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Experimental Evaluation of Mass Change Approaches for Scaling Factors Estimation

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ABSTRACT

In operational modal analysis the input forces are unknown so the modes shapes are obtained with an arbitrary normalization. In some applications the mass normalized modes shapes are required and the scaling factors have to be estimated. A way of obtaining these factors is to modify the modal properties of the structure by introducing changes in the mass or stiffness; mass change is usually the most convenient way to modify the structural properties in a controlled manner. Different strategies and approaches to perform the mass change method have been proposed in recent years. In this work the scaling factors of a 15 tonnes concrete slab strip are obtained using several equations proposed by different authors and the results are compared.

The results show that the scaling factors can be estimated with reasonable accuracy using the different equations if a good mass change strategy is applied.

Nomenclature

Φ	Mass normalized mode shape
Ψ	Arbitrary normalized mode shape
α	Scaling factors
[K]	Stiffness Matrix
$[M_0]$	Mass Matrix
$[M_1]$	Spatial distribution of added masses
β_1	Ratio of added mass
ω_0	Natural frequencies
ω_1	Modified natural frequencies
k	Terms of the diagonal of the projection matrix

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1. Introduction

In operational modal analysis only arbitrary normalized modes shapes can be obtained [1], e.g. maximum component of unity or arbitrary length. If the modal masses are required the scaling factors must be estimated. The scaling factors are constants that relate each mass normalized mode shape with the correspond arbitrarily normalized one. That is:

$$\Phi = \alpha \, \Psi \tag{1}$$

where Φ is the mass normalized modal shape, Ψ is the correspond arbitrary normalized modal shape and α is the scaling factor.

A way to estimate the scaling factors is to modify the dynamic behavior of the structure changing the stiffness, the mass or both and perform a new operational modal testing and analysis. These methods based on dynamic modification use both the modal parameters of the unmodified and modified structure, so if the modal parameters are related the scaling factors can be obtained.

In recent years, some different approaches [2, 3, 4, 5, 6, 7] and strategies [8, 9, 10] in the mass change method have been proposed. The method consists of modifying the dynamic behavior of the structure by attaching masses to several points on the structure where the mode shapes are known. The accuracy obtained in the scaling factor estimation depends on both the accuracy obtained in the modal parameter identification and the mass change strategy used to modify the dynamic behavior of the structure. The mass change strategy refers to the definition of the magnitude, the location and the number of masses to be attached to the structure.

In this paper, the scaling factors of a 15 tonnes concrete slab strip by means of Operational Modal Analysis (OMA) are estimated with the mass change method using different proposed approaches and strategies and the results are compared. The results are also compared with the estimated scaling factors using classical modal analysis (CMA).

2. Mass Change Method - Theory

The mass change method consists of performing operational modal analysis on both the original and the modified structure. The modification is carried out by attaching masses to several points of the structure where the mode shapes of the unmodified structure are known. The user selects the number, the magnitude and the location of the masses. To facilitate the mass modification and the calculation of the scaling factors, lumped masses are often used, so that, the mass change matrix becomes, in general, diagonal.

Different approaches to determine the scaling factors can be derived mainly from the eigenvalue equations of both the original and the modified structure. The classical eigenvalue equation in case of no damping or proportional damping is given by:

$$[M_0]\Phi_0 \cdot \omega_0^2 = [K] \cdot \Phi_0 \tag{2}$$

where Φ_0 is the mode shape, ω_0 the natural frequency, $[M_0]$ the mass matrix and $[K]$ the stiffness matrix. If a mass change is made so that the new mass matrix is given by $[M_0 + \beta\, M_1]$, then the eigenvalue equation becomes:

$$[M_0 + \beta M_1]\Phi_1 \cdot \omega_1^2 = [K] \cdot \Phi_1 \tag{3}$$

where Φ_1 and ω_1 are the mode shape and the natural frequency of the modified problem, respectively, M_1 is a matrix of arbitrary rank describing the spatial distribution of the mass modification and β is a scaling parameter with respect to which the mass change magnitudes are computed. Combining equations (1), (2) and (3) and after several assumptions a number of different expressions can be derived to estimate the scaling factors. A general form of these expressions can be summarized in the following formula [7]:

$$\alpha_0^2 \cong \frac{\omega_0 - \omega_1}{a_1} \frac{1}{\beta_1} \frac{a_3}{(\Psi_0^T [M_1] a_2)} \quad (4)$$

where the group of parameters (a_1, a_2, a_3) are different in each expression, e.g.:

($\omega_0/2$, Ψ_0, 1)	in the expression from Parloo et al. [2],	(5)
(ω_1, Ψ_0, 1)	in the expression from Brincker et al. [5]	(6)
(ω_1, Ψ_1, 1)	in the expression from Aenlle et al. [6], and	(7)
(ω_1, Ψ_1, κ)	in the Bernal projection expression [7].	(8)

The parameter β_1 is taken so that the [ΔM] mass change matrix that it is used in eqs. (5), (6) and (7), can be obtained by means of the relation [ΔM] = β_1 [M_1]. In the projection approach, eq. (8), the main idea is to express the mode shapes of the modified state on the basis of the unmodified modes shapes [7]. To introduce this projection the vector of constants κ is used, which is obtained from the diagonal of the matrix $[q]= [\Psi_0]^{-1} [\Psi_1]$.

For the above described methods the following notation will hereafter be used: α_{00P} for eq. (5), α_{00} for eq. (6), α_{01} for eq. (7) and α_{01B} for eq. (8).

If only the modified modes shapes are used in eq. (7), that is, the parameters (a_1, a_2, a_3) are given by (Ψ_1^T M_1 Ψ_1), the α_{11} scaling factor is obtained and a new approach can be used combining α_{00} and α_{11}.

$$\alpha^* = (\alpha_{00} + \alpha_{11})/2 \quad (9)$$

It can be observed from eqs. (5) to (9) that different results from the scaling factors would be expected in each expression since different parameters are used in each one. For example, in eqs. (5) and (6) the unmodified modes shapes of the structure are used, so if the mode shapes change significantly, when adding the masses to the structure, the results would be expected to be less accurate. On the other hand, the eqs. (7), (8) and (9) take into account both the modified and unmodified modes shapes. Eq. (7) is expected to be better for small changes in the modes shapes and eqs. (8) and (9) are expected to be better for large changes in the mode shapes.

To obtain good results with the aforementioned equations it is important to take into account that different strategies can be used. These strategies are related to the mass magnitude and the number and location of the masses in the experiment [8, 9, 10].

3. Experimental Program

In this work, several OMA measurements using different mass strategies are done in a concrete slab. In each mass configuration the natural frequencies and mode shapes are estimated and then, the scaling factors are estimated with the mass change method using the different expressions. For comparing the estimated scaling factors obtained with OMA, CMA measurements are also done in the slab and the scaling factors are calculated from the FRF's.

3.1. Structure and Test Configurations

The structure used for the experiments is a simply supported in-situ cast post-tensioned slab strip of span 10.8 m. Its total length is 11.2 m, which includes 0.2 m overhangs over the knife edge supports. It has a width of 2 m, depth of 0.275 m and weighs approximately 15 tonnes. There is a known non-linearity in the slab at one of the supports, where exist a gap between the knife-edge support and the slab. There is also a potential for friction at the supports. Due to these effects the frequencies can change by 2-3% depending on the amplitude of excitation.

In both modal analysis, OMA and CMA, 21 Honeywell QA750 accelerometers were used. They were located at degrees of freedom (DOFs) 4 to 24, as indicated in Figure 1, and were installed on leveled Perspex base plates. For digital data acquisition, a Data Physics DP730 spectrum analyzer (Mobilyzer II) with 24 input channels and 4 output channels (24-bit resolution) was used. The Mobilyzer was controlled by a laptop PC connected to it via an Ethernet connection. Figure 2 shows images of the slab strip during the measurements, both with and without additional mass blocks present. The sampling rate and acquisition time used were 128 Hz and 15 minutes, respectively.

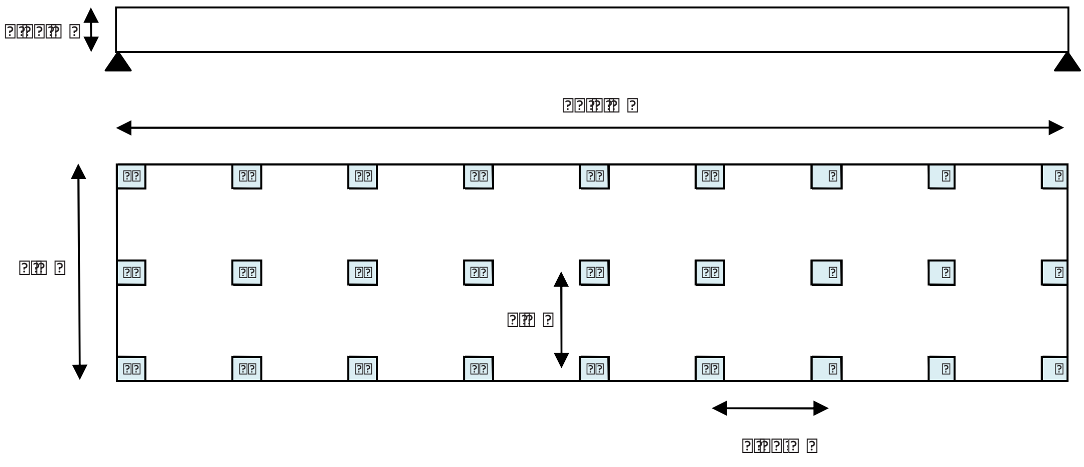

Figure 1: Slab dimensions and test grid

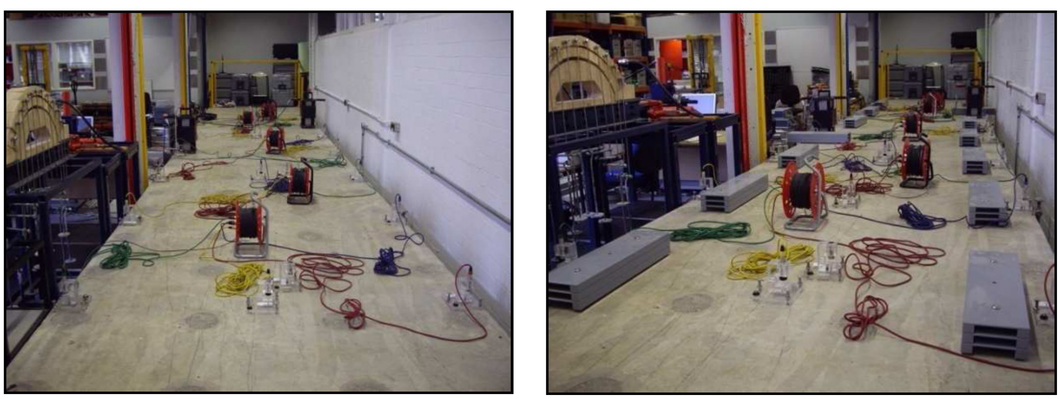

Figure 2: Concrete slab strip

To obtain the modal properties of the concrete slab from OMA, the Enhanced Frequency Domain Decomposition (EFDD) and Stochastic Subspace Iteration (SSI) techniques were used, both implemented in the ARTeMIS Extractor software. Since the natural excitation level is too low (the slab is in a very quiet lab) and due to the non-linearities, artificial excitation is used in the tests to avoid variations in frequencies that cannot be related with the mass change configurations. Two APS Dynamics Model 113 shakers were used as source of excitation. The slab was driven with uncorrelated random signals from both shakers with the same excitation level for all the experiments. The shakers were located at DOFs 15 and 19.

In CMA, the modal properties were estimated from Frequency Response Function (FRF) data calculated using both the excitation and response time histories with the same configuration of shakers and accelerometers than the OMA analysis, that is, shakers located at DOFs at 15 and 19 and accelerometers located at DOFs 4 to 24. To obtain the modal parameters from FRF data, the ME'scopeVES software was used in a MIMO modal analysis with two inputs and 21 outputs.

The first four significant natural frequencies of the concrete slab, obtained from OMA and CMA, are presented in Table 1.

Table 1: First four natural frequencies of the slab

	Mode	1: Bending	2: Bending	3: Torsion	4: Bending
OMA	Freq. (EFDD) [Hz]	4.497	16.844	25.717	37.600
	Freq. (SSI) [Hz]	4.499	16.835	25.767	37.612
CMA	Freq. (FRFs) [Hz]	4.499	16.845	25.750	37.612

3.1. Mass configuration and strategies

The quantity of mass reduces the aforementioned gap between the slab and its support and therefore affects the non-linearity introduced in the dynamic properties of the slab. To minimize this effect the same quantity of mass, 1050 kg, is used in all the experiments. There is one exception, where a mass of 600 kg was used to examine the difference between different changes in mass magnitude. The different mass configurations used in the measurements are presented in Table 2, which were selected to compare the different expressions described in paragraph 2 as well as different mass change strategies [10]. As the mass magnitude is always the same, except for case 3b, the strategies are related to the location and number of masses used in the experiments. The results are expected to be of decreasing accuracy from case 1 to case 5, since a more uniform distribution of masses would be expected to perform better over a range of modes. Cases 1 and 2 try to optimize the first four modes of the structure, cases 3 and 3b try to optimize modes 1 and 4 of the structure whereas cases 4 and 5 are strategies for which a significant mode shape modification is expected, hence allowing the comparison of the different expressions that take into account the modified mode shapes.

Table 2: Mass configurations

Case	№ of Masses	Mass per DOF	Mass change		DOF's with Mass
1	21	50	1050 Kg	7 %	All except [1 2 3 25 26 27]
2	14	75	1050 Kg	7 %	[4 6 7 9 10 12 13 15 16 18 19 21 22 24]
3	6	175	1050 Kg	7 %	[7 9 13 15 19 21]
3b	6	100	600 Kg	4.3 %	[7 9 13 15 19 21]
4	3	350	1050 Kg	7 %	[7 13 19]
5	4	---	1050 Kg	7 %	[7 (425 kg) 13 (150 kg) 19 (200 kg) 21 (275 kg)]

4. Results

The FRF's of some channels (MIMO analysis) and the EFDD singular value decomposition (OMA) are presented in Figure 3. The first four natural frequencies of the slab were presented in Table 1.

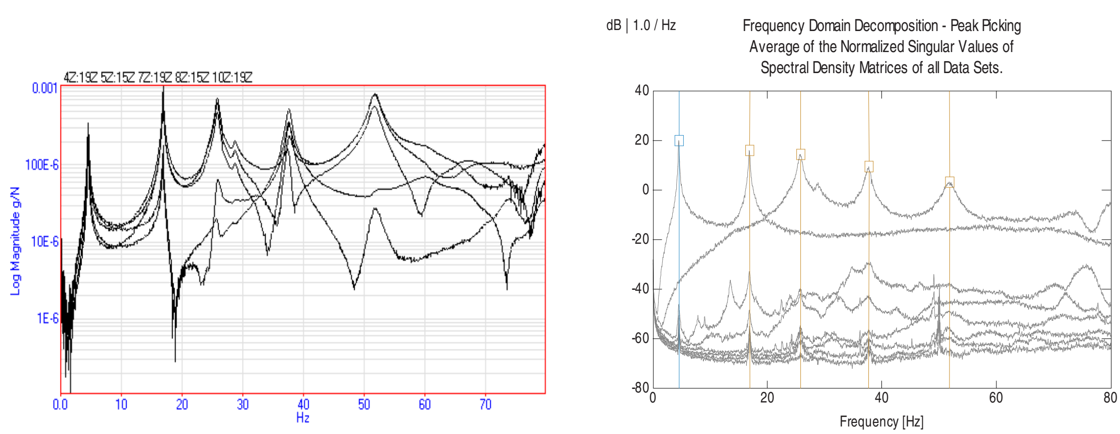

Figure 3: FRFs and SVD

The scaling factors obtained with CMA are presented in Table 3. These are obtained from two different data sets, m00 and m00b, without masses on the slab. In both data sets the inputs are uncorrelated random and the shakers are in the same location. As two reference inputs are used the strongest reference is used for each mode for the residue estimates.

It is important to remark that although the modal masses, and hence the scaling factors, were obtained directly from the FRF's, there are several factors such as the location of the shakers, lines used in the FRF's or the fit model used in the analysis that can result in variations of 10-20% in the results. In order to compare the results with those from OMA through the mass change method, the average of the estimated scaling factors provided by CMA is used for each mode, see Table 3.

Table 3: Scaling factors from CMA analysis

	Data Set						
	m00			m00b			
	Number of Lines in the FRFs						
MODE	1024	2048	4096	1024	2048	4096	Average
1	0.01236	0.01242	0.01260	0.01320	0.01240	0.01315	0.01269
2	0.01250	0.01255	0.01290	0.01288	0.01270	0.01310	0.01277
3	0.01440	0.01370	0.01430	0.01440	0.01360	0.01565	0.01434
4	0.01322	0.01317	0.01325	0.01370	0.01366	0.01373	0.01345

4.1. Symmetric mass change configurations

The scaling factors, corresponding to cases 1, 2 3, and 3b, obtained from EFDD analysis and using the expressions described in paragraph 2, are presented in Table 4. The column $\Delta\omega$ is the frequency shift that is obtained for each mode in the different configurations and the MAC is the modal assurance criteria between the unmodified and modified mode shapes.

It can be seen that in cases 1 and 2, where the attached masses are uniformly distributed across the slab, the scaling factors are estimated with a good accuracy and the different expressions provide similar results. As was expected, the changes in mode shapes are very small.

There is a significant difference in the third mode shape between cases 1 and 2. This mode is the first torsion mode of the slab at 25.72 Hz. Due to the gap in one of the slab supports, there is a rocking motion at 28.5 Hz that, depending of the mass configuration, can interfere in the stability of the torsion mode. In Figures 3a and 3b are presented the SSI stabilization diagram for the configurations with 14 and 21 masses respectively. In these figures can be seen that the first singular value and how the magnitude of the rocking motion is higher in the configuration with 21 masses. This effect can reduce the accuracy in the estimation for this particular mode. On the other hand, the configuration of 14 masses is a better strategy for the torsion mode (masses on both sides of the slab). In table 4 it can be observed that high frequency shift and MAC are obtained with this mass distribution, so that better results are expected.

Table 4. EFDD scaling factors

EFDD	Case 1: 21 MASSES 50 kg Each DOF's [All except supports]						
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01244	0.01241	0.01241	0.01206	0.01244	0.01269	4.11328	0.99998
2	0.01278	0.01278	0.01278	0.01241	0.01278	0.01277	3.87354	0.99993
3	0.01249	0.01251	0.01250	0.01218	0.01247	0.01434	3.35588	0.99687
4	0.01354	0.01328	0.01354	0.01307	0.01354	0.01345	4.61459	0.99928
EFDD	Case 2: 14 MASSES 75 kg Each DOF's [4 6 7 9 10 12 13 15 16 18 19 21 22 24]							
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01263	0.01259	0.01259	0.01222	0.01263	0.01269	4.25529	0.99998
2	0.01312	0.01313	0.01313	0.01271	0.01312	0.01277	4.18566	0.99991
3	0.01424	0.01430	0.01430	0.01356	0.01423	0.01434	6.33416	0.99937
4	0.01377	0.01354	0.01378	0.01325	0.01352	0.01345	4.98406	0.99965
EFDD	Case 3: 6 MASSES 175 kg Each DOF's [7 9 13 15 19 21]							
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01228	0.01220	0.01220	0.01185	0.01228	0.01269	4.66487	0.99987
2	0.01358	0.01353	0.01353	0.01305	0.01358	0.01277	5.10335	0.99993
3	0.01683	0.01679	0.01678	0.01562	0.01677	0.01434	9.39348	0.99783
4	0.01875	0.01839	0.01863	0.01741	0.01857	0.01345	9.34624	0.98862
EFDD	Case 3b: 6 MASSES - 100 kg Each DOF's [7 9 13 15 19 21]							
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01251	0.01270	0.01260	0.01213	0.01270	0.01269	3.04230	0.99986
2	0.01231	0.01255	0.01246	0.01120	0.01255	0.01277	2.85250	0.99968
3	0.01269	0.01290	0.01280	0.01187	0.01290	0.01434	3.52880	0.99836
4	0.01400	0.01413	0.01401	0.01346	0.01412	0.01345	3.26300	0.99866

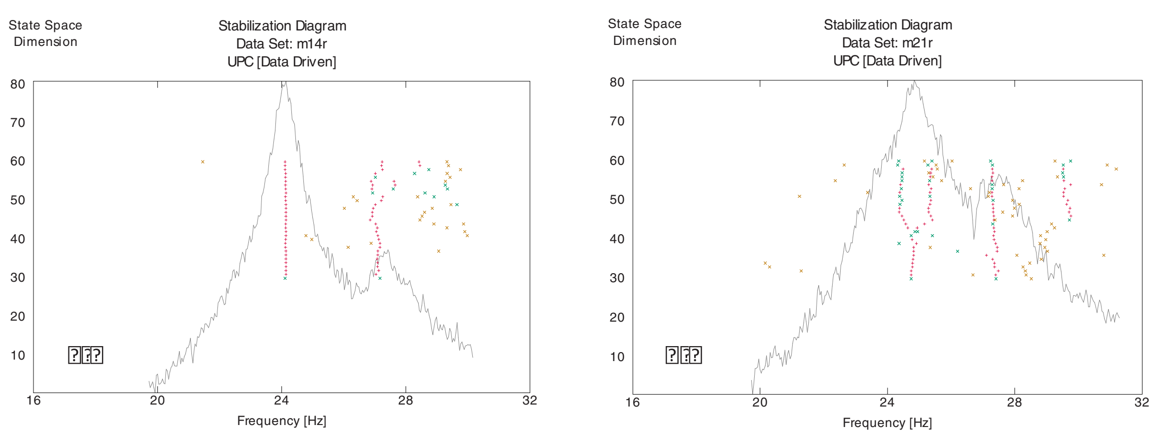

Figure 3a and 3b: SSI stabilization diagram for 21 and 14 masses configurations.

In cases 3 and 3b 6 masses are located at the same position but with different mass magnitude. The results present quite significant differences. In case 3 the scaling factor corresponding to the first mode are acceptable but in the values corresponding to the other three modes are too high. In case 3b, which has the same mass locations but less mass magnitude, the accuracy in the results improves. On one hand, it is not a good strategy to optimize many modes simultaneously with only a few heavy masses. On the other hand, the gap and the friction in the supports change depending on the location and the magnitude of the masses, so that, the non linear effects are mass dependent. So, although better results are expected when the ratio ω_0/ω_1 is higher [10], in this particular case, the dynamic behavior change of the slab is too much and the results don't present good accuracy. That is, although the mode shapes do not change significantly the dynamic behavior of the structure is quite different, as shown in Figure 4. Anyway, in both cases the results obtained for the different mass change methods are similar, indicating that none of them improve the results when a poor strategy is used.

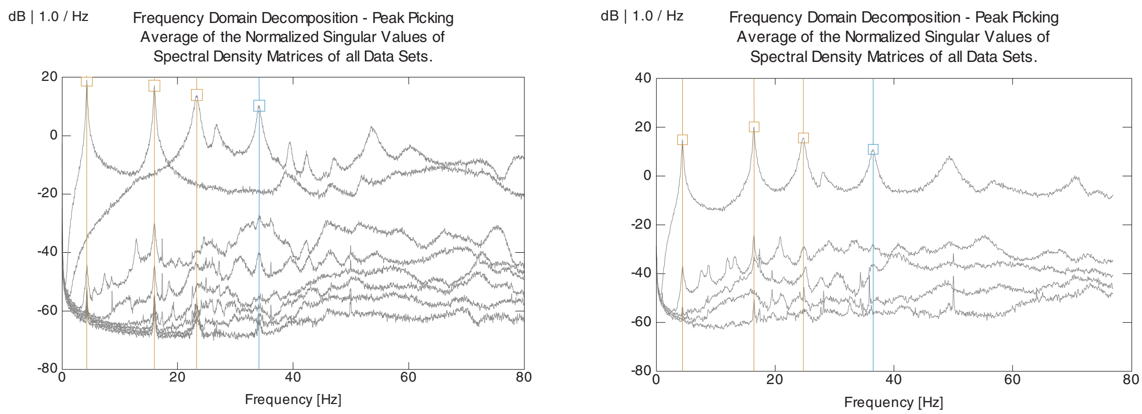

Figure 4: SVD of cases 3 (left) and 3b (right)

4.2. Non-symmetric mass configurations

Cases 4 and 5 correspond to two non-symmetric mass configurations. The results are presented in Table 5 for both cases with EFDD analysis. It can be seen that some mode shapes change significantly. As in case 3, there is a lot of concentrated mass on the slab and therefore there are important changes in the dynamic behavior of the structure due to the non linear effects. As expected, the results from α_{00P} and α_{00P} are the most inaccurate and α_{00P}, in particular, gives lower values than the other approaches. Although expressions α_{01}, α^* and α_{01B} use the modified mode shapes, the results are still not particularly good. In α_{01B} (the projection approach) the κ factor that is introduced to improve the results is not enough due to large changes in mode shapes, where the new modified mode shape can be a projection of more than one un-modified mode shape. Nevertheless, α_{01B} improves the results compared with α_{01} and α^*.

Table 5: Non-symmetric mass configurations (EFDD)

EFDD	Case 4: 3 MASSES - 350 kg Each DOF [7 13 19]							
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01219	0.01216	0.01216	0.01176	0.01216	0.01269	3.69273	0.99992
2	0.01455	0.01430	0.01430	0.01393	0.01440	0.01277	8.30466	0.97431
3	0.01862	0.02190	0.02192	0.01703	0.02027	0.01434	8.72393	0.99300
EFDD	Case 5: 4 MASSES - DOF's [7 (425 kg) 13 (150 kg) 19 (200 kg) 21 (275 kg)]							
Mode	α_{00}	α_{01}	α^*	α_{00P}	α_{01B}	αFRF's	$\Delta\omega$ [%]	MAC
1	0.01452	0.01452	0.01451	0.01378	0.01452	0.01269	6.51488	0.99957
2	0.01382	0.01421	0.01415	0.01291	0.01417	0.01277	8.23031	0.99104
3	0.01592	0.01751	0.01731	0.01487	0.01737	0.01434	8.81142	0.96573
4	0.01781	0.01832	0.01724	0.01651	0.01749	0.01345	9.12996	0.86864

5. Conclusions

- Different mass change expressions have been applied to estimate the scaling factors of a 15 tonnes concrete slab. The results show that all the methods estimate the scaling factors with good accuracy when a good mass change strategy is used.

- The approach of Parloo et al. [2] is the one that estimates the scaling factors with the largest errors for all the mass change configurations.

- The scaling factors obtained directly from the FRF's present the same or even higher uncertainties that the scaling factors obtained by means of the mass change method.

- The Bernal projection approach [7] provides better results when mode shapes change significantly.

Acknowledgements

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