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Structural Dynamics of a Frame Including Axial Load Effects

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Nomenclature

ω_i	Natural Frequency
$[M]$	Mass Matrix
$[K]$	Stiffness Matrix
$[K_G]$	Geometric Stiffness Matrix
E	Young's Modulus
I	Area Moment of Inertia
A	Cross Sectional Area
L	Length of Beam Element
ρ	Density
y_i	Transverse Displacement
ϕ_i	End Rotation
m	Mass
J	Moment of Inertia
P	Axial Load
q_i	Displacement Vector
$\ddot{q}_i$	Acceleration Vector

ABSTRACT

This paper considers the free vibration of a plane, rectangular, portal frame consisting of slender members. Natural frequencies and mode shapes are influenced by the addition of mass at the corners of the frame. The members are sufficiently slender that axial effects occur, and may ultimately lead to buckling. The results from both theoretical and experimental studies are presented.

Introduction

A recurring theme in the design of structures is to consistently push for them to be thinner and lighter. This is especially true in the case of the aerospace industry, where any weight savings translates into better performance and efficiency. However, with any design goal, there are trade-offs that need to be taken into consideration. As structures become lighter and more slender, loadings can start to have an effect on dynamic behavior as well as static behavior. For frame structures, axial loads on the members can affect the stiffness, and in turn, the natural

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frequencies of the frame. Several studies have investigated the effects of applied axial loads to simple frame structures. Lieven and Greening [1] conducted modal tests on a frame with an induced forced due to the shortening of a turnbuckle in one of the members. In their study they also utilized numerical matrix methods to predict mode shapes and natural frequencies. Similarly, Mead [2] analytically investigated the variation of natural frequencies and mode shapes of a planar frame structure that was subjected to externally applied axial loads.

The experimental setup considered in this paper is a simple portal frame structure with slender members. Instead of an external load being directly applied to the structure itself, axial loads are induced by adding masses to the corners of the frame. The masses themselves have an effect on the dynamics of the structure in addition to degrading the stiffness by adding a compressive axial load to the columns. Modal tests were conducted to find the variation in the natural frequencies as mass was added to the frame. Numerical simulations were then ran in ANSYS and FRAME3DD finite element packages to model and gain further insight from the behavior of the structure [3][4].

Theory

Frames are essentially an assembly of beams connected at the ends, and can be modeled by solving a system of partial differential beam equations whose boundary conditions are determined by the joints of the frame. These systems of equations generally produce very large transcendental equations that are only tractable when the structure has a very simple geometry and several other simplifying assumptions can be made [5]. In order to analyze more complex structures, matrix methods have been developed that discretize frame structures into a set of beam elements and lumped masses [6]. When the mass and stiffness properties are discretized, a matrix set of equations describing the behavior of the system can be written as

$$[\mathbf{M}]\ddot{q}_i + [\mathbf{K}]q_i = 0, \tag{1}$$

where the mass and stiffness matrices are the global mass and stiffness matrices for the entire structure, with the q_i vector consisting of displacement and rotational degrees of freedom.

The global mass and stiffness equations are assembled from local beam mass and stiffness matrices, where $q_i = [y_k \;\; \phi_k \;\; y_m \;\; \phi_m]^T$ is a vector of end displacements and rotations. The displacements and rotations for a single beam element are shown in Fig. 1.

Figure 1: Original and deformed beam with local coordinates. *After* [7].

The local mass, $[\mathbf{M}]$, and stiffness, $[\mathbf{K}]$, matrices for a beam element of a frame are

$$[\mathbf{M}] = \frac{\rho A L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \tag{2}$$

and

$$[\mathbf{K}] = \frac{EI}{L^2}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}. \tag{3}$$

When compressive axial loads are applied to beams their stiffness is degraded by the load. This is called a 'geometric stiffness' effect and, in the static sense, is the mechanism that is responsible for buckling. In order to account for this in the matrix stiffness method, a separate geometric stiffness matrix,

$$[\mathbf{K_G}] = \frac{P}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}, \tag{4}$$

is factored into the traditional beam stiffness formulation [8].

By adding in the axial load effects on the stiffness of the system, a geometric stiffness matrix augments the traditional stiffness matrix. Equation (1) now becomes

$$[\mathbf{M}]\ddot{q}_i + [\mathbf{K} - \mathbf{K_G}]q_i = 0. \tag{5}$$

Equation (5) can either be used in a time marching algorithm to produce dynamic simulations, or if q_i is assumed to be simple harmonic motion, $\ddot{q}_i = -\omega^2 q_i$, the equation

$$([\mathbf{K} - \mathbf{K_G}] - \omega_i^2[\mathbf{M}])q_i = 0 \tag{6}$$

can be solved as a general eigenvalue problem to attain the natural frequencies and mode shapes of the structure.

Experimental Setup

The experimental laboratory setup consisted of a simple portal frame comprised of polycarbonate beam members. The polycarbonate beams had a cross section of dimensions, 2.554 cm x 0.154 cm, and a density of $\rho = 1157$ kg/m^3. A schematic and picture of the setup is shown in Fig. 2. For the experiments conducted, the bottom supports of the columns were clamped and the corners were connected at 90 degrees using angled aluminum fixtures. The base fixtures for the columns could also be converted to pinned connections in order to allow the bottom of the columns to freely rotate. The column length, L_1, was 15.24 cm and the length of the cross beam of the frame, L_2, measured 45.72 cm. Threaded rods were attached to the aluminum brackets such that masses could be added to the two corners.

Modal data were taken using an Ometron VH300+ laser doppler vibrometer, and an Endevco 2302-50 modal impact hammer. The data were recorded with a Dell laptop computer with PULSE data acquisition software and a Brüel & Kjær type 3109 input module. All of the hammer impact hits and vibration measurements took place in the plane of the frame.

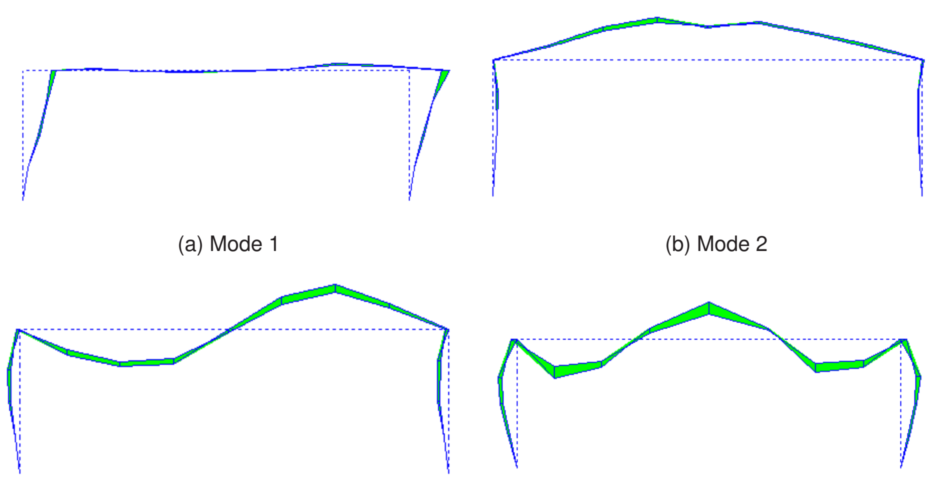
Figure 2: Schematic of experimental frame setup and picture of physical laboratory model

Simulations

Simulations of the frame were ran in both the commercial finite element software package ANSYS, and FRAME3DD, an open source frame analysis software package. The frame was modeled using 2-D beam elements, with fixed end supports for the columns. The experimentally added mass was modeled as a lumped mass added to the nodes at the top two corners of the frame. The simulations also allowed for the option of adding a lumped inertia term to the corners nodes. The geometric stiffness effects from the masses were modeled as a force in FRAME3DD, in addition to the lumped mass. The geometric stiffness was accounted for in ANSYS by applying an acceleration to the frame equal to that of gravity, creating an equivalent force on the columns from the lumped masses.

Results

The experimental frequency response functions were loaded into and analyzed using ME'scopeVES [9]. Figure 3 shows the experimental mode shapes for the lowest four modes of the frame. Figure 4 shows the four lowest calculated mode shapes from ANSYS.

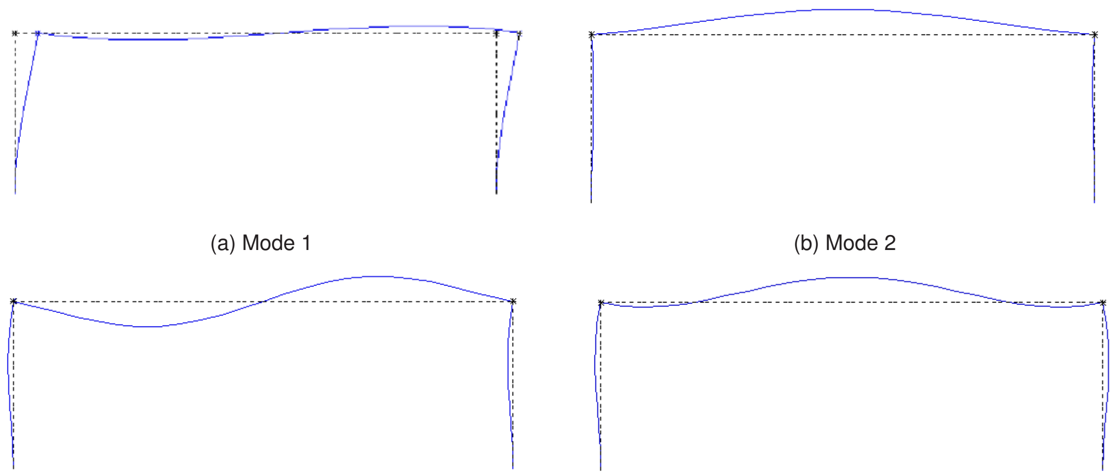


(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

Figure 3: Experimentally determined mode shapes

The experimentally determined mode shapes all corresponded with those that were predicted by the simulations from ANSYS and FRAME3DD. The mode shapes also retained their ordering, both experimentally and in simulation, as mass was added to the frame.

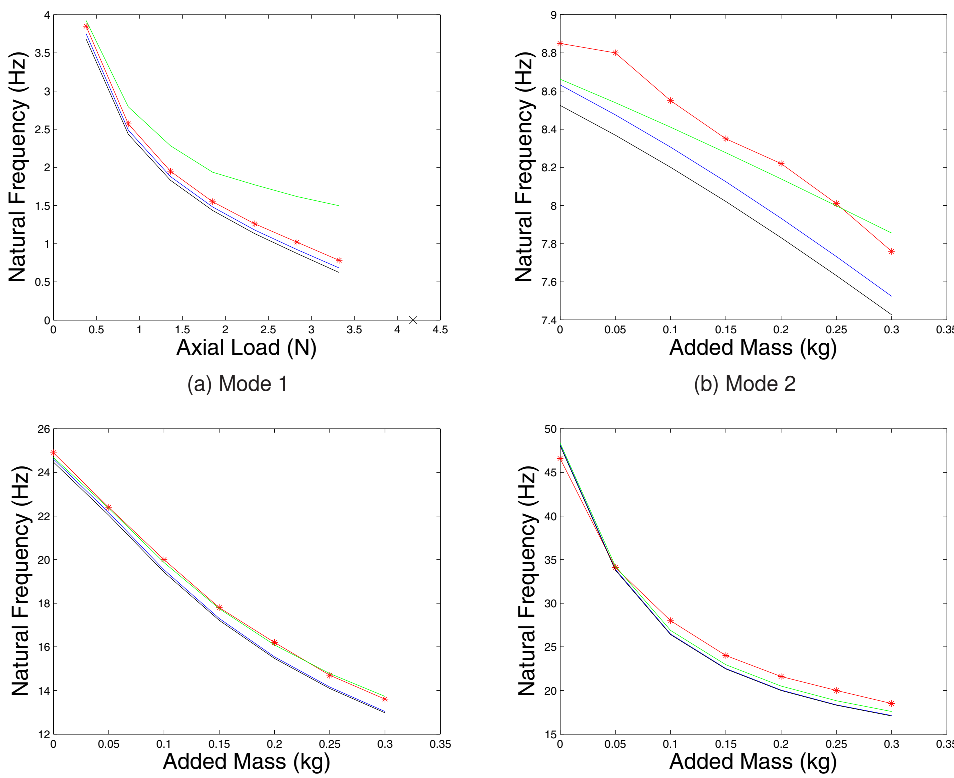


(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

Figure 4: Numerically determined mode shapes from ANSYS

Figure 5 shows the variation of the natural frequencies of the first four modes of the frame with increasing axial load due to added mass. Both ANSYS and FRAME3DD programs produce very similar results for all four sets of natural frequencies. Experimentally the structure was found to be very lightly damped, and damping was not considered in the numerical results. Additionally, initial geometric imperfections that are bound to be present in experimental systems were also not included in the numerical results.



(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

Figure 5: Natural frequency variations with added mass. Red - experimental data. Blue - Frame3DD simulation results. Black - ANSYS simulation results. Green - ANSYS simulation results without geometric stiffness.

An interesting point to investigate with this structure is what factors contribute to this change in natural frequency. Unlike several of the other studies on geometric stiffness effects, mass and rotational inertia were being added to the frame structure which can also have an effect on the natural frequencies of the system. Using numerical simulations, the effects of the axial loading can be separated from those of mass and rotational inertia. The green trace in Fig. 5 shows the results of ANSYS simulations where the natural frequencies are calculated without any geometric stiffness effects, i.e. assuming negligible axial loading effects on the lateral stiffness of the columns. Mode 1, which is very similar to the static buckling mode, was the most affected by neglecting the axial loading. Figure 5a shows the natural frequency plotted versus the axial load on each column, which is caused by gravity acting on the masses added to the corners, $P = mg$. The independently calculated static buckling load from ANSYS is shown with an 'X' at an axial load value of 4.185 N. Modes 2-4 are primarily affected by the mass and show little change in their behavior from neglecting the axial loading. Figures 5b-5d show the variation of the natural frequency of modes 2-4 with the amount of mass, m, added to each corner of the frame.

Conclusions

A planar frame with slender, flexible members was investigated both experimentally and numerically to attempt to characterize a change in modal behavior with added mass and increased axial load. Natural frequency was shown to vary due to not only geometric stiffness effects but to added mass and rotational inertia present in the experimental system as well. Additionally, the change in dynamic behavior due to axial loading did not change all of the modes of vibration in the same way. The loading did cause a significant difference in the prediction of the fundamental frequency of the frame as mass was added, but did not have much of an effect on the higher modes.

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