Chapter 15
Identification of a Class of Hybrid Dynamic Systems

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Abstract The behavior of hybrid dynamic systems (HDS) switches between several modes with different dynamics involving both discrete and continuous variables in the course of time. Their identification aims at finding an accurate model of the system dynamics based on its past inputs and outputs. The identification can be achieved by two steps: the clustering and the regression. The clustering step aims at the estimation of the mode (discrete state) of each input–output data point as well as the switching sequence among these modes. The regression step determines the sub-models controlling the dynamic (continuous states) in each mode. In Pattern Recognition (PR) methods, each mode is represented by a set of similar patterns forming restricted regions in the feature space, called classes. A pattern is a vector built from past inputs and outputs. In this chapter, we propose to use an unsupervised PR method to realize the clustering step of the identification of switched linear HDS. The determination of the number of modes as well as the switching sequence does not require any information in advance about the modes, for example, their distribution, their shape, . . . , or their number.

15.1 Introduction

Pattern recognition (PR) is the study of how machines can learn from experience to make sound decisions about the categories or classes of patterns of interest. In statistical PR methods [10], patterns are described as random variables, from which

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M. Sayed-Mouchaweh and E. Lughofer (eds.), *Learning in Non-Stationary Environments: Methods and Applications*, DOI 10.1007/978-1-4419-8020-5_15,

class densities can be inferred. These variables carry discriminating information about patterns. They are called features, which are usually quantitative observations, or measurements, about patterns. Therefore, a pattern is represented by a set of d features so it can be viewed as a d-dimensional feature vector in the feature space.

PR involves two stages: preprocessing and classification. The aim of the preprocessing is to find features in such a way that patterns belonging to different classes occupy different regions of the feature space. The classification stage is a mapping of a pattern from the feature space into the decision one. The latter is defined by a set of predefined classes. This mapping is achieved using a classifier. The latter is a method or algorithm which generates a class membership function in order to classify unlabeled incoming patterns into one of the predefined classes. Depending on the information available for classifier training, one can distinguish between supervised [21] and unsupervised [4, 11, 12] learning. In the first case, called also classification, there exists a set of patterns with their class assignment or label, called learning set. The goal of supervised learning is to learn a set of membership functions that allows the classification of new patterns into one of the existing classes. The problem of unsupervised learning, also called clustering, arises if clusters', that is, classes, memberships of available patterns, and perhaps even the number of clusters, are unknown. In such cases, a classifier is learned based on similar properties of patterns: patterns belonging to the same cluster should be as similar as possible and patterns belonging to different clusters should be clearly distinguishable. Hence, the goal of clustering is to partition a given set of patterns into clusters based on their similarity.

One of the applications of PR is the identification of hybrid dynamic systems (HDS). The latter are characterized by the interaction between continuous time dynamics and discrete events or logic rules [6,15]. The identification aims at obtaining an accurate model of the system dynamics based on its past inputs and outputs. The problem of obtaining a model of a hybrid system from a given set of input–output data has attracted, since few years, the attention of several researchers. Many models have been proposed to describe them as piecewise autoRegressive with exogenous inputs (PWARX), switched AR (SAR), switched ARX (SARX), switched nonlinear ARX (SNARX) and PW nonlinear ARX (PWNARX) ones [2,14,22].

Generally, the identification is divided into two steps: clustering and regression. In the clustering step, the discrete modes, that is, classes, that each input–output data point belongs to as well as the switching sequence among these modes are estimated. The regression step aims at finding the models governing the continuous dynamic in each mode.

In this chapter, we propose an approach to achieve the clustering step of the identification of switched HDS described by SAR or SARX models. The latter are a particular class of HDS [14]. In this approach, the number of discrete modes, classes, and the switching sequence among them are estimated using an unsupervised PR method. This estimation is achieved without the need to any prior information about these modes, for example, their shape or distribution, or their number.

This chapter is organized as follows. In Sect. 15.2, the principles of the proposed approach are detailed. Then, its performance is evaluated using two examples. The

first one is an example of HDS, modeled as SARX, switching among 3 modes. The second example exploits the acoustic signals used to detect a leak in the steam generator unit of the nuclear power generator prototype fast reactors (PFR). These signals record the noises in the steam generator unit. The latter functioning, described as SAR model, switches between two modes (normal and faulty representing a leak) in several time instants. The advantages and the drawbacks of the proposed approach according to the ones of literature are discussed in Sect. 15.4. We finish the chapter with a conclusion and the future work.

15.2 Proposed Approach for the Identification of HDS

In this section, we present an approach to achieve the clustering step of the identification of switched HDS. This approach determines the number of modes or classes, $i = 1,\ldots,c$ and the switching sequence $\lambda_j, j = 1,\ldots,N$ using a historic of N observations of the system input u_j and output $y_j, j = 1,\ldots,N$.

The proposed approach is based on two phases: the feature space construction and the modes estimation ones. The first phase aims at finding the features, based on the input–outputs data points, leading to well separate the modes in the feature space. The second phase uses the unsupervised fuzzy pattern matching (FPM) as a clustering method to determine the number of modes and to learn their membership functions. The performances of the mode estimation phase are evaluated by the closeness of the number of modes and of the time instants of switching among them to the real ones. These performances depend on the discrimination power of the feature space. Better the modes are separated in the feature space, better the modes estimation is. Figure 15.1 illustrates these phases of the proposed approach.

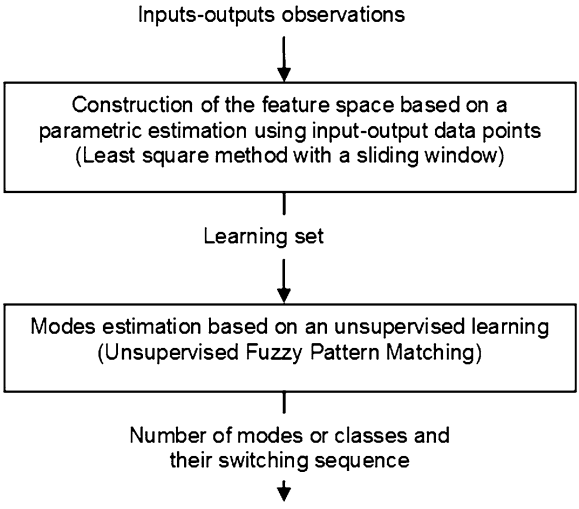

Fig. 15.1 Phases of the proposed approach

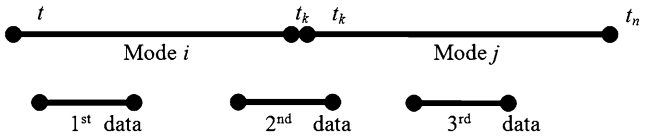

Fig. 15.2 Illustration of the feature space construction

15.2.1 Feature Space Construction Phase

Let $[u_0\ y_0, u_1\ y_1, \ldots, u_{k-1}\ y_{k-1}, u_k\ y_k]$ be the set of the system past and actual input–output observations, where $u_k \in \mathbb{R}$ and $y_k \in \mathbb{R}$ are, respectively, the input and output at time step k. The goal of this phase is to build the feature space from past inputs and outputs. In the obtained feature space, the pattern at the actual time step k is defined as follows: $x_k = \Phi(u_{k-1}, y_{k-1})$, where Φ is the mapping from the observation space, that is, input–output space, to feature one. In this chapter, we propose to use the Least Square Method (LSM) with a sliding window [17] to estimate the parameters of the continuous dynamic of each mode. In this method, the parameters about the system dynamic modes are estimated at instant t so that it minimizes the sum of the squares of the differences between the output of the system and the one of the prediction model over a sliding window of t measurements. These parameters are then used as features to represent the different modes or classes.

Hence, in order to construct the feature space, we propose to estimate the sub-models parameters using a sliding window as shown in Fig. 15.2. The latter shows clearly that two cases are possible: the first one corresponds to the use of the data points of the current mode (i.e., the first and the third data sets) and the second one corresponds to the case when the data points of the current and the successor modes are used to estimate the parameters (i.e. the second data set). Thus, the parameters estimation procedure will provide different sets of parameters. Some of these sets represent the real modes of the switching system (for example the parameters sets estimated using the first and the third data sets) and other sets represent some biased models (e.g. the second data set in Fig. 15.2). The latter case will be distinguished in the modes estimation phase as belonging to two different modes.

15.2.2 Modes Estimation Phase

In order to determine the number of modes contained in the learning set $X = [x_N, \ldots, x_k, x_{k-1}, \ldots, x_1]^T$ and to learn their membership functions, we use unsupervised FPM which is a developed version of the original supervised FPM [7]. The proposed unsupervised FPM has an agglomerative characteristic. Thus, it does not require any prior information about the number of classes. The classes' membership functions are constructed sequentially with the patterns arrival. According to the

ratio $r = \frac{L}{U+L}$ of the number L of labeled points to the one U of unlabeled points, the proposed method can be totally supervised, $r = 1$, or totally unsupervised, $r = 0$. The functioning of unsupervised FPM is divided into detection, adaptation and fusion steps.

15.2.2.1 Classes Detection Step

Let $x = (x^1, x^2, \ldots, x^d) \in \mathbb{R}^d$ be a given pattern vector in a feature space constituted of d parameters or attributes. Statistically, the features are assumed to have a probability density function (pdf) conditioned on the pattern class. Thus, a pattern vector x belonging to the class C_i is viewed as an observation drawn randomly from the class-conditional probability function $p(x/C_i)$. Each attribute is divided into equal intervals defining the bins of the histogram according to this attribute. This histogram is used to estimate the conditional probability density for the class that x is driven from. Let $X^j_{\min}$ and $X^j_{\max}$ be, respectively, the lower and upper borders of the histogram according to the attribute j. These borders can be defined as the minimal and maximal values of all the patterns of the learning set X according to each attribute or parameter. Let h be the number of histogram bins, then each bin, according to the attribute j, has the larger:

$$\Delta^j = \frac{X^j_{\max} - X^j_{\min}}{h}, \quad j \in \{1, 2, \ldots, d\}. \tag{15.1}$$

Thus, the limits of these bins are defined as follows:

$$\begin{aligned} b^j_1 &= [X^j_{\min}, X^j_{\min} + \Delta^j], \quad b^j_2 = [X^j_{\min} + \Delta^j, X^j_{\min} + 2\Delta^j] \\ b^j_h &= [X^j_{\min} + (h-1)\Delta^j, X^j_{\max}], \quad j \in \{1, 2, \ldots, d\}. \end{aligned} \tag{15.2}$$

Generally, the histogram or the distribution of probability

$$\left\{ p^j_i(b^j_{ik}), i \in \{1, 2, \ldots, c\}, j \in \{1, 2, \ldots, d\}, k \in \{1, 2, \ldots, h\} \right\}$$

for a class C_i according to the attribute j is determined by calculating the probability $p^j_i(b^j_{ik})$ of each bin b^j_{ik}:

$$p^j_i(b^j_{ik}) = \frac{n^j_{ik}}{N_i}, \tag{15.3}$$

where n^j_{ik} is the number of points of the class C_i which are in the bin b^j_{ik} and N_i is the total number of points of the class C_i. The resulting distribution of probability is transformed into a distribution of possibility

$$\left\{ \pi^j_i(b^j_{ik}), i \in \{1, 2, \ldots, c\}, j \in \{1, 2, \ldots, d\}, k \in \{1, 2, \ldots, h\} \right\}$$

by using the transformation of Dubois and Prade [9]:

$$\pi_i^j(b_{ik}^j) = \sum_{z=1}^{h} \min(p_i^j(b_{iz}^j), p_i^j(b_{ik}^j)). \tag{15.4}$$

A membership function can be generated by considering the possibility distribution numerically equal to the fuzzy membership function [23]. The possibility distribution is more adapted than the probability one to estimate membership functions in the case of data infected by noises and uncertainties related to the features estimation [9]. Finally, the density of possibility Π_i^j of the class C_i according to the attribute j is obtained by a linear interpolation of the bins centers of the histogram of possibility.

The first incoming pattern x will be considered as the point prototype of the first class: $C_1 \leftarrow x, c \leftarrow 1$. If x is located in the bin $b_k^j, k \in \{1,2,\ldots,h\}$, then the probability histogram of C_1 according to the attribute j is: $\pi_1^j = \{p_{11}^j = 0, p_{12}^j = 0, \ldots, p_{1k}^j = 1, \ldots, p_{1h}^j = 0\}$. The possibility histogram will then be computed using (15.2). Since there is just one pattern, the possibility histogram is equal to the probability one. The possibility density of the class C_1 is obtained by a linear linking between the center of the bin b_k^j, which has the height 1, and the ones of its left b_{k-1}^j and right b_{k+1}^j neighbors, which have both at present the height 0. Generally, if $C = \{C_1, C_2, \ldots, C_c\}$ is the set of learned classes at present. Let x be a new incoming pattern which is not assigned to any of the learned classes (membership rejection). The detection strategy is defined as follows:

$$\pi_i(x) = 0,\ \forall i \in \{1,2,\ldots,c\} \Rightarrow c \leftarrow c+1, C_c = \{x\}, \pi_c = \left\{\pi_c^1, \ldots, \pi_c^j, \ldots, \pi_c^d\right\}. \tag{15.5}$$

15.2.2.2 Classes Adaptation Step

The local adaptation step aims at updating the classes' possibility densities after the classification of each new pattern in order to take into account the information carried by the new classified patterns in the class.

Let x' be a new pattern classified in the class $C_i, \forall i \in \{1,2,\ldots,c\}$. This classification is obtained by a projection of the pattern on the possibility density Π_i^j of the class C_i according to each attribute j and then merging the values according to all attributes using the aggregation operator "minimum." The point x will be assigned to the class for which it has the highest membership value. If the membership value $\pi_i(x')$ of x' to the class C_i is different of zero, then this pattern will be assigned to the class C_i and the possibility densities of this class according to each attribute will be updated. The goal is to take benefit of the information carried by the new classified pattern for the classification of the next incoming ones. To establish an incremental update of possibility densities, let $p_i^j = \left\{p_{i1}^j, p_{i2}^j, \ldots, p_{ik}^j, \ldots, p_{ih}^j\right\}$ and $\pi_i^j = \left\{\pi_{i1}^j, \pi_{i2}^j, \ldots, \pi_{ik}^j, \ldots, \pi_{ih}^j\right\}$ define, respectively,

the probability and possibility histograms of the class C_i according to the attribute j. Let $p_i'^j = \left\{p_{i1}'^j, p_{i2}'^j, \ldots, p_{ik}'^j, \ldots, p_{ih}'^j\right\}$ and $\pi_i'^j = \left\{\pi_{i1}'^j, \pi_{i2}'^j, \ldots, \pi_{ik}'^j, \ldots, \pi_{ih}'^j\right\}$ define, respectively, the updated probability and possibility histograms of the class C_i according to the attribute j after the assignment of x' to the class C_i. Let us suppose for the simplicity that: $p_{ih}^j < p_{i(h-1)}^j < \ldots < p_{i1}^j$, then these new probabilities can be computed incrementally by [21]:

$$x'^j \in b_k^j, \forall k \in \{1, \ldots, h\} \Rightarrow p_{ik}'^j = p_{ik}^j * \frac{N_i}{N_i + 1} + \frac{1}{N_i + 1}$$

$$p_{iz}'^j = p_{iz}^j * \frac{N_i}{N_i + 1}, \forall z \in \{1, \ldots, h\}, z \neq k. \tag{15.6}$$

Then the new possibilities can be computed using Dubois and Prade transformation defined by (15.2). Thus, the local adaptation step is defined as follows:

$$\pi_i(x') = \max_{z \in \{1,\ldots,c\}} (\pi_z(x')) \Rightarrow C_i \leftarrow \{C_i, x'\}, \pi_i' = \left\{\pi'^1, \pi'^2, \ldots, \pi'^j, \ldots, \pi'^d\right\}. \tag{15.7}$$

The flow chart of the detection and local adaptation steps of unsupervised FPM is presented in Fig. 15.3.

15.2.2.3 Classes Merging Step

The occurrence order of incoming patterns influences the final constructed clusters. This may lead to obtain several different partitions or number of clusters. Thus, several clusters can represent the same functioning mode. These clusters must be merged into one cluster to obtain one partition and one membership function. This fusion can be done using a similarity measure. The latter measures the overlap or closeness between constructed clusters. There are different similarity measures in the literature. Most of them are based on the computation of the degree of overlapping of clusters or the distance between clusters' centers. The clusters overlapping degree is based on the number of ambiguous patterns, belonging to several clusters, and their membership values to these clusters. If the number of these ambiguous patterns is large enough and their membership values to several clusters are high, then these clusters cannot be considered as heterogeneous anymore and must be merged. An interesting similarity criterion which takes into account at the same time the number of ambiguous patterns as well as their membership values is defined by [12]:

$$\delta_{iz} = 1 - \frac{\sum\limits_{x \in C_i \vee x \in C_z} |\pi_i(x) - \pi_z(x)|}{\sum\limits_{x \in C_i} \pi_i(x) + \sum\limits_{x \in C_z} \pi_z(x)}. \tag{15.8}$$

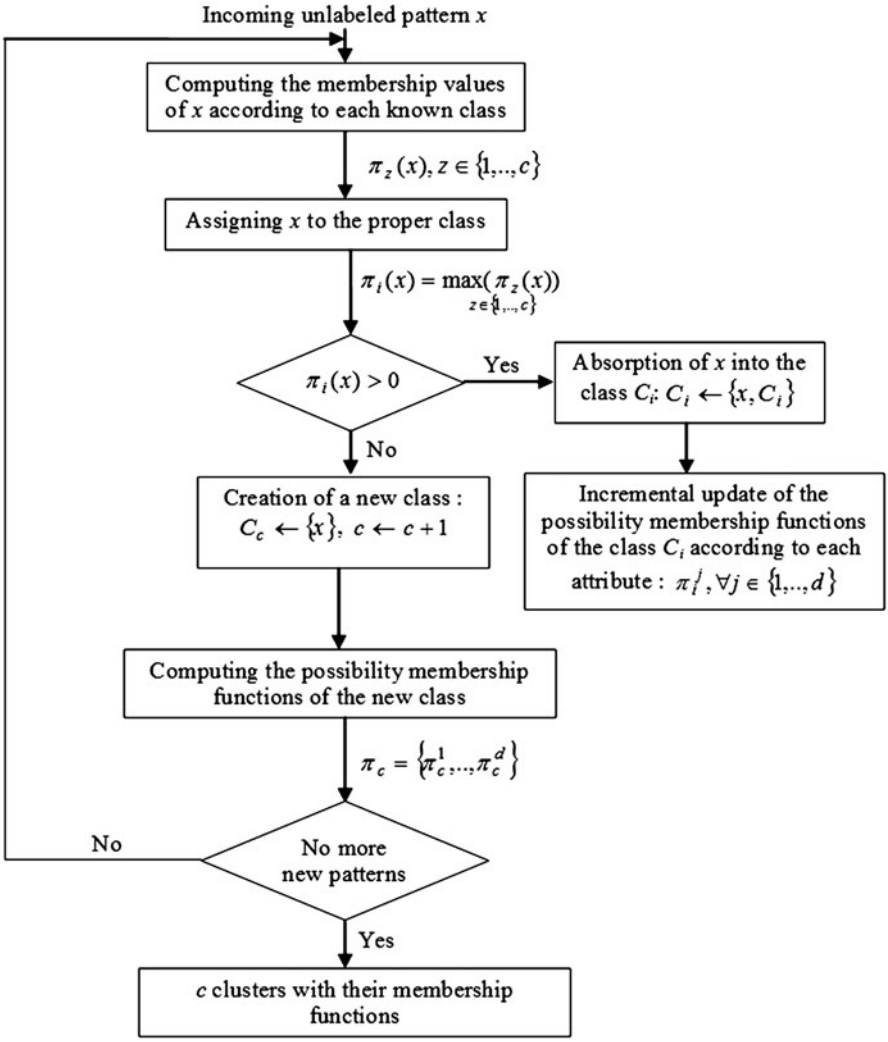

Fig. 15.3 Flow chart of unsupervised FPM for the detection and local adaptation steps

Where $\pi_i(x)$ and $\pi_z(x)$ are, respectively, the membership values of x to C_i and C_z. δ_{iz} is the similarity measure between the two classes. More the similarity value is close to 1, more the two classes are similar and must be merged. Figure 15.4 shows the values of this similarity measure according to the closeness of two Gaussian classes.

The clusters are merged when this measure reaches a predefined threshold. In general, a value of the similarity measure greater than 0.1 is enough to merge two clusters. Indeed, starting from this value, two clusters begin to be partially overlapped in the feature space. Figure 15.5 shows the flow chart of unsupervised FPM for the merging step.

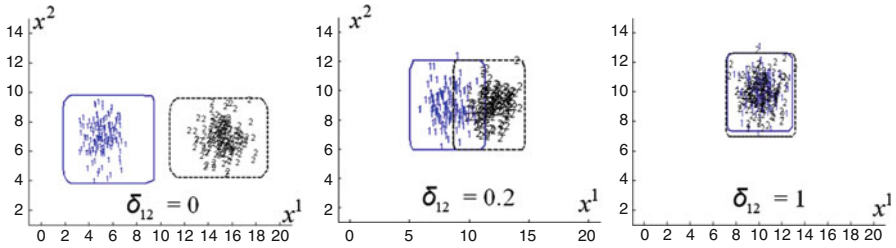

Fig. 15.4 Similarity measure between two classes

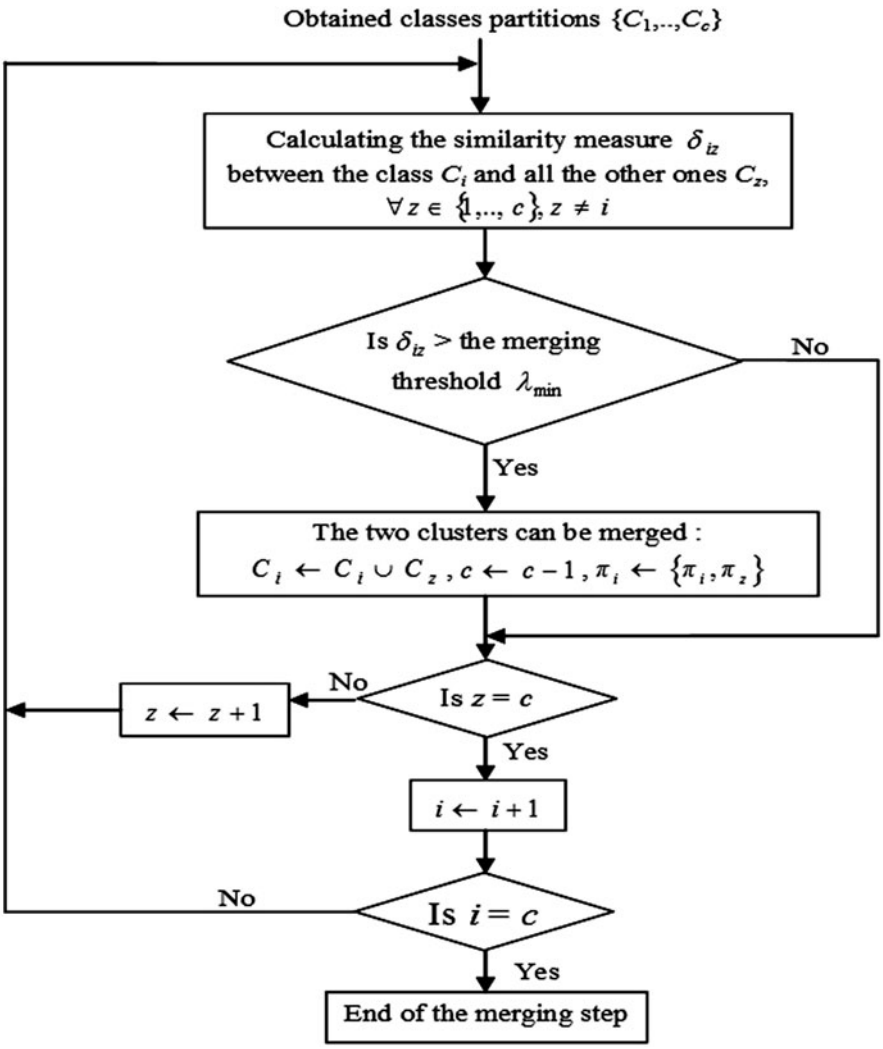

Fig. 15.5 Flow chart of unsupervised FPM for the merging step

15.3 Experimental Results

15.3.1 Simulation Example

In order to illustrate and to show the effectiveness of the proposed approach, let us consider the switched HDS described by Fig. 15.6. Where modes 1, 2, and 3 are, respectively, described by the following discrete-time transfer functions:

$$F_1(z) = \frac{z+0.5}{2z^4+0.5z^3+0.8z^2-0.3z+0.9} \quad 0 \le t < t_{12} = 900 \ \wedge \ 3{,}800 \le t \le 5{,}000 \tag{15.9}$$

$$F_2(z) = \frac{z+0.9}{z^4-0.8z^3+0.13z^2-0.16z+0.45} \quad 900 \le t < t_{23} = 2{,}500 \tag{15.10}$$

$$F_3(z) = \frac{z+0.2}{z^4-0.4z^3+0.29z^2-0.65z-0.2} \quad 2{,}500 \le t_{31} < 3{,}800. \tag{15.11}$$

On the other hand, the output of the system is defined by:

$$y_c(k) = \sum_{i=1}^{3} p_{i,k} y_i(k), \tag{15.12}$$

where k represents the time index, $i = \{1,2,3\}$ represents the index of the local mode, $y_c \in \mathbb{R}$ is the output of the system, $y_i(k) \in \mathbb{R}$ is the output of the local model i, and $p_{i,k}$ is the weight associated to $y_{i,k}$.

Note that for each time step k, the weights verify the following conditions:

$$p_{i,k} \in \{0,1\}, \quad \sum_{i=1}^{n} p_{i,k} = 1. \tag{15.13}$$

In mode 1, the system switches to mode 2, at the time instant $t_{12} = 900$. Then when the system is in the second mode, it switches to the third one at the time instant $t_{23} = 2{,}500$. Finally, the system switches to mode 1 at the time instant $t_{31} = 3{,}800$.

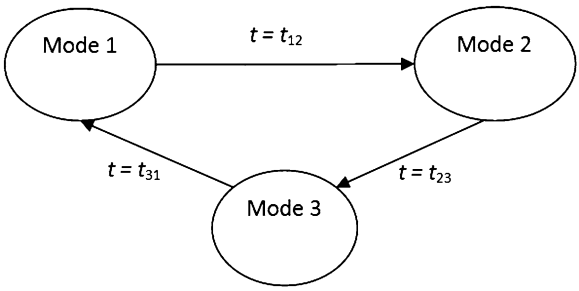

Fig. 15.6 Switching sequence for the simulation example

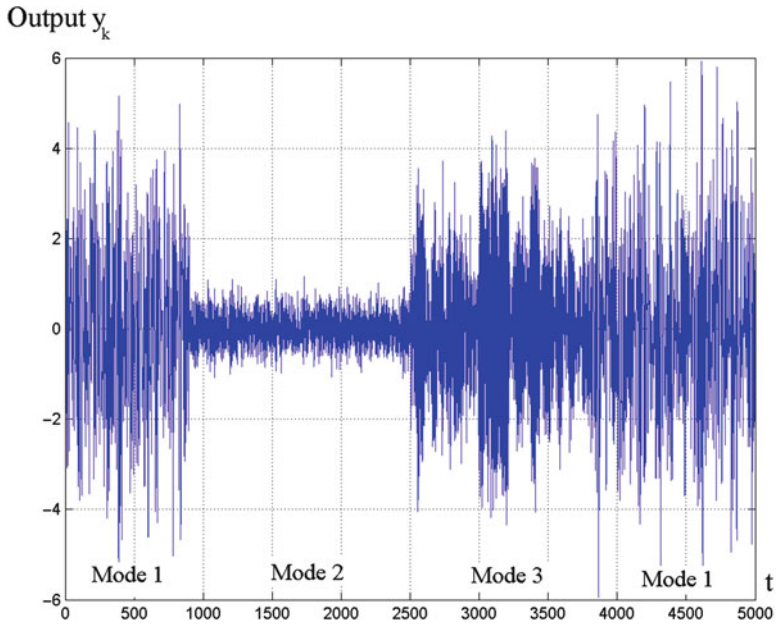

Fig. 15.7 Signal output of the simulation example of Fig. 15.6

Table 15.1 Similarity measure between the classes obtained by the application of the proposed approach on the learning set of the example of Fig. 15.6

Class	1	2	3	4	5	6
1	–	0	0	0	0	0
2	0	–	0	0	0	0
3	0	0	–	0.17	0	0.21
4	0	0	0.17	–	0.33	0
5	0	0	0	0.33	–	0
6	0	0	0.21	0	0	–

In order to simulate the system, an input–output identification data set has been generated. The output signal contains 5,000 data points and is generated using a pseudo random binary sequence (PRBS) as an input. Figure 15.7 shows the output signal of this example with the course of time. The feature estimation using the LSM, developed in Sect. 2.1, with a sliding window of 100 data points shifted by one time unit are used to determine the 4,900 patterns. This allows obtaining the learning set X containing 4,900 patterns in a feature space of 6 parameters. The number of parameters depends on the transfer function order. We have applied unsupervised FPM on the learning set X of the example of Fig. 15.6. Six classes are obtained. The similarity measures between these classes are shown in Table 15.1. If the fusion threshold δ is equal to 0.17, then we can obtain the following three classes: $\{C_1\}, \{C_2\}, \{C_3\} \leftarrow \{\{C_3\} \cup \{C_4\} \cup \{C_5\} \cup \{C_6\}\}$.

Figure 15.8 shows the distribution of the patterns of the learning set in each of the obtained three classes according to their occurrence time. Table 15.2 shows the switching time obtained for each mode or class. We can see that the error of time

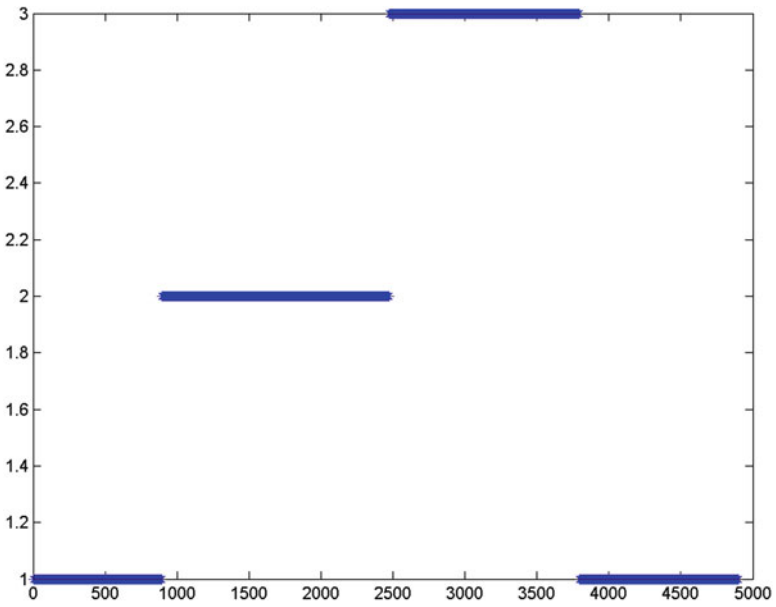

Fig. 15.8 Data points classes according to their occurrence time for the example of Fig. 15.6

Table 15.2 Switching time between the modes of the example of Fig. 15.6 obtained by the proposed approach

	Real switching time	Estimated switching time	Estimation error
t_{12}	900	901	1
t_{23}	2,500	2,495	5
t_{31}	3,800	3,794	6

switching is small (six time steps in the worst case). The proposed approach has a low computational complexity and low learning or classification time which depend both on the number of attributes and not on the number of patterns in the data set. This classification time for each pattern of the learning set is equal to 3.4×10^{-4} s using a computer with Pentium 4 2.8 GHz.

However, the performance of the proposed approach depends on the separability between the different modes, that is, classes, in the feature space. These different modes occupy separated regions in the feature space if their parameters are properly estimated. This needs a suitable size of the sliding time window in order to include enough of input–output data points. Thus, in order to find a suitable time window size, the proposed approach is applied using several time window sizes. We choose the smallest window size which minimizes the similarity measure (maximizes the separability) between the obtained classes. For the example of Fig. 15.6, we can notice that mode 3 has a very close behavior to the one of mode 1. While mode 2 has a clear different behavior of the one of other modes. Thus, the sliding window must have a sufficient size to well separate modes 1 and 3. Figure 15.9 shows the

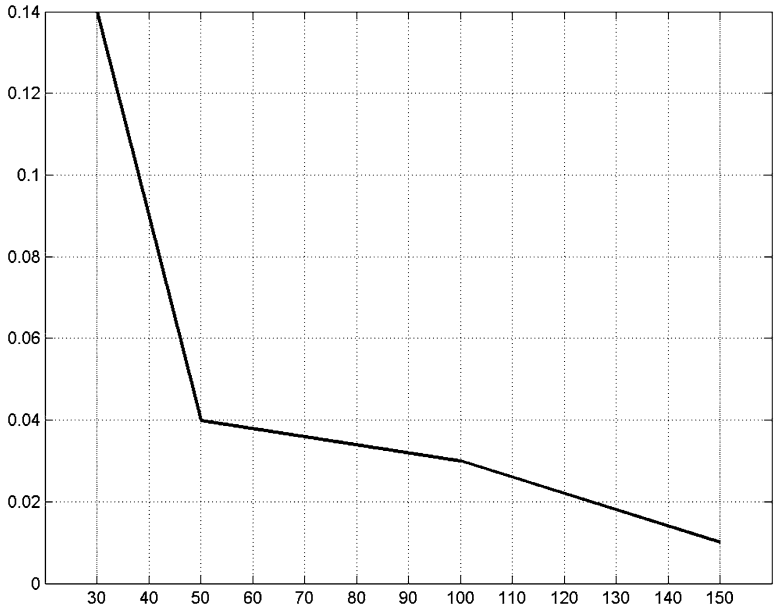

Fig. 15.9 Similarity measure between modes 1 and 3 for different sizes of the sliding window used to estimate the parameters of each mode

similarity measure between modes 1 and 3 for different sizes of the sliding window. We can notice that a sliding window of 30 input–output data points is not sufficient to enough separate modes 1 and 3. Indeed, the similarity measure indicates that these two modes belong to the same class or mode. Thus, in this case, modes 1 and 3 are merged into one mode and unsupervised FPM provides two modes which is an erroneous result. While a sliding window of 50 points or plus is sufficient to well separate the classes and to obtain the right number of modes. Figures 15.10 and 15.11 show the values of all the patterns of the learning set X according to each attribute for two sliding windows of sizes 30 and 100, respectively. We can observe that, for the case of a window size of 100 points, features 3 and 4 separate well modes 1 and 3, and the other features separate mode 2 from the other modes. In the case of a window of 30 points, mode 1 has a very close behavior to the one of mode 3 according to each one of the 6 features. This entails to consider these two modes as one mode by unsupervised FPM.

15.3.2 Application Example

PFR are used to produce nuclear power from nuclear fuel. They are cooled by metal liquid sodium. Indeed, water is difficult to use as a coolant for a fast reactor because collisions with the hydrogen nuclei in water quickly remove most of the kinetic

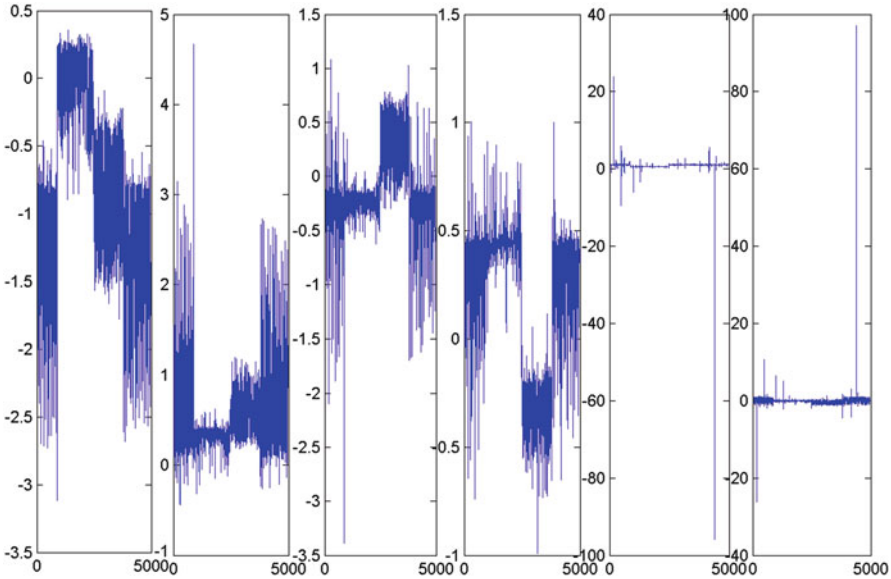

Fig. 15.10 Classes features estimated using a sliding window with 30 input–output data points

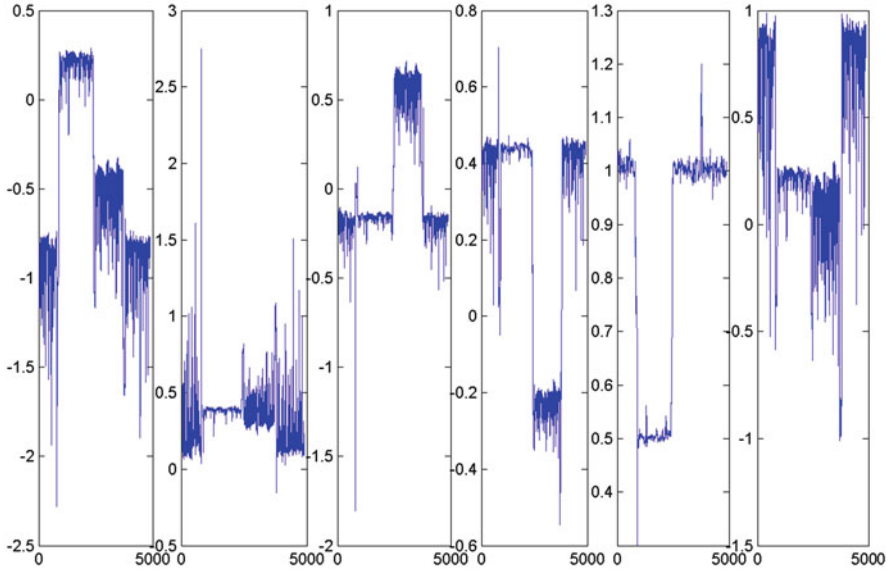

Fig. 15.11 Classes features estimated using a sliding window with 100 input–output data points

energy from the neutrons [8]. In contrast, sodium atoms are much heavier than both the oxygen and hydrogen atoms found in water, and therefore the neutrons lose less energy in collisions with sodium atoms. Sodium also does not need to be pressurized since its boiling point is higher than the reactor's operating temperature. However, a disadvantage of sodium is its violent chemical reactivity, which requires special precautions. If sodium comes into contact with water it explodes.

Actually, in-sodium hydrogen detectors are used in PFR to detect steam leaks in steam generator. However, they have long detection times of the order of two minutes. This is because hydrogen needs to transport from the leak site to the detector location [19]. Therefore, leaks will grow to a large leak which may cause serious damage through an explosion due to an increase of in-sodium gas pressure.

This limitation can be avoided using acoustic signals recording the background noises in the steam generator. Thus, the objective of this application is to design an acoustic leak detection to detect at early stage (faster than the hydrogen detectors) the reactions sodium/water. This acoustic leaks detection can be used as a supplementary tool besides the hydrogen detectors to detect steam leaks.

The available acoustic signals were recorded using data from background noise measurements on the steam generator from the end-of-life of PFR at United Kingdom. In these experiments, argon was injected into sodium, and acoustic noises were measured. Indeed, experimental results have shown that steam and argon injections give similar acoustic noise output at a given mass flow rate [20]. Figure 15.12 shows an acoustic signal recorded in response to an injection command. The signal records the noises resulting of the injection of argon in the steam generator unit of PFR. This injection simulates a fault occurred by a leak in the steam generator unit. Thus, the functioning of the steam generator unit switches between two modes (normal: non-injection and faulty: argon injection) in several time instants as it is shown in Fig. 15.12. The signal is sampled at the frequency 2,048 Hz.

We apply the proposed approach on the acoustic signal of Fig. 15.12. The signal is considered as the output of switched autoregressive (SAR) system. Therefore, the feature space is defined by the estimated parameters (coefficients) ai of the model AR. AIC criterion [1] has been used in order to select the order of the AR dynamic model. $d = 152$ is the AR model order which minimizes AIC criterion (Fig. 15.13). These features change with time. In order to capture this change, these features are calculated during a sliding time window. The latter size must include a sufficient number of data points in order to properly estimate the parameters of each mode. We have tested several sizes of time window. We have selected the one which maximizes the discrimination power between the different modes in the feature space, that is, obtaining compact and separated classes. This experimentation leads to select a sliding window with an initial length $\Delta t_{\mathrm{f}} = 8,192$ data points and a shift length Δt_{s} equal to 2,048 data points. Therefore, to define a pattern in the feature space, a time window containing 8,192 data points is required. Consequently, to determine the functioning mode (injection or non-injection), a delay time of 4 s is needed.

It is useful to reduce the feature space defined by the coefficients of the dynamic parametric model AR (d). The reduction operation aims at keeping the

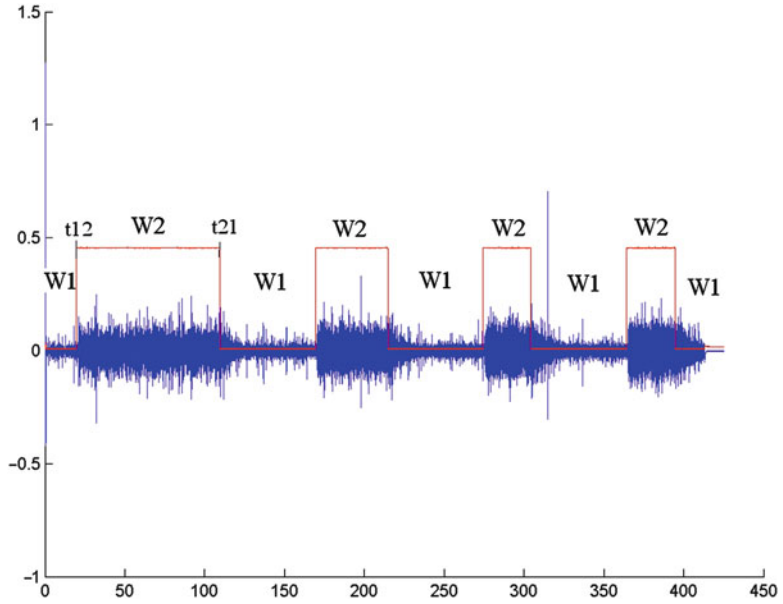

Fig. 15.12 Acoustic signal in response to argon command signal. W1: non-injection class, W2: injection class, t12: time of switching from W1 to W2, t21: time of switching from W2 to W1

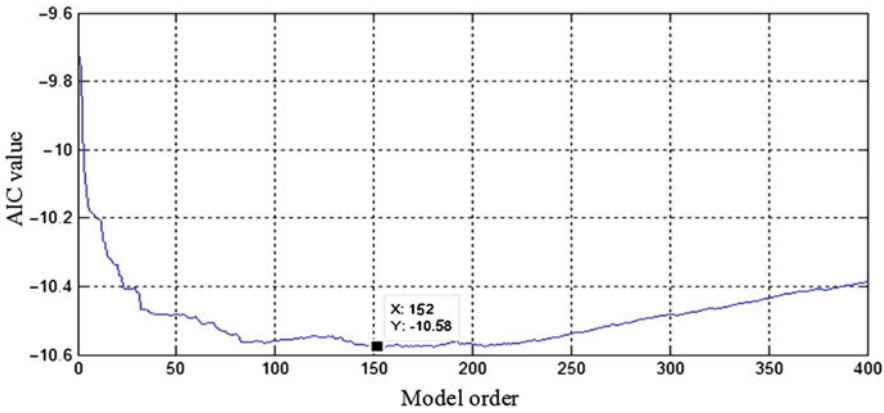

Fig. 15.13 AR model order selection based on AIC criterion

distinguishing features leading to separate as well as possible the different classes. We used the principal component analysis (PCA) to extract from the set of features the ones which are uncorrelated. 13 parameters have been selected because they are independent and conserve about 82% of the complete inertia carried by the 152 parameters. Then, we selected from this set of independent features the ones which have a combination leading to obtain the lowest error of classification. Two independent and discriminative AR model coefficients (coefficients 3 and 5) were conserved to define the feature space.

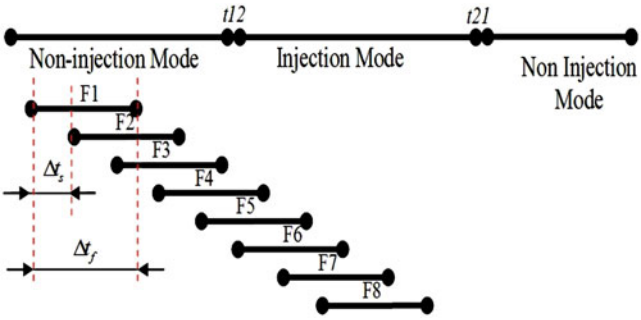

Fig. 15.14 Estimation of the feature space parameters using a sliding time window

Table 15.3 Similarity measure between the three obtained classes for the example of Fig. 15.12

Class	1	2	3
1	–	0.4	0
2	0.4	–	0.17
3	0	0.17	–

Feature space parameters represent the estimated coefficients of SAR model. These parameters, features, change with time. In order to capture this change, these features are calculated during a sliding time window (Fig. 15.14). However, the sliding window size must include a sufficient number of data points to properly estimate the parameters of each mode. We have tested several sizes of time window. We have selected the one which maximizes the discrimination power between the different modes in the feature space, that is, obtaining compact and separated classes. This experimentation leads to select a sliding window with a fixed length $\Delta t_f = 8{,}192$ data points and a shift length Δt_s equal to 2,048 data points. We apply the proposed approach on the acoustic signal of Fig. 15.12. The obtained results of clustering are shown in Table 15.3. Table 15.4 shows the switching times between the modes for the example of Fig. 15.12. The similarity, obtained by unsupervised FPM, between C_1 and C_2, is very important (equal to 0.40). Thus, the classes C_1 and C_2 must be merged. These two classes correspond to the non-injection class. The class C_3 corresponds to the injection class. The similarity value between C_2 and C_3 indicates that the class C_2 is a transitory one between C_1 and C_3.

Table 15.4 shows the switching time obtained for each mode, class. We can notice that the activation of the injection command several times leads to increase the delay required to detect the switching from injection to non-injection classes. This is due to the fact that the time required to allow the attenuation of the excitation resulted by the injection command increases with the number of activation of the injection command. However, there is no time delay to detect the switching from non-injection to injection classes. This advantage is very useful in a critical system

Table 15.4 Switching times between the modes for the example of Fig. 15.12 obtained by the proposed approach

	Real switching time	Estimated switching time	Estimation error
t_{12}	19	19	0
t_{21}	112	119	7
t_{12}	169	169	0
t_{21}	215	226	11
t_{12}	275	275	0
t_{21}	305	314	9
t_{12}	363	363	0
t_{21}	395	411	16

as the nuclear reactor since the transition from non-injection to injection modes simulates a leak in the steam generator. This leak must be detected as soon as possible to avoid an explosion.

15.4 Discussion and Related Work

Identification approaches of the literature are generally divided into clustering-based, Bayesian, bounded-error, algebraic-geometric, and optimization-based ones. Each of these approaches has its own advantages and drawbacks according to the assumptions needed on the number of modes, their order, the computational complexity, the dwell time in each mode, the achievable performance, the possibility to achieve on-line and/or off-line identification, etc.

In [13], the authors proposed a clustering-based approach that partitions the regressor space into regions on which a linear local model is valid. Then, it estimates each model parameters by standard least squares regression tools. However, this approach requires the knowledge of the ARX sub-models orders, and it stills suboptimal since the convergence depends strongly on the initialization step. In [5], a clustering-based method using the evidential theory is proposed. This approach supposes that each input–output data point is a cluster, that is, model. Then, the evidential theory is used for grouping data points that are more likely to have been generated by the same mode. The advantage of this approach is that it does not require the number of modes to be known a priori. However, the number of modes obtained by this approach depends strongly on a tuning parameter which is the number of neighbors. Moreover, the models order needs to be known a priori.

In [16], a Bayesian approach is proposed. This approach treats the parameters to be identified as random variables described with their pdfs. The data classification problem is posed as the problem of computing the a posteriori pdf of the model parameters, and the data are clustered in a suboptimal way using Bayesian inferences. However, this approach needs a priori knowledge about: the number of the modes, the ARX sub-models order and the pdfs of the parameters. In [3], a bounded-error approach is proposed. The main feature of this approach is to impose

that the identification error is bounded by a given bound for all the samples in the data set. The approach consists of three main steps: initialization, refinement, and region estimation. At the initialization step, the estimation of the number of sub-models, data classification, and parameter estimation are performed simultaneously by partitioning a set of linear complementary inequalities derived from data into a minimum number of feasible subsystems. Then, a refinement procedure is proposed in order to reduce misclassifications and to improve parameter estimates. Region estimation is finally performed via two class or multi-class linear separation techniques. However, this approach requires the orders of the ARX sub-models to be known a priori. Moreover, the bound needs to be properly adjusted in the identification procedure in order to find the desired trade-off between model complexity and fit quality.

In [22], the authors proposed a solution for the identification of noiseless PWARX models with unknown and different orders. The presented algorithm is based on an algebraic approach in which homogeneous polynomials are used to realize a segmentation of the regression space into regions that correspond to the discrete states. [18] proposed an identification approach that considers the plant as a nonlinear black-box and uses feedforward neural networks to predict the continuous outputs of the given HDS. In the same context, [2] proposed an on-line identification that uses an adaptive growing and pruning radial basis function neural network.

In this chapter, we considered the identification of Switched linear autoregressive (SAR) and switched linear autoregressive with exogenous inputs (SARX) models. In this class of HDS, the system switches arbitrary from one mode to another one. The proposed approach is a clustering-based one. It does not require the knowledge about the number of modes, the model parameters, and the switching sequences. In addition, this approach can be used to achieve both off-line and on-line identification. This is possible thanks to the low classification time and to the agglomerative and recursive character of the proposed approach. Finally, this approach overcomes the problem of initialization thanks to the use of a similarity measure to merge the clusters belonging to the same mode.

15.5 Conclusions

In this chapter, a clustering-based approach is proposed for the identification of switched linear autoregressive (SAR) and Switched linear autoregressive with exogenous inputs (SARX) models of hybride dynamic systems (HDS). The goal is to determine the number of modes as well as the switching sequence among them. The estimation of the number of modes is achieved using unsupervised FPM. The latter is based on a competitive agglomerative technique which allows the detection of new clusters sequentially without the need to any prior information about these clusters, that is, modes, or their number. Then, the clusters membership functions are refined sequentially with the assignment of new unlabeled patterns. Since the order of patterns' occurrences can be different according to the switching

sequence and there is no information about the clusters positions, several different portioning or clusters can be obtained. Thanks to the use of a similarity measure, the clusters, which are close to each other so that they cannot be considered anymore heterogeneous, are merged. The complexity and the computation time of the proposed approach are low and depend only on the dimension of the feature space. However, the proposed approach requires a discriminate feature space in order to separate the classes or the modes. Thus, a LSM with a sliding window is used to estimate the parameters of each mode or class. This estimation enhances the discrimination among classes. However, the sliding window must include enough of output data points in order to well separate the modes which have close dynamic behavior. We are developing this approach to be operant for the other classes of HDS as the piecewise autoregressive exogenous (PWARX), switched nonlinear ARX (SNARX), and PW nonlinear ARX (PWNARX) ones. In addition, we aim at relaxing the prior knowledge of the order of the transfer function for the construction of feature space. The goal is to be able to realize the identification of HDS containing subsystems or modes of different orders.

Acknowledgements This work is supported and integrated in the TEESC project. The authors would like to thank the scientific interest group surveillance, safety and security of the big systems.

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