Optimal second order reduction basis selection for nonlinear transient analysis

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Abstract Effective Model Order Reduction (MOR) for geometrically nonlinear structural dynamics problems can be achieved by projecting the Finite Element (FE) equations on a basis constituted by a set of vibration modes and associated second order modal derivatives. However, the number of modal derivatives generated by such approach is quadratic with respect to the number of chosen vibration modes, thus quickly making the dimension of the reduction basis large. We show that the selection of the most important second order modes can be based on the convergence of the underlying linear modal truncation approximation. Given a certain time dependency of the load, this method allows to select the most significant modal derivatives set before computing it.

1 Introduction

Several advanced structural systems often exploit nonlinear geometrical effects rather than avoiding them. Large displacements are unavoidable for aeronautics and aerospace structures for which the minimum weight is a key design factor. It is therefore important to consider nonlinearity even in the early design stage by appropriate modeling practice. However,the nonlinear dynamic analysis of large finite element (FE) models is an onerous task, and effective Model Order Reduction (MOR) techniques are widely welcomed.

The vibration mode superposition has become a standard practice for linear dynamics problems. However, its straightforward extension to geometrical nonlinear dynamic analysis poses some problems. In principle, to reduce the number of degrees of freedom, several vibration modes can be extracted at certain dynamic equilibrium states and used to project the dynamic set of equations on. This approach

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bears the drawback of recomputing the modal basis too frequently to preserve accuracy,and the effectiveness of the method is often lost [6].

A major issue in nonlinear structural applications is the bending-stretching coupling arising from the finite out-of-place displacements a slender or thin-walled structure exhibits during operation. Typically, low frequency vibration modes are bending dominated and they do not contain the proper membrane displacement contribution that is necessary to accurately represent the effect of the nonlinearity. Various workarounds have been proposed. A selection of modes, dual or companion modes, has been proposed, relying on membrane static displacements induced by specified transverse displacements directly related to the bending modes used. These modes, representing the membrane behavior resulting from bending are considered in the implicit condensation strategy (using companion modes) presented by [3] and the full modeling approach usingdual modes introduced by [7]. Both methods use bending modes in the basis. The implicit condensation method uses only bending modes in the basis and extracts membrane modes through an assumed quadratic functional form, while the full modeling method using dual modes adds modes to the basis that directly describe the membrane response. A different procedure using dynamic analysis to calculate companion modes is proposed by [8]. They found that very good results are obtained using a basis formed by a combination of bending vibration modes and membrane vibration modes. Forming the optimal basis could require the extraction of many modes, and also the selection of the most significant ones is an issue. Moreover, for more complicated structures, a categorization in bending modes and membrane modes might not be possible.

Past contributions [4, 5] have outlined the potential of including higher order modes to enrich the modal basis and avoid expensive recomputing of the modal basis during the time integration. Essentially, the modal derivatives are calculated by differentiating the eigenvalue problem or alternatively by using finite differences [10]. An interesting application of modal derivatives can be found in the computer-graphics world, [1] although the attention is more focused on a realistic representation of the deformation rather than on the actual accuracy of the reduced solution. Methods based on higher order modes are a natural extension of linear modal superposition analysis. Once a good modal basis has been formed for the underlying linear system, the corresponding second order modes are directly generated from this basis and provide the displacement contributions that are essential to represent the effects of the nonlinearity.

The number of modal derivatives is quadratic with respect to the size of the size of the generating set of vibration modes. However, not all the modal derivatives are relevant to the accuracy of the reduced response. We present in this contribution a simple and effective selection criterion that allows to choose the most significant modal derivatives before computing the actual shapes. The criterion is based on the convergence of the modal truncation of the underlying linearized problem.

2 Reduced equations of motion

We consider here the discretized N FE nonlinear dynamic equations of motion of a general tridimensional structure. For this study, we do not consider damping and we assume that the external force term can be expressed as a constant shape scaled by a time dependent function. The governing FE system of equations, together with the boundary conditions, writes:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}\varphi(t) \\ \mathbf{u}(0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \end{cases} \tag{1}$$

where $\mathbf{u}$ is the generalized displacement vector, $\mathbf{M}$ is the mass matrix, $\mathbf{g}(\mathbf{u})$ is the nonlinear force vector and $\mathbf{f}$ is the applied load shape multiplied by the time function $\varphi(t)$. The initial conditions for the displacement and the velocity vector are indicated with $\mathbf{u}_0$ and $\dot{\mathbf{u}}_0$, respectively. We further assume that the nonlinearity of $\mathbf{g}(\mathbf{u})$ is caused by geometrical effects only, i.e. when the displacements are so large that a linear kinematic model does not hold. This is typically the case of thin-walled structures, which can undergo large displacements while staying in the elastic range of the material.

In practical applications, the system of N equations 1 is usually large. The number of unknowns can be reduced to M, with $M << N$, by projecting the displacement field $\mathbf{u}$ on a suitable basis Ψ of time-independent vectors, as:

$$\mathbf{u} = \Psi\mathbf{q} \tag{2}$$

where $\mathbf{q}(t)$ is the $M \times 1$ vector of modal amplitudes. The governing equations can then be projected on the chosen basis Ψ in order to make the residual orthogonal to the subspace in which the solution $\mathbf{q}$ is sought. This results in a reduced system of M non-linear equations:

$$\Psi^T\mathbf{M}\Psi\ddot{\mathbf{q}}(t) + \Psi^T\mathbf{g}(\Psi\mathbf{q}) = \Psi^T\mathbf{f}\varphi(t) \tag{3}$$

or, equivalently,

$$\hat{\mathbf{M}}\ddot{\mathbf{q}}(t) + \hat{\mathbf{g}}(\Psi\mathbf{q}) = \hat{\mathbf{f}}\varphi(t) \tag{4}$$

We refer to the numerical solution of the full model as the *full* solution, while the solution of the reduced model will be called *reduced* solution. The key of a good reduction method is to find a suitable basis Ψ that is able to reproduce the full solution with a good, hopefully controlled, accuracy.

3 Reduction basis

We discuss is this section how to form the reduction basis Ψ. We propose a basis of vibration modes calculated around a given equilibrium position $\mathbf{u}_{eq}$ enriched by the so-called modal derivatives (MD). The two contributions will be separately discussed.

3.1 Vibration modes

Let us consider a static equilibrium position $\mathbf{u}_{eq}$ when the applied load is constant and given by $\mathbf{f}\varphi_{eq}$. We can then linearize the system of equations 1 around such configuration assuming that the motion $\tilde{\mathbf{u}}$ around $\mathbf{u}_{eq}$ is small, i.e. $\mathbf{u} = \mathbf{u}_{eq} + \tilde{\mathbf{u}}$, $\ddot{\mathbf{u}} = \ddot{\tilde{\mathbf{u}}}$. The linearized dynamic equilibrium equations become:

$$\mathbf{M}\ddot{\tilde{\mathbf{u}}} + \mathbf{K}_{eq}\tilde{\mathbf{u}} = \mathbf{f}\tilde{\varphi}(t) \tag{5}$$

where $\tilde{\varphi}(t)$ is a small load variation from φ_{eq}. The tangent stiffness matrix $\mathbf{K}_{eq}$ is defined as:

$$\mathbf{K}_{eq} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}}\bigg|_{\mathbf{u}=\mathbf{u}_{eq}} \tag{6}$$

the associated eigenvalue problem to equation 5 writes:

$$\left(\mathbf{K}_{eq} - \omega_i^2 \mathbf{M}\right) \Phi_i = \mathbf{0}, \quad i = 1, 2, \dots, N \tag{7}$$

and its solution provides N vibration modes (VMs) Φ_i and associated vibration frequencies ω_i^2.

3.2 Modal derivatives

The projection of the governing equations on a reduction basis formed by a reduced set of VMs is a well-known technique for linear structural dynamics. The main advantage of this technique is that the resulting reduced model consists of a system of uncoupled equations that can therefore be solved separately. As discussed in the introduction, several attempts has been made to extend the vibration modes projection for nonlinear analysis. The main limitation of such approach lies in the fact that the vibration basis changes as the configuration of the system changes. It is therefore required to upgrade the basis during the numerical time integration to account for the effect of the nonlinearity.

For thin-walled structural applications as the one considered in this contribution, the system is usually characterized by significant nonlinear bending-stretching coupling effects that are usually not captured by a reduction basis formed by linear vibration modes only.

Since the VMs change with respect to the configuration, a natural way of accounting for the main effect of nonlinearity is to compute their derivatives with respect to the configuration around the reference state $(\mathbf{u}_{eq}, \varphi_{eq})$ at which the modes are calculated. The direction for the derivatives are provided by the VMs chosen for the linear analysis. We want to compute the modal derivatives (MDs) Φ_{ij}:

$$\Phi_{ij} = \frac{\partial \Phi_i}{\partial q_j} \tag{8}$$

where q_j is the amplitude associated to Φ_j. In other words, we would like to know how a certain mode Φ_i changes if the structure is displaced according to the shape described by mode Φ_j.

A way to proceed is to differentiate the eigenvalue problem 7 with respect to the modal amplitudes.

$$\left[\mathbf{K}_{eq} - \omega_i^2 \mathbf{M}\right] \frac{\partial \Phi_i}{\partial q_j} + \left[\frac{\partial \mathbf{K}_{eq}}{\partial q_j} - \frac{\partial \omega_i^2}{\partial q_j}\mathbf{M}\right] \Phi_i = \mathbf{0} \tag{9}$$

It has been shown by [10] and [5] that the terms associated to the mass can be neglected. The derivative $\frac{\partial \omega_i^2}{\partial q_j}$ is zero since the frequency variation should not be an odd function of the modal amplitude. The term $\omega_i^2 \mathbf{M} \frac{\partial \Phi_i}{\partial q_j}$ is related to the inertial forces associated to the second order mode Φ_{ij}. Since these terms are typically featuring in-plane deformations (at least for thin-walled structures), their contribution is negligible. Numerical experiments have shown that the neglecting of mass related terms does not change the results appreciatively. By doing so, the problem 9 becomes:

$$\mathbf{K}_{eq} \frac{\partial \Phi_i}{\partial q_j} = -\frac{\partial \mathbf{K}_{eq}}{\partial q_j} \Phi_i \tag{10}$$

The right-hand side pseudo-force can be calculated at element level an then assembled. It can be shown that the modal derivatives are symmetric, i.e. $\Phi_{ij} = \Phi_{ji}$. Therefore, given M vibration modes, $R = M(M+1)/2$ MD can be calculated. Note that the matrix of coefficients can be factorized once for all and only the right-hand sides need to be computed. This can be done at element level and subsequently assembled, see for details [11].

3.3 Projection basis

Once a set of M VMs Φ_i are calculated by solving the eigenvalue problem 7, the MDs Φ_{ij} can be generated by solving the linear problems 10. A reduction basis including both VMs and MDs can be formed, as:

$$\Psi = [\Phi_i \ \Phi_{ij}] \ , \ i,j = 1,\ldots,M, \ \ \Phi_{ij} = \Phi_{ji} \tag{11}$$

This complete reduction basis provides a very accurate reduced model, as shown in the numerical example. However, the number of MDs that can be generated from a subset of M VMs is quadratic with respect to M. For complex problems when a rather large VMs basis is required, the number of MDs quickly becomes large. We would like to find a simple selection criterion that allows to form the best reduced MDs set for a given problem. Specifically, we would like to answer the following questions:

1. Given a certain time dependent load, is it possible to find the best P MDs, where $P < R$?
2. Can the selection be based on the convergence of the underlying linear dynamics problem, i.e. the convergence of the modal truncation based on the chosen M VMs?
3. Can the most significant MDs be selected before computing them?
4. Given a reduction basis, can an error bound be devised?

We show in this contribution that the first three questions can be answered. The selection criterion is discussed in the following section.

4 Selection criterion

Suppose we extracted a set of VMs $\Phi_i, \ i = 1,2,\ldots M$ with $M \ll N$, together with the associated frequencies ω_i. The approximation of the response $\mathbf{u}(t)$ of the linear system 5 is given by [2]:

$$\mathbf{u}(t) = \sum_{i=1}^{M} \frac{\Phi_i \Phi_i^T \mathbf{f}}{\mu_i} \int_0^t \frac{\sin(t-\tau)}{\omega_i} \varphi(\tau) d\tau = \sum_{i=1}^{k} \alpha_i \theta_i(t) \tag{12}$$

where $\mu_i = \Phi_i^T \mathbf{M} \Phi_i$ is the modal mass associated to the i^{th} VM. We dropped the $\tilde{}$ for clarity. The convergence of the modal truncation approximation thus depend on two distinct contributions, namely a *quasi-static* contribution and a *spectral* contribution. The first, associated to the spatial factors α_i:

$$\alpha_i = \frac{\Phi_i \Phi_i^T \mathbf{f}}{\mu_i} \tag{13}$$

implies that the load shape $\mathbf{f}$ has to be nearly orthogonal to the $N-M$ modes left out of the approximation. The spectral type is determined by the convergence to zero of the convolution products 12 when progressing in the eigenspectrum of the system. The temporal factors $\theta_i(t)$ depend in general on the frequency content of the system and of the applied load. Let us consider here two relevant cases:

- for a step load $\varphi(t) = 1$ for $t \geq 0$:

$$\theta_i(t) = \frac{1-\cos(\omega_i t)}{\omega_i^2} \tag{14}$$

- for a harmonic load $\varphi(t) = \cos(\Omega t)$:

$$\theta_i(t) = \frac{\omega_i \sin(\Omega t) - \Omega \sin(\omega_i t)}{\omega_i(\omega_i^2 - \Omega^2)} \tag{15}$$

We can now assume that the M VMs selected for a good accuracy for the linear problem 5 will interact when the nonlinearity is considered. However, if the nonlinearity is mild, they will still roughly contribute to the solution as indicated by 12. We can then further assume that the contribution of the MDs is of second order, and their mutual relevance could be indicated by:

- : for step load:

$$b_{step}^{ij} = \frac{|\alpha_i||\alpha_j|}{\omega_i^2 \omega_j^2} \tag{16}$$

- for harmonic load:

$$b_{harm}^{ij} = |\alpha_i||\alpha_j|\beta_i\beta_j \tag{17}$$

where the generic term $\beta_k(\Omega)$ is:

$$\beta_k(\Omega) = \left|\frac{1}{(\omega_k^2 - \Omega^2)}\right| + \left|\frac{\Omega}{\omega_k(\omega_k^2 - \Omega^2)}\right|$$

The factors b_{step}^{ij} and $b_{harm}^{ij}(\Omega)$ indicate the relative amplitude of the MD mode Φ_{ij} given the contribution of Φ_i and Φ_j to the linear solution. Note that $b_{harm}^{ij}(0) = b_{step}$. The selection of the best MDs subset can be based on the magnitude of the b_{ij} coefficients: the most relevant MD will be the ones with the highest values of the corresponding b_{ij} coefficients. This simple criterion is heavily based on the convergence properties of the linear reduced problem. This implicitly assumes that the nonlinearity effect is, in a mathematical sense, a second order effect. How far this concept can be stretched is a problem-dependent issue. Moreover, the actual interaction between the modes is not investigated: we are simply assuming that two VMs that bear a relevant part of the reduced solution are likely to interact a lot. The big advantage of this method lies in the fact that the most important contributions can be estimated prior the actual calculation of the MDs. In the following section we show the potential of this method by mean of a numerical example.

5 Numerical Example

We consider here a short cantilever plate modeled with triangular shell elements. The geometrical nonlinearity is due to the von-Karman kinematic model adopted [11]. The full model consists of 546 degrees of freedom. The load is applied at one corner of the free edge in order to induce both bending and twisting modes in the response. The geometric and material properties are reported in Figure 1 together with the tip displacement component when the load is applied statically. The markedly nonlinear behavior is evident. The nonlinear static FE equations are solved with the normal flow algorithm as proposed by [9].

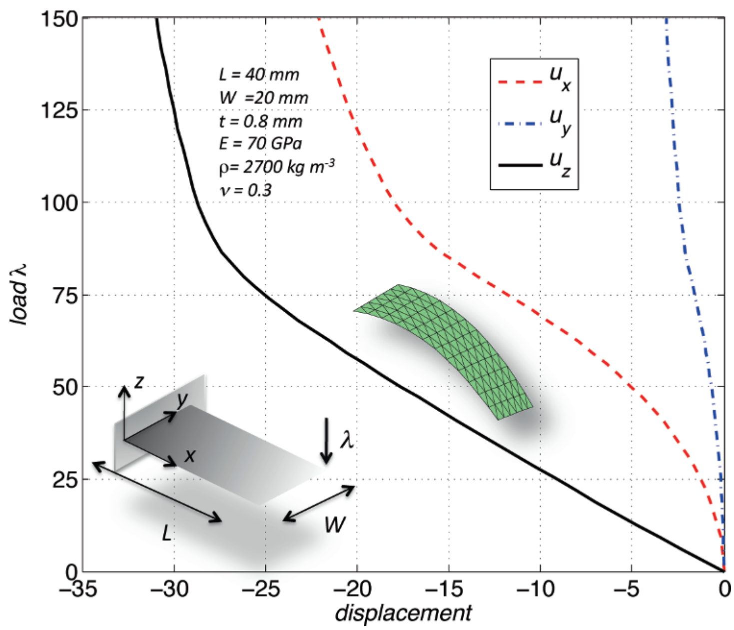
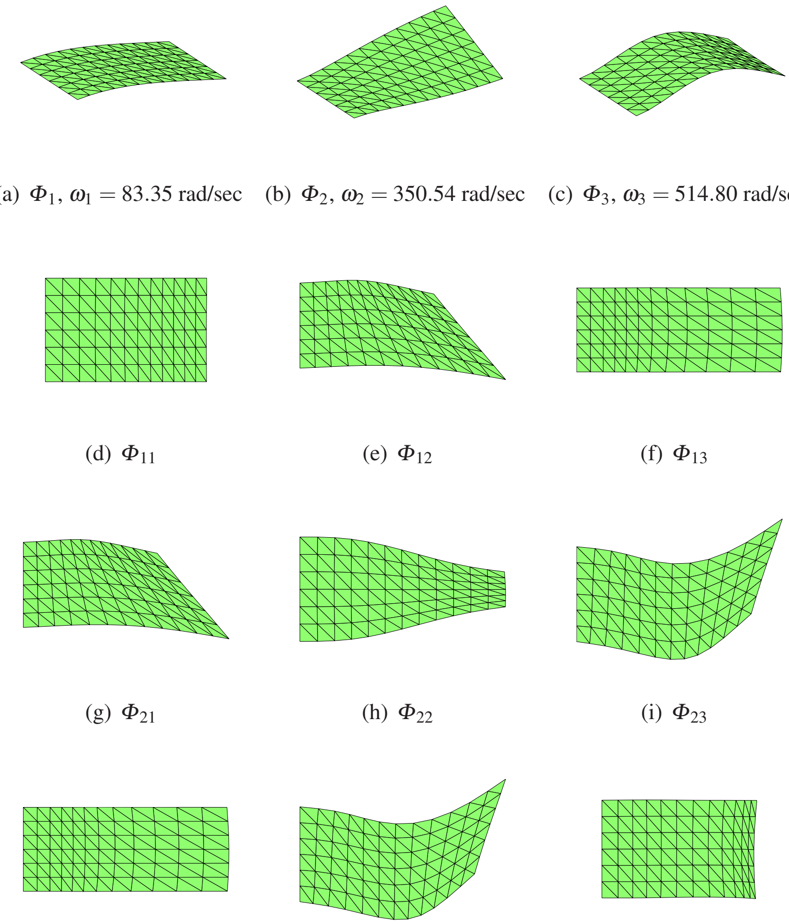


Fig. 1 Static nonlinear response of the short cantilever. The three components of the displacement at the point where the load is applied are shown. The load is applied is the vertical direction throughout the entire analysis. The stiffening is due to the axial stretching caused by the bending deflection.

We first consider the case of a dynamic step load of a magnitude of 30 N. For all the reduced analysis, a basis formed with the first 10 VMs is considered. The VMs are calculated around the initial undeformed configuration, i.e $\mathbf{u}_{eq} = \mathbf{0}$.This generates $R = 55$ second order MDs Φ_{ij}. To better illustrate the concept of MDs, the first three VMs and the corresponding MDs are shown in 5. The results of the selection criterion for the step load are shown in the bar plot on Figure 3. The b_{step} value has been raised to 0.25 power in order to better show the smaller contributions on the



(a) Φ_1, $\omega_1 = 83.35$ rad/sec (b) Φ_2, $\omega_2 = 350.54$ rad/sec (c) Φ_3, $\omega_3 = 514.80$ rad/sec

(d) Φ_{11} (e) Φ_{12} (f) Φ_{13}

(g) Φ_{21} (h) Φ_{22} (i) Φ_{23}

(j) Φ_{31} (k) Φ_{32} (l) Φ_{33}

Fig. 2 The first three vibration modes (VMs) and the corresponding modal derivatives (MDs). The VMs are out-of-plane modes, featuring bending and torsion. For this specific application, the MDs are in-plane only, and they are represented top-view to better show the non-uniform in-plane deformation.

bar plot. Rather intuitively, the MD Φ_{11} is the more important. The cross terms relating the first two VMs to all the other VMs are comparable to the diagonal entries, indicating the importance of the interaction between the retained VMs. Mode 6 does not give any contribution since it consists of a bending motion in the plane of the plate.

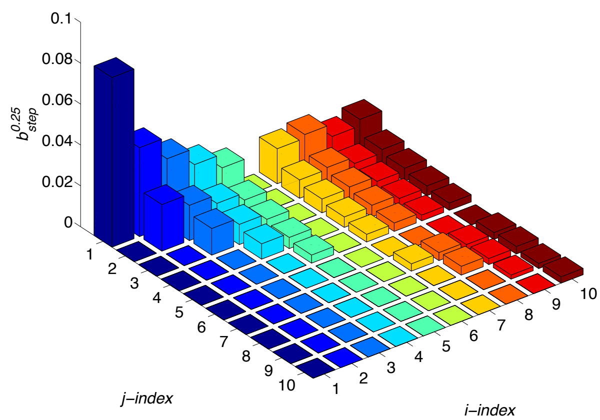


Fig. 3 Values of b_{step} relative to the first $M = 10$ vibration modes. The emphasis is put on the first mode and its interaction with all the other modes. The values for b_{step} are raised to 0.25 power to better highlight the relative contribution of all the terms. Mode 6 does not participate since it features in-plane lateral bending. (The lower diagonal part of the symmetric b_{ij} is set to zero for display purposes)

The nonlinear FE dynamic equations are solved with the Implicit Newmark time integration scheme. The results for the dynamic analysis are shown in Figure 4 for the vertical component of the displacement at the loaded point. The nonlinear reduced model with a basis formed by the first 10 vibration modes and all the possible second order MDs ($M = 10$, $R = 55$) yields a very good approximation of the full nonlinear response. Also, a reduced analysis with the basis formed by VMs only ($M = 10$) is shown (case (c)). The result is overly stiff and clearly wrong, and apparently much worse than the linear analysis. This might be deceiving, since only the vertical displacement is monitored here. In fact, unlike the nonlinear model, the linear analysis will not reproduce any axial contribution. A stiffening axial force is generated by the bending-only vibration modes through the nonlinear bending-stretching coupling. Since no axially dominated second order modes are included in the basis, there is no way to alleviate the resulting membrane tension. The outcome is the shown overly stiff response. Several reduced responses generated by randomly choosing a set of $P = 20$ second order MDs to enrich the $M = 10$ VMs basis are also plotted. None of these responses provide the same accuracy as the one resulting from the proposed selection criterion.

Similar results are shown for the case of harmonic load, see Figure 6. The frequency of the applied load Ω is set as 450 rad/sec, an intermediate value between the first and the second eigenfrequency of the system.The outcome of the selection criterion b_{harm} as per equation 17 is shown in Figure 5, while the load amplitude is 60 N. In this case, the criterion places more emphasis on the second and the third VM and their interaction. Also the interaction of the first VMs with all the other modes is rather important. The slight discrepancy between the full nonlinear

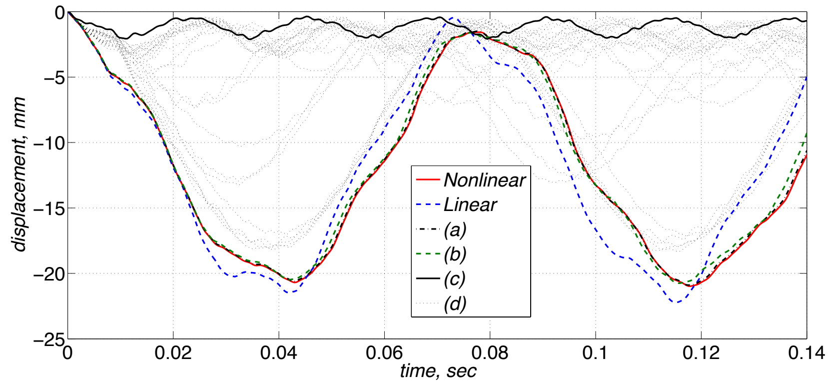


Fig. 4 Dynamic response at the loaded tip for a step load of magnitude $\varphi(t) = 30$ N. The u_z component is shown. (a) : basis formed with all the possible second order modes $M = 10$, $R = 55$; (b) : basis formed with the best 20 second order fields, $M = 10$, $P = 20$; (c) : basis formed with vibration modes only, $M = 10$, $P = 0$; (d) : basis formed with several random choices of $P = 20$ second order modes.

response and the complete reduced basis ($M = 10$ and $R = 55$) might be due to a too reduced underlying basis of VMs. The response of the reduced system with the proposed selection is rather close to the full response and much more accurate than the responses generated with a random choice of the second order fields. The reason for the overly stiff response of the nonlinear reduced system when using vibration modes only is the same as explained for the case of step load.

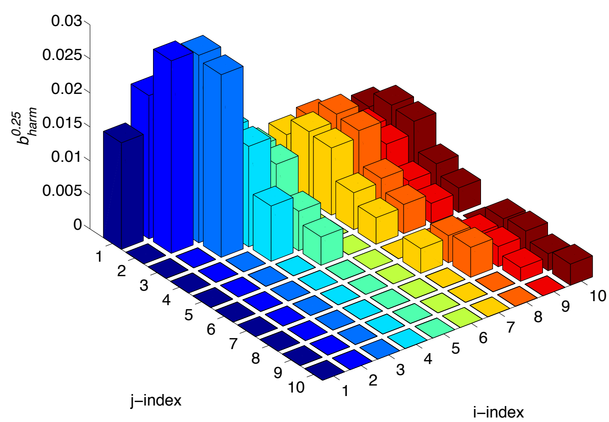


Fig. 5 Values of b_{harm} relative to the first $M = 10$ vibration modes and a forcing frequency Ω of 450 rad/sec. The emphasis is in this case on the second and the third mode, and their interaction with all the other modes.(The lower diagonal part of the symmetric b_{ij} is set to zero for display purposes)

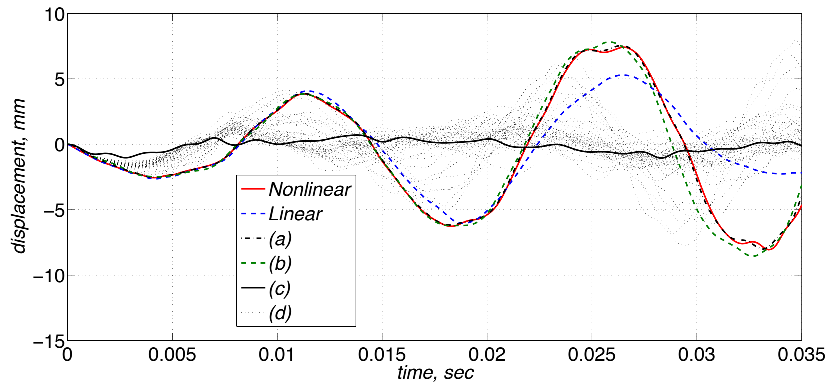


Fig. 6 Dynamic response at the loaded tip for a harmonic load $\varphi(t) = 60\cos(450t)$ N. The u_z component is shown. (a) : basis formed with all the possible second order modes $M = 10$, $R = 55$; (b) : basis formed with the best 20 second order fields, $M = 10$, $P = 20$; (c) : basis formed with vibration modes only, $M = 10$, $P = 0$; (d) : basis formed with several random choices of $P = 20$ second order modes.

6 Conclusions and Discussion

We presented a simple and effective criterion to select the most significant second order modal derivatives for the MOR of geometrically nonlinear structural analysis. The criterion is based on the convergence of the vibration modes truncation for the underlying linearized dynamic problem . Given a certain first order modal basis which is converged in a linear sense, the selection criterion looks at the spatial and spectral properties of the eigenspectrum and the applied load. In this way, the most relevant second order modes can be selected before their actual calculation. Numerical results confirmed the effectiveness of the proposed approach.

The power of this simple yet effective method lies on the fact that the second order modes enrichment can be seen as higher order expansion of the solution. Therefore, the convergence properties of the base linearized problem should naturally provide a guideline also for the higher order expansion.

Yet, the number of second (most significant) order modes required for a given accuracy is in general not known. Future work will focus on the derivation of an error bound as function of the selected second order modes as well as the spectral properties of the linearized system.

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