Independent Modal Space Control Technique for Mitigation of Human-Induced Vibrations in Floors

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ABSTRACT

Significant levels of vibration in civil engineering floors under human-induced excitations often cause annoyance to their occupants. Various Active Vibration Control (AVC) strategies have been investigated in the past for mitigating the effect of such vibrations in some problematic floors; for example, control laws making use of acceleration feedback and velocity feedback schemes. The research presented in this paper aims to demonstrate that the use of the Independent Modal Space Control (IMSC) approach, previously tried and implemented in marine applications, can be invaluable to isolating and controlling specific modes of vibration in civil engineering floors.

This approach may prove attractive particularly in floors with very close modes of vibrations where only certain modes prove to be problematic under human-induced excitations. The IMSC technique is implemented in a reduced order model (ROM) of a laboratory structure and two sets of studies with this technique are presented here. In the first study, only the first mode of vibration of the laboratory structure is targeted, while in the second study, both the first and second vibration modes of the laboratory structure are targeted.

1 INTRODUCTION

Civil engineering structures, particularly floors, have become prone to human-induced vibrations as a result of advancements in structural technology enabling design of light and slender structures often spanning long distances. Many contemporary floor structures also comprise more open-plan layouts with less inherent damping thereby increasing their susceptibility to such vibrations (Bachmann 1992, Nyawako and Reynolds 2007).

There has been significant research in recent years into passive, semi-active and active control of vibrations in civil engineering structures (Bachman and Weber 1995, Setareh 2002, Koo et al 2004, Hanagan et al 2003a, Diaz and Reynolds 2009). As pertains to active vibration control of these structures, there have been rigorous studies aimed at trying to derive suitable control laws for different scenarios, ranging from controlling wind/earthquake induced vibrations in civil engineering structures to controlling human-induced vibrations. Some of the control laws that have been investigated and implemented in the past for mitigating human induced vibrations in floors include direct velocity feedback (DVF), direct acceleration feedback (DAF), Compensated Acceleration Feedback (CAF), and Response Dependent Velocity Feedback schemes (Hanagan et al 2003b, Hanagan et al 2000, Diaz and Reynolds 2010, Nyawako and Reynolds 2009).

Notwithstanding the above mentioned active vibration control laws, the potential benefits of other categories of active control approaches widely used in the mechanical and aerospace sectors are being investigated for mitigation of human induced vibrations in civil engineering structures. Among these categories is the Independent Modal Space Control technique (IMSC), a modal control approach. This technique has been extensively used in the marine industry for developing isolation systems that improve the crew and passenger comfort (Daley et al 2004). Its attractiveness stems from the fact that it can be invaluable for isolating and controlling target modes of vibration in civil engineering structures.

The basic idea of the IMSC approach is that by looking at frequency response functions of structural systems, for example of floors, designers can detect troublesome modes or groups of modes. Desired structural behaviours in terms of modal damping

ratios and frequencies can be defined and a modal control strategy can then be designed to adjust the closed-loop behaviour in some suitable way (Inman 2001, Daley et al 2004, Fang et al 2003). Performances that can be realised are dependent on the number of sensor and actuators available as well as their dynamics. It has widely been observed that an IMSC implementation typically requires the number of actuators to be equal to that of modelled modes (Nguyen 1991).

The work presented here explores the possibility of realising an IMSC controller for mitigating human induced vibrations in floors. A structural model and input forces used in analytical simulations is introduced. A brief overview of the IMSC strategy used in this analytical work is shown and some results of analytical simulations for two different IMSC controllers setting are presented. IMSC controller 1 aims to target only the first mode of vibration of the reduced order model (ROM) of the laboratory structure while IMSC controller 2 aims to target both the first and second modes of vibration of the ROM. Some results and conclusions are finally presented.

2 STRUCTURAL MODEL AND INPUT FORCES

2.1 Structural Model

The laboratory structure is a simply-supported in-situ cast post-tensioned slab strip of span 10.8m. Its total length is 11.2m, which includes 200mm overhangs over the knife-edge supports. It has a width of 2.0m, depth of 275mm, and weighs approximately 15 tonnes. The first and second modes of vibration have natural frequencies of 4.55 Hz and 17.02 Hz with modal damping ratios of 0.4 % and 0.2 %, respectively. The first mode is particularly prone to excitation by the second and third harmonics of walking excitation (Reynolds 2000).

The IMSC controller design presented in these studies is formulated from a ROM of the laboratory structure. This ROM is developed based on uncontrollability and unobservability at node points of vibration modes of the laboratory structure (Seto and Mitsuta 1992). The mode order is chosen as two here and the node points of the third bending mode have been chosen as the locations of the masses for the lumped parameter system as shown in Fig. 1.

Fig. 1 Laboratory structure grid, mode shapes and 2-DOF lumped mass model

By making use of a mass normalised scaling for the mode shapes at the chosen locations of the third bending mode, i.e. $\left[\Phi\right]^T \left[M \right] \left[\Phi\right] = [I]$, the mass and stiffness matrices of the physical co-ordinates can be derived from Eqs. (1) and (2). Equation 3 is often used to obtain convergence i.e. diagonalisation of the mass and stiffness matrices of the physical co-ordinates.

$$
M = \left(\Phi^T\right)^{-1}\Phi^{-1} = \left(\Phi\Phi^T\right)^{-1} \qquad M^{-1} = \left(\Phi\Phi^T\right)
$$
 (1)

$$
K = \left(\Phi^T\right)^{-1} \Omega^2 \Phi^{-1} \tag{2}
$$

$$
\begin{bmatrix} \delta \phi_{11} & \delta \phi_{21} & \delta \phi_{12} & \delta \phi_{22} \end{bmatrix}^T = \begin{bmatrix} \frac{\partial \varepsilon_r}{\partial \phi} \end{bmatrix}^T \left[\begin{bmatrix} \frac{\partial \varepsilon_r}{\partial \phi} & \frac{\partial \varepsilon_r}{\partial \phi} \end{bmatrix}^T \right]^{-1} (-\varepsilon_r) \tag{3}
$$

By applying Eqs. (1), (2) and (3) to the mass-normalised mode shapes of the laboratory structure derived from a Finite Element model, the following physical parameters of the 2-DOF lumped mass model in Eqs. (4) and (5) are obtained. $M_1 = M_2 = 5596.3Kg$; $K_1 = K_3 = 0.4565e + 7N/m$; $K_2 = 2.9876e + 7N/m$. The assumed Rayleigh damping matrix is as shown in Eq. (6) .

Mass matrix (Kg)
\n
$$
M = \begin{bmatrix} 5596.3 & 0 \\ 0 & 5596.3 \end{bmatrix}
$$
\n
$$
K = \begin{bmatrix} 3.4441e + 7 & -2.9876e + 7 \\ -2.9876e + 7 & 3.4441e + 7 \end{bmatrix}
$$
\n(4)

Damping matrix (Ns/m)
$$
C = \begin{bmatrix} 1896.5 & -616.7 \\ -616.7 & 1896.5 \end{bmatrix}
$$
 (6)

2.2 Input Forces for Analytical Studies

Human movement is often characterised by rhythmical body motions such as walking and running. Such motions induce dynamic loads into the structures they occupy and may result in significant resonant, transient, steady-state or impulsive responses (Bachmann et al 1995).

The disturbance forces considered here are walking force time histories obtained from treadmill walking tests and a random excitation signal with a frequency span of $0 - 40$ Hz. These are shown in Figs. 2a and 2b for 10s durations.

Fig. 2 Walking force time history and random excitation force time history for analytical simulations

3 INDEPENDENT MODAL SPACE CONTROL (IMSC) STRATEGY

For a discrete set of measurements, \overline{x} , the equation of motion for the 2-DOF lumped mass model of the laboratory structure shown in [Fig. 1](#page-1-0) can be determined as shown in Eq. 7. The parameters M, C, K are as shown in Eqs. 4 and 5. f denotes the vector of applied forces. For the IMSC approach, a decoupled independent modal description of a structure is often necessary. The decoupled independent modal space description outlined in Eq. 8, which is derived from Eq. 7 can be obtained from the transformation shown in Eq. 9 (Daley 2004, Inman 2001). ϕ is an orthonormal matrix of the eigenvectors of $M^{-(1/2)}$ KM^{-(1/2)}

$$
[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = f \tag{7}
$$

$$
\underline{\ddot{\eta}} + \Delta \underline{\dot{\eta}} + \Omega \underline{\eta} = \underline{f}^m \tag{8}
$$

$$
\underline{\eta} = \phi^T M^{1/2} \underline{x} \tag{9}
$$

Where:

$$
\Lambda = \begin{bmatrix} 2\varsigma_1 \omega_1 & & \\ & \ddots & \\ & & 2\varsigma_i \omega_i \end{bmatrix}, \Omega = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_i^2 \end{bmatrix}, f^n \text{- modal force},
$$

Λ - damping ratios, Ω - spectral matrix

Thus, the problem is transformed from a MIMO (multiple-input multiple-output) control design problem in Eq. (7) into a series of multiple independent SISO (single-input single-output) control problem in Eq. (8). A general independent modal controller can now be defined as shown in Eq. (10). The closed loop description of each mode takes the form of Eq. (11), which enables the damping and frequency of each mode to be manipulated independently. The configuration of the vector of modal forces, f^m can be set depending on the number of modes to be controlled whilst taking into account the number of sensors and actuators available as well. In the work presented here, two IMSC controllers are designed as explained in section 1.4.

$$
\underline{f}^{m}(s) = r_{m}(s) - G_{m}(s)\underline{\eta}(s)
$$
\n(10)
\nWhere:
\n
$$
G_{m}(s) = \begin{bmatrix} g_{1}(s) & 0 & \cdots & 0 \\ 0 & g_{2}(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & g_{m}(s) \end{bmatrix}
$$
\n
$$
\eta_{i}(s) = \frac{1}{s^{2} + 2\varsigma_{i}\omega_{i}s + \omega_{i}^{2} + g_{i}(s)} r_{m,i}(s)
$$
\n(11)

Where:

Since the controller in Eq. (10) is defined in the modal space, it cannot be implemented directly and a transformation is
necessary from the measured input signals. The matrix transformation required for extracting the first two bending modes of
the laboratory structure at the locations pre-determined can be determined by making use of the rows of the matrix
$$
\phi^T M^{1/2}
$$
.
This enables Eq. (10) to be re-arranged in the physical domain as shown in Eq. (12) and this can now be implemented
directly. A vector $y(s)$ contains the sensor measurements and vector $f(s)$ is the local demand force.

$$
f(s) = M^{1/2} \phi r_m(s) - M^{1/2} \phi G_m(s) \phi^T M^{1/2} y(s)
$$
 (12)

The local demand forces can now be determined from the linear displacements, velocities and accelerations at each actuator location as noted in Eq. (12). For the work presented in this paper, once the local demand forces are evaluated, an inverse actuator model (i.e. an inverse model for the APS Dynamics Model 400 Electrodynamic Shakers) is used to calculate the desired control voltage signal which is then transmitted to the actuators. A typical global processing stage is demonstrated in [Fig. 3.](#page-4-0)

Fig. 3. Typical global control processor (after Daley et al. 2004)

4 ANALYTICAL SIMULATIONS

The analytical simulations presented in this work cover two IMSC controller designs

- a) IMSC Controller 1 This controller aims to increase the damping in the first mode of vibration of the ROM of the laboratory structure by up to 20 times while not engaging the second mode of vibration.
- b) IMSC Controller 2 This controller aims to increase the damping in both the first and second modes of vibration of ROM of the laboratory structure by up to 20 times.

4.1 IMSC Controller 1

The settings of this controller are set so as to target an increase in damping of the $1st$ mode only by up to 20 times whilst not engaging the second mode of vibration. Figs. 4a and 4b show the uncontrolled and controlled acceleration responses of the laboratory structure model to a walking excitation force noted in section 1.2. [Fig. 4c](#page-5-0) illustrates the control force whilst [Fig.](#page-5-0) [4d](#page-5-0) shows the point mobility frequency response function for the uncontrolled and controlled structural model. The peak 1s running RMS acceleration responses for the uncontrolled and controlled ROM are 0.44 m/s² and 0.088m/s², respectively. This follows the recommendation of ISO 2631:1997, for which the peak 1s running RMS is defined as the Maximum Transient Vibration Value (MTVV).

4.2 IMSC Controller 2

In this second controller design, the settings are tuned to achieve an increase in damping of both the $1st$ and $2nd$ modes of the ROM structural model by up to 20 times. Figs. 5a and 5d show the uncontrolled and controlled acceleration responses of the laboratory structure model to a walking excitation force noted in section 1.2. [Fig. 5c](#page-6-0) illustrates the control force and [Fig. 5d](#page-6-0) shows the point mobility frequency response function for the uncontrolled and controlled structural model. The peak 1s running RMS acceleration responses for the uncontrolled and controlled ROM are 0.44 m/s² and 0.054m/s², respectively.

5 RESULTS AND CONCLUSIONS

Tables 1 and 2 demonstrate the vibration mitigation performances of the two IMSC controller structures outlined in section 1.4. The benefits of the IMSC controller design technique in meeting the desired vibration mitigation performances can clearly be seen from these results.

Table 1. Peak 1s running RMS acceleration for uncontrolled and controlled laboratory

Case	Uncontrolled (ms^{-2})	Controlled $(ms-2)$	$\%$ red.
IMSC Controller 1	0.44	0.088	800%
IMSC Controller 2	0.44	0.054	877%

structure model under walking excitation for IMSC Controllers 1 and 2

Table 2. Attenuations in vibration at target modes of vibration

(4.55 Hz and 17.02 Hz) for IMSC Controllers 1 and 2			
Case	Attenuation Mode 1 (dB)	Attenuation Mode 2 (dB)	
IMSC Controller 1	10.0	0.0	
IMSC Controller 2	10.0	13.0	

Based on this work, the modal control technique is seen as a potential and attractive method for controlling human induced vibrations in civil engineering floor structures. As previously noted by some researchers, its benefit is that civil engineering designers can now evaluate frequency response functions of floors, detect a troublesome mode or groups of modes and design a modal control strategy to adjust the closed loop behaviour in some suitable way. The effects of the controller designer on other modes can also be evaluated to determine its robustness.

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