# Responding to Students: Enabling a Significant Role for Students in the Class Discourse

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**Abstract** This is a case study of a highly regarded high-school mathematics teacher in Israel. It examines the kinds of responses to students' talk used repeatedly by the teacher, directing and shaping the classroom discourse, during different parts of the lesson. The main data source included 21 h of observations in two of this teacher's classrooms. Analysis of the video-taped lessons showed that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. This was mainly due to the teacher's responsiveness to students. The most common teacher response was elaborating. Accompanying talk occurred considerably less, and the teacher rarely expressed puzzlement or opposition when responding to students' talk. The chapter demonstrates how the teacher combined her attention to students' talk, with the goal of making progress on the main topic.

**Keywords** Teacher responsiveness · Classroom discourse · Instructional decisions · Expertise in math teaching · Elaborating talk · Accompanying talk

## Introduction

Expertise in mathematics teaching is frequently associated in the literature with devoting considerable class time to solving problems, proposing and justifying alternative solutions, critically evaluating alternative courses of action, leading to different methods of solving problems, not necessarily anticipated by the teacher ahead of time (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2001; Even & Lappan, 1994; National Council of Teachers of Mathematics, 2000). Expertise in teaching mathematics is often linked to encouraging students to make conjectures, explain their reasoning, validate their assertions, discuss and question their own thinking and the thinking of others, and argue about what is mathematically true (Collins, Brown,

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& Newman, 1990; Even & Tirosh, 2002; Wood, Williams, & McNeal, 2006). Hence, expertise in mathematics teaching implies, among other things, a significant and influential role for students in the class discourse.

To enable a significant and influential role for students in the class discourse the mathematics teacher needs to play the role of diagnostician ("Images of Expertise in Mathematics Teaching" in the chapter by Russ, Sherin, & Sherin, this book). Research and professional rhetoric suggest that awareness to, and understanding of, students' mathematics learning and thinking are central to good teaching (e.g., Barnett, 1991; Even, 1999; Even & Markovits, 1993; Fennema et al., 1996; Llinares & Krainer, 2006; National Council of Teachers of Mathematics, 1991; Scherer & Steinbring, 2006). Consequently, the development of such awareness and understanding has become part of the curriculum of teacher education for both prospective and practicing teachers in recent years (e.g., Even, 1999, 2005a; Markovits & Even, 1999; Fennema et al., 1996; Tirosh, 2000).

Yet, improving teachers' understanding of what their students say, write or do still leaves the problem of how teachers may use this understanding to make better instructional decisions. How they may encourage and enable a significant and influential role for students in the class mathematics discourse, while, as river guides ("Images of Expertise in Mathematics Teaching" in the chapter by Russ et al., this book), respond to the students, to the context, and to what occurs in the moment (Berliner, 1994). This is not an easy task, as research suggests (Chazzan & Ball, 1999; O'Connor, 2001; Simon, 1997; Wood, 1994). For example, Even (2005b) illustrates the difficulties teachers encounter when facing the need to address students' mistakes, even after the teachers developed rich and profound understandings of the nature and sources of these mistakes. Ball (1993) describes the challenge of responding to students who present novel ideas that are not in line with standard mathematics, even in the case of an expert teacher with deep disciplinary understandings. Research suggests that expert teachers are better than novice teachers at productively altering the direction of their lesson in response to students' questions or comments (Brown & Borko, 1992). Yet, as Ball's study shows, responding to students' talk and action is problematic even for expert teachers.

A review of the literature provides limited information on the ways teachers attend and respond to students during mathematics lessons. Most studies have been conducted as part of intervention programs, involving a small number of lessons. Moreover, information on the ways teachers respond to students' talk and action during mathematics lessons is often derived from studies that do not specifically focus on that, but rather on class discourse (Even & Schwarz, 2003; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Resnick, Salmon, Zeitz, Wathen, & Holowchak, 1993; Sherin, 2002), patterns of interaction (Bauersfeld, 1988; Wood, 1994; Voigt, 1995; Lobato, Clarke, & Ellis, 2005), and teaching strategies (Fraivilling, Murphy, & Fuson, 1999). Missing are studies that focus purposely on teachers' responses to students' talk and action during a relatively long period of regular mathematics lessons. Our study focuses on this.

The chapter examines how a teacher who has a reputation of encouraging a significant and influential role for students in the class discourse, responds to students' mathematical talk in class. The chapter examines the kinds of responses used repeatedly by the teacher, directing and shaping the classroom discourse, during different parts of the lesson.

## Methodology

This is a case study of an experienced high-school mathematics teacher, highly regarded by her colleagues and other members of the mathematics education community in Israel. In addition to teaching high school mathematics, she has been a central member of several curriculum development teams, was a member of the national syllabus committee for junior-high mathematics, and has served as educator for prospective and practicing mathematics teachers. In her various roles she regularly sought for innovations in content and ways of teaching, and systematically reflected on her own teaching and the learning processes of her students. In numerous formal and informal conversations she often expressed the importance she attributed to being attentive and responsive to students and to encouraging students to take a significant role in the lesson.

The main data sources include observations of the teaching of mathematics in two of this teacher's classes. One of the classes the teacher taught was a 9th grade class and the other a 10th grade class; both in the high-school where she regularly taught mathematics – an academic oriented Jewish religious girl school. The 9th grade class was composed of lower-achieving students whereas the 10th grade class was composed of high-achieving students.

The second author observed 9 lessons in the 9th grade class and 8 lessons in the 10th grade class (the length of each lesson ranged between 36 and 88 min). Total time of observation was 21 h: about 10.5 h in each class. About one-half of the observed lessons in the 9th grade class were on functions; the rest were on geometry. Similarly, about one half of the observed lessons in the 10th grade class were on analysis; and the rest were on geometry. This research design enabled us to examine the nature of the teacher's ways of attending to students' talk and action during a rather long period of regular mathematics lessons, in a variety of settings: different classes, and when teaching different mathematical topics.

All 17 observed lessons were videotaped; notes were taken during and after each observation, and informal conversations were often held with the teacher. At the end of the data collection period, an 80-min long semi-structured interview was held with the teacher. The interview focused on her way of teaching, students' participation in the lessons, her response to students' talk, and differences in response in different settings. Later on, two additional semi-structured interviews were held with the teacher, focusing on her way of teaching, on the structure of a typical lesson of hers, and on the teaching sequence in each observed lesson.

Of the 17 observed lessons, 16 lessons consisted of whole-class work, small group/individual work, and class organization; one lesson consisted of small group/individual work and class organization only. Detailed data analysis of the lessons included only the talk during whole-class work, which comprised more than one-half of all lesson time – close to 12 h. The interviews and observations of the small group/individual work were used to support or downplay interpretations and to provide additional information about the teacher's responsiveness to students.

Following Even and Schwarz (2003), analysis of teacher responsiveness included an examination of the occurrence of four kinds of teacher responsiveness: *Accompanying* talk refers to talk in which the teacher attended to a student's talk without elaboration, typically acknowledging that she follows the student's talk. *Elaborating* talk refers to talk in which the teacher elaborated utterances and expressed deeper cognitive involvement. *Opposition* refers to talk in which the teacher explicitly expressed disagreement and objection. *Puzzlement* points to talk expressing confusion, perplexity or bewilderment.

Teacher responsiveness to students may be related to the purpose of the lesson segment. Thus, data analysis focused also on identifying the purposes of different components of the teaching sequence in each lesson. The coding we used for this is based in part on the coding system developed in the TIMSS-Video Study (Hiebert et al., 2003), but was modified to fit with the teacher's view, as indicated in interviews and conversations with her, and with the observational data. Thus, we combined two categories from the TIMSS-Video Study's coding system (Hiebert et al., 2003) – "Introducing new content" and "Practicing new content" – into one category, "Work on the main topic", because this category fits better with the class practice and with the teacher's description of the structure of her lessons. We also added to the TIMSS-Video Study's coding system the category "Extending beyond the main topic" because the teacher explicitly stated in a conversation that she often does that intentionally. The resulting coding system for this study includes four main categories. The first three categories center on mathematical work; the last one on class organization:

- *Work on the main topic*: focuses on introducing, investigating, extending, and deepening the main topic of the lesson.
- *Reviewing content introduced previously*: focuses on reminding students of, and clarifying, content learned earlier in the lesson, in previous lessons, or in lower grades.
- *Extending beyond the main topic*: focuses on extending and enriching students' knowledge and understanding of mathematics.
- *Class organization*: focuses on mathematical organization (e.g., distributing materials or homework assignments) or on non-mathematical work (e.g., disciplining students).

Finally, we examined what kinds of responses characterized each of the first three lesson components.

## Responsiveness to Students in the Lessons

Analysis of the data suggested that teacher responsiveness to students characterized the mathematical work during whole-class work sessions. Almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. The teacher was attentive and responsive to different kinds of students' talk, including students' questions, answers, hypotheses, claims, remarks, mistakes, etc. The nature of the mathematical activity triggered by, built or followed on, students' utterances varied, and included, for example, discussing students' answers, investigating students' hypotheses, clarifying concepts critical for work on assigned tasks, strengthening previously learnt materials, answering students' queries, explaining the nature of mathematics and the work of mathematicians, etc.

Overall, two kinds of teacher response – elaborating and accompanying – were used repeatedly by the teacher, whereas opposition and puzzlement seldom occurred. The most common response was elaborating; accompanying occurred less frequently. Nonetheless, both elaborating and accompanying talk occurred during every lesson that included whole-class work.

Analysis of the data shows several similarities and some differences in the use of the four kinds of teacher responsiveness among the three lesson components. Below we describe and exemplify the kinds of responses practiced by the teacher during each lesson component: work on the main topic, reviewing content introduced previously, and extending beyond the main topic.

## *Work on the Main Topic*

Work on the main topic comprised of introducing, investigating, extending, and deepening the main topic of the lesson. This kind of activity occurred during every lesson, and most of the total lesson time was devoted to it. Usually, the teacher initiated this kind of mathematical work. Typically, it involved collaborative whole-class work built on students' small group/individual problem solving.

All four kinds of responses were enacted by the teacher when working on the main topic. The most common response was elaborating; accompanying occurred less frequently. Teacher opposition and puzzlement occurred only a small number of times. Below are illustrations of the different kinds of teacher responsiveness to students when working on the main topic.

### Opposition During Work on the Main Topic

The teacher seldom expressed disagreement or objection to students' ideas when working on the main topic. When she did, it was when students' suggestions severely deviated from the main point. One of these rare events occurred when she introduced the topic of similarity of polygons. The teacher started the lesson by asking the 10th grade students to explain the meaning of similarity in everyday life. The first students' suggestions were all closely tied to the mathematical notion of similarity: "Same angles but not the same sides" or "The ratios between the sides are equal". The teacher repeatedly rejected these suggestions, emphasizing that she was looking for something not in the mathematical world: "You explain it from a mathematical point of view. I'd like a description from everyday life."

### Elaborating Talk During Work on the Main Topic

A common teacher behavior was to elaborate students' talk and express deep cognitive involvement in students' suggestions. For example, as part of the work on the topic of similarity of polygons, following the previous exchange on what similarity might mean in everyday life, yet before being given the formal definition of similarity of polygons, the 10th grade class students were assigned the task to imagine that they were using a camera or a photocopy machine. They were asked then to draw polygons that would be similar to the ones in Fig. 1 (drawn on triangular lattice), and to find the angle measures of the original and the new polygons.

After small group/individual work, a whole class work began, focusing first on the angle measures of the given shapes. The angles of the triangle in Fig. 1a were easily found, based on the fact that it is an equilateral triangle. But the triangle in Fig. 1c was a challenge. One student suggested that the top angle is a right angle based on its appearance. As a result, a discussion on whether one can be sure of that arose, eventually rejecting this method. This discussion was characterized by the teacher elaborating students' ideas and expressing profound cognitive involvement in their suggestions:

S: There is an angle of 90°.
T: How do you know? You see. Is it allowed?
S: I don't know.
S: Is seeing allowed?
T: Seeing is allowed but you cannot decide based on seeing.

Work on finding the angle measures continued, led by the teacher who kept using elaborating talk throughout this discussion. A student suggested that the left base-angle is 60°, based on what they found regarding the equilateral triangle in Fig. 1a and the problem of whether "seeing" is allowed in mathematics (i.e., is a valid

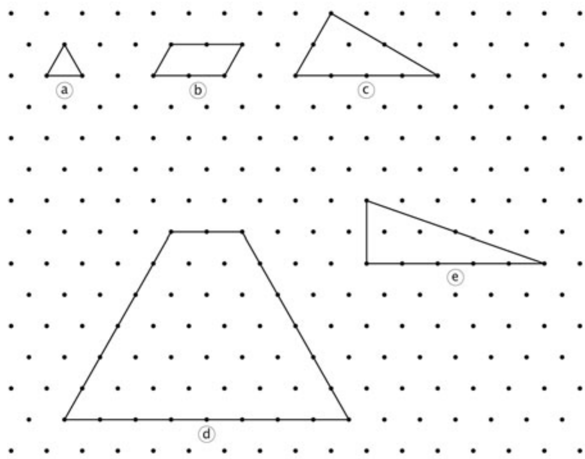


**Fig. 1** Shapes used in the similarity activity

tool for determining mathematical truths) emerged again, distinguishing between the nature of "seeing" in each case:

S: There is also a 60° angle. It's like the angle of triangle 1a.
T: Right. There is a 60° angle here. I agree. It's like the angle of triangle 1a.
S: Can we do that? Do you allow us to do that?
T: What? What did we do?
S: According to the dots.
T: According to the dots we determined that
S: Ah, then everything will be much easier.
T: Sure... This is allowed, to "see" that there is an equilateral triangle here. What is the difference between "seeing" that this is a 90° angle and between "seeing" that there is an equilateral triangle?
S: Here it is 90° [incomprehensible] as if you see it. And here you can base it. You know that their distance is equal [distances between dots on the triangular lattice].
T: It is given to us that the distances are equal, so actually it is not based on "seeing". We decided that this is an equilateral triangle based on what is given. It is given to us that the distances are equal, and it is given to us, and it means that the triangle is an equilateral triangle. In contrast, here when I look at the angle and it looks like 90°. But maybe it is 91° So, can someone continue and show...

A student then added the following construction (see Fig. 2a) and claimed that the right base-angle is 30°. She argued that the side AC is an angle bisector because it is both a height and a median in an isosceles triangle. The teacher requested a justification: "You claim that this is in the middle. Who wants to say, again it is a bit 'seeing' and a bit, I'd like a clear strong explanation, why is it in the middle?" Attempting to prove this, another student suggested to complete the triangle into a rhombus, and added the following construction (see Fig. 2b). From here it was straightforward for other students to point out that the diagonals of a rhombus form right angles at their intersection and bisect each other. Thus, they concluded that the side AC is indeed an angle bisector and the right base-angle is 30°. Trying in vain to use the Pythagorean theorem in order to prove that the top angle is 90°, students eventually suggested using the angle sum of a triangle property.

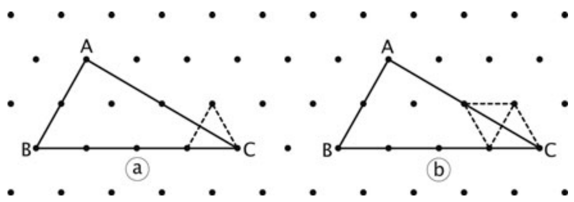


**Fig. 2** Finding angle measures of the triangle in Fig. 1c

This episode of finding the angle measures of the triangle in Fig. 1c exemplifies how work on the main topic that was led by the teacher, depended on, and was responsive to, students' talk. It illustrates how work on the main topic comprised of the teacher elaborating students' suggestions and taking part in developing ideas students suggested. The teacher was attentive to students' ideas, and embraced their suggestions as a starting point for mathematical examinations. Thus, problems were mostly solved according to students' proposals and suggestions. The teacher adopted students' suggestions even when they were unproductive or mistaken, spending the time needed to examine their potential and adequacy.

In addition to making students' proposals and suggestions part of the content dealt with in the lesson, the teacher was also attentive to students' requests regarding the issues and topics to be dealt with, and often accepted their requests. For example, after finding the angle measures of the polygons in Fig. 1a–c the teacher shifted the focus of the discussion in the 10th grade class to the formal definition of similarity of polygons and to properties of similarity of different polygons. Then, just before the end of the lesson, the teacher started to explain the homework assignment when a student interrupted her, requesting to complete the task of finding the angle measures of the triangle in Fig. 1e. The teacher accepted this request and the rest of the lesson time was devoted to discussing this. Even though it was the end of the lesson, the teacher responded to students' ideas using elaborating talk. For example, following a student's remark that the triangle was not a 30-60-90° triangle, the teacher asked the students whether they were certain of that. After students responded positively she asked them to explain this claim. Finally the class discussed the two explanations suggested by the students and another one suggested by the teacher.

The discussions outlined above about similarity evolved after the teacher formally opened up a whole-class discussion, asking the class to suggest how to find the angle measures of the polygons in Fig. 1, orchestrating a collaborative whole-class problem solving session. Yet, there were quite a few instances when work on the main topic that comprised teacher's elaborating students' suggestions occurred rather spontaneously. An example for that is an episode taken from a series of lessons on the quadrilateral family in the 9th grade class. During one of the lessons, the class worked on determining which quadrilaterals have reflective symmetry. The students were asked to fold a paper in half, and cut out different quadrilaterals (parallelogram, trapezoid, kite, rectangle, rhombus, square) using the fold line as the line of symmetry. After several unsuccessful attempts to cut a parallelogram that is not a rectangle, according to these instructions, one student noticed that another student "succeeded" to cut such a parallelogram (in fact that student did not use the fold line as the line of symmetry). Astonished, the student inquired, "How did you do it?" The teacher overheard the conversation. She picked up the cut paper, presented it to the whole class, declared that one student succeeded in the task, and asked the class how the student managed to cut out the parallelogram. Eventually, the class discovered that the student did not use the fold line as the line of symmetry.

**Accompanying Talk During Work on the Main Topic**

Another common teacher behavior was to attend to a student's talk without elaboration, typically acknowledging that she followed the student's talk. There were a few times when the teacher used only accompanying talk; yet, more often she combined accompanying with elaborating talk.

Occasionally, when students gave correct answers, or when gathering students' thoughts as a starting point for work on the main topic, the teacher attended to students' ideas without elaboration. The following illustration of using this type of accompanying talk is taken from an episode that occurred after the teacher expressed disagreement with the suggestions that the 10th grade students proposed for the meaning of similarity in everyday life because they were all closely tied to the mathematical notion of similarity. A student then proposed something different:

S: The same shape but smaller.
T: The same shape but smaller.
The teacher then asked the students to give her examples from everyday life for similar shapes:
S: Perhaps Babushka [a Russian nested doll – Matryoshka doll]?
T: Ah, Babushka, Babushka dolls.

As can be seen, the teacher used in these short excerpts accompanying talk, basically repeating the student's words: "The same shape but smaller", "Ah, Babushka, Babushka dolls."

Another example for accompanying talk that is not embedded in elaborating talk is taken from an activity that followed the activity described above of cutting out different shapes using the fold line of a paper as the line of symmetry. The teacher asked the 9th grade students to report which shapes they succeeded to cut out. The list on the board included the following shapes: circle, square, rectangle that is not square, rhombus that is not square, parallelogram that is not rectangle or rhombus, trapezoid, and kite.

T: Okay, then out of all these – which ones did you succeed at [cutting out]?
S's: Circle, square, rectangle that is not square,
T: [marks on the board each shape the students mention, holding her marker by the next shape on the list: rhombus that is not square].
S: Parallelogram, no, parallelogram I didn't succeed.
S's: Rhombus, trapezoid, rhombus that is not square.
T: [continues to mark each shape the students mention. Eventually all shapes are marked but the parallelogram]. Okay.

Later in the lesson, a student raised again the case of the parallelogram, and the teacher responded by opening up a discussion regarding whether a parallelogram has a line of symmetry. This time, as she often did, the teacher used accompanying talk combined with elaborating talk. The following excerpt illustrates this. It occurred when the teacher drew a parallelogram with a straight line parallel to one pair of its sides, and asked the students to prove that the two adjacent angles of the parallelogram on opposite sides of the line are not equal to each other.

S: I think that the adjacent angles need to be 180°.
T: Very good.
S: [incomprehensible] Never mind.
T: We said that this and this it's 180°. If they are equal then what?
S: 90°.
T: And what will the parallelogram be then?
S: A rectangle.
T: If one is acute then what happens to the other one?
S: The other one is obtuse so that there is 180°.

As can be seen in this excerpt, the teacher first used accompanying talk: "Very good" and "We said that this and this it's 180°" which basically repeats a student's idea. But then she began to use elaborating talk, and actively participated in the construction of the proof.

### Puzzlement During Work on the Main Topic

Even though whole-class work on the main topic comprised of immense students' participation, and teacher attention and responsiveness to students characterized by-and-large the mathematical work during whole-class work sessions, the teacher rarely expressed confusion when responding to students' talk. One of these unusual episodes where the teacher's response reflected puzzlement occurred during a lesson in the 9th grade class that centered on exploring relationships among rectangles that have a fixed perimeter or a fixed area. When examining whether a fixed perimeter implies a fixed area the teacher phrased the problem as: "If the perimeters of two rectangles are equal then the areas are equal: Is this claim correct?" A student interpreted this as if the problem was whether there exists a rectangle whose perimeter equals its area. For a few seconds the student and the teacher expressed puzzlement until another student pointed out the reason for confusion. The teacher responded rather astonished:

T: No! No, no, this is not what I meant!
S: Then what did you mean?
T: Not that the perimeter equals the area.
S: Then?
T: Rather that I have two rectangles [draws two rectangles on the board]... Does the fact that the perimeter of this one equals the perimeter of that one mean that the area of this one equals the area of that one? Not that the perimeters equal the areas.

The teacher, who sensed that she misunderstood what a student had said, insisted in the episode described above on clarifying the confusion. However, there was one time when the teacher acknowledged that she was puzzled by a student's talk, yet she chose not to clarify this confusion. It happened when the students' talk was not at the heart of the main point: When collecting students' suggestions for rectangles with a fixed area, a student provided a long complicated explanation on how she found, without using a calculator, that 6 is the other dimension of a rectangle whose area is 15 squared units and one of its dimensions is 2.5. The teacher acknowledged that she was attentive, but confusedly concluded: "Okay, I didn't really understand what you said" and continued with the lesson.

## *Reviewing Content Introduced Previously*

Reviewing content introduced previously comprised of reminding students of, and clarifying, content learned earlier in the lesson, in previous lessons, or in lower grades. This kind of mathematical activity occurred during most of the lessons. It tended to be rather short and only a small part of the total lesson time was devoted to it. Reviewing content introduced previously rarely occurred as a teacher initiative during the observed lessons. In those few times that it did, it occurred at the beginning of a lesson, and served as a means for the teacher to collect information regarding students' readiness for the planned work on the main topic. Nonetheless, reviewing content introduced previously occurred almost always as a teacher's response to students' queries or requests that emerged during work on the main topic.

Two out of the four kinds of responses examined – elaborating and accompanying – were performed by the teacher when reviewing content introduced previously. The most common response was again elaborating; accompanying occurred less frequently. Like in the case of work on the main topic, the teacher led the review – triggered by students' queries and requests – building on students' active participation. Thus, the activity depended on, and was responsive to, not only the student's initial talk that initiated the review, but often also to on-going students' talk. Below are illustrations for elaborating talk and accompanying query used by the teacher when reviewing content introduced previously.

### Elaborating Talk During Reviewing Content Introduced Previously

To signal the end of the small group/individual work in the 10th grade class, regarding the polygons in Fig. 1, as a transition to a whole class discussion, the teacher said: "Girls, start to talk about, about the relationships between sides and angles. Is there any connection between this and similarity?" The first student's response was: "What the heck is similarity anyhow?" The teacher responded by restating the idea she presented before the small group/individual work: "We didn't define it yet. But we understand it as some kind of enlargement or reduction by a photocopy machine." Expressing deep involvement in the student's query, the teacher aimed to make sure that they found common ground. She drew on the board two parallelograms, one of which was derived from the other by reducing only one pair of opposite sides, asking the students to determine whether the two are similar to each other. After a collaborative examination the class concluded that reduction (or enlargement) by a photocopy machine (i.e., similarity) reduces (or enlarges) all sides of a shape in the same proportion.

Another example for the teacher's use of elaborating talk when reviewing content introduced earlier in response to students' queries or requests is taken from a concluding lesson on the quadrilateral family in the 9th grade class. In a previous lesson the teacher defined a rhombus as a parallelogram with one pair of equal adjacent sides, and the class worked on the rhombus properties and relationships with other members of the quadrilateral family. In the concluding lesson, after some work on finding characteristics of a rhombus based on the definition and previous work on parallelograms, one of the students pointed to one of the rhombus characteristics found by the class – that all sides are equal – and questioned why the teacher said previously that a rhombus is a parallelogram with one pair of equal adjacent sides, whereas all the sides are equal. The teacher promised to address it later. The class finished the planned work on finding rhombus characteristics, and the teacher returned to the student's query regarding what a rhombus is: a parallelogram with one pair of equal adjacent sides (as defined in a previous lesson) or a parallelogram in which all the sides are equal (as concluded in the current lesson). Attending to the student's confusion, the teacher responded by reviewing the definition of a rhombus, clarifying the distinction between the definition that was introduced in a previous lesson and the rhombus attributes found in the current lesson:

> [The student] said that if we know that a rhombus has four equal sides, then why did we begin by saying such a thing [points to the definition of a rhombus on the board: a parallelogram with two equal adjacent sides]? Does anyone have an idea?
> [Pause]
> Okay, let me tell you. We could have said that a rhombus is a parallelogram with four equal sides, right? [But] in definitions we try to say as little as possible. That means, I don't want to say everything I know about a rhombus as its definition. I say as little as possible in the definition, and all the rest I can prove by myself. In other words, we managed to prove, based on the fact that we knew that this pair is equal, we managed to prove that all sides are equal. This we managed to prove.

The teacher used elaborating talk also when reviewing content introduced in previous school years in response to students' queries or requests. For example, during an analysis lesson the teacher asked her 10th grade class to find the dimensions of a square box (i.e., a right square prism) made from a 60 cm long string with the maximum volume. After the presentation of the problem, a student questioned whether a square box could also be a cube. Even though the students have already studied these shapes in lower grades, the teacher assessed that other students may also be confused regarding the distinction and relationship between the two, and decided to clarify it. She did not answer succinctly, but rather elaborated the distinction:

> Okay. Let us first clarify some concepts. What is a square box? I am sketching and sketch with me. . . A square box is a box with a square base. It doesn't have to be a cube. . . Make it high enough so that it doesn't look like a cube.

And the teacher continued to explain that square box is not synonym to cube. Rather, it denotes a whole family of boxes in which a cube is only one special case.

### Accompanying Talk During Reviewing Content Introduced Previously

Occasionally when reviewing content introduced previously the teacher used accompanying talk, by and large combining it with elaborating talk. When the teacher initiated the review it typically served as a means for collecting information regarding students' readiness for the planned work on the main topic. In such cases the teacher often first attended to students' ideas without elaboration, typically acknowledging that she followed the student's suggestions. For example, when starting the topic of the quadrilateral family the teacher started the work by reviewing what a parallelogram is:

T: Who remembers what a parallelogram is? Raise your hands. Who remembers what a parallelogram is?
S: A quadrilateral with two opposite parallel sides.
T: A quadrilateral with two opposite parallel sides.

As can be seen, the teacher used in this short excerpt accompanying talk, basically repeating the student's words: "A quadrilateral with two opposite parallel sides."

However, reviewing content introduced previously occurred almost always as a teacher's response to students' queries or requests that emerged during work on the main topic. In such cases when the teacher used accompanying talk she often combined it with elaborating talk. For example, a short time after the teacher clarified the distinction between square box and cube, a student announced that volume was difficult for them. The teacher attended to this statement of difficulty and reviewed the relevant content, which again had been already studied in a lower grade. She started by unpacking the sources of difficulty, "Do you know how to calculate the volume of a box?" After students responded saying "No", the teacher reminded them of a problem on which they worked in the previous school year, when they were in the 9th grade. That problem dealt with finding the dimensions of an open box constructed from a square cardboard sheet, which can hold the largest amount of chocolate. The teacher drew a square box on the board, and together with the class calculated its volume by making reference to filling the box with chocolate:

T: What is the volume? The number of 1 cm $\times$ 1 cm $\times$ 1 cm chocolate cubes that fill. . . How many would fill the first layer? . . .How many would fill the whole box?... Who wants to tell me how one calculates the volume of a square box or a non-square?
S: The area of the base times the height.

T: The area of the base times the height. Is it clear why?... Is there anyone who doesn't understand? The area of the base gives the number of cubes in the first layer.

As can be seen in this excerpt, there was a time when the teacher used accompanying talk, basically repeating the student's words: "The area of the base times the height". But before and after this she used elaborating talk to remind the students of the formula for calculating volume of boxes, which was needed in order to solve the problem of maximum volume. While doing that, the teacher focused on explicating the meaning of volume of a box by making connections to a problem on which the class worked in the previous school year. She exhibited a systematic way of filling the box with one-unit chocolate cubes – layer after layer – reflecting the structure of the formula, emphasizing the meaning of a volume of a box as the number of one-unit chocolate cubes that would completely fill it.

## *Extending Beyond the Main Topic*

Extending beyond the main topic comprised of extending and enriching students' knowledge and understanding of mathematics. Extending beyond the main topic occurred during most lessons, tended to be rather short, and a rather small part of the total lesson time was devoted to it. This kind of mathematical work was usually triggered by students' talk.

Three kinds of responses were enacted by the teacher when extending beyond the main topic. The most common response was elaborating; accompanying occurred less frequently, and teacher puzzlement occurred once. Below are illustrations of the different kinds of teacher responsiveness to students when extending beyond the main topic.

### Elaborating Talk During Extending Beyond the Main Topic

The teacher often expressed deep cognitive involvement in students' suggestions, even when it meant deviating from the main topic of the lesson. Sometimes it implied working on new mathematical content. For example, as part of the work on the angle measures of the triangle in Fig. 1c, a student suggested to check whether it is a right triangle, inquiring whether when one side of a triangle is one-half of a second side, it implies that it is a "pretty" triangle: the name used in this class for a right triangle with angle measures of 30-60-90°. As a response, the teacher deviated from the main topic and made this query an object of examination for the whole class. Led by the teacher, the class unpacked the student's query, clarifying what the givens are in the implied conjecture. The teacher called students' attention to the fact that the conjecture is close to be the converse of a theorem they had proven in a previous lesson: In a right triangle, the side opposite the 30° angle is one-half of the hypotenuse. The class continued to work on rephrasing the conjecture, using more formal terms. Finally, the teacher assigned as homework checking whether the conjecture was correct.

In the above episode, extending beyond the main topic comprised of work on new mathematical content: phrasing, and proving or refuting a conjecture regarding right triangles. The episode below had a similar nature. This episode occurred during one of the lessons in the 9th grade class, which centered on finding all rectangles that have a fixed perimeter or a fixed area. When working on the case of a fixed perimeter of 16 units, one of the students found the $4 \times 4$ rectangle, which is also a square. She then noticed that the perimeter of this square and its area are equal to each other, and raised the question whether this is true for all squares. Later on, when collecting all students' suggestions for rectangles with a fixed perimeter of 16 units on the board, the teacher pointed to the $4 \times 4$ rectangle, and repeated the student's question:

> [The student] asked this question, and I want you to examine this question: I have a square. I saw that the perimeter and the area result in the same numbers. Is it true for all squares in the world that their area and perimeter are the same?

Another student pointed to the $2 \times 2$ square, showing that its area does not equal its perimeter. The teacher then explicated that this was a counter example, because it showed that the claim is not true for all squares. By doing that, the class not only worked on new mathematical content: proving or refuting a student's conjecture regarding the equality between a square's area and perimeter, but the teacher also explained an important general mathematical principle, of refutation by a counter example.

Thus, as this episode illustrates, in addition to work on new mathematical content, there were times when extending beyond the main topic comprised of developing students' understanding about general norms and conventions in the discipline of mathematics. The episode described earlier regarding the definition of a rhombus in the 9th grade class also exemplifies this. In that episode, the teacher was attentive to a student's confusion regarding what a rhombus is: either a parallelogram with one pair of equal adjacent sides or a parallelogram in which all the sides are equal. Attending to the student's confusion, the teacher responded by reviewing the definition of a rhombus. Yet, she used the student's question also as a vehicle for explaining the minimalism principle of mathematical definitions, deepening students' understanding of mathematical norms and conventions beyond the main topic:

> In definitions we try to say as little as possible. That means, I don't want to say everything I know about a rhombus as its definition. I say as little as possible in the definition, and all the rest I can prove by myself.

The teacher continued to explain that mathematical definitions are not like dictionary definitions, which include as many characteristics as possible about the defined words (concepts).

### Accompanying Talk During Extending Beyond the Main Topic

Occasionally, the teacher used only accompanying talk; but more often she combined accompanying with elaborating talk. This happened, for example, in the above illustration of elaborating talk, when a student suggested checking whether the triangle in Fig. 1c is a "pretty" triangle (i.e., a right triangle with angle measures of 30-60-90°):

S: If one side is one-half of the second, can we say that this is a "pretty" triangle?
T: If a triangle has one side that is one-half of the second, I am repeating the question, if a triangle has one side that is one-half of the second, does it imply that the triangle is a right triangle?
S: And one angle is 60°
T: And one angle is 60°
. . .
T: If in a right triangle. . . there is an angle of 60° [incomprehensible] and the ratio between the two sides that are not opposite the 60° angle. . .
S: She [another student] said: Can we say, the sides that include the angle?
T: The sides that include the angle. Great phrasing. And the sides that include the angle: What about them?
S: Their ratio
T: And their ratio is 1–2. We succeeded to phrase it better. Earlier we said that one [side] is one-half of the other, and now that the ratio is 1–2. Then
S: The triangle is a right triangle.
T: Very true. Then the triangle is a right triangle.
As can be seen, the teacher used in these short excerpts accompanying talk. She basically repeated the students' words, leading the class to unpack the student's question, clarifying what the givens are in the implied conjecture. Yet, this accompanying talk was integrated with elaborating talk, situating the conjecture as "almost" the converse of a theorem they had proven in a previous lesson:
T: I repeat the theorem. It's a converse of a theorem. . . It's somewhat converse, it's not really converse, but it's almost converse.

### Puzzlement During Extending Beyond the Main Topic

The teacher response reflected puzzlement only once during whole-class work that extended the main topic. It occurred when the teacher asked the 10th grade class whether the fact that a theorem in mathematics is true implies that the converse of that theorem is true as well. A student interpreted this question as if the teacher was referring to a specific theorem with which the class dealt a few minutes earlier. For a few seconds the student and the teacher expressed puzzlement until another student pointed out the reason for confusion. The teacher explained to the whole class the source of confusion and called students' attention to the potential problematic use of language:

> See how problematic language can be. I think of one thing and [the student] thinks of another. And we try to communicate. It's really a deaf persons dialog.

## Conclusion

This chapter examined how an experienced high-school mathematics teacher, who had a reputation of encouraging a significant and influential role for students in the class discourse, responded to students' mathematical talk in class. The chapter examined the kinds of responses used repeatedly by the teacher, directing and shaping the classroom discourse.

Analysis of the lessons showed that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. This was true in general, and also during each lesson component (work on the main topic, reviewing content introduced previously, and extending beyond the main topic). Moreover, the teacher was attentive and responsive to different kinds of students' talk.

For example, the teacher made a student's mistake regarding a parallelogram's line of symmetry an object for mathematical exploration for the whole class, and made use of it to discuss an important mathematical topic. She incorporated an examination of the student's mistaken cut parallelogram into a public discussion, revealing what the student did wrong, concluding eventually that a parallelogram does not have a line of symmetry. The teacher attended to the student's work, and acknowledged its value, even though it was wrong, by asking the whole class to examine its validity, and by showing how work on mistakes can advance understanding.

On another occasion, the teacher answered a student's specific question about what a rhombus is, reviewing content introduced in a previous lesson. Yet, she did not deviate from the main teaching sequence at a point that could have been confusing for the class. Instead, she acknowledged the importance of a student's question by promising to respond to it later.

Still in a different occurrence, the teacher attended to a student's hypothesis, and acknowledged its value by asking the whole class to examine its validity. By attending to the student's hypothesis, the teacher deviated from the main topic and made the student's conjecture an object of examination, asking the class to prove or refute the conjecture regarding the equality between a square's area and perimeter. She then exploited the opportunity and made use of a student's answer not only to respond to the student's original hypothesis, but also to extend students' knowledge beyond the topic at stake, and explained an important general mathematical principle, of refutation by a counter example. She signaled that raising hypotheses is a valued activity, and used the opportunity to extend the problem solving activity the class has been already doing. By expecting the other students to participate in the problem solving process, and by using a solution suggested by a student, the teacher indicated also that students' input counts.

The finding that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk was mainly due to the teacher's responsiveness to students. The most common teacher response was elaborating. The teacher constantly elaborated students' utterances and expressed profound cognitive involvement in what students said. In addition to being the most common teacher response to students in general, elaborating was also the most common teacher response during each of three different lesson components. When working on the main topic the teacher embraced and elaborated students' ideas both as a starting point for mathematical examinations and throughout the work. She embraced students' suggestions for whole-class examination when they were productive and correct and also when they were unproductive or mistaken, and she took an active part in developing students' ideas. Although reviewing content introduced previously and extending beyond the main topic rarely occurred as a teacher initiative, but rather were comprised mainly of the teacher's response to students' queries, remarks or requests that emerged during work on the main topic, the teacher's elaborating talk then was similar in nature to that during work on the main topic. Here too the teacher seldom responded succinctly, but rather provided elaborated reviews or extensions (triggered by students' queries and remarks), building on students' active participation.

Responding to students using accompanying talk occurred considerably less than elaborating talk. Yet, the teacher often acknowledged that she followed the student's talk by attending to a student's talk without elaboration. Typically the teacher combined brief accompanying talks in much longer elaborating response; in general, and also during each of three different lesson components. Occasionally, though, accompanying talk was used not as a component of elaborating talk. Sometimes this happened when students provided a correct answer, and the teacher then repeated the student's answer without elaboration and quickly returned to work on the main topic. A few other times it occurred as a teacher initiative when she gathered students' thoughts, hypotheses or solutions, as a starting point for whole-class work.

The teacher rarely expressed puzzlement or confusion when responding to students' talk. Because almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk, this reflects an utter sensitivity, awareness and knowledge about students, and about their thinking and ways of talking. Teacher puzzlement regarding students' talk occurred a small number of times during work on the main topic and once when extending the main topic, but not when reviewing content introduced earlier. Yet, there seems to be no relationships between the occurrence of teacher puzzlement and the nature of the lesson component. It appears that the teacher expressed confusion whenever she could not follow students' talk regardless of the part of the lesson in which it occurred. Yet, she consistently insisted on clarifying the confusion unless it was extremely remote from the main issue.

Teacher responsiveness to students in the form of opposition also seldom occurred. The teacher expressed disagreement or objection to students' ideas a few times during work on the main topic. In contrast with her practice during most of the observed time, in those few events, when students' ideas deviated considerably from the main point, the teacher did not embrace or follow on students' suggestions, queries and remarks. Instead, she pointed out what they should focus on. Yet, this response is quite different from the more common teacher practice of objection to students' wrong answer (Resnick et al., 1993).

This chapter examined the kinds of responses used repeatedly by an experienced high-school mathematics teacher, directing and shaping the classroom discourse, during different parts of the lesson. The chapter presents the ways in which the teacher encouraged and enabled a significant and influential role for students in the class mathematics discourse, while as river guide ("Images of expertise in mathematics teaching" in the chapter by Russ et al., this volume), responded to the students, to the context, and to what occurred in the moment (Berliner, 1994). The chapter demonstrates how the teacher combined her attention to students' talk, with the goal of making progress on the main topic. She was sensitive to students' difficulties in regard to content learned previously, but devoted only a short time to reviewing content introduced previously, using these episodes to enhance understanding. She also exploited opportunities to extend beyond the main topic, developing understanding of the nature of work in mathematics and the nature of the discipline.

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