Chapter 12
Case Study: Child Development (Sociology)

12.1 Overview

Starting point is the data matrix made of six indicators describing different aspects of child development and 21 nations, mainly of Europe. A ranking of the 21 nations is based on a composite indicator. Using several methods, we try to assess the construction of the composite indicator of UNICEF. Both the concordance analysis (Chapter 10) and the canonical order (Chapter 9) support the construction of the UNICEF. The comparison with the canonical order is naturally more detailed and we find some rank inversions. With the help of the local partial order model (Chapter 9), we explain these. Without partial order, there is no reason to define separated subsets. The partial order constructed from the six indicators and the 21 nations shows several separated subsets (see Chapter 5). Most striking is the separation between {It, Pt} and the residual set of nations. Which indicators explain this separation and with which values? The partial order identifies the indicators "family" and "education" as the responsible ones. When an aggregation to a composite indicator is performed, the single indicators lose their individuality as they are just summands contributing to the value of the composite indicator. Similarly, one can construct new orders (called the m^r orders, see Chapter 7) which also do not take into account the individuality of the indicators. The resulting partial orders contain pretty long chains of nations which allow unambiguous ranking of many nations without crunching the indicators into one composite indicator. There is a natural question about as to how much the composite indicator-based ranking can be improved by changing the value of an indicator. We do not perform a complete analysis but illustrate the methodological steps with Ireland and Denmark. We show that Ireland can improve its ranking position best if it has better indicator values in "education."

R. Brüggemann, G.P. Patil, *Ranking and Prioritization for Multi-indicator Systems*,
Environmental and Ecological Statistics 5, DOI 10.1007/978-1-4419-8477-7_12,

12.2 Basic Information, Data Matrix, and Results of UNICEF Study

This study is based on a report of UNICEF (Innocenti Research Centre, Report Card 7, 2007). A comprehensive assessment of well-being of "children and young people in 21 nations of the industrial world is given" (cited from the report).

The study provided 40 different indicators which are aggregated through several interim steps into six main indicators (Table 12.1).

From the data matrix (see Table A.7), UNICEF defines an index with equal weights for all six indicators, i.e.

$$\Gamma(x) = \Sigma(1/6)^* R_i(x), \text{ weight vector} = (1/6, 1/6, \ldots, 1/6) \tag{12.1}$$

where $R_i(x)$ is the rank by the ith indicator of nation x.

For example, Belgium (Be) has the following data (Table 12.2)

$$\Gamma(\text{Be}) = (1^*7 + 1^*16 + 1^*1 + 1^*5 + 1^*19 + 1^*16)/6 = 10.7$$

From Γ, the following ranking, O_Γ, is deduced (from the worst to the best):

(UK, US, Hu, Au, Pt, Fr, Cz, Pl, Gr, Ca, De, Be, Ire, It, No, Su, Es, Fi, Dk, Sw, Ne)

Table 12.1 Six indicators and their background information

Indicator	Abbreviation	Background information
Material well-being	wb	Relative income poverty, households without jobs, reported deprivation
Health and Safety	hs	Health at birth, immunization, mortality
Educational well-being	ed	Aspirations, achievements, participation
Family and peer relationships	fa	Family structure, family relations, peer relations
Behaviors and risks	br	Risk behavior, experience of violence, health behavior
Subjective well-being	sub	Health, personal well-being, school well-being

Table 12.2 Section out of Table A.7

		wb	hs	ed	fa	br	sub
		Material well-being	Health and safety	Educational well-being	Family and peer relationships	Behaviors and risks	Subjective well-being
Belgium	Be	7	16	1	5	19	16

The top third subset is Ne, Sw, Dk, Fi, Es, Su, No.
The middle third subset is It, Ire, Be, De, Ca, Gr, Pl.
The bottom third subset is Cz, Fr, Pt, Au, Hu, US, UK.

12.3 Motivating the Use of Partial Order

This is a detailed UNICEF study, where the peculiarities of any single nation are commented, and there is a broad discussion on how the six indicators were obtained.

Now any politician of a nation may see that his nation is “good” with respect to some indicator, even if his nation receives a bad overall position in the ranking. Thus Italy is good in the indicator “family,” “fa.” Naturally the question arises: Is not “family,” “fa,” more important than the others, say “ed,” education, and give “fa” a higher weight? UNICEF, however, used the same weight for each indicator. Hence, the summation (Eq. (12.1)) implies that good points may compensate bad points and vice versa. If, however, such compensation is allowed, then questioning the uniform weight of any indicator in the index is indeed justified. Thus Italy would get a better overall position if the weight for “fa” would get a higher value. Such procedure would however make politicians of other nations unhappy. Poland, for example, is good in education and would therefore like to see this indicator given a higher weight.

There is another issue: The need for defining a composite indicator (like that in Eq. (12.1)) is understandable. However, as the UNICEF report indicates, there is in some of the six indicators a high degree of conceptual overlapping. For example, the indicator “he,” health, is one of the six indicators. However, the indicator health is also partly present in the indicator “sub,” “subjective well-being.” This kind of conceptual overlap means that health is more pronounced in the ranking, because it appears two times, once explicitly in “health” and once implicitly in “sub.” Hence, it may be a good idea to keep the six indicators separated, but simultaneously analyzed rather than composited.

12.4 Partial Order Analysis

12.4.1 Aims of Partial Order Analysis

1. Can we find chains of nations, where the status of one country is comparably fixed with respect to some others? There may be a real need to improve child well-being in one or more of the six attributes for such countries which worry about their poor position.
2. Are there countries whose positions will severely depend on how the six indicators are combined? They may initialize an update of the study to show how different weight vectors can influence their positions.

12.4.2 *Hasse Diagram of 21 Nations Based on Six Indicators*

In Fig. 12.1, the Hasse diagram is shown based on the entries of Table A.7. We rearranged the table so that the "good" nations are on the top of the Hasse diagram. The information base, IB, is {wb, hs, ed, fa, br, sub}. In order to include USA, the missing value in the indicator "sub" was given the mean value taken from the 20 nations.

12.4.2.1 Structural Characterization

There are six components, four of which are trivial, i.e., are isolated elements (see Chapter 2). One nontrivial component consists of the majority of nations $X_1 = \{\text{Ne, Sw, No}, \ldots,\}$ and the other one consists of Italy and Portugal $X_2 = \{\text{It, Pt}\}$. The set ISO (see Chapter 2) contains the isolated nations Be, Ca, Pl, and Au. How different components are related with the data matrix is studied below. UK, Ne, and Dk are articulation points (Chapter 5). Removing, for example, UK from the data matrix would make Su an isolated element. Removal of Ne from the data matrix would generate two new components. Su is an example of a "loosely" connected object. Isolated and loosely connected ones need our attention because of their peculiar data profiles. The tool to find out the peculiarity is the search for antagonistic indicators.

12.4.2.2 Level

The Hasse diagram provides three levels. Generally speaking, this low number of levels means that there are many incomparabilities. Large $U(x)$ sets (see Chapter 3) in turn indicate that many nations have some indicator values which make them better and some other indicator values which make them worse than others. Here are the three levels (see Chapter 5):

level$_3$ = {Ne, Su, No, Sw, Fi, Dk, It, Be, Ca, Pl, Au},
level$_2$ = {Ire, Hu, Es, De, Cz, Fr, Pt}, and
level$_1$ = {Gr, UK, US}.

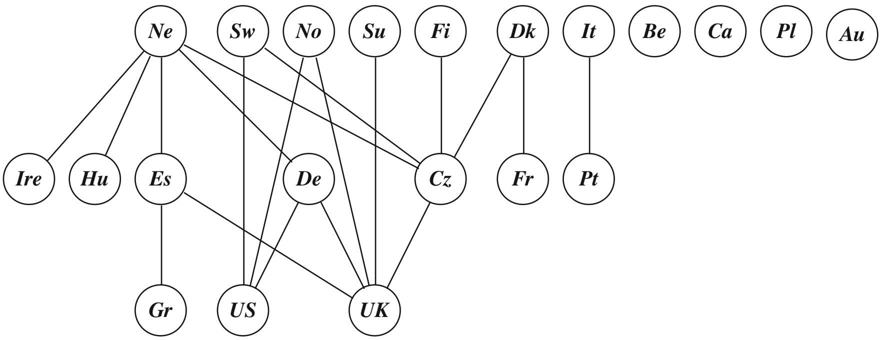

Fig. 12.1 Hasse diagram of 21 nations, IB = {wb, hs, fa, ed, br, sub}

There is the set of isolated elements ISO = {Be, Ca, Pl, Au} and a nontrivial component Pt $\leq$ It.

12.4.2.3 Concordance Analysis

In Chapter 2, we have learned that the drawing of a Hasse diagram is not unique. Therefore, we must examine two scenarios using concordance analysis (see Chapter 10):

Scenario H: The objects are given the highest level possible.
Scenario L: The objects are given the lowest level possible.

In Table 12.3, the number of objects related to scenario H is in bold, while that of scenario L is in italic letters. The concordance indices, con, referring only to the main diagonal (because of only three subsets) are as follows:

Scenario H: con $= 10/21 = 0.48 > T = 0.333$ (d is the dimension of concordance matrix $= 3$)
Scenario L: con $= 12/21 = 0.57 > T = 0.333$ (d is the dimension of concordance matrix $= 3$)

Thus the level provided by partial order theory corresponds pretty well to the classification by means of Γ.

Table 12.3 Concordance analysis for scenarios *H* and L

	Subsets in UNICEF		
Subsets in partial order	Ne, Sw, Dk, Fi, Es, Su, No	It, Ire, Be, De, Ca, Gr, Pl	Cz, Fr, Pt, Au, Hu, US, UK
Ne, Sw, No, Su, Fi, Dk, It, Be, Ca, Pl, Au	**6**	**4**	**1**
Ne, Sw, Dk, Fi	*4*	*0*	*0*
Ire, Hu, Es, De, Cz, Fr, Pt	**1**	**2**	**4**
Es, De, Cz, Su, No, It	*3*	*2*	*1*
Gr, US, UK	**0**	**1**	**2**
Ire, Hu, Gr, US, UK, Fr, Pt, Be, Ca, Pl, Au	*0*	*5*	*6*

12.4.2.4 Chains

The maximal chain length equals 3. There are eight chains of maximal length 3 and eight chains of maximal length 2. The identification of chains is an important tool to find the invariant mutual order among the nations. Let us take one three-element chain like UK < De < Ne. As discussed in Chapter 7, the relative ranking of the elements of a chain will be invariant relative to the weight vector. Thus Germany, De, is in all indicators worse than the Netherlands. Clearly, for Germany, it is not possible to suggest any weight vector to get a better ranking result than Netherlands. Germany must improve the situation for children, leading to better values for the indicators. In Section 12.9.2, we will have more discussion on this.

12.4.2.5 Antichains

There are more and longer antichains than chains, indicating that incomparability is the dominant factor. Isolated elements or weakly connected elements like Su or Fr, or the chain Pt < It, will have large $U(x)$ sets and large ranking intervals. Following Chapter 3, large ranking intervals imply high influence of weights on the final ranking positions (see Eq. (3.19)).

12.5 Indicator Set

12.5.1 Attribute-Related Sensitivity

In Fig. 12.2, the values $W(X, \mathrm{IB}, \mathrm{IB} - \{q_i\})$, $i = 1, \ldots, 6$ (for details, see Chapter 4), are shown.

Figure 12.2 shows that the most important indicator is fa, "family and peer relationships," and the least one is br, "behavior and risks." If indicator fa is deleted from the data matrix, then the Hasse diagram (Fig. 12.3) is obtained.

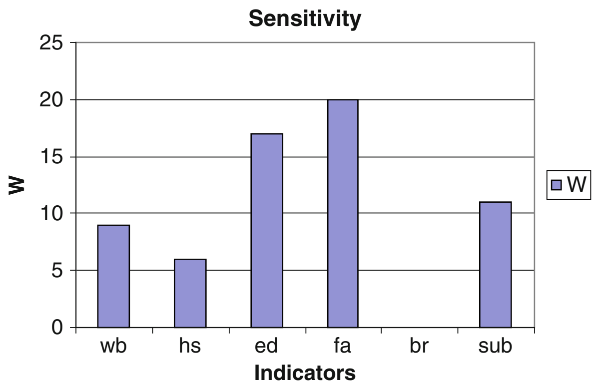

Fig. 12.2 Attribute-related sensitivity of the six indicators to the Hasse diagram

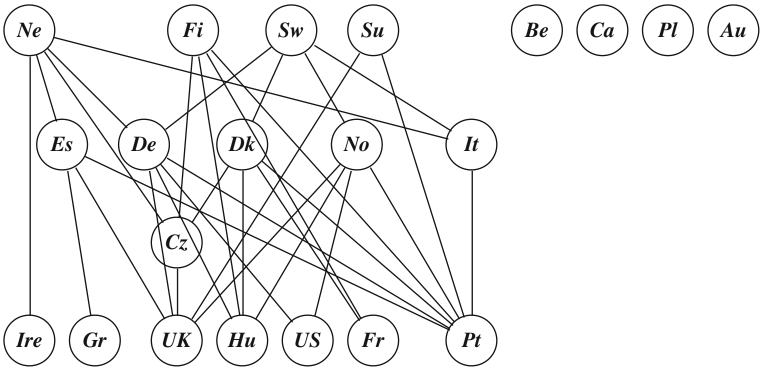

Fig. 12.3 Effect of deleting indicator fa from the data matrix

We note the following:

- The longest chain contains UK $\leq$ Cz $\leq$ Dk $\leq$ Sw, hence four levels can be identified.
- The four isolated elements Be, Ca, Pl, and Au remain.
- The component Pt $\leq$ It is now connected and Italy "moves down" because its strong indicator "fa" is deleted from the data matrix and its weaker indicators prevail.
- Su, Switzerland, is still "loosely" connected (Chapter 5), and all other proper maximal elements (Ne, Fi, and Sw) are covering four or more nations.

Some other nations "move down" too, which were originally in $level_3$ or $level_2$ (Fig. 12.1). For example, in Fig. 12.3, we find Dk $\leq$ Sw. Hence, with respect to all five indicators wb, hs, ed, br, and sub, Dk has worse values than Sw. Now look at Fig. 12.1: Here Dk is incomparable to Sw and is an element of the first level. Only in the indicator "family," Denmark is definitely better than Sw. Therefore, adding the indicator fa to the data matrix makes Denmark incomparable to Sw.

12.5.2 Ambiguity, Cumulative Ambiguity, and Minimum Rank Graphs

According to Chapter 4, we calculate CAM to be 0.881. As CAM can only vary between 0 and 1, this means adding new indicators to the data matrix will not change the Hasse diagram much: New indicators may break some of the comparabilities making the Hasse diagram (Fig. 12.1) still more flat. We conclude that on the one hand, the given six indicators provide a sufficient diverse picture and on the other hand, deleting some indicators from the data matrix may change the Hasse diagram (as can be seen when Fig. 12.1 is compared with Fig. 12.3). We see that adding indicators ed and sub to the column of "fa" leads to CAM = 0.75, which is 85% of the final value of 0.88. The partial order due to {fa, ed, sub} contains 85% of the incomparable pairs of the partial order found in Fig. 12.1. The remaining indicators he and br may be seen as fine-tuning the partial order, because their contribution to the final CAM value is rather small.

12.5.2.1 Cumulative Ambiguity Graph

In Fig. 12.4, the cumulative ambiguity graph (Chapter 4) is shown.

12.5.2.2 Minimum Rank Graph

As explained in Chapter 4, we start with the most important indicator fa and add successively the less important ones.

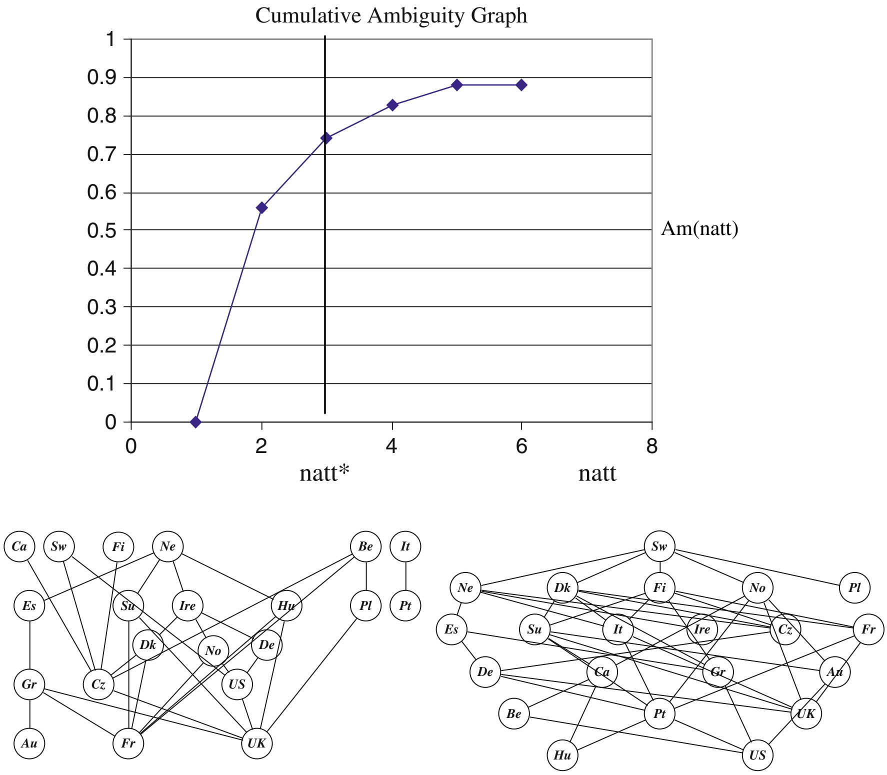

Fig. 12.4 Am(natt) and the decomposition of IB at natt* = 3 (see Chapter 4). (*Bottom) LHS*, $IB^{(1)} = \{fa, ed, sub\}$; *RHS*, $IB^{(2)} = \{wb, hs, br\}$

The indicator fa ranks the nations from worst to best as follows:

fa: UK < US < Cz < Ca < Fi < Au < Sw < Pl < De < Fr < Gr < No < Dk < Es < Ire < Hu < Be < Su < Ne < Pt < It

The result of adding the next indicator, ed, reduces the length of the maximal chain.

Italy

Instead of 20 successors in the chain, induced by "fa," Italy has now only one successor, namely Pt. Adding indicator ed reduces the minimum rank and increases the number of possible positions depending on the actual selection of weights.

Belgium

Due to the indicator fa alone, Belgium has 16 successors. Adding the indicator ed, the number of successors of Be remains the same. The minimum rank graph summarizes the effect of successively adding the indicators to the data matrix. Figure 12.5 provides the minimum rank graph for Belgium and Italy.

12.5.3 Antagonism

Let us select two subsets $X_1 = \{\text{It, Pt}\}$ and $X_2 = X - X_1 - \text{ISO}$.

Through software WHASSE, we find the set of antagonistic indicators AIB = {ed, fa}. The indicators ed and fa completely separate the two subsets It, Pt on the one hand and Ne, Sw, Su, ..., US on the other hand (see Fig. 12.6), $\text{Sep}(X_1, X_2, \text{AIB}) = 1$).

Once again the indicator "family," fa, plays a distinctive role: Together with ed, it separates X_2 {Italy, Portugal} from X_1, i.e., from 15 other nations.

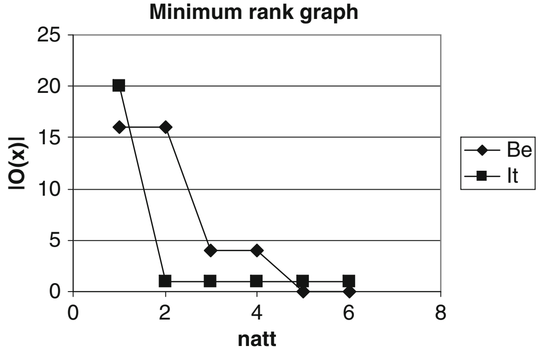

Fig. 12.5 Minimum rank graph of Be and It

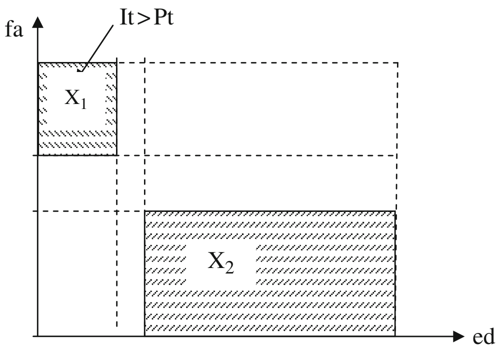

Fig. 12.6 Relation of Italy, It, and Portugal, Pt, to the other non-isolated nations X_2 (not scaled)

12.6 Partial Orders Based on Rank Orders of Attributes

The idea is to try to find orders which are not based on an averaging over all indicator values but retain the information about their disparity. The Hasse diagram due to the m^3 order (Chapter 7) is shown in Fig. 12.7).

In contrast to Fig. 12.1, we now get pretty long chains. For example

US < UK < Cz < Ire < Es < Sw < Ne or US < UK < Hu < Ca < Es < Sw < Ne.

Thus a comparison of nations on the basis of their worst, median, and best ranks is much simpler. We can even identify a greatest (Netherlands) and a least element (USA) (see Chapter 2). We may discard the median (being used as a fine trigger)

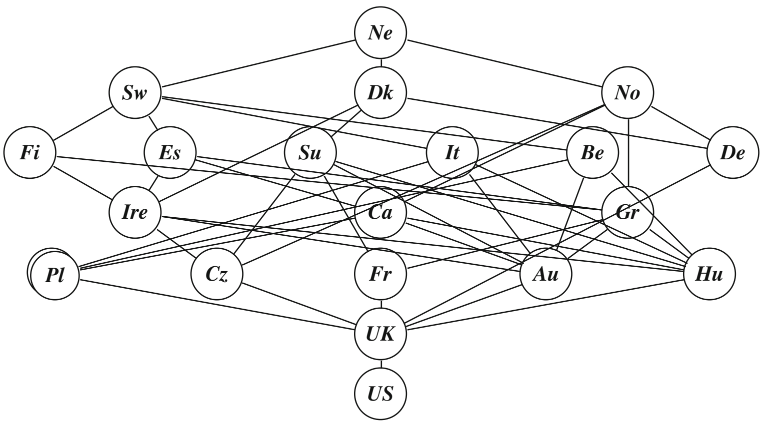

Fig. 12.7 Hasse diagram of m^3 order. Pl ≅ Pt

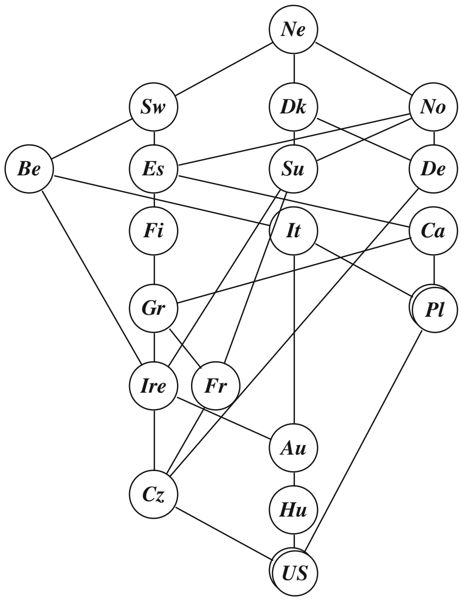

Fig. 12.8 Hasse diagram of the m^2 order (equivalent: {US, UK} and {Pl, Pt})

and examine the Hasse diagram, based on the m^2 order alone. Instead of the individual indicators we now compare nations on the basis of their worst and their best indicator values simultaneously (Fig. 12.8).

What do we see in Fig. 12.8?

1. Gr $<_{m^2}$ Fi: The worst indicator value of Gr is less than the worst indicator value of Fi. The best indicator value of Gr is less than the best indicator value of Fi.
2. Ne is at the top of the m^2 order. From this we know, independent of how large actually the interval [min(q(Ne)), max(q(Ne))] is, that all other min values as well as all other max values are less than or equal to those of Netherlands.
3. Es $||_{m^2}$ Su: The disparity due to m^2 in indicator values of these two nations is different.

Furthermore, we explained in Chapter 7 the role of incomparabilities in m^2 order. There we learned that incomparability in m^2 implies an inclusion order of the [min(q), max(q)] intervals. Hence an antichain like Be, Es, Su, De is a chain in the $\subseteq$ order according to increasing intervals [min(x), max(x)] (Fig. 12.9).

Germany, De, has the smallest interval. Therefore, Germany has a pretty sharp distribution of the six indicator values. Any variation of the weight vector for index calculation would lead to only a small variation in Γ. No political influence on the weight vector will help Germany to get a better position than Su or Es or Be. Belgium has the greatest interval within this subset of countries. This fact means

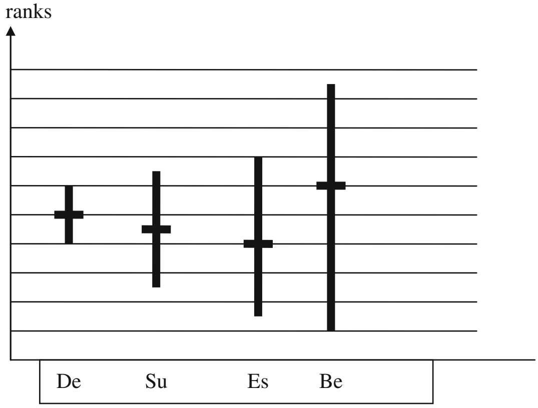

Fig. 12.9 Analysis of one of the antichains of diagram Fig. 12.8. The medians were taken from the data matrix

that Belgium has many choices in the selection of weight vectors to improve its ranking position.

12.7 Linear Orders

12.7.1 Where Are We?

The partial order analysis began with the data matrix, as can be found in Table A.7.

The Hasse diagram was rather flat because of many incomparabilities. Nevertheless, a sensitivity analysis could be performed and we showed how to find out why there are separated subsets (for example, {It, Pt} vs other nations). A decision about a mutual ranking among nations is difficult to make because of the shortness of chains. We know that Gr < Es < Ne without crunching the six indicators into a composite indicator. However, there is no answer for a mutual ranking between, say, Es and Cz. A step forward in decision making is to analyze the m^2 or m^3 orders where much of individual information about the indicators is lost but the information about disparities among their values retained. With this, many longer chains can be obtained, and we found that, for example, Es > Cz without worrying about weights. We continue this line of argumentation and derive weak orders among the nations based on purely order theoretical information as can be found in the Hasse diagram of Fig. 12.1.

12.7.2 Averaged Height Estimated by the Local Partial Order Model

The essential parameters of this model are U(x), the set of successors, $S(x)$, and the set of predecessors, $P(x)$ (see Chapters 3 and 9). In Table 12.4, the characteristics of the LPOM model are summarized.

Table 12.4 Characteristics of the 21 nations w.r.t. LPOM taken from Fig. 12.1

Nation	U	S	P	Nation	U	S	P
Ne	12	8	0	Es	17	2	1
Su	19	1	0	De	17	2	1
No	18	2	0	Cz	15	1	4
Sw	17	3	0	Ire	19	0	1
Fi	18	2	0	Hu	19	0	1
Dk	17	3	0	Gr	18	0	2
It	19	1	0	UK	11	0	9
Be	20	0	0	US	16	0	4
Ca	20	0	0	Fr	19	0	1
Pl	20	0	0	Pt	19	0	1
Au	20	0	0				

The LPOM model finds the following weak order:

(UK < US < Cz < Gr < Fr ≅ Pt ≅ Ire ≅ Hu < Au ≅ Pl ≅ Ca ≅ Be < De ≅ Es < It ≅ Su < Fi ≅ No < Dk ≅ Sw < Ne)

The top seven are Ne, Sw, Dk, No, Fi, Su, and It. Compared with the top seven of the UNICEF, Italy is included, whereas Spain is located in the middle group. In its simplest form, LPOM generates many ties. Therefore, we also apply a canonical order which, however, will result in some ties too because of the inherent symmetry of the Hasse diagram (Fig. 12.1).

12.7.3 Canonical Order

Following the lattice theoretical method explained in Chapter 9, the ranking is as follows:

O_{poset}: (UK < US < Gr < Cz < Pt < Fr < Ire ≅ Hu < Ca ≅ Au ≅
≅ Pl ≅ Be < De < Su < Es < No < Fi < It < Sw < Dk < Ne)

In comparison to the order obtained by the composite indicator of UNICEF, there are some rank inversions such as Hu vs Gr, Au vs Cz, or It vs Su.

We apply the method explained in Section 10.2 to see quantitatively the degree of coincidence between the two orders (canonical order and UNICEF ranking). The partial order analysis shows that 27 incomparable pairs appear between O_Γ and O_{poset}, therefore $d_{\text{coinc}} = 1–(27/210) = 0.87$. The Spearman correlation index (see Section 10.3) is 0.88. Both numbers indicate that the general trend is rather well coincident for both approaches.

12.7.4 Analysis of Rank Inversions: An Example

Why is in the canonical order Su < It, whereas in the UNICEF ranking Su > It? By application of LPOM, where Su ≅ It, we can outline the reason.

In Fig. 12.10, the rank-ordered rows, qo, of the data matrix of Su and It (see Chapter 7) are shown.

Averaging due to the UNICEF method leads clearly to Su > It. The high value of Italy in the indicator fa cannot compensate all the low values in the other indicators. Why we arrive at Su < It in the canonical order? In Fig. 12.11, a scheme according to the LPOM approach is shown.

In the Hasse diagram (Fig. 12.1) approximately Su and It have the same order theoretical configuration. Both have only one successor and no predecessor. However, Italy has 19 nations which can take all of the three positions in the $S - x - P$ chain (see Chapter 9), whereas Switzerland, Su, has only 6. Italy tends to get a higher position in the weak order of canonical order because there are more nations to realize lower positions than for Switzerland.

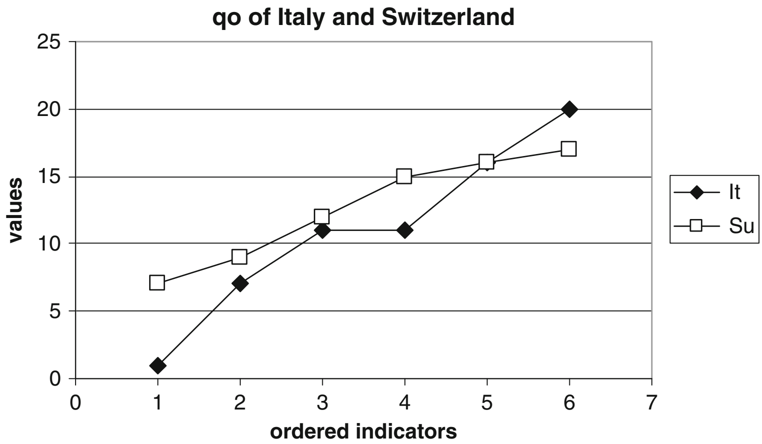

Fig. 12.10 Rank-ordered attribute values qo(It) and qo(Su), see Chapter 7

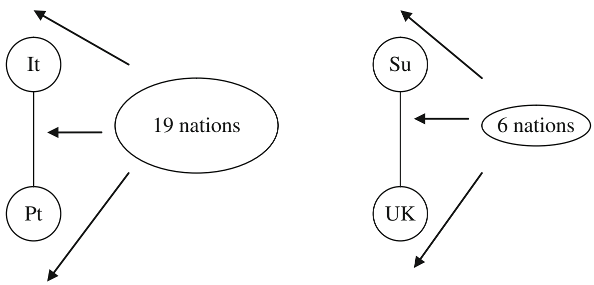

Fig. 12.11 Scheme to explain Su < It in canonical ranking (see text)

12.8 Sensitivity of the Single Nations to Indicators

So far we have discussed partial orders, having complete set X in mind. Now we take the point of view of a stakeholder who would like to learn more about his country $x \in X$.

One question is: What is the sensitivity of a single nation x to the indicators? For an answer, we have to select the singleton $\{x\}$ for X' in Eqs. (4.2) and (4.4). The results are shown in Table A.8. We see, for example, that wb is important for Ne, fa for Sw, and De, ed for Su, and It and sub for Dk ($W(\{\text{Ne}\}, \text{IB}, \text{IB-}\{\text{wb}\}) = 5$, $W(\{\text{Sw}\}, \text{IB}, \text{IB-}\{\text{fa}\}) = 7$, etc.). Figure 12.12 illustrates the influence of wb on Ne by displaying the down sets.

Deleting indicators from the data matrix is certainly not possible when a ranking of nations is ahead. However, a high sensitivity of a nation to an indicator R_i may motivate to improve the value of $R_i(x)$. The question is, the improvement of which indicator R_i is most helpful. Here we perform an attribute value sensitivity study (see Section 6.6) which may be more straightforward.

12.9 Attribute Value Sensitivity

12.9.1 Preliminaries

What happens if a certain indicator value is changed by a unit, Δ, in particular 1? For which indicator out of the six will this have the best effect in terms of partial order?

If there is a change in partial order, what consequences can be drawn for linear orders?

The six indicators of this study are ranks. Therefore, to keep the computational effort tractable, a simulation study needs to be performed in the following.

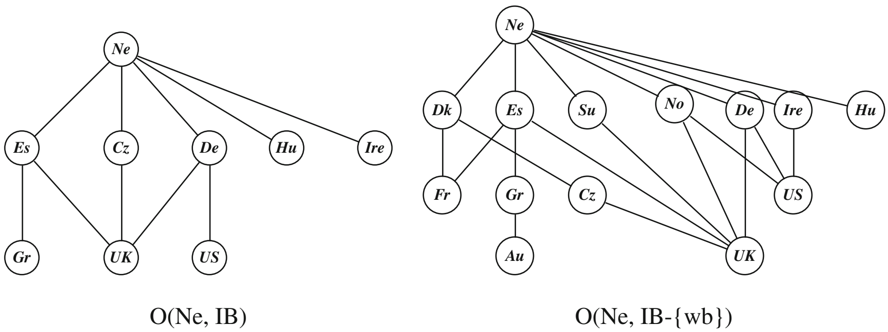

Fig. 12.12 Down sets of Ne according to different information bases

Let x be the nation of interest and $R_i(x)$ the rank of the ith indicator and y that nation for which $R_i(y) = R_i(x) + 1$. Simulating now a change for nation x in R_i, by Δ, we consider the following:

(1) x: change of ranks according to $R_i(x) \rightarrow R_i(x) + \Delta$
(2) y: change of ranks according to $R_i(y) = R_i(x) + \Delta \rightarrow R_i(x) = R_i(y) - \Delta$.

Now, with the example of Ireland, we study Δ=1 for all single indicators one after another (Section 12.9.2), and with the example of Denmark, we study the influence of changing Δ on certain indicators (Section 12.9.3).

12.9.2 Ireland

The question is: Which kind of improvement of one of the six indicators might be the most efficient one (Table 12.5)?

12.9.2.1 Changes in Hasse Diagram

Table 12.5 shows that changing the indicator values of ed or br by $\Delta = 1$ affects the position of Ireland in the Hasse diagram: By changing the ranks for ed or br, Ireland becomes an isolated element.

12.9.2.2 Changes in Linear Orders O_Γ and O_{poset}

O_Γ: While $|U|$ is increasing, Ireland becomes an isolated element. Indeed the ranking interval of 20 and following the lines of Section 3.7, Ireland has the chance to get the top position in O_Γ through an appropriate weight vector. The lowest possible rank in O_Γ is 1 and the highest possible rank is 21.

O_{poset}: As several other nations are isolated elements, Ireland becomes equivalent to many others in the final weak order. As there is no weight vector which can be varied, Ireland's ranking position as an isolated element is slightly better!

Table 12.5 Simulation of the ranks of Ireland. The term "standard" refers to the Hasse diagram, Fig. 12.1

Indicator	$\|P\|$	$\|S\|$	$\|U\|$	What happens
Standard	1	0	19	(See Fig. 12.1)
wb	1	0	19	–
hs	1	0	19	–
ed	0	0	20	Ire is an isolated element in the Hasse diagram
fa	1	0	19	–
br	0	0	20	Ire is an isolated element in the Hasse diagram
sub	1	0	19	–

12.9.2.3 Consequences for Management

As the indicator br may be the outcome of the situation of the child, which is also partially described by other indicators, the recommended activity should concentrate on the indicator ed. In fact, the indicator educational well-being has three more basic perspectives (see Table 12.1):

- achievement at age 15 with:
  1. average achievement in reading literacy
  2. average achievement in mathematical literacy
  3. average achievement in science literacy
- *aspiration*: percentage aged 15–19 remaining in education and
- *participation*: the transition to employment with
  1. percentage aged 15–19 not in education, training, or employment
  2. percentage of 15-year olds expecting to find low-skilled work

Therefore the activities to give Ireland better positions should be concentrated on educational well-being with all its aspects.

12.9.3 Denmark

We select three indicators:

(1) sub, the indicator where Denmark has its worst score
(2) ed, an indicator where Denmark gets a middle position and
(3) wb, an indicator where Denmark has the maximal score.

In the first two cases, we examine what happens if Δ is increased step by step.

In the third case, we reduce the indicator wb step by step. Once again we apply the two-step procedure as explained in Section 12.9.1.

The results are listed in Table A.9.

In Fig. 12.13, the threshold values of $|\Delta_u|$ and $|\Delta_d|$ (see Fig. 6.11) are shown.

Figure 12.13 shows, for example, that one has to invest more than five points into the indicator ed until a change in terms of partial order characteristics appears (in contrast to sub, where already two points are sufficient).

The study shows that

- improving sub by one point does not change the Hasse diagram; however, by a change of two points, Denmark gets four successors and consequently $|U|$ decreases.
- improving ed by five points does not change the characteristics of Denmark. When Denmark gets in ed, the rank 20 $|S(\text{Dk})|$ increases by one unit.

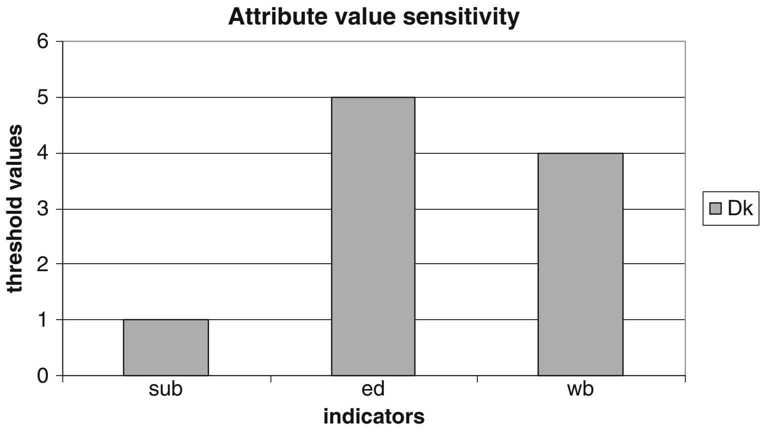

Fig. 12.13 Threshold values of $|\Delta_u|$ and $|\Delta_d|$ of Dk

- a change for the worse in indicator wb, material well-being, by four points does not change the Hasse diagram. If, however, the rank of wb is changed to the value 13, Denmark loses one successor and $|U|$ increases by one.

12.10 Summary and Commentary

This chapter dwells on two main lines of arguments:

1. How far can we verify the results of a study based on weight vectors by alternative methods?
2. Do we find additional insights by partial order?

We show that the results of UNICEF based on a weight vector are pretty coincident. The coincidence of levels, provided by partial order theory with three subsets derived from the UNICEF ranking, can be questioned because of the rather rough classification into three states. However, the linear orders derived from more sophisticated partial order techniques do not differ much from those based on an index. The partial order confirms the UNICEF results although they are based on an equal weighting of the indicators. If weights are considered as uncertain, then the application of Eq. (3.19) allows to check which nation would have a chance to get better ranking positions just by proposing other weighting schemes. Germany, for example, has little possibilities to improve its ranking position. The m^2 order shows this fact unambiguously.

Does an index hide important results? This second question can be answered with yes. When the partial order is considered, then its network of cover relations is of concern. This network of lines has much to do with "where is an object and why is it, where it is." Therefore, the importance of indicators for the Hasse diagram is to be considered and the indicator "family" is the most important one, whereas the indicator "behavior and risk" is least important.

Reference

UNICEF (2007). Child poverty in perspective: An overview of child well-being in rich countries. Innocenti report Card 7. Florence: UNICEF Innocenti Research Centre.