Reflections and Future Prospects

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Abstract This concluding chapter summarizes what can be learnt from this book concerning the concept and nature of expertise, and how expertise is theoretically conceptualized and empirically measured. The chapter discusses differences between the Eastern and Western perspectives on expertise, and exemplifies their different orientations of teaching towards the subject of mathematics and the individual students. Furthermore, we discuss and analyze the current state-of-art research on expertise and possible research directions for the future.

Keywords Professional knowledge of teachers · Expertise · Novice-expert-model · Cultural differences

It is now common knowledge that teacher expertise in mathematics instruction varies individually and affects teaching performance. However, there is still very limited understanding of the nature of teacher expertise in mathematics instruction. As teachers and teaching have become recognized as a vital part for enhancing students' academic achievement, understanding the nature of teachers' expertise is an unavoidable issue. In fact, with ever-increasing emphasis in current worldwide educational efforts to improve students' mathematics learning, those who care about finding ways of improving mathematics classroom instruction and teacher education have stressed the importance of knowing and understanding what is needed for making and developing expert-like mathematics. Towards this end, this book makes a much-needed and important contribution to the international community of mathematics education and teacher education.

Understanding and evaluating teacher expertise has been a perplexing issue in many education systems for years. Taking an international perspective to examine teacher expertise that is appreciated in the East and the West should help advance our understanding of the issue. For example, existing cross-national studies have

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revealed remarkable differences between Japan, Hong Kong, and the United States in mathematics classroom instruction (Hiebert et al., 2003; Stigler & Hiebert, 1999), between Mainland China and the United States in teachers' knowledge of mathematics for teaching (e.g., Ma, 1999). In particular, Ma's study (1999) revealed that Chinese elementary teachers had a profound understanding of the fundamental mathematics they teach, whereas US teacher participants lacked a strong knowledge base in mathematics. It is in the same spirit of expanding possibilities for learning from many more education systems that this book succeeds in helping readers develop a better understanding of teacher expertise that is valued in different systems.

In this concluding chapter, we will discuss what we can learn from this book and provide our prospects for research development in the future. In particular, consistent with several issues being highlighted in the introduction chapter, we will focus on (1) some similarities and differences between Eastern and Western perspectives on teachers' expertise or its specifications and (2) cultural differences in viewing expert teachers and teaching. Based on these summaries and discussions, we will then suggest directions for future research and the development of teaching practice with regard to teacher expertise in mathematics instruction.

Conceptualizing and Specifying Teacher Expertise

In her fundamental chapter on theoretical perspectives and trends in research on expertise, chapter "Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise" by Chi (this book) develops an overall framework for describing and conceptualizing the nature of expertise from a general perspective. She emphasizes that in contrary to what had been presumed so far, the amount of knowledge is not an important aspect for discriminating experts from novices in solving knowledge-lean tasks: the domain-relevant knowledge and its structure are actually very important for successfully solving complex and knowledge-rich tasks. Especially the representation of the domain-relevant knowledge seems to be a distinctive feature of expertise. Chi points out that teacher experts will generally have a normatively correct and deeper representation of the topics they teach in comparison to novice teachers, who will have a more shallow or incomplete representation of the teaching topics and other important aspects.

A more concrete approach to elaborate on the construct of teacher expertise is the approach by Blömeke (2002), who describes the importance and impacts of (subject-related) knowledge on teacher expertise and its development. She describes teachers with expertise as experts, can develop mental models of the situation in the classroom by referring to previous knowledge especially under enormous time pressure. These models allow them to select the relevant information out of a huge amount of information, to process it, and to come to decisions of different kinds, to recognize problems and to react sensitively and successfully from a pedagogical point of view. On the level of the cognitive processes involved, this means the inclusion of components of expertise such as rapid judgment in the situation, combining and structuring of events observed in the classroom intercourse into a few categories,

and the willingness to change the course of classroom interaction when necessary. This description has the following consequences on the teaching profession: it is consensus that teachers with expertise have a more adequate level of knowledge; that they are able to structure the teaching-and-learning-processes more adequately and in a goal oriented manner; that they see the teaching-and-learning-process in a holistic way, combining the subject's teaching requirement, the organization of the teaching-and-learning-process, and the students' needs as a whole using more abstract concepts. Additionally, the knowledge of teachers with expertise is more coherent and organized according to the teaching situation, not according to single students (Bromme, 1992). Teaching expertise includes the transfer of learned rule-knowledge to more complex if-then-rules, taking information on the teaching situation into consideration.

Consistent with the above conceptualization of teacher expertise by Chi (this book) and Blömeke (2002), teachers' sound subject knowledge has been taken as an important part of their expertise in mathematics instruction in many chapters of this book. In fact, it becomes a common feature, which is highlighted by contributions from the East and the West in one way or another. Yet, there are also notable differences both across and within the East and West in terms of the ways of specifying knowledge as part of teacher expertise. In general, contributions from the East tend to focus on teachers' instructional practices to discuss and specify teachers' expertise in a more holistic way. Thus, teachers' knowledge as part of their expertise is identified by analyzing teachers' instructional practices, and is not taken as a stand-alone facet but rather as an integrated aspect of what teachers are capable of doing. Possible differences across the contributions from the East can be found in the nature of instructional practices that have been analyzed, varying from classroom teaching (chapter "Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics" by Li, Huang, & Yang, this book; chapter "Expertise of Mathematics Teaching Valued in Taiwanese Classrooms" by Lin & Li, this book; chapter "Exploring Korean Teacher Classroom Expertise in Sociomathematical Norms" by Pang, this book), textbook use (chapter "The Japanese Approach to Developing Expertise in Using the Textbook to Teach Mathematics" by Takahashi, this book), to mathematics department heads' perceptions (chapter "Perceptions of School Mathematics Department Heads on Effective Practices for Learning Mathematics" by Lim-Teo, Chua, & Yeo, this book). In contrast, contributions from the West likely examine and analyze teachers' knowledge as an important, yet stand-alone aspect of teacher expertise. Teacher expertise is regarded in an analytical way as containing different components, including knowledge, beliefs, and teaching performance. Even further, chapter "Teacher Expertise Explored as Mathematics for Teaching" by Simmt (this book) conceptualizes and specifies teacher expertise as mathematics for teaching, which differs from the type of mathematics that mathematicians use and talk about. The differences in conceptualizing teacher expertise and the nature of mathematics knowledge in teacher expertise suggest different perspectives and approaches that can possibly be used to examine teacher expertise.

Apart from the central importance of structured knowledge as a key aspect of expertise, Chi describes three constructs emphasized in current research on expertise

in her chapter. In particular, the first construct is *deliberate practice*, which is defined as "expanding intentional efforts to achieve further improvement through focused, concentrated, well-structured, programmatic, and goal-oriented practice. Moreover, the goals of practice are set to go beyond one's current level of achievement, and evaluated by identification of errors, and so on." (chapter "Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise" by Chi, this book) This construct is specifically crafted to explain why some teachers can become real experts but others cannot. This question is of great interest to cognitive psychologists and educational researchers. Yet, the construct itself does not provide a detailed account for the development of structured knowledge in expertise growth.

There is a general consensus that the development of expertise is a tedious learning process, which may last for ten years. The learning theories which serve as a basis to account for expertise development are more and more orientated towards constructivism and emphasizing the domain-specificity of the knowledge. For example, Blömeke adapts the model of Neuweg (1999) in order to describe the development from novice to expert teachers (Blömeke, 2002, p. 81; own translation). This model describes the development of the teachers' expertise concerning various aspects identified as being important for teachers' expertise in various approaches. For example, how the teaching situation is perceived or how the behavior of a teacher is determined. With this model a detailed description of a possible development from novices to experts is provided referring to the different theoretical models on the development of expertise (Fig. 1).

Concerning the development of expertise, Blömeke (2002) describes teachers at the beginning of their teaching practice as being in transition from competence level to mastery level, which means in detail to broaden the achieved competencies through the development of everyday routines. These everyday routines allow the experts to perceive the classroom situation as a whole and not only as consisting of

	level	level	competence	level	level
Considered elements	Context-free	Context-free and *situational*	Context-free and situational	Context-free and situational	Context-free and situational
Sense for the essentials	No	No	Worked out	*Immediate*	Immediate
Perception of the whole situation	Analytical	Analytical	Analytical	*Holistic*	Holistic
Determination of the behavior	By rules	By rules and guidelines	Through extensive planning	Through limited planning	*Intuitive*

Fig. 1 Model on the development of expert teachers (Blömeke, 2002)

single students. This enables the experts to recognize the essential aspects of a situation immediately without long analyses and it allows experts to teach competently without having planned each detail of a lesson carefully in advance. This aspect is emphasized in the study by Shimizu (2008) where novice teachers were compared to experienced teachers.

In her framework, Blömeke (2002) emphasizes the necessity of content-related knowledge and refers to the classification developed by Shulman (1985), which comprises subject matter content knowledge, pedagogical content knowledge, pedagogical, and curricular knowledge. As discussed above, the necessity of high levels of knowledge in these areas as a prerequisite for expertise is emphasized in the various chapters of this book.

Likewise, Schoenfeld and Kilpatrick (2008) developed a framework for proficiency in teaching mathematics, consisting of a set of dimensions, which they consider indispensable for expertise in mathematics teaching and which are strongly related to teaching and learning processes. They name broad and in-depth, sound knowledge of school mathematics, different heuristic strategies and meta-cognitive control strategies as well as a growing competence to reflect teaching-and-learning processes as dimensions of expertise. Knowledge of school mathematics, which should be both deep and broad, plays a central role in their theoretical approach. The breadth of knowledge covers the multiple ways of conceptualizing the relevant mathematics as well as of representing it in various ways, of understanding the key aspects of each topic, and of seeing connections to other topics at the same level. The deepness refers to knowledge on the curricular origin and further conceptual development of the content. Schoenfeld and Kilpatrick (2008) describe the outcome of this kind of knowledge: it allows proficient teachers to prioritize and organize content in such a way that students are introduced to basic/important ideas and not lost in an abundance of details. Furthermore, this knowledge allows teachers to respond flexibly to questions posed by students. This kind of knowledge is called "knowledge of mathematics for teaching" by Ball, Thames, and Phelps (2007) and according to Schoenfeld and Kilpatrick (2008) it involves "more than 'just' knowing the mathematics in the curriculum" (p. 322). Proficient teachers or expert teachers can respond more flexibly to the students' questions than novices. Furthermore, expert teachers are able to craft and manage learning environments, are able to develop classroom norms, and support classroom discourse in the sense of teaching for understanding. To summarize, teachers with expertise show a greater consciousness towards mathematical learning processes and their content as well as towards the development of the students' thinking processes than novices, which is emphasized by Llinares and Krainer (2006) as well.

Cultural Differences

The aforementioned differences in conceptualizing teacher expertise between the East and the West may also be linked to the unspoken difficulty of identifying expert teachers between these two cultures. After having taken a careful read of

the corresponding chapters in this book, readers can notice that not every chapter focuses on expert teachers. In fact, identifying expert teachers can pose a bigger challenge to researchers in the West than in the East, as teaching is regarded a private practice in the West but not in the East (Kaiser & Vollstedt, 2007; Li & Li, 2009). Thus, it is more understandable that researchers in the West take a more hypothetical approach to conceptualize teacher expertise, which is regarded as being necessary to be an expert teacher. In contrast, it is relatively easier for researchers in the East to first identify those teachers who are expected to have expertise and then analyze their expertise in a holistic or analytical way. It presents a procedure similar to many studies on expertise in psychology. Cross-cultural differences in teaching practice and people's views of teaching practices suggest an important dimension when examining and understanding teacher expertise in different cultural contexts.

In different chapters describing an Eastern or Western background, another apparent remarkable difference is the description on the various roles of expert teachers. Chapter "Images of Expertise in Mathematics Teaching" by Russ, Sherin, and Sherin (this book) develop four metaphors of expertise in their chapter:

- the role of teachers as diagnosticians, which refers mainly to the teacher's ability to interpret students' thinking and students' strategies;
- the role of teachers as conductors, shaping the classroom discourse and using classroom norms for communicating about mathematical ideas;
- the role of teachers as architects selecting cognitively demanding tasks;
- the role of teachers as river guides, which involves improvisation, deciding on the spot how to unfold the lesson;

This description of expertise clearly focuses on the learning process and the individual student, his or her learning and the organization of learning processes in order to promote the students' learning.

A comparison to the different aspects of expertise from an Eastern perspective shows clear differences. Yang (2010) differentiates in his study on expert teachers in China multiple roles, which have to be played by an expert teacher:

- expert in teaching, i.e., organizing good teaching processes;
- researcher, i.e., conduct teaching research and publish papers in professional and academic journals;
- teacher educator, i.e., mentor non-expert teachers and facilitate non-expert teachers' professional development;
- scholar, i.e., an expert teacher should have profound knowledge base in mathematics and other areas;
- expert in examination, i.e., have the ability to pose examination problems;
- exemplary model for students and colleagues.

Similar descriptions are developed by Li, Huang, and Yang in their study on expert teachers in this book, in which they describe that expert teachers should serve as moral role models who stand for culturally valued moral characteristics and expertise for others to follow. They continue with the function of expert teachers as

researchers and elaborate that it is important to engage in research and write scientific papers in order to be identified as expert teachers. Being an expert teacher needs to contribute to the improvement of other teachers' academic level and teaching ability, which includes the teacher educator role. From a Western perspective, it is surprising that scientific research is widely required, especially as this means that expert teachers must have written a monograph or more than three research papers published in journals at the provincial level or beyond. They add that an expert teacher should be the leader of the teaching subject at the municipal or county level, who has shown high quality teaching with public and exemplary lessons, and who should have won a prize at a teaching contest at the national level. According to Huang and Li (2009) the teacher promotion system, commonly practiced in China, provides a platform for teachers to value and pursue mathematics classroom instruction excellence. Yang (2009) emphasizes that in contrast to Western culture, where the policy of closed classroom doors is followed (Kaiser & Vollstedt, 2007), the classroom teaching of Chinese mathematics teachers is open for colleagues' observation, studies and discussions, mainly based on the Teaching Research Groups.

Comparing Eastern and Western perspectives on expertise as related to expert teachers' roles, one can describe the Eastern perspective on teacher expertise as more holistic, aiming for a systemic change of the teaching-and-learning processes in school by strengthening teachers as researchers and developing expertise in scientific work. Furthermore the holistic view in the East is accompanied by the public recognition of expert teachers, who are responsible for the development of mathematics education on a broader basis including not only teachers, but curricular aspects as well. The Western perspective is clearly focused on the teaching-and-learning process within the classroom, where experienced teachers shall display their expertise especially in interactions with the students. Characteristic for the Western approach to expertise is the focus on the individual student, who is put into the centre of reflections and actions; the promotion of learning processes of individual students is a major goal of the classroom activities.

Therefore, the differences between the Eastern and the Western approaches concern the different foci on levels of change: while in the Eastern conception a change on a systemic level is desirable, the Western conception refers to changes on the local level. These differences with systemic change focusing on groups of actors in the Eastern conception and the local change with a focus on the individual student relate to strong cultural differences, which are described by cultural psychology as orientation towards collectivistic oriented countries in contrast to individualistic oriented countries. In collectivistic oriented countries, societal actions are seen as commitment against social networks, whereas in individualistic oriented countries the conviction that societal action is a result of freely negotiated contracts is dominant (Hofstede, 1980, 2001; Mascolo & Li, 2004). Transferring this differentiation to individualistic and collectivistic orientation towards education implies (cf. Triandis, 1995) that in collectivistic oriented countries the role of social relations is more strongly emphasized in the learning process: according to this theoretical approach students rather learn due to their commitment towards their teachers, their family, and the social group, who conversely have the responsibility to provide

every necessary support. Failing in school is in this social paradigm attributed to a lack of effort and the required changes rather aim for higher efforts of the students and not for a change of the schooling framework to the benefit of the individual student. In individualistic oriented countries, students are more strongly seen as autonomous subjects, who learn on an individual basis, mainly independent from other individuals. Lacking learning success is explained by referring to inadequate social conditions such as too difficult tasks, poor explanation skills of the teachers or, in general, poor lessons and faults by the teacher. The changes required refer to a change in these social conditions such as changing the teaching styles or the lesson structure, the tasks, or even the school system, but do only seldom refer to the individual student.

These differences in the cultural and psychological paradigms underlying Eastern and Western educational approaches seem to be adequate to explain at least partly the strong holistic focus on expertise in Eastern countries by embracing the professional development of whole teacher groups and in general the educational system, including work on curricular aspects. In addition, the strong individualistic orientation of Western cultures, which expects teachers to provide effective learning environments, good classroom management and so on, leads to a conceptualization of expertise which focuses on the individual student's teaching-learning-processes. Cai, Wang, Wang, and Garber (2009) confirm these results from another perspective: namely effective mathematics teaching from the teachers' perspective. They report that most Asian teachers are more mathematics content-oriented, they emphasize that an effective teacher should understand the content thoroughly and organize teaching in well-structured lessons. This is in strong contrast to teachers from America and Europe, who tend to be more person-oriented and emphasize that an effective teacher should be passionate about mathematics, he or she should have good listening skills and provide enough room and time for students to learn for understanding.

Although there are these strong culturally-based differences between Eastern and Western countries, the cross-national comparative results of the OECD-study TALIS (Teaching and Learning International Survey) point out significant commonalities as well as differences in the teachers' beliefs about the nature of teaching and learning between the participating countries that reveal a few unexpected results (Schmidt et al., 2007). For example, Western countries emphasize the individual student, on the other hand – rather unexpectedly – countries sharing Confucian traditions follow the same constructivist ideas.

To summarize, there are apparently strong cultural differences concerning the description of expertise in mathematics education. These might explain the different ways of implementing expertise in mathematics education.

Areas of Future Research Directions

The above discussion highlights several aspects to be learned from this book. While we can learn much from reading the book, many more questions can actually be

raised about teacher expertise. Here we would like to share three areas of further research with the readers.

The first area relates to teacher expertise itself. The aforementioned similarities and differences in the conceptualization of expertise between the East and West suggest some important aspects that need further examination. In particular, although sound subject knowledge is commonly regarded as being an important part of teacher expertise, it remains unclear what exactly expert or experienced teachers know about mathematics. Further research is needed to examine the expert teachers' level of knowledge about school mathematics, and to find out whether it is important for them to also know advanced mathematics. Different from Ma's study that compares Chinese and US teachers' mathematics knowledge, we suggest to examine subject knowledge in expertise as a consistently changing and dynamic body of knowledge. The nature of mathematics subject knowledge in teacher expertise may vary dramatically between novice and expert teachers, and it is important for us to know and understand what kind and level of structured knowledge expert teachers need and how it is developed.

In Chi's description of three current constructs on expertise research, the second construct is *adaptive expertise*. Chi (this book) describes adaptive expertise as "the notion of knowing not only how to execute or apply a procedural skill, but an adaptive expert is one who also has conceptual understanding of that skill". She emphasizes that adaptive experts understand the procedures or the skills in a profound way, so that they are able to generalize their skills to other non-routine problems. In order to acquire a conceptual understanding, it seems to be necessary to reflect and self-explain the solution of the problem during the problem solving process, which leads to a deeper understanding. This emphasis and high importance of reflection and metacognition is in accordance with new trends in mathematics education, which stress the necessity of metacognition for higher-order thinking processes. Chi's discussion suggests adaptive expertise in mathematics instruction as an important area of research studies. In mathematics education research, a growing number of studies investigates how to help students develop adaptive expertise in problem solving (e.g., Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009), which might help to develop similar studies for teachers. However, much remains unclear about the nature of adaptive expertise in mathematics instruction. Research on adaptive expertise becomes especially important as it can help to understand what is needed for being a real expert teacher and not a routine expert or just an experienced teacher.

Looking into areas in which further research is needed, we take the development of expertise and its wider promotion as one more area of research. As a continuation of this book, Yeping Li and Ruhama Even will edit a special issue of ZDM – The International Journal on Mathematics Education at the end of 2011. The issue on "Approaches and Practices in Developing Teachers' Expertise in Mathematics Instruction" begins with the observation that while educational research has dramatically increased its emphasis on teachers and teaching practice over the past few decades (e.g., Sikula, 1996; Townsend & Bates, 2007), the need for improving teachers' expertise has emerged ever-increasingly in various ways. This includes

the need for practicing teachers' continuous knowledge and practice development in mathematics and pedagogy, teachers' training for undertaking and implementing changes in the curriculum and instruction, and teachers' professional promotion. While various approaches and practices (e.g., lesson study in Japan, teaching research group and apprenticeship practice in China, and video case based learning in the US) have been generated and implemented to address different needs across educational systems, much remains to be learned about specific approaches and practices that have been developed and used effectively. Knowing and understanding effective approaches and practices for developing practicing teachers' expertise in mathematics instruction have become especially important to those who care about the ways of improving mathematics classroom instruction and broad teacher professional education. Li and Even emphasize, that until today, researchers have not come to a consensus on how to define and assess teachers' expertise. In contrast, as described in this chapter, distinct differences between the various approaches common in the Eastern and the Western debate exist (Lappan & Li, 2002). This themed issue is proposed as a sequel to this collaborative book publication on teachers' expertise in mathematics instruction, for the international mathematics education community to develop and share relevant research in the much-needed topic area of approaches and practices utilized to develop such expertise. Hopefully, this book will serve as a starter for rich and extensive debates on the definition and development of expertise, how to promote it, and will lead to reflective ideas on its further embedding in joint cross-cultural endeavors on expertise in mathematics teaching.

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