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Model Updating by Combining Substructure Energy Functions and Subset Selection

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ABSTRACT

Since a reliable finite element model has broad applications in structural response prediction, control, health monitoring and condition assessment, model updating techniques are often applied to improve the performance of an analytical model for its intended use. The sensitivity-based iterative model updating approach becomes preferable because of its physically meaningful results, however, the selection of updating parameters remains a difficult problem to handle. In this paper, a synthesized approach, which combined substructure energy functions and subset selection, was proposed recognizing that the former can be used to locate the area containing dominant modeling errors and the latter to ensure the least number of effective parameters be chosen to reduce the modal residue adequately. This synthesized method can potentially reduce the burden of calculation but have more credible results. Its effectiveness was demonstrated systematically by numerical simulation with encouraging results.

NOMENCLATURE

K, K_j	stiffness matrix, substructure stiffness matrix;
M, M_j	mass matrix, substructure mass matrix;
$\omega, \omega_A, \omega_E$	circular frequency, analytical circular frequency, experimental circular frequency;
ψ, ψ_A, ψ_E	mode shape vector, analytical mode shape vector, experimental mode shape vector;
N	number of measured mode shape;

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$\Delta\prod_j^s, \Delta\prod_j^k$ strain energy, kinetic energy functions of the J th substructure;

S sensitivity matrix;

b modal parameter residue between measured and analytical results;

θ physical parameters vector for updating;

Φ, Φ_E mode shape matrix, experimental mode shape matrix;

Φ_{UA} analytical mode shape components corresponding to unmeasured degrees of freedom;

Φ_{MA}, Φ_{MA}^+ analytical mode shape components corresponding to measured degrees of freedom and its pseudoinverse matrix;

$\lambda, \lambda_{i,j}$ eigenvalue, the derivative of the i th eigenvalue with respect to the j th physical parameter;

$\psi, \psi_{i,j}$ eigenvector, the derivative of the i th eigenvector with respect to the j th physical parameter;

$K_{,j}$ the derivative of stiffness matrix with respect to j th physical parameter;

$M_{,j}$ the derivative of mass matrix with respect to j th physical parameter;

W weight matrix;

1. INTRODUCTION

Since a reliable finite element model has broad applications in structural response prediction, control, health monitoring and condition assessment, model updating techniques are often applied to improve the performance of an analytical model for its intended use. Model updating is a sequential process which including several steps, and the selection of parameters for updating is probably the most important task [1]. The parameters should be chosen which associated with features of the model which are in doubt for physical reasons, thus these recognized uncertainties can be corrected. So parameter selection means modeling error localization. However, this is a nontrivial task. It needs structural analysts' application of considerable physical insights. Several parameter selecting methods have been proposed during the past decades, and they have relation to parameterization method used in model updating.

In direct model updating, the potential updating parameters are individual terms of stiffness and mass matrixes. Lallement and Piranda proposed a method of balancing the eigenvalue equation $[\mathrm{K}-\omega^2\mathrm{M}]\psi=0$, in which the analytical eigensolution $(\omega^2,\psi)=(\omega^2_{\mathrm{A}},\psi_{\mathrm{A}})$ is replaced by eigenvalues and properly expanded eigenvectors from experiments $(\omega^2_E,\psi_{\mathrm{E}})$, to give a localization matrix $[\mathrm{K}-\omega^2{}_{\mathrm{E}}\mathrm{M}]\psi_{\mathrm{E}}=\mathrm{L}$ [2]. The dominant modeling errors are reflected by those degrees of freedom associated with large values of $q_i=\sum_{h=1}^{n} p_h L^2_{ih}, i=1,\cdots,n$, where P_h is weighting scalars.

In iterative model updating, one parameterization method is substructure parameter, which is associated with individual finite element or groups of finite elements, the other parameterization method adopts physical parameters, such as Young's modulus, mass density and so on. With regard to the former case, Link and Santiago [3] proposed the use of energy functions based upon the substructures to locate regions which possibly have modeling error. In the latter case, Lallement and Piranda proposed a method named best subspace to choose those updating parameters which can reduce the discrepancy more effectively between analytical and experimental results [2]. Selecting updating parameters from all possible ones is in fact a subset selection, and the theory of which has been well established in the statistical literature [4]. And the best subspace method adopts a

forward subset selection approach. Fritzen studied QR orthogonalisation strategy in subset selection, and compared it with Efroymson's criterion method, saying that both performed successful [5] [6]. Golub and Van loan described SVD-based subset selection procedure [7]. Friswell extended the subset selection method to circumstance of multiple testing data [8]. In subset selection, all kinds of methods try to select a subset of parameters which are least in correlation thus reduce the residue most effectively.

Updating parameters should be selected with the aim of correcting modeling errors. Subset selection method which is not based upon physical insights choose the parameters which most effectively improve the correlation of analytical and experimental results, However, this is not a sufficient condition to assure that these parameters must have dominant errors. Substructure energy functions is a method based upon physical insights with the potential to find modeling error regions, but not all parameters in error regions are suited to be updated due to the property of sensitivity-based model updating approach. These two methods can complement each other to obtain potentially the parameters selected not only based upon physical insights but with high effectiveness in updating process.

In this paper, a synthesized approach based on the discussion above was proposed which begins with the use of substructure energy function aiming at obtain the likely regions with modeling errors, then only the physical property parameters in these regions are filtered by subset selection, the best subspace method is used in subset selection for example, with parameters obtained as final updating parameters. Section 2 outlines some important theory used in this synthesized approach. A simulated cantilever beam example is used to demonstrate the properties of the method in section 3, coming to conclusions in section 4.

2. THEORETICAL BACKGROUND

In this synthesized approach, the measured mode shapes are firstly expanded by the system equivalent reduction expansion process (section 2.3), and the likely regions with modeling error are obtained according to the theory of substructure energy functions (section 2.1), then physical parameters in these likely error regions are filtered by the subset selection theory (section 2.2), finally, those parameters chosen are used to be updated by a sensitivity-based model updating approach (section 2.5).

2.1 Substructure Energy Functions

Link and Santiago proposed the use of energy functions based upon the substructure stiffness and mass matrixes [3]. The substructure strain energy function can be expressed as,

$$\Delta \Pi_j^s = \sum_{h=1}^{N} (\psi_{ah} - \psi_{Eh})^T K_j (\psi_{ah} - \psi_{Eh}) \tag{1}$$

And substructure kinetic energy function as,

$$\Delta \Pi_j^K = \sum_{h=1}^{N} (\psi_{ah} - \psi_{Eh})^T M_j (\psi_{ah} - \psi_{Eh}) \omega_{Eh}^2 \tag{2}$$

Poorly modeled regions are believed to be detected by large residual substructure strain energy function or kinetic energy function values. Since in both residual energy functions, full experimental eigenvectors are obtained from a few component values of mode shape corresponding to places with sensors by modal expansion process in which the analytical eigenvectors with errors are used in most practices, the detected regions with modeling error can just be likely.

2.2 Subset selection

In sensitivity-based model updating, the following equation is obtained,

$$S \cdot \theta = b \tag{3}$$

where S is the sensitivity matrix and contains the first order derivatives of modal residual between experimental and analytical results with respect to the candidate updating parameters. It is assumed that only a subset of these parameters are non-zero. Lallement and Piranda proposed a subset selection procedure, commonly known as forward subset selection, to produce a sub-optimal solution [2].

Firstly, a single column is sought which can best represent the residual vector b. If the columns of S are given by S_j, namely $S = [S_1\ S_2\ \ ...\ \ S_p]$, then the parameter which minimizes the residual

$$J = \left\| b - S_j \tilde{\theta}_j \right\|_2^2 \tag{4}$$

is selected, where $\tilde{\theta}_j$ is the least squares estimate of the j th parameter, with the expression of

$$\tilde{\theta}_j = S_j^T b \,/\, S_j^T S_j \tag{5}$$

Substituting equation (5) into (4), The result is obtained that the j th parameter is selected that maximizes

$$(S_j^T b)^2 \,/ (S_j^T S_j) \tag{6}$$

Since b is a constant vector,

$$\cos^2 \phi_j = (S_j^T b)^2 \,/ (S_j^T S_j)(b^T b) \tag{7}$$

is maximized equivalently when the j th parameter is selected, where ϕ_j is the angle between the vectors b and S_j. In other words, the j th parameter is selected that minimizes the angle between the vector b and S_j.

Then, the forward subset selection method starts to select the second parameter, together with the already selected one, to make a combination of two columns of S, which constitutes the best sub-basis for the representation of b. Let j_1 represents the first parameter selected and the corresponding column of S be S_{j1}. To select the subsequent parameter following j_1, the vector b and columns of S must be transformed to be orthogonal to vector S_{j1}. Thus the columns of S and vector b are replaced with

$$S_j \rightarrow S_j - S_{j1}(S_{j1}^T S_j \,/\, S_{j1}^T S_{j1}) \tag{8}$$

$$b \rightarrow b - S_{j1}(S_{j1}^T b \,/\, S_{j1}^T S_{j1}) \tag{9}$$

In this transformed problem, the second parameter is similarly obtained $j_2 (\neq j_1)$ as the first one , which minimizes the angle between the vector b and S_j. So far, an iterative process is produced to find a subset collection from all candidate parameters, provided a stop rule is specified.

2.3 Modal Expansion Process

In model updating, problems arise from comparing measured experimental modal data with numerical analytical modal data because of incompleteness. There are fewer freedoms of response measured by sensors in

experiments than those calculated in analysis. Aiming to make this comparison done, one way is to reduce the number of degrees-of-freedom (DOFs) of analytical model, the other is to expand the DOFs of measured modal vectors. In this paper, the updating parameters are defined in the analytical model, which is to be updated, thus, the measured incomplete mode shapes vectors are expanded to the full set of DOFs of the analytical model.

Several modal expansion techniques are available [1], and in terms of preservation of eigeninformation, the system equivalent reduction expansion process (SEREP) is adopted here considering its improved accuracy resulting from preserving the dynamics before and after the process in the least-squares sense [9]. The expanded mode shape matrix is given by

$$\tilde{\Phi} = T \cdot \Phi_E \tag{10}$$

where the transformation matrix T is given by

$$T = \left[\frac{\Phi_{MA}}{\Phi_{UA}}\right] \cdot \Phi_{MA}^{+} \tag{11}$$

2.4 Eigenpair Sensitivities

To obtain sensitivities of eignvalues and eigenvectors with respect to physical parameters, the method proposed by Fox and Kapoor's [10] is used in this paper. The expression of sensitivity of eigenvalue is

$$\lambda_{i,j} = \psi_i^T [K_{,j} - \lambda_i M_{,j}] \psi_i \tag{12}$$

And the sensitivity of eigenvector is

$$\psi_{i,j} = \sum_{k=1}^{N} a_{ijk} \psi_k \tag{13}$$

where coefficients a_{ijk} is determined by

$$a_{ijk} = -\psi_l^T [K_{,j} - \lambda_i M_{,j}] \psi_i / (\lambda_i - \lambda_l) \quad l \neq i \tag{14}$$

$$a_{iji} = -(\psi_i^T M_{,j} \psi_i / 2) \tag{15}$$

2.5 Sensitivity-based model updating methodology

In sensitivity-based finite element model updating using measured modal parameters, the identification of structural parameters is formulated in an optimization problem where the structural parameters are sought so that the updated finite element model can reproduce as closely as possible the measured modal parameters [11]. The objective function measuring the residual of modal data (natural frequencies (NF) and mode shapes) between experiment and numerical analysis is expressed by

$$MinJ_X = \varepsilon^T W \varepsilon = \left\| W^{1/2} (\tilde{z} - z(\theta) \right\|_2^2 \tag{16}$$

where ε is the output error of the modal parameters, $\tilde{z}$ and $z(\theta) \in R^n$ are the experimental and analytical modal vectors with $n = n_f \cdot (n_m + 1)$, n_f and n_m are the numbers of measured modal frequencies and measured coordinates for each mode respectively.

The solution to equation (16) is obtained from a sensitivity-based iterative optimization approach. Beginning with

an initial estimate of updating parameters, the solution for updating parameters at the Kth iteration step is obtained by the expressions of

$$\theta^{K+1} = \theta^K + \Delta\theta^K \tag{17}$$

$$\Delta\theta^K = [S^T S]^{-1} S^T (\tilde{z} - z(\theta^K)) \tag{18}$$

where S is sensitivity matrix obtained by the theory described in section 2.4.

3. NUMERICAL EXAMPLE

In this illustrative numerical example, a refined FE reference model is built to represent the true behavior of the experimental structure (see Figure 1), and mode shape data with partial DOFs is obtained by a limited number of sensor measurements from this reference model. Accordingly, a FE updating model with a relatively coarse FE mesh is built which has modeling errors to be updated (see Figure 1). The structural type used is a 1D cantilever beam with the following properties: density 2,500 Kg/m^3 and elastic modulus 3.0e+10N/㎡; length 6.4 m, cross section area $b \cdot\ h = 0.2 \cdot\ 0.3m^2$. Matlab is used for building the finite element model and performing the whole numerical calculation process.

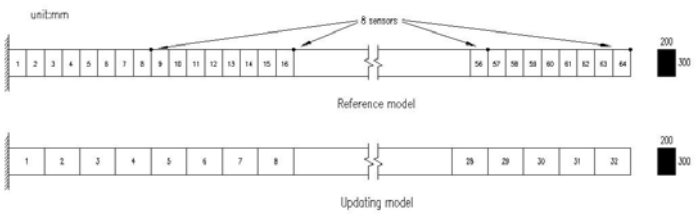

Figure 1: The reference model and updating model

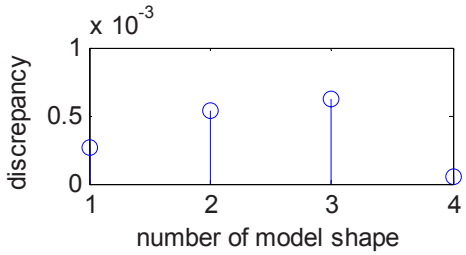

Figure 2: The first four NF discrepancy between reference model and theoretical value

The reference model(RM) has 64 Euler-Bernouli beam elements, and the updating model(UM) 32 elements. Modeling errors are simulated with reduction of elastic moduli E_i of 40% and 50% of the 8th and 18th elements respectively in updating model. Eight equally spaced responses (representing sensor measurements) are obtained from the reference model, and only first five modes with 9 DOFs (eight from sensors and one from fixed end of cantilever) are used in the model updating process to represent realistic situation.

The reference model is firstly inspected whether it can represent the true physical structure. The first four natural frequencies are compared to their theoretical values [12], with the result of the largest discrepancy below 0.1 percent (see Figure 2). Thus the reference model can be assumed to represent the experimental data accurately, and the discretization error ignored. Before updating, the discrepancy between the reference model and updating model are reflected by natural frequencies discrepancy percent J and modal assurance criteria(MAC) values (see Table 1), where

$$J = \left| f_{ref} - f_{upd} \right| / f_{ref} \cdot\ 100\% \tag{19}$$

$$MAC = \left| \Phi_E^T \Phi_A \right|^2 / \left(\left| \Phi_E^T \Phi_E \right| \cdot \left| \Phi_A^T \Phi_A \right| \right) \tag{20}$$

Table 1. The First Five Natural Frequencies (NF) and MAC Comparisons before and after Updating.

Modes	NF-RM (Hz)	NF-UM (Hz)(before)	NF-UM (Hz)(after)	J(%) (before)	J(%) (after)	MAC (before)	MAC (after)
1	4.0947	3.9947	4.0875	2.44	0.18	1.0000	0.9989
2	25.6650	24.8689	25.6395	3.10	0.10	0.9998	1.0000
3	71.8840	70.7533	71.8963	1.57	0.02	0.9987	1.0000
4	140.9275	136.0643	140.9898	3.45	0.04	0.9988	1.0000
5	233.1025	229.9906	233.9547	1.34	0.36	0.9985	1.0000

In order to use the substructure energy function to localize the regions with modeling errors, the updating model is organized into 16 substructures with every substructure composed of two contiguous FE elements, and the first five mode shapes are expanded using SEREP method. In this example, structural mass is assumed unchanged, and substructure strain energies of all 16 substructures are calculated and normalized to the greatest one of them (see Figure 3).

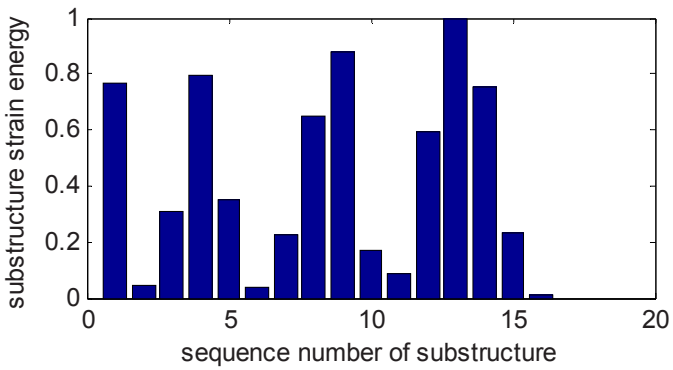

Figure 3: Normalized substructure strain energy

Four substructures with largest strain energy are taken to be likely regions with modeling errors, which are sequentially the 13^{th} , 9^{th} ,4^{th} and 1^{st} substructure. Thus the moduli of 1, 2, 7, 8, 17, 18, 25, 26 elements of updating model will be provided as candidate parameters to be filtered by subset selection. After implementing a best subspace subset selection, the first four elements chosen are 2, 17, 18, 8 elements. Based on the results of these two methods, the moduli of 17,18,8 elements are finally chosen to be updated by a sensitivity-based approach with good results shown in the Table 1 and Table 2.

Table 2. The Values of updating parameters before and after Updating. Unit: N/㎡

Element	Before updating	After updating	True values	discrepancy
8	1.80e+10	2.97e+10	3.00e+10	-1.00%
17	3.00e+10	3.07e+10	3.00e+10	+2.33%
18	1.50e+10	2.95e+10	3.00e+10	-1.67%

If the updating parameters are chosen by subset selection alone as usually, all of moduli of 30 elements in updating model will be candidate parameters, leading to that not only greater quantity of calculation, especially in step-wise regression subset selection [8], but also relatively large number of elements will be finally used in order to include the two true error parameters. From this example, the fact that it is very hard to accurately choose the updating parameters is also discovered. In order to obtain good updating result, one needs make trade-offs between using more updating parameters hoping to contain all parameters with errors and the negative influence by those

parameters which should not be included indeed in practice.

4. SUMMARY

The selection of updating parameters is a very important question in model updating, but it seems that there is no an exact method to solve it. It is recommended to use several different methods and only to pick out those parameters which have been selected by more than one technique, and the approach proposed here is a try. From the results of this synthesized approach composed of two complementing methods, one can potentially be more sure of the feasibility of the rightness in choosing updating parameters and reduce the burden of calculation though it is still very difficult to obtain the exact parameters with modeling error.

5. ACKNOWLEDGMENTS

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