System Identification of a Three-Story Infilled RC Frame Tested on the UCSD-NEES Shake Table

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ABSTRACT

A 2/3-scale, three-story, two-bay, infilled RC frame was dynamically tested on a shake-table. The test objectives were to assess the seismic performance of existing, non-ductile, infilled RC frames and provide data for the evaluation of newly developed analytical methods predicting the behavior of such structures. The shake-table tests were designed to induce damage on the structure progressively through scaled earthquake records of increasing intensity. Between the earthquake records, the response of the frame to low-amplitude ambient vibration and white-noise base excitation tests was measured. At these low levels of excitations, the structure can be considered as a quasi-linear system with parameters depending on the damage state. The deterministicstochastic subspace identification method based on system input and output signals has been used to estimate the modal parameters of the test structure at its various damage states. The identification is conducted considering the white-noise base excitation and the resulting structural response measured by accelerometers. The study has quantified the decrease of natural frequencies and the increase of structural damping at progressive damage states. The identified modal parameters have been used for damage identification of the infilled frame in a companion paper.

Introduction

Changes in dynamic characteristics of structures have been used extensively for the purpose of condition assessment and damage identification of structures. In recent years, vibration-based structural health monitoring based on modal parameters has attracted increasing attention in the civil engineering research community as a potential tool to develop methods through which structural damage can be identified at the earliest possible stage. Extensive literature reviews on damage identification based on changes in vibration characteristics have been provided by Doebling et al. [1, 2] and Sohn et al. [3]. The study presented in this paper focuses on the experimental modal analysis of a three-story infilled reinforced concrete (RC) frame at different damage levels due to a sequence of shake-table tests.

The frame was a 2/3-scale model of a prototype structure designed to represent non-ductile RC frames built in California in the 1920s. It was tested at the University of California San Diego (UCSD) in November 2008, using the Large High Performance Outdoor Shake-table, which is part of the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES). The testing sequence is part of a collaboratory project involving researchers from Stanford University, the University of Colorado at Boulder and UCSD. The goals of the research team are to evaluate the seismic performance of older infilled RC frame buildings and to develop analytical methods and retrofit techniques for this type of structures. This two-bay, three-story frame was the largest structure of this type ever tested on a shake-table. The shake-table tests were designed to induce damage on the structure progressively through a sequence of scaled earthquake records. Between the earthquake excitations, the RC frame was subjected to low-amplitude ambient vibration and white-noise base excitation tests. At these low levels of excitations, the structure can be considered as a quasi-linear system with its parameters depending on the damage level. The deterministic-stochastic subspace identification (DSI) method, based on system input and output signals, has been used to estimate the modal parameters (natural frequencies, damping ratios, and mode shapes) of the frame at its various damage states as well as those of the undamaged structure. The identification method adopted in this study uses the white-noise base excitation and the corresponding structural response measured at different damage states of the structure. Recently, the DSI method was successfully applied by Moaveni et al. [4] for system identification of a seven-story building slice tested on the UCSD-NEES shake-table. It is worth noting that during the application of white-noise base excitations with the shake-table, the

table response was distorted by the system dynamics and, thereby, deviated significantly from a white noise/broadband excitation. Therefore, the application of output-only system identification methods to the whitenoise test data will result in modal parameters with significant estimation error.

Test Setup

Three-Story Infilled RC Frame

The prototype structure was designed by Stavridis and Shing [5] to represent a structure built in California in the 1920s era. The design only considered the gravity loads in accordance with the engineering practice of that era. However, the design was based on contemporary construction materials which were used for the construction of the test specimen. The plan view of the prototype structure and the elevation of an external frame are presented in Figure 1. The structural system consists of a non-ductile RC frame, which is infilled with three-wythe unreinforced masonry walls on the exterior. Such structural system can be found in many existing buildings in the western United States, including pre-1930s buildings in California. Moreover, this type of construction is common in many regions of the world with high seismicity such as the Mediterranean and Latin America.

Fig. 1 Plan view of the prototype structure (left) and Elevation view of an exterior frame (right)

The specimen tested on the outdoor shake-table at UCSD is shown in [Figure 2](#page-2-0). It corresponds to the external frame of the prototype structure with all its dimensions scaled by 2/3 and infill walls consisting of two wythes of brick units. The scaling factors were determined according to the similitude requirement [6] and are summarized in Table 1. From this point on, the ground motion levels mentioned in the paper are with respect to the full-scale prototype structure. As shown in [Figure 2](#page-2-0), two steel towers were secured on the shake-table on the north and south side of the structure to prevent a potential out-of-plane collapse of the structure during severe shaking. However, they did not interact with the structure during the tests as they were placed with a 0.75 inch gap from the specimen. Further details regarding the design of the specimen and the shake-table tests can be found in [7].

Fig. 2 Front view (left) and side view of the specimen (right)

Instrumentation Layout

The specimen and the steel towers were instrumented with an extensive array of sensors. They included 135 strain gages placed in the longitudinal and transverse reinforcing steel bars of the first story beams and columns where damage was anticipated, 71 string pots and Linear Variable Differential Transformers (LVDTs) to measure relative deformations of the structural members, and 59 uniaxial accelerometers. Since each accelerometer could only measure the acceleration along one direction of motion, metal cubes were attached on the structure to mount the accelerometers and obtain measurements along X, Y and Z directions, with X being the direction of motion, Y the out-of-plane direction, and Z the vertical direction. Six additional accelerometers were mounted at the middle and at the west end of the foundation slab of the specimen. The recordings from the middle accelerometer are considered as the input excitation for the structure. In this study, measured response data from 9 longitudinal (3 per floor), 9 vertical (3 per floor) and 6 transverse (2 per floor) acceleration channels, as shown in Figure 3, have been used to identify the modal parameters of the test structure. The channels have been named as LF-D, in which L is the location on the floor and can be W for West, C for Center, or E for East, F is equal to 1, 2, or 3 and denotes the floor level the accelerometer is located, and D denotes the direction of acceleration measured and can be X, Y, or Z. It should be mentioned that Channel C1-X (C: center of floor; 1: first floor; X: measuring in X direction) did not function properly during all the tests and Channel E3-X did not function correctly during Tests 5 and 9 (see [Table 2](#page-3-0)). However, there is sufficient redundancy in instrumentation that this does not present a problem.

Fig. 3 Locations and directions of accelerometers on each floor which are used in this study

The measured acceleration responses were sampled at 240Hz resulting in a Nyquist frequency of 120Hz, which is higher than the modal frequencies of interest in this study (< 60Hz). Before applying the system identification method to the measured data, all the absolute acceleration time histories were band-pass filtered between 0.5Hz and 70Hz using a high order (1024) Finite Impulse Response (FIR) filter.

Dynamic Tests Performed

A sequence of 44 dynamic tests was applied to the test structure including ambient vibration tests, free vibration tests, and forced vibration tests (white noise and seismic base excitations). The main goal of the testing sequence was to gradually damage the structure by subjecting it to seismic ground motions of increasing amplitude. Between the earthquake records, low-amplitude white noise base excitation tests as well as ambient vibration tests were performed, so that the modal parameters of the structure can be identified at the different damage states. The ground motion accelerograms were obtained by scaling time and amplitude of the acceleration time history recorded at the Gilroy 3 station during the 1989 Loma Prieta earthquake using the scaling factors shown in [Table 1](#page-1-0). For structures with a natural frequency close to the frequency of the infilled frame studied here, 67% of the Gilroy 3 motion corresponds to a design basis earthquake (DBE) for Seismic Design Category D. Moreover, for this structure, the 100% level of the Gilroy 3 motion corresponds to a maximum considered earthquake (MCE). Between the earthquake tests, recordings were obtained from ambient vibration as well as low-amplitude whitenoise base excitation tests for the purpose of modal analysis. In this study, eight damage states in the testing sequence have been analyzed including the initial state of the undamaged structure. Damage state S0 is defined as the undamaged (baseline) state of the structure before its exposure to the first seismic excitation, while damage states S1 to S7 correspond to the state of the structure after exposure to the earthquake ground motions of increasing amplitude. The design basis earthquake (67% of the Gilroy) was applied twice on the shake-table and visible cracks were observed after the structure was exposed to this earthquake for the second time. Table 2 summarizes the dynamic tests used in the present study and the corresponding damage states of the test structure. It should be noted that the ambient vibration data are not used for the system identification of the infilled frame structure because the signal to noise ratio of the measured ambient acceleration response is very low.

| Test No. | Test Date | Test Description | Damage State | | |
|----------|----------------------------|-------------------------|----------------|--|--|
| 5 | 11/3/2008 | 0.03g RMS WN, 5 min | S0 | | |
| 8 | 66 | 20% Gilroy EQ | | | |
| 9 | $\mathfrak{c}\mathfrak{c}$ | 0.03g RMS WN, 5 min | S ₁ | | |
| 12 | 11/6/2008 | 40% Gilroy EQ | | | |
| 13 | $\mathfrak{c}\mathfrak{c}$ | 0.03g RMS WN, 5 min | S ₂ | | |
| 21 | 11/10/2008 | 67% Gilroy EQ (DBE) | | | |
| 25 | 11/12/2008 | 0.04g RMS WN, 5 min | S ₃ | | |
| 26 | | 67% Gilroy EQ (DBE) | | | |
| 27 | 66 | 0.04g RMS WN, 5 min | S4 | | |
| 28 | 66 | 83% Gilroy EQ | | | |
| 29 | $\mathfrak{c}\mathfrak{c}$ | 0.04g RMS WN, 5 min | S5 | | |
| 33 | 11/13/2008 | 91% Gilroy EQ | | | |
| 35 | 66 | 100% Gilroy EQ (MCE) | | | |
| 36 | $\mathfrak{c}\mathfrak{c}$ | 0.04g RMS WN, 5 min | S6 | | |
| 40 | 11/18/2008 | 120% Gilroy EQ | | | |
| 41 | 66 | 0.04g RMS WN, 5 min | S7 | | |

Table 2. Dynamic tests used in this study (WN: white noise base excitation; EQ: earthquake base excitation)

System Identification of the Infilled Frame

In the study presented here, the deterministic-stochastic subspace identification (DSI) method, an input-output system identification method, has been used to estimate the modal parameters of the infilled frame at various damage states. Output-only system identification methods have been successfully applied for system identification of linear systems [4, 8]; however these methods are based on the assumption of a broadband (ideal white-noise) input excitation. In this experiment, the white noise base excitation inputs have been significantly modified by the shake-table system dynamics and the table outputs are far from broadband signals. Therefore, the application of output-only system identification methods to these "white noise" base excitation test data will result in large estimation errors in the modal parameters. [Figure 5](#page-4-0) shows the Fourier Amplitude Spectrum (FAS) of the unfiltered "white noise" base excitation acceleration measured on the shake-table during Test 9 (damage state S1). The largest peak in the FAS of the input acceleration can be due to the effects of the oil column resonance, which is around 11.5Hz. The mechanical characteristics of the shake-table are available in [9].

Review of Deterministic-Stochastic Subspace Identification Method

The deterministic-stochastic state-space model for linear time-invariant systems can be written as

$$
\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{w}(k)
$$

\n
$$
\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{v}(k)
$$
\n(1)

where **A**, **B**, **C** and **D** are state-space matrices, $u(k)$ and $y(k)$ denote the input and output vectors, respectively, and $\mathbf{x}(k)$ is the state vector. In the deterministic-stochastic model, the process noise $\mathbf{w}(k)$ represents disturbances (small unmeasured excitations) and modeling inaccuracies, while the measurement noise $v(k)$ models the sensor inaccuracies. However, in the data-driven stochastic subspace identification (SSI-DATA) method, an output-only method, both noise terms (**w** and **v**) also implicitly include the input excitation since it is impossible to distinguish the input information from the noise terms. Considering the assumptions that the deterministic input $u(k)$ is uncorrelated with both the process noise $w(k)$ and the measurement noise $v(k)$, and that both noise terms are not identically zero, a robust identification algorithm was developed by Van Overschee and de Moore [10] to identify the state-space matrices in the combined deterministic-stochastic system. Similar to SSI-DATA, robust numerical techniques such as QR factorization, SVD, and least squares are used in this method. At each damage state, the DSI method has been applied to 5-minute long filtered input-output data records with a sampling frequency of 240Hz. The measured input (shake-table acceleration) and the absolute longitudinal (7 channels during Tests 5 and 9; 8 channels during other tests) and vertical (9 channels) acceleration response data were filtered between 0.5 to 70Hz using a 1024 order FIR filter. For each dynamic test, an input-output Hankel matrix was formed including 20 block rows with 17 or 18 rows each (1 input and 16 or 17 output channels) and 71,962 columns using the filtered data.

Modal Identification Results

Modal parameters of the test structure were identified using the DSI method outlined above based on the inputoutput data measured from low-amplitude (0.03g and 0.04g RMS) white-noise base excitation tests at various damage states (S0, S1, S2, S3, S4, S5, S6, and S7). [Figure 6](#page-5-0) shows in polar plots the complex-valued mode shapes of the four most significantly excited modes of the test structure identified based on data from Test 5 (damage state S0). The four most significant vibration modes identified at this damage state consist of the first two longitudinal (1-L, 2-L), the second torsional (2-T) and the second coupled longitudinal-torsional (2-L-T) modes. The torsional modes were excited probably due to imperfections in the construction of the specimen which could induce eccentricities. An additional source of excitation of torsional modes was the interaction between the specimen and the shake-table which could amplify the imperfection of the shake-table and result in input excitation along the Y direction. The real parts of these mode shapes are illustrated in [Figure 7](#page-5-0). It should be noted that the six measurements perpendicular to the infill wall plane (W1-Y, W2-Y, W3-Y, E1-Y, E2-Y, and E2-Y) are also used to plot the mode shapes in [Figures 6](#page-5-0) and [7](#page-5-0). However, from [Figure 7](#page-5-0) it is observed that the longitudinal mode shapes have negligible out-of-plane components. The polar plot representation of a mode shape provides information on the degree of non-classical (or non-proportional) damping [11] characteristics of that mode. If all components of a mode shape (each component being represented by a vector in a polar plot) are collinear, that vibration mode is classically damped. The more scattered the mode shape components are in the complex plane, the more the system is non-classically (non-proportionally) damped in that mode. However, measurement noise (low signal-to-noise ratio), estimation errors, and modeling errors can also cause a truly classically damped vibration mode to be identified as non-classically damped. From [Figure 6,](#page-5-0) it is observed that 1-L, 2-L and 2-L-T modes at damage state S0 are identified as almost perfectly classically damped while some degree of non-proportional damping is identified for 2-T mode.

Fig. 6 Polar plot representation of complex mode shapes at damage state S0

Fig. 7 Vibration mode shapes of the infilled frame at damage state S0

The natural frequencies and damping ratios of these four most significantly excited modes are given in [Table 3](#page-6-0) for all damage states. Modal Assurance Criterion (MAC) values [12] were also computed to compare each complexvalued mode shapes identified at each damage states with the corresponding mode shape identified at damage state S0. It should be noted the modal parameters of the second torsional mode (2-T) are identified based on the longitudinal (7 channels during Tests 5 and 9; 8 channels during other tests) and transverse (6 channels) measurements while the other three modes are identified based on longitudinal and vertical (9 channels) data. This is due to the fact that the transverse components of longitudinal mode shapes are negligible while these components are small in the 2-L-T mode shape. From [Table 3](#page-6-0), the following observations can be made. First, the identified natural frequencies decrease consistently with increasing level of damage, while the identified damping ratios increase. Second, the reduction in the natural frequencies of the two longitudinal modes with damage is much more significant than for the 2-T and 2-L-T modes. Similar observation can be made on the level of increase in the identified damping ratios. Third, the MAC values between the identified mode shapes at each damage state with their counterpart identified at S0 also have a monotonically decreasing trend with increasing damage. The high MAC values (close to one) for damage states S1 to S4 indicate that there is little change in the identified mode shapes at these damage states. The identified natural frequencies and mode shapes of the two longitudinal modes have been used in a companion paper for damage identification of the infilled frame at its various damage states.

The modal parameters presented in [Table 3](#page-6-0) provide useful insight on the structural performance and the damage sustained due to the seismic excitations. The results show that the structure practically remained elastic after the low level earthquakes and the first noticeable deterioration of the structural properties occurred after the 67% of Gilroy, which corresponds to a design basis earthquake (DBE) for this structure. At S3, the inspection of the physical specimen revealed the first visible cracks in the infill panel with a window at the first story, as shown in [Figure 8\(a\)](#page-6-0). This observation is consistent with the fact that the modal parameters at S3 show the first noticeable differences compared to the modal parameters at S0. The differences are mainly depicted in the drop of the first mode frequency and the increase of the damping ratio for that mode. However, the cracks were insignificant with respect to the structural integrity of the specimen and did not alter the structural properties drastically. Significant change in the structural properties is noted at damage state S6, which occurred after subjecting the structure to a

maximum considered earthquake (MCE). At this stage, significant cracks developed in both bays of the first floor, and the columns also cracked as illustrated in Figure 8(b). Although these cracks did not appear to be critical for the structure, they caused a natural frequency reduction of more than 50%, which would be translated to a reduction of the stiffness to approximately one fourth of its initial value. Moreover, the damping value increased from the initial value of 2.0% to 15.7% due to the energy dissipated by the sliding along the fractured mortar joints.

| Damage State: | | S ₀ | S ₁ | S2 | S ₃ | S ₄ | S ₅ | S ₆ | S7 |
|-----------------------|----------------|----------------|----------------|-----------|----------------|----------------|----------------|----------------|-----------|
| Mode 1-L | Frequency [Hz] | 18.18 | 18.11 | 17.99 | 16.74 | 15.93 | 14.78 | 8.47 | 5.34 |
| | Damping [%] | 2.0 | 2.4 | 1.9 | 3.3 | 3.8 | 6.1 | 15.7 | 15.6 |
| | MAC | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.98 | 0.80 | 0.71 |
| Mode 2-T [*] | Frequency [Hz] | 21.16 | 21.02 | 21.32 | 20.77 | 20.16 | 19.69 | 18.20 | 17.39 |
| | Damping [%] | 1.5 | 1.5 | 1.3 | 1.5 | 1.8 | 1.5 | 1.5 | 1.6 |
| | MAC | 1.00 | 1.00 | 1.00 | 0.99 | 0.98 | 0.99 | 0.95 | 0.68 |
| Mode 2-L | Frequency [Hz] | 41.22 | 41.09 | 41.56 | 40.21 | 38.56 | 35.50 | 27.34 | 22.57 |
| | Damping [%] | 1.1 | 1.0 | 1.0 | 1.4 | 3.0 | 4.4 | 4.8 | 4.2 |
| | MAC | 1.00 | 1.00 | 1.00 | 0.99 | 0.96 | 0.92 | 0.74 | 0.67 |
| Mode 2-L-T | Frequency [Hz] | 57.81 | 57.35 | 57.96 | 56.25 | 54.64 | 52.65 | 45.98 | 43.33 |
| | Damping [%] | 1.1 | 1.3 | 1.0 | 0.7 | 1.2 | 2.1 | 2.1 | 2.7 |
| | MAC | 1.00 | 1.00 | 1.00 | 0.97 | 0.92 | 0.87 | 0.54 | 0.29 |

Table 3. Modal parameters of the infilled frame identified at different damage states

* *Mode 2-T is identified based on longitudinal and transverse measurements while the other three modes are identified based on longitudinal and vertical measured data*

The identified first-mode natural frequency at S6 coincides with the frequency corresponding to the peak of the acceleration response spectrum of the scaled Gilroy motion. When the motion amplitude was increased to 120% of the original record, the spectral acceleration at that frequency was 2.87 g. Therefore, significant damage was induced by the earthquake and severe shear cracks developed at mid-height in the middle column. These cracks further reduced the frequency to less than one third of its initial value, although it did not further increase the damping value, which was already high. [Figure 9](#page-7-0) shows the observed damage at this state.

Fig. 8 Crack pattern at damage states S3 (a) and S6 (b)

Fig. 9 Crack pattern at damage states S6 in the solid infill and the middle column (left), and the infill with opening and the east column (right)

Conclusions

A 2/3-scale, three-story, two-bay, infilled RC frame was tested on the UCSD-NEES shake-table in November 2008. The test objective was to assess the seismic performance of infilled frames and provide data for the evaluation of newly developed analytical methods. The shake-table tests were designed to induce damage on the structure progressively through scaled earthquake records. At various levels of damage, low-amplitude ambient vibration and white-noise base excitations were applied to the frame which responded as a quasi-linear system with parameters evolving as a function of structural damage. The white-noise base excitation tests were used for the system identification of the infilled frame structure in this study. The deterministic-stochastic subspace identification method, an input-output system identification method, has been applied to estimate modal parameters of the frame at various damage states.

The results of this system identification study indicate that: (1) the identified natural frequencies decrease consistently with increasing level of damage indicating a reduction of the stiffness, while the identified damping ratios increase; (2) the reduction in the natural frequencies of the two longitudinal modes with damage is much more significant than that for the 2-T and 2-L-T modes; similar observation can be made on the level of increase in the identified damping ratios; (3) the MAC values that relate the identified mode shapes at each damage state to those of the undamaged state decrease with increasing damage. The modal parameters identified at various damage states in this study provide the input for damage identification of this infilled frame using a sensitivitybased finite element model updating strategy which is the topic of a companion paper [13].

Acknowledgements

The shake-table tests discussed in this paper were supported by the National Science Foundation Grant No. 0530709 awarded under the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES) program. Input from other collaborators at Stanford University and the University of Colorado at Boulder, and a Professional Advisory Panel (PAP) throughout this study is gratefully acknowledged. The panel members are David Breiholz, John Kariotis, Gregory Kingsley, Joe Maffei, Ron Mayes, Paul Murray, and Michael Valley. Finally, the authors would like to thank the technical staff at the Englekirk Structural Engineering Center of UCSD and Ioannis Koutromanos for their assistance in the shake-table tests. However, the opinions expressed in this paper are those of the authors and do not necessarily represent those of the NSF, the collaborators, or PAP.

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