# Chapter 1
# Why Prioritization, Why Ranking

## 1.1 Motivating Issues and Situations

Let us begin with examples.

### *1.1.1 Chemicals*

Chemicals can be as harmful to humans and the environment as they are useful. Therefore, it appears rather clear that only those chemicals should be used in the market that do not have an adverse impact on humans and the environment. The list of chemicals in the market of the European Union between 1971 and 1981 (EINECS list, http://chemicalwatch.com/927) contains 1,00,000 chemicals and almost 1,000 chemicals newly enter the market yearly, see, e.g., Bruggemann and Drescher-Kaden (2003), van Leeuwen et al. (1996), and Ahlers (1999). How do we find out whether they are hazardous? There are many time-consuming and expensive investigations necessary to perform a risk assessment. Hence the question is: With which chemicals to begin at first? Thus ranking is needed to give the more involved investigations a reasonable operating sequence (Newman, 1995). Once accepted that a ranking is needed, we discover that there is no intrinsic property of a chemical which tells us that it is hazardous. Still worse, one needs to know the hazard of chemicals in different scenarios. Hence, several aspects of a chemical need to be simultaneously considered. And thus the final and central question arises: How to rank chemicals characterized by several attributes?

Examples related to chemicals are given in Chapter 11.

### *1.1.2 Child Well-Being*

In a report of UNICEF, a ranking of 21 rich nations was performed with respect to child well-being. For this purpose, 40 attributes were identified characterizing each country. From these 40 attributes, 6 were constructed by which the countries were ranked. It is clear that each of these six rankings need not be the same. Therefore, a

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composite indicator was defined, giving each of the six indicators the same weight. How far is this justified? What influence does this kind of aggregation have on the final result? Italy, for example, could get a better position if the indicator "family" would get more weight on the index. How can we analyze the role of weights? We discuss this in more detail in Chapter 12.

### *1.1.3 Regional Pollution*

Geographical sites can be ranked with respect to their pollution. The natural question is then: What constitutes pollution, and how to measure it? For example, the Environmental Protection Agency (EPA) in Baden-Wuerttemberg, Germany, performed over years a careful monitoring study and included many possible targets, like herb layer and tree leaves. One may think of highly polluted regions as "high spots or hot spots." How to find them becomes an issue if there is a joint pollution by several chemical elements (see Chapter 11).

### *1.1.4 Integrity of Watersheds*

Scientists of the Atlantic Slope Consortium (ASC) developed three levels of indicators to describe the health of watersheds. The indicators of the three levels increase in quality and accuracy of the data as well as the amount of cost and efforts needed to obtain the data. An important question is about how well level one or level two indicators perform compared with level three indicators. Partial order may help with this question (Chapter 14).

### *1.1.5 Surface Water Management Strategies*

High concentration of nutrients such as phosphorus or nitrogen in surface waters is of much concern for environmental protection agencies. What could be done to improve the situation? Clearly one has to study the release paths by which nutrients enter the surface waters. Then one has to develop strategies to control these and limit the emissions into surface waters. How well do such strategies work? In Chapter 11, we analyze 15 management strategies developed to reduce the concentration of nitrogen in the surface waters of the river Elbe basin. Each strategy is characterized by eight indicators according to the path through which nitrogen enters the surface water. Is there a best strategy? Can we compare, for example, rural and technical strategies? Unfortunately they turn out to be incomparable when partial order is applied. However, we use tools to facilitate the comparison without having to develop weights for all the eight indicators.

## 1.2 Composite Indicators

Composite indicators are applied and constructed everywhere. Saltelli et al. (2008) characterize the issue of composite indicators as follows: "Composite indicators tend to sit between advocacy (when they are used to draw attention to an issue) and analysis (when they are used to capture complex multidimensional phenomena)." The construction of composite indicators (OECD, 2008) can be described in the following six steps:

(1) Aim: What is to be indicated?
(2) Do we have a measure for that aim? If not already measurable, one mostly needs a set of several scalar indicators as proxies to describe the aim. Depending on the inherent complexity of the aim and the information available, the set of indicators may be pretty large or small.
(3) How to select these indicators which can serve as a basis for construction of the composite indicator?
(4) If the initial set of indicators is large, then it is convenient as an interim step to aggregate them as per commonalities. These interim aggregations are called pillars, from which the composite indicator is built. How do we construct the pillars? How far do we accept contextual overlapping, i.e., that one indicator describes partially the same aspects as another? Beyond this, orientation aspects are assessed.
(5) How to obtain the composite indicator from the pillars? If conceptual simplicity prevails at this stage, one may combine the values of the pillars by a weighted sum.
(6) Since weights often come under scrutiny and controversy, it is a good practice to test the composite indicator for its robustness and sensitivity to results in response to varying weights.

For any of these six steps, concepts and methods are available to assess and use them. Besides expert judgments, univariate methods and multivariate methods are available. It is imperative that composite indicators deliver not only rankings but also satisfactory metrics.

## 1.3 What Does Partial Order Offer with the Composite Indicator Given?

In order to understand why and where partial order can be of help with the above steps, we provide a preliminary explanation of partial order: Partial order theory is a discipline associated with discrete mathematics and its subdiscipline, graph theory. In the case of a suitable binary relation between two objects, partial order theory is the theory by which objects, characterized by multiple indicators, can be compared and ordered, (see Chapter 2). The order relations can be displayed as in graph theory. Therefore, partial order and graph theories have many common topics.

Using composite indicators, objects of interest can be compared and an important application is to deduce through the scalar values of the composite indicators a ranking of the objects. Partial order as the theory of order is applied to the set of objects and it delivers insights into the six steps which result in ranking of objects. Besides helpful scientific insights into the above six steps, partial order renders results which help clarify the roles and consequences of indicators and their weights.

Let us now describe what partial order offers for the six steps:

(a) We can derive a measure by which the set of indicators can be checked for its appropriateness and completeness as a proxy for a non-measurable nevertheless important aim. We call this measure an “ambiguity graph” and introduce it in Chapter 4.
(b) The composite indicator depends on the functional form and especially if a linear combination is selected, it depends on the weights. Independent of whether there is uncertainty about the functional form or about the weights, partial order can derive subsets of objects whose relative rankings are invariant with respect to the functional form selected or the weights. In this connection, a key concept in partial order theory is that of a chain. We introduce the concept in Chapters 2 and 3. It appears almost everywhere in the monograph.
(c) Because of the averaging process in the weighted sum, the individual role of a single indicator cannot be easily traced back. Partial order theory offers some tools to overcome this difficulty. These tools are developed within the context of stepwise aggregation: Start with an indicator, add the next, see what happens until the composite indicator is finally attained. In Chapter 7, devoted to stepwise aggregation, several tools are explained, such as “comparability acquisition profile” (Patil, 2001).
(d) The averaging process, mentioned in (c), may affect objects in different ways if weights are uncertain. Uncertainty concerning the weight values results in a rank interval indicative of ambiguity in the ranks of objects. Partial order theory provides an upper limit for the ranges of ranks of objects (“rank ambiguity”) due to a set of possible weight vectors (Bruggemann et al., 2001; Patil and Taillie, 2004). We discuss this point in Chapter 3 and we revisit this ambiguity concept in Chapter 7. There, we conduct a Monte Carlo simulation in changing the weights and in observing the corresponding rank frequency distributions for the objects in response to the varied weights.
(e) The crucial role and consequence of weights are well known. Therefore, we offer a method to deduce weights from the data matrix alone, where the rows are defined by the objects and the columns by the indicators (Patil, 2001). We discuss this method in a case study about watersheds in Chapter 14.
(f) Partial order theory also offers several methods to obtain linear orders of objects (with or without ties). Therefore, one of these partial order methods could be selected if a ranking is wanted that does not need to weight the indicators. One may even compare the ranking due to a composite indicator with that obtained from partial order theory if there are no uncertainties. We discuss how to obtain a ranking from partial order in Chapter 9 and how to compare different partial orders in a rather general setting in Chapter 10.

**Table 1.1** Data matrix with indicators of different scaling levels

| | $q_1$ (continuous in concept) | $q_2$ (linguistic description) | $q_3$ (ordinal indicator) |
|---|---|---|---|
| $x_1$ | 0.3 | Good | 2 |
| $x_2$ | 0.35 | Medium | 3 |
| $x_3$ | 0.2 | Bad | 1 |

(g) Partial order can work even if the data matrix consists of indicators of different scaling levels as shown below. This kind of a situation occurs often in scientific fields where quantitative measures are difficult to obtain (Table 1.1).

Chapters 3, 4, 7, 9, and 10 and to some extent Chapter 14 help enlighten with steps (1)–(6) to construct composite indicators.

## 1.4 What Does Partial Order Offer More Generally?

Partial order, in its own right, delivers results, some of which may not be restrictive to steps (1–6) but nevertheless help understand the impacts of indicators on the objects in a multi-indicator system.

Such results are important, especially when crunching the indicators into a composite indicator is not an option, as it was the case in a study of pollution in a state of Germany. There the measurements of single indicators were so expensive that an averaging into a composite indicator was seen to be too disadvantageous (see Chapter 11).

The Hasse diagram, which is a graph theoretical visualization of a partially ordered object set, is an ideal tool if the number of objects is not too large. Striking feature in Hasse diagrams is the concept of incomparability, which appears if the order of objects due to one indicator contradicts the order of another indicator. Chains can be easily identified from a Hasse diagram. If the Hasse diagram is too messy to get chains by inspection, software tools help find chains. Many concepts can be motivated just by discussing them within a simple Hasse diagram but are still valid even when the Hasse diagram loses its visual appeal, because it is too complex and messy. These important concepts follow.

## 1.5 Important Questions and Concepts Involving Partial Order

(a) *Where is an object, and why is it where it is?* This question aims at the position of an object in a Hasse diagram, identifies its minimum rank, its maximum rank, and the objects which are incomparable to the object under consideration. Subsets of objects can be characterized by their pattern of indicator values as shown in Fig. 1.1. There a Hasse diagram is constructed of regions in a state in

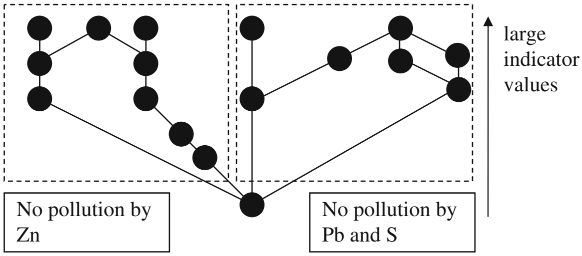


**Fig. 1.1** Example of a Hasse diagram, where parts of it can be characterized by certain indicator values

Germany, polluted by lead (Pb), cadmium (Cd), zinc (Zn), and sulfur (S). The vertices are representing the regions and the lines comparabilities.

In many cases, formal concept analysis (Chapter 8), which is part of partial order theory, provides a powerful visualization where the information about the indicator values and the positions of objects is simultaneously available. Formal concept analysis allows a "symmetric analysis" of the data matrices (Annoni and Bruggemann, 2008).

(b) *Which indicators influence the positions of objects?* Although this question is not in the foreground of the construction of composite indicators, the sensitivity of indicators to a Hasse diagram and hence to the objects in it throws also a light on the selection and interpretation of the indicators. For example, in a study about fish communities in wetlands, it turns out that the indicator describing the population of a certain fish species has a high impact on the Hasse diagram displaying the impacts of the indicator on the positions of the objects. This is directly related to the strategies of that fish species to survive under competition and bad water quality.

(c) *How to model ordinally – a fuzzy approach in partial order?* Partial orders appear in many facets, depending on how the order relation is defined. As we use partial order to rank objects described by a tuple of indicator values, we must use an appropriate order relation, the product order (Chapter 2). Often data matrices contain indicators continuous in concept so that even small numerical differences can influence the partial order. In Chapter 6 we discuss several techniques to perform "ordinal modeling," i.e., how far we can ignore numerical differences which seem to be too small for being interpreted as an order relation. An important concept therefore is that of fuzzy partial order. A fuzzy membership function is introduced which describes as to how to rate an object above or below another even when their data profiles crisscross. We apply fuzzy partial order to a data matrix concerning the effects of biomanipulation and we see how biological competitions among phytoplankton species suppress the basic information about biomanipulation.

So, our answer to "why partial order?" can be summarized as follows.

Partial order delivers analytical tools to better interpret and understand how from a multi-indicator system a composite indicator is built and what can be said about relative rankings of objects without the need of specifying weights or even the functional form of aggregation.

This monograph describes the partial order concepts, methods, and tools within the first ten chapters and applies them to the case studies in subsequent five chapters. We now consider some choice examples of interesting issues and questions.

## 1.6 Pertinent Issues and Questions

### *1.6.1 Weights and Indicator Values*

Could Italy improve its ranking just by scrutinizing the weights? We render in Chapter 12 how the indicators of interest can be found and that in the case of Italy a higher weight for the indicator "family" would improve the position of Italy in the final ranking. Can Germany do the same and try to be better in the final ranking than Netherlands? Our analysis shows: No chance! Germany must improve its values of the indicators. Change in weights will not help. Does Germany have a chance to get a better ranking position in comparison to some other nations? Yes, since Germany is incomparable to some other nations, changing weights would influence the final ranking position of Germany relative to these nations. Must we rely on different trials of weights to see what can happen? No, since we can show that the possible ranking interval of each object depends on a simple characteristic of the partial order as discussed in Chapter 3.

### *1.6.2 Problems with Averaging*

Within a study of bridge stability crossing channels, bridge 17 got a worse evaluation in comparison with bridge 57. As proxies for "bridge stability," indicators are defined which have influence on the bridge stability, such as channel alignment, local channel characteristics, and bank stability.

Analysis by partial order tools shows that because bridge 17 is incomparable to bridge 57, bridge 17 must have at least one indicator where bridge 17 is better than bridge 57, which is classified as an "excellent" bridge/stream system. We can now identify which property makes bridge site 17 better than bridge site 57: It is the channel alignment. Although the ordinal way of consideration of objects implies loss of some information, we see that some important information is not lost. The remaining information is crucial. It is, however, unavailable if only the composite indicator is considered.

### *1.6.3 Associations and Implications of Indicators*

During the investigation of the environmental performance indicator (EPI), the interest lies in how differently nations of different regions of the world are ranked. As

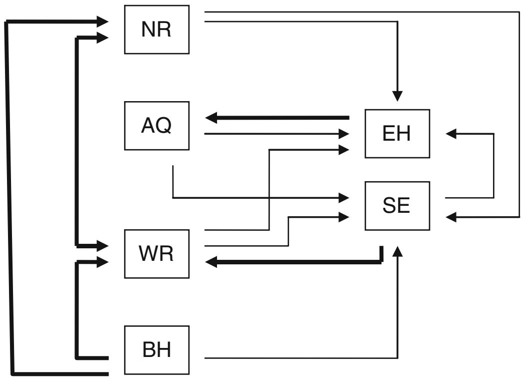


**Fig. 1.2** Association and implication network among the indicators of the EPI study. The *simple arrows* represent associations or implications found for nations of the EU and the *bold arrows* represent those of the nations of ASEAN

examples, we selected ASEAN and EU nations. Partial order applied to the nations as objects does not yield useful results. There is too high a degree of incomparabilities. This throws light on why weights are needed, but at the same time on problems with them. On the one hand, without weights a ranking would mean to fight losing battles because of too many incomparabilities. However, on the other hand, any incomparability implies a compensation: Good values in one indicator may average out bad values of other indicators and vice versa in getting a composite indicator.

So instead, we studied the association and implication structure of the indicators of the ASEAN group and EU and we found rather different associations (see Fig. 1.2, where we present the results as a joint network).

Note that in ASEAN the resource aspect is more pronounced (natural resources are associated with water resources and vice versa), whereas in the EU, this striking fact is not observed. Details, see Chapter 15.

### *1.6.4 Prioritization and Ranking for a Subset of Objects ("Hot Spots")*

We may not always be interested in a comparative analysis of all objects of an object set. Instead we may want to put our fingers on those objects which have, for example, high values in some indicators. Such objects may serve as candidates for a more detailed scrutiny. It could be, for example, because we want to study them further by using more information about them. A very simple transformation applied to any single indicator may do that job. However, we have a multi-indicator system and a partial order in our hand and we must ask: Is the transformation compatible with the partial order and how does the simultaneous application of the transformation affect the objects? In Chapter 6, we show that the transformation is compatible with the partial order and derive an equation which estimates the fraction of relevant and irrelevant objects.

### 1.6.5 How Do We See the Role of Indicators in Terms of Single Objects?

How does the minimum rank of an object vary with the cumulation of indicators in a canonical sequence? In Chapter 4, we see that some characteristic partial order quantities of an object, such as its number of incomparable elements or the number of elements below it, vary with the canonical sequence and apply these ideas to child well-being (Chapter 12), bridge stability (Chapter 13), and watersheds (Chapter 14). As can be expected, some objects will vary strongly, whereas some others may not.

### 1.6.6 Proximity Analysis

For a robustness study of a composite indicator relative to weights, a distance measure is needed. We introduce proximity analysis (Chapter 10) and apply it to watersheds, where three levels of sophistication are considered. It is of interest to know as to how far indicators of a level of low degree of sophistication can serve as proxies for indicators of a level of high sophistication. We show that the low-cost indicators (level one) are better proxies for high-cost indicators (level three) than are level two indicators.

### 1.6.7 What to do with supervised classification?

Often the object set is partitioned into disjoint subsets using external information. We need to compare the resulting classes with each other. In order to do this, we employ concepts of dominance and separability (Restrepo et al., 2008). Whereas dominance is conceptually a generalization of an order relation, separability measures the degree of incomparabilities among two disjoint object subsets (Chapter 5).

*Dominance*: A typical question is: What are the dominances among European nations, classified due to their geographical positions, when, for example, the Human Environment Index (HEI) is considered. The dominance analysis shows that nations of south Europe are dominating almost all the others. This kind of a question with its potential to simplify complex Hasse diagrams serves as an attractive application of dominance analysis.

*Separability*: If the separability gets by definition its maximum value of 1, a natural question arises as to which indicators are responsible for the fact that no object of one object set is comparable with an object of another object set. If, for example, it turns out that regions of high agricultural density are separated from those of high industrial activity when pollution is of concern, then the question is: Which pollution indicators are responsible for the separatedness? We introduce an important concept, namely that of antagonistic

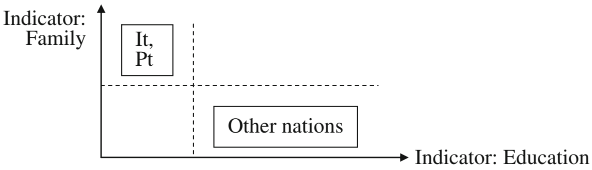


**Fig. 1.3** Indicators "family" and "education" are antagonistic with respect to the set {Italy (It), Portugal (Pt)} and the set of other nations

indicators: A minimum set of indicators needed to explain the separation of two object subsets. In Chapter 12, child well-being, the indicators "family" and "education" are antagonistic in their role of explaining the separation of two subsets of nations. Italy and Portugal are good in "family" whereas less good in "education." For the multitude of other nations, the reverse is true (Fig. 1.3).

### *1.6.8 Visualization in Multi-indicator Systems*

Statistics provides powerful methods to visualize even large data matrices. Does partial order with its focus on comparison provide visualization tools? An important graphical representation is the Hasse diagram. However, Hasse diagrams lose in general their appealing charm if the number of objects is too large. Myers and Patil (2010) developed visualization alternatives. Another well-known visualization tool is POSAC (partial order scalogram analysis with coordinates) described in Chapter 3. POSAC is applied on Internet sources about drinking water quality in Germany (Chapter 11), where partial order dimension analysis is also of help. In Chapter 14, watershed evaluation, weights are derived from the data matrix using POSAC.

## 1.7 Organization of Our Book

This monograph focuses on partial order and its applications in different scientific fields:

- We first explain what a partial order is and then provide a graphical display, called a Hasse diagram, where the objects to be ranked are positioned in a network-like graph.
- We discuss how far we can help with the selection of attributes helpful for ranking.
- We show the intricate role of attributes and the positions of objects in the Hasse diagram.
- We find data-driven rankings without the intervention of stakeholders.

Finally, we will focus on different examples.

The monograph consists of four parts:

(I) a basic theoretical part;
(II) illustrative case studies;
(III) live case studies;
(IV) appendix with data matrices and additional material.

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