Compositional Verification of Untimed Properties for a Class of Stochastic Automata Networks

Nihal Pekergin and Minh-Anh Tran

Abstract We consider Stochastic Automata Networks whose transition rates depend on the whole system state but are not synchronised and are restricted to satisfy a property called *inner proportional*.We prove that this class of SANs has both product form steady-state distribution and product form probability over untimed paths. This product form result is then applied to check formulae that are equivalent to some special structure that we call *path-product* of sets of untimed paths. In particular, we show that product form solutions can be used to check unbounded Until formulae of the Continuous Stochastic Logic.

1 Introduction

Probabilistic model checking is an extension of the formal verification methods for systems exhibiting stochastic behaviour. The system model is usually specified as a state transition system, with probabilities attached to transitions, for example, Markov chains. A wide range of quantitative performance, reliability, and dependability measures can be specified using temporal logics such as Continuous Stochastic Logic (CSL) defined over Continuous Time Markov Chains (CTMC) [1] and Probabilistic Computational Tree Logic (PCTL) defined over Discrete Time Markov Chains (DTMC) [8]. To perform model checking by numerical analysis we need to compute transient-state or steady-state distribution of the underlying CTMC. The numerical model checking has been studied extensively and numerous algorithms [2] have

N. Pekergin (✉) · M.-A. Tran
LACL, University of Paris-Est Créteil Val de Marne, 61 avenue Général de Gaulle, 94010 Créteil, France
e-mail: nihal.pekergin@u-pec.fr

M.-A. Tran
e-mail: minh-anh.tran@u-pec.fr

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been devised and implemented in different model checkers. Despite the considerable works in the domain, the numerical Markovian analysis still remains a problem.

Different approaches have been applied to overcome the state space explosion problem. Data structures which lead to compact representations of large models such as Binary Decision Diagrams (BDD) and Multi Terminal Binary Decision Diagrams (MTBDD) with efficient manipulation algorithms have been applied to consider large models. This approach is called symbolic model checking [3]. Another approach is based on the state space reduction techniques. The idea here is to have a representation of the underlying model in a reduced-size model, which is called abstraction in model checking [10]. The large models can also be analysed by decomposition which means that sub-systems are analysed in isolation and then the global behaviour is deduced from these solutions. This is called as compositional model checking [4, 6].

The goal of this paper is to present a model checking approach which is able to take advantage of product form solutions. It is worth pointing out that product form solutions play an important role in calculating stationary distributions of Markov chains in performance evaluation [1], but, on the contrary, are believed to have no significant use in model checking. In this paper, we study a subclass of Stochastic Automata Networks (SANs) without synchronisations, which have product form steady-state distributions [7]. In the subclass, there is no synchronisation, all transition rates are functional and restricted to satisfy a property that we call *inner proportional*. This class remains large enough, for example, to generalise competing Markov chains [5]. We profit from product form solutions of this class to perform the CSL model checking for the untimed Until path and the steady-state formulae.

The rest of the paper is organised as follows: Sect. 2 gives a brief introduction for CSL and then introduces the class of SANs for which the compositional model checking is performed. Section 3 proves the product form solution for the steady-state and Sect. 4 provides the product form over untimed paths.

2 Framework and Model

CSL Model checking: In this paper, we consider the steady-state and untimed Until formulae of CSL model checking. We briefly give the syntax and semantic for these operators and we refer to [1, 2] for further information. Model $\mathcal{M}$ is a time-homogeneous CTMC with infinitesimal generator Q taking values in a set of states S. AP denotes a finite set of atomic propositions, and $L : S \to 2^{AP}$ is the labelling function which assigns to each state $s \in S$ the set $L(s)$ of atomic propositions $a \in AP$ those are valid in s. A path through $\mathcal{M}$ can be finite or infinite. A finite path σ of length n is a sequence of states: $\sigma = s^0, s^1, \ldots, s^n$ with transition rates $Q(s^i, s^{i+1}) > 0$. We denote by $paths_s$ the set of all paths starting from state s. Let p be a probability threshold and $\triangleleft$ be an arbitrary operator in the set $\{\leq, \geq, <, >\}$.The syntax of CSL is defined by :

$$\phi ::= true \mid a \mid \phi \wedge \phi \mid \neg\phi \mid \mathcal{P}_{\triangleleft p}(\phi\, \mathcal{U}\, \phi) \mid \mathcal{S}_{\triangleleft p}(\phi)$$

The expression $\mathcal{P}_{\lhd p}(\phi_1\, \mathcal{U}\, \phi_2)$ asserts that the probability measure of paths satisfying $\phi_1\, \mathcal{U}\, \phi_2$ meets the bound given by $\lhd p$. The path formula $\phi_1\, \mathcal{U}\, \phi_2$ asserts that ϕ_2 will be satisfied at some time $t \in [0, \infty)$ and that at all preceding time ϕ_1 holds. $\mathcal{S}_{\lhd p}(\phi)$ asserts that the steady-state probability for ϕ-states meets the bound $\lhd p$. We present briefly the semantics of these formula where $\models$ is the satisfaction operator:

$s \models true$	for all $s \in S$
$s \models a$	iff $a \in L(s)$
$s \models \neg\phi$	iff $s \not\models \phi$
$s \models \mathcal{P}_{\lhd p}(\phi_1\, \mathcal{U}\, \phi_2)$	iff $\mathbb{P}^{\mathcal{M}}(s, \phi_1\, \mathcal{U}\, \phi_2) \lhd p$
$s \models \mathcal{S}_{\lhd p}(\phi)$	iff $\sum_{s \mid s \models \phi} \pi^{\mathcal{M}}(s) \lhd p$

where $\mathbb{P}^{\mathcal{M}}(s, \phi_1\, \mathcal{U}\, \phi_2)$ denotes the probability measure of all paths σ starting from s ($\sigma \in paths_s$) satisfying $\phi_1\, \mathcal{U}\, \phi_2$, i.e., $\mathbb{P}^{\mathcal{M}}(s, \phi_1\, \mathcal{U}\, \phi_2) = \mathbb{P}\{\sigma \in paths_s \mid \sigma \models \phi_1\, \mathcal{U}\, \phi_2\}$; $\pi^{\mathcal{M}}(s)$ denotes the steady-state probability of state s of the chain $\mathcal{M}$. In the case $\mathcal{M}$ is ergodic, the steady-state distribution is independent of the initial state, then the steady-state formula is satisfied or not whatever the initial state.

SANs with local, functional and inner proportional transitions: Consider a network of N interacting stochastic automata $A_1, A_2, \ldots A_N$ where

- Transitions are *local*, i.e., not synchronised: it is forbidden to have two events occurring at the same time in two different automata.
- Transition rates of each automaton depend on the state of the whole system. Such transitions are also called *functional* transitions.
- For any automaton, transition rates are restricted to be *inner proportional*: they may depend on the state of all other automata; however, the proportion between two arbitrary transition rates of this automaton remains independent from the state of other automata.

In this work, we note

- $s = (s_1, s_2, \ldots, s_N)$ the state vector, $s_{-k} = (s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_N)$ the state vector without component s_k.
- $\mathcal{S}_k$ the set of states of automaton A_k, $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_N$ the set of all system states, and $\mathcal{S}_{-k} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_{k-1} \times \mathcal{S}_{k+1} \times \cdots \times \mathcal{S}_N$ the set of states of all automata other than A_k.
- $Q_k^s : \mathcal{S}_k \times \mathcal{S}_k \to \mathbb{R}$ the infinitesimal generator of A_k when the system is in state s. More precisely, transition rates of A_k are functions of the state vector s_{-k}.

In state s, the total outgoing rate from automaton A_k is $-Q_k^s(s_k, s_k) = \sum_{s'_k \neq s_k} Q_k^s(s_k, s'_k)$.

Property 1 ***(Characterisation of inner proportional transitions)*** *The transition rates of A_k are inner proportional if and only if there exists a state-dependent factor $\alpha_k : \mathcal{S}_{-k} \to \mathbb{R}$ and an infinitesimal generator $Q_k : \mathcal{S}_k \times \mathcal{S}_k \to \mathbb{R}$ which does not depend on the vector s_{-k} such that*

$$Q_k^s(s_k, s'_k) = \alpha_k(s_{-k}) Q_k(s_k, s'_k) \qquad \forall s_k, s_{k'} \in \mathcal{S}_k, \forall s_{-k} \in \mathcal{S}_{-k}. \tag{1}$$

Matrix Q_k is the infinitesimal generator of the *representative automaton* of A_k (or A_k in isolation). In isolation, the total outgoing rate from state s_k is given by $-Q_k(s_k, s_k) = \sum_{s'_k \neq s_k} Q_k(s_k, s'_k)$.

3 Product Form Solution for the Steady-State

First of all, SANs with local, functional and inner proportional transitions form a subclass of SANs without synchronisations considered in [7]. In this work, Fourneau et al. considered SANs where transitions are local and functional, but not necessarily inner proportional. They denote by F_k the set of infinitesimal generators of A_k. This notation F_k denotes the set $\{Q_k^s : s \in \mathcal{S}\}$ in our model. Theorem 6 of [7] states that a SAN with local and functional transitions has a product form steady-state distribution if for any automaton A_k there exists a probability distribution π_k that verifies the following equation

$$\pi_k Q = 0 \qquad \forall Q \in F_k. \tag{2}$$

In the view of Property 1, for inner proportional transitions, F_k is given by

$$F_k = \{\alpha_k(s_{-k}) Q_k : s_{-k} \in \mathcal{S}_{-k}\},$$

where α_k is a real-valued function of s_{-k} and Q_k is the representative infinitesimal generator of automaton A_k. Thus, a distribution π_k satisfies Eq. (2) for all infinitesimal generators of A_k if it satisfies the following Eq. (3) for only Q_k.

Theorem 1 *If for each automaton A_k in isolation there exists a probability distribution π_k such that*

$$\pi_k Q_k = 0, \tag{3}$$

then the steady-state distribution of the system has the following product form

$$\pi(s) = C \prod_{k=1}^{N} \pi_k(s_k). \tag{4}$$

This product-form solution can be used to check the steady-state formula $\mathcal{S}_{\triangleleft p}(\phi)$ to see if the sum of steady-state probabilities of states satisfying ϕ meets the bound p or not. In the following, two applications of this product form result will be illustrated.

Example 1 Generalised competing Markov chains. We extend the system of competition between concurrent processes over a number of shared resources [4, 5]. The extension is that common resources are no longer limited to be mutually exclusive and strong blocking but may be used by different components at the same time. In other words, transition rates may not be switched off to zero but are only reduced by some factor when common resources are shared. Transition rates are local, functional and inner proportional: when a component shares some common resources with others, its transition rates are reduced by some factor α, which might be a function of the states of all other resources. More precisely, the transition rate matrix of component k is of the form $Q_k^s = \alpha_k(s_{-k})Q_k$. Thus, Theorem 1 applies to this system of generalised competing Markov chains.

Example 2 Multiclass queue with proportional state-dependent rates In this example, Theorem 1 is applied to a multiclass queue with state-dependent arrival rates and service rates. Consider a queue of N classes of customers where customers of each class arrive according to a variable-rate Poisson process. Let x_k be the number of class-k customers, $x = (x_1 \dots x_N)$ be the system state. For class k, suppose that service requirements follow an exponential distribution of parameter μ_k and the arrival rate $\lambda_k(x_{-k})$ is a general function of the vector x_{-k} composed of numbers of customers of other classes. Besides, suppose that the service effort $\Phi_k(x_{-k})$ allocated to class k is also a function of x_{-k}. Thus, class k is characterised by state-dependent arrival rate $\lambda_k(x_{-k})$ and departure rate $\mu_k\Phi_k(x_{-k})$.

SAN representation. Let us describe each class by an automaton. First, transition rates of each automaton are arrival rate and departure rate of the corresponding class. Therefore, these transition rates are functional. Second, if two events of two different classes are not allowed to occur at the same time, transition rates are local. Finally, if the ratio between arrival rate $\lambda_k(x_{-k})$ and departure rate $\mu_k\Phi_k(x_{-k})$ is equal to a constant λ_k/μ_k for any class k, transition rates are inner proportional. The system is a SAN with local, functional and inner proportional transitions. Thus, Theorem 1 applies and gives us an example of state-dependent multiclass queue with product form solutions w.r.t. classes.

4 Product Form Solution for Untimed Paths

In this section, we refer to a transition as a *k-move* if it corresponds to an event of automaton A_k, and we consider an arbitrary starting state $s = (s_1 \dots s_k \dots s_N)$. Product form solution for untimed paths is based on the following key result which is a direct consequence of the inner proportional characterisation (Property 1).

Property 2 *Conditioning on the event E_k^j that the first k-move happens at jth transition, the probability of the event $Obs(s_k, s_k')$ of observing the move (s_k, s_k') at this first k-move depends neither on the index j nor on the state of other automata:*

$$\mathbb{P}\left(Obs(s_k, s_k') \mid E_k^j\right) = \frac{Q_k(s_k, s_k')}{-Q_k(s_k, s_k)} \qquad \forall k, j. \tag{5}$$

Assumption In the rest of the work, we suppose that automaton A_k will make a move in the future with probability one if the total outgoing rate of its representative automaton is strictly positive. This assumption states that the system will make a *k-move* and the first *k-move* will happen at jth transition with some finite index j.

Property 3 *Conditioning that the total outgoing rate of the representative automaton of A_k is strictly positive, automaton A_k will make a move in the future and the probability of observing (s_k, s'_k) at the first k-move does not depend on the state of other automata and is given by:*

$$\mathbb{P}\left(Obs(s_k, s'_k) \mid -Q_k(s_k, s_k) > 0\right) = \frac{Q_k(s_k, s'_k)}{-Q_k(s_k, s_k)} \qquad \forall k. \tag{6}$$

In this part, we are interested in checking if state s satisfies an untimed formula ϕ. If the formula corresponds to a set of untimed paths, the work consists in calculating probabilities conditioned on the set of untimed paths starting with s. Let us denote this set by $U^{(s)}$. Besides, for the representative automaton of A_k, let $U_k^{(s_k)}$ be the set of untimed paths starting with s_k.

Definition 1 For any untimed path $\sigma = (s^0, s^1, s^2, \ldots)$, its k-projection is

$$proj_k(\sigma) = (s_k^{i_0}, s_k^{i_1}, s_k^{i_2}, \ldots), \quad 0 = i_0 < i_1 < i_2 < \cdots$$

such that any two consecutive system states s^j, s^{j+1} whose kth components are the same, i.e., $s_k^j = s_k^{j+1}$, are projected into a unique state s_k^j.

Thus, a k-projection of a path is defined such that repeated states are deleted.Consider an arbitrary starting state $s = (s_1, s_2, \ldots, s_N)$ and a finite untimed path $\sigma_k = (s_k, s_k^1, \ldots, s_k^l)$ of A_k in isolation. We say that the k-projection of an untimed path σ starts with σ_k if

$$proj_k(\sigma) = (s_k, s_k^1, \ldots, s_k^l, \ldots).$$

In the rest of the paper, the notation $proj_k(\sigma) = \sigma_k$ indicates that the k-projection of σ starts with σ_k. For example, consider $\sigma_k = (s_k, s'_k)$ of length 1. Property 3 gives the probability that the k-projection of σ starts with (s_k, s'_k), i.e., automaton A_k will make a move and the first *k-move* corresponds to (s_k, s'_k).

Theorem 2 *Conditioned on starting states s and s_k respectively, the probability of observing an untimed path σ whose k-projection starts with σ_k is equal to the probability of observing σ_k in the representative automaton of A_k:*

$$\mathbb{P}\left(\sigma : proj_k(\sigma) = \sigma_k \mid U^{(s)}\right) = \mathbb{P}\left(\sigma_k \mid U_k^{(s_k)}\right). \tag{7}$$

In the following we shall consider sets of untimed paths. We first introduce the notion of *path-product* over these sets.

Definition 2 For all $k = 1 \ldots N$, let U_k be a set of untimed paths σ_k in $\mathcal{S}_k$. The path-product of the sets $U_1 \ldots U_N$ is defined by the set of untimed paths σ in $\mathcal{S}$,

$$U \equiv \{\sigma : proj_k(\sigma) = \sigma_k, \sigma_k \in U_k, k = 1 \ldots N\}, \text{ we note } U = \odot U_k.$$

For example, the set of untimed paths starting with state s is the path-product of the sets of untimed paths starting with state s_k of automaton A_k for all $k = 1 \ldots N$, that is, $U^{(s)} = \odot U_k^{(s_k)}$.

Theorem 3 *Let s be an arbitrary starting state, U_k be a set of finite untimed paths starting with s_k in $\mathcal{S}_k$ for any automaton A_k, and U be the path-product $\odot U_k$. We have the following product form*

$$\mathbb{P}\left(\sigma \in U \mid U^{(s)}\right) = \prod_{k=1}^{N} \mathbb{P}\left(\sigma_k \in U_k \mid U_k^{(s_k)}\right). \tag{8}$$

Theorem 3 is important as it gives us a compositional method to check any formulae that is equivalent to a path-product of sets of single component untimed paths. In particular, we shall consider global unbounded Until formulae in the sequel.

Single component unbounded Until formulae: One consequence of Theorem 2 is the following result which provides a compositional method to check any single component untimed formula ω_k.

Theorem 4 *For any system state $s = (s_1, \ldots, s_k, \ldots, s_N)$ and for any single component untimed formula ω_k, the satisfaction of ω_k by the whole system is equivalent to its satisfaction by component k:* $\quad s \models \omega_k \Longleftrightarrow s_k \models \omega_k$.

Thus, one may simply check if state s_k verifies formula ω_k for automaton A_k in isolation instead of working with global state s. For example, one may remove all functional interactions and only needs to pay attention to the corresponding isolated chain (or isolated class) in the model of generalised competing Markov chains (or multiclass queue with proportional state-dependent rates respectively).

Global unbounded Until formulae: Let $U^{(\phi \mathcal{U} \psi)}$ be the set of all untimed paths σ that satisfy the Until formula $(\phi\, \mathcal{U}\, \psi)$. The probability that s satisfies $(\phi\, \mathcal{U}\, \psi)$ is the following probability:

$$\mathbb{P}\left(\sigma \in U^{(\phi \mathcal{U} \psi)} \mid U^{(s)}\right) = \mathbb{P}\left(\sigma \in U^{(s)} \cap U^{(\phi \mathcal{U} \psi)} \mid U^{(s)}\right).$$

Theorem 5 *If $U^{(s)} \cap U^{(\phi \mathcal{U} \psi)}$ is a path-product of the form*

$$U^{(s)} \cap U^{(\phi \mathcal{U} \psi)} = \odot U_k \tag{9}$$

where U_k is some set of finite untimed paths σ_k of automaton A_k in isolation for all k, the probability that s satisfies the formula $\phi\, \mathcal{U}\, \psi$ has the following product form

$$\mathbb{P}(s \models \phi\,\mathcal{U}\,\psi) = \prod_{k=1}^{N} \mathbb{P}\left(\sigma_k \in U_k | U_k^{(s_k)}\right). \tag{10}$$

This is a direct consequence of Theorem 3 applied to the set $U = U^{(s)} \cap U^{(\phi\mathcal{U}\psi)}$. The idea of this result is to decompose the Until formula probability into separated components. However condition (9) seems to be sophisticated. Let us illustrate it by considering a concrete Until formula in the following.

Application of the compositional approach: Consider the multiclass queue described in Example 2 with batch Poisson arrivals [9]: For each class, arrivals of batches follow a Poisson process, where the batch size is a positive integer random variable. The Poisson parameter and the random variable for the batch size are functional, i.e., may depend on the state of all other classes.Suppose that arrival rates, batch sizes, departure rates depend on the system state such that inner proportional property holds : $Q_k^s = \alpha_k(s_{-k})Q_k \quad \forall k$. With this extension, the SAN remains local, functional and inner proportional.

In a multiclass queue, we are often interested in the number of customers of each class. The logic formulae are to compare this number of customers to a threshold or a composition of these formulae. For each class k, let M_k be a threshold. We have a failure (overload) for class k if its number of customers reaches M_k. Whenever this happens the class stays at state M_k forever by convention, that is, $Q_k(M_k, s_k) = 0 \ \ \forall s_k \neq M_k$. On the contrary, the system functions properly if there exists a class k such as its number of customers does not exceed a threshold m_k. We are interested in verifying the Until formula $(\phi\,\mathcal{U}\,\psi)$ where

$$\begin{cases} \phi = \phi_1 \vee \cdots \vee \phi_N, & \phi_k = \{x_k \leq m_k\} \\ \psi = \psi_1 \wedge \cdots \wedge \psi_N, & \psi_k = \{x_k \geq M_k\}. \end{cases} \tag{11}$$

Condition ψ means failure of all classes, on the contrary, condition ϕ means that the system functions with at least one class. Let us remark that this Until formula is different from the steady-state probability of being in ψ-states, we consider indeed the probability to reach ψ-states passing through ϕ-states.

Consider the probability $\mathbb{P}(s \models \phi\,\mathcal{U}\,\psi)$ for an arbitrary state s. In order to use the above compositional approach, we shall determine the corresponding sets $U^{(s)}$, $U^{(\phi\mathcal{U}\psi)}$ and their intersection. First of all, $U^{(s)}$ is composed of untimed paths that begin with s. This set is simply the path-product of sets $U_k^{(s_k)}$ of untimed paths that begin with s_k for each component k, i.e., $U^{(s)} = \odot U_k^{(s_k)}$. Second, $U^{(\phi\mathcal{U}\psi)}$ is composed of finite untimed paths that satisfy the Until formula $(\phi\,\mathcal{U}\,\psi)$. Lastly, the intersection of the two sets is given by the following set of finite untimed paths $U = \{\sigma : \sigma\ starts\ with\ s, \sigma \models \phi\,\mathcal{U}\,\psi\}$. Replacing $(\phi\,\mathcal{U}\,\psi)$ by its definition described by Eq. (11), we obtain $U = \{\sigma : proj_k(\sigma) = \sigma_k, \sigma_k\ starts\ with\ s_k,\ \sigma_k \models \phi_k\,\mathcal{U}\,\psi_k, k = 1 \ldots N\} = \odot U_k$, where $U_k = \{\sigma_k : \sigma_k\ starts\ with\ s_k, \sigma_k \models \phi_k\,\mathcal{U}\,\psi_k\}$. As a result, Theorem 5 can be applied and the probability that s satisfies $(\phi\,\mathcal{U}\,\psi)$ is given by

$$\prod_{k=1}^{N} \mathbb{P}\left(\sigma_k \in U_k \mid U_k^{(s_k)}\right) = \prod_{k=1}^{N} \mathbb{P}\left(s_k \models \phi_k\, \mathcal{U}\, \psi_k\right).$$

In this example, the global Until formula can be decomposed into single component Until formulae. Instead of calculating the probability that some starting state satisfies a global Until formula, one only needs to calculate the product of corresponding single component probabilities.

5 Conclusion

In this paper we prove the product form solutions for the steady-state distribution of a class of SANs which generalises competing Markov chains. We perform the verification of the untimed Until and the steady-state formulae for this class of models through the product form solutions. In the last years, the common points for the performance evaluation and the quantitative model checking have been emphasised by many authors. Product form solutions have been largely used in performance evaluation and we think that it would be interesting to look for classes of models that can be efficiently model checked by means of product form solutions.

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