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A non-destructive technique for the health monitoring of tie-rods in ancient buildings

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ABSTRACT. A technique is developed to identify in-situ the tensile force in tie-rods which are used in ancient monumental masonry buildings to eliminate the lateral load exercised by vaults and arcs. The technique is a frequency-based identification method that allows to minimize the measurement error and that is of simple execution. In particular, the first natural frequencies of the tie-rod are experimentally identified by measuring the FRFs with instrumented hammer excitation. Then, a numerical model, based on the Rayleigh-Ritz method, is developed for the axially-loaded tie-rod by using the Timoshenko beam theory retaining shear deformation and rotary inertia. Non-uniform section of the rod is considered since this is often the case for hand-made tie-rods in old buildings. The part of the tie-rod inserted inside the masonry wall is also modeled and a simple support is assumed at the extremities inside the walls. The constraints given to the part of the tie-rod inserted inside the masonry structure are assumed to be elastic foundations. The tensile force and the stiffness of the foundation are the unknown. In some cases, the length of the rod inside the masonry wall can be also assumed as unknown. The numerical model is used to calculate the natural frequencies for a given set of unknowns. Then, a weighted difference between the calculated and identified natural frequencies is calculated and this difference is minimized in order to identify the unknowns, and in particular the tensile force. An estimation of the error in the identification of the force is given. The technique has been tested on six tie-rods at the central vault of the famous Duomo of Parma, Italy.

Keywords: Tie-rods, frequency-based identification method, Rayleigh-Ritz method.

INTRODUCTION

Tie-rods were often used in ancient monumental masonry buildings to eliminate the lateral load exercised by the vaults and arcs. They give a fundamental contribution to the structural equilibrium. As a consequence of foundation settlements, the tensile force on tie-rods can surpass the yield strength of the material, which is not particularly high since old-time metallurgy was not able to obtain high-strength rods. Also corrosion can play a role in decreasing the strength of ancient tie-rods. For these reasons, it is important to identify the tensile force in tie-rods of masonry building, especially in case of evident deformation of arcs and vaults, in order to substitute them in dangerous cases.

Unfortunately there is no non-destructive technique for a direct in-situ measurement of the force on the rod. Several techniques have been proposed for an indirect measurement of the force. Blasi and Sorace [1, 2] introduced a technique based on a combination of static and dynamic identification. In particular, they modeled the tie-rod as a simply supported Euler beam with two identical rotational springs of unknown stiffness at the edges. The two unknowns, i.e. the tensile force and the stiffness of the two identical rotational springs, were identified by two equations: (i) a static equation giving the central deflection of the rod under a given load, and (ii) a dynamic equation giving the fundamental natural frequency of the rod. The method was tested in laboratory giving good results. Anyway, it requires two different in-situ experiments (measurement of the central static deflection under static load and of the fundamental natural frequency) and can give results with significant error in case of measurement error since the two unknowns are determined by only two data points.

Briccoli Bati and Tonietti [3] introduced a single static test to identify the force. It requires the measurement (i) of three vertical displacements under a concentrated static load, and (ii) of the strains variations at three sections of the rod. Also in this case good agreement with laboratory experiments has been found.
Fully dynamical identification of cable tension force has been recently proposed by Kim and Park [4]. It allows to identify the tension force, flexural rigidity and axial rigidity of the cable from measured natural frequencies. Anyway this technique is not immediately applicable to tie-rods since they cannot be modeled as cables and present uncertain constraints due to the portion of the rod inserted into the masonry wall or column.
In the present study, a technique is developed to identify in-situ the tensile force in tie-rods in ancient monumental masonry buildings. The technique is based on a frequency-based identification method that allows to minimize the measurement error and that is of simple execution. In particular, the first natural frequencies of the tie-rod are experimentally identified by measuring the FRFs with instrumented hammer excitation; four to six natural frequencies can be easily identified with a simple test. Then, a numerical model, based on the Rayleigh-Ritz method [5], is developed for the axially-loaded tie-rod by using the Timoshenko beam theory retaining shear deformation and rotary inertia. Non-uniform section of the rod is considered since this is often the case for hand-made tie-rods in old buildings. The part of the tie-rod inserted inside the masonry wall is also modeled and a simple support is assumed at the extremities inside the walls. The constraints given to the part of the tie-rod inserted inside the masonry structure are assumed to be elastic foundations. The tensile force and the stiffness of the foundation are the unknown parameters. In some cases, the length of the rod inside the masonry wall can be also assumed as unknown. The numerical model is used to calculate the natural frequencies for a given set of unknowns. Then, a weighted difference between the calculated and identified natural frequencies is calculated and this difference is minimized in order to identify the unknowns, and in particular the tensile force. An estimation of the error in the identification of the force is given. The technique has been tested on the six tie-rods at the ancient Dome of Parma, Italy, famous for the frescos painted by the master Francesco Mazzolla, called *Il Parmigianino*, in 1500.

2. ANALYTICAL MODEL

The tie-rod is modeled as a simply supported Timoshenko beam; the supports are assumed at the beam edges inside the masonry wall and the portion of the beam inside the wall is subjected to an elastic Winkler foundation simulating the interaction between the beam and the wall, as shown in Figure 1. An isotropic beam of length L and non-uniform prismatic section A is considered. A Cartesian coordinate system (x, y, z) is assumed on the beam where the x-axis is coincident with the centroidal axis and the y and z axes are coincident with the principal axes of the root cross-section. It is assumed that the centroidal axis is coincident with the elastic axis so that bending-torsion coupling is negligible [6]. The analysis is here limited to the x-y plane, and the kinematic displacements u, v, along the x and y axes respectively, are given by

$$u(x,y,t) = -y\ \theta(x,t)\,, \qquad v(x,y,t) = v(x,t)\,, \tag{1a,b}$$

where v is the transverse displacement and of the centroidal axis and θ is rotation of the cross-section about the positive z-axis. By using equation (1), the nonzero strain components of the beam are

$$\varepsilon_{xx} = -y\frac{\partial\theta}{\partial x}\,, \qquad \gamma_{xy} = \frac{\partial v}{\partial x} - \theta\,. \tag{2a,b}$$

The flexural displacement of the simply supported beam is expanded by using the following series of admissible functions

$$v(x) = \sum_{i=1}^{N} a_i \sin\left(\frac{i\pi x}{L}\right). \tag{3}$$

The rotation of the cross-section is expanded as

$$\theta(x) = \sum_{i=1}^{N} b_i\, i \cos\left(\frac{i\pi x}{L}\right)\ . \tag{4}$$

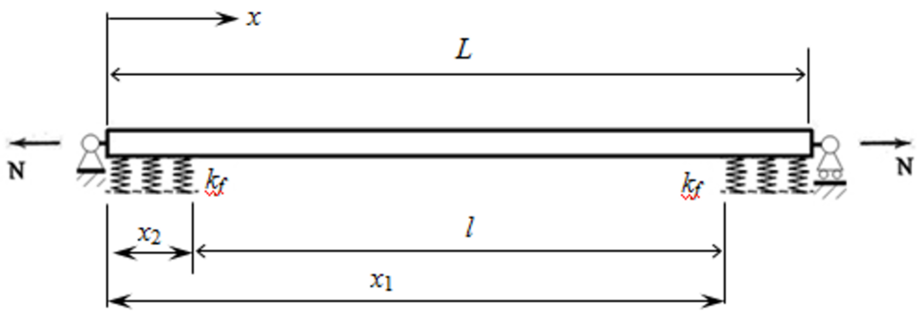

Figure 1. Analytical model of a tie-rod.

In equations (3) and (4) the same number of terms N has been used for the expansions of v and θ; in general, this number of terms can be different. The potential strain energy of the beam is given by [6]

$$V_B = \frac{1}{2}\int_0^L \int_A \boldsymbol{\sigma}^{\mathrm{T}} \boldsymbol{\varepsilon} \,\mathrm{d}A\,\mathrm{d}x \ , \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the strain vector and $\boldsymbol{\sigma}$ is the stress vector. By using Hooke's stress-strains relationships, equation (5) can be rewritten as [6, 7]

$$V_B = \frac{1}{2}\int_0^L \left[EJ\left(\frac{\partial \theta}{\partial x}\right)^2 + k\,GA\left(\frac{\partial v}{\partial x} - \theta\right)^2 \right] \mathrm{d}x \ , \tag{6}$$

where EJ and kGA are the flexural rigidity and the shear rigidity, respectively, with E being the Young modulus, J the second moment of inertia of the beam cross-section about the y-axis, k the shear coefficient [8] and G is the shear modulus. In particular, J, k and A are functions of x. The potential energy associated with the axial load F (positive is for traction) is expressed by

$$V_F = \frac{1}{2} F \int_0^L \frac{\partial^2 v}{\partial x^2} \mathrm{d}x = \frac{F\pi^2}{4L} \sum_{i=1}^{N} i^2 a_i^2 \ . \tag{7}$$

The potential energy associated with the elastic foundation is given by

$$V_W = \frac{1}{2}\int_{x_1}^{x_2} k_f v^2 dx = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_i\, a_j \int_{x_1}^{x_2} k_f \sin\left(\frac{i\pi x}{L}\right)\sin\left(\frac{j\pi x}{L}\right)\mathrm{d}x \ , \tag{8}$$

where k_f is the stiffness of the foundation, assumed uniform for simplicity, and the portion of the beam inserted in the walls is comprised between x_1 and x_2. In particular, x_1 = 0 for the left edge; for the right edge, x_2 = L. Both energies associated to elastic foundations at the left and right ends of the beam must be included. The global potential energy is

$$V = \frac{1}{2}\int_0^L \left[EJ(x)\left(\frac{\partial \theta}{\partial x}\right)^2 + k(x)GA(x)\left(\frac{\partial v}{\partial x} - \theta\right)^2 + F\frac{\partial^2 v}{\partial x^2} \right]\mathrm{d}x + \frac{1}{2}k_f\left[\int_0^{x_2} v^2\,\mathrm{d}x + \int_{x_1}^{L} v^2\,\mathrm{d}x\right] . \tag{9}$$

The kinetic energy of the beam is given by

$$T_B = \frac{1}{2}\int_0^L \int_A \rho \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 \right] \mathrm{d}A\,\mathrm{d}x \, , \tag{10}$$

where ρ is the mass density. By using equations (1a,b) and integration over the cross-section A, equation (10) can be rewritten as [6]

$$T_B = \frac{1}{2}\int_0^L \left[\rho J \left(\frac{\partial \theta}{\partial t} \right)^2 + \rho A \left(\frac{\partial v}{\partial t} \right)^2 \right] \mathrm{d}x \ . \tag{11}$$

The reference kinetic energy of the beam, i.e. the maximum kinetic energy divided by ω^2, is given by

$$T_B^* = \frac{1}{2}\int_0^L \left[\rho J(x)\theta^2 + \rho A(x) v^2 \right] \mathrm{d}x \ . \tag{12}$$

By introducing the following vectorial notation

$$\mathbf{q}^{\mathrm{T}} = \left(a_1, \ldots, a_N, b_1, \ldots, b_N \right)^{\mathrm{T}} \ , \tag{13a}$$

$$V = \mathbf{q}^{\mathrm{T}} \mathbf{K} \mathbf{q} \ , \tag{13b}$$

$$T_B^* = \mathbf{q}^{\mathrm{T}} \mathbf{M} \mathbf{q} \ , \tag{13c}$$

the natural circular frequencies ω of the tie-rod are obtained by solving the following eigenvalue problem

$$\mathbf{K} - \omega^2 \mathbf{M} = 0 \ . \tag{14}$$

The shear coefficient k in equations (6, 9) for rectangular cross-section of Timoshenko beams is given by [8]

$$k = -\frac{2(1+\nu)}{\left[\frac{9}{4a^5 b} C_4 + \nu \left(1 - \frac{b^2}{a^2} \right) \right]} \ , \tag{15a}$$

where

$$C_4 = \frac{4}{45} a^3 b \left(-12a^2 - 15\nu a^2 + 5\nu b^2 \right) + \sum_{n=1}^{\infty} \frac{16\nu^2 b^5 \left[n\pi a - b \tanh\left(\frac{n\pi a}{b} \right) \right]}{\left(n\pi^5 \right)(1+\nu)} \ , \tag{15b}$$

ν is the Poisson's coefficient, 2*a* is the depth of the beam in *y*-direction and 2*b* is the width of the beam in *z*-direction; *a* and *b* are functions of *x*.

3. IDENTIFICATION METHOD

In order to identify the axial force *F* and the stiffness k_f of the elastic foundation, the weighted difference between the calculated and identified natural frequencies is introduced

$$err_{RMS}\left(F, k_f \right) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \delta_i^2} \ , \tag{16}$$

where *M* is the number of the natural modes included in the identification process (5 to 10) and δ_i is the difference between the *i*-th computed and experimentally measured natural frequency. The function given in equation (16) is minimized over the full set of the useful values of the two unknowns (*F*, k_f), defined in the pre-processing, giving finally the identification of the axial force *F*. This minimization process requires the calculations of a large set of eigenvalue problems (14). The advantage of a self-developed optimized codes for the calculations of the natural frequency of the beam is fundamental versus a commercial FEM code. In the present case, the self-developed program has been validated by comparison to the ANSYS commercial program, and the results obtained are practically coincident. In particular, the first two mode shapes of one of the tie-rods studied in Section 4 are shown in Figure 2; they have been obtained by the ANSYS program and are coincident to those computed by the self-developed Rayleigh-Ritz program. However, since the numerical program implementing equation (14) uses a largely reduced size of the model with respect to the FEM commercial program, the time saving is huge.

The minimization process of function (16) can be implemented versus three unknowns F, k_f and the length of the elastic foundation, in case that the length of the tie-rod inserted into the masonry wall is unknown.

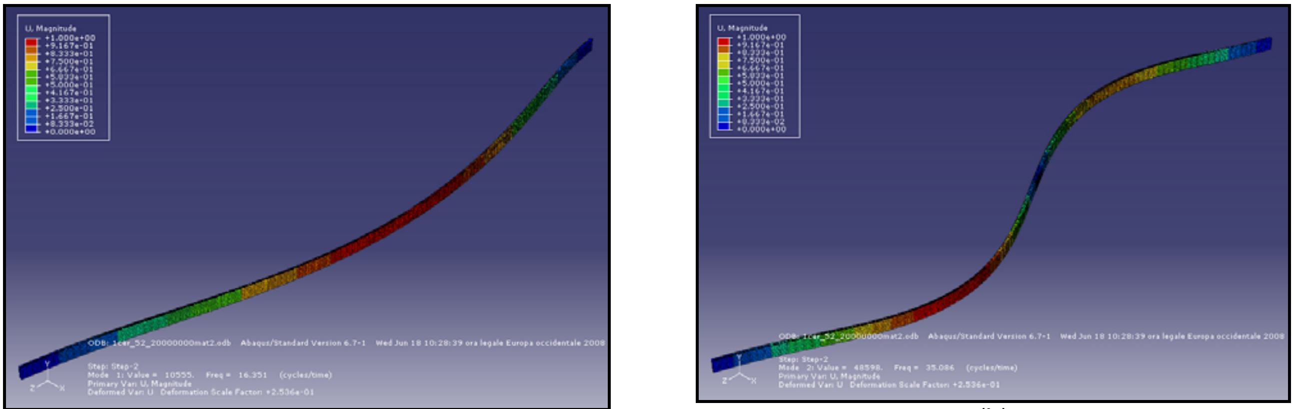
(a) (b)

Figure 2. Numerical simulation of a tie-rod dynamics showing (a) mode I and (b) mode II

4. EXPERIMENTS AND IDENTIFICATION AT THE CASTLE OF FONTANELLATO

Experimental measurement of the natural frequencies of the six tie-rods of the central vault of the ancient Dome of Parma, Italy (see Figures 3 and 4), was performed in September 2008. The used instrumentation is composed by an accelerometer *B&K* 4370, an instrumented hammer *B&K* 8202, two charge amplifiers *B&K* 2635 and a *Pimento* 8 channel system by *LMS* for data acquisition and experimental modal analysis.

The tests have been conducted in two days, and an articulated aerial platform has been used to reach the 6 tie-rods at 21 meters of height. The tie-rods have a circular cross-section of 40 mm in diameter; all tie-rods have been changed in '50 because the rupture of one of them.

Experimental tests, showed in figure 4, have been made putting ne or two accelerometer on the rod, and gently hammering it by hand. One of the six measured Frequency Response Functions (FRFs) is shown in Figure 5 for the tie-rods "3": eight peaks are clearly visible that identify the first eight natural vibration modes of the tie-rod.

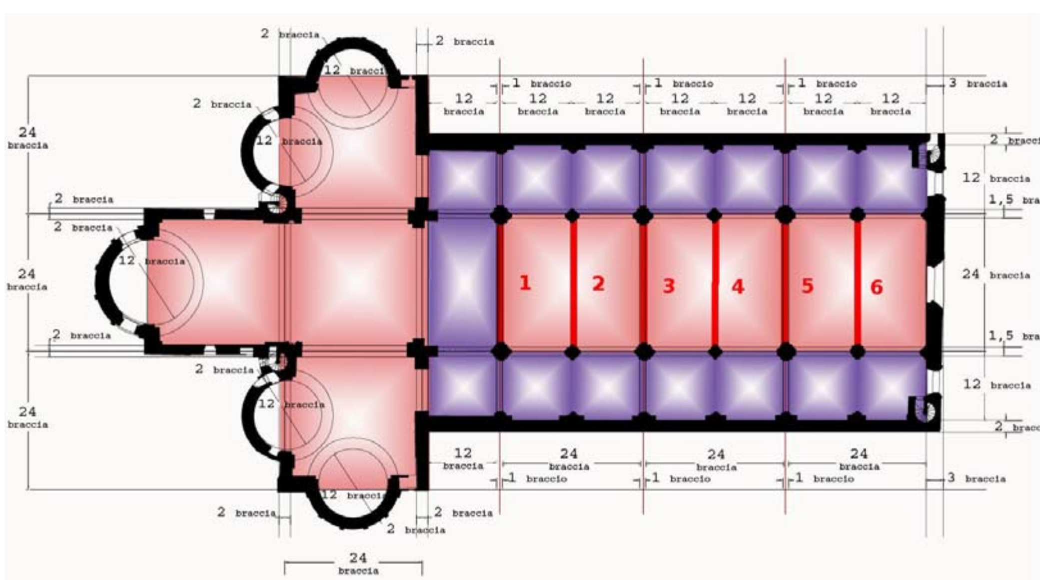

Figure 3. The Dome of Parma and the 6 tie-rods at 21 meters height.

Elaboration of experimental data has been conducted in conjunction with the numerical calculation of the axial load effect on the vibration modes of each tie-rods. The assumed material characteristics are: $E = 193\times10^9$ N/m^2, $G = 74.8\times10^9$ N/m^2, $\rho = 7870$ kg/m^3, $\nu = 0.29$. From $M = 2$ to $M = 6$ natural modes are used in the identification process, and $N = 15$ terms in the expansions (3) and (4). The effect of the axial force F on the natural frequencies is increase the computed natural frequencies in the direction of higher frequency. Otherwise, the effect of the stiffness k_f of the elastic foundation changes the slope of the curve. Therefore, the two unknowns have two different type of effect on the dynamics of the tie-rod, simplifying the identification process.

Figure 4. Experimental testing at the Dome of Parma in 2008.

The results of the identification process are given in Table 1. Results are presented for identification using M = 2 to 6 modes and show a convergence of the natural frequencies both experimentally and numerically determined.

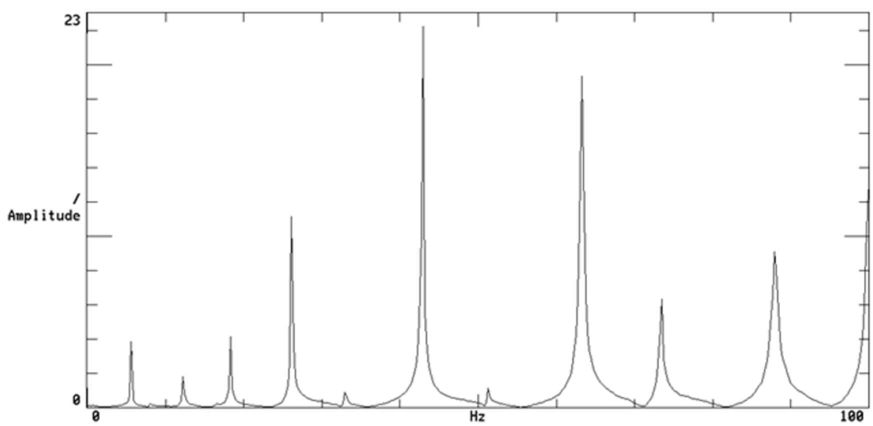

Figure 5. Natural modes of tie-rods “3”.

Table 1. Experimental and computationally determined natural frequencies of the six tie-rods of Parma Dome.

Tie-rod	Freq. Nat. 1 [Hz]	Freq. Nat. 2 [Hz]	Freq. Nat. 3 [Hz]	Freq. Nat. 4 [Hz]	Freq. Nat. 5 [Hz]	Freq. Nat. 6 [Hz]
	Measured					
1	5,9	13,0	19,0	27,3	34,4	44,7
2	6,1	13,4	19,8	28,3	35,2	45,4
3	5,6	12,2	18,3	26,1	33,0	43,0
4	6,6	14,4	21,2	29,8	37,4	47,4
5	6,8	15,4	22,0	31,7	39,1	50,3
6	6,6	13,7	20,3	28,6	35,9	45,4
	Simulated					
1	5,9	12,7	18,6	26,9	33,6	43,6
2	6,1	13,1	19,2	27,6	34,6	44,6
3	5,6	12,2	17,9	26,1	32,9	43
4	6,6	14,1	20,6	29,5	36,6	47
5	6,8	14,4	21,2	30,1	37,7	48,3
6	6,6	13,8	20,5	28,8	36,2	46

Verification of the mechanical resistance of the tie-rods is made by the simple formula of axial stress calculation in a uniform cross-section beam:

$$\sigma_A = \frac{F}{A_{res}}, \qquad (17)$$

where A_{res} is the design area of the metric M40 bolt in the medium section of the tie-rod. Results are shown in graphical form in figure 6. The stress state of at least 3 of the 6 tie-rods is over the admissible design stress, that for this steel is 160MPa.

5. CONCLUSIONS

The fully dynamic identification method allows to determine with a simple experiment the axial force on tie-rods. Since the experimental apparatus is compact and no fixed and accurate reference is necessary, the technique is particularly suitable for in-situ measurement on monumental buildings. The technique presented here has the advantage of using redundant data with respect to the unknowns in the identification process, minimizing the measurement and modeling errors.

Moreover, it allows to estimate the accuracy in the estimation of the axial force acting on the tie-rod: the experimental testing at the Parma Dome showed a dangerous state of the tie-rods of the central vault.

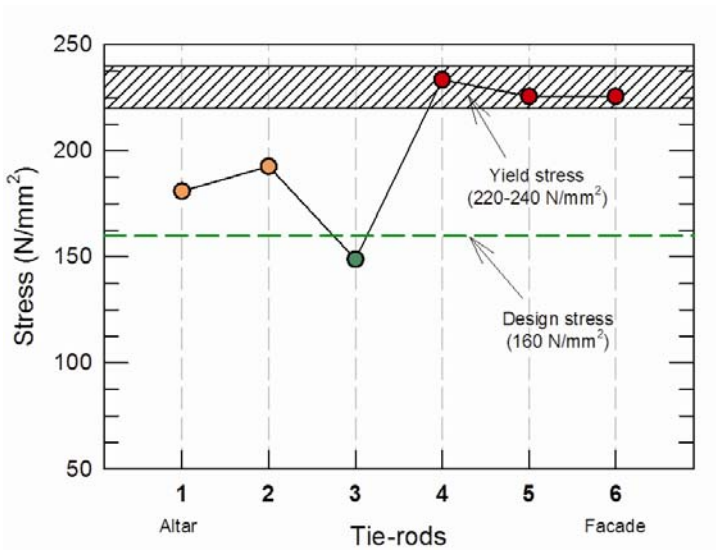

Figure 6. Stress state in the six tie-rods of Parma Dome.

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