Chapter 6
Bond Graphs and Inverse Modeling for Mechatronic System Design

Wilfrid Marquis-Favre and Audrey Jardin

Abstract This chapter is concerned with the design of mechatronic systems on dynamic and energy criteria. Compared to the traditional trial–error–correction approach a methodology is presented that drastically decreases the number of simulation iterations and ensures more relevant solutions with respect to the specifications. Moreover, early in the design stages, this methodology enables to check if the design problem is well posed before any simulation. This verification is possible according to the structural analysis concept that points out the characteristic properties of the design models independently of the parameter numerical values. Also, the methodology is based on model inversion that uses straightforwardly the information written in the specifications. Finally, because of its ability to represent multidisciplinary physical systems, to acausally describe a model and to easily undertake a structural analysis, and to visualize the results of this analysis, the bond graph language is well dedicated to this methodology. In this chapter topics like design model validity, specifications validity, structural analysis, technological component specifications, selection and validation, and open-loop control determination will be discussed.

Keywords Inverse model · Structure analysis · Power line · Causal path · Bicausality · Component specification · Sizing validation

6.1 Introduction

This chapter presents the use of the bond graph language for inverse model-based design and, in particular, a methodology concerning the sizing[1] of mechatronic

W. Marquis-Favre (✉)
Laboratory Ampère, Institut National des Sciences Appliquées de Lyon (INSA),
University of Lyon, Villeurbanne Cedex, France
e-mail: wilfrid.marquis-favre@insa-lyon.fr

[1] The term "sizing" is used here to equally designate the process of choosing off-the-shelf components or of specifying brand new components.

W. Borutzky (ed.), *Bond Graph Modelling of Engineering Systems*,
DOI 10.1007/978-1-4419-9368-7_6,

systems on dynamic and energy criteria. In the design cycle this methodology takes place between, on the one hand, the step of the functional analysis for the definition of the product concepts and, on the other hand, the step of the geometric definition of the designed components for the prototype manufacture. The output of the previous step in this design cycle defines the specifications for the methodology, and in return, the results of the methodology furnish the data for the next step. Based on model inversion and applied in the context of the bond graph language, this enables the designer to directly use the specifications data in order to determine what is unknown in his design problem (the component sizes in the system to design).

Figure 6.1 presents, in a simplified sketch, the methodology for sizing the components of a mechatronic system. The theoretical material used in the methodology is presented in Section 6.2. Section 6.3 introduces four criteria, bicausality, and the notion of analysis levels that guide the application of the methodology phases and the search of a design solution. Finally, Section 6.4 goes into the details of the different phases of this methodology. They are chronologically ordered from the validity checking of the design model and of the specifications, the component specification and selection, the selected component validation, until the open-loop control determination.

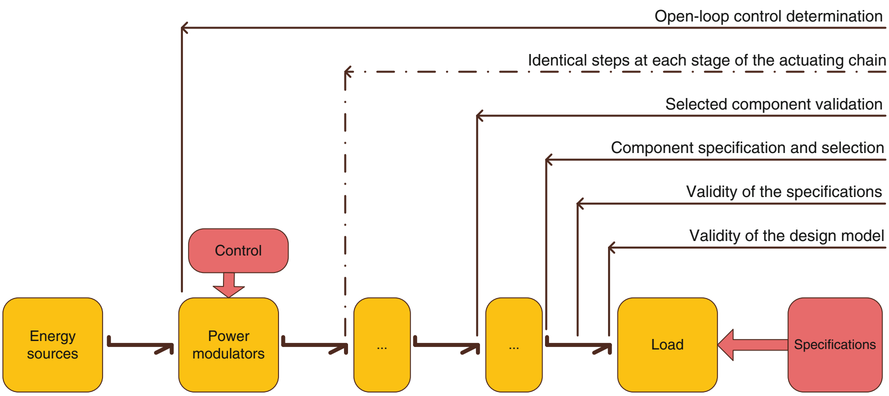


Fig. 6.1 Phases in the sizing process methodology

6.2 Theoretical Concepts

This section presents the theoretical material required for the methodology concepts and the proof of its effectiveness. A very brief review of model inversion is first recalled. Then the definitions of relative orders, orders of zeros at infinity, and essential orders are presented. These notions are also reviewed in the bond graph language for defining structural analysis in this framework. In particular the concepts of power lines and causal paths are defined. They will be used for checking the structural criteria of invertibility and differentiability.

6.2.1 Model Inversion

The basic concepts of inversion are now presented in the context of state-space representation.

6.2.1.1 Direct Model

A direct model corresponds to the physical way the associated system behaves. It enables the physical outputs to be calculated from data given about the physical inputs and the parameters (Fig. 6.2). In the bond graph language the direct model is obtained by assigning a preferential integral causality to the acausal representation.

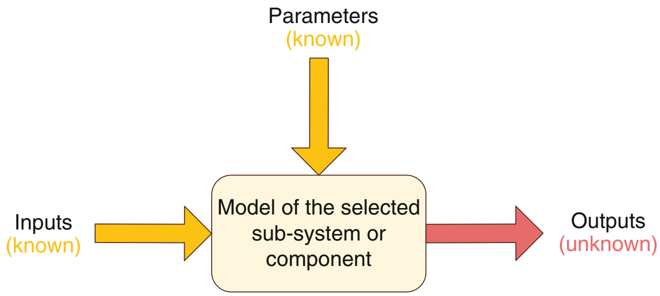


Fig. 6.2 Organization of quantities for a direct model

In the case of a square linear time-invariant (LTI) system Σ, the state-space model can be expressed by

$$\Sigma : \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{cases} \tag{6.1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{y} \in \mathbb{R}^m$ denote, respectively, the input and output vectors, and **A**, **B**, **C**, and **D** are, respectively, $(n \times n)$, $(n \times m)$, $(m \times n)$, and $(m \times m)$, constant matrices.

6.2.1.2 Inverse Model

The inverse model corresponds to a re-organization of the equations where the input and output roles are exchanged: inputs become outputs and vice versa (Fig. 6.3).[2]

---

[2] Here inversion is considered between inputs and outputs exclusively. It could also be envisaged between parameters and outputs. This would correspond, in this case, to the objective of a parameter synthesis.

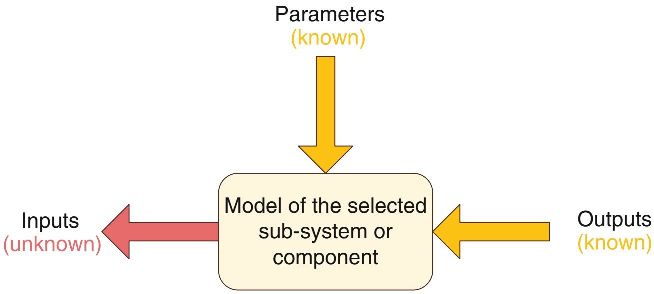


Fig. 6.3 Organization of quantities for an inverse model

Assuming that it exists, the inverse model denoted Σ^{-1} of Σ is expressed by

$$\Sigma^{-1} : \begin{cases} \dot{\mathbf{z}}(t) = \mathbf{A_{inv}}\mathbf{z}(t) + \mathbf{B_{inv}}\mathbf{y}_\alpha(t) \\ \mathbf{u}(t) = \mathbf{C_{inv}}\mathbf{z}(t) + \mathbf{D_{inv}}\mathbf{y}_\alpha(t) \end{cases} \tag{6.2}$$

where $\mathbf{z} \in \mathbb{R}^n$ is the state vector, $\mathbf{y}_\alpha \in \mathbb{R}^m$ denotes the vector resulting from differential and algebraic operations on $\mathbf{y}$, and $\mathbf{A_{inv}}$, $\mathbf{B_{inv}}$, $\mathbf{C_{inv}}$, and $\mathbf{D_{inv}}$ are, respectively, $(n \times n)$, $(n \times m)$, $(m \times n)$, and $(m \times m)$ constant matrices.

The inverse model presented in (6.2) is of full rank in the sense that its state vector $\mathbf{z}$ has the same dimension n as that of the vector $\mathbf{x}$. However, it was proved that there exists an inverse model of minimal order where the state vector has a dimension less than n [46]. This feature will be of great interest in the building of the inverse model from a bond graph representation.

One of the main characteristics of an inverse model is the presence of the output derivatives in the equations (vector $\mathbf{y}_\alpha(t)$). This will be discussed in more detail in the following sections. In particular, in structural analysis, the notion of essential orders enables the necessary minimal number of output time differentiations to be anticipated before the construction of the inverse model. This notion will be translated into the bond graph language.

The key principle for obtaining the inverse model from the direct one is to successively differentiate the outputs with respect to time until the inputs appear in the expression of the output derivatives. Then, from this transformation of the model, the aim is to express the inputs in terms of the outputs by inverting these equations if possible. The condition for the existence of this inversion will also be discussed in the following sections.

Model inversion was discussed in 1963 by Zadeh and Desoer [57] and by Weiss [53] in the context of functional reproductibility. Brockett and Mesarović [5, 6] established the first necessary and sufficient condition of invertibility and an algorithm of inversion for LTI single-input/single-output (SISO) models. Youla and Dorato [12, 56] dealt with multi-input/multi-output (MIMO) models. They set a simpler criterion of invertibility and proposed a new algorithm for inversion. In 1969 Silverman [47] went back over the SISO case and proved that Brockett's algorithm is generally applicable to discrete systems and to linear parameter varying (LPV) models. His work was the basis for a number of other works like those of Sain and Massey [45], Porter [38], or Willsky [54]. To name a few, other contributions about inversion were from Rosenbrock and van der Weiden [44] for their system matrix approach; Hirschorn [21], Singh [48], Nijmeijer [35, 36], and Fliess [14] for their work on nonlinear models; Tan and Vandewalle [50] on singular systems; and Perdon et al. [37] for their work on periodic systems.

Finally, inversion has already been used in robotics for sizing manipulators, by Potkonjak and Vukobratović [39, 52] who introduced criteria based on power, transient power (first time derivative of power), and power jerk (second time derivative of power) and by Dequidt et al. [9, 10] who proposed a selection method of high-performance motion servomechanisms.

6.2.2 Concepts of Structural Analysis

This section presents the key concepts of structural analysis which are used in the sizing methodology. They are defined in the framework of LTI models. These concepts furnish qualitative information to the designer about his model as well as tools that help him in his design process. Structural analysis does not depend on the numerical values of the model parameters. Thus, it is carried out before any simulation and enables time to be saved in the design process by detecting, as soon as possible, whether the design problem is well defined or not. A practical result in the context of the bond graph sizing methodology based on inverse models is to state structural criteria for a model to be invertible and for output specifications to be sufficiently differentiable.

6.2.2.1 Relative Order

The relative order, denoted n'_i, associated with the output y_i of the system Σ, corresponds to the minimal number of times that it is necessary to time differentiate this output to make one component of the input vector $\mathbf{u}$ appear (6.3) [5, 7, 11]. This relative order can also be determined from the transfer matrix [5, 8] or from the notion of the infinite zero in row [5, 11]:

$$n'_i = \begin{cases} 0 & \text{if } \mathbf{d}_i \neq \mathbf{0} \\ \inf_{k\in\mathbb{N}^*} & \{k|\mathbf{c}_i\mathbf{A}^{k-1}\mathbf{B} \neq \mathbf{0}\} \end{cases} \tag{6.3}$$

The relative order indicates that the output y_i will appear with a time derivative of order n'_i at least in the inverse model. Depending on the model, this derivative order can be higher and then defined by the essential order of this output (see Section 6.2.2.3). It can be shown that the difference between the relative order and the essential order is related to the notion of the dynamic extension that must be introduced for a model to be decouplable by a static feedback [19]. The determination of the relative order from a bond graph representation will be shown in Section 6.2.3.

6.2.2.2 Structure at Infinity

Introduced in 1982 by Vardulakis [51] the Smith–McMillan factorization at infinity of a transfer matrix $\mathbf{T}(s)$ of rank r enables the structure at infinity of a system $\Sigma : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ (6.1) to be characterized (6.4)[3]:

$$\mathbf{T}(s) = \mathbf{B}_1(s)\boldsymbol{\Lambda}(s)\mathbf{B}_2(s) \tag{6.4}$$

where

- $\mathbf{B}_1(s)$ and $\mathbf{B}_2(s)$ are biproper matrices[4];
- $\boldsymbol{\Lambda}(s) = \begin{pmatrix} \boldsymbol{\Delta}_\infty(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$ with $\boldsymbol{\Delta}_\infty(s) = \begin{pmatrix} s^{-n_1} & & \\ & \ddots & \\ & & s^{-n_r} \end{pmatrix}$

  and $n_1 \leq n_2 \leq \cdots \leq n_r$.

The integer $n_i \geq O$ (resp. ≤ 0) is the order of the ith zero (resp. pole) at infinity of the corresponding system Σ. This concept is used to define the essential order and will also be defined in the bond graph language.

6.2.2.3 Essential Order

The notion of essential order was first introduced by Commault et al. [7] to solve the problem of decouplability by static feedback. The essential order n_{ie} of the output y_i of a system Σ is the maximal order of its time derivatives appearing in the inverse model [7, 20]. For the system $\Sigma : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$, assumed invertible, the essential order of output y_i is calculated by (6.5)

$$n_{ie} = \sum_{j=1}^{m} n_j - \sum_{j=1}^{m-1} \overline{n}_{ij} \tag{6.5}$$

where

- n_j is the order of the jth zero at infinity of Σ;
- $\overline{n}_{ij}$ is the order of the jth zero at infinity of $(\mathbf{A}, \mathbf{B}, \overline{\mathbf{C}}_i, \overline{\mathbf{D}}_i)$ when

---

[3] $\mathbf{T}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$ with $\mathbf{I}$ the $(n \times n)$ identity matrix.

[4] A rational matrix $\mathbf{B}(s)$ is biproper if and only if it is proper and its inverse is also proper. A matrix $\mathbf{B}(s)$ is proper if and only if all its elements are rational fractions with the degree of their denominator greater than that of their numerator [42].

$$\overline{\mathbf{C}}_i = \left(\mathbf{c}_1^T \; \mathbf{c}_2^T \; \ldots \; \mathbf{c}_{i-1}^T \; \mathbf{c}_{i+1}^T \; \ldots \; \mathbf{c}_m^T \right)^T$$

$$\overline{\mathbf{D}}_i = \left(\mathbf{d}_1^T \; \mathbf{d}_2^T \; \ldots \; \mathbf{d}_{i-1}^T \; \mathbf{d}_{i+1}^T \; \ldots \; \mathbf{d}_m^T \right)^T$$

and $\forall k \in \{1, \ldots, m\}$, $\mathbf{c}_k$ (resp. $\mathbf{d}_k$) is the kth row of $\mathbf{C}$ (resp. $\mathbf{D}$).

The concept of essential order is presented in the bond graph language in the next section.

6.2.3 Structural Analysis Concepts in Bond Graph

The previous concepts of structural analysis are now reviewed in the context of the bond graph language. First, the notions attached to power lines and causal paths are defined and then used for determining the relative orders, the orders of the zeros at infinity, and the essential orders. All the following definitions are given for the bond graph representation of an LTI system $\Sigma : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$.

6.2.3.1 Power Line Concepts

Three definitions are given about the power line concepts. A power line characterizes the way energy flows between two points in a system. So, talking about inverse models (here implicitly between the inputs and the outputs), the input/output (I/O) power line concept is defined. Finally, the invertibility criteria presented in Section 6.3.1 lead to introduce the notion of disjoint power lines.

Definition 6.1 (Power line) In an acausal bond graph representation, a power line between two components is a series of power bonds and multiport elements connecting these two components [33, 55].

Definition 6.2 (Input/output (I/O) power line) An input/output (I/O) power line starts from a modulated element and goes to a detector (De or Df element).

Definition 6.3 (Disjoint power line) Two power lines are said to be disjoint only if there is no power in common [34].

6.2.3.2 Causal Path Concepts

While the power line is an acausal concept, i.e., it does not require any organization of the model equations, the causal path needs a causality assignment in the bond graph representation. Its definition is first recalled. Then the length and the order of a causal path are introduced, and finally, both different and disjoint causal paths are defined. The latter concepts, as for the power line, will be used in the invertibility criteria. The concept of different causal paths will also be used to characterize the structure at infinity of a model from its bond graph representation.

Definition 6.4 (Causal path) In a causal (or bicausal) bond graph representation, a causal path is a series of effort and flow variables successively related according to the model causality assignment [34, 55].

Definition 6.5 (Input/output (I/O) causal path) An input/output (I/O) causal path starts from a modulated element and goes to a detector (De or Df element).

Definition 6.6 (Causal path length) In a causal (or bicausal) bond graph representation, the length, denoted $\ell_k(v_i \rightarrow v_j)$, of a causal path k between a variable v_i and another variable v_j is defined as the number of energy storage elements in integral causality along this causal path [41].

Definition 6.7 (Causal path order) In a causal (or bicausal) bond graph representation, the order, denoted $\omega_k(v_i \rightarrow v_j)$ (or the generalized length), of a causal path k between a variable v_i and another variable v_j is defined as the difference between the number of energy storage elements in integral causality and the number of those in derivative causality along this causal path [2, 15].

Definition 6.8 (Different causal path) In a bond graph representation in preferential integral causality, two causal paths are said to be different if they have no energy storage element in integral causality in common [40, 41].

Definition 6.9 (Disjoint causal path) In a causal or (bicausal) bond graph representation, two causal paths are said to be disjoint only if they have no variable in common [34]. This translates into a graphical disjunction of these two causal paths in the bond graph representation.

6.2.3.3 Structure at Infinity

This section gives the procedures that enable the output relative orders, the number and the orders of the zeros at infinity, and the essential orders of a system Σ to be determined directly from a bond graph representation. These procedures use the concepts defined in the previous sections.

Procedure 1 (Output relative order (Fotsu-Ngwompo [15] and Wu and Youcef-Toumi [55])) *In a bond graph representation in preferential integral causality of a system Σ, the relative order n'_i of the output y_i (and so the ith infinite zero order in row) is determined by $\omega_{i_{min}}$, the minimal order a causal path (Definition 6.7) can have between the output y_i and any inputs.*[5]

Procedure 2 (Number of zeros at infinity (Jardin [24] and Sueur and Dauphin-Tanguy [49])) *In a bond graph representation in preferential integral causality of a system Σ, the number r of zeros at infinity is determined by the maximal number*

[5] In the case of several causal paths between the same I/O pair having the same minimal order and of which the sum of their gain is equal to zero, the relative order of the studied output can be greater than n'_i. The gain of a causal path is determined by the product of the gains of all the elements contained in the path.

of paths a set of disjoint I/O causal paths (Definition 6.9) may have for the bond graph representation. When the model is invertible the number r is equal to m.

Procedure 3 (Orders of zeros at infinity (Jardin [24] and Sueur and Dauphin-Tanguy [49])) *In a bond graph representation in preferential integral causality of a system Σ, the orders n_i $(i \in \{1, \ldots, r\})$ of the zeros at infinity are determined by*

$$\begin{cases} n_1 = L_1 \\ n_i = L_i - L_{i-1} \end{cases} \tag{6.6}$$

where L_i is the minimal length a set[6] *of i different I/O causal paths (Definition 6.8) can have.*[7]

The essential orders of the outputs, used in the differentiability criterion (see Section 6.3.2), can be determined from a causal bond graph [1, 13, 24] but this requires manipulation of different causal bond graph representations. Instead a straightforward procedure has been established using a bicausal bond graph representation. The procedure is now given:

Procedure 4 (Output essential order (El Feki et al. [13] and Jardin [24])) *In a bicausal bond graph representation of a system Σ, the essential order n_{ie} of the output y_i of a system Σ can be expressed by*

$$n_{ie} = - \min_{j \in \{1,\ldots,m\}} \left\{ \omega_{ji} \right\} \tag{6.7}$$

where ω_{ji} represents the minimal order a causal path (Definition 6.7) can have between the double source associated with y_i and the double detector associated with u_j.

All the material is now available to present the criteria to check the invertibility and the differentiability of a model and then to present the different phases of the sizing methodology.

6.3 Criteria for Inversion and Analysis Levels

This section defines the criteria that will be used in the bond graph sizing methodology based on model inversion. Then bicausality is presented as a tool for determining the inverse model directly from a bond graph representation. As seen in Procedure 4, bicausality also enables the essential orders to be determined.

---

[6] By extension the length of a set of causal paths is the sum of the lengths of the causal paths (Definition 6.6) constituting the set.

[7] As for the relative orders, the orders of the zeros at infinity may be affected by the possible existence of causal paths between the same I/O pair having the same length and of which the sum of their gain is equal to zero.

6.3.1 Invertibility Criteria

First, a series of criteria concerns the invertibility checking of a model. An approach based on different I/O causal paths (see Definition 6.8) and the system matrix determinant[8] has been proposed in [40]. Here the approach based on disjoint I/O causal paths (see Definition 6.9) is presented [15, 24, 25]. It uses two structural criteria which, if not verified, enable the inversion process to be stopped early in the procedure. A third criterion is formulated at a behavioral level. This level is called behavioral in the sense that it requires analytical developments based on the constitutive and conservation laws in the bond graph representation.

Criterion 1 (Acausal) *In the acausal bond graph representation, if no set of disjoint I/O power lines (Definition 6.3) exists then the model is not invertible.*

Criterion 2 (Causal) *In the bond graph representation in preferential integral causality, if no set of disjoint I/O causal paths (Definition 6.9) exists then the model is not invertible.*

Criterion 3 (Junction structure solvability) *In the bicausal bond graph representation, if for all the sets of disjoint I/O power lines and for all the sets of disjoint I/O causal paths retained for the bicausality assignment, a non-solvable junction structure appears then the model is not invertible.*

The latter criterion corresponds to checking if the equations of the corresponding model are solvable. In practice it is generally sufficient to detect possible causal loops[9] and to verify that they are not algebraic,[10] and if they are, to verify that they have no unitary gain.

Remark To a large extent these criteria can be applied in the case of nonlinear models. In that case it has to be further checked that the constitutive laws "touched" by inversion in the bond graph representation (elements passed through by bicausality or of which causality changes with respect to the causal representation) are invertible in the domain of definition of the involved variables.

6.3.2 Differentiability Criterion

The criterion given now aims at verifying that the output specifications in the sizing problem are mathematically in adequacy with the structure of the inverse model.

---

[8] The system matrix of a model $\Sigma : (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ is defined by $\mathbf{P}(s) = \begin{pmatrix} s\mathbf{I} - \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & \mathbf{D} \end{pmatrix}$ with $\mathbf{I}$ the $(n \times n)$ identity matrix [43].

[9] It is recalled that a causal loop is a closed causal path.

[10] Algebraic causal paths and algebraic causal loops have constant gains.

Criterion 4 (Specification differentiability) *In order to simulate an inverse model each output specification must have a time differentiation order greater than or equal to* n_{ie}*, the essential order (Section 6.2.2.3 and Procedure 4) of the corresponding specified output* y_i *in the system* Σ.

In fact if the specifications in a sizing problem based on the approach of model inversion do not verify this criterion, unit pulses may appear when inverting the equations which is not physically feasible.

6.3.3 Bicausality Assignment Procedure

6.3.3.1 Bicausality

Bicausality is the extension of causality for obtaining the inverse model equations directly from a bond graph representation. The way bicausality is assigned in a bond graph depends on the results of the invertibility criteria and of the structural analysis in terms of I/O power line (see Definition 6.2) sets and I/O causal path (see Definition 6.5) sets.

The principle for assigning bicausality lies on the different mathematical combinations of the adaption of two conjugate power variable pairs when two subsystems are physically connected. This adaption is expressed by two implicit equations between the efforts and the flows on both the subsystem ports (Fig. 6.4).

Causality, by comparison, corresponds to the physical principle postulating that a subsystem cannot impose both the conjugate power variables to the other subsystem to which it is connected. From this constraint the only two possibilities of causal assignment are given in Fig. 6.5 causal bond graphs with their corresponding causal equations.

Bicausality breaks this physical principle and accepts that a subsystem "imposes" both the conjugate power variables to the other subsystem to which it is connected. In fact, mathematically speaking, this corresponds to exploiting the two last combinations of Fig. 6.4 implicit equations thus giving Fig. 6.6 bicausal equations.

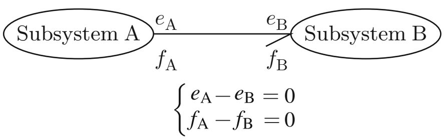


Fig. 6.4 Bond graph representation of two physical subsystem connection

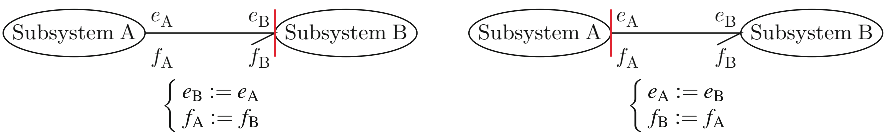
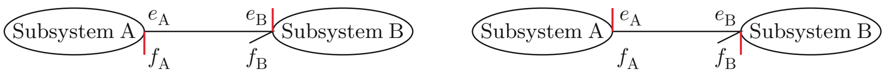


Fig. 6.5 Causal bond graphs of two physical subsystem connections



$$\begin{cases} e_B := e_A \\ f_B := f_A \end{cases} \qquad \begin{cases} e_A := e_B \\ f_A := f_B \end{cases}$$

Fig. 6.6 Bicausal bond graphs of two physical subsystem connections

The graphic trick to represent bicausality in a bond graph breaks the causal stroke into two half strokes each dedicated to the assignment of one of the two conjugate power variables (here the flow variable is on the half arrow side and the effort variable on the opposite side). The assignment rule remains in agreement with the one of causality since a flow is "imposed" on the subsystem far from the flow-dedicated half stroke while an effort is "imposed" on the subsystem closed to the effort-dedicated half stroke [18] (Fig. 6.6).

Now two new elements are required to assign bicausality in a bond graph representation. The first element, a double source, "initializes" this assignment by imposing both the conjugate power variables at one port of the model, while the second one, a double detector, "receives" both the conjugate power variables at another port of the model. In the context of I/O inverse model, the double sources (resp. double detectors) replace the detectors (resp. the modulated elements) carrying the outputs (resp. the inputs). Concerning the double source, distinction must be done with respect to both types of detectors which they are substituted with. For an effort (resp. flow) detector, the replacing double source assigns both a specified effort (resp. flow) and a null flow (effort). The bond graph representations of the two types of double sources with their respective bicausality assignment are displayed, respectively, in Figs. 6.7 and 6.8.

A procedure for bicausality assignment is now given. It uses the criteria previously defined and will be applied to determine the inverse model in the sizing methodology. The input of this procedure is the acausal bond graph representation of the model of a physical system. In the very first step a set of disjoint I/O causal paths of minimal order is searched. This guarantees an inverse model of minimal order (see Section 6.2.1.2). The interest of the preceding is that, on the one hand,

$$f := 0 \qquad \begin{cases} e := e_{specified} \\ f := 0 \end{cases}$$

Fig. 6.7 Bond graph representation of a double source replacing an effort detector

$$e := 0 \qquad \begin{cases} e := 0 \\ f := f_{specified} \end{cases}$$

Fig. 6.8 Bond graph representation of a double source replacing a flow detector the inverse model obtained has a dynamic part of minimal dimension and, on the other hand, that the specified outputs appear in the equations with their minimal time differentiation order [15, 24].

Procedure 5 (Bicausality Assignment (Fotsu-Ngwompo [15] and Jardin [24]))

1. *In the bond graph representation in preferential integral causality, choose a set of disjoint I/O power lines associated*[11] *with a set of disjoint I/O causal paths of minimal order. If such sets do not exist then the model is not invertible (criteria 1 and 2), and the procedure stops.*
2. *In the acausal bond graph representation, replace the modulated elements (resp. detectors) associated with inputs (resp. outputs) by double detectors DeDf (resp. double sources SeSf).*
3. *For each element of which causality is imposed (sources, elements with non-invertible constitutive laws) assign it and propagate it through the junction structure taking into account the causality constraints of 0- and 1-junctions, TF- and GY-elements.*
4. *Along each power line chosen at step 1 propagate bicausality from the double source to the double detector and propagate causality through the junction structure taking into account the causality constraints of 0- and 1-junctions, TF- and GY-elements. If at this step causal conflicts or non-solvable causal loops appear, repeat the previous steps with another set of disjoint I/O power lines. If none of them solves the problem of causal conflicts or non-solvable causal loop appearance then the model is not invertible (Criterion 3) and the procedure stops.*
5. *For the energy storage elements assign a preferential integral causality if possible and propagate it through the junction structure as previously.*
6. *If some R-elements remain not causally determined then assign a causality to one and propagate as previously. Repeat this step until all the R-elements are causally determined.*
7. *If the bond graph is not completely causally determined assign a causality on a bond and propagate it as previously. Repeat this step until all the bond graph is causally determined.*

6.3.4 Notion of Analysis Levels

One of the main interests of structural analysis is to provide the designer with qualitative information on his model and to help him take decisions in the design process. The structural feature of the analysis, i.e., independent of any numerical value and, thus, before the numerical simulation, enables time to be saved by detecting in early

[11] A set of causal paths is said to be associated with a set of power lines if each junction belonging to the power lines is also passed through by at least one of the associated causal paths [33].

stages if the design problem is ill-posed. A practical objective of structural analysis is to verify the mathematical conditions of model inversion. Depending on the answer the designer will be able to question his design model or his specifications.

The bond graph language offers different levels of information depending on what one decides to read in the bond graph representation. For instance, the graphic reading leads to retrieval of some properties of the model structure independently of the mathematical forms of the phenomenon constitutive laws. A deeper reading enables "behavioral" properties to be obtained in the sense that, in addition to the graphic reading, the mathematical forms of the phenomenon constitutive laws and the way the phenomena are reticulated in the model clarify some properties.[12] Finally, if the numerical values of the parameters are introduced an even deeper analysis is possible either analytically or by simulation.

This inclusion of information levels is interesting from a chronological point of view in a design process. In fact, if a property is not verified at a level, it is not at the successive level at all. For instance, concerning non-invertibility of a model, if it is detected at the earliest stage (power lines – Criterion 1, or causal paths – Criterion 2, or junction structure solvability – Criterion 3), the designer will not spend time to go further in his design problem which will be known as ill-posed.

Moreover, this strategy has the advantage to provide the designer with a guide in his design process. In fact, if he assesses his model as sufficiently faithful to the studied system, the results of the analysis levels (bond graph structure, behavioral

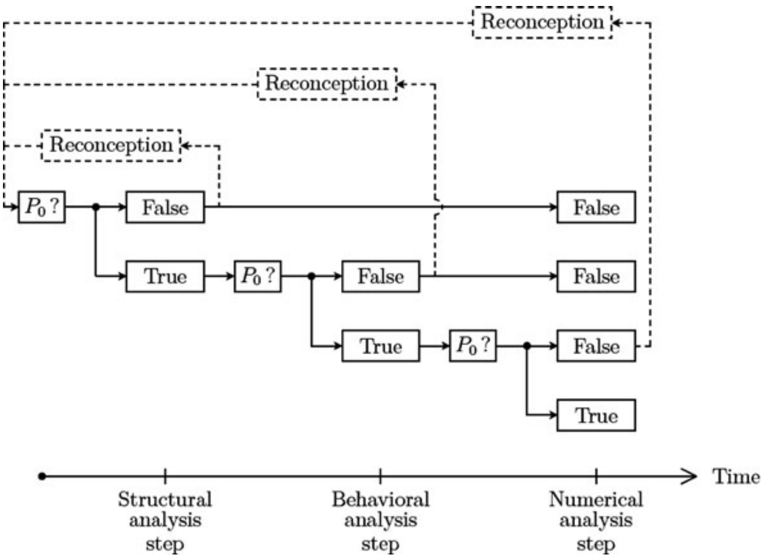


Fig. 6.9 Different analysis levels for the design process

[12] Note 5 illustrates this distinction between the information at the graphic structure level and the one at the behavioral structure level.

structure, numerical) can be extrapolated to the system design at the corresponding level (Fig. 6.9) [24]. For instance, if a property (invertibility or other) is not verified at the bond graph structure level, then the energy architecture of the system must be questioned. If it is verified at the bond graph structure level but not at the behavioral structure one, the designer can try to identify if it is due to a certain coupling between components in the system and if adding a phenomenon a priori neglected or eliminating one a priori not preponderant would solve the problem. At that point it must be emphasized that the bond graph representation offers an ideal tool to locate where the design possibilities are in the physical system. Finally, if a property is also verified at the behavioral structure level but not at the numerical one then the designer can work out the values for the parameters not yet fixed in the physical system or in the specifications.

6.4 Phases of the Sizing Methodology

The different phases of the sizing methodology are now presented. They are chronologically organized according to Fig. 6.1. For each phase care is taken to clearly present the objective, the inputs, and the unknowns of the posed problem. Steps of the methodology are then listed and the example (the same for all the phases) of an automotive application illustrates the methodology.

6.4.1 Validity of the Design Model

Today, in particular for complex systems like in mechatronics, design is largely based on virtual prototyping where the model has a central role. Here, a design model refers to a model that is at the basis of a synthesis work for what is to be designed. Early in the design process it is important to validate it. In an approach based on model inversion checking the validity of design models consists of checking their invertibility. The problem position of this phase is summarized in Table 6.1.

Table 6.1 Problem position of methodology phase 1 for the validity of the design model

Phase objective	To validate the model used in the successive phases of design
Inputs of the problem	• The load that the system to design actuates • The load inputs[a] • The specified outputs of the load
Unknowns of the problem	The system to design
Problem posed	To test the structural invertibility of the model

[a] To give the load inputs or outputs signifies that the quantities that play these roles in the model are identified but their time evolution, except if explicitly mentioned, is not necessarily known or given

Methodology – Phase 1 (Validity of the design model)

1. *Model the load.*[13]
2. *Build the acausal bond graph representation.*
3. *Apply Criterion 1 with respect to the inputs and the specified outputs of the problem.*
4. *If it is verified, assign the preferential integral causality to the bond graph representation, else the model is not structurally invertible and the phase stops.*
5. *Apply Criterion 2.*
6. *If it is verified, assign bicausality according to Procedure 5 to the bond graph representation, else the model is not structurally invertible and the phase stops.*
7. *Apply Criterion 3.*
8. *If it is verified, test the invertibility of the constitutive laws touched by bicausality or by a change of causality with respect to the initial causality assignment, else the model is not structurally invertible and the phase stops.*
9. *If one of the previously mentioned laws is not invertible the model is not invertible and the phase stops.*

The conclusion of this phase is that either the conditions are passed and the next phase can be carried out or they are not and the designer must question his design model by detecting at which step the invertibility test failed.

Example The illustrating application, sketched in Fig. 6.10, is an automotive vehicle in a braking situation on a straight trajectory. The model considered is planar and constitutes a chassis and two axles. The vehicle has longitudinal, heave, and pitch motions. Front and rear axles are composed each of a wheel and a suspension acting only vertically. The heave and pitch evolutions are supposed sufficiently small to consider a linear model with constant distances (wheelbases and mass center height). The road is assumed flat. The braking situation starts from an initial velocity and a constant deceleration distributed on both the axles is applied at a certain time after.

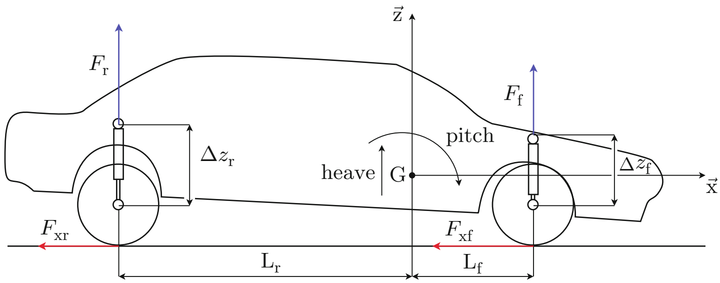


Fig. 6.10 Sketch of the automotive vehicle

[13] The modeling step consists here of setting up the physical hypotheses of the model.

In the context of riding comfort, the design problem is here to size both the front and rear suspensions with respect to given specifications on the heave and pitch behaviors for different decelerations and different front/rear braking distributions. To fulfill the requirements active suspensions are envisaged. The specified outputs are, respectively, the heave z and the pitch angle φ of the vehicle, and the inputs are both front F_f and rear F_r forces exerted by the suspensions. Table 6.2 summarizes the parameters for the vehicle considered as the load of the design problem and Fig. 6.11 shows three braking situations corresponding, respectively, to the three cases of deceleration: 0.3, 0.5, and 0.8 g. For each one three front/rear distributions (expressed in percentage of the specified deceleration) are studied: 93/69, 100/38, and 100/20.

The application of phase 1 requires the acausal bond graph representation given in Fig. 6.12. The acausal structural analysis gives eight I/O power lines between the suspension forces (F_f,F_r) and the heave and pitch velocities ($\dot{z}$, $\dot{\varphi}$).[14] Combining these I/O power lines, two sets of disjoint power lines exist; thus the first criterion is verified. Figure 6.12 shows one of them.

Then the preferential integral causality is assigned to give the causal bond graph representation in Fig. 6.13. The causal structural analysis results in four I/O causal paths, each of length 1, and two sets of I/O disjoint causal paths of length 2 which is minimal. The second criterion is also verified and Fig. 6.13 displays the set associated with that of Fig. 6.12 power lines.

Finally, the bicausal bond graph representation is obtained in Fig. 6.14. It has been obtained, on the one hand, from the set of the disjoint I/O causal paths of minimal length equal to 2 and, on the other hand, from the set of the associated disjoint

Table 6.2 Parameters of the automotive vehicle

Chassis	• Mass: $M_v = 1700\,\text{kg}$ • Moment of inertia around mass center: $I_v = 450\,\text{kg m}^2$ • Front wheelbase from mass center: $L_f = 1\,\text{m}$ • Rear wheelbase from mass center: $L_r = 1.7\,\text{m}$ • Mass center height: $h = 0.55\,\text{m}$
Front axle	• Wheel mass: $M_w = 33.7\,\text{kg}$ • Tire stiffness: $k_{tf} = 44{,}400\,\text{Nm}^{-1}$ • Tire structural damping: $b_{tf} = 1348\,\text{N}^{-1}(\text{ms})^{-1}$
Rear axle	• Wheel mass: $M_w = 33.7\,\text{kg}$ • Tire stiffness: $k_{tr} = 50{,}000\,\text{Nm}^{-1}$ • Tire structural damping: $b_{tr} = 1280\,\text{N}^{-1}(\text{ms})^{-1}$
Initial conditions	• Vehicle velocity: $V_{x0} = 36\,\text{ms}^{-1}$ • Front tire deflection: $Q_{f0} = -0.13\,\text{m}$ • Rear tire deflection: $Q_{r0} = -0.24\,\text{m}$

[14] All the analysis steps were undertaken with MS1 [23].

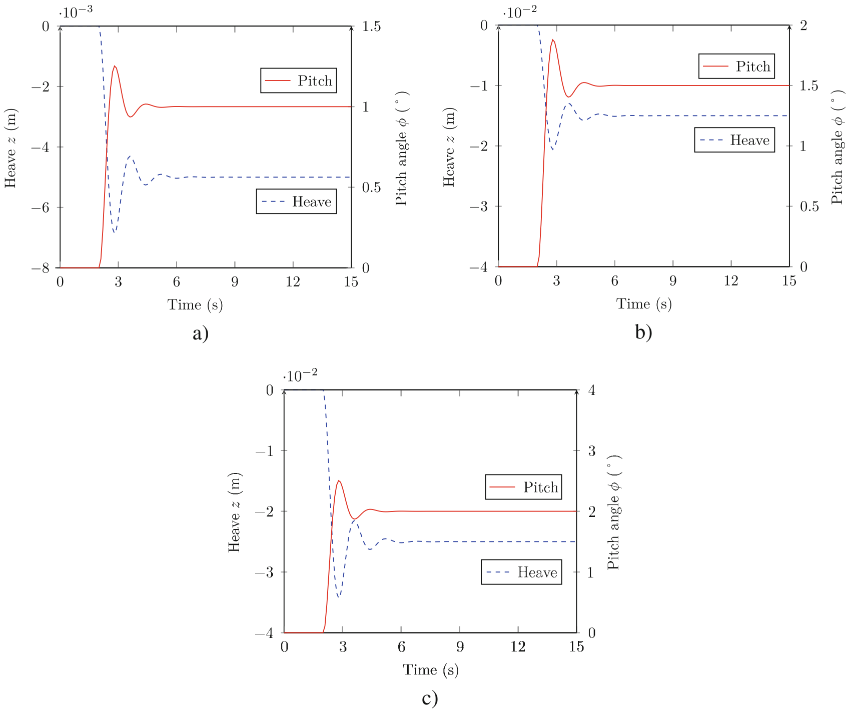


Fig. 6.11 Specifications of the design problem for a deceleration of **(a)** 0.3 g **(b)** 0.5 g **(c)** 0.8 g

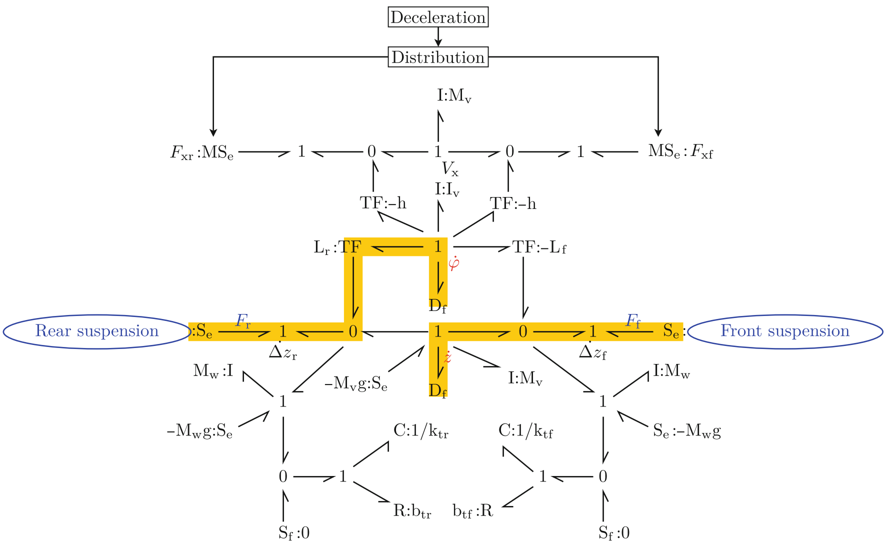


Fig. 6.12 Acausal bond graph representation of the automotive vehicle model

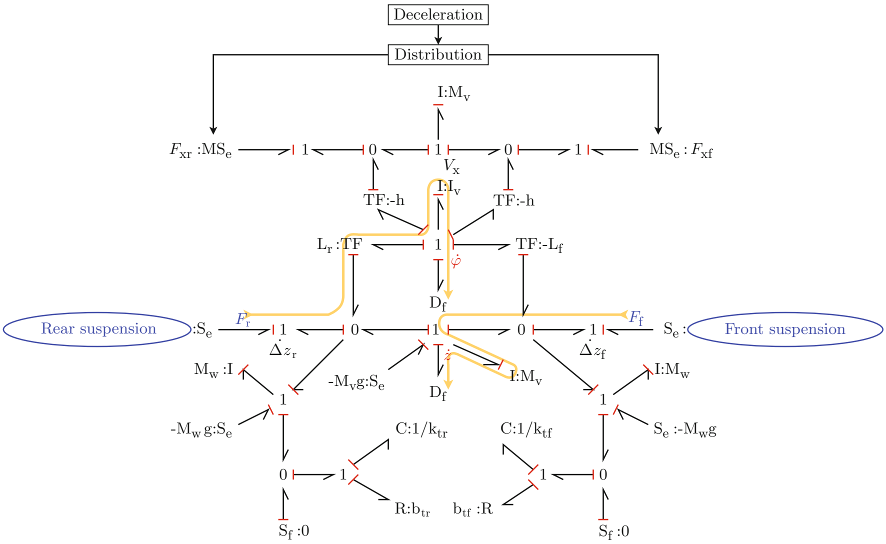


Fig. 6.13 Causal bond graph representation of the automotive vehicle model

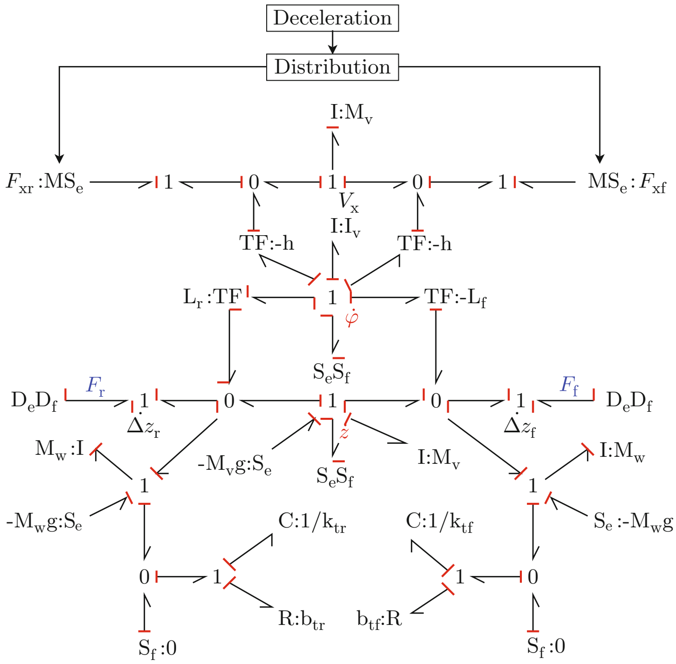


Fig. 6.14 Bicausal bond graph representation of the automotive vehicle model

I/O power lines. It can be easily verified in this bicausal bond graph representation that the junction structure is solvable. In conclusion of this phase the design model of the load (vehicle without its suspensions) is validated with respect to the pair of inputs (F_f,F_r) and the pair of outputs ($\dot{z}$, $\dot{\varphi}$). The design process can be pursued.

6.4.2 *Validity of the Specifications*

Once the design model is validated the next question concerns the specifications, in particular, the mathematical form of the specified outputs. The problem position of this phase is summarized in Table 6.3.

Methodology – Phase 2 (Validity of the specifications)

1. *Model the load.*
2. *Build the acausal bond graph representation.*
3. *Assign bicausality according to Procedure 5 to the bond graph representation with respect to the inputs and the specified outputs.*[15]
4. *For a specified output determine its essential order according to Procedure 4.*
5. *Apply Criterion 4.*
6. *If it is not verified, the phase stops, else repeat the previous two steps for all the outputs.*

If one or more specifications do not verify the differentiability criterion they must be first questioned, but the design model may also be discussed at this stage. On the contrary, if the design model is considered faithful to reality, the non-verification of this differentiability criterion signifies that the inputs are not capable of producing the specifications on the outputs. Unit pulses would appear which are physically not feasible.

Table 6.3 Problem position of the methodology phase for the validity of the specifications

Phase objective	To validate the specifications for the successive phases of design
Inputs of the problem	• The load that the system to design actuates • The load inputs • The specified outputs of the load • The mathematical definition of the specified outputs
Unknowns of the problem	The system to design
Problem posed	To test the necessary time derivative order for the specified outputs

[15] If the previous phase has been executed the different invertibility criteria have been verified and the bicausality is already assigned.

Example Inspection of Fig. 6.14 bicausal bond graph representation shows that among all the causal paths from the outputs $\dot{z}$ and $\dot{\varphi}$ to the inputs F_{f} and F_{r}, the lower orders are -1 for both outputs. Thus their essential orders are equal to 1 and the specifications for heave and pitch velocities must be at least C^1 functions. Compared to the data given in Fig. 6.11 it can be concluded that the specifications verify the differentiability criterion and the methodology phases can be pursued.

6.4.3 Component Specification and Selection

Once the well posedness of the design problem is checked in terms of the design model and the specifications, the next phase is to specify the component directly connected to the load (or augmented by the part of the system already sized in previous design steps). It uses the inverse model of the load in simulation (Fig. 6.15).

Contrary to the previous phases this one contributes directly to the system synthesis. In fact it consists of determining the specifications at the outputs of the components connected to the load (or augmented by the part of the system already sized in previous design steps) straight from the specifications given for the load outputs. The problem position is summarized in Table 6.4.

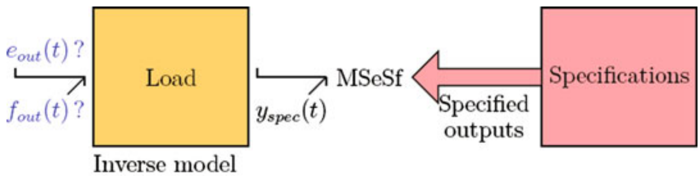


Fig. 6.15 Simplified sketch of the component specification phase

Table 6.4 Problem position of the methodology phase for the component specification and selection

Phase objective	To specify a component to size in the system to design
Inputs of the problem	• The load that the system to design actuates (or augmented by the part of the system already sized) • The load inputs (or of the part of the system already sized) • The specified outputs of the load • The mathematical definition of the specified outputs • The manufacturer's documentation of components
Unknowns of the problem	The component to size in the system to design
Problem posed	To calculate the specifications at the outputs of the component to design from the specifications of the load outputs

Methodology – Phase 3 (Component specification and selection)

1. *Model the load (possibly with the part of the system already sized).*
2. *Build the acausal bond graph representation.*
3. *Assign bicausality according to Procedure 5 to the bond graph representation with respect to the inputs and the specified outputs.*
4. *Simulate the obtained inverse model from the bicausal bond graph.*
5. *Compare the calculated conjugate power variables at the double detector ports to the data of the manufacturer's documentation.*

One practical result of this phase is a selection of components (if any exists) that satisfy the specifications, the so-called backward transported. If none has been selected the designer gets precise specifications to launch the design of a brand new component.

The approach based on the inverse model in the selection of components has several advantages. This enables, in one simulation run, relevant information to be obtained in the selection process. In fact the backward transportation takes into account the dynamic feature of the specifications. It clearly shows possible oversizing margins or, if manufacturer's data limits are crossed over, the amplitude and duration over the limits since the curves obtained are time parametrized. The duration over the limits can even be used in the case of sizing based on intermittent operation. Also, the simulation results are obtained in a completely independent way from what is still unknown in the system to design and from the control inputs of the system. Finally, the approach does not necessitate to take any a priori option on the component technology. Thus technology comparison can also be undertaken in an easy way. This selection phase can be summarized in Fig. 6.16.

Example The bicausal bond graph representation of the automotive example is given in Fig. 6.14. Figures 6.17, 6.18, and 6.19 show, for the different specified

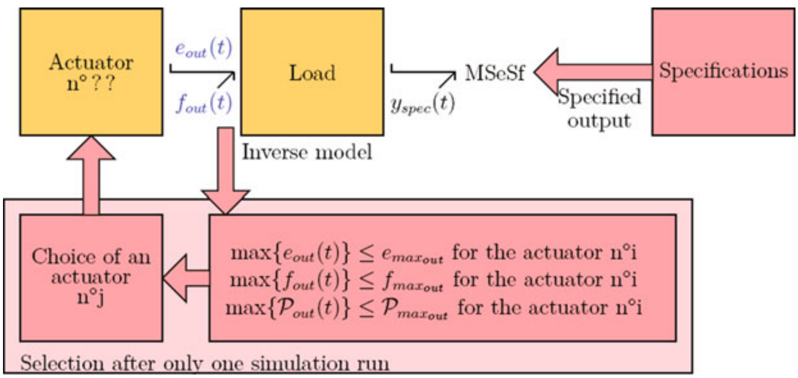


Fig. 6.16 Simplified sketch of component selection phase

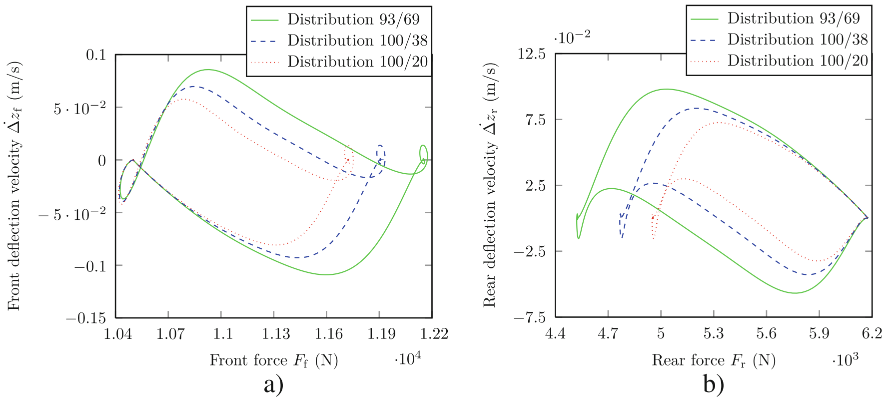


Fig. 6.17 Simulation results of the specifications backward transported for a deceleration of 0.3 g. **(a)** front suspension and **(b)** rear suspension

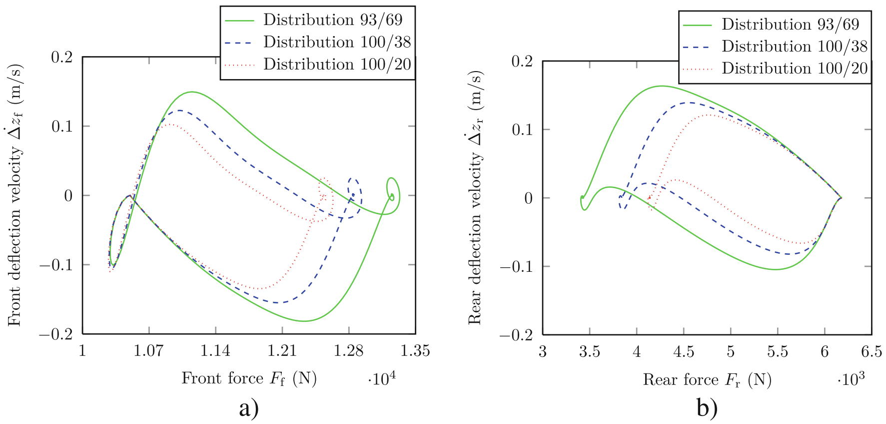


Fig. 6.18 Simulation results of the specifications backward transported for a deceleration of 0.5 g. **(a)** front suspension and **(b)** rear suspension

decelerations, the results in effort/flow frames of the conjugate power variables $(F_f, \Delta\dot{z}_f)$ and $(F_r, \Delta\dot{z}_r)$ required, respectively, for the front and rear suspensions,[16] where Δz_f and Δz_r denote their deflection. It also gives in this way information about power demands.

Examples of manufacturer's component limits are superimposed on Fig. 6.19 curves and show that they both satisfy the backward transported specifications.

[16] All the simulations were undertaken with MS1 and the solver ESACAP [22].

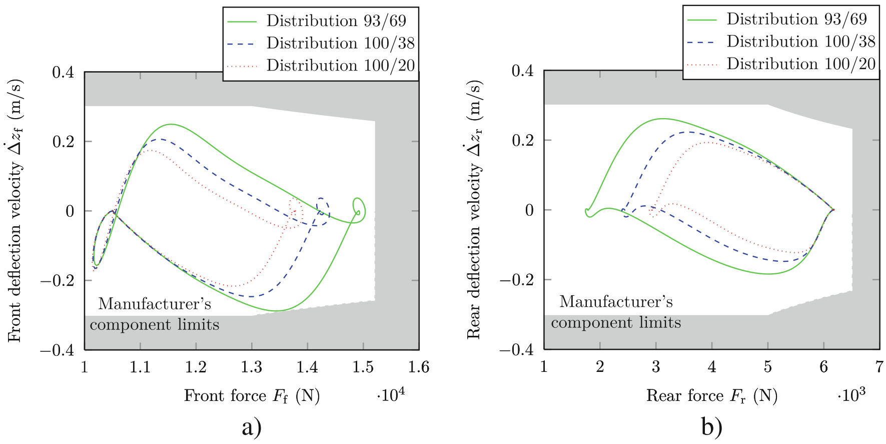


Fig. 6.19 Simulation results of the specifications backward transported for a deceleration of 0.8 g. **(a)** front suspension and **(b)** rear suspension

6.4.4 Selected Component Validation

Starting from the previous component selection the next phase is to completely validate these components by taking into account their dynamic behavior and their parameters. This necessitates to model them and to reconsider the two former phases, but this time on the set constituted by the load and the component that is being validated. Then the inverse model of this set, fed with the load output specifications, can be simulated. This simulation furnishes the conjugate power variables at the set inputs and anywhere in the model, in particular, the variables of the component that is being validated (Fig. 6.20). The problem position of this phase is summarized in Table 6.5.

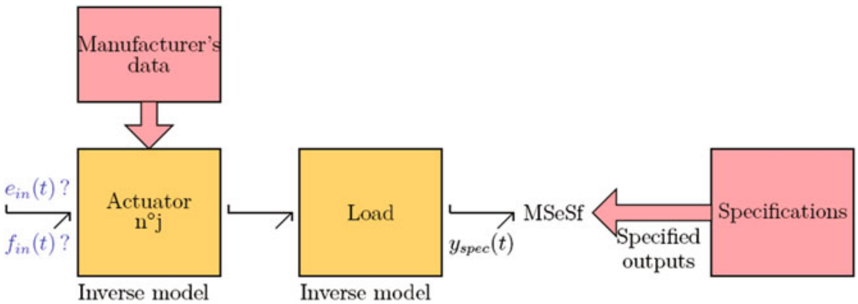


Fig. 6.20 Simplified sketch of the selected component validation phase

Table 6.5 Problem position of the methodology phase for the selected component validation

Phase objective	To validate a selected component in the system to design
Inputs of the problem	• The load that the system to design actuates (or augmented of the part of the system already sized) • The component being validated (model, parameters, manufacturer's data) • The inputs of the component being validated • The specified outputs of the load • The mathematical definition of the specified outputs
Unknowns of the problem	The rest of the system to design
Problem posed	To calculate the variables of the component being validated, in particular its inputs, from the specifications of the load outputs

Methodology – Phase 4 (Selected component validation)

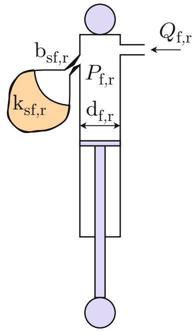
1. *Model the load (possibly with the part of the system already sized).*
2. *Build the acausal bond graph representation.*
3. *Assign bicausality according to Procedure 5 to the bond graph representation with respect to the inputs and the specified outputs.*
4. *Simulate the obtained inverse model from the bicausal bond graph.*
5. *Compare the calculated conjugate power variables at the double detector ports and the variables of the component being validated to the data of the manufacturer's documentation.*

For the component being validated if any variable is over the manufacturer's data limits then the component is not validated and taken away from the previous selection. From the results it is possible to know precisely the reason (variables, amplitude, duration, etc.) why a component is not validated. If no component is validated, like for the previous phase, it will be possible to clearly specify the design of a brand new component.

On the contrary, in the case when several selected components are validated, either other criteria like mass, dimension, and cost can be introduced to achieve the sizing process or all the validated components can be kept for the successive phases. In particular, the latter case provides degrees of freedom for the successive sizing phases.

Example In the set of the selected components obtained from the previous phase, the example of an active hydraulic suspension equipped with a sphere is taken for illustrating the validation phase (Fig. 6.21). A linear model of this component is considered with an equivalent stiffness, a dissipation through the orifice between the sphere and the chamber, and an ideal transduction between the hydraulic and the translational domains. The suspensions are supposed to be fed with volume flow rates considered now as the new control inputs of the overall system. The acausal

Fig. 6.21 Sketch of the active hydraulic suspension



bond graph representation is given in Fig. 6.22a and Table 6.6 shows the parameters of the selected components. They replace the effort sources associated with the forces F_f and F_r in Fig. 6.12 bond graph representation.

Then bicausality assignment (Fig. 6.22b) involves the verification of the invertibility and differentiability criteria. This step, not presented here, is supposed executed.

Figure 6.23 shows, in the case of a deceleration of 0.8 g, the simulation results concerning the input conjugate power variables (pressures and volume flow rates) (P_f, Q_f) and (P_r, Q_r), respectively, for the front and rear suspensions. The superimposition of the manufacturer's data limits indicates that the rear suspension is validated but not the front one. If, in the previous phase, another component was selected, the

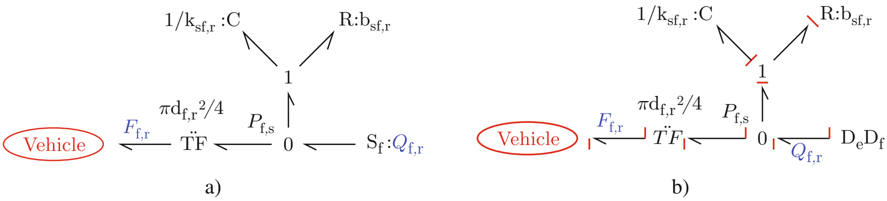


Fig. 6.22 (**a**) Acausal and (**b**) bicausal bond graph representations of the active hydraulic suspensions

Table 6.6 Parameters of the active hydraulic suspensions

Front suspension	• Sphere equivalent stiffness: $k_{sf} = 2.27 \times 10^{10}$ Pa m^{-3} • Viscous damping coefficient: $b_{sf} = 5 \times 10^9$ Pa (m^3 s^{-1})$^{-1}$ • Piston diameter: $d_f = 19.5$ mm • Initial sphere hydraulic volume: $V_{sf0} = 1.549 \times 10^{-3}$ m^3
Rear suspension	• Sphere equivalent stiffness: $k_{sr} = 3.36 \times 10^{10}$ Pa m^{-3} • Viscous damping coefficient: $b_{sr} = 4.3 \times 10^9$ Pa (m^3 s^{-1})$^{-1}$ • Piston diameter: $d_r = 21.3$ mm • Initial sphere hydraulic volume: $V_{sf0} = 0.516 \times 10^{-3}$ m^3

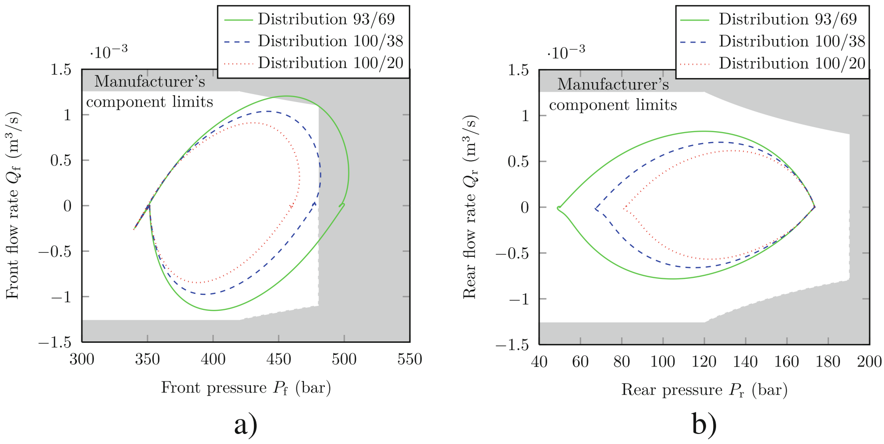


Fig. 6.23 Simulation results of the validation phase for the (**a**) front suspension and (**b**) rear suspension

validation phase can be repeated. If none other exists the design of a new suspension can be envisaged.

6.4.5 Open-Loop Control Determination

The last phase presented concerns the determination of the open-loop control when all the power components of the system to design have been sized (Fig. 6.24). The problem position is summarized in Table 6.7.

Methodology – Phase 5 (Open-loop control determination)

1. *Model the load and the components of the actuating system.*
2. *Build the acausal bond graph representation.*
3. *Assign bicausality according to Procedure 5 to the bond graph representation with respect to the control inputs and the specified outputs.*
4. *Simulate the obtained inverse model from the bicausal bond graph.*

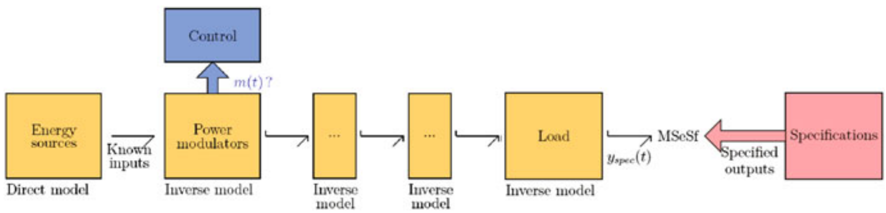


Fig. 6.24 Simplified sketch of the open-loop determination phase

Table 6.7 Problem position of the methodology phase for the open-loop control determination

Phase objective	To determine the open-loop control to follow the specified outputs
Inputs of the problem	• The load and the sized components of the system actuating the load (models, parameters) • The control inputs • The specified outputs of the load • The mathematical definition of the specified outputs
Unknowns of the problem	The time functions of the control inputs
Problem posed	To calculate the control inputs from the specifications of the load outputs

The simulation of the inverse model furnishes the ideal inputs to follow the specifications on the outputs. Then a control law synthesis can be started on this basis to increase the performance of the system. An interesting use of this approach is also the determination of power assistance laws. For instance, in the domain of automotive applications the determination of the power steering assistance (hydraulic or electrical) is of prime importance in the vehicle design [29, 30].

Example On the automotive example, if the volume flow rates Q_f and Q_r are considered as the control inputs and if both suspensions in the previous phase are conserved through the non-validation of the front one, Fig. 6.25 extracts the simulation already obtained in Fig. 6.23, but now displayed with respect to time.

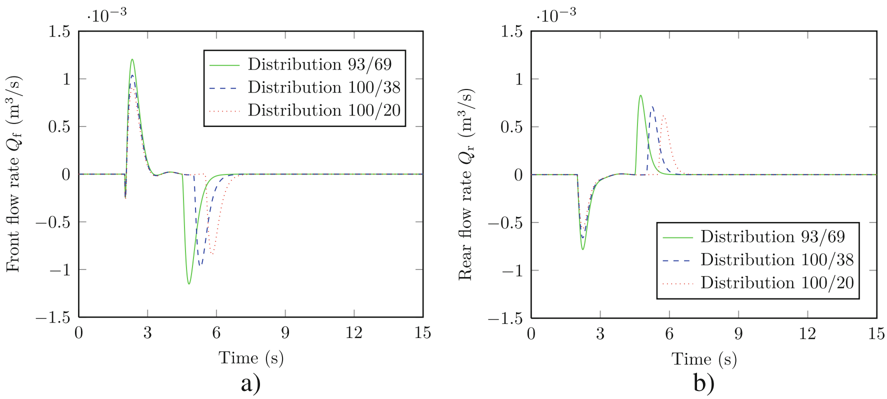


Fig. 6.25 Simulation results of the open-loop control determination for the (a) front suspension and (b) rear suspension

6.5 Conclusion

This chapter presented the use of the bond graph language for a sizing methodology based on model inversion. It first gave the theoretical material for the manipulated concepts and their translation or determination in a bond graph representation. Then the tools for testing the existence of an inverse model and for organizing the equations were presented. Finally, the phases of the methodology were proposed. These phases were chronologically presented; however, depending on the design problem, one can directly come to the one of one's own interest.

One of the main advantages of the approach based on model inversion is the fact that it poses the design problem in such a way that the specifications can be straightforwardly used. In fact, at each phase, the model manipulation needs no information about what is still unknown. Thus this approach saves time in the design process by decreasing the number of calculus iterations. Furthermore, it brings more insight into the design, in particular by increasing the pertinence of the component sizing (dynamic specifications, oversizing margins, amplitude and duration over the manufacturer's data limits, etc.). If a solution exists for the design problem the designer has the guarantee to have it in the selection phase. Also, when no component satisfies the load output requirements, the designer still gets precise specifications for the design of new components.

The bond graph language shows to be well adapted for the approach presented. The different phases of the methodology get the benefits of all the bond graph features (energy based, multidomains, multitechnologies, graphic, etc.). Moreover, the different levels of information lead to different levels of analysis for the designer. The first levels are structural and enable detection of whether or not the design problem is ill-posed in an early stage of the design process. Also, still depending on the information level, the analysis can guide the designer to question his problem and, in particular, if a solution is to be searched at a structural, behavioral, or parameter level in the system to design.

On the basis of the concepts developed in the former sections, the latter section showed a series of design problems. The used approach can also be interesting for problems like system architecture synthesis and comparison [28], parameter synthesis [16], equilibrium or steady-state position determination [4], or the coupling of model inversion with dynamic optimization [24, 26, 27, 32]. Finally, the approach was used in the domain of active systems [31], in industrial applications like in aeronautics for electro-hydraulic actuators [17] or in automotive for electric power steering and suspension systems [29, 30], and for classic and hybrid power trains [3, 28].

References

1. Bertrand, J.M. *Analyse structurelle et commande par découplage entrée-sortie des modèles bond graphs*. PhD thesis, Université des sciences et technologies de Lille, Villeneuve-d'Ascq, France, 1997. 164 p.

2. Bertrand, J.M., Sueur, C., Dauphin-Tanguy, G. On the finite and infinite structures of bond-graph models. In *IEEE International Conference on Systems, Man and Cybernetics*, volume 3, pages 2472–2477, Orlando, FL, October, 1997.
3. Bideaux, E., Laffite, J., Derkaoui, A., Marquis-Favre, W., Scavarda, S., Guillemard F. Design of a hybrid vehicle powertrain using an inverse methodology. In *Conference Proceeding of Power Transmission and Motion Control*, Bath, Great Britain, September, 7–9 2005.
4. Bideaux, E., Marquis-Favre, W., Scavarda, S. Equilibrium set investigation using bicausality. In *Mathematical and Computer Modelling of Dynamical Systems. Special Issue on Bond Graph Modelling*, volume 12, pages 127–140, April–June, 2006.
5. Brockett, R.W. Poles, zeros, and feedback: State space interpretation. *IEEE Transactions on Automatic Control*, 10(2):129–135, April, 1965.
6. Brockett, R.W., Mesarović, M.D. The reproducibility of multivariable systems. *Journal of Mathematical Analysis and Applications*, 11:548–563, 1965.
7. Commault, C., Descusse, J., Dion, J.M., Lafay, J.F., Malabre, M. New decoupling invariants: the essential orders. *International Journal of Control*, 44(3):689–700, September, 1986.
8. Dauphin-Tanguy, G. *Les bond graphs*. Traité IC2 Information – Commande – Communication. Hermès Sciences, Paris, 2000.
9. Dequidt, A. *Contribution à une approche interdisciplinaire de la conception des systèmes mécaniques commandés – application aux axes des machines*. PhD thesis, Université de Valenciennes, Valenciennes, France, 1998.
10. Dequidt, A., Castelain, J.-M., Valdès, E. Mechanical pre-design of high performance motion servomechanisms. *Mechanism and Machine Theory*, 35(8):1047–1063, August, 2000.
11. Descusse, J., Dion, J.M. On the structure at infinity of linear square decoupled systems. *IEEE Transactions on Automatic Control*, 27(4):971–974, August, 1982.
12. Dorato, P. On the inverse of linear dynamical systems. *IEEE Transactions on Systems Science and Cybernetics*, 5(1):43–48, January, 1969.
13. El Feki, M., Jardin, A., Marquis-Favre, W., Krähenbühl, L., Bideaux, E., Thomasset, D. Determination of essential orders from a bond graph model. *Journal of Dynamic System Measurement and Control*, 2010, (Submitted).
14. Fliess, M. A note on the invertibility of nonlinear input-output differential systems. *System & Control Letters*, 8(2):147–151, December, 1986.
15. Fotsu-Ngwompo, R. *Contribution au dimensionnement des systèmes sur des critères dynamiques et énergétiques – approche par Bond Graph*. PhD thesis, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 1997.
16. Fotsu-Ngwompo, R. Développement d'une méthodologie de dimensionnement des éléments d'un système : application au cas d'une suspension hydropneumatique. Technical report, Laboratoire d'Automatique Industrielle, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 1998.
17. Gandanegara, G., Roboam, X., Charles Maré, J., Sareni, B. Synthesis of an electro hydrostatic actuator from bicausal bond graphs. In *Proceedings of the International Conference on Bond Graph Modeling and simulation*, pages 71–78, San Diego, CA, January, 2007.
18. Gawthrop, P.J. Bicausal bond graphs. In *Proceedings of the International Conference on Bond Graph Modeling and simulation*, pages 83–88, Las Vegas, NV, January 1995.
19. Gilbert, E.G. Decoupling of multivariable systems by state feedback. *SIAM Journal of Control*, 7(1):50–63, February, 1969.
20. Glumineau, A., Moog, C.H. Nonlinear morgan's problem: Case of $(p + 1)$ inputs and p outputs. *IEEE Transactions on Automatic Control*, 37(7):1067–1072, July, 1992.
21. Hirschorn, R.M. Invertibility of multivariable nonlinear control systems. *IEEE Transactions on Automatic Control*, 24(6):855–865, December, 1979.
22. http://www.ecs.dtu.dk/esacap.htm. ESACAP Simulation Program.
23. http://www.lorsim.be. MS1 software.
24. Jardin, A. *Contribution à une méthodologie de dimensionnement des systèmes mécatroniques : analyse structurelle et couplage à l'optimisation dynamique*. PhD thesis, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 2010.

25. Jardin, A., El Feki, M., Marquis-Favre, W., Thomasset, D., Bideaux, E. Use of structural analysis in a bond graph-based methodology for sizing mechatronic systems. In *Proceedings of the 7th edition of France-Japan, 5th Europe-Asia Congress on Mechatronics*, Le Grand Bornand, France, May, 2008.
26. Jardin, A., Marquis-Favre, W., Thomasset, D. Représentation bond graph de problèmes d'optimisation dynamique : extension aux systèmes à stockages d'énergie non-linéaires. In *Proceedings of the CIFA Conference*, Bucarest, Roumania, September, 3–5 2008.
27. Jardin, A., Marquis-Favre, W., Thomasset, D. Bond graph sizing of mechatronic systems: Coupling of inverse modelling with dynamic optimization. In *The 6th Vienna International Conference on Mathematical Modelling*, pages 1929–1938, Vienna, Austria, February, 11–13 2009.
28. Laffite, J. *Aide à la conception et au dimensionnement énergétique et dynamique de systèmes mécatroniques par une approche inverse : application aux chaînes de transmission automobiles*. PhD thesis, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 2004.
29. Méchin, O. *Conception des systèmes de la liaison au sol : contribution à la définition du cahier des charges*. PhD thesis, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 2003.
30. Méchin, O., Marquis-Favre, W., Scavarda, S., Ferbach, P. Sizing of an electric power steering system on dynamic and energetic criteria. In *Proceedings of the International Conference on Bond Graph Modeling and Simulation*, pages 137–147, Orlando, FL, January, 2003.
31. Miled, F. *Contribution à une méthodologie de conception des systèmes dynamiques actifs*. PhD thesis, Université de Franche-Comté et Université de Technologie de Belfort-Montbéliard, France, 2003.
32. Mouhib, O., Jardin, A., Marquis-Favre, W., Bideaux, E., Thomasset, D. Optimal control problem in bond graph formalism. *Simulation Modelling Practice and Theory*, 17(1):240–256, January, 2009.
33. Ngwompo, R.F., Bideaux, E., Scavarda, S. On the role of power lines and causal paths in bond graph-based model inversion. In *Proceedings of the International Conference on Bond Graph Modeling and Simulation*, pages 78–85, New Orleans, LA, January, 2005.
34. Ngwompo, R.F., Scavarda, S., Thomasset, D. Physical model-based inversion in control systems design using bond graph representation, part 1: theory. *Proceedings of ImechE, Part I: Journal of Systems and Control Engineering*, 215(2):95–103, 2001.
35. Nijmeijer, H. Invertibility of affine nonlinear control systems: a geometric approach. *System & Control Letters*, 2(3):163–168, October, 1982.
36. Nijmeijer, H. Right invertibility for a class of nonlinear control systems: a geometric approach. *System & Control Letters*, 7(2):125–132, April, 1986.
37. Perdon, A.M., Conte, G., Longhi, S. Invertibility and inversion of linear periodic systems. *Automatica*, 28(3):645–648, May, 1992.
38. Porter, W.A. Decoupling of and inverses for time-varying linear systems. *IEEE Transactions on Automatic Control*, 14(4):378–380, August, 1969.
39. Potkonjak, V., Jaksic, N. Contribution to a computer-aided choice of d.c. motors for manipulation robots. *Robotica*, 4:37–41, 1986.
40. Rahmani, A. *Etude structurelle des systèmes linéaires par l'approche bond graph*. PhD thesis, Université des Sciences et Technologies de Lille, Lille, France, 1993.
41. Rahmani, A., Sueur, C., Dauphin-Tanguy, G. Formal determination of controllability/observability matrices for multivariable systems modelled by bond graph. In *Proceedings of IMACS/SICE International Symposium of Robotics, Mechatronics and Manufacturing System*, pages 573–580, Kobe, Japon, September, 1992.
42. Richard, J.P. *Algèbre et analyse pour l'automatique*. Number ISBN 2-7462-0198-4 in Systèmes automatisés : information – Commande – Communication. Hermès Sciences, Paris Science, 2001.
43. Rosenbrock, H.H. *State-space and multivariable theory*. Thomas Nelson and sons LTD, Great Britain, 1970.

44. Rosenbrock, H.H., van der Weiden, A.J.J. Inverse systems. *International Journal of Control*, 25(3):389–392, March, 1977.
45. Sain, M.K., Massey, J.L. Invertibility of linear time-invariant dynamical systems. *IEEE Transactions on Automatic Control*, 14(2):141–149, April, 1969.
46. Silverman, L.M. Properties and application of inverse systems. *IEEE Transactions on Automatic Control*, 13(4):436–437, August, 1968.
47. Silverman, L.M. Inversion of multivariable linear systems. *IEEE Transactions of Automatic Control*, 14(3):270–276, June, 1969.
48. Singh, S.N. A modified algorithm for invertibility in nonlinear systems. *IEEE Transactions on Automatic Control*, 26(2):595–598, April, 1981.
49. Sueur, C., Dauphin-Tanguy, G. Poles and zeros of multivariable linear systems: a bond graph approach. In G. Dauphin-Tanguy, P.C. Breedveld, editors, *IMACS Transactions, Bond Graph for Engineers*, pages 211–228. Elsevier Science, North-Holland, 1992.
50. Tan, S., Vandewalle, J. Inversion of singular systems. *IEEE Transactions on Circuits and Systems*, 35(5):583–587, May, 1988.
51. Vardulakis, A.I.G., Limebeer, D.J.N., Karcanias, N. Structure and Smith-MacMillan form of a rational matrix at infinity. *International Journal of Control*, 35(4):701–725, April, 1982.
52. Vukobratovic, M., Katic, D., Potkonjak, V. Computer-assisted choice of electrohydraulic servosystem for manipulation robots using complete mathematical models. *Mechanism and Machine Theory*, 22(5):431–439, 1987.
53. Weiss, L. On a question related to the control of linear systems. *IEEE Transactions on Automatic Control*, 9(2):176–177, April, 1964.
54. Willsky, A.S. On the invertibility of linear systems. *IEEE Transactions on Automatic Control*, 19(3):272–274, June, 1974.
55. Wu, S.T., Youcef-Toumi, K. On relative degrees and zero dynamics from physical system modeling. *Journal of Dynamic Systems, Measurement, and Control*, 117(2):205–217, 1995.
56. Youla, D.C., Dorato, P. On the inverse of linear dynamical systems. Technical Report PIBMRI-1319-66, Polytechnic Institute of Brooklyn, New York, NY, March, 1966.
57. Zadeh, L.A., Desoer, C.A. Linear systems theory: The state space approach. In W. Linvill, L.A. Zadeh, and G. Dantzia, editors, *Series in Systems Science*, page 628. McGraw-Hill, New York, NY, 1963.