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Examination of Validity for Viscoelastic Split Hopkinson Pressure Bar Method

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ABSTRACT: The examination of the accuracy for a split Hopkinson pressure bar method with viscoelastic input and output bars (viscoelastic SHPB method), which is one of the methods of evaluating the dynamic properties for viscoelstic materials, is executed. The key-point for accurately determining the dynamic properties of low impedance materials lies in how to measure the moderate size of reflected and transmitted waves. Moreover, the loading at both ends of the specimen should be equal each other in this method. Polymethyl methacrylate (PMMA) bars are used as the input and output bars. The viscoelastic properties of PMMA are approximated to a 3-element solid model in advance through preliminary wave propagation experiments. The same model is used as the mechanical properties of specimens, and the value of the instantaneous modulus is changed. The shape of waves at the boundary and the gage position of the input and output bars as well as the specimen are analyzed using the Elementary theory. The stress distribution in the specimen is also examined. It is found that the dynamic properties of viscoelastic materials can be evaluated with good precision by viscoelastic SHPB method when the ratio of the input / output bars and the specimen of mechanical impedance is about 10-20%. Then, the viscoelastic SHPB tests are actually performed. The complex compliance of the specimens which represent the characteristics of materials is obtained. The dynamic properties of the specimen satisfied the condition obtained by the previously mentioned analysis can be evaluated with high accuracy.

1. INTRODUCTION

The history of split Hopkinson pressure bar method (SHPB method) is very old. It was developed to examine the impact plasticity of metallic materials by Kolsky in 1949 [1]. The SHPB method consists of holding a thin cylindrical specimen of a test material between input and output bars made of elastic metal. The applications to polymers or composite materials have been extended, and the SHPB method with viscoelastic input and output bars is widely used as a technique of evaluation of the dynamic properties of their low impedance materials. When polymeric materials are used for input and output bars, it becomes necessary to correct measured waveforms to take into account the attenuation and dispersion generated by the wave propagation. Wang *et al*. [2] first researched a

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viscoelastic SHPB technique. Much research has been reported [3]-[5] about viscoelastic SHPB technique with low impedance materials at high strain rate. However, it is not clear that the accuracy of the viscoelastic SHPB method depends on the materials used. Therefore, the applicable condition of the specimens which can be evaluated by the viscoelastic SHPB method is clarified by both the experiments and the numerical results in this work.

2. WAVE PROPAGATION IN VISCOELASTIC BARS

Based on the Elementary theory, a longitudinal strain pulse $\bar{\varepsilon}(x,\omega)$ propagating in a thin viscoelastic bar can be written as following expression in the frequency domain: [6]

$$\bar{\varepsilon}(x,\omega)=\bar{\varepsilon}(0,\omega)\cdot\exp\{-(\alpha+ik)\}x\ , \tag{1}$$

where x, ω and i are the coordinate along the rod axis, angular frequency and imaginary unit, respectively. The strain waves of arbitrary points can be calculated by Eq.(1). The attenuation coefficient α and the wave number k are the functions of ω, and are related to the complex compliance as

$$\left.\begin{aligned} k^2-\alpha^2&=\rho\omega^2 J_1^*(\omega) \\ 2\alpha k&=\rho\omega^2 J_2^*(\omega) \end{aligned}\right\}, \tag{2}$$

where ρ is the material density. The complex compliance $J^*(\omega)$ is defined by the ratio of strain to stress as

$$J^*(\omega)=J_1^*(\omega)-iJ_2^*(\omega)=\frac{\bar{\varepsilon}(x,\omega)}{\bar{\sigma}(x,\omega)}. \tag{3}$$

When a 3-element solid model shown in Fig.1 is used especially, the real part $J_1^*(\omega)$ and the imaginary part $J_2^*(\omega)$ of the complex compliance are expressed as follows:

$$\left.\begin{aligned} J_1^*(\omega)&=\frac{1}{E_1}+\frac{E_2}{E_2^{\ 2}+(\omega\eta_2)^2} \\ J_2^*(\omega)&=\frac{\omega\eta_2}{E_2^{\ 2}+(\omega\eta_2)^2} \end{aligned}\right\}. \tag{4}$$

3. VISCOELASTIC SPLIT HOPKINSON PRESSURE BAR METHOD

3-1 Correction of Waveform

Figure 2 shows a viscoelastic SHPB setup. A specimen is sandwiched between viscoelastic input and output bars. Strain gages ① and ② are situated on the input and output bars, respectively. A compressive wave generated by an impact of a striker bar propagates along the input bar toward the specimen, and is measured as an incident pulse $\varepsilon_i^{①}(t)$ by the strain gage ①. Some part of the incident pulse is reflected at the bar / specimen interfaces, and is measured as a reflected pulse $\varepsilon_r^{①}(t)$. also by the strain gage ①. The remaining wave penetrates the specimen and is measured as a transmitted pulse $\varepsilon_t^{②}(t)$ by the strain gage ②. Let $\varepsilon_i^{①}(\omega)$, $\varepsilon_r^{①}(\omega)$ and $\varepsilon_t^{②}(\omega)$ be the Fourier transforms of the measured on the strain gages $\varepsilon_i^{①}(t)$, $\varepsilon_r^{①}(t)$ and $\varepsilon_t^{②}(t)$, respectively.

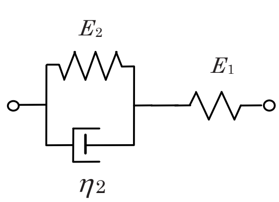

Fig.1 3-element viscoelastic model

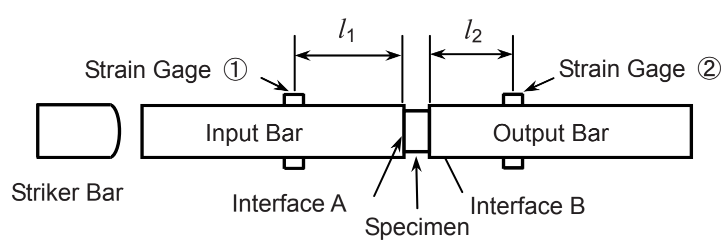

Fig.2 Schematic drawing of SHPB set-up

Based on Eq.(1), the following equations can be derived for correcting the individual strain pulses to be the waveforms at the interface A or B (bar / specimen interfaces):

$$\bar{\varepsilon}_i^{\,\mathrm{A}}(\omega) = \bar{\varepsilon}_i^{\,①}(\omega)\exp\{-(\alpha + ik)l_1\}\ , \tag{5}$$

$$\bar{\varepsilon}_r^{\,\mathrm{A}}(\omega) = \bar{\varepsilon}_r^{\,①}(\omega)\exp\{(\alpha + ik)l_1\}\ , \tag{6}$$

$$\bar{\varepsilon}_t^{\,\mathrm{B}}(\omega) = \bar{\varepsilon}_t^{\,②}(\omega)\exp\{(\alpha + ik)l_2\}\ , \tag{7}$$

where l_1 indicates the distance between the interface A and the strain gage ①, while l_2 denotes the distance between the interface B and the strain gage ②. The signs in exponential terms of Eqs.(6) and (7) must be pluses because reflected and transmitted pulses should be corrected in the opposite directions of their wave propagation. Making use of Eq.(5) to (7), the incident and reflected pulses can be corrected to be those at the interface A, while the transmitted wave can be corrected to be the value at the interface B. By applying the Fourier inverse transformation to Eqs.(5) to (7), corrected waveforms of incident, reflected and transmitted pulses in the time domain can be obtained as $\bar{\varepsilon}_i^{\,\mathrm{A}}(t)$, $\bar{\varepsilon}_r^{\,\mathrm{A}}(t)$ and $\bar{\varepsilon}_t^{\,\mathrm{B}}(t)$, respectively.

<u>3-2 Stress and Strain of Specimen</u>

In a conventional SHPB method using elastic bars, stress σ_S and strain rate $\dot{\varepsilon}_S$ of a specimen are given by

$$\sigma_S(t) = \frac{E\{\varepsilon_i^A(t) + \varepsilon_r^A(t) + \varepsilon_t^B(t)\}A}{2A_S}\ , \tag{8}$$

$$\dot{\varepsilon}_S(t) = \sqrt{\frac{E}{\rho}} \cdot \frac{\varepsilon_i^A(t) - \varepsilon_r^A(t) - \varepsilon_t^B(t)}{l}\ , \tag{9}$$

where E , ρ and A denote Young's modulus, density and cross-sectional area of the input and output bars, respectively. l and A_s show length and cross-sectional area of the specimen. Applying the Fourier transformation, and replacing the Young's modulus E by reciprocal of the complex compliance $J(\omega)$ of the input and output bars [7], we find the following equations for obtaining stress and strain of the specimen in the frequency domain:

$$\bar{\sigma}_s(\omega) = \frac{\{\bar{\varepsilon}_i^A(\omega) + \bar{\varepsilon}_r^A(\omega) + \bar{\varepsilon}_t^B(\omega)\}A}{2J(\omega)A_s} \tag{10}$$

$$\bar{\varepsilon}_s(\omega) = \frac{\bar{\varepsilon}_i^A(\omega) - \bar{\varepsilon}_r^A(\omega) - \bar{\varepsilon}_t^B(\omega)}{(\alpha + ik)\,l} \tag{11}$$

When the stress in the specimen is uniform, the loads at the interface A and B are equal.

$$\bar{\varepsilon}_i^{\,\mathrm{A}}(\omega) + \bar{\varepsilon}_r^{\,\mathrm{A}}(\omega) = \bar{\varepsilon}_t^{\,\mathrm{B}}(\omega)\,. \tag{12}$$

Then, the stress and strain of the specimen are given from substituting Eq.(12) into Eq.(10) and (11).

$$\bar{\sigma}_s(\omega) = \frac{A\bar{\varepsilon}_t^{\,\mathrm{B}}(\omega)}{A_s J^*(\omega)} \tag{13}$$

$$\bar{\varepsilon}_s(\omega) = -\frac{2\bar{\varepsilon}_r^{\,\mathrm{A}}(\omega)}{(\alpha + ik)l_s} \tag{14}$$

It is found that the stress and strain of the specimen can be calculated by the transmitted and reflected waves, respectively. When the Fourier inverse transformation is applied to Eq.(13) and (14), the stress and strain of the specimen in the time domain $\sigma_s(t)$ and $\varepsilon_s(t)$ can be obtained.

3-3 Evaluation of Viscoelastic Properties

The complex compliance of a test material can be calculated as the ratio of strain to stress of the specimen:

$$J_s^*(\omega) = J_{s1}^*(\omega) - iJ_{s2}^*(\omega) = \frac{\bar{\varepsilon}_s(\omega)}{\bar{\sigma}_s(\omega)} \tag{15}$$

4. ANALYSIS OF WAVEFORMS USING ELEMENTARY THEORY

4-1 Reflection and Transmission at Interface

Consider uniform elastic bars Ⅰ and Ⅱ which contact at the interface as shown in Fig.3 [8]. When the incident wave $\varepsilon_i(t)$ reaches the interface, some part of wave is reflected to the medium Ⅰ as $\varepsilon_r(t)$ and the rest is transmitted to the medium Ⅱ as $\varepsilon_t(t)$. Assuming the force equilibrium and continuity of particle velocity at the interface, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ are obtained in terms of $\varepsilon_i(t)$. Then, by applying the Fourier transform to their equations, the following equations for a viscoelastic medium in the frequency domain are obtained.

$$\bar{\varepsilon}_r = \frac{A_{\mathrm{II}}\rho_{\mathrm{II}}c_{\mathrm{II}} - A_{\mathrm{I}}\rho_{\mathrm{I}}c_{\mathrm{I}}}{A_{\mathrm{I}}\rho_{\mathrm{I}}c_{\mathrm{I}} + A_{\mathrm{II}}\rho_{\mathrm{II}}c_{\mathrm{II}}}\bar{\varepsilon}_i, \tag{16}$$

$$\bar{\varepsilon}_t = \frac{2A_{\mathrm{I}}\rho_{\mathrm{I}}c_{\mathrm{I}}}{A_{\mathrm{I}}\rho_{\mathrm{I}}c_{\mathrm{I}} + A_{\mathrm{II}}\rho_{\mathrm{II}}c_{\mathrm{II}}} \cdot \frac{c_{\mathrm{I}}}{c_{\mathrm{II}}}\bar{\varepsilon}_i, \tag{17}$$

where A_{I} and ρ_{I} represent the cross section of the bar and the material density correspondent to the medium Ⅰ, respectively. c_{I} is the functions of ω, and denotes the propagation velocity of medium Ⅰ. Similarly, A_{II}, ρ_{II} and c_{II} correspond to the medium Ⅱ. Fractions in right sides of Eq.(16) and (17) are defined as the coefficients of reflection $R = \bar{\varepsilon}_r / \bar{\varepsilon}_i$ and the coefficients of transmission $T = \bar{\varepsilon}_t / \bar{\varepsilon}_i$, respectively.

4-2 Calculation of Wave Propagation in Viscoelastic SHPB Method

Figure 4 shows the calculated values of the strain wave propagation in input / output bars and a specimen in the frequency domain. Let the reflectivity and transmittance when a strain wave propagates from the input / output bars to the specimen be R and T, and let them when a strain wave propagates from the specimen to the input / output bars be R' and T', respectively. When the same material is used for input / output bars, R' is equal to R by Eq.(19). Based on Eq.(1), the terms of waveform change after propagating in the distance l_1, l_2 and l_s are expressed by follows, respectively:

$$\left.\begin{aligned} B_1 &= \exp\{-(\alpha + ik)l_1\} \\ B_2 &= \exp\{-(\alpha + ik)l_2\} \\ B_s &= \exp\{-(\alpha_s + ik_s)l_s\} \end{aligned}\right\}, \tag{18}$$

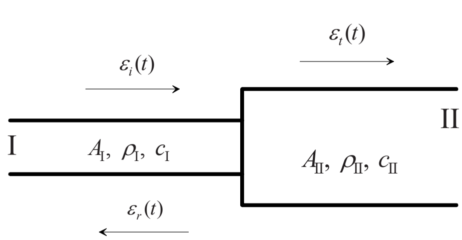

Fig.3 Propagation of strain waves at interface

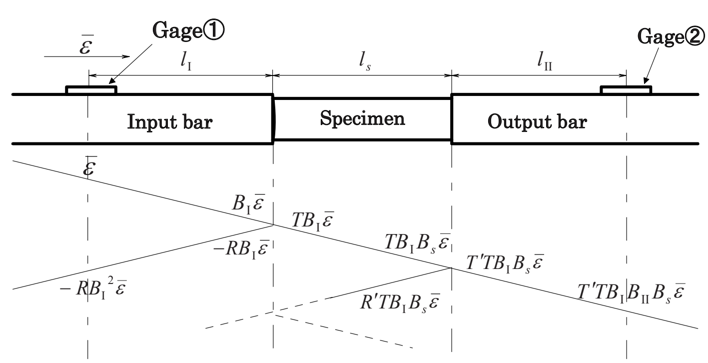

Fig.4 Propagation of predicted wave on viscoelastic SHPB method using Elementary theory

where α and α_s indicate the attenuation coefficient of input / output bars and a specimen, k and k_s denote the wave number of input / output bars and a specimen, respectively.

A compressive wave at the strain gage ① in the frequency domain assumed to be $\bar{\varepsilon}_i^{①}(\omega)$ as shown in Fig.4. The wave reaches the interface A as $B_1\bar{\varepsilon}_i^{①}(\omega)$, and part of it is reflected as $-RB_1\bar{\varepsilon}_i^{①}(\omega)$ and the rest is transmitted as $-TB_1\bar{\varepsilon}_i^{①}(\omega)$. Thus, the wave propagates repeating the reflection and transmission. Consequently, after propagating enough, the superposed wave signal $\bar{\varepsilon}_r^{①}(\omega)$ and $\bar{\varepsilon}_t^{②}(\omega)$ measured by the strain gage ① and ② can be calculated as follows:

$$\bar{\varepsilon}_r^{①}(\omega) = -RB_1^{\,2}\bar{\varepsilon}_i^{①}(\omega)\sum_{m=1}^{\infty}(1+TT'R^{2(m-1)}B_s^{\,2m})\,. \tag{19}$$

$$\bar{\varepsilon}_t^{②}(\omega) = TT'B_1B_2B_s\bar{\varepsilon}_i^{①}(\omega)\sum_{m=1}^{\infty}\{1+(RB_s)^{2m}\}\,. \tag{20}$$

Moreover, the stress $\bar{\sigma}^{\mathrm{A}}(\omega)$ and $\bar{\sigma}^{\mathrm{B}}(\omega)$ at the interface A and B can be calculated as following forms:

$$\bar{\sigma}^{\mathrm{A}}(\omega) = \frac{B_1\bar{\varepsilon}_i^{①}(\omega)}{J^*}\left\{1-R\sum_{m=1}^{\infty}(1+TT'R^{2(m-1)}B_s^{\,2m})\right\}, \tag{21}$$

$$\bar{\sigma}^{\mathrm{B}}(\omega) = \frac{TT'B_1B_s\bar{\varepsilon}_i^{①}(\omega)}{J^*}\sum_{m=1}^{\infty}\{1+(RB_s)^{2m}\}\,, \tag{22}$$

where J^* is the complex compliance of the stress bars. Applying the Fourier inverse transformation to Eq.(19), (20) and Eq.(21), (22), the strain and the stress in the time domain can be obtained, respectively.

4-3 Reflected Wave and Stress at Both Ends of Specimen

The input and output bars in the present SHPB method assumed to be made of PMMA. The viscoelastic behavior of PMMA was examined by preliminary experiments, and was identified by 3-element solid model $E_1 = 4.17$ GPa, $E_2 = 4.40\times10^1$ GPa and $\eta_2 = 2.25$ MPa・s shown in Fig.1. The mass density of PMMA is 1180 kg/m^3. The length and the diameter of the input / output bars are 1800mm and 20mm. The length and the diameter of the specimen are 7.5mm and 15mm, respectively. That is, the slenderness ratio of specimen $l_s / d_s = 0.5$ which is widely used in SHB method. The distance l_1 and l_2 are 700mm and 50mm. Figure 5 indicates the strain wave at the gage position ① as the initial condition. This is the waveform measured in the actual experiment by using the striker with the diameter of 15mm and the length of 100mm. The velocity of the striker bar is about 10m/s. Based on the

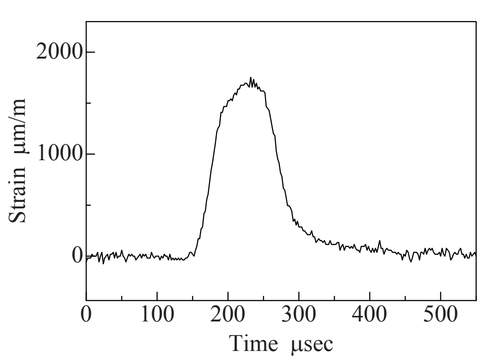

Fig.5　Incident strain wave on strain gage ①

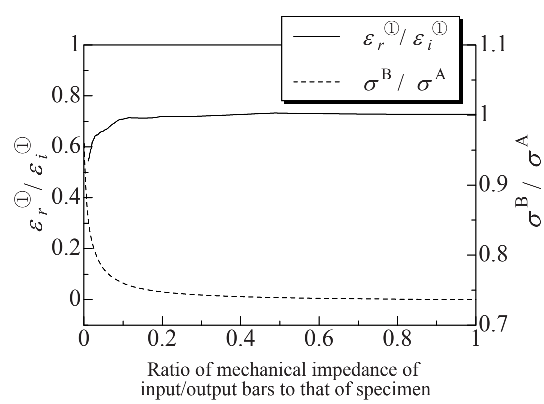

Fig.6　Proportion of reflected waves at interface A and stress at both interfaces

previously mentioned calculation method, reflected wave and stress at both ends of specimen are examined by changing the length and the characteristic value of the specimen. The specimen is also approximated to a 3-element solid model, and the parameter of E_1 is only changed.

Figure 6 shows the reflectivity at the interface A (solid line) and the stress distribution in the specimen (dotted line). The horizontal axis denotes the ratio of the characteristic values of the input / output bars to that of the specimen, the left vertical axis indicates the ratio of the peak value of $\varepsilon_r^{①}$ to that of $\varepsilon_i^{①}$ and the right vertical axis is the ratio of the peak value of σ^{B} to that of σ^{A}. The smaller the characteristic value of the specimen, the bigger the reflected wave is measured. The specimen of which the characteristic value is small has to be used in order to measure the bigger reflected wave. On the other hand, the smaller the characteristic value of the specimen, the bigger the difference of the load at both interfaces of the specimen is. The specimen of which the characteristic value is big has to be used in order to displace the specimen uniformly.

It is necessary for the Viscoelastic SHPB Method to use the experimental condition in which the specimen is deformed uniformity and the reflected wave of suitable size are measured. When the ratio of the characteristic values of the stress bar to that of the specimen is about 10%, $\sigma^{\mathrm{B}} / \sigma^{\mathrm{A}} \cong 0.99$ and the error is about 1%. If the ratio of the characteristic values is lower than 10%, the error suddenly grows and the difference of the load at both interfaces of the specimen generates. In addition, when the ratio of the characteristic values is about 20%, $\varepsilon_r^{①} / \varepsilon_i^{①} \cong 0.03$. If the ratio becomes lower than 20%, the value of $\varepsilon_r^{①} / \varepsilon_i^{①}$ gradually grows and bigger reflected wave comes to be obtained. Consequently, the appropriate experimental conditions of the Viscoelastic SHPB Method is 10～20% in the ratio of the characteristic values of the stress bar to that of the specimen.

5. VISCOELASTIC SPLIT HOPKINSON PRESSURE BAR TEST

The same sizes as the input / output bars and specimen in the analysis are used in the viscoelastic test. A polypropylene is tested as the specimen of one example. Figure 7 shows the experimental strain histories. Using Eqs.(5) to (7), the measured incident, reflected and transmitted strain pulses are corrected to be those at the interface A or B in the frequency domain. The stress and strain of the specimen is calculated by Eqs.(10) and (11). Then, the complex compliance of the specimen is obtained by Eq.(15) as shown in Fig.8. The white and black plots in the figure show J_{s1}^* and J_{s2}^*. The vertical bar indicates the standard deviation. Frequency dependence of the complex compliance can be approximated as the solid lines depicted in the figure by use of the 3-element viscoelastic model. Viscoelastic parameters of the model determined by a graphical method are also given in the

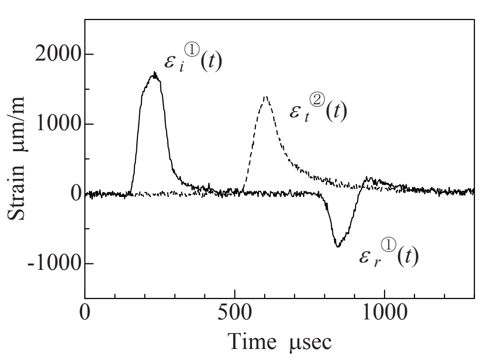

Fig.7 Typical strain records on polypropylene

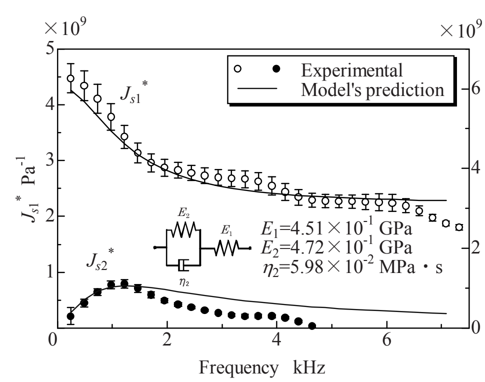

Fig.8 Complex compliance of experiment and model's prediction

figure. It is found that the 3-element viscoelastic model provides reasonable estimation of the dynamic behavior of the material. E_1 of the input / output bars and the specimen are 4.17 GPa and 4.51×10^{-1}GPa, respectively. The ratio of each E_1 is about 11%, and its value meets the requirement obtained by the analysis. Consequently, the dynamic properties of the specimen in the condition determined by the analysis can be evaluated at high accuracy in the viscoelastic SHPB method.

6. CONCLUSIONS

The conclusions obtained from the present study are summarized as follows:

・ The technique for calculating the waveforms in the viscoelastic SHPB method was proposed using the wave propagation analysis based on the Elementary theory.

・ It was clarified that we have to use the experimental condition in which the ratio of the characteristic values of the input / output bars to that of the specimen is about 10～20% in the viscoelastic SHPB method.

・ It was found that the viscoelastic properties of the specimen in the condition determined by the analysis can be evaluated at high accuracy in the actual viscoelastic SHPB tests.

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