

# Chapter 1

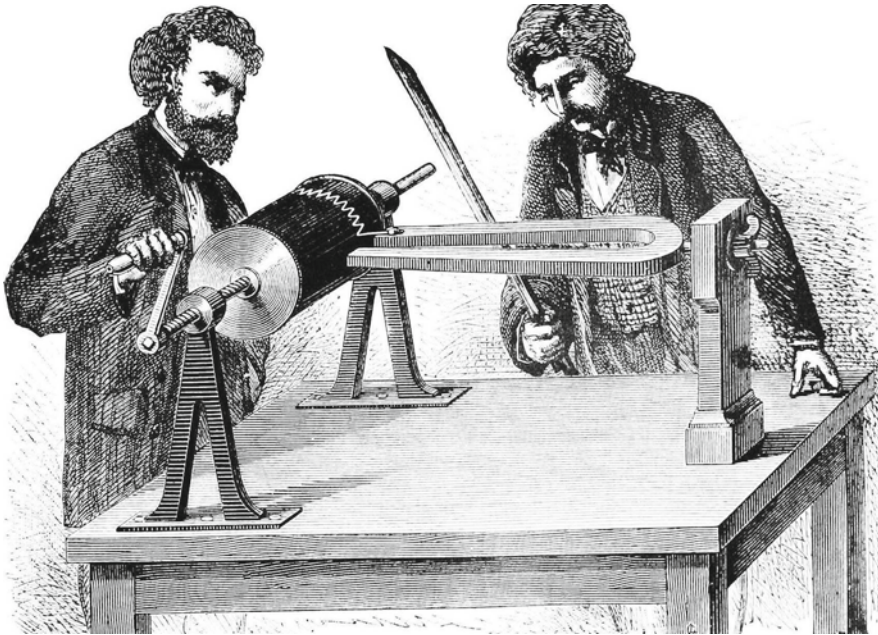
## Introduction

Mechanical vibrations or, in other words, oscillations about an equilibrium point are present in countless real-life situations. If one thinks about vibrations, their positive and useful nature may come to mind: the sound of musical instruments emitted as a result of vibration, the mechanical feedback of cellular phones and game consoles or the action of loudspeakers. Engineering practice actively utilizes mechanical vibrations as well, for example to transport objects, to separate materials or to compact surfaces.

Unfortunately, unlike in the above examples, most often vibrations are highly undesirable. Mechanical vibrations may be simply a nuisance just as the decreased ride comfort in automobiles, but can also be disastrous and life threatening as the vibration of buildings due to earthquake events. Unwanted vibration may decrease product performance, damage quality, cause economic or critical safety problems.

Practicing engineers and scientists are constantly working to create more complex theoretical foundations for understanding vibration phenomena and to have better tools to analyze, measure and eliminate it. Just as in the case of many other fields of science, the greatest push for the development of modern vibration analysis was the direct result of the work of Isaac Newton. More specifically, vibration dynamics can be described through his ideas known as “Newton’s three laws of motion” [67] and of course the introduction of calculus. The tools of the trade are constantly being developed just as well. Early examples of vibration analysis hardware range from mechanical contraptions acting as signal generators to learn the frequency range of audible sound [97], to devices like the one illustrated in Fig. 1.1 recording the vibratory motion of a tuning fork excited with a bow [71]. While the means for vibration analysis and measurement have come a long way since, the ultimate goals of scientists are still the same.

Although all physical systems have some inherent natural physical damping, in some cases the level is simply not satisfactory. Vibration attenuation techniques are often utilized to increase the energy dissipation of systems and structures. In this way the response of a structure driven at resonant frequencies may be greatly decreased. Vibration attenuation is conventionally carried out by passive means and techniques, taking advantage of the physical properties of the system itself and vibration



**Fig. 1.1** Graphical observation of a vibrating tuning fork [71]

phenomena. Passive vibration insulation methods, such as the use of Helmholtz resonators, dampers, shock absorbers and others are an effective way to dampen unwanted oscillations [4, 7, 24, 41, 99]. The traditional engineering approach to avoid the undesirable effects of mechanical vibrations is to alter mass, stiffness and damping properties of structures with respect to the initial configuration. While this is the most straightforward and simple method, unfortunately it has a significant drawback: an inevitable weight increase. . . In certain situations, this is entirely unacceptable. In addition, design and geometry alterations may not be always viable. Passive treatments are usually acceptable for higher frequency vibrations, but for low frequency, they tend to get bulky and expensive.

Probably the most widely known cautionary tale and textbook example of the power of mechanical vibrations is the collapse of the Tacoma Narrows Bridge in 1940 [8]. The slender and elegant structure which was the third longest suspension bridge in its time, was however posed with an extreme tendency to wind-induced aeroelastic flutter and the resulting vibration [88]. Although several measures were implemented to control the vibration response of the structure—including tie-down cables and hydraulic shock absorbers at the bridge towers—none of them were effective [91]. Figure 1.2 illustrates the moments of collapse of the Tacoma Narrows Bridge,<sup>1</sup> after the central span was excited to vibrate in its

<sup>1</sup> Courtesy of the Division of Work & Industry, National Museum of American History, Behring Center, Smithsonian Institution.



**Fig. 1.2** A cautionary tale and textbook example of severe resonance effects: the collapse of the Tacoma Narrows Bridge due to wind induced aeroelastic flutter and the resulting resonance [25]

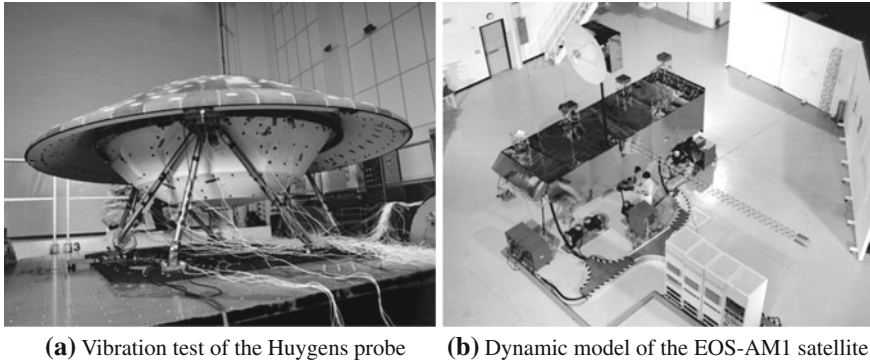
second resonant<sup>2</sup> twisting mode by complex wind induced aeroelastic fluid-structure interaction.

## 1.1 What is Active Vibration Control?

Active vibration control (AVC) can be an effective substitute to traditional approaches, introducing exceptional damping levels to mechanical structures, which are very difficult to attenuate by traditional methods. Active vibration control employs actuators to utilize external force effects on the vibrating mechanical system in order to dissipate energy. The actuators are driven by control systems, which gain feedback from sensors assessing the levels of displacement, velocity or acceleration by direct or indirect methods. The information gathered by the sensors and the ultimate action of actuators is connected by a controller strategy, which determines the behavior of the controller and ultimately the controlled plant. Usually, AVC systems are highly

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<sup>2</sup> Note that the Tacoma Narrows Bridge collapse is strictly speaking not the best example of forced resonance, but a different concept of *self-excitation* [8].



**Fig. 1.3** The Huygens probe [23] is shown instrumented with many acceleration sensors, undergoing vibration testing in (a), while (b) shows the full-scale dynamic model of the EOS-AM1 satellite as a part of Controls-Structures Interaction (CSI) program dedicated to reducing vibrations in spacecraft through structure optimization coupled with active and passive vibration control [66]

integrated with the controlled plant and may be regarded as one complex mechatronic unit. Active vibration control is also commonly referred to as active vibration attenuation (AVA), active vibration damping (AVD) and active vibration isolation (AVI) in the literature. These terms are interchangeable and we will prefer the phrase “active vibration control” to denote the concept.

Active vibration control is no longer a distant technology concept existing only in the realm of experimental laboratories and abstract academic studies. With the advent of cheaper and better sensors, actuators and computing hardware, AVC systems are starting to emerge not only in high-tech applications but in everyday consumer products as well. More and more components employing different AVC technologies are being developed for the aviation, aeronautic, automotive and building industries. Some proposed applications of active vibration control include vibration control of large flexible space structures [39, 59, 72], fixed wing surfaces of commercial and military aircraft [19, 27, 28], blade surfaces and other parts of rotary wing aircraft [9, 47, 54], automobile suspensions [13, 63, 80], satellites [30], antenna systems [1, 82], precision manufacturing techniques [21, 92, 98], robotic arms and manipulators [14, 37, 96], optical systems [53, 65, 68], earthquake protection systems [48, 73, 76] and many others. Vibration reduction efforts are illustrated on spacecraft featured in Fig. 1.3<sup>3,4</sup> [23, 66]. A closely related field to active vibration control is active noise control (ANC). ANC systems implemented in aircraft may for example moderate the noise impact of helicopters, short takeoff and landing (STOL) or vertical takeoff and landing (VTOL) aircraft, since these tend to be used more closely to the densely populated areas [77].

<sup>3</sup> Courtesy of NASA.

<sup>4</sup> Courtesy of DASA and the European Space Agency (ESA).

## 1.2 The Choice of Strategy in Active Vibration Control

There is one very important and often overlooked aspect of active vibration attenuation systems: the *control strategy* itself. There is an overwhelming selection of literature reviewing the types of actuators that can be employed to dissipate energy in AVC systems. Similarly, the range of available devices used for sensing vibrations, along with their optimal placement is often reviewed in great detail. However, hardware is not the only aspect of AVC systems that is constantly evolving. There have been immense developments in the field of control theory in the past couple of decades. These developments are being gradually transferred to everyday use, resulting in advanced algorithms, which should enhance the overall AVC strategy and thus the dynamic behavior of the plant.

The obvious and simple control strategies for vibration damping have been already thoroughly investigated [34, 40]. Most of the available literature lists fairly simple feedback controllers, with strategies resting on classical control theory adapted for vibration attenuation applications. The utilization of direct position feedback control [2, 43, 55, 76, 79] and velocity feedback control [27, 28, 61, 81, 100] is common and used extensively in experimental AVC applications. Proportional integrating derivative (PID) controllers have proved their worth over the years in numerous industrial applications, and due to the simplicity of the strategy and the analogy with position, velocity and acceleration feedback [34] have also found their place in vibration control [3, 18, 29, 35, 36]. These are very well established, albeit somewhat limited control strategies for the modern active vibration applications integrated in high-tech products.

Despite some advantages, it is time to get past position feedback and similar conventional methods and utilize the more progressive results of control theory. This is not only valid for AVC but for all technical fields. Unfortunately, the industry and commercial users are responding very slowly to the advancement of modern control theory. And in fact, who can blame them? If a primitive proportional controller works well in an application, why would one want to change it? The answer is simple: in addition to ensuring a *basic satisfactory functionality*, novel approaches have much more to offer. Traditional controllers implemented in AVC systems often do not provide the necessary maximal performance; their setup merely involves a series of trial-and-error experiments. Control methods based on established parameter tuning approaches such as a properly tuned proportional integral derivative controller may provide an increased performance. However, we have to realize that the performance of such tuned PID controllers may still not be the best possible for the given situation. Moreover, input and output constraints may be required because of safety or economic considerations. Because real world processes and actuators have inherent boundaries, control moves must be often *constrained*. To give an example commonly encountered in active vibration control, let us note that piezoelectric actuators are especially prone to failure through depolarization, if the maximal allowable voltage level is exceeded. Other advanced intelligent materials used for actuation have clearly defined breakdown limits as well, which must be respected in the interest



of preserving the functionality of the AVC system in the long run. Traditional systems solve this problem by including saturation limits for the outputs. For instance, a PID controller may compute an input to the system that is simply not realizable because of the physical limits of the actuator. In this case, the input is clipped to the allowable level, raising the question of performance once more. Could we use a strategy that handles constraints with a greater performance? Furthermore, as it will be later demonstrated, the inclusion of such process constraints unfortunately may also render the originally stable formulation *unstable*. Because of their inherent properties, such poorly designed controllers then carry a potential to react on a disturbance in an unstable way, resulting in dramatic consequences. So to summarize our discussion, what can be gained using more advanced control strategies?

- increased performance
- guaranteed stability
- constraint handling

One might argue that a linear quadratic (LQ) controller provides an optimal performance for the given settings. This is true, since LQ is one of the fundamental optimization-based approaches. Why would we need anything more advanced or complicated if LQ is optimal? Linear quadratic controllers [21, 33, 45, 62], the so-called  $\mathcal{H}_\infty$  (read: H-infinity) [9, 15, 42] and other simple optimization-based approaches yielding a fixed feedback law very often act as control strategies in active vibration control systems. While the optimality of the performance of this control system might be seemingly solved, we are still posed with the problem of finite actuator capabilities. An optimal boundary control problem with applied voltage acting as the control input is considered by Lara et al. for an AVC problem [52]. The method of Lara et al. to solve the problem of actuator constraints is to penalize control effort in the control law. Another example of a similar approach is presented by Dong et al. who used an LQ law penalizing control input heavily to prevent the overload of the actuators [26]. Clearly, this is not the best solution for a real application. This is because such a heavy penalization of the control input results in a very conservative and therefore *suboptimal* strategy, which still cannot guarantee the feasibility of the process constraints. Since the fulfillment of constraints cannot be guaranteed, saturation limits are still enforced somewhere within the loop. As it will be later demonstrated, this again raises the question of the so important controller stability.

Currently the only control technique, which can deal with constraints and their effect on future control actions is model predictive control (MPC). Model predictive control not only handles constraints well, but also does this while maintaining an optimal control process. Moreover, thanks to special formulations all this is ensured while providing a guarantee of system stability and the fulfillment of constraints. Optimality, constraint handling and stability guarantees come with a steep price though: the price of heavy computational requirements.

### 1.3 The Role of Model Predictive Control in Active Vibration Control

Model predictive control is an advanced method of process control, based on an optimization procedure that has to be performed real-time in between sampling instants [60, 78]. The optimization process minimizes a numeric indicator of control quality, called the *cost function*. This cost function consists of future predictions of the possible outcomes of outputs, states and inputs based on a mathematical model of the underlying dynamics of the controlled plant. The optimization process is *constrained*, which means it takes into account the boundaries of inputs, outputs or states given as process constraints. It is also possible to formulate this complex problem in a such way that the control process is guaranteed to remain *stable* at all times, while the fulfillment of constraints called *feasibility* is also ensured. As one would imagine, this online optimization is a sophisticated process and as such requires considerable computing power and relatively long execution times. Model predictive control has been utilized in engineering practice for decades already, although mostly in slow sampling applications like in the petrochemical industry [74]. The petrochemical industry was the first to recognize the merits of predictive control, such as its performance and constraint handling—while its implementation was also possible due to the slow dynamics of the processes measured in several tens of minutes.

Due to the fast dynamics of vibration phenomena, active vibration attenuation applications require much higher sampling speeds when compared to petrochemical processes. The requirement of fast sampling can render the computationally intensive online MPC optimization task intractable in real-time. To put it in another way, in many cases the computation of the next system input to the actuators would take much more time than is available. Creating an MPC formulation that guarantees the stability of the control process at all times complicates the formulation even further. It not only increases the necessary execution times for the real-time optimization process, but also limits the pool of viable system states from which it is possible to steer the plant into equilibrium. As it will be later demonstrated, this problem is especially prevalent if the controlled system is subjected to a range of disturbances, which exceed the possibilities of the actuating elements.

The above-mentioned limitations did not stop academic researchers from implementing MPC to vibration control systems in its traditional formulation. One of the best examples of this is the work of Wills et al. who have used traditional dual-mode infinite horizon quadratic programming (QP)-based MPC to control a vibration attenuation system in real-time<sup>5</sup> with impressive speeds [93, 94]. Model-based predictive control is considered by Blachowski et al. for the attenuation of guyed antenna mast vibrations [10]. Other examples of model predictive control applied to active vibration control systems also exist [37, 64, 75, 96, 101], while the main limitation of these works is the lack of proper a priori stability and feasibility guarantees. This

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<sup>5</sup> Claimed to be the current fastest quadratic programming-based fully optimal MPC application by Dr William Heath on a seminar titled “*Robustness of input constrained model predictive control*”. 26th Oct. 2007, University of Oxford, Oxford.

work attempts to justify this drawback by considering only such MPC formulations that do guarantee stability and feasibility. As it turns out, a whole class of problems in vibration control needs a bit of special attention: these are *lightly damped vibrating mechanical systems*, which due to their dynamic characteristics make the implementation of stabilized MPC particularly difficult [85].

One way to overcome the issues characterized above is to use a highly optimized optimization solver [31, 32], with the possible combination of utilizing powerful computing hardware. The alternative method is to consider an MPC formulation, which due to its unique formulation may considerably save on computational time. Recent academic research shed light on numerous computationally efficient MPC variants [5, 6, 17, 49, 89, 90]. One of these *computationally efficient* MPC methods is often referred to as multi-parametric MPC (MPMPC) or explicit MPC [51, 70]. Explicit MPC, which is renowned for very short achievable sampling periods, will be tested against other formulations in this book. MPMPC is based on the idea of precalculating control moves for the piecewise-affine polytopic regions of the state-space, and applying them from a look-up table online [5, 6]. Despite its obvious advantages, a priori stability and feasibility guarantees and increasing problem dimensionality may render the offline controller computation intractable for certain types of practical engineering problems, such as the ones encountered in AVC. One of the other computationally efficient MPC methods we concentrate on in this work is called Newton–Raphson’s MPC (NRMPC). The algorithm created and subsequently improved by Kouvaritakis and Cannon et al. [16, 49, 50, 56] uses a formulation which sacrifices a small level of optimality to arrive at a final online algorithm which can be evaluated in very little time.

## 1.4 Model Predictive Vibration Control of Flexible and Lightly Damped Mechanical Systems

One may think of a very simple active vibration damping example which illustrates a whole class of real-life applications. A cantilever beam clamped at one end having the other free and equipped with piezoelectric transducers may represent a smart helicopter rotor blade. The vibration of a rotor blade in flight is undesirable since it decreases performance and increases fuel consumption, thus desirable to be eliminated by embedded piezoelectric actuation. Another obvious application is the vibration damping of large space structures. Such structures may be antenna masts, solar panels, aircraft wings and manipulation arms. These applications are researched currently by NASA, Boeing and other institutions [10, 11, 22, 44, 69]. An example of a helicopter rotor blade<sup>6</sup> utilizing active vibration damping is illustrated in Fig. 1.4.

While the clamped active cantilever beam can be considered as an oversimplified laboratory demonstration example, it is entirely sufficient to represent the dynamic characteristics of large, flexible vibrating systems with very little damping [20].

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<sup>6</sup> Courtesy of The Boeing Company.





**Fig. 1.4** A *smart* rotor blade undergoes whirl tower tests at The Boeing Company facility in Mesa, Arizona, USA [12]. For the purposes of control system design, the dynamic behavior of flexible and lightly damped structures such as active rotor blades can be effectively emulated by much simpler laboratory models

In fact, such *flexible and lightly damped* mechanical systems are widely recognized to have unique properties in the eye of the control engineer, and have been treated as such in many academic works [9, 46, 54, 57, 58, 79, 83, 95]. These vibrating mechanisms have common dynamic properties: in case piezoelectric actuation or other not too powerful actuation method is considered, the range of expected disturbances resulting in displacements is significant in comparison with the maximal static effect of actuators. Application of MPC algorithms on such and similar structures may be difficult, since these properties in combination with the stability requirement and fast sampling necessitates very long prediction horizons [85]. Increasing problem dimensionality to these levels makes the application of QPMPC unlikely for the heavy online computational requirements, while application of MPMPC is implausible because of controller complexity and calculation time. Investigating the possibility to use stabilized model predictive controlled vibration attenuation on flexible systems therefore may shed light yet unforeseen obstacles, but it can also bring the advanced research fields of smart materials, vibration attenuation and model predictive control closer.

In fact, this is what this monograph attempts to carry out: combining existing technologies into a complex synergistic unit; pairing up knowledge from the fields of mechanics, dynamics, systems design, electronics, control engineering, computer science, mechanical engineering in a hands-on experimental environment. Mechatronics,<sup>7</sup> as the definition goes, is a very good descriptor of the ambitions presented in this book. One may concentrate on a given problem area, like the predominant subject of MPC controller here, but must never ignore the big picture—the complex mechatronic unit. Vibration attenuation by piezoceramics; stable and efficient MPC have been all around for quite some time in the engineering and research community, while blending these and more may perhaps bring new insights onto the surface.

## 1.5 About the Book

The following passages briefly review the structure of this book, its scope, limitations and some assumptions used in the upcoming chapters. The following section acts as a supplement to reading the table of contents, characterizing the logical distribution of information in the text. To those readers already familiar with active vibration control or control theory, a brief guide is given in [Sect. 1.5.2](#) to skipping certain parts of the monograph. While we have tried our best to include every important aspect of predictive control applied to vibration attenuation, we cannot cover everything. We present our excuses in [Sect. 1.5.3](#).

### 1.5.1 Structure of This Book

In order to isolate clearly the dominant fields of engineering considered in this book, we have separated it into three distinct parts and collected the respective chapters accordingly. The first of these, Part I, reviews active vibration control and its related topics. The objective of Part II is to introduce model predictive control to the reader, who already has some fundamental knowledge of control theory. Finally, Part III presents a collection of chapters, which have a common goal of using the MPC strategy in active vibration control.

This introduction is followed by Part I, which consists of four chapters. The first of these deals with the fundamentals of vibration dynamics. The motion of the simple spring-mass-damper system is analyzed and gradually expanded to include more advanced topics, such as the vibration mechanics of multiple degree of freedom systems and distributed parameter systems. [Chapter 3](#) then reviews some of the modern intelligent materials that are either already commonly used in AVC or have the potential to emerge into commercial products. These materials include shape memory alloys, electro and magnetostrictive actuators, electro and magnetorheological dampers, piezoceramics and electrochemical materials. The fundamental strategies

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<sup>7</sup> The term *mechatronics* was coined by Tetsuro Mori, a senior engineer at Yaskawa, in 1969.

used in active vibration damping are covered in [Chap. 4](#). This is not merely a primer into the basics of control theory but rather a view presented from the field of AVC with an extensive collection of application examples. Controllers ranging from simple position and velocity feedback to more advanced ones such as genetic algorithms are briefly covered in [Chap. 4](#). Finally, [Chap. 5](#) divulges more specific topics concerning the construction and properties of our simple AVC laboratory demonstrator. Here the demonstrator assembly is presented along with its mechanical properties, a FEM analysis and some details on the components utilized in the design. To summarize, we have covered in Part I:

- vibration dynamics and modeling
- advanced materials
- algorithms in vibration control
- construction of a laboratory device

The next part gives an up-to-date review of the model predictive control strategy and some of its novel formulations which can be essential for its implementation in AVC. Part II consists of three distinct chapters, each dealing with a different slice of the MPC problem. The first one in [Chap. 6](#) is aimed at the reader with no prior experience with MPC. Here we present the fundamentals of predictive control, and a step-by-step discussion which ultimately results in the so-called dual-mode constrained MPC formulation. The next chapter, that is [Chap. 7](#), expands the problem of MPC and includes the issue of stability. In addition to reviewing the conditions for a stable MPC control course, we will summarize a formulation that provides stability and constraint feasibility guarantees a priori, while it also draws from a maximal possible pool of states. The second part of our book is finished by [Chap. 8](#), which reviews some well accepted computationally efficient MPC formulations such as multi-parametric explicit MPC, and certain novel formulations such as the Newton–Raphson MPC method. To briefly summarize the contents, Part II features:

- introduction to MPC
- stability of MPC
- computational efficiency of MPC

Part III consists of a collection of four chapters. The first of these reviews the existing and possible applications of model predictive vibration control. [Chapter 9](#) lists applications ranging from simple laboratory demonstration devices such as the one presented here, to more advanced ones including automotive, aeronautics and civil engineering. This chapter not only lists a selection of AVC application fields, but reviews the possible improvements and challenges brought on by the implementation of the MPC strategy. The following [Chap. 10](#) deals with the implementation of computationally efficient MPC controllers in active vibration control. The topics covered include the implementation of traditional dual-mode stabilized MPC using specialized solver software and the deployment of MPMPC and the formulation of an NRMPC algorithm. The final two chapters of our book cover the simulation results and experimental results of using MPC on our demonstration example. [Chapter 11](#) points out numerous issues encountered in the application of MPC in AVC. These

include the requirement of large horizons, problems with the invariance and optimality of the NRMPC approach and the simulation comparison of different algorithms. Finally, [Chap. 12](#) presents the experimental results of using model predictive control on the AVC demonstrator. The different computationally efficient predictive control formulations are contrasted with regard to their damping performance and timing properties in different excitation situations. The outside excitation used in the experiments includes an initial displacement, frequency domain tests using a chirp signal and different random excitations. The final part of this book thus covers the following topics:

- applications of MPC in AVC
- implementation of the MPC strategy in AVC
- problems and issues with MPC in AVC, simulation studies
- experimental study of MPC in AVC

The book ends with two additional chapters, summarized in the Appendix. Both these chapters list the detailed instructions and may be interesting to those who are attempting to gain further information about the algorithm implementation details or finite element modeling of structures actuated by piezoceramics. Appendix A gives step-by-step instructions on the modeling of the AVC demonstrator featured in this book in the ANSYS finite element modeling environment. The topics included here are the definition of geometry, meshing, definition of boundary conditions and finally modal and harmonic analyses. Appendix B is essentially a supplement to [Chap. 10](#), and gives even more details to those interested in the implementation particulars of MPC algorithm. Thus to summarize, the appendix contains:

- guide to analyze active structures using piezoceramics in ANSYS
- algorithm and programming particulars of the MPC implementation

### ***1.5.2 Do I Have to Read the Whole Book?***

Yes. This short and concise answer is meant mainly for our own students. For the rest of the readers this matter is a little more complex, and depends on what kind of previous knowledge one possesses from the different fields of science and engineering utilized in this work. If the reader has absolutely no or minimal experience in vibration mechanics and control theory, we recommend to read the book from the beginning to the end. Hopefully, Part I of our book may convey some new information or view related to AVC to those who are already acquainted with control theory, vibration mechanics or active vibration control as well.

In case one is familiar with the fundamentals of vibration mechanics, and has some idea of control theory concepts such as linear quadratic control, state-space systems but has not been extensively involved with the MPC strategy, we recommend to start with Part II. This is also true for readers who have been involved with active vibration control in some way. The predictive control method is elaborated in a detailed way,

thus the discussion should be sufficient to introduce one to the world of MPC. In case the reader is also familiar with the details of model predictive control and its stable formulations, we may suggest starting with the very last chapter of Part II, that is [Chap. 8](#). This is where we take a look at some of the computationally efficient MPC formulations, which will be at the center of attention later on.

It would be burdensome (and probably useless as well) to list all the permutations of science fields which appear in this book in more or less detail. However, to summarize our effort in giving hints to simplify the reading of this book, we list the chapters that may be *left out* in case one is already familiar with:

- basic vibration mechanics: [Chap. 2](#)
- smart materials: [Chap. 3](#)
- basic control theory: [Chap. 4](#)
- construction of AVC laboratory devices: [Chap. 5](#)
- active vibration control: [Chaps. 2, 3, 4, 5](#)
- intermediate control theory, including basic MPC: [Chaps. 2, 6](#)
- advanced control theory, including stabilized MPC: [Chaps. 2, 6, 7](#)
- MPC including stabilized and computationally efficient: [Chaps. 2, 6, 7, 8](#)
- MPC algorithm implementation: [Chap. 10](#), [Appendix A](#)
- experimental hardware implementation: [Chap. 5](#), [Appendix A](#)
- finite element modeling: [Appendix A](#)

### ***1.5.3 The Scope and Limitations of This Work***

It is always subjective and dependent on one's personal interest whether a sub-problem is sufficiently elaborated in a book utilizing knowledge from multiple scientific and engineering fields. A control theorist may desire more rigorous and detailed mathematical proofs including uncertainty and robustness analysis while discarding the rest of the work. A computer scientist could focus solely on programming details, a mathematician on optimization aspects, while a person invested in mechanics and dynamics may want to see a better finite element model and ignore the rest. One must however take into account the inherent multidisciplinary nature of this work, and see the big picture. After all this is what *mechatronics* is about: relating common fields of engineering by taking relevant pieces of knowledge and fusing it with a versatile and complex application.

To explore the properties of all major control strategies and algorithms used in active vibration attenuation would be an overly ambitious enterprise, and it has been done by other scientists before in works discussing the general aspects of AVC [[34](#), [40](#), [73](#)]. Similarly, the analysis and measurement of vibration dynamics is a vast area of scientific interest, definitely deserving detailed attention [[4](#), [24](#), [38](#), [41](#)]. Given the multidisciplinary character of this work, various scientific and engineering fields are involved in shaping the final contribution of the book. One must not expect a thorough and complete summary of all involved theoretical knowledge, rather a

selected cross-section of subjects providing fundamental ideas. This monograph thus rather concentrates on describing *predictive vibration control* as a whole, and it is not meant to be a textbook on subjects like vibration mechanics, finite element analysis or control theory. If interested, one may refer to the abundance of excellent books and other literature listed in the bibliography sections for more information.

On the other hand, several people who have seen the book manuscript at some point of its progress have reminded us that maybe there is too much attention devoted to implementation details. Implementation details such as the exhaustive account on the construction of the laboratory device, a complete guide to its finite element analysis and an elaborate review of controller algorithm implementations. We cannot count the occasions when we have decided to eliminate these parts from the book and then changed our minds subsequently. Our final decision was to include these details in the monograph, and not without good reason. We have been inexperienced with the construction of laboratory devices for active vibration control, not even mentioning the several hours spent with unfruitful FEM simulation attempts. Time spent on such matters could have been better used concentrating on more important aspects of the problem, thus we decided to save the trouble for others and leave these portions of the text as inspiration and help. Our final decision seems to be justified by the legions of e-mails from engineering students and researchers who have found portions of our previous work on the Internet and requesting information about construction of similar laboratory devices or the FEM analysis of vibrating structures with piezoelectric actuation.

The implementation of the computationally efficient Newton–Raphson MPC control strategy on a physical system has been not carried out in other known publications so far, therefore it duly deserved our detailed attention. The NRMPC algorithm extension presented by Kouvaritakis et al. in [50] was meant to enhance process optimality. While this extension is described in theory and evaluated in simulation for the AVC example here, in our experience the optimality increase gained is not significant in the case of lightly damped systems. The experimental tests featured in this monograph thus do not compare the damping performance of different NRMPC variations, only the base algorithm with optimized prediction dynamics [16].

Experimental trials performed with the NRMPC code left no doubt about the computational efficiency of the algorithm, even when related to other very efficient approaches such as explicit MPC (MPMPC). The use of high order state-space models to generate predictions would increase the bandwidth coverage of an AVC system by including dynamics describing resonant modes beyond the first one. Trials exciting higher bandwidth dynamics would not only provide more insight into the vibration damping capabilities of the investigated algorithms, but would also show timing properties with larger problem dimensionality. As our practical tests have sufficiently demonstrated, the use of simple second order prediction models bring enough difficulty into the implementation of stabilized model predictive control in active vibration control. The implementation of multi-parametric programming-based explicit MPC with long horizons and large model orders is hardly viable as of today while unfortunately the suboptimality of NRMPC with high order prediction models turned out to be disappointing as well. As the dominant dynamics of lightly



damped systems is sufficiently described by simple second order prediction models, our focus in this book was rather on such simple mathematical representations of vibration dynamics.

Reviewers of conference papers published based on our efforts summarized in this book [84–87], often pointed out the lack of direct comparison between NRMPC and more traditional control strategies like PID, positive position feedback, energy- or passivity- based methods. Avoiding contrasting NRMPC, MPMPC or QPMPC with much simpler methods is however not caused by our ignorance or lack of interest. A constrained MPC algorithm with stability guarantees provides numerous advantages over saturated LQ for instance, but it may not match its simplicity of implementation. Comparing control methods with diametrically different degrees of complexity and distinctive philosophy would be unfair, just as matching no control at all to a new control method is biased. Novel MPC controllers should be contrasted against other MPC methods, or algorithms with similar benefits and drawbacks.

We have to note that the aim of this book is not to prove that constrained MPC algorithms are better than traditional algorithms with saturation limits. The constraint handling feature of MPC is not the same as cutting off inputs at minimal or maximal boundaries by simple hard saturation limits. One might think that the effect is eventually the same, but MPC has much more to offer. Although the performance of saturated LQ controllers in SISO systems may match that of the constrained MPC methods, the performance advantages of predictive control are more evident with increasing plant complexity. This is not all, as the introduction of the nonlinear saturation function to closed-loop control may affect stability—possibly leading to serious problems. All of this is solved by the constraint handling capability of stabilized MPC. Due to its advantages over classical control methods with hard saturation limits, industrial users have long recognized the merits of predictive strategies. What is beneficial for a plant with slow dynamics can also be beneficial for a system with fast dynamics.

The type of constraints considered in the simulations and experiments on the AVC demonstrator will be limited to *input constraints*. Although one of the major advantages of MPC over other methods is the ability to handle state and output constraints, a purely input constrained MPC controller still offers optimal performance and stability, unlike hard-saturated strategies. Neither of the examined MPC methods would require fundamental changes to implement state or output constraints, therefore these will be only discussed in the chapters concentrating on theoretical aspects. The inclusion of state constraints would not change the fundamental issues of MPC in fast sampling systems, therefore the upcoming discussions will examine different topics. Our aim is not to argue in favor of MPC in comparison with traditional methods, rather explore the yet undiscovered field of active vibration control via model predictive techniques.

### 1.5.4 Assumptions and Objectives of Part III

While Parts I and II of our book concentrate on reviewing the fundamentals of active vibration control and model predictive control, Part III will combine these two fields into *predictive vibration control* and treat it as one compound topic. The discussion is essentially centered on a series of simulations and experiments for which we have to make certain assumptions. These assumptions will then define *what* we want to perform and *how* we would like to do that.

In order to emulate the mechanical properties of large, lightly damped flexible vibrating systems, a small-scale laboratory dynamic model has been created. This laboratory model is essentially a clamped cantilever beam with bonded piezoelectric actuators and a laser triangulation-based feedback system. The main objective of Part III is to investigate the possible ways to utilize model predictive control to minimize the vibration levels of flexible and lightly damped systems (modeled by this device); while ensuring constraint feasibility and guaranteed stability. Since MPC formulations without proper stability guarantees have already been implemented on active vibration control systems, a particular attention will be devoted to use formulations ensuring stability and feasibility. Here, we take a practical view on analyzing damping performance provided by various computationally efficient and stable MPC algorithms and assessing the limitations imposed by both on and offline computation requirements.

The strategy implemented on this particular AVC demonstrator has to be stabilizing and constrained model predictive control. Stability is to be guaranteed a priori and this algorithm has to be efficient enough to allow for general applications in active vibration control. The highly flexible nature of the physical system may cause additional issues with the size of the region of attraction, as it is suggested later on.

The MPC controller applied to the experimental device must minimize the deflection measured at the beam tip, that is, minimizing the vibration amplitudes resulting from the first resonant mode. To emulate the difference between actuator capabilities and expected structural deformations, a large range of allowable beam tip deflections, thus a large region of attraction in the stabilized MPC law is considered. While the piezoelectric actuators supplied with voltages meeting polarization limits may generate only a static deflection approximately in the range of  $\pm 0.2$  mm, beam resonance measured at the tip in the first mode easily exceeds  $\pm 10$  mm resulting in a deflection angle of  $[\sim 1.5^\circ]$ . The region of attraction defined by the MPC law must be able to cover this area, thus states measured in this deflection range must be included in the set of all feasible states.

The MPC strategy must minimize the first vibration mode, which is located at approximately 8 Hz. The sampling rate necessary to cover this frequency by a second order model is a relatively modest 100 Hz. The state-space model describing beam dynamics is limited to second order, in the interest of bounding the size of the online optimization problem. This is necessary to include the computationally least efficient traditional dual-mode quadratic programming-based MPC in the comparison. In fact,

QPMPC will act as a benchmark to compare timing properties and process optimality expressed in damping performance.

Part III of this book compares the vibration damping performance, real-time execution timing properties and implementation possibilities of the following MPC controllers offering guaranteed stability and constraint feasibility, in both simulation and experiments:

- dual-mode quadratic programming-based MPC (QPMPC)
- multi-parametric programming-based precomputed explicit MPC (MPMPC)
- multi-parametric programming-based precomputed minimum time suboptimal explicit MPC (MT MPMPC)
- Newton–Raphson’s computationally efficient suboptimal MPC (NRMPC)
- and finally a saturated linear quadratic (LQ) controller serving as a basis of comparison both for damping performance and for timing

All of the above defined algorithms must cover the same region of attraction, running on the same implementation software, utilize the same linear time-invariant state-space prediction model, identical state observers, penalization and other applicable settings. The algorithms shall be verified in various situations both in the time and frequency domain.

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