Chapter 2
Linear, Nonlinear, and Complex Systems

Outline: In this chapter readers will be introduced to basic ideas and definitions of system theory, nonlinearity, nonlinear deterministic systems, and complexity. It will include examples and some hints to statistical and geometrical methods. This chapter is not essential for the clinical part of the book, but it is meant to offer a deeper understanding of the concepts of time series analysis, especially for nonlinear methods. We therefore recommended reading it.

Linear Systems

A linear system is simply something that can be defined completely by one or more linear equations. We have summarized some (mathematical) definitions around systems in Table 2.1. As an example consider a bucket into which water flows. If the amount of water per time unit is always the same, the amount of water in the bucket can be described with help of a linear equation. The equation can be solved analytically. It is possible to calculate the amount of water at any time if you know the beginning value (the amount of water in the bucket at $t=0$).

If you describe a system with the help of values taken at different intervals, you have a time series. Time series consist of a set of data and are necessarily discrete (not continuous). The linear numerical description of time series data consists of a first-power mathematical equation. This equation has therefore no exponents and describes a line in a Cartesian two-dimensional graphical system:

$$f(x) = a + bx. \tag{2.1}$$

A given amount of input stimulus x produces a proportional corresponding magnitude in output response y. The stimulus produces a response independent of initial conditions. To describe a linear system, statistics are appropriate, the stimuli being the independent, and response the dependent variable (Schumacher 2004).

G. Ernst, *Heart Rate Variability*,
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Table 2.1 Definitions

A *system* is a collection of variables interacting with each other to accomplish some purpose (McGillem and Cooper 1974).
A *dynamic system* is a system that evolves over time by accepting, then operating on, an original signal to produce a new set of signals (Strogatz 1994).
Signals represent the means by which energy is propagated through a system and may depict any variable within a system (McGillem and Cooper 1974).
A *time series data set* is a collection of observations (data points) made sequentially over time (Chatfield 1989).

The Eq. (2.1) is in fact the simplified form of a differential equation. A time series, however, can also be described by one or more difference equations. A difference equation describes a system stepwise. It returns value at time step 1, 2, 3, and so on. You obtain a numerical solution in a difference equation if you start with an initial value, calculate it according to the equation, reaching so the first result r_1. You put this result again into the equation, obtaining so the next result $r_{2.}$ This process can be repeated infinitely and is called *iteration*.

$$f\left(x_{n+1}\right) = a + bx_n. \tag{2.2}$$

Difference equations were important for the discovery of mathematical chaotic systems, which will be explained later in this chapter.

Linear power spectrum techniques, which transform time series into frequency-domain data, are considered as linear signal analysis too. All power spectrum analysis techniques (like fast Fourier transformation or autoregressive modelling) transform a time series data set into its frequency components by decomposing the original signal into a series of sinusoidal waves analogous to a prism separating light into its corresponding colors.

Nonlinear Systems

A nonlinear system is mathematically defined as a 2nd- or higher-power system, that is, the independent variable in the mathematical equation contains an exponent. In a linear system, the variables produce an output response; whereas, in a nonlinear system the variables contribute to the output response. Although a linear system can be decomposed into its component parts, in a nonlinear system the parts interfere, cooperate, or compete with each other. A small change can alter the nonlinear system dramatically because the initial condition of all variables along with the input stimulus influences the output response (Strogatz 1994). Nonlinear dynamic systems theory allows for the mathematical reconstruction of an entire system from one known variable since the reconstructed dynamics are geometrically similar to the original dynamics.

Probably the simplest form of a nonlinear equation is

$$f(x) = x^2. \tag{2.3}$$

If you show a linear system in a graphical form, you see a (straight) line. Any nonlinear system will show a (more or less complicated) curve. A line has always the same slope at any point, a curve, however, has different slopes, maxima and minima.

These kinds of equations can in principle be solved analytically. We can calculate at any point the value of $f(x)$, but also the slope, global and local maxima and minima, or the position function. But in most cases, nonlinear systems cannot be solved analytically. Why are nonlinear systems so much harder to analyze than linear ones? The essential difference is that linear systems can be broken down into parts. Then each part can be solved separately and finally recombined to get the answer (Strogatz 1994). The problem here is that in the real world we do not find systems where variables act independently. It would be possible to describe the behavior of the heart rate over time if respiration would not have an effect on preload, blood pressure not on afterload, volume not on heart rate, and so on. In reality, most systems have parts that interact in one way or another, and this makes it necessary to describe such systems mathematically on a nonlinear way.

Chaos Theory

The misleading expression "chaos theory" describes the properties of nonlinear deterministic systems. It is a specialized sub-theory of nonlinear systems that describes the behavior of a system with few variables over time when the variables of the time step $n+1$ are dependent on the variables at time step n (compare Eq. (2.2)). The process of turning the result of one time step into the independent variable of the next time step is called iteration. In contradiction to the associations related with chaos, a chaotic system is directly dependent on its initial conditions, but the terminal state of the system after infinite time steps can vary considerably. With methods and algorithms of chaos theory it is possible to distinguish between stochasticity (real independent changes without any rule) and chaos (changes dependent on the conditions before). In fact, most biological time series are based on a combination of these two elements. The robustness of a chaotic system seems often to be dependent on stochasticity (also often called "noise"). This means that a physiological system, which is considerably deterministic, can possibly only be stable if some real random fluctuations are part of it.

Among many investigators and pioneers who paved the way of modern mathematical chaos theory was the meteorologist E. Lorenz and the ethologist R. May. Lorenz modelled atmospheric convection in terms of three differential equations and described their extreme sensitivity to the starting values used for their calculations. May showed that even simple systems (in this case interacting populations) could display very "complicated and disordered" behavior. Among other pioneers in the field were D. Ruelle and F. Takens. They related the still mysterious turbulence of fluids to chaos and were the first to use the name "strange attractor." Soon thereafter M. Feigenbaum revealed patterns in chaotic behavior by showing how the quadratic map switches from one state to another via period doubling. The term "chaos" had been already introduced by T.- Y. Li and J. Yorke during their analysis of the same map. Several Russian mathematicians like A. Kolmogorov and Y.G. Sinai have also contributed to the characterization of chaos, its relation to probabilistic laws, and information theory (Faure and Korn 2001).

There is no simple powerful and comprehensive theory of chaotic phenomena, but rather a cluster of theoretical models, mathematical tools, and experimental techniques. Chaos theory is a specialized application of dynamic system theory. Nonlinear terms in the equations of these systems can involve algebraic or more complicated functions and variables and these terms may have a physical counterpart, such as forces of inertia that damp oscillations of a pendulum, viscosity of a fluid, nonlinear electronic circuits, or the limits of growth of biological populations, to name a few. Since this nonlinearity renders a closed form of the equations impossible, investigations of chaotic phenomena try to find qualitative and quantitative accounts of the behavior of nonlinear differentiable dynamical systems. Qualitative approaches include the use of state spaces or phase spaces to characterize the behavior of systems on the long run, or to describe fractals as pattern of self-similarity.

Phase space is a mathematical and abstract construct with orthogonal coordinate directions representing each of the variables needed to specify the instantaneous state of a system, such as velocity and position (of ,e.g., a pendulum) or pressure and volume changes (e.g., of a lung connected to a respirator). Common for variables is that they are time dependent. Time itself is not represented as coordinate, but on the phase space curve itself. Typically, a phase space starts at a certain point and the system goes through a finite (or infinite) time length. The system might be end at a certain point, which is often called an attractor or a limit point. A limit point for instance is the point where a pendulum finally ends. In the absence of friction, however, the pendulum moves on the same way for infinite time, which leads to a limit circle that describes a stable oscillation. A normal attractor shows a kind of equilibrium, either with or without movement of the system. A system can possibly never reach equilibrium. But beyond attractors or limit cycles, chaotic systems can also reach a kind of equilibrium without moving on the same track again. This is described by the term "strange attractor" that is shown by curves in state space that never repeat but are similar to each other. Limit points are in addition distinguished with regards to local stability. An attractor is regarded as locally stable when perturbations are damped over time, whereas they are seen as unstable if small perturbations increase over time. Locally unstable attractors are also called repellors. A third class of equilibrium points is saddle points that are attractors from some regions, but repellors for other regions.

A physical system can undergo transitions if some of the parameters are disturbed. Perturbations can cause the system to oscillate until it finally returns and ends at the same end point. Consider a stress response of the body. Systemic-released adrenaline and synaptically released noradrenaline results in an increased heart rate. The system will eventually adapt, catecholamines will be eliminated, and (if the stress becomes chronic) receptors will be internalized. At the end, the system will return to a kind of equilibrium.

The amount of perturbations a system is able to tolerate without coming into transition to another state correlates with its robustness. Most systems tend to be robust to most perturbations. The cardiac system can be perturbed in many ways. The blood volume can be increased or decreased, the concentrations of electrolytes can change with some consequences for frequency and rhythm patterns, the rhythm itself can be perturbed by the vegetative nervous system, but in most cases the heart rhythm as signal returns eventually to its basic values, the system is robust. But some quite small perturbations can change the system dramatically. This can lead to a transition to, for instance, atrial fibrillation or asystole. It is typical for systems to be generally robust but sensitive to some probably small perturbations.

Transitions can be showed in logistic maps. These usually two-dimensional maps show a final value of a measured or observed parameter after finite (or infinite) iterations (nothing other than the attractor) dependent on a control parameter (the independent value). The classical logistic map is derived from the already named population studies. The logistic equation is a first order difference equation of the form:

$$x_{n+1} = kx_n\left(1 - x_n\right) \tag{2.4}$$

where x is the dependent value of the system and k is the independent factor. In population biology, x was a relative value between 0 and 1, where 1 represents the maximal possible population in an area and 0 extinction. k represents the growth factor: the higher k is, the faster the population grows. It turns out that for low values of k, the initial population settles down to a stable size that will reproduce itself each year. As k increases, the first unstable fix point appears. The successive value of the population x oscillates in a 2-year circle between two alternate numbers. For increasing values of k, a cycle repeats every 4 years, 8 years, and so on. This is called a period doubling or cascade. Finally, the behavior becomes chaotic; at this stage wild fluctuations hide very effectively the simplicity of the underlying rule (Fig. 2.1).

The cardiac cycle represents a deterministic system in which the RR-distance depends partially on the RR-distances of the last heartbeats. But there are only few mere deterministic systems. Usually, as stated earlier, systems have both deterministic and stochastic elements. Stochastic elements again represent either other complex systems that might be partially deterministic in nature (pseudostochasticity) or might represent gradually real stochastic systems (consequences of quantum fluctuations). This stochastic element is often called "noise" and is often of high importance. It has been repeatedly shown that noise is essential for the stability of artificial and real neural networks. Reducing the "noise" leads to a breakdown of the system, whereas a certain amount of stochasticity leads to stability and rhythmicity. Noise in neuronal communication increases the efficacy of the signal recognition.[1]

[1] For a larger discussion, see (Rieke et al. 1999).

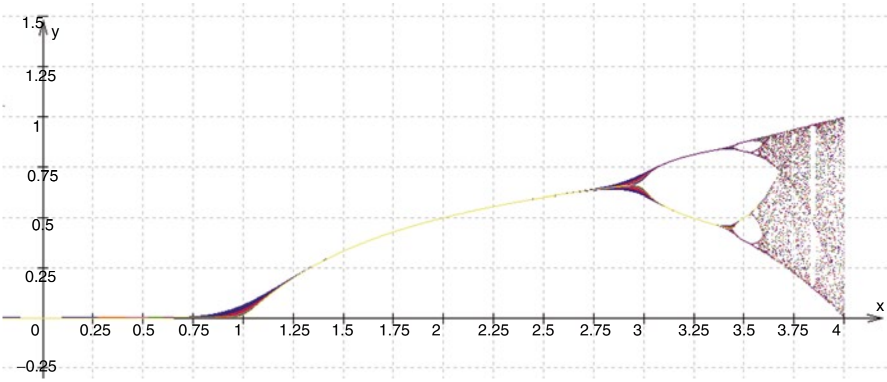

Fig. 2.1 Logistic map of the equation $x_{n+1}=kx_n(1-x_n)$ (also called bifurcation diagram)

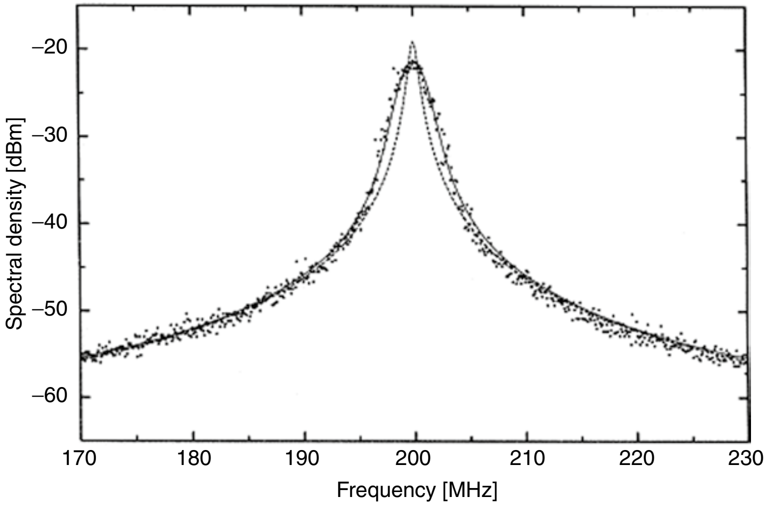

Fig. 2.2 White noise

Noise (stochasticity) is differentiated in white, brown, and pink noise. White noise is a random signal with a flat power spectral density. In other words, the signal's power spectral density has equal power in any frequency band, having a given bandwidth. White noise is considered analogous to white light, which contains all frequencies. Brown noise,[2] also called red noise, is the kind of noise produced by random Brownian motion. Its spectral density is $1/f^2$ denoting more energy at lower frequencies. Pink noise is defined as a signal with a frequency spectrum proportional to the reciprocal of the frequency. It is called pink noise for being intermediate between white noise and brown noise (Figs. 2.2, 2.3, and 2.4).

[2] It is not called after the color but in honor of Robert Brown, the discoverer of Brownian motion.

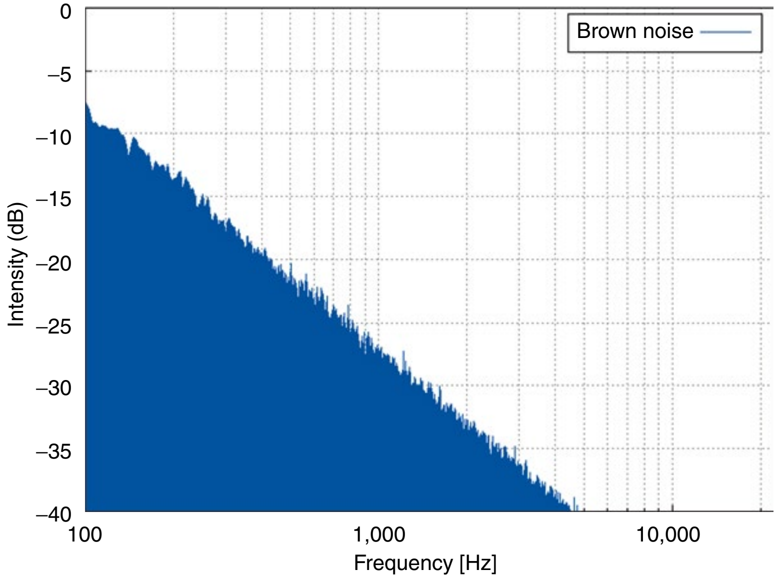

Fig. 2.3 Brown (red) noise

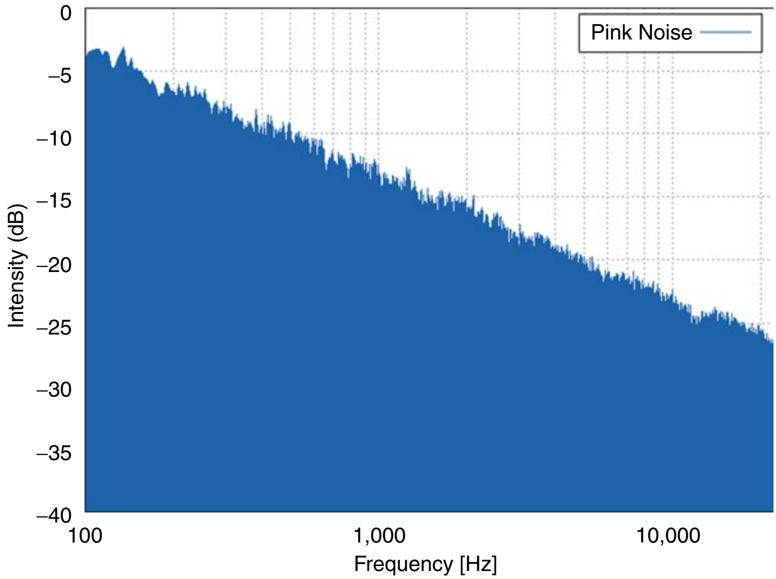

Fig. 2.4 Pink noise

An older linear tool for examining time series is Fourier analysis, specifically *FFT* (fast Fourier transform). FFT transforms the time domain into a frequency domain and examines the series for periodicity. The analysis produces a *power* *spectrum*, the degree to which each frequency contributes to the series. If the series is periodic, then the resulting power spectrum reveals peak power at the driving frequency. Plotting log power versus log frequency:

- *White noise* (and many chaotic systems) has zero slope.
- *Brown noise* has slope equal to −2.
- *1/f (Pink) noise* has a slope of −1.

1/*f* noise is interesting because it is ubiquitous in nature, and it is a sort of *temporal fractal*. In the way a fractal has self-similarity in space, 1/*f* noise has self-similarity in time. Pink noise is also a major player in the area of *complexity*.

Several attempts have been made to quantify chaos (this means to describe the amount of deterministic behavior if there is something that might resemble a strange attractor). Some of them are based on the assumption that strange attractors fulfill the condition satisfying the "ergodic" hypothesis, which proposes that trajectories spend comparable amounts of time visiting the same regions near the attractor.

The *Lyapunov exponent* is used frequently. It is a measure of exponential divergence of nearby trajectories in the state space. Otherwise stated, it depends on the difference between a trajectory and the path it would have followed in the absence of perturbation. Assuming two points x_1 and x_2 initially separated from each other by a small distance δ_0, and at time *t* by distance δ_t, then the Lyapunov exponent λ is determined by the relation

$$\delta_{x(t)} = \delta_{x(0)} e^{\lambda t} \tag{2.5}$$

where λ is positive if the motion is chaotic and equal to zero if the two trajectories are separated by a constant amount as, for example, if they are periodic (a limit cycle).

Entropy is a quantity that comes originally from thermodynamics. It describes the amount of disorder in a given system (this is a rather simplified description. A probably better verbal approach is to term it as the number of degrees of freedom of a system). A chaotic system can be considered as a source of information. It makes prediction uncertain due to the sensitive dependence on initial conditions. Any imprecision in our knowledge of the state is magnified as time goes by. A measurement made at a later time provides additional information about the initial state. From a macroscopic point of view, the second law of thermodynamics tells us that a system tends to evolve toward the set of conditions that has the largest number of accessible states compatible with the macroscopic conditions. In a phase space, the entropy of a system can be written as

$$H = -\sum_{i=1}^{n} p(i) \log \; p(i) \tag{2.6}$$

where *p* is the probability that the system is in state *i*. In practice one has to divide the region containing the attractor in n cells and calculate the relative frequency (or probability *p*) with which the system visits each cell. Entropy has a special significance in time series and we shall revisit the methodology in the Chap. 4. The prototype is the Kolmogorov–Sinai entropy or Shannon entropy. In heart rate variation approximate entropy and more recently sample entropy are used.

Where Lyapunov exponent and entropy focus on the dynamic of trajectories in the phase space, *dimension* emphasizes the geometric features of attractors. Traditionally, dimension is understood in the classic Cartesian way. A dimension is a parameter (or measurement) required to define the characteristic of an object. In mathematics generally, dimensions are the parameters required to describe the position and relevant characteristics of any object within a conceptual space – where the number of dimensions of a space are the total number of different parameters used for all possible objects considered in the model. An even more abstract perspective generalizes the idea of dimensions in the terms of scaling laws. The so-called Hausdorff dimension is an extended nonnegative real number associated to metric space. To define the Hausdorff dimension for a given space X, we first consider the number $N(r)$ of circles of radius r which are required to cover X completely. Clearly, as r gets smaller, $N(r)$ gets larger. Roughly, if $N(r)$ grows the same way as $1/r^d$ as r is squeezed down to zero, then we say X has the dimension d. Related methods include the box-counting dimension, also called Minkowski–Bouligand dimension.

Fractals are irregular geometric objects. An important (defining) property of a fractal is *self-similarity*, which refers to an infinite nesting of structure on all scales. Strict self-similarity refers to a characteristic of a form exhibited when a substructure resembles a superstructure in the same form. Heart rate on the frequency domain (see time-domain analysis) is fractal in nature and measures of fractality have been used to characterize the amount of nonlinearity (see fractal analysis).

Nonlinear statistic tools have been introduced in the last decades. Return maps, also called Poincaré plots, have been used to distinguish between stochastic systems or deterministic systems (Clayton 1997). Briefly, return maps plot a point in a Cartesian system where x is the current value of the time series and y is the next point of the time series. This is repeated for the next pair of values. Stochastic time series show a distribution like in Figs. 2.1 and 2.5.

If we look at a time series produced with the already known logistic equation $x_{n+1} = kx_n(1 - x_n)$ with a k of 3.99, this time series looks graphically highly stochastic (Fig. 2.6).

A return map, however, reveals the deterministic properties of this time series (Fig. 2.7).

Complexity

Complex systems are sometimes positioned between simple systems and stochastic systems. One approach uses the idea of predictability. A system may be predictable (we know how it will develop over a certain time range) or may not be predictable (we know definitely that we don't know how the system will develop over a certain time range). Highly predictable and highly unpredictable systems are *simple*, since the method of forecasting is so straightforward (Crutchfield 2002). But most interesting systems are between those extremes. Interest in them arose because complex

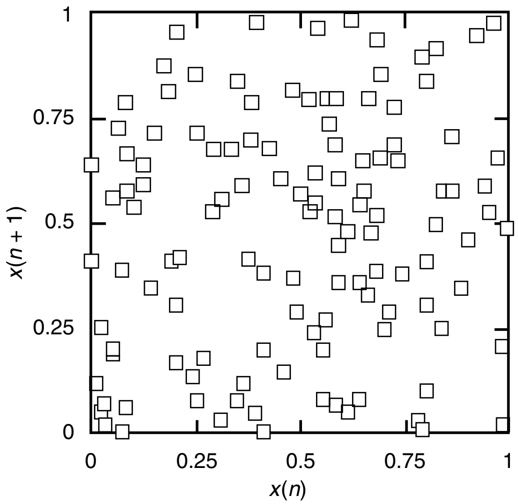

Fig. 2.5 Return map of a stochastic time series (From Clayton (1997))

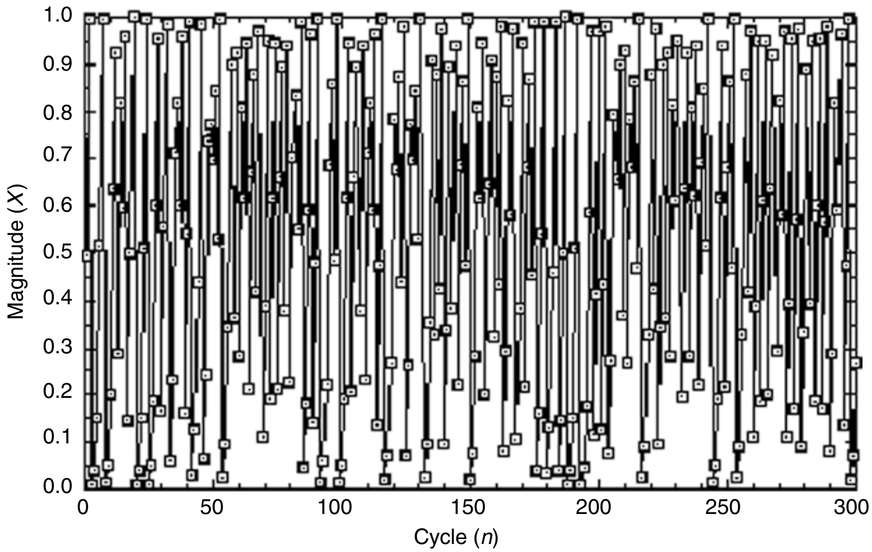

Fig. 2.6 Time series of $x_{n+1} = 3.99\, x_n(1 - x_n)$ (From Clayton (1997))

systems seem to be sensitive to some small perturbations, but at the same time complex systems can be quite resistant to other perturbations, which makes them robust and adaptable (Holt 2004).

There exist several different definitions of complex systems. At the present time, the notion of complex system is not precisely delineated yet. The idea is somewhat fuzzy and it differs from author to author. Main approaches include:

- The number of components in the system (the system's *dimension*)
- The degree of *connectivity* between the components

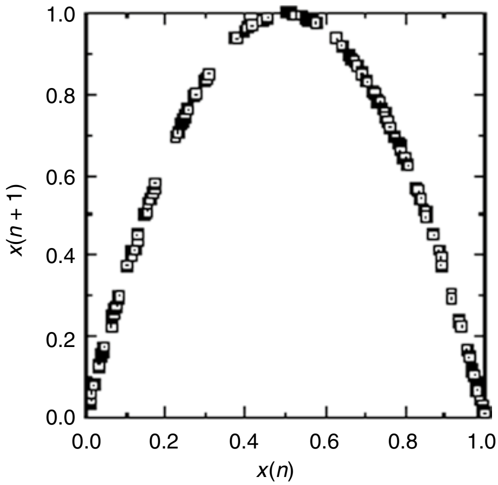

Fig. 2.7 Return map of $x_{n+1} = 3.99\, x_n(1 - x_n)$ (From Clayton (1997))

- The dynamic properties and *regularity* of the system's behavior
- The information content and *compressibility* of data generated by the system (Holt 2004)

But there is fairly complete agreement that the "ideal" complex systems, those that we would like most to understand, are the biological ones and especially the systems having to do with people: our bodies, our groupings, our society, and our culture. Lacking a precise definition, it is possible to convey the meaning of complexity by enumerating what seem to be the most typical properties. Some of these properties are shared by many non-biological systems as well.

Complex Systems Contain Many Constituents Interacting Nonlinearly

Nonlinearity is a necessary condition for complexity, and almost all nonlinear systems whose phase space has three or more dimensions are chaotic in at least part of that phase space. This does not mean that all chaotic systems are complex. For one thing, chaoticity does happen with very few constituents; complexity does not.

The Constituents of a Complex System Are Interdependent

Here is an example of interdependence. Consider first a non-complex system with many constituents, say a gas in a container. Take away 10 % of its constituents, which are its molecules. Nothing very dramatic happens. The pressure changes a little or the volume or the temperature or all of them. But on the whole, the final gas looks and behaves much like the original gas. Now, do the same experiment with a complex system. Take a human body and take away 10 %, let's just cut out a leg! The result will be rather more spectacular than for the gas.

A Complex System Possesses a Structure Spanning Several Scales

Take the example of the human body again. Scale 1: head, trunk, limbs, and the macroscopic scale; Scale 2: blood vessels, nerves, and tissue level; Scale 3: cells and communications between individual cells; Scale 4: intracellular, genome, proteonome, and translational processes; Scale 5: biological chemistry, enzymatic processes, and physical chemistry. At every scale we find a structure. Different scales influence each other. This is an essential and radically new aspect of a complex system, and it leads to the fourth property.

A Complex System Is Capable of Emerging Behavior

Emergence happens when you switch the focus of attention from one scale to the coarser scale above it. A certain behavior, observed at a certain scale, is said to be emergent if it cannot be understood when you study, separately and one by one, every constituent of this scale, each of which may also be a complex system made up of finer scales. Thus, the emerging behavior is a new phenomenon special to the scale considered, and it results from global interactions between the scale's constituents. The combination of structure and emergence leads to *self-organization*, which is what happens when an emerging behavior has the effect of changing the structure or creating a new structure. There is a special category of complex systems that was especially created to accommodate living beings. They are the *complex adaptive systems*. As their name indicates, they are capable of changing themselves to adapt to a changing environment. They can also change the environment to suit themselves. Among these, even narrower categories are *self-reproducing*: they know birth, growth, and death. Needless to say, we know very little that is general about such systems considered as theoretical abstractions. We know a lot about biology. But we don't know much, if anything, about other kinds of life, or life in general.

Let us return now to the relationship between complexity and chaos. They are not at all the same thing. When you look at an elementary mathematical fractal, it may seem to you very "complex", but this is not the same meaning of complex as when saying "complex systems." The simple fractal is chaotic; it is not complex. Another example would be the simple gas mentioned earlier: it is highly chaotic, but it is not complex in the present sense. We already saw that complexity and chaos have in common the property of nonlinearity. Since practically every nonlinear system is chaotic some of the time, this means that complexity implies the presence of chaos.

But the reverse is not true. Chaos is a very big subject. There are many technical papers. Many theorems have been proved. But complexity is much, much bigger. It contains lots of ideas that have nothing to do with chaos. Chaos is basically pure mathematics, and by now it is fairly well known. Complexity is still almost totally unknown. It is not really mathematics, but more like theoretical physics. The field of chaos is a very small subfield of the field of complexity. Perhaps the most striking difference between the two is the following. A complex system always has several scales. While chaos may reign on scale n, the coarser scale above it (scale $n-1$) may be self-organizing, which in a sense is the opposite of chaos. Therefore, let us add a fifth item to the list of the properties of complex systems.

Complexity Involves Interplay Between Chaos and Non-chaos

Many people have suggested that complexity occurs "at the edge of chaos" (Kauffman 2002), but this is not entirely clear. Presumably it means something like the following: imagine that the equations of motion contain some "control" parameter that can be changed depending on the environment (e.g., temperature, concentration, intensity of some external factor like sunlight). We know that most nonlinear systems are not 100 % chaotic: they are chaotic for some values of the control parameter and not chaotic for others. Then there is the edge of chaos, i.e., the precise value of the control for which the nature of the dynamics switches. It is like a critical point in phase transitions. It is the point where the long-range correlations are most important. Perhaps complex systems, such as biological systems, manage to modify their environment so as to operate as much as possible at this edge of chaos place, which would also be the place where self-organization is most likely to occur. It makes sense to expect self-organization to happen when there are strong long-range correlations. Finally, there is one more property of complex systems that concerns all of us very closely, which makes it especially interesting. Actually, it concerns all social systems, all collections of organisms subject to the laws of evolution. Examples could be plant populations, animal populations, other ecological groupings, our own immune system, and human groups of various sizes such as families, tribes, city states, social or economic classes, sports teams, Silicon Valley dotcoms, and of course modern nations and supranational corporations. In order to evolve and stay alive, in order to remain complex, all of the above need to obey the following rule.

Complexity Involves Interplay Between Cooperation and Competition

Once again this is interplay between scales. The usual situation is that competition on scale n is nourished by cooperation on the scale below it (scale $n+1$). Insect colonies like ants, bees, or termites provide a spectacular demonstration of this. For a sociological example, consider the bourgeois families of the nineteenth century of the kind described by Jane Austen or Honoré de Balzac. They competed with each other toward economic success and toward procuring the most desirable spouses for their young people. And they succeeded better in this if they had the unequivocal devotion of all their members, and also if all their members had a chance to take part in the decisions. Then of course there is war between nations and the underlying patriotism that supports it. Once we understand this competition–cooperation dichotomy, we are a long way from the old cliché of “the survival of the fittest,” which has caused so much damage to the popular understanding of evolution (Baranger).

Monitoring, Predicting, and Managing Complex Systems

The wish to monitor complex systems can have several reasons. The conditions of complex systems might reflect their robustness or fragility. This can mirror robustness against perturbations from outside the system, but also robustness against internal oscillations. As described, complex systems can move to a point where a transition occurs. Several forms of transitions have been described in theoretical models and also partially observed in real-world systems (Scheffer et al. 2009). Monitoring complex systems has to be done over time. Changes of surrogate parameters might describe that the system approach a possible threshold – a so-called tipping point – where the systems shifts abruptly from one stage to the next.

It is well known that it is not possible to predict the state of any iterative system beyond certain iterations. At the same time it is known that any system has a finite number of states of equilibrium or quasi-equilibrium that it can reach. This is not necessarily contradictory. The non-predictability of a system regards first the impossibility to predict certain variables. It was originally recognized in meteorology – that even the best computer using the best model is not able to forecast the weather more than some days in advance. But on the other hand, rhythmicity leads to predictability. We know that usually winter is cooler than summer, rain falls in springtime even if we are not able to predict exactly a day’s temperature or the days when it will rain. The predictability in complex systems can mean that the number of possible states is known, but in the beginning, the attractor the system will be going toward is not yet known.

Illness interpreted within a complex systems paradigm can be described as a system being in equilibrium (an attractor state that means health) that is perturbed by an external or internal event. This perturbation is big enough to cast the system out of equilibrium. Then eventually it moves back to the same basin of attraction (equilibrium in health) or to another basin of attraction (chronic illness or death). The direction of the system (and the velocity of changes) might be more interesting as the state of the system itself at a certain point of time. A systems dynamic approach can be to monitor the system and in particular the system changes (using special variables that represent a system state) and to react fast according to these changes. Part of this theory is that early reactions in beginning changes might require less measures or even minimal measures in difference to a system which is already far in the direction of another basin of attraction.

In nonlinear systems, big perturbations might only have small effects, but in the right moment, a small perturbation may be enough to cause a system change (Scheffer et al. 2009). If we assume that the latter situation can be defined, it should be possible either to perturb the system in an adequate manner, pushing it over the tipping point, or conversely to avoid a transition by using countermeasures when the system is evolving near transition points. It is important to recognize, however, that there is not only one kind of transition. In models, critical thresholds for transitions correspond to bifurcations (Kuznetsov 1995). Particularly relevant are catastrophic bifurcations that occur after passing a critical threshold when a positive feedback propels the system through a phase of directional change toward a contrasting state (Scheffer et al. 2009). Other classes of bifurcations occur when one kind of attractor is exchanged with another, e.g., a terminal cycle against a strange (chaotic) attractor.

With help of models it is possible to identify clues that may be associated with a system near a transition point. One of the most important clues has been discussed as a “critical slowing down” phenomenon (Wissel 1984). “Critical slowing down” has been observed in very distinct phenomena, as in cell-signaling pathways (Bagowski and Frrell 2001), ecosystems (Scheffer et al. 2009), and climate (Lenton et al. 2008). Close to the bifurcation points, the exchange rates of the system around the equilibrium become zero. This implies that as the system approaches such critical points, it becomes increasingly slow in recovering from small perturbations (Scheffer et al. 2009). This slowing can begin already far from the tipping point and increases as the tipping point is approached (Van Nes and Scheffer 2007). In real systems this phenomenon could be tested by inducing small perturbations that are not sufficient to drive the system over the transition point and then by measuring the rates of change. Otherwise it can be possible to observe the effects of usually always existing natural perturbations on the exchange rates.

Slowing down can lead to an increase in autocorrelation in fluctuation patterns. This can be shown mathematically (Scheffer et al. 2009). The reason is that in case of a reduced exchange rate, the system at point b is more and more similar to the system at one point a in the past, the system has a memory of itself, so to say. This autocorrelation phenomenon can be measured with help of the frequency spectrum of the system (Livina and Lenton 2007). Another consequence can be increased variance – as eigenvalue approaches zero, the impacts of shock do not decay and their accumulating effects increase the variance of the state variable (Scheffer et al. 2009). Another possibility is to look at the asymmetry of fluctuations (Guttal and Jayaprakash 2008). This is not necessarily a result of critical slowing down. It has rather something to with an approaching unstable attractor from one side in the state space. Also flickering can occur, if the system is near a system shift, being alternately attracted by two basins of attraction. This has been discussed as an alarming sign before phase transitions, e.g., in models of lake eutrophication (Carpenter and Brock 2006).

In conclusion, in the last years several interesting approaches to predict system transitions have been proposed. However, sophisticated ideas to manage complex systems are either lacking or only theoretical. Regarding complex social systems, scientists are rather skeptical about managing theories (Willke 1999).

Summary

- Linear systems are only a special condition. Most systems are only linear if they are simplified. Most biological systems are nonlinear in nature.
- In principle, systems consist of stochastic and deterministic elements. It is possible, but not always easy to analyze systems in order to quantify the fraction of determinism. Determinism means simply that the behavior of a system over time is dependent on its history.
- Nonlinear deterministic ("chaotic") systems show robustness, which is partially dependent on stochastic noise. This robustness is with respect to some kinds of perturbation. On the other hand, nonlinear deterministic systems can be highly sensitive to certain other perturbations, leading to fast disintegration of the system
- Complex systems are nonlinear systems, where their parts interact nonlinear and where there exist different interacting scales. Complex system show emergent behavior, they can change from a disordered to an ordered state and vice versa.

Further Readings

Many excellent introductions to nonlinear and complex systems have been published in the last years. Important ideas and materials of this chapter were obtained from Strogatz (1994), Clayton (1997), Faure and Korn (2001), Kauffman (2002), and Baranger.

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