Chapter 1
Introduction

Ay! There are times when the great universe
Like cloth in some unskilful dyers' vat
Shrivels into a hand's-breadth, and perchance
That time is now! Well! Let that time be now.
Let this mean room be as that mighty stage
Whereon kings die, and our ignoble lives
Become the stakes God plays for.

Oscar Wilde: A Florentine Tragedy

The universe, ultimately, is to be described by quantum theory. Quantum aspects of all there is, including space and time, may not be significant for many purposes, but are crucial for some. And so a quantum description of cosmology is required for a complete and consistent worldview. At any rate, even if we were not directly interested in regimes where quantum cosmology plays a role, a complete physical description could not stop at a stage before the whole universe is reached. Quantum theory is essential in the microphysics of particles, atoms, molecules, solids, white dwarfs and neutron stars. Why should one expect this ladder of scales to end at a certain size? If regimes are sufficiently violent and energetic, quantum effects are non-negligible even on scales of the whole cosmos; this is realized at least once in the history of the universe: at the big bang where the classical theory of general relativity would make energy densities diverge.

One might ask a quantum theory of *what* should be considered. The classical universe is described by general relativity, which may be quantized on its own if its degree of freedom, space–time geometry, is seen as fundamental. Alternatively, general relativity might be seen as ultimately being a phenomenological continuum theory, much as in hydrodynamics. By itself, it would not reveal what the fundamental, microscopic degrees of freedom should be. Nonetheless, general relativity serves as a crucial guideline in constructing a quantum theory of gravity, for it is to be reproduced as the semiclassical limit on a certain range of scales, including those on which we currently probe the universe. This by itself is challenging enough a task owing to the existence of many mathematical consistency conditions. Most

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current theories indeed point to the presence of new microscopic entities, and they provide insights into some of their properties—be they described as strings, loops or something else. Irrespective of what exactly is realized, general relativity must be extended for it is singular; and quantum theory must play a role.

Quantum gravity applies to many situations, most importantly early-universe cosmology and black holes. Cosmology complicates and simplifies considerations at the same time. It comes with severe conceptual problems of how to interpret the wave function of the whole universe, with all observers having to be situated within the system. Despite many activities for several decades, a proper understanding of this situation remains a challenge. But there is also an advantage in this context: the cosmological principle, which states the assumption of homogeneity on large scales and is by now well justified by extensive galaxy maps, reduces the number of degrees of freedom. One obtains a technically simpler framework, which is helpful for testing existing general theories but also provides possible physical insights.

Much of quantum gravity is part of mathematical physics due to the heavy tools required. But one should keep in mind that the objective is quite different from usual mathematical edifices: quantum gravity at present is not constructed on firm ground; it rather grows toward a certain, vaguely formulated aim. Some principles must certainly exist and be used, but there are no generally accepted axioms from which one could start, stepping ahead theorem by theorem. This situation often makes developments, even crucial ones, appear fuzzy. Nevertheless, progress is clearly visible by models becoming more and more controlled and realistic and, put the other way, by several developments having been ruled out with later progress.

Accordingly, the focus in this book will not be so much on specific cases, unless they illustrate key features, but rather on a general framework which, at the current stage in time and in the author's personal opinion, summarizes distinctive properties of quantum cosmology. This guide through the scaffolding should provide readers interested in working on those problems with a quick route to the construction site, discussing tools and stating open issues to provide an entrance into this rather messy field. Keep in mind that, while the final building is likely to follow the shape and height already indicated, the scaffolding itself eventually will have to be torn down. But before the building stands, the reader is advised not to pay too much attention to all kinds of details: one should not measure carpets before the walls are set down.

There are two main instances in which quantum physics arises: quantum dynamics and quantum geometry. Quantum dynamics includes the usual conceptual problems of measurements, observables and the role of quantum variables such as fluctuations and correlations. Especially in quantum cosmology it also, ultimately, requires one to understand the meaning and the arrow of time. As general issues, all this is rather insensitive to the specific realization of quantum space–time, and can thus be found and analyzed already in the Wheeler–DeWitt formulation of quantum cosmology started in the 1960s. Quantum geometry, on the other hand, depends more sensitively on the quantization framework used. Here, the structure of space and time on their smallest, possibly atomic scales and their refinement in the course of time is crucial. The most specific such realization so far has been made within the framework of loop quantum gravity, which will be introduced and used throughout this book. But all general aspects, whenever applicable, will be discussed with as small a number of ingredients from a loop quantization as possible.

We will begin the exposition with a rather detailed discussion of quantum theory in the context of cosmology. This introduction will show why an atomic understading of space–time structure is relevant, a specific form of which is then provided using the methods of loop quantum gravity. In this canonical quantization one starts with a "kinematical" quantization of spatial geometry, already illustrating the discrete nature. Dynamics then shows how such atomic structures change in time according to a quantum Hamiltonian. At this stage, control over atomic quantum space–times will be gained.

An analysis of dynamical equations in general is complicated, especially if gravity is involved which requires self-interaction and non-linearity. Part II of this book will introduce the key tool to a manageable analysis: effective descriptions. They will first be applied to quantum cosmology in the Wheeler–DeWitt form, and then to loop quantum cosmology which introduces additional non-linearities. At this stage, we will see the first intuitive mechanism of resolving singularities by repulsive quantum forces. At the same time it will become clear that quantum dynamics, not just quantum geometry, is highly relevant for understanding the big bang. With these results, a discussion of what the actual meaning of resolving singularities might be will be given.

The third part extends constructions and results from isotropic models to several less symmetric cases, first to anisotropic ones which also include models of the Schwarzschild black-hole interior. Here we will see the first applications to black hole dynamics. (There are other important applications of quantum gravity to black holes, mainly in the context of black-hole thermodynamics. They are not part of this book since those methods differ considerably from what one uses in quantum cosmology. This line of research has so far provided scant insight about the dynamics.) General black-hole models, including a phase of gravitational collapse and possibly one of Hawking evaporation, require inhomogeneous geometries. Spherically symmetric ones are the simplest among those and will be discussed in detail. Also here, results about singularity resolution are available, but still inconclusive. By similar constructions one can describe models such as Einstein–Rosen waves or those of Gowdy type, which do have local gravitational degrees of freedom. They provide further interesting examples of singularities and possible resolutions, but investigations have only just begun. Inhomogeneities can also be introduced as perturbations on a homogeneous background space–time, which brings us back to applications in cosmology.

Part IV is a discussion of the typical mathematical issues involved in quantum cosmology: properties of difference equations, the derivation and use of physical Hilbert spaces, and general aspects of effective descriptions.[1]

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