}X_{ijk})\leq\overline{F}_1\right\}\geq\delta_1^U \tag{90.17}$$

$$\Leftrightarrow L\left(\frac{\sum_{i\in\Phi}\sum_{j\in\Psi}Ed_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(ed_{ijk}+ew_{ijk})X_{ijk}-\overline{F}}{\sum_{i\in\Phi}\sum_{j\in\Psi}\alpha_{ij}^{Ed}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\alpha_{ijk}^{ed}+\alpha_{ijk}^{ew})X_{ijk}}\right)\geq\delta_1^U \tag{90.18}$$

$$\Leftrightarrow \frac{\sum_{i\in\Phi}\sum_{j\in\Psi}Ed_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(ed_{ijk}+ew_{ijk})X_{ijk}-\overline{F}}{\sum_{i\in\Phi}\sum_{j\in\Psi}\alpha_{ij}^{Ed}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\alpha_{ijk}^{ed}+\alpha_{ijk}^{ew})X_{ijk}}\leq L^{-1}\delta_1^U \tag{90.19}$$

$$\Leftrightarrow \sum_{i\in\Phi}\sum_{j\in\Psi}Ed_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(ed_{ijk}+ew_{ijk})X_{ijk}$$
$$-L^{-1}\delta_1^{(U)}\left(\sum_{i\in\Phi}\sum_{j\in\Psi}\alpha_{ij}^{Ed}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}(\alpha_{ijk}^{ed}+\alpha_{ijk}^{ew})X_{ijk}\right)\leq\overline{F}_1. \tag{90.20}$$

This completes the proof.

From Lemma 90.1 and its proof, apparent that the possibilistic constraint can be transformed into crisp one. So that, the remaining result can be easily calculated according to Lemma 90.1 as follow,

$$Pos\left\{\sum_{i\in\Phi}\sum_{j\in\Psi}\widetilde{Ed}_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\widetilde{ed}_{ijk}X_{ijk}\leq ED^U\right\}\geq\delta_2^U$$

is equivalent to the following equation:

$$\sum_{i\in\Phi}\sum_{j\in\Psi}Ed_{ij}Y_{ij}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}ed_{ijk}X_{ijk}$$
$$-L^{-1}\delta_2^U\left(\sum_{i\in\Phi}\sum_{j\in\Psi}\alpha_{ij}^{ED}+\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\alpha_{ijk}^{ed}\right)\leq ED^U. \tag{90.21}$$

$$Pos\left\{\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\widetilde{ew}_{ijk}X_{ijk}\leq EW^u\right\}\geq\delta_3^U$$

is equivalent to the following equation,

$$\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}ew_{ijk}X_{ijk}-L^{-1}(\delta)_3^U\sum_{i\in\Phi}\sum_{j\in\Psi}\sum_{k\in\Omega}\alpha_{ijk}^{ew}\leq EW^U. \tag{90.22}$$

$$
\begin{cases}
\min F_1^* = \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} Ed_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(ed_{ijk}+ew_{ijk})X_{ijk} \\
\qquad -L^{-1}\delta_1^{(U)}\left(\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\alpha_{ij}^{Ed}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}(\alpha_{ijk}^{ed}+\alpha_{ijk}^{ew})X_{ijk}\right) \\
\max F_2 = \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} p_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} P_{ijk}X_{ijk} \\
\text{s.t.}
\begin{cases}
\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} p_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} P_{ijk}X_{ijk} \geq \bar{f}_{ep} \\
\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}\theta_{ijk}X_{ijk} \geq \overline{Y}_{EC} \\
\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} Ed_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} ed_{ijk}X_{ijk} \\
-L^{-1}\delta_2^U\left(\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\alpha_{ij}^{ED} + \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}\alpha_{ijk}^{ed}\right) \leq ED^U \\
\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} ew_{ijk}X_{ijk} - L^{-1}(\delta)_3^U\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega}\alpha_{ijk}^{ew} \leq EW^U \\
\begin{cases}
\max H_1 = \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} c_k\theta_{ijk}X_{ijk} - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} f(Y_{ij}) - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} f(X_{ijk}) \\
\qquad - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} C_{ijk} - hij\left(Y_{ij} - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} X_{ijk}\right) \\
\max H_2 = \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} X_{ijk} \geq \overline{D}_k \\
\text{s.t.}
\begin{cases}
\sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} c_k\theta_{ijk}X_{ijk} - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi} f(Y_{ij}) - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} f(X_{ijk}) \\
- \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} C_{ijk} - hij\left(Y_{ij} - \sum\limits_{i\in\Phi}\sum\limits_{j\in\Psi}\sum\limits_{k\in\Omega} X_{ijk}\right) \geq \bar{f}_p \\
\sum\limits_{i\in\Phi} t_{ij}Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{k\in\Omega} t_{ijk}X_{ijk} + \sum\limits_{i\in\Phi}\sum\limits_{k\in\Omega} C_{ijk}Z_{ijk} - hij\left(Y_{ij} + \sum\limits_{i\in\Phi}\sum\limits_{k\in\Omega} X_{ijk}\right) \\
\leq \overline{PC}_j^U \\
Y_{ij} - \sum\limits_{i\in\Phi}\sum\limits_{k\in\Omega} X_{ijk} \leq IV_j^U.
\end{cases}
\end{cases}
\end{cases}
\end{cases}
\tag{90.23}
$$

90.4 Case Study

In this section, a practical example in Pakistan is considered to show the whole process of the modeling, that is proposed as follow.

90.4.1 Background Review

Pakistan is the sixth largest country in term of natural stone resources, especially marble and granite. Pakistan has major deposit of high quality marble and granite in a wide range of colors, shades and patterns. Almost all provinces in Pakistan have natural stone resources. Initial estimation indicates 166 billion tons of marble and wide rang of granite are reserved across Pakistan. It can be processed into many kinds of useful products mainly include marble products, granite sand, granite slabs, man made slabs, nano calcium carbonate, natural building material, artificial stone products and etc. Although Pakistan is rich in stone resources, the stone industry is not well developed. There are some reasons that are mentioned below.

- Stone plants have old and poor quarrying technique. At present most of stone quarries are operating with old technique of blasting and do not have basic machinery and equipment. They explore stone mine in a disorder way and it results in waste of natural stone resources and also causes air pollution, water pollution and vegetation deterioration.
- On the processing side there are very few units with a complete rang of machinery and equipment capable of processing stone in according to international standard. Most of stone plants have old technology and skill its result in lower production rate, waste of natural stone resources and environment pollution.
- Stone plants are producing many common products. They haven't produced high valuable products. It means that stone industry doesn't provide high economic growth and enough employment.
- Government has no long term policy about exploring stone mine and also no strategy to prevent environment by the pollution of stone industry.

Table 90.1 Parameters of granite for every stone plant j which explores stone mine i

Granite stone plant	Parameters					
	$\widetilde{Ed}_{ij}$ (kg/m^3)	P_{ij} (Person)	t_{ij} (PKR/m^3)	h_{ij} (PKR/m^3)	IV_{ij}^U (M m^3)	$\overline{PC}_j^U$ (M PKR)
Master	(32.7,33.2,34.5)	120	120.45	7.50	2.7	25000
Shabir	(33.8,35.4,37.2)	133	115.34	6.85	1.3	24500
Hanam	(34.5,35.8,36.9)	122	118.23	6.50	1.8	26000
Norani	(29.6,31.7,34.5)	112	110.55	5.80	1.2	22300
Mir	(31.7,33.5,37.3)	105	112.70	4.6	.8	22000

90.4.2 Data and Computation

Pakistan Stone Development Company (PASDEC), a public limited company and subsidiary of Pakistan Industrial Development Corporation, has initiated many projects to uplift the existing set-up of marble and granite sector. Despite over 166 billion tons of reserves, more than 70 types of marble and granite available, 1,225 operational mines and more than 2,000 processing factories, the marble and granite industry in Pakistan is quite underdeveloped. Pakistan's marble and granite industry is determined to achieve one goal: extraction of squared blocks. In the long run the industry's vision is to establish itself among the socially responsible and globally competitive dimension stone industry in the world. For this Pakistan is taking necessary steps to overhaul the value chain. Next it plans to invest in more advanced processing capabilities to build on the upgraded raw materials.

Table 90.2 Parameters of marble for every stone plant j which explores stone mine i

Granite stone plant	Parameters					
	$\widetilde{Ed}_{ij}$ (kg/m^3)	P_{ij} (Person)	t_{ij} (PKR/m^3)	h_{ij} (PKR/m^3)	IV_{ij}^U (M m^3)	$\overline{PC}_j^U$ (M PKR)
Master	(24.7,25.2,26.5)	105	118.45	10.50	1.7	28000
Shabir	(24.8,25.4,27.2)	115	116.34	11.85	1.5	25000
Hanam	(24.5,25.8,26.9)	122	120.23	9.50	1.	26000
Norani	(25.6,26.7,27.5)	100	125.55	13.80	1.2	29000
Mir	(25.7,26.5,27.3)	105	119.70	10.6	1.8	27000

Table 90.3 Parameters for every granite stone product

Parameter	Granite stone products			
	Marble chips	Marble blocks	Marble slabs	Marble tiles
c_k	12000 (PKR/ton)	400 (PKR/m^2)	550 (PKR/m^2)	650 (PKR/m^2)
$\overline{D}_k$	12.5×10^{10} (ton)	3.5×10^5 (m^2)	5.5×10^8 (m^2)	10.5×10^{12} (m^2)

Table 90.4 Parameters for every marble stone product

Parameter	Marble stone products			
	Nano calcium carbonate	Granite slabs	Granite sand	Man mad composite slabs
c_k	12000 (PKR/ton)	400 (PKR/m^2)	550 (PKR/m^2)	650 (PKR/m^2)
$\overline{D}_k$	12.5×10^{10} (ton)	3.5×10^5 (m^2)	5.5×10^8 (m^2)	10.5×10^{12} (m^2)

All the emission coefficients of the stone dust are considered triangular fuzzy numbers listed in Tables 90.1 and 90.2 by the expert's advice, so it is easy to convert the fuzzy model into crisp form. As the demand and the price of the marble and granite stone products sharply increase, the government requires that their output from all the plants should at least satisfy the basic market demand $\overline{D}_k (k \in \Omega = 1, 2, \cdots, w)$, is a future forecasted demand of granite and marble products by stone plants which are found in Tables 90.3 and 90.4. The unit price of every stone products can be found in Tables 90.3 and 90.4. The upper limitations of the inventory and production for every stone plant are also listed in Tables 90.1 and 90.2. The possibilistic level δ_j^L that plant j want to obtain the minimum emissions can be found in Tables 90.1 and 90.2. Since every plant has the different capacities in controlling the emissions, the fixed and unit variable cost, emission coefficients and constant costs are different from each other, which can be found in Tables 90.5 and 90.6. The transform rate θ_{ijk} and the lower limitation of the product k in plant j are also listed in Tables 90.5 and 90.6.

Table 90.5 Parameters for granite stone product *k* which is produced stone plant *j* by stone mine *i*

Stone plant	Granite stone products	Parameter					
		P_{ijk}	C_{ijk}	t_{ijk}	θ_{ijk}	$\widetilde{ed}_{ijk}$	$\widetilde{ew}_{ijk}$
Master	NCC	4	230	850	5.0	(4.50,6.45,7.23)	(15.50,16.50,17.90)
	GSl	2	310	260	235.20	(25.42,26.65,29.23)	(12.2,13.90,14.90)
	GSa	2	45	130	105	(28.56,30.2,33.15)	(0.56,0.90,1.20)
	MmS	3	380	190	25.5	(2.65,3.90,5.20)	(3.20,3.85,4.25)
Shabir	NCC	3	240	870	5.50	(3.90,5.10,6.42)	(4.35,4.95,5.25)
	GSl	1	290	255	240.50	(24.23,26.10,27.95)	(25.69,26.24,27.56)
	GSa	1	60	135	2.00	(30.45,32.50,34.65)	(0.78,0.98,1.45)
	MmS	4	370	195	27.00	(2.90,3.75,4.60)	(3.70,4.41,5.45)
Hanam	NCC	3	235	880	4.70	(4.65,5.90,7.10)	(4.30,5.10,6.25)
	GSl	2	300	265	220.00	(25.90,26.5,27.56)	(25.36,26.45,27.10)
	GSa	2	55	145	1.00	(27.90,28.60,29.65)	(0.70,1.10,1.75)
	MmS	3	380	185	24.00	(2.50,3.70,4.20)	(3.45,3.95,4.58)
Norani	NCC	4	245	855	5.00	(4.65,5.45,6.87)	(4.65,5.14,5.85)
	GSl	2	285	270	230.5	(25.23,26.40,27.90)	(25.45,26.10,27.23)
	GSa	2	65	140	0.00	(27.90,28.70,29.89)	(0.58,0.97,1.25)
	MmS	3	390	170	24.5	(2.60,3.95,4.59)	(3.85,4.15,4.89)
Mir	NCC	3	240	865	4.5	(4.60,5.70,6.75)	(4.30,5.10,5.95)
	GSl	1	320	265	215.00	(25.90,26.75,27.89)	(25.20,26.45,27.10)
	GSa	1	50	140	1.00	(28.45,29.74,30.56)	(0.50,095,1.45)
	MmS	3	365	135	26.00	(2.56,3.87,4.50)	(3.25,3.95,4.58)

Table 90.6 Parameters for marble stone product *k* which is produced stone plant *j* by stone mine *i*

Stone plant	Marble stone products	Parameter					
		P_{ijk}	C_{ijk}	t_{ijk}	θ_{ijk}	$\widetilde{ed}_{ijk}$	$\widetilde{ew}_{ijk}$
Master	MBCs	3	250	650	55.0	(14.50,16.45,17.23)	(15.50,16.50,17.90)
	MBBs	4	180	470	5.50	(15.42,16.65,19.23)	(8.2,9.90,10.90)
	MBSs	3	150	250	20	(18.56,20.2,21.15)	(9.56,10.90,11.20)
	MBTs	3	240	550	25.5	(25.65,26.90,27.20)	(26.20,27.85,28.25)
Shabir	MBCs	4	260	640	50.50	(13.90,15.10,16.42)	(14.35,15.95,16.25)
	MBBs	4	170	465	4.50	(14.23,16.10,17.95)	(8.69,9.24,10.56)
	MBSs	3	145	245	22.00	(18.45,19.50,20.65)	(10.78,11.98,12.45)
	MBTs	4	250	560	27.00	(24.90,25.75,26.60)	(26.70,27.41,28.45)
Hanam	MBCs	3	255	660	54.70	(14.65,15.90,17.10)	(14.30,15.10,16.25)
	MBBs	4	175	460	3.00	(15.90,16.5,17.56)	(8.36,9.45,10.10)
	MBSs	3	145	250	21.00	(17.90,18.60,19.65)	(10.70,11.10,12.75)
	MBTs	3	250	565	24.00	(25.50,26.70,27.20)	(26.45,27.95,28.58)
Norani	MBCs	4	260	656	52.00	(14.65,15.45,16.87)	(14.65,15.14,15.85)
	MBBs	3	165	460	2.5	(15.23,16.40,17.90)	(8.45,9.10,10.23)
	MBSs	3	140	250	20.00	(17.90,18.70,19.89)	(10.58,11.97,12.25)
	MBTs	3	250	565	24.5	(25.60,26.95,27.59)	(26.85,27.15,28.89)
Mir	MBCs	4	240	665	50.5	(14.60,15.70,16.75)	(14.30,15.10,15.95)
	MBBs	4	165	455	4.00	(15.90,16.75,17.89)	(8.20,9.45,10.10)
	MBSs	3	140	264	21.00	(18.45,19.74,20.56)	(10.50,11.95,12.45)
	MBTs	2	270	570	26.00	(25.56,26.87,27.50)	(26.25,27.95,28.58)

90.5 Conclusion

In this paper, we have developed a bi-level multi-objective optimization model with fuzzy coefficients and possibilistic constraints under the fuzzy environment. In the model, the government was considered as the leader level for minimizing the emissions of the stone dust and the waste water and maximizing the employment and economic growth and stone plants were considered as the followers level for maximizing the profit, sustainable development and minimizing the emissions. In this paper, the developed model has been converted into the crisp equivalent one to deal with some special fuzzy numbers. At the end, a practical case was presented for the proposed model.

References

1. Al-Jabri M, Sawalh H (2002) Treating stone cutting waste by flocculation-sedomentation. In: Proceeding of the Sustainable Enviromental Sanitation and Water Services Conference, 28th WEDC conference, Calcutta, India
2. Nasserdine K, Mimi Z, Bevan B et al (2009) Environmental management of the stone cutting industry. Journal of Environmental Management 90:466–470
3. Almeida N, Branco F, Santos JR (2007) Recycling of stone slurry in industrial activities: Application to concrete mixtures. Building and Environment 42:810–819
4. CH2MHILL (2002) Herbon industerial waste water controle and pretreatment fesibility study. Task 10, unpublisheed report
5. Montero MA, Jordan MM, Almendro-Candel MB et al (2009) The use of a calcium carbonate residue from the stone industry in manufacturing of ceramic tile bodies. Applied Clay Science 43:186–189
6. Ammary BY (2007) Clean production in stone cutting industries. International Journal of Enviroment and Waste Managment 1:106–112
7. Algin HM, Turgut P (2008) Cotton and limestone powder wastes as brick material. Construction and Building Materials 22:1074–1080
8. Hebhoub H, Aoun H, Belachia M et al (2011) Use of waste marble aggregates in concrete. Construction and Building Materials 25:1167–1171
9. Pearce DW, Warford JJ (1993) World without end: Economics, environment and sustainable development. Oxford University Press/World Bank, Oxford
10. Grossman GM, Krueger AB (1991) Environmental impacts of a North American free trade agreement. Discussion Paper No. 158, Woodrow Wilson School, Princeton University, Princeton, NJ
11. Ryu JH, Dua V, Pistikopoulos EN (2004) A bi-level programming framework for enterprise-wide process networks under uncertainty. Computers and Chemical Engineering 2:1121–1129
12. Takama N, Umeda T (1980) Multi-level, multi-objective optimization in processing engineering. Chemical Engineering Science 36:129–136
13. Teresa W, Som S, Mengqi H (2011) The application of memetic algorithms for forearm crutch design: A case study. Mathematical Problems in Engineering, Doi:10.1155/2011/162580
14. Bellman RE, Zadeh LA (1970) Decision-making in a fuzzy environment. Management Science 17:141–164.
15. Shih HS, Lai YJ, Lee ES (1996) Fuzzy approach for multi-level programming problems. Computers and Operations Research 23:73–91

16. Singh A, Lou HH (2006) Hierarchical Pareto optimization for the sustainable development of industrial ecosystems. Industrial and Engineering Chemistry Research 45:3265–3279
17. Arora SR, Gupta R (2009) Interactive fuzzy goal programming approach for bi-level programming problem. European Journal of Operational Research 194:368–376
18. Dubois D, Prade H (1978) Operations on fuzzy numbers. International Journal of System Sciences 9:613–626
19. Lim SR, Park JM (2008) Cooperative water network system to reduce carbon footprint. Environmental Science and Technology 42:6230–6236
20. World Commission on Environment and Development (WCED) (1987) Our common future. Oxford University Press, New York