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A Comparison of Direct Velocity, Direct and Compensated Acceleration Feed-back Control Systems in Mitigation of Low-frequency Floor Vibrations

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NOMENCLATURE

x	Displacement vector (n×1)	K	Stiffness matrix (n×n) (N/m)
$\dot{x}$	Velocity vector (n×1)	k	Modal stiffness (N/m)
$\ddot{x}$	Acceleration vector (n×1)	ζ_n	Modal damping ratio
F	Force vector (n×1)	ω_n	Natural modal frequency (rad/s)
M	Mass matrix (n×n)	g	Feed-back gain
M	Modal mass(kg)	$G_A(j\omega)$	Actuator's transfer function
C	Damping matrix (n×n)	$G_s(j\omega)$	Sensor's transfer function
C_m	Modal damping	$G(j\omega)$	Plant's transfer function

ABSTRACT

Numerous control systems, passive, active and semi-active, have been developed in past research to mitigate undesirable vibrations in civil engineering structures. Active vibration control (AVC) is emerging as a realistic option for mitigation of human-induced vibrations in office floors. Some AVC control laws commonly associated with vibration mitigation of human-induced office floor vibrations that have been investigated within this research include direct velocity feedback (DVF), direct acceleration feedback (DAF) and compensated acceleration feedback (CAF). The research presented in this work evaluates the bandwidths of effectiveness of the vibration mitigation performances of each of the aforementioned AVC control laws for idealised single-degree-of-freedom (SDOF) models of a floor structure with frequencies ranging from 1 Hz to 20 Hz. Modal masses and damping ratios of these SDOF floor representations have been kept constant. It was found that DVF performs best in low frequency floors while DAF performs well at much higher frequency floors. CAF performs quite well in both low and high frequency floors used in these studies.

Keywords: Active control, compensator, human induced vibrations.

1. Introduction

Recent advancements in construction materials and design technologies have led to lighter and more slender structural elements that are more appealing from an architectural point of view. Slender floor structures, for example, being a consequence of such advancements, often do not meet the desired vibration serviceability requirements. Problems have been reported in numerous structures like office floors, shopping malls, airports concourses and restaurants to name but a few (1).

There has been a vast amount of research in the past to mitigate human induced vibrations in floors. Passive technologies were the typical first generation approach. They employed some mechanical elements and devices to dissipate part of the energy in floors and thereby reduce the vibration levels. These technologies do not need any external energy sources and are quite reliable. However, they have limited effectiveness in some instances and have to be tuned to specific dominant modes of vibration (2).

To improve the efficiency of passive techniques, semi-active control was developed. With these techniques, some features of passive vibration mitigation systems can be adjusted in real-time to increase their vibration mitigation effectiveness for a broad spectrum of disturbances. Examples of such properties that can be adjusted include the fluid viscosity of viscous dampers as seen in magneto-rheological (MR) and electro-rheological (ER) dampers. However, there are still a number of issues with regards to their effectiveness in various scenarios, particularly for mitigation of human induced vibrations in floor structures.

Active vibration control (AVC) has been developed to make vibration mitigation, for example of floors under human induced vibrations, to be more efficient and adaptive, although it is not the ultimate solution and has some potential drawbacks in itself. An active control system in simplest form comprises of a sensor to monitor the structural response, a control algorithm to compute the required control signal and an actuator to introduce the control force (3).

Within AVC studies several control laws exist, typical examples of which make use of feedback loops. Some of the AVC control laws that have been implemented by researchers for mitigation of human induced vibrations in floors include DVF, DAF and CAF. Each possesses some advantages and disadvantages, which make a proper selection of which one to implement in a particular application complicated.

This paper investigates the vibration mitigation performance of DVF, DAF and CAF control laws on some idealised SDOF floor slabs with variable dynamic properties i.e. frequencies ranging from 1 Hz to 20 Hz. The modal masses and modal damping ratios have been kept constant in all the models. This would give an indication of what type of control system can be implemented in a floor structure of given modal properties. Actuator dynamics and sensor dynamics have been considered in all the cases, where the sensor dynamics refers to the dynamics of the integration circuit. Root locus studies are used to evaluate optimal control gains that are implemented in the analytical studies. Analytical studies comprise of heel-drop tests making use of an idealised triangular pulse and walking excitation measured from treadmill tests. For heel-drop excitation, the time for the response to decay to 5% of its peak is used as the performance indicator. For walking excitation, the peak value of the 1s running RMS (MTVV) has been as the performance indicator. Some root locus plots are presented to elaborate on the reasons for the vibration mitigation performances attained.

2. Dynamics of Control System Components

2.1. Floor Models

In this research, SDOF models as shown in Fig. 1 and with linear, constant coefficient differential equation taking the form of Eq. 1, have been selected to represent the dominant modes of some 20 idealised simply supported floors of different modal properties. The dominant modes of these floors have been assumed to have natural frequencies ranging from 1 Hz to 20 Hz as highlighted in Table 1. The modal masses and damping ratios of each of these models have been assumed to be constant. This aids in evaluating the possible bandwidths of efficiency of each of the AVC control laws noted in the abstract for vibration mitigation of floors of different modal properties. For a collocated sensor/actuator pair in each idealised floor representation, the transfer functions of each of these models can be represented by Eq. 2,

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{1}$$

$$G(s) = \frac{\chi_i s^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{2}$$

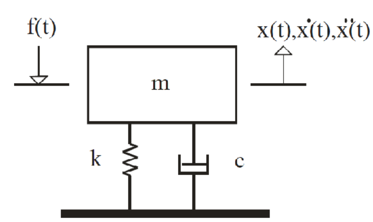

Figure 1 – SDOF Model

where m is the modal mass, c is modal damping and k is the modal stiffness, $s = j\omega$, ω is the frequency, χ_i , ζ_i and ω_i are the inverse of the modal mass, the damping ratio and natural frequency associated with the i^{th} mode, respectively.

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Table 1 – Properties of System SDOF Models

2.2. Actuator Dynamics

The proof-mass actuator considered in this research is an APS dynamics Model 400 mechanical shaker. It has a 30.4 kg reaction mass and its estimated natural frequency is 1.8 Hz. The dynamics can closely be modelled by the linear third-order model in Eq. 3. $K_A>0$, and ξ_A and ω_A are, respectively the damping ratio and natural frequency of the actuator and the pole at $-\varepsilon$ provides the low-pass characteristics of this actuator (2).

$$G_A(s) = \left(\frac{K_A s^2}{s^2+2\xi_A\omega_A s+\omega_A^2}\right)\left(\frac{1}{s+\varepsilon}\right) \tag{3a}$$

$$G_A(s) = \left(\frac{12600s^2}{s^3+62.16s^2+728.6s+6573}\right) \tag{3b}$$

2.3. Sensor Dynamics

The sensor dynamics i.e. the dynamics of the integrator circuit used in these studies can be modelled by the filter in Eq. 4.

$$G_S(s) = \left(\frac{s}{s^2+8s+100}\right) \tag{4}$$

3. Control Laws

Three different control laws used for mitigation of human-induced floor vibrations in previous research include DVF, DAF and CAF (2). The vibration mitigation performance bandwidths of these control laws for human induced vibrations are investigated in this AVC research on some idealised floors whose modal properties have been presented in section 2.1. The human excitation forces that have been used in the analytical studies, briefly described in section 4, comprise of an idealised heel-drop excitation force (1) as well as real walking excitation forces obtained from treadmill tests at The University of Sheffield.

A voltage saturation level of V_L= 0.5V has been introduced in the analytical studies. In principle, this is the saturation voltage that would be implemented in the experimental implementation of the AVC system. The three control laws noted previously will now be briefly introduced.

3.1. Direct Velocity Feedback (DVF)

With DVF control law, for a collocated sensor and actuator control system, sensor outputs (velocity) are multiplied by pre-selected control gains and these fed back to the collocated actuators as expressed in Eq. 5.

$$F_c = -g.\dot{y}_s \qquad (5)$$

Where $\dot{y}_s$ is the collocated velocity output signal, g is the control gain and F_c is the control voltage that is fed to the actuator amplifier to drive the actuators. The control set-up for DVF control law is presented in Fig. 2. This figure incorporates a voltage saturation stage.

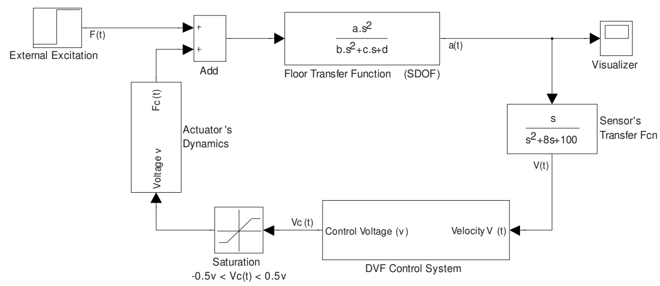

Figure 2 - DVF Scheme (with saturation Voltage-limiter)

3.2. Direct Acceleration Feedback (DAF)

For DAF control law, also considering a collocated sensor and actuator set-up, sensor outputs (acceleration measurements in this case) are multiplied by pre-selected control gains to generate the control signal that is then fed to the actuator amplifier to drive the actuators. This is presented in Eq. 6 and with the control set-up being presented in Fig. 3.

$$F_c = -g.\ddot{y}_s \qquad (6)$$

Where $\ddot{y}_s$ is the collocated acceleration output, g is the control gain and F_c is the control voltage signal to amplifier.

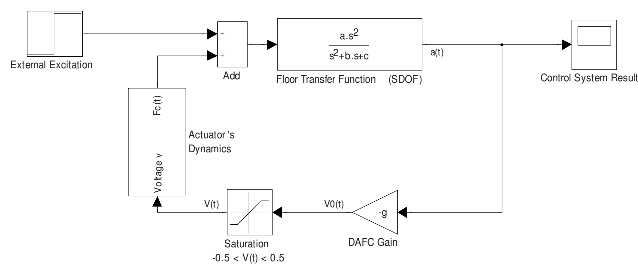

Figure 3 - DAF scheme

3.3. Compensated Acceleration Feedback (CAF)

To increase the stability of DAF system and make it more amendable to the introduction of significant damping, a phase-lag compensator can be designed. The CAF set-up as designed in a previous research (2) is represented in Fig. 4.

The compensator takes the form of $\frac{s+a}{s}$. By increasing the constant 'a', the compensator acts as an integrator $\frac{a}{s}$, and similarly, by selecting small values for the constant, 'a', the compensator behaves more like DAF. In this paper, constant 'a' = 55. This is applicable for the frequency bandwidth of interest in these studies presented.

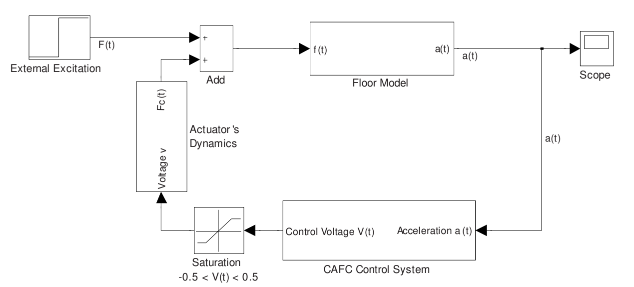

Figure 4 – CAFC system using phase-lag compensator

4. Human Excitations Forces

The excitation forces used in the analytical studies presented in this paper include an impulsive excitation force and a real walking force time history obtained from treadmill walking tests.

4.1. Heel-drop Force

The heel-drop excitation force can be idealised as a triangular pulse with a peak force of 2670 N, reducing to zero in a time of 0.05 s, which corresponds to an impulse of 67 Ns (1). This is shown in Fig. 5.

4.2. Walking Excitation

Walking force time histories obtained from treadmill walking tests at The University of Sheffield were used in the analytical studies. The walking force specifications are: Body weight: 886 N, Walking Speed = 1.2085 m/s equivalent to 1.7 Hz pacing rate (and further assuming a 70 cm step length). This is shown in Fig. 6.

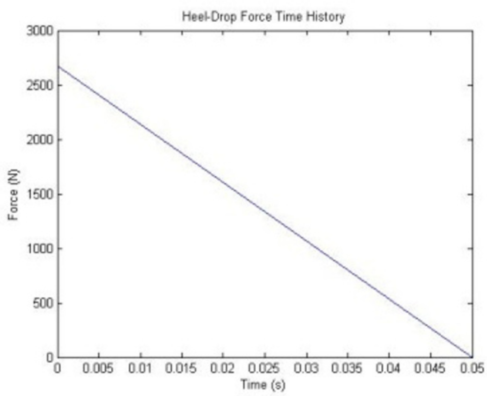

Figure 5 - Heel-drop force time history

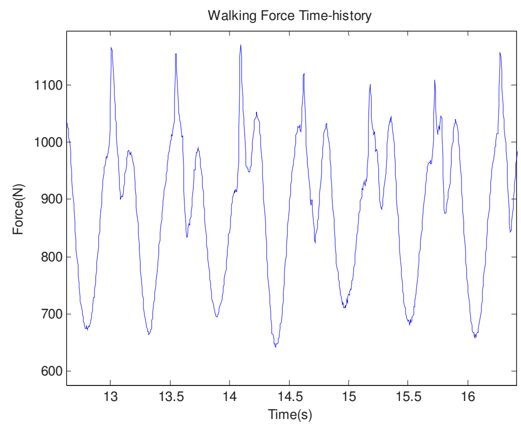

Figure 6 – Part of walking force time history

5. Simulation Results

The three techniques chosen to evaluate the efficiencies of DVF, DAF and CAF control schemes for vibration mitigation of the idealised floor systems noted earlier include:

(a) A study of the damping introduced by the control systems (DVF, DAF and CAF) into the primary structure by a study of the root locus studies of the closed-loop systems.

(b) A study of the MTVV, described as the peak of the 1s running RMS of the uncontrolled and controlled (DVF DAF and CAF) responses for walking excitation only.

(c) A study of the time it takes for the uncontrolled and controlled responses to decay to 5% of the initial peaks for impulsive excitation only.

5.1. Damping Ratio

The closed-loop systems for each of the setups in Figs. 2, 3 and 4 can be described by the transfer function representation in Eq. 7. This incorporates the primary structure, actuator and sensor (integrator circuit) dynamics. $G(j\omega)$ is the transfer function of the floor, $G_A(j\omega)$ is the transfer function of the actuator, $G_s(j\omega)$ is the transfer function of the sensor and $G_c(j\omega)$ is the transfer function of a compensator, if any.

$$G_T(j\omega) = G(j\omega)G_A(j\omega)G_s(j\omega)G_c(j\omega) \qquad (7)$$

By making use of root locus studies for the transfer function representation in Eq. 7 for each of the idealised structural models, optimum gains were evaluated as presented in section 5.4 as well as maximum damping introduced into each of the structural models.

[Fig. 7](#) shows a plot of the damping ratio introduced into the structural models for DVF, DAF and CAF control laws for optimum gains. DVF exhibits the best performance at low frequencies while CAF exhibits better performance at higher frequencies. The performance here is with reference to the amount of damping introduced into the structural models for each of the control laws.

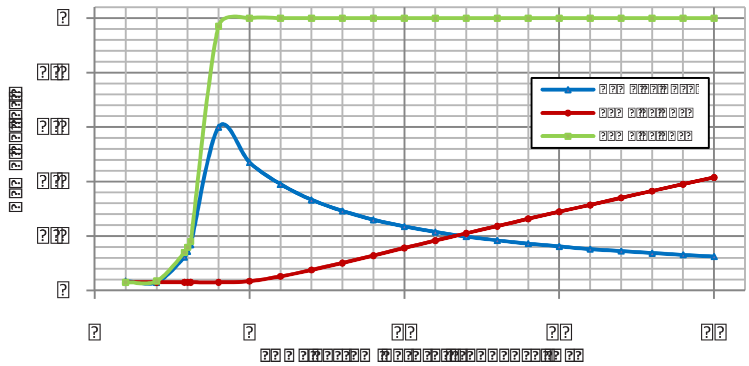

Figure 7 - Structural damping ratios for DVF, DAF & CAF control for each of the idealised structural models

5.2. MTVV Studies (For Walking Excitation Only)

The optimum gains evaluated in the root locus studies in the previous section and presented in section 5.4 are implemented in these analytical studies for DVF, DAF and CAF control laws. The MTVV is defined as the peak of the 1s running RMS acceleration response of both the uncontrolled and controlled structures. A typical plot of responses of an open and closed loop system and the corresponding 1s running RMS acceleration are shown in Fig. 8.

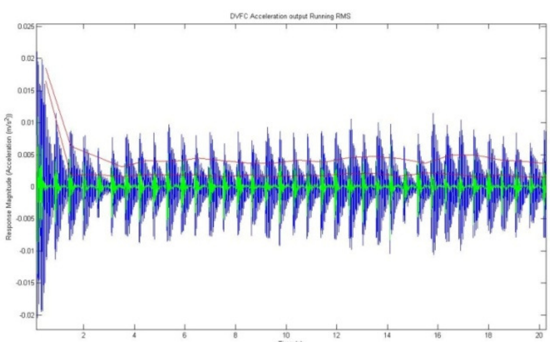

Figure 8 - A typical open-loop and closed-loop system acceleration response and the corresponding 1 sec running RMS accelerations

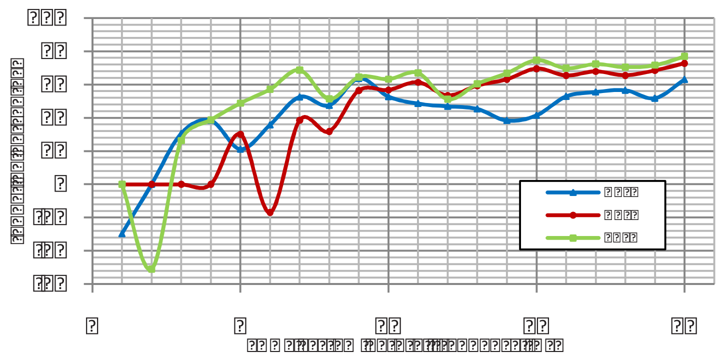

Figure 9 – Acceleration MTVV Reduction of DVFC, DAFC and CAFC systems

The trends of the reduction in MTVV of the closed loop (for DVF, DAF and CAF) responses in comparison with the open-loop system responses are plotted in Fig. 9. Again, it can be see that DVF exhibits better vibration mitigation performances at low frequencies and CAF exhibits better vibration mitigation performances at high frequencies.

5.3. Reduction in Decaying Time (For Heel-drop Excitation Only)

The optimum gains determined from the root locus studies in section 5.1 presented in section 5.4 are used in the analytical studies in this section for each of the idealised structural models. The time for the responses to decay to within 5% of the initial peak is used to evaluate the vibration mitigation performance of DVF, DAF and CAF for each of the structural models subjected to an impulsive excitation force. Fig. 10 shows a typical open-loop and closed-loop response. Fig. 11 shows the vibration mitigation performances for each of the control laws implemented on each of the idealised structural models as percentage improvements. Again, it can be seen that DVF performs best at low frequencies while CAF performs better at higher frequencies.

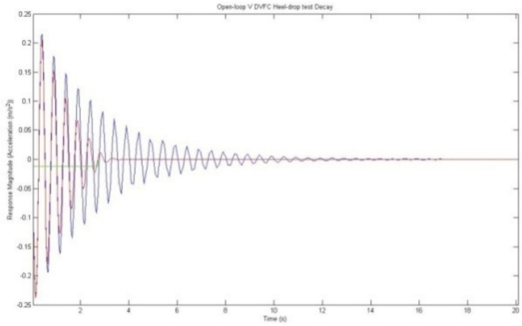

Figure 10 - Typical open-loop and closed-loop acceleration responses to an impulsive excitation

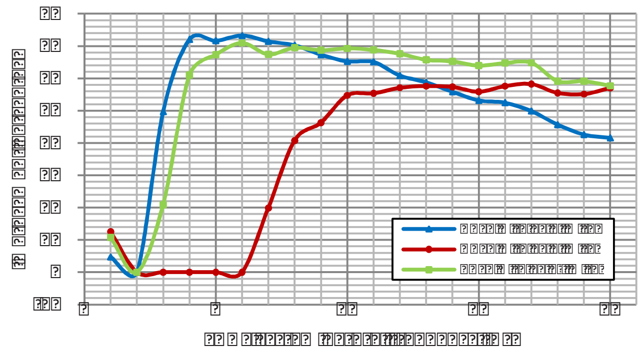

Figure 11 - Decrease of Reduction Time of Peak Responses to 5% of its Value from Open-loop to Closed-loop system (Heel-drop Test)

5.4. Optimum Gains

The optimum gains used in the above analytical studies that have been determined from the root locus studies are presented in Fig. 12 for DVF, DAF and CAF control laws.

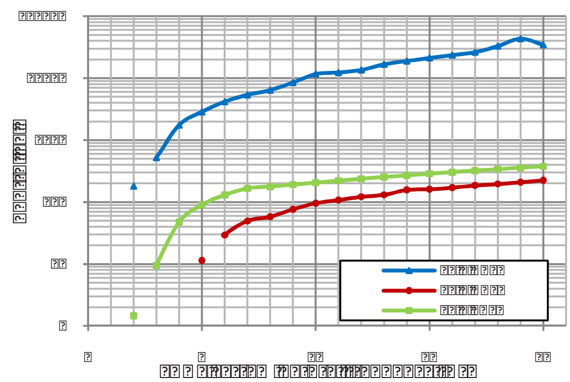

Figure 12 - Optimal gains for DVF, DAF & CAF control laws

6. Results from Analytical Studies

6.1. DVF Control Law

From Figs. 7, 9, and 11, it can be seen that DVF is more effective in predominantly low frequency floors i.e. between 3-10 Hz. Fig. 7 exhibits a sharp peak between 3-4 Hz and there is a gradual decrease in higher frequencies. DVF does not have any significant effect on the very low-frequency floors i.e. with natural frequencies ranged 1-3 Hz as a result of the actuator dynamics lying within this range.

Some root locus plots of some of the idealised floor models in Figs. 13, 14, 15, 16 and 17 aid in justifying the observed vibration mitigation performances with DVF control law.

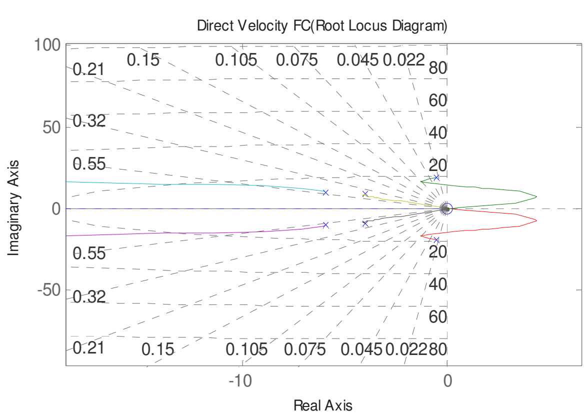

Figure 13 - Root locus plot for structural model with f = 3 Hz for DVF control law

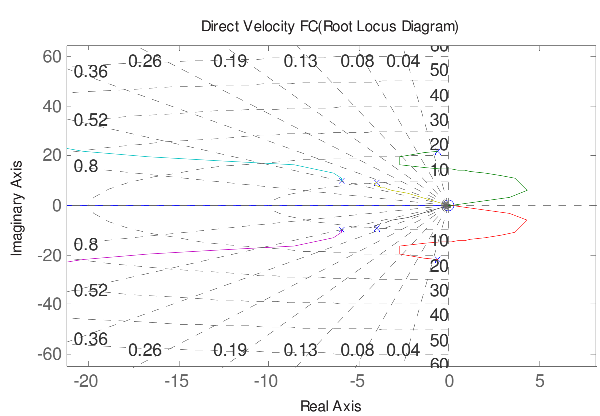

Figure 14 - Root locus plot for structural model with f = 3.5 Hz for DVF control law

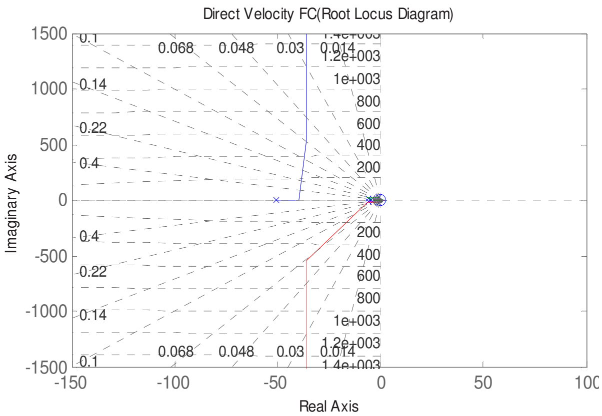

Figure 15 - Root locus plot for structural model with f = 3.8 Hz for DVF control law

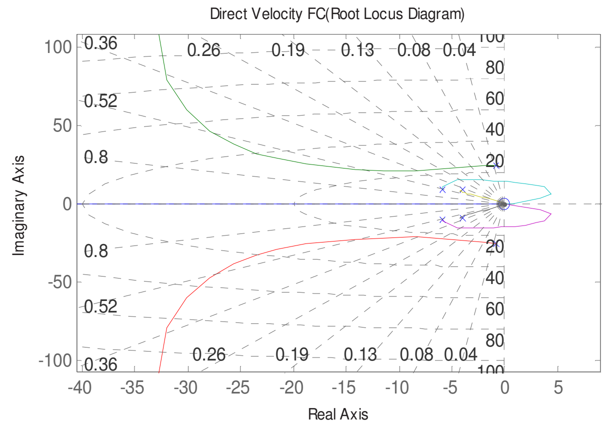

Figure 16 - Root locus plot for structural model with f = 4 Hz for DVF control law

The structural models comprise of a pair of very low-damped poles and a zero in the origin. A pair of high-damped poles represents the actuator dynamics and a pole in ε = -50.26 represents the low-pass element of the actuator. An obvious pole flipping between floor and actuator transfer function poles can be seen about f = 4 Hz, and the loci suddenly gain a departure angle very close to the negative real axis after 4 Hz which implies higher structural damping can be attained after this frequency. This accounts for the sharp peak between 3.5-4 Hz in Fig. 7.

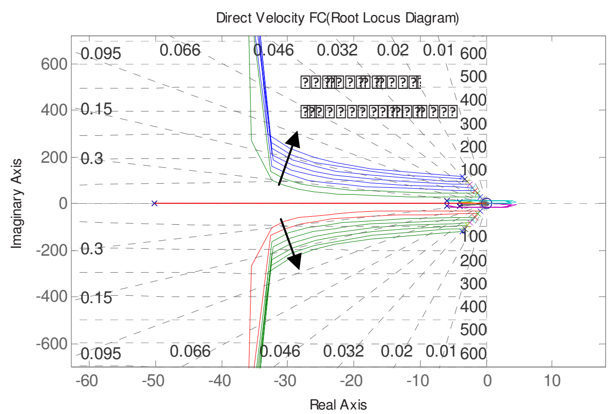

Figure 17 - DVFC system Rlocus diagram evolution for f = 5 Hz to f = 20 Hz

6.2. DAF Control Law

From Fig. 7, it can be seen that DAF control law does not offer any significant influence on the dynamic properties of the idealised models with natural frequencies below 5 Hz. After 5 Hz, however, the damping introduced into the system gradually increases almost linearly. This can be explained considering the results from the root locus studies presented in Figs. 18, 19, 20 and 21.

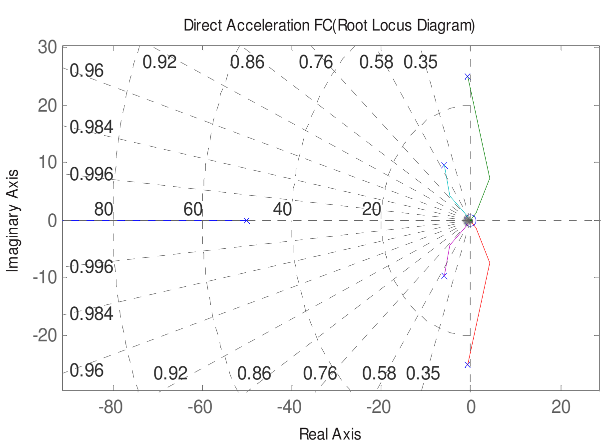

Figure 18 – f = 4 Hz DAFC system Rlocus diagram

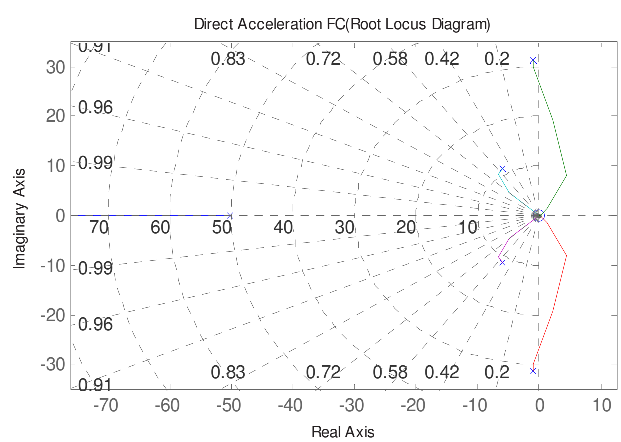

Figure 19 – f = 5 Hz DAFC system Rlocus diagram

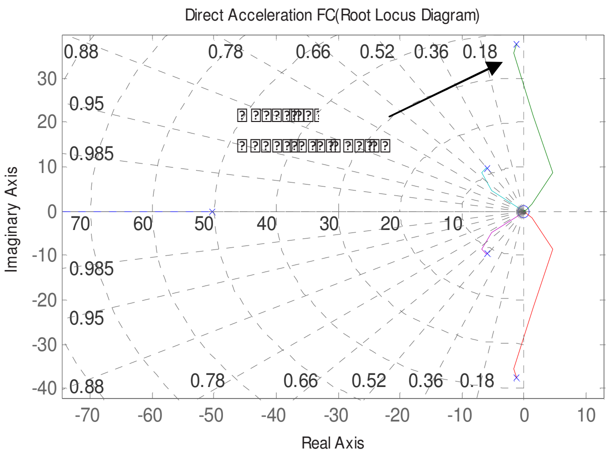

Figure 20 – f = 6 Hz DAFC system Rlocus diagram

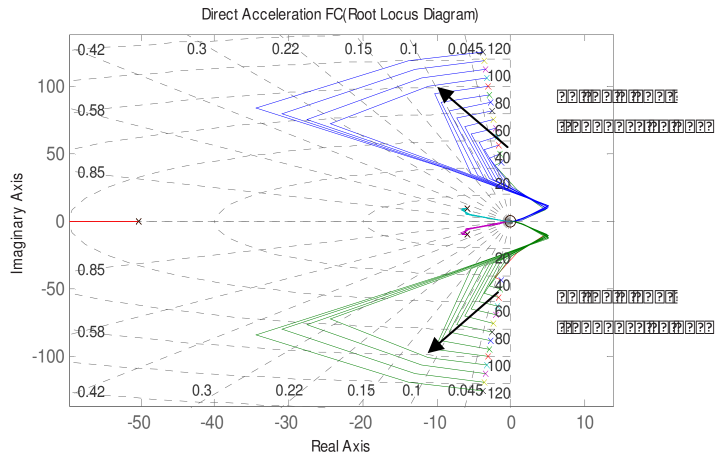

Figure 21 – Evolution of f = 7 to 20 Hz DAFC system Rlocus diagram

No pole flipping occurs in these closed loop systems. For idealised structural models with modal properties between 1 Hz to 5Hz, the departure angles of the loci of models are located in the right half plane towards the positive real axis. This indicates a near-unstable scenario with a very low damping value. This positive departure angle introduces much less damping than the open loop damping of the structural models. For idealised structures with natural frequencies above 5 Hz, the departure angles of the loci gradually change clockwise from positive to negative values. A gradual evolution of the root locus plot for structural models above 7 Hz can be seen in Fig. 21.

6.3. CAF Control Law

CAF control law is an integrated version of DVF and DAF control schemes. It aims at utilising the advantages of DVF in lower frequencies and DAF in higher frequencies. This can be seen in Fig. 7. The zero of the compensator at '–a' improves systems stability by inducing higher damping as well as attracting the system poles after pole flipping. Figs. 22, 23, 24, 25, 26 and 27 highlight the evolution of root locus plots of the closed-loop systems for the idealised structural models noted earlier.

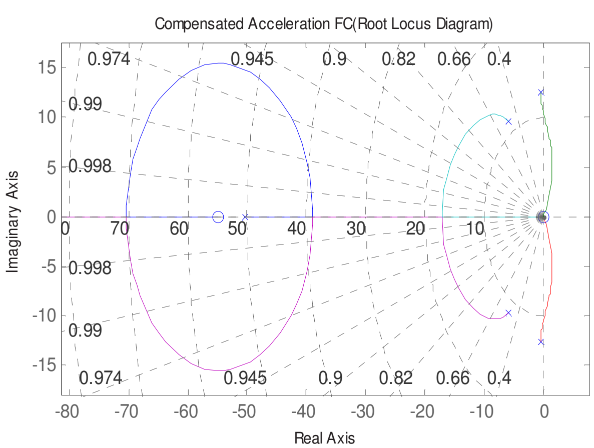

Figure 22 - Root locus plot of system model with f = 2 Hz

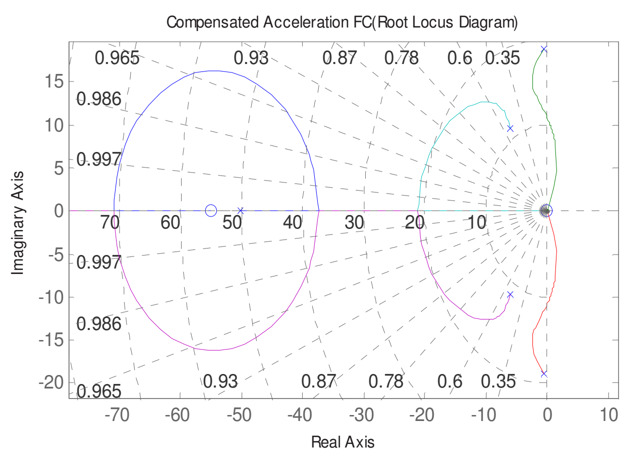

Figure 23 - Root locus plot of system model with f = 3 Hz

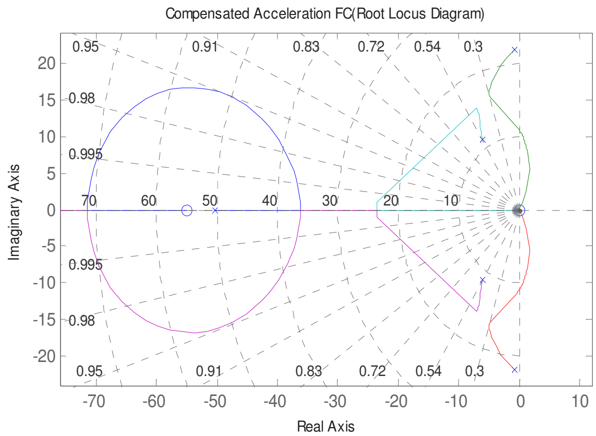

Figure 24 - Root locus plot of system model with f = 3.5 Hz

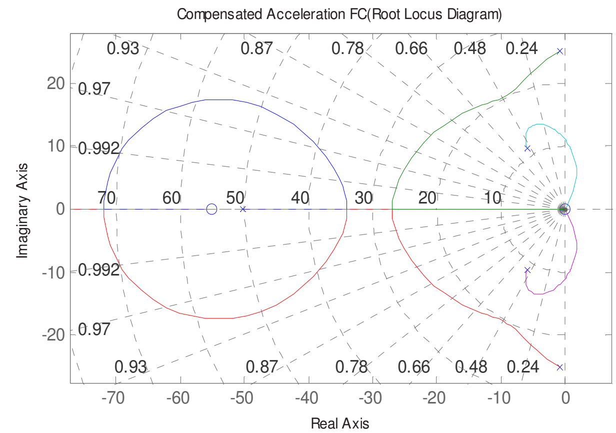

Figure 25 - Root locus plot of system model with f = 4 Hz

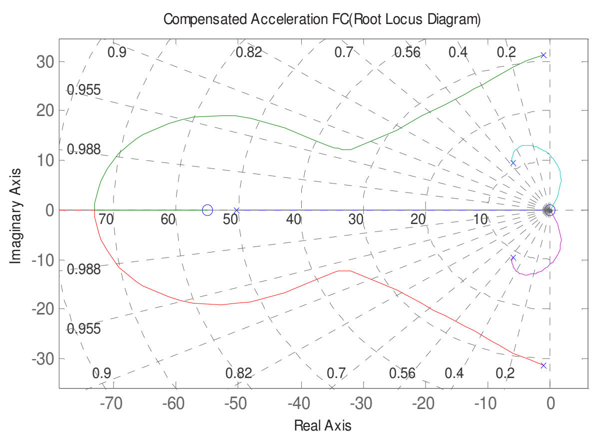

Figure 26 - Root locus plot of system model with f = 5 Hz

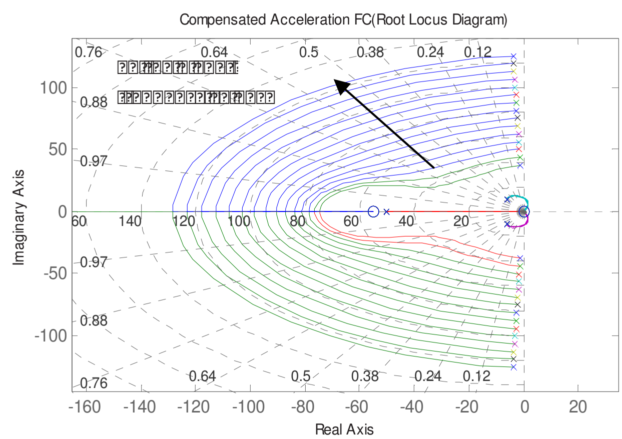

Figure 27 - Root locus plots of system models with f = 6 - 20 Hz

Two pole flipping phenomena can be seen in Figs. 25 and 26. The first one happens between f = 3.5 Hz to f = 4 Hz, where the new floor loci show higher stability and damping. The second one happens between f = 4 Hz and f = 5 Hz where phase-lag compensator zero at '-50.26' attracts one of the system poles that makes the system highly stable and damped.

7. Conclusion

DVF performs best for low frequency structural models while DAF has a tendency to perform better at higher frequencies. CAF acts quite well in both in mid-range and high frequency floors; there is still a problem with implementations in low-frequency floors. Finding proper compensators for mitigation of vibrations in very low frequency structural models is an exciting challenge and an ongoing area of research.

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