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Gabriele Kaiser
Editors

Expertise in Mathematics Instruction

An International Perspective

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Editors
Prof. Yeping Li
Department of Teaching, Learning and Culture, College of Education and Human Development
Texas A&M University
College Station, TX 77843-4232, USA
yepingli@tamu.edu

Prof. Gabriele Kaiser
Faculty of Education, Psychology, Human movement, Didactics of Mathematics
University of Hamburg
Von-Melle-Park 8
Hamburg 20146, Germany
gabriele.kaiser@uni-hamburg.de

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To our families –
Aimee, Andrew, Melinda, and Jianrong
&
Sarah and Thomas
– for their love and support

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Contributors

Irina Andreitz Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, irina.andreitz@uni-klu.ac.at

Michelene T. H. Chi Department of Psychology, Arizona State University, Tempe, AZ, USA, michelene.chi@asu.edu

Kwee Gek Chua National Institute of Education, Singapore, Republic of Singapore, kweegek.chua@nie.edu.sg

Ruhama Even Department of Science Teaching, Weizmann Institute of Science, Rehovot, Israel, ruhama.even@weizmann.ac.il

Orly Gottlib Orot Israel College, Rehovot, Israel, paulog@netvision.net.il

Rongjin Huang Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX, USA, hrj318@gmail.com

Gabriele Kaiser Faculty of Education, Psychology, Human Movement, Didactics of Mathematics, University of Hamburg, Hamburg, Germany, gabriele.kaiser@uni-hamburg.de

Eckhard Klieme German Institute for International Educational Research, Frankfurt, Germany, klieme@dipf.de

Konrad Krainer Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, konrad.krainer@uni-klu.ac.at

Yeping Li Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX, USA, yepingli@tamu.edu

Suat Khoh Lim-Teo National Institute of Education, Singapore, Republic of Singapore, suatkhoh.teo@nie.edu.sg

Pi-Jen Lin Graduate Institute of Mathematics and Science Education, National Hsinchu University of Education, Hsinchu, Taiwan, linpj@mail.nhcue.edu.tw

Johannes Mayr Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, johannes.mayr@uni-klu.ac.at

Vilma Mesa School of Education, University of Michigan, Ann Arbor, MI, USA, vmesa@umich.edu

Florian H. Müller Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria, florian.mueller@uni-klu.ac.at

JeongSuk Pang Department of Elementary Education (Mathematics Education), Korea National University of Education, Cheongwon-gun, Chungbuk, South Korea, jeongsuk@knue.ac.kr

Christine Pauli Institute of Education, University of Zurich, Zurich, Switzerland, cpauli@ife.uzh.ch

Kurt Reusser Institute of Education, University of Zurich, Zurich, Switzerland, reusser@ife.uzh.ch

Rosemary S. Russ School of Education and Social Policy, Northwestern University, Evanston, IL, USA, r-russ@northwestern.edu

Alan H. Schoenfeld School of Education EMST, University of California, Berkeley, CA, USA, alans@berkeley.edu

Bruce Sherin School of Education and Social Policy, Northwestern University, Evanston, IL, USA, bsherin@northwestern.edu

Miriam Gamoran Sherin School of Education and Social Policy, Northwestern University, Evanston, IL, USA, msherin@northwestern.edu

Edward A. Silver School of Education, University of Michigan, Ann Arbor, MI 48109, easilver@umich.edu

Elaine Simmt University of Alberta, Edmonton, AB, Canada, esimmt@ualberta.ca

Akihiko Takahashi School of Education, DePaul University, Chicago, IL, USA, atakahas@depaul.edu

Svenja Vieluf German Institute for International Educational Research, Frankfurt, Germany, vieluf@dipf.de

Yudong Yang Center for Teachers' Development, Shanghai Academy of Educational Sciences, Shanghai, China, mathedu@163.com

Joseph Kai Kow Yeo National Institute of Education, Singapore, Republic of Singapore, kaikow.yeo@nie.edu.sg

About the Authors

Irina Andreitz earned a master degree in Psychology. Since 2007 she has worked as a researcher at the Institute of Instructional and School Development at the University of Klagenfurt. Her main research areas include learning motivation, teachers' self-determined motivation, evaluation of teacher professional development and educational development programs.

Michelene Chi is a cognitive science researcher interested in issues of learning. Her research has focused on ways of optimizing students' learning, especially in domains of sciences. Her early research investigated children's competence as a function of knowledge and differences between novices and experts. More recently, she has focused on the origin of misconceptions, and explored approaches to teaching emergent, robustly misconceived processes. She has also published many articles on how students learn from generating self-explanations, from being tutored, from collaborating, and from observing and overhearing tutorial dialogues. Recently she introduced a framework that can differentiate students' learning activities as active, constructive or interactive. Two of her papers have been ranked #1 and #7 most highly cited articles published by the journal *Cognitive Science*. Michelene Chi is currently a Professor in the Department of Psychology at Arizona State University, Tempe, AZ.

Kwee Gek Chua is currently a Lecturer at Mathematics and Mathematics Education Academic Group of the National Institute of Education (NIE), Singapore. She is a mathematics teacher educator involved in the training of pre-service secondary and primary mathematics teachers. Prior to joining NIE in 2009, she was in the teaching service of Ministry of Education (MOE) Singapore for 35 years, where she served in various capacities: as a primary and secondary school mathematics teacher, a mathematics head of department, a Research and Evaluation officer with MOE, and a teaching fellow at NIE. Her research interests include the impact of teachers' behavior on students' achievement and attitude, mathematical pedagogical content knowledge of pre-service and practicing teachers and their professional development.

Ruhama Even is a Professor at the Weizmann Institute of Science and the Rudy Bruner Chair of Science Teaching. She heads the Mathematics Group in the

Department of Science Teaching and directs the Teaching Certification program at Weizmann's Feinberg Graduate School. Her main research interests include education and professional development for mathematics teachers and teacher educators, and the interactions among mathematics curriculum, teachers, and classrooms. She has been a member of the International Committee of the International Group of Psychology of Mathematics Education (PME) and co-chair of ICMI Study 15 on the professional education and development of teachers of mathematics. Ruhama Even earned her Ph.D. in Mathematics Education from Michigan State University in 1989.

Orly Gottlib is a Lecturer on Mathematics and Pedagogy at the Orot Israel College and PhD student at the Kreitman School of Advanced Graduate Studies, Ben Gurion University. Her research focuses on teachers' responsiveness to utterances of students in mathematics lessons.

Rongjin Huang is a Mathematics Educator who had been working as a secondary mathematics teacher and university faculty in China for many years. He is currently continuing his further study at Texas A&M University. He is a team member of the Learner's Perspective Study which is a well-known international classroom research project. His interests include mathematics classroom research with a focus on Chinese pedagogy of mathematics, and mathematics teacher education. He has completed several research projects funded by The University of Hong Kong and University of Macau and his work has been extensively published in articles and book chapters internationally. He has also published several books. He organized and chaired several topic study sessions at various regional and international professional conferences such as EARCOME 3 & 4, and ICME-11.

Gabriele Kaiser is a full Professor in Mathematics Education at the Faculty of Education of the University of Hamburg since 1998. She holds a master's degree as a teacher for mathematics and humanities, which she completed at the University of Kassel in 1978. She has taught in school from 1979–2000. She completed her doctorate in mathematics education in 1986 on applications and modelling and her post-doctoral study (so-called "Habilitation") in pedagogy on international comparative studies in 1997, both at the University of Kassel. Her post-doctoral study was supported by a grant of the German Research Society (DFG). From 1996 to1998 Gabriele Kaiser worked as a guest professor at the University of Potsdam.

Her areas of research include modelling and applications in school, international comparative studies, gender and cultural aspects in mathematics education and empirical research on teacher education. She has received grants from the German Research Society (DFG) in order to support this research. At present she is Editor-in-Chief of the journal *ZDM – The International Journal on Mathematics Education* (formerly *Zentralblatt fuer Didaktik der Mathematik*), published by Springer and Editor of a monograph series "Advances in Mathematics Education" published by Springer too. Since July 2007 she has served as president of the International Study Group for Mathematical Modelling and Applications (ICTMA), an ICMI affiliated Study Group.

Eckhard Klieme is Full Professor of Educational Science at Goethe University and Director of the Center for Research on Educational Quality and Evaluation at the German Institute for International Educational Research (DIPF) in Frankfurt am Main (since 2001). His research areas include educational effectiveness and educational assessment. Eckhard Klieme received a Diploma in Mathematics and a Doctorate in Psychology from The University of Bonn, Germany. He has been the principal investigator for research projects on mathematics and science instruction (including TIMSS-Video/Germany, the Swiss-German "Pythagoras Study", and ongoing studies on formative assessment) as well as for nation-wide school evaluation studies in Germany (citizenship education, language education, after school programs). He is also involved in international studies such as TALIS and PISA, where he currently serves as chair of the International Questionnaire Expert Group.

Konrad Krainer is Full Professor at the University of Klagenfurt. He started his professional career as a mathematics teacher and research assistant. Konrad Krainer took leadership in several international scientific boards, including the European Society for Research in Mathematics Education and the Education Committee of the European Mathematical Society. He was Associate Editor of the *Journal of Mathematics Teacher Education* (1998–2009) and is Co-Editor of the *International Handbook of Mathematics Teacher Education* (2008). He leads the Institute of Instructional and School Development and the Austrian National Initiative IMST (Innovations in Mathematics, Science and Technology Teaching, 2000–2012). His research interests are mathematics teacher education, school development and educational system development.

Yeping Li is Full Professor of Mathematics Education at Texas A&M University. He is interested in examining issues related to mathematics curriculum and teacher education in various education systems, and understanding how factors related to mathematics curriculum and teachers may come together in shaping effective classroom instruction that is valued in different cultures. His research projects were supported by various funding agencies, such as the Spencer Foundation. His work has been published extensively in many journals internationally. He has been serving as an Associate Editor for the *Journal of School Science and Mathematics*, and a guest editor for other journals including the *International Journal of Educational Research* and *ZDM – The International Journal on Mathematics Education*. He organized and chaired many group sessions at various prominent national and international professional conferences, such as ICME-10 in 2004 and ICME-11 in 2008. He received his Ph.D. in cognitive studies in education from the University of Pittsburgh, USA.

Suat Khoh Lim-Teo is an Associate Professor and the current Dean/Academic at the National Institute of Education, Singapore. She was formerly the Head of the Mathematics and Mathematics Education Academic Group (2000–2004) and an Associate Dean in charge of pre-service programs (2003–2006). She has been a mathematics teacher educator for more than 20 years, teaching various methods and content courses in pre-service as well as in-service programs. She has also been the

President of the Association of Mathematics Educators, Singapore (1998–2000), a Vice-President of the Singapore Mathematical Society (2001–2004) and a member of the International Program Committee for ICME10 (Denmark, 2004). While her earlier research interests were in various aspects of mathematics learning, her recent focus is in the area mathematics teacher education, particularly in the development of mathematics pedagogical content knowledge of primary school teachers.

Pi-Jen Lin is a Professor at the National Hsinchu University of Education, Taiwan. Her research focuses on mathematics instruction, pre-service and in-service teachers' professional development. Aside from this, she has taken the responsibility of the international studies such as TIMSS2003, TIMSS2007, TIMSS2011 at the fourth grade level and TEDS-M2008 for primary level in Taiwan. Lin has authored or co-authored over 100 publications in national or international journals or book chapters and has lectures and presentations in national or international conferences or institutes. She has served on several national commissions including curriculum advisory committees at national and local levels. She is the editor of several national journals such as the *Chinese Journal of Science Education*. Most recently, she served as a member of the International Committee of the International Group of Psychology of Mathematics Education (PME30–PME33).

Johannes Mayr is Full Professor at the University of Klagenfurt. He started his professional career as an elementary school teacher, later on he worked as a teacher educator at the Teacher Training College of the Diocese of Linz, Austria (1975–2005). He is co-ordinator of the web-based program Career Counselling for Teachers (CCT). His main research areas are teacher education, school development, classroom climate, and classroom management.

Vilma Mesa is Assistant Professor of Education at the University of Michigan. She investigates the role that resources play in developing teaching expertise in undergraduate mathematics. Currently she is investigating the nature of mathematics teaching in community colleges, attending to how instruction is organized, delivered, and sustained. She has conducted several analyses of textbooks and has been involved in evaluation projects that study the impact of innovative teaching practices in mathematics for students in science, technology, engineering, and mathematics fields. She has a B.S. in computer sciences and a B.S. in mathematics from the University of Los Andes in Bogotá, Colombia, and a master's and a Ph.D. in mathematics education from the University of Georgia.

Florian Müller earned a master degree in Educational Studies and Educational Psychology at the University of in Munch, Germany and a Ph.D. in Education. He worked as a lecturer and researcher from 1995 to 2006 at the University of Bundeswehr in Munich and in 2004 as a guest researcher at the University of Cape Town (South Africa), Institute of Psychology. Since 2006 he has been a member of the Faculty of Interdisciplinary Studies of the University of Klagenfurt; and Associate Professor since 2009. He is associate editor of the *Journal of Educational Research* – online. His main research lies in the following areas: self-determined student and teacher motivation, interest, teacher professional development, and teaching and learning in higher education. His main practical activities are basic and advanced teacher training and further education in academic instruction.

JeongSuk Pang is an Associate Professor of mathematics education at Korea National University of Education (KNUE), South Korea. While her publications have been diverse covering various aspects of elementary mathematics education, she is particularly interested in the analysis of the mathematics classroom culture from a socio-cultural perspective and professional development both for pre-service and in-service teachers. With a fund from Korea Research Foundation, she recently developed case-based pedagogy for teachers to learn how to teach mathematics. In 2009, in addition to receiving the best professor award at KNUE, Pang was also awarded first place in a competition to develop "classroom-friendly" instructional methods to educate pre-service teachers. She has been actively involved in developing new mathematics curriculum and its concomitant textbook series. She has been serving as an international director of the Korea Society of Educational Studies in Mathematics and is a member of the International Committee for PME (2008–2012).

Christine Pauli studied education, psychology and philosophy at the University of Berne, Switzerland, after teacher training and 10 years of professional experience as an elementary school teacher. She received a Ph.D. in Educational Psychology from the University of Zurich, Switzerland. She is now a Senior Researcher at the Institute of Education, University of Zurich. Her main research interests are video-based studies on teaching and learning, classroom discourse and teacher education.

Kurt Reusser is Professor of Educational Psychology and Education at the University of Zurich (since 1993). After training and serving as an elementary school teacher (1966–1972), he received his M.A. (1977) and Ph.D. (1984) from the University of Berne (Hans Aebli). After 6 years educating teacher candidates at a teacher training college, he was a postdoctoral fellow (1984–1986) at the University of Colorado, Boulder, working with Walter Kintsch. He received his post-doctoral Habilitation in Educational Psychology in 1990. His research and publications have focused on cognitive and instructional psychology, problem solving, the work of Jean Piaget, general didactics, teacher learning and teacher education and video-based research on mathematics teaching and learning (principal investigator of a series of video studies including the TIMSS 1999 video study).

Rosemary S. Russ is a Research Assistant Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. Her research involves examining K-16 science and mathematics learning, as evidenced in classrooms and clinical interviews, to understand student and teacher epistemology and cognition. In addition, she is interested in understanding clinical interviews as discourse interactions. She draws on methodological traditions from qualitative education research, cognitive science, and conversation analysis. She has coauthored several book chapters on student and teacher cognition and recent articles appear in *Science Education* and *Issues in Teacher Education*. Russ is a 2010 National Academy of Education/Spencer Postdoctoral Fellowship finalist.

Alan Schoenfeld is the Elizabeth and Edward Conner Professor of Education and Affiliated Professor of Mathematics at the University of California at Berkeley. He is a Fellow of the American Association for the Advancement of Science and an inaugural Fellow of the American Educational Research Association, and a Laureate of the education honor society Kappa Delta Pi. He has served as President of the American Educational Research Association and as the vice president of the National Academy of Education. In 2008 he was given the Senior Scholar Award by AERA's Special Interest Group for Research in Mathematics Education.

After obtaining his Ph.D. in mathematics from Stanford in 1973, Schoenfeld turned his attention to issues of mathematical thinking, teaching, and learning. His work has focused on problem solving (what makes people good problem solvers, and how can people get better at it?), assessment, teachers' decision-making, and issues of equity and diversity. His most recent book, *How We Think*, provides detailed models of human decision making in complex situations such as teaching.

Bruce Sherin is an Associate Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. His research focuses primarily on conceptual change in science, particularly as it occurs within novel technology-based learning environments. In recent research, he has begun to investigate the use of techniques from computational linguistics for the analysis of natural language data. In addition, with Miriam Sherin, he is exploring the use of novel technologies for the study of teacher cognition. He currently serves on the editorial boards of the journals *Cognition and Instruction* and the *Journal of the Learning Sciences*. In addition, he is an executive editor of the *International Journal of Computers for Mathematical Learning*.

Miriam Gamoran Sherin is an Associate Professor of Learning Sciences in the School of Education and Social Policy at Northwestern University. Her interests include mathematics teaching and learning, teacher cognition, and the use of video for teacher learning. Sherin's forthcoming book *Mathematics teacher noticing: Seeing through teachers' eyes* will be published in 2011 by Routledge. Recent articles appear in *Journal of Teacher Education*, *Teaching and Teacher Education,* and *Journal of Mathematics Teacher Education*. In 2001, Sherin received a postdoctoral fellowship from the National Academy of Education and in 2002 she was awarded a Career Grant from the National Science Foundation. In April, 2003, Sherin received the Kappa Delta Pi/American Educational Research Association Division K Award for early career achievements in research on teaching and teacher education.

Edward A. Silver is the William A. Brownell Collegiate Professor of Education and Professor of Mathematics at the University of Michigan. His scholarly interests include the study of mathematical thinking, especially mathematical problem solving and problem posing; the design and analysis of intellectually engaging and equitable mathematics instruction for students; innovative methods of assessing and reporting mathematics achievement; and effective models for enhancing the knowledge of teachers of mathematics. He served as editor of the *Journal for Research in Mathematics Education* from 2000–2004, and he is currently serving as co-editor of

The Elementary School Journal. He received his doctorate in mathematics education from Teachers College, Columbia University.

Elaine Simmt is Professor of Secondary Mathematics Education at the University of Alberta, Canada. She began her career as a secondary school teacher of mathematics and physical science. She completed doctoral studies in mathematics education under the supervision of Tom E. Kieren. Simmt's research explores teaching and learning as understood through enactivsm and the frame of complexity theory. Most recently that work has led to her to the study of mathematics for teaching. In that work she and Dr. Brent Davis (University of Calgary) applied organization principles from complex adaptive systems in their study of the emergent learning systems found in mathematics classrooms and professional development sessions. Site-based research has triggered Simmt's interest in research methods and designs suitable for studying the complex and contextual phenomena of teaching and learning.

Akihiko Takahashi is currently the Director of the Asia-Pacific Mathematics and Science Education Collaborative and an Associate Professor of mathematics education at DePaul University in the United States. He teaches mathematics teaching and learning, and mathematics for prospective teachers. He also provides workshops and seminars for practicing teachers using ideas from the U.S. and Asian countries. He was an elementary teacher in Japan before becoming an educator of mathematics teachers. During his elementary teaching career, he was nationally active in mathematics lesson study and mentored 200 pre-service teachers. He received his Ph.D. from the University of Illinois at Urbana-Champaign; his dissertation research focused on internet use in mathematics education.

Svenja Vieluf received a Diploma in Psychology from University of Mainz, Germany. She is currently a research assistant at the Center for Research on Educational Quality and Evaluation at the German Institute for International Educational Research (DIPF) in Frankfurt am Main Her research interests include international comparative educational research, instructional quality, cross-cultural research methods, and migration and education. She was involved in the analysis and reporting of the OECD-Teaching and Learning International Survey and is now affiliated with the international consortium for PISA 2012.

Yudong Yang is currently Associate Professor and deputy director of the Center for Teachers' Development in Shanghai Academy of Educational Sciences since 2006, and Deputy Secretariat of Shanghai Committee of School Mathematics Teaching since 2008. His major research interests are in mathematics teacher education, especially in-service teacher's professional learning in school-based teaching environment and he is also a founding member of the WALS (World Association of Lesson Study). In his recent research project sponsored by the Ministry of Education of China, he focused on how to drive classroom teaching and learning by a series of questions originated from primitive mathematics ideas.

Joseph Kai Kow Yeo is an Assistant Professor in the Mathematics and Mathematics Education Academic Group at the National Institute of Education, Singapore. As a teacher educator, he is involved in training pre-service and in-service mathematics teachers at primary and secondary levels and has also conducted numerous professional development courses for teachers in Singapore. Before joining the National Institute of Education in 2000, he held the post of Vice Principal and Head of Mathematics Department in secondary schools. His research interests include mathematical problem solving in the primary and secondary levels, mathematics pedagogical content knowledge of teachers, mathematics teaching in primary schools and mathematics anxiety.

Part I
Introduction and Research Perspectives

Expertise in Mathematics Instruction: Advancing Research and Practice from an International Perspective

Yeping Li and Gabriele Kaiser

Abstract Expertise in mathematics instruction, as commonly recognized, varies from one teacher to another and also affects their teaching performance. Studies on expertise in mathematics instruction are thus important, albeit long overdue, to reveal its specifics. To advance relevant research and practice for the improvement of teacher expertise in mathematics instruction, this book takes a unique approach to present new research from multiple education systems in the East and West. In this introduction chapter, we highlight the background of this book project, three important issues probed in this book, and the book's content structure and overview.

Keywords Eastern culture · Expert teacher · International perspective · Mathematics instruction · Teacher expertise · Western culture

Introduction

There is a general consensus on the importance of having and developing expertise in a professional field. Experts' masterful performance in many fields, such as sports, medicine, mathematics, and music often amazes us. Efforts to pursue excellence in different fields have led not only to the better quality of work and performance, but also to the on-going quest about the nature of expertise that helps distinguish experts as they are from many others. Examining and knowing the nature of expertise also helps us understand what it may take for a novice to become an expert in that field. It is now commonly acknowledged that experts are knowledgeable about what they do and they have a more structured knowledge than non-experts (e.g., "Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise" in the chapter by Chi, this book). Yet, much still

Y. Li (✉)
Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX 77843-4232, USA
e-mail: yepingli@tamu.edu

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remains to be understood about the nature of expertise, especially in those fields that often present complex and not well-structured tasks such as classroom instruction. Classroom teaching presents itself as such a task that relates to numerous factors and no well-defined algorithm is available to guarantee a successful solution. Nevertheless, the quality of classroom instruction has continually been taken as a key factor contributing to students' learning. Examining and understanding the nature of teacher expertise in mathematics instruction is certainly not a trivial task for educational researchers, and is also imperative to those who seek ways to help teachers improve the quality of their classroom instruction.

This book reflects the ever-increasing interest and effort in improving students' learning of mathematics through enhancing teachers' quality and their teaching. While examining and learning about teachers' classroom instruction and their expertise have long been the interest of educational researchers and psychologists albeit mainly in the West (e.g., Borko & Livingston, 1989; Leinhardt, 1989; Leinhardt & Greeno, 1986; Livingston & Borko, 1990; Swanson, O'Connor, & Cooney, 1990), there is a lack of systematic studies on teachers' expertise in mathematics instruction. The initial development of this book project relates not only to the importance and needs of further research on mathematics teachers' expertise, but also to recent international studies that documented distinct, yet often praised, teachers' instructional performance and their knowledge in several high-achieving educational systems in East Asia. In particular, this book was initiated and motivated with the following two reasons:

First, this book presents an extension of a recent ZDM thematic issue on exemplary mathematics instruction in East Asia (Li & Shimizu, 2009). As the thematic issue of ZDM focused on exemplary mathematics instruction in six high-achieving education systems in East Asia (i.e., China, Hong Kong, Japan, Singapore, South Korea, and Taiwan), relevant studies illustrated what Asian teachers may do in carrying out their culturally valued lesson instruction but not the kind of expertise that is needed to make exemplary teaching performance possible. With limited knowledge now available about Asian teachers' expertise in mathematics instruction, this book thus contains a collection of studies on teachers' expertise in mathematics instruction in five out of the same six high-achieving education systems in East Asia (i.e., China, Japan, Singapore, South Korea, and Taiwan).

Second, this book was also inspired by the well-publicized Ma's work that compared selected Chinese and US elementary teachers' knowledge in mathematics (Ma, 1999). Ma revealed the dramatic differences in elementary teachers' knowledge in mathematics between China and the United States, which led to further questions about the nature of expertise that may help connect or distinguish teachers' instructional performance between the East and West. Therefore, this book is taking an international perspective to include two sets of chapters that focus on teachers' expertise in mathematics instruction in the West and East, respectively. The international perspective should allow us to reflect on teacher expertise that is valued for developing high-quality classroom instruction in different education systems. Taken together, these intended extensions allow the book to make unique contributions to the much-needed study of teachers' expertise in mathematics instruction.

Indeed, the book presents a new scholarship in addressing what appears to be a rather traditional topic in educational and psychological research. Although examining and understanding expertise is not a new endeavor, studying teachers' expertise in mathematics instruction is a challenging task. First of all, the challenge lies not only in the complexity of mathematics instruction practices that do not have a commonly agreed-upon effectiveness, but also in the array of factors contributing to classroom instruction that go beyond cognition. Rather than staying away from such a challenging task, this book's contributors undertook the challenge to develop or adopt different perspectives and methods in examining various aspects of teachers' expertise in mathematics instruction. Second, taking an international perspective in this book is unique in that it presents not only an advantage, as mentioned above, but also a challenge. The challenge is embedded in the nature of teaching as a cultural activity (Stigler & Hiebert, 1999). What teachers do in mathematics classrooms is fundamentally influenced by specific cultural values. Examining teachers' expertise in mathematics instruction thus calls for extra caution in understanding and interpreting teachers' expertise that contributes to culturally-valued instructional practice in a specific system and cultural context. Therefore, what we can expect to learn from this book will differ from a typical book on expertise in many ways, and is irreplaceable due to the very nature of the task in focus.

Initiating and editing of this themed book also builds upon our ongoing research interests in mathematics classroom instruction and mathematics teachers' knowledge (e.g., Blömeke, Kaiser, Lehmann, & Schmidt, 2008; Huang & Li, 2010; Kaiser, Luna, & Huntley, 1999; Li & Shimizu, 2009). As editors of this themed book, we contribute from our own extensive experiences in mathematics education research and practices in the East and West. At the same time, we got intensive insights into the nature of expertise and its successful practices in the East and West. We are therefore convinced that the chapters in this book are valuable sources of information for international readers to learn and reflect upon possible similarities and differences in teachers' expertise that is needed to develop culturally valued instructional practices.

Examining and Understanding Expertise in Mathematics Instruction in an International Context

As a cultural activity, mathematics teaching is situated in a specific cultural setting and also presents unique challenges to teachers in that culture. TIMSS classroom video studies (Hiebert et al., 2003; Stigler & Hiebert, 1999) have prompted further interests and studies about specific teaching practices that are formed and nurtured in a specific education system such as China (Fan, Wong, Cai, & Li, 2004), several education systems in East Asia (e.g., Li & Shimizu, 2009; Lim, White, & Kaur, 2008), or different cultural traditions around the world (Clarke, Keitel, & Shimizu, 2006). However, neither TIMSS video studies nor some other existing studies on classroom instruction aimed to analyze and discuss what teachers need to know and be able to do in each participating education system. Much remains to be understood about the nature of teachers' expertise that is valued in different education systems in the East and West. Included in this book, the majority of individual chapters can provide readers with its specifics about teacher expertise valued in one education system. With these chapters being put together as a collection, this book provides readers a platform to cross-examine and reflect on different aspects and issues of teacher expertise that are specified and discussed in different education systems. While readers can surely learn something beyond individual chapters through reading the book, here we would like to highlight three issues that are important for the broad readership in mathematics education and teacher education internationally.

The Issue of Identifying and Selecting Teachers with Expertise

In order to study teachers' expertise, it is a common approach to examine what expert teachers know and are able to do while implementing mathematics teaching. These studies were carried out either with a comparison to novices or without such a comparison. However, identifying and selecting expert teachers is not a task that is based on a commonly-accepted approach across different studies (e.g., Berliner, 1986, 2001). While some researchers may rely on teachers' educational background and their years of experience, others may emphasize their students' academic performance and administrators or peers' recommendations. In fact, there is often a lack of clear reference to teachers' performance in classroom instruction or their knowledge when identifying expert teachers. The situation is especially acute in the West, where teachers' instructional practices are not made public for scrutiny and discussion (e.g., Kaiser & Vollstedt, 2008). The lack of commonly used criteria in evaluating teachers and their teaching led researchers to make their own selection with different criteria in the past. Researchers' judgement and determination of different selection criteria, as often practiced in the West, do pose an inherent difficulty when so-called or assumed expert teachers who may not possess expected expertise are selected for studying their expertise. Thus, rather than solely relying on researchers' judgement and decisions, some contributors of this book used different approaches in identifying and selecting expert teachers. Expert teachers can be those who have been pre-identified as meeting specific certification requirements in the United States (e.g., "Coordinating Characterizations of High Quality Mathematics Teaching: Probing the Intersection" in the chapter by Silver & Mesa, this book), or those who have obtained an advanced rank as meeting specific professional requirements including classroom teaching in China (e.g., "Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics" in the chapter by Li, Huang & Yang, this book). The use of such alternative approaches enables researchers to focus on those teachers who have already been identified and valued as expert teachers in a specific education system.

As it is generally acknowledged that classroom instruction is a complex and cultural activity (Stigler & Hiebert, 1999), being an expert teacher for doing what is culturally valued instructional practice is likely a culturally-related judgement in different education systems. The identification and selection of expert teachers remains to be a challenge to the fields of mathematics education and teacher education. Nevertheless, the use of alternative approaches by some researchers in this book provides us a direction for possible methodology changes in studying teacher expertise especially in an international context.

The Issue of Specifying and Analyzing Aspects of Teachers' Expertise in Mathematics Instruction

Acknowledging the importance of teacher expertise does not provide specific suggestions or approaches for conceptualizing and studying teacher expertise. Because studies on teachers' expertise are not a new endeavor in the realm of educational research, previous studies on teacher expertise can provide us with some hints for the aspects of teacher expertise that have typically been focused on.

In the United States, many researchers took a personal expertise perspective to examine individual expert teachers' knowledge, their teaching practices, or teachers' knowledge development from novice to expert (e.g., Berliner, 1986, 2001; Borko & Livingston, 1989; Leinhardt & Smith, 1985). Over the years, different approaches have been developed for examining teachers' expertise. In particular, Sherin, Sherin, and Madanes (2000) indicated that two main different approaches have been developed to conceptualize teachers' expertise. One is a cognitive modeling approach that focuses on classroom instruction process, and the other is a knowledge system perspective that tends to specify knowledge components of teachers' expertise. Moreover, the development and exhibition of teachers' expertise is also associated with their beliefs and views of what can be counted as effective/good teaching. Thus, past research has developed a repertoire of methodologies that can possibly be used in studying teachers' expertise in mathematics instruction. Given the diverse aspects of teacher expertise that can possibly be focused on, we expect to find different aspects and approaches being taken by contributors of this book. For example, some contributors focused on teachers' practices in classroom instruction (e.g., "Responding to Students: Enabling a Significant Role for Students in the Class Discourse" in the chapter by Even & Gottlib, this book; "Expertise of Mathematics Teaching Valued in Taiwanese Classrooms" in the chapter by Lin & Li, this book), some focused on teachers' knowledge and/or beliefs (e.g., "Teacher Expertise Explored as Mathematics for Teaching" in the chapter by Simmt, this book), while others took a combination of both instructional practices and knowledge (e.g., "Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics" in the chapter by Li et al., this book; "Cross-Nationally Comparative Results on Teachers' Qualification, Beliefs, and Practices" in the chapter by Vieluf & Klieme, this book).

At the same time, some researchers examined the traditional teaching practices of expert teachers in the context of current educational changes (e.g., Schoenfeld, 1988). This presents a holistic perspective that takes into account the large social and cultural setting and related changes in valuing certain educational practices demonstrated with specific expertise in mathematics and pedagogy for teaching. Intuitively, results from this type of research pose a similar question and challenge in understanding what is valued as teachers' expertise in different system and culture settings, as teaching is now commonly acknowledged as a cultural activity. Although we will further discuss the cultural issue in the next section, we want to remind readers about possible social-cultural influences on teacher expertise that are valued and examined in different chapters of this book.

The Issue of Understanding Expertise in Mathematics Instruction that is Valued in Different Cultures

Understanding and evaluating teacher expertise has been a perplexing issue in many education systems. By taking an international perspective, this book provides us with a unique opportunity to better understand the nature of teacher expertise that may be viewed and valued differently across the East and West. Indeed, taking an international perspective has helped us to learn a great deal about our own educational policy and practices in mathematics curriculum (e.g., Leung & Li, 2010; Li & Kulm, 2009), teachers' classroom instruction (e.g., Clarke et al., 2006; Li & Shimizu, 2009; Stigler & Hiebert, 1999), and teachers' knowledge (e.g., Ma, 1999; Sullivan & Wood, 2008). It is in the same spirit that the chapters of the book offer insight into teacher expertise that is valued in other system and cultural contexts.

At the same time, possible differences in viewing and valuing teacher expertise would also place a unique challenge for conducting cross-cultural examinations of teacher expertise. Thus, this book was not proposed as a collection of cross-cultural studies. Instead, this book contains a collection of studies of teacher expertise within individual education systems in the East and West, respectively. Correspondingly, studies on teacher expertise in individual education systems are grouped into two separate sections with one for education systems in the East and the other for the West. With this grouping we offer insight not only into teachers' expertise of Eastern and Western cultures, but also collective differences and similarities between the East and West regions (i.e., across the two sections).

In addition, this book did not place or pre-specify any specific conception of teacher expertise. In this way, our contributors were given much flexibility in identifying and examining what is valued in teacher expertise in different education systems. The collection of individual studies from the East and West should help to provide a glimpse of the nature of teacher expertise in mathematics instruction that is also of interest to cognitive psychologists, and to explain what is valued for and in mathematics classroom instruction in the East and West. Finally, without prespecifying a conception of teacher expertise, this book can also help raise questions and issues for mathematics education researchers to guide critical examination of what can be learned from other education systems.

Overview of the Book

The book contains four parts. The first part provides an introduction and related research summaries. It is structured as containing three chapters, with the first chapter as this introduction chapter to this book including its organization and content overview, the second chapter to provide an overview of related theoretical perspectives and methodological approaches, and the third chapter to provide a review of related research on this topic in the field of mathematics education. The second part contains a series of five chapters that examined teacher expertise valued in a Western setting. Correspondingly, the third part contains a similar set of five chapters as Part II but with research focusing on an Eastern setting. Part IV is a part for reporting a large cross-national study related to teacher expertise and commentary chapters. Two commentary chapters are included to draw together research reported in Parts II and III. While one is to reflect on teacher expertise, the other is to reflect on what we can learn from this international collaborative effort and possible research directions for the future.

This book structure allows readers to get relevant information about the three issues highlighted in the above section "Examining and Understanding Expertise in Mathematics Instruction in an International Context". In particular, because each chapter in Parts II and III tends to focus on those (expert) teachers and their expertise valued in a specific education system in the East or West, we expect that readers can gain much information about the first and second issues (see section "Examining and Understanding Expertise in Mathematics Instruction in an International Context") from reading individual chapters. However, the third issue of understanding teacher expertise in mathematics instruction that is valued in different cultures is not always stated explicitly in each chapter. Thus, the book's structure of separate parts for the Western and Eastern regions (i.e., Parts II and III) should assist readers when reading and reflecting on possible similarities and differences on teacher expertise both within and across the Western and Eastern regions.

Part I: Introduction and Related Research Summaries

Three chapters included in this part aim to provide a general background about this book and relevant research on expertise. The chapter written by Michelene Chi provides an overview of psychological studies of expertise. With a focus on the changes in theoretical perspectives and methodological approaches, Chi outlines some major developments in psychological studies over the years. Although the concept of expertise has always been related to knowledge, it took years of research development to learn the importance of knowledge especially structured knowledge.

Now, psychological researchers pay close attention to the acquisition of expertise. Some current constructs include deliberate practice, adaptive expertise, and team expertise. A new idea about the acquisition of expertise is also proposed as the construct of a perspective shift. Nevertheless, Chi indicates that many questions still remain to be explored about expertise and its acquisition.

Different from Chi's overview of psychological studies on expertise, Russ, Sherin, and Sherin focus on the concept of teaching expertise that are emerged in the study of mathematics teaching. Thus, these researchers take a historical perspective to trace the study of mathematics teaching in an attempt to capture emerged images of teachers in mathematics teaching. In particular, four images have been identified: mathematics teachers as diagnosticians of students' thinking, conductors of classroom discourse, architects of curriculum, or river guides who are flexible in the moments of teaching. The identification of these images helps us not only understand specific expertise that may be required behind different images, but also guide further efforts in identifying and positioning possible new images in mathematics teaching.

Part II: Understanding and Examining Teacher Expertise in a Western Setting

There are five chapters that report on the study of teacher expertise in a Western setting. These five chapters present diverse perspectives and approaches employed to examine various aspects of teacher expertise in different education systems. While the first three chapters make a close connection with teachers' classroom instruction in studying teacher expertise, the remaining two chapters conceptualize teacher expertise more in terms of knowledge or structured components.

In the first chapter, Silver and Mesa probed different approaches and their intersections in characterizing high quality mathematics teaching. By taking three different views of exceptional mathematics teaching, the researchers examined empirically how lesson instruction and teachers' commentaries on lessons submitted by a group of teachers obtained the NBPTS (US National Board for Professional Teaching Standards) certification may be similar or different from those by a group of teachers who were not awarded the NBPTS certification. Their analyses identified some strong interactions between the NBPTS view of accomplished teaching and the effective use of cognitively demanding tasks in the mathematics classroom, but not with the expected use of innovative pedagogical strategies to engage students. Through these three different views, the study certainly enables us to develop a better understanding about those NBPTS certified teachers' instructional performance. At the same time, the results also illustrate those teachers' strengths and weakness in selected aspects of teacher expertise in mathematics instruction.

Pauli and Reusser developed the chapter, expertise in Swiss mathematics instruction, through drawing together data and findings from several video studies on mathematics teaching in Switzerland. The researchers proposed a profile of teacher expertise that is associated with different components of a didactic triangle (i.e., content, teacher, and students). With the model of the didactic triangle, the researchers aimed to identify possible strengths and weaknesses in expertise that Swiss mathematics teachers in general (not just expert teachers) may have. In particular, the researchers pointed out that Swiss mathematics teachers have particular strengths in the culture of communication, support, and relationships that mainly connects teacher and students in the didactic triangle, and positive but less strong in connecting content and students. They further suggested that Swiss teachers need to improve their didactics of mathematics that connects teacher and content.

The chapter written by Even and Gottlib focuses on an experienced high school mathematics teacher's classroom practices in responding to students. Different from other chapters in the second part, the researchers provided a detailed analysis of the teacher's classroom instruction. The identification and selection of this experienced teacher was due to her reputation of involving and engaging students in the class discourse. The teacher's extensive involvement in some curriculum committees at the national and local levels also suggests her extensive knowledge in mathematics curriculum and instruction. The detailed analyses of her teaching in both a lower-achieving 9th grade class and a high-achieving 10th grade class revealed how the teacher developed her instruction as building upon students' talk. Developing communication and relationships with students is apparently taken as an important component for making effective instruction possible in this classroom. Behind the teacher's sensitivity about students' talk and her skills in identifying and developing learning opportunities for students, the researchers illustrated some important aspects of teacher expertise that are valued in a mathematics classroom in Israel.

Four researchers from Austria, Müller, Andreitz, Krainer and Mayr, contributed this chapter to document the effects of a research-based learning approach (a four-semester program of "Pedagogy and Subject Didactics for Teachers") on teachers' professional development. In addition to surveying teacher participants' motivation, learning strategies, and satisfaction with course, the researchers employed multiple scales to capture possible changes in the program participants' interests, competencies, and knowledge. The substantial changes in the multiple scales of competence and knowledge show not only the effectiveness of the program, but also several aspects that are valued in Austria as teaching job-related expertise. In particular, the use of a video task for teaching related analysis in the program confirms the idea that teachers' active participation and practices are essential for their professional development.

In her chapter, Simmt focused on the nature of mathematics that teachers need to work with to conceptualize teacher's expertise as mathematics for teaching (MFT). The model of MFT is further specified as a multi-layered and nested knowledge. In this way, Simmt highlights the knowledge nature of mathematics teacher's expertise and its structure. The model is then used to illustrate teachers' MFT and its changes through analyzing the actions and interactions of a group of mathematics teachers in a professional development session.

Part III: Understanding and Examining Teacher Expertise in an Eastern Setting

Similar to Part II, Part III also includes five chapters that individually present different studies on teacher expertise in five different education systems in East Asia. Different from Part II, all five chapters in this part tend to connect with teachers' instructional practices when addressing the issue of teacher expertise. Yet, the diversity is evident in terms of their selection of focal aspects and use of different perspectives across these five chapters.

The chapter, written by Li, Huang and Yang, aimed to characterize teacher's expertise through analyzing teachers' lesson instruction, their lesson design and reflections. The researchers focused on five selected expert teachers who are officially recognized with the teacher ranking system in China. A prototype view of teaching expertise was used to identify six similarity-based central tendencies in mathematics instruction that are shared among these expert teachers. The content of teacher expertise is thus not pre-defined but revealed through teachers' instructional practices in this study. Moreover, the researchers included a case analysis of one expert teacher's lesson instruction to provide rich descriptions and illustrations of the prototype of these teachers' teaching expertise. The findings help not only to illustrate the complexity of mathematics teaching expertise, but also to inform of the aspects of teacher's expertise that are important for developing culturally valued mathematics instruction in China.

Takahashi pointed out that "teaching the textbook" is taken as different from "using the textbook to teach mathematics" in Japan. The distinction has been used in Japan to differentiate and classify teachers into three levels in terms of their extents of using textbooks for teaching. To take a closer look at possible knowledge and expertise requirements behind these distinctions, Takahashi surveyed a small group of teachers who were pre-classified as belonging to these three different levels. The results reveal the differences in knowledge and expertise among these teachers in three levels, and also provide a glimpse of the type of expertise in "using textbooks to teach mathematics" that is valued and practiced in Japan.

Three researchers from Singapore took an alternative approach to examine what is valued in Singapore's mathematics instruction. Other than examining teachers' lesson instruction directly, Lim-Teo, Chua and Yeo conducted a survey and interviews of primary schools' mathematics department heads on their perceptions of effective practices for learning mathematics. The results reveal that mathematics department heads value those instructional practices that enhance conceptual learning and pupil motivation to learn. Although the results are not necessarily aligned with the general perception of typical instructional practices in Singapore, the study revealed much expected changes in what is valued in mathematics instruction in Singapore.

Taking students' engagement in meaningful discourse as an important instructional practice, Pang compared and contrasted more successful and less successful teachers in carrying out such practice in Korea. The comparison focused on the ways two selected teachers lead to the development of unequally successful mathematics classrooms. As the two classes established similar social participation patterns but a different quality of mathematical discourse, the results suggest important differences in teacher expertise that can contribute to the establishment of different sociomathematical norms in these two classrooms.

Focusing on the case of Taiwan, Lin and Li also adopted the prototype view of teaching expertise to explore similarity-based family resemblance in expert teachers' mathematics instruction. Three expert teachers were identified and selected using a set of criteria that are valued in Taiwan. Those expert teachers' lesson instruction was analyzed in terms of: selecting and sequencing problems for and in classroom instruction, selecting and sequencing students' solutions for the whole-class discussion, asking questions and responding to students during the class discussion, and transitioning from one activity to another. The common features of these expert teachers' lesson instruction were thus revealed and further illustrated with an expert teacher's instructional practices.

Part IV: Researching and Reflecting on Teacher Expertise in an International Context

This last part includes three chapters that do not place a focus on teacher expertise solely in the East or West. The chapter contributed by Vieluf and Klieme draws on data from the OECD-Teaching and Learning International Survey (TALIS) collected from 23 countries. The researchers aimed to gain an overview of cross-national similarities and differences in selected measures on teacher qualification, beliefs, and practices. Their results reveal some global similarities in broad terms, a finding consistent with the understanding of global similarities in schools and instructional organization. At the same time, their results suggest the importance of examining and understanding cross-national differences in profiles and constructs of teacher quality. The researchers pointed out that more cross-cultural research on teacher expertise and teacher quality, both qualitative and quantitative, is needed.

The last two chapters are reflections on teacher expertise and related research. The chapter contributed by Schoenfeld addresses two important and related issues: value-based variations in conceptualizing and measuring teacher expertise and the development of teacher expertise itself. He first highlights possible variations in teachers' and researchers' beliefs and values about what are "important" in the act of teaching. Such differences directly relate to different ways to conceptualize and consequently measure teacher expertise. Schoenfeld argues that it is ultimately important to link teacher expertise with students' enhanced performance, although this has been a serious challenge to all researchers. He then shifts the focus to teachers' lesson instruction itself, and hypothesizes that the development of teacher expertise should bear a direct connection to those aspects (i.e., teacher's resources, goals, and orientations) that will lead to the improvement of teacher's instructional performance.

In the concluding chapter by Kaiser and Li it is summarized what we can learn from this book concerning the concept and nature of expertise, how it is theoretically described and empirically measured. The chapter summarizes the differences between Eastern and Western perspectives on expertise and exemplifies their different orientation towards the teaching subject mathematics or the individual students. Furthermore the paper analyses our knowledge of the factual situation referring to current studies and describes possible research directions for the future.

Significance and Limitations

By taking an international perspective, this book aims to provide a unique platform for mathematics educators and teacher educators worldwide to develop a better understanding about teacher expertise in mathematics instruction. The significance of this scholarly work lies in its timely importance of developing and promoting research on mathematics teachers' quality and instruction. Similar to the case of mathematics teaching, it is not surprising for us to learn some similarities as well as differences in aspects of teacher expertise that are valued in different system and cultural contexts. Gaining such knowledge from this book is necessary not only for understanding the development of culturally-valued teaching performance in different education systems, but also for identifying aspects of teacher expertise for improvement.

At the same time, we realize that it is impossible for the book to address all the questions related to teacher expertise. In fact, we sincerely hope that this book can stimulate further study and discussion of teacher expertise and its development in different education systems. For example, although the book contains chapters about teacher expertise from the East and West respectively, it is not clear whether different aspects of teacher expertise valued in the East or West may link to students' enhanced learning ("Reflections on Teacher Expertise" in the chapter by Schoenfeld, this book). Nor is it clear which aspect of teacher expertise may be more important than others. Equally, if not more important, mathematics educators and teacher educators would also be interested in learning about effective approaches and practices used for developing teacher expertise in different education systems ("Reflections and Future Prospects" in the chapter by Kaiser & Li, this book). Indeed, the richness of the topic itself suggests that the book can well be a starting point for developing the much needed research on this topic in the future.

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Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise

Michelene T. H. Chi

Abstract This chapter begins by briefly overviewing the early approaches, perspectives, and findings in the expertise research. Basically, the approaches have first focused on exceptional experts, then studies evolved into studying expert performance relative to novices, with emphases on differences in their strategies of searching for a solution, the structure of knowledge, and finally in representation. Then three constructs emphasized in current research on expertise are described. These constructs are ideas about deliberate practice, adaptive expertise, and team expertise. The last section of the chapter proposes a new perspective for understanding the acquisition of expertise, which is the idea of a perspective shift. Interleaved throughout the chapter is discussion of how the acquisition of expertise can be facilitated and/or accelerated.

Keywords Expertise trends · Perspective shift · Theoretical models

Research on expertise has spanned several decades. Because so many chapters and edited volumes have been written about expertise (see for example, Ericsson, Charness, Feltovich, & Hoffman's 2006, *Cambridge Handbook of Expertise and Expert Performance*), the goal of this chapter is not to review the many studies on expertise. Instead, the first part of this chapter overviews very briefly the evolution of the research focus and perspectives for the last four or so decades. The second part of this chapter highlights the new constructs that are currently being explored about expertise. The final section offers a new idea for how the acquisition of expertise might be facilitated, the construct of a perspective shift.

Retrospective for the Past Three Decades

Researchers and lay people have always been fascinated by experts and exceptional individuals. In the early days, exceptional individuals have been identified as those

M.T.H. Chi (✉)
Department of Psychology, Arizona State University, Tempe, AZ, USA
e-mail: michelene.chi@asu.edu

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individuals who are generally recognized and acknowledged by the public as great people, such as popular composers (Kozbelt, 2004) and scientists who made great discoveries (Chi & Hausmann, 2003), and so on. Studying exceptional individuals has been referred to as an absolute approach (Chi, 2006).

Studying Exceptional Experts

There were four types of studies of exceptional individuals. One type of studies described how they went about making their discoveries, by studying their notes and diaries. These studies tried to capture when a discovery was made and under what circumstances. The goal was to try and capture the cognitive processes underlying their discoveries (Nersessian, 1992; Tweney, 1989).

A second type of studies looked at the societal and environmental conditions that may have led to their superiority, such as their age of onset, their productivity profile, and their parental influences (Lehman, 1953). A third type of studies tacitly assumed that there is some innate talent or mental capacity to their greatness (Simonton, 1977), so such studies might investigate differences in their cognitive structures, such as that exceptional individuals might have a larger memory capacity (Pascual-Leone, 1978).

A final type of studies looked at how exceptional individuals perform in the tasks in which they excel. For example, one might document and marvel at how a single chess master can play many different games with many different players while blindfolded (Binet, 1894), or how a great physician can diagnose a disease accurately and quickly (Elstein, Shulman, & Sprafka, 1978; Barrows, Norman, Neufeld, & Feightner, 1982; Neufeld, Norman, Barrows, & Feightner, 1981). In general, when only exceptional individuals are being examined, it is difficult to validate or refute hypotheses about how they became experts.

A Difference in Search Strategies

By the early seventies, the study of expertise introduced two new perspectives. One new perspective is methodological, in that expertise studies introduced the relative approach (Chi, 2006). A relative approach contrasts the performance of a more advanced individual (referred to as the experts) with the performance of a more novice individual. There are several advantages to the relative approach. First, the relative approach makes the tacit assumption that a novice can become an expert, because an expert is no longer viewed as a uniquely exceptional individual. Rather, an expert is someone who is relatively more advanced, as measured in a number of ways, such as academic qualifications, years of experience on the job, consensus among peers, assessment based on some external independent task, or assessment of domain-relevant content knowledge. Second, a relative approach also frees up the constraint of making sure that the level of expertise across studies are defined in exactly the same identical way, since a relative approach can tell us in what ways an expert excels over a novice, even without equating the index of expertise across studies. Third, a relative approach defines expertise by the experts' knowledge, and not by any innate hardwired capacity.

The second perspective that was introduced in the seventies was theoretical, due to the advent of computers. This new perspective – an information processing approach, required a task analysis, that is, the decomposition of a complex task such as problem solving, into three components: (a) the relevant background knowledge, (b) the problem solving strategies or ways of searching through the space of all possible moves. and (c), understanding or representing the problem in terms of a space of all possible moves. To elaborate, the first component of relevant background knowledge refers to the amount of knowledge one has, indexed in some objective way. So for instance, an expert might have more knowledge because s/he has taken four algebra courses, whereas a novice might be someone who is just starting to take algebra.

The second component of problem solving strategies can be explained more easily after we define the third component – the representation of a problem. The representation of a well-defined problem consists of its elements, all the permissible operators that can operate on the steps of the problem, the constraints on the operators, and the goal of a problem. A representation of a problem usually refers to knowing the elements in the problem, the allowable operators, the constraints on the operators and the goal. The degree to which one has a complete representation of all the components of a problem essentially is a measure of how well a student understands a problem, because knowing the elements, the permissible operators, the constraints on the operators and the goal, allows one to generate a complete representation (or problem space) of all the permissible moves. Essentially it means being able to represent the entire problem space of solution steps.

To illustrate, suppose a learner is asked to solve an algebra equation $5X + 2X + 10 = 31$ for X. What is the representation of such a problem? A representation consists of the elements, the permissible operators, the goal and so forth. Figure 1 is a partial problem space of some of the permissible moves for this problem. The permissible operators in this problem are moving numbers from one side to the other side of the equal sign, adding, subtracting, multiplying and dividing; and the goal is finding X. More specifically, the space of all possible moves are: moving the 10 to the right of the equal sign (see the first step in the last column of Fig. 1), subtracting 10 from 31, putting parenthesis around $(5 + 2)$ then multiply by X, and so forth. However, it is not permissible to decouple the X from the 2, as in making an operation such as $2(X + 10)$ from $2X + 10$. These types of student errors can typically be characterized as errors in not knowing the constraints on the operators. In any case, representing the problem means knowing all the possible moves, knowing the elements, the constraint, and so forth. Successfully solving this well-defined problem can be conceived of as finding the right path that leads to the correct solution.

The second component of a representation refers to the problem solving strategies of how one searches the problem space of all possible moves. Looking at the problem space shown in Fig. 1, one can search from top-down (or forward strategy), starting from the given equation and moving toward the goal of finding X, or one

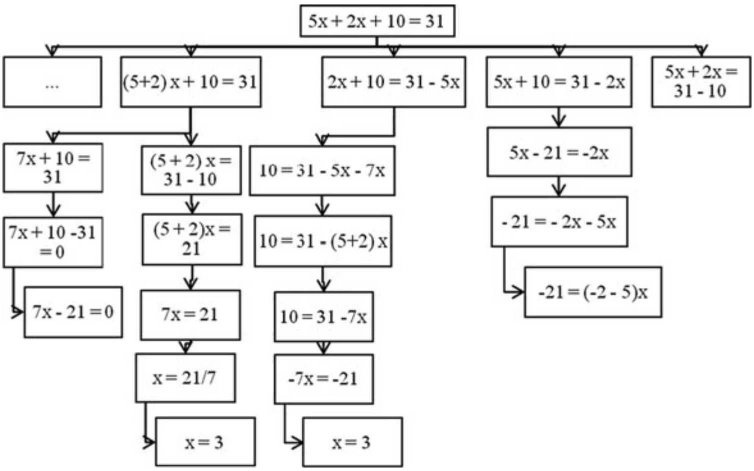

Fig. 1 A partial search space

can search bottom-up in the figure (meaning a backward strategy), starting with the goal, and working backward. Alternatively, an efficient way to search is to create a sub-goal so that it reduces the portion of the space that has to be searched. Suppose one sets a sub-goal of grouping all the X-terms. Such a sub-goal would eliminate taking the second and third path at the first level of search.

Using this knowledge-search strategy-representation framework, it was typically assumed back in the seventies, that the first and third components of problem solving – knowledge and its representation, were not significant factors that differentiated experts from novices because the problems used in problem solving research were often knowledge-lean puzzle-type problems, such as the Tower of Hanoi. For the Tower of Hanoi, the elements are the disks, the operators are the moves by each disk, and the constraints are rules such as that a larger disk is not permitted to be set on top of a smaller disk. These elements, operators and constraints are often in fact given in the problem statement, so that a complete representation can be easily generated without applying any other background knowledge. For example, for the Tower of Hanoi problem, the goal is to move a stack of three disks, one at a time, from the first peg to the last peg; and the constraints on the operators is that only one disk can be moved at a time, and a larger disk may not be put on top of a smaller disk. As can be seen in Fig. 2, it is quite simple to generate a problem space of solution steps for the Tower of Hanoi problem. (Fig. 2 shows the complete problem space of all possible moves.) Thus, understanding such a problem in the sense of representing the entire problem space is not a difficult task. Therefore, solving such a problem becomes an issue of searching for the optimum path through the problem space of different solution steps. Little background knowledge is needed in order to know how to begin to solve such a puzzle, since these puzzle-like problems required little knowledge that is not already given. In short, it is not surprising that problem solving research back then focused on the strategies by which the problem space was being searched.

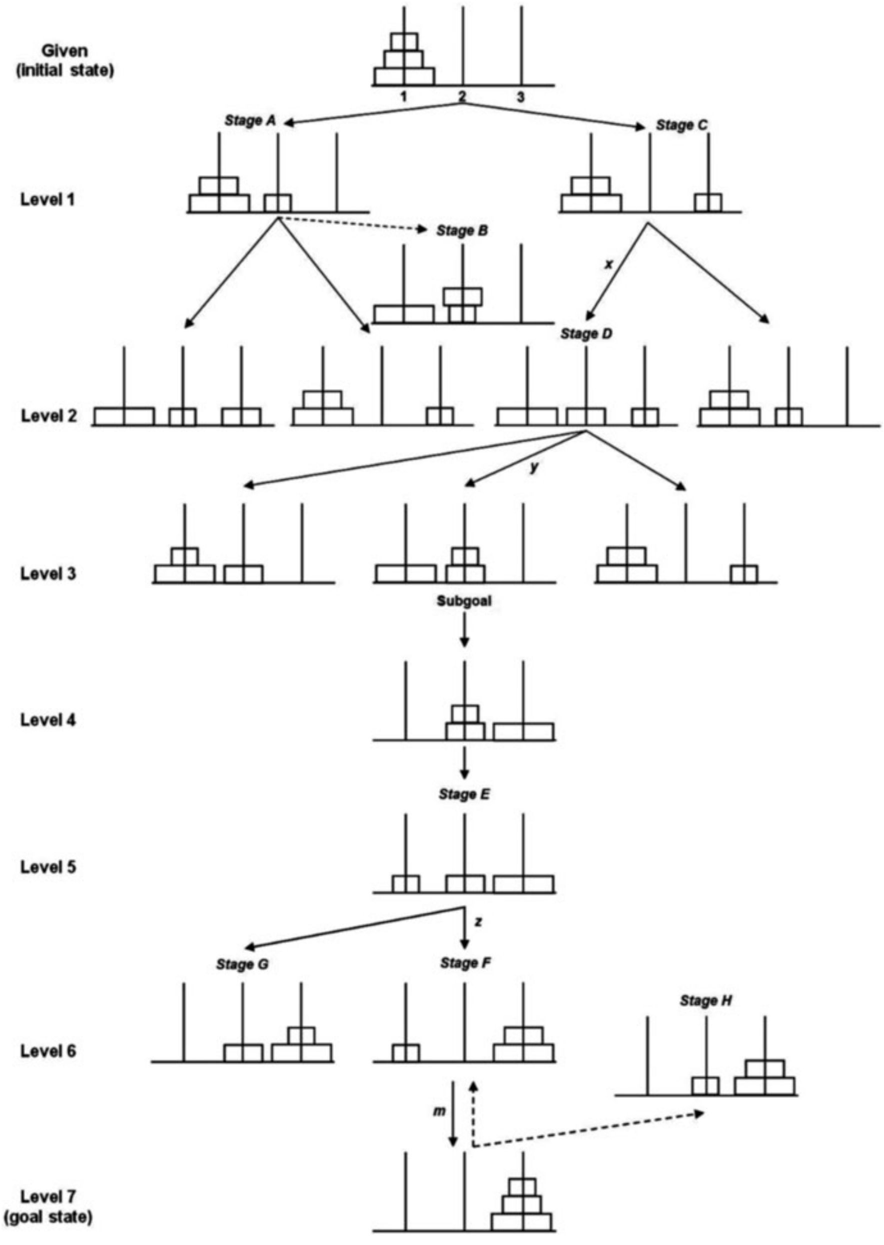

Fig. 2 A complete search space for the Tower of Hanoi problem

A Difference in the Structure of Knowledge

When researchers began to study problem solving beyond puzzle problems and focused instead on academic disciplines, such as mathematics and physics, they carried over the assumptions of solving puzzle problems. That is, they continued to ignore potential differences in representation. Therefore, the findings of such studies continued to conclude that expert and novice physics problem solvers differed in their problem solving performance primarily in the way they search their problem space. Figure 3a depicts the view that experts' superior knowledge may have dictated a difference in their search strategies in that their strategies might be superior to the novices' strategies. This approach was fostered by the work of Simon and Simon (1978). Figure 3a also shows a question mark in terms of whether or

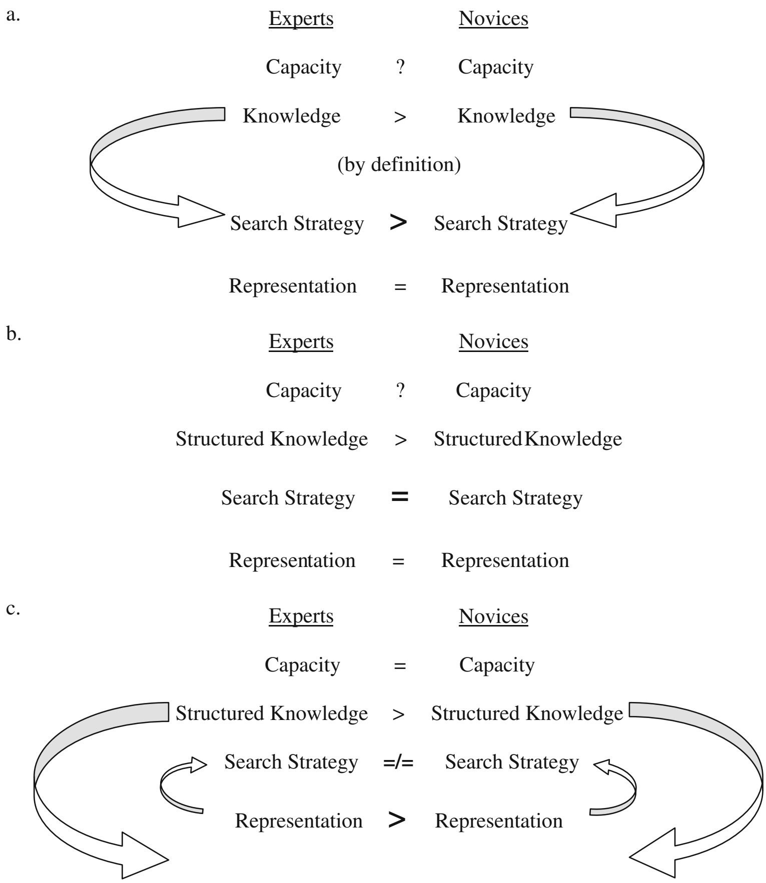

Fig. 3 Assumptions about differences in problem solving components between experts and novices

not experts' innate capacity is any different from novices, as there was no direct evidence as yet.

The idea that experts and novices differed primarily in their search strategies violated some findings in the chess literature. In non-toy and knowledge-rich domains, such as chess, it became apparent that search strategies per se did not differ significantly between experts and novices. For example, deGroot (1966) found that Master chess players searched the representation of all possible chess moves only to a depth of two or three levels, much as novice players would. Therefore, a competing assumption was that experts and novices have similar search strategies. Moreover, the representation of all possible chess moves continue to be assumed to be equivalent between experts and novices since they can be easily generated, once a player knows what are the allowable moves. These alternative set of assumptions are depicted in Fig. 3b.

From Fig. 3b, it seems that the only remaining difference between experts and novices is the knowledge component. It did not seem adequate to simply claim that experts had more knowledge. The relevant question remained: how does an expert's greater knowledge facilitate their superior performance, in terms of any kind of measures, such as speed, efficiency, search strategies, and so forth. The classic study by Chase and Simon (1973) on chess expertise basically proposed that what differed between experts and novices was not merely the amount of knowledge in a specific domain, but more importantly, how that knowledge is structured. Moreover, they refuted the idea of an innate difference in mental structures. For example, they showed that both experts and novices can recall about the same number of chess pieces and their locations if the chess pieces were randomly placed on a chessboard, suggesting that their memory capacity for chess piece locations were the same. However, if the chess pieces were placed in the context of meaningful plays, then the experts far outperformed the novices in recalling the location and identity of the chess pieces. These two types of studies put to rest the ideas that exceptional individuals have better mental capacities and more superior search strategies. Instead, these studies highlighted the importance of structured domain-relevant knowledge, as indicated in Fig. 3b.

How is domain knowledge structured? The Chase and Simon work began to capture what is the structure of greater knowledge in the chess domain. One analysis of structure was the idea of "chunks", which is a cluster of related pieces that are often placed in proximity on a chessboard. Thus these chess chunks were visual patterns. The concept of "chunks" can of course be extended to many other domains. For instance, a 3-digit number such as 100 is an important chunk or a meaningful unit to an adult, but perhaps not to a child (Chi, 1976). The concept of the structure of knowledge was important because it attempted to explain how greater knowledge can have a bearing on task performance. In the context of memory for chess board pieces, it explained how recall was a function of the size of chunks, and therefore, even if experts and novices could recall the same number of chunks, experts' chunk structures were larger, therefore accounted for their superior recall in terms of pieces. Many other studies followed in identifying and capturing the structure of domain knowledge.

A Difference in Representation

Beyond the context of recall of chess pieces, how might knowledge influence performance in more academic domains such as problem solving in mathematics or physics? In attempting to answer this question, researchers in the early eighties turned to the third component of problem solving. The third component is the component of the representation of a problem. It turns out that when the domain is not a toy domain but an academic domain, representing a problem is quite difficult, and expert and novice problem solvers focused on different elements within a problem when representing it. Chi, Feltovich, and Glaser (1981) found, for instance, that when given the same description of a physics problem to solve, advanced graduate students represented the deep principle-based aspects of a common routine physics problem whereas novice students represented the superficial surface elements of a problem, such as whether it described an inclined plane, a pulley, or friction. This representational difference can be captured by looking at what problems novices and experts considered to be similar. Figure 4a depicts the diagrams of two pairs of problems that novices considered to be similar; notice that their judgments are based on similarity in the concrete elements describe in the problem situations, such as round disks or inclined planes. Experts, on the other hand, tended to consider problems to be similar if they are governed by the same underlying principles. Figure 4b shows two pairs of problems advanced physics students considered to be similar even though they have dissimilar surface or concrete elements; but they do share similar deep principles, such as problems solvable requiring a consideration of energy, or "work is lost somewhere," or by Newton's Second Law.

The finding of representational differences between experts and novices has immediate and far-reaching implications. The immediate implication for expertise research was that such representational differences obviously dictated why experts and novices appeared to search the problem space differently. The difference reflects a difference in their representations, so it is not the case that experts and novices have the same problem space to search, as is commonly assumed back then in the problem solving literature, especially for knowledge-lean problems. In other words, the differences between experts and novices in their representations of the same problem dictated and resulted in different searches in their problem spaces. Essentially, this refuted the assumption made in earlier expertise research that the problem representation of experts and novices were the same, which was a legitimate assumption for toy domains but not for knowledge-rich academic domains. Thus, the origin of search differences that were uncovered by studies such as Simon and Simon's (1978), is their representations as a function of prior knowledge, and not in a difference in search strategies per se. Figure 3c depicts the assumptions of this revised view that knowledge differences allowed experts and novices to represent a given problem differently, which in turn then dictated the kind of search strategies they would use for solving the problem, which may or may not be the same. Since the eighties, representational differences between experts and novices have since been

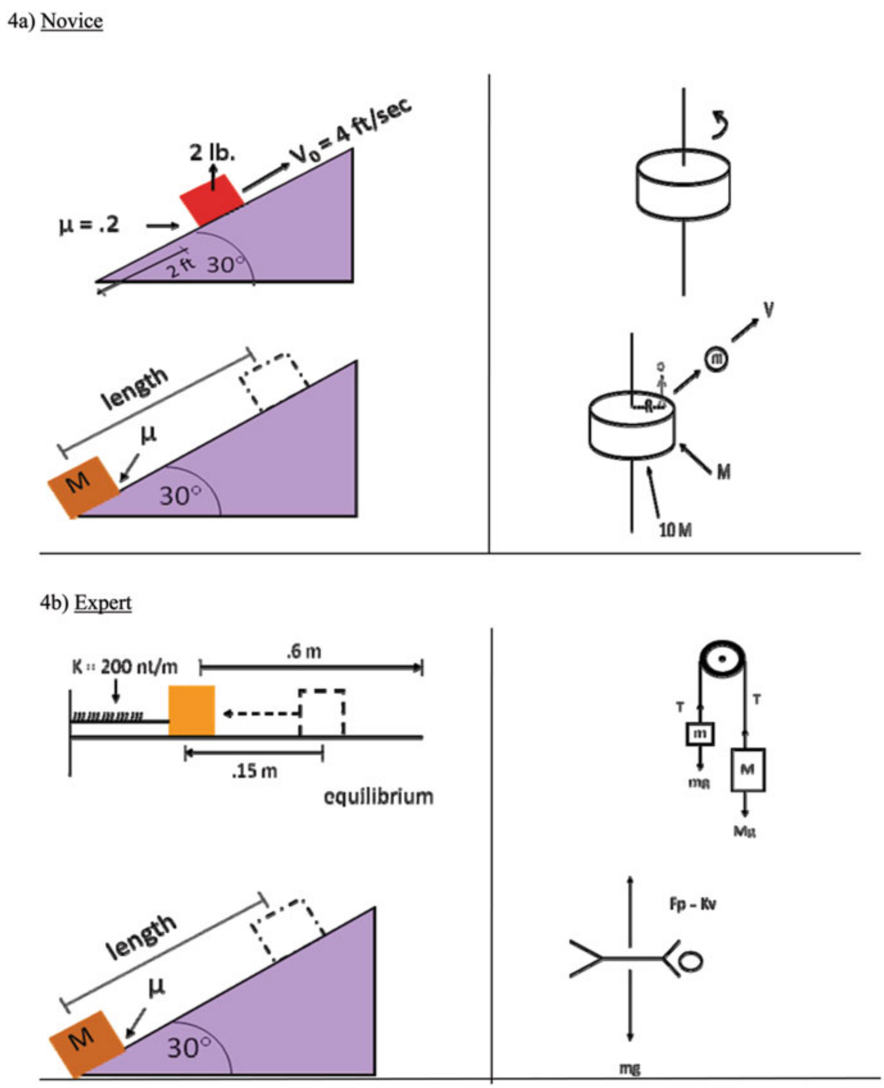

Fig. 4 Pairs of problems that novices (**a**) and experts (**b**) considered to be similar

replicated in many studies and many domains. The idea that experts and novices differ in the depth of their representations was characterized in many subsequent studies on expertise in many different domains.

A far-reaching implication of representational differences between experts and novices is that this means that teachers will generally have a normatively correct and deeper representation of a topic or concept they are teaching, whereas novice students will have a naïve, shallow, and incomplete representation. The consequence of such lack of correspondence between the representations of teachers and students will undoubtedly lead to misunderstanding of a teacher's explanations. We have shown consistently in tutoring work that students have difficulty learning from hearing a tutor's explanations, whereas they learn better when the tutor scaffolds them (Chi, Roy, & Hausmann, 2008). This inefficiency of explanations may be caused by the lack of correspondence in representations.

Issues of Training

The focus on academic domains also brought to fore the idea that expertise should be an attainable skill that novices should aspire to attain. Therefore, changes in the conception of expertise in the literature also led to research that took much more of a relative approach, in that, one should contrast more expert-like performers with less expert-like performers, and not necessarily focus on the performance of exceptional experts. Therefore, many studies could simply contrast advanced students with less advanced students, since such contrasts could potentially inform us on ways to advance a novice student to be more skillful.

The critical question remains as to how one becomes an expert in the sense of being able to represent a problem deeply. Little progress had been advanced to understand this difficult issue. Although some attempts have been made to directly teach novice students the way experts categorize problems or to directly teach them to relate key words or explicit cues with one of the deep physics principle (Dufresne, Gerace, Hardiman, & Mestre, 1992), it doesn't appear as if this kind of training can accelerate or shortcut the achievement of expertise readily, which is typically claimed as requiring 10 years of practice, at minimum. In other words, to be more specific, when a novice reads a physics problem statement, such as that

> A block of mass M1 is put on top of a block of mass M2. In order to cause the top block to slip on the bottom one, a horizontal force F1 must be applied to the top block. Assume a frictionless table, find the maximum horizontal,

the explicit words in the problem statement itself does not elicit the relevant deep physics principles. However, it is not the case that novices cannot identify the relevant and important key words: In fact, novices can identify the relevant and important key words in a problem statement quite adequately, as shown in Chi, Glaser, and Rees (1982, Study 8). The issue is that the key words themselves do not lead novices to make further inferences as they do for experts. In our data, we found that a keyword such as "frictionless" would lead an expert to infer that there are "no dissipative forces", which in turn led the expert to further infer that it's a "Conservation of Momentum" problem. In short, the key words themselves do not directly evoke the correct underlying principles; instead, intermediary or secondary cues are first derived from the key words. If this is true, then it is not clear how we can teach students to directly associate key words with the underlying physics principles, and expect deep understanding, without also teaching them how to derive the secondary cues from the keywords. If we must teach them how to derive the secondary cues from the keywords, then such instruction may not necessarily accelerate the acquisition of expertise.

Another example can illustrate the potential flaw of this intervention approach of directly teaching the relationship between the keywords and the principle. In a study of 32 expert physicians in four different specialties (cardiologists, hematologists, infectious disease specialists, and internists), we presented them with individual patient cases and asked them to diagnose the disease of the patient cases and give reasons for their diagnoses (Hashem, Chi, & Friedman, 2003). We then coded the number of cues in the cases that they used to come up with their diagnoses. We found that when a case matches the physicians' specialty so that they have expertise (such as a blood disease case diagnosed by a hematologist), they tended to use multiple cues in the case statement to come up with the diagnoses. However, when the case does not match their specialty (so that they are more novice), then they tended to use only single cues to come up with the hypothesized diagnoses. Presumably, using multiple cues is more accurate and physicians with more expertise in a case were able to use multiple cues. Table 1 shows the frequency with which they used single cues versus multiple cues as a function of whether the cases matched or did not match their specialties. With respect to the training question raised above, does this mean that we can accelerate the acquisition of expertise by teaching physicians to use multiple cues? It does not seem obvious that one can accelerate the association between cues and hypothesized diagnoses by telling physicians what the cues are, since presumably they were taught the cues already. Perhaps expertise involves not only the detection of individual cues within a case, but in addition, perhaps the acquisition of expertise requires the development of knowledge of the interaction of multiple cues and their relationship to a specific diagnosis.

In summary, this section raced through three decades of work on expertise by highlighting the underlying assumptions and conclusions of the different theoretical and methodological perspectives and approaches to the study of expertise. Expertise was always defined as having more knowledge, but knowledge originally played a very minor role. Instead, expertise was defined by one's ability to search efficiently and effectively. In light of new evidence, it became clear that expertise did not necessarily result in more efficient searches, rather expertise can be defined as having more structured knowledge. Structured knowledge in turn dictated how experts represented a to-be-solved problem. Thus, the differences in the representation between

Table 1 The use of single or multiple cues as a function of the match between the case to-be-diagnosed and the physician's specialty (data taken from Hashem et al., 2003)

Cues and Specialty		
	Cases match their specialty	Cases outside their specialty
Single cues	29	190
Multiple cues	61	45

experts and novices dictated how they searched. Finally, we still have no obvious insights about how expertise can be taught, or how we can accelerate the acquisition of expertise.

The Current Constructs

Many questions remain about expertise, such as how to accelerate and facilitate its acquisition. Three new constructs have been introduced and emphasized in the last decade. The first construct is the idea of deliberate practice, attempting to answer the question of how some individuals reach elite status of expertise and others remain mediocre. The second construct is the idea of adaptive expertise, exploring the notion of a more innovative expert, one who is not rigid and conventional. The third construct is the idea of a team, group, or system-level expertise, bringing forth new challenges in understanding how an expert team can be construed, since an expert team does not appear to be composed of expert individuals, measured either in terms of a team's performance or learning. These three constructs are explored briefly in this section.

Deliberate Practice

Deliberate practice is a construct advanced primarily by Ericsson (Ericsson & Lehmann, 1996). The construct was introduced to account for the fact that not all experts achieve elite status, some remain mediocre in the sense that some individuals are satisfied in reaching an acceptable level of performance and continue in maintaining that level of performance with minimal effort for years on end. In understanding how some individuals reach elite status, Ericsson proposed the construct of "deliberate practice." The assumption is that those experts who reach elite status are the ones that engage in deliberate practice, even though they spend about the same amount of time practicing as non-elite experts.

Deliberate practice is defined as expanding intentional efforts to achieve further improvement through focused, concentrated, well-structured, programmatic, and goal-oriented practice. Moreover, the goals of practice are set to go beyond one's current level of achievement, and evaluated by identification of errors, and so on. For example, elite figure skaters spent more time on challenging jumps than less elite skaters; the interpretation of this kind of practice is that they intentionally attempt to achieve more challenging jumps in order to improve and move themselves up in their level of expertise (Deakin & Cobley, 2003). They seek challenges because they view failures as opportunities to improve. Deliberate practice is contrasted with mindless performance or playful engagement (p. 15), or "merely executing proficiently during routine work" (Ericssson, 2006, p. 683, Chapter 38, *Handbook*). As Ericsson (2006, p. 691) puts it, "Those select group of individuals who eventually reach very high levels do not simply accumulate more routine experiences of domain-related activities, but extend their active skill-building period for years or even decades." For example, musicians who are the more elite experts are the ones who concentrate on practicing with the intention of achieving beyond the level that they are currently capable of performing (Ericsson, Krampe, & Tesch-Romer, 1993). It is as if they are always reaching beyond their "zone of proximal" achievement.

Deliberate practice does involve many other players as well. It involves a coach or a teacher who designs the targeted practice task, who continually guides, monitors, and gives feedback to the expert in performing the task. Family members also play a huge role in helping their children develop elite expertise. According to Ericsson (2006), parents of elite experts are actively involved in helping them find a good teacher, helping them with their practice, spending large amounts of money for equipment, driving them to lessons, sometimes even relocating to be closer to a specific teacher or training opportunities. These parental involvement and sacrifices are reminiscent of parents of immigrant families, resulting in high success rates of immigrant children on measures such as college completion, but it is not clear whether children from immigrant families also achieve elite status. If not, then these parental factors may only guarantee success, but not necessarily elite expertise.

It is very difficult to say whether deliberate practice is the result of some personality or individual attributes, such as motivation or persistence, or whether it is the nature of the designed deliberate practice task that is critical for achieving elite status. For example, elementary and secondary students seem to fall into two types: intrinsically motivated versus extrinsically motivated (Dweck, 2000). Intrinsically motivated students persist through challenging tasks by adopting high-quality learning behaviors, while extrinsically-motivated students tend to adopt tasks and behavior that may produce rewards or satisfies the requirements without worrying about whether they have actually learned. In short, one type of learner might be more likely and inclined to engage in deliberate practice to achieve elite status.

If the hypothesis is true that some experts achieve elite status because of motivational or other reasons rather than the nature of deliberate practice itself, then we should see that having the guidance and help of a coach in designing tasks for students will not succeed with all students, because these alternative factors may come into play. Some related evidence might be interpreted in this context. In the Chi et al. (2008) study, an expert tutor guided 10 students individually in solving physics problems. These 10 students were asked to read and learn the relevant materials from which the to-be-solved physics problems were taken. After their independent unguided learning, they took a pre-test, so the pre-test in essence assessed how well they could learn on their own. All 10 students had similar background knowledge about physics. The hatched bars in Fig. 5 show the amounts the students could learn on their own (pre-test) and the dark bars show how much more the tutor could help the students gain. As Fig. 5 shows, not surprisingly, there is a difference in how much the sample of 10 students could learn on their own. What is surprising is that the poorest three students gained the least amount whereas both the intermediate students and the best students gained substantially more. What this data tells us is that the same tutor could not design guidance and feedback that

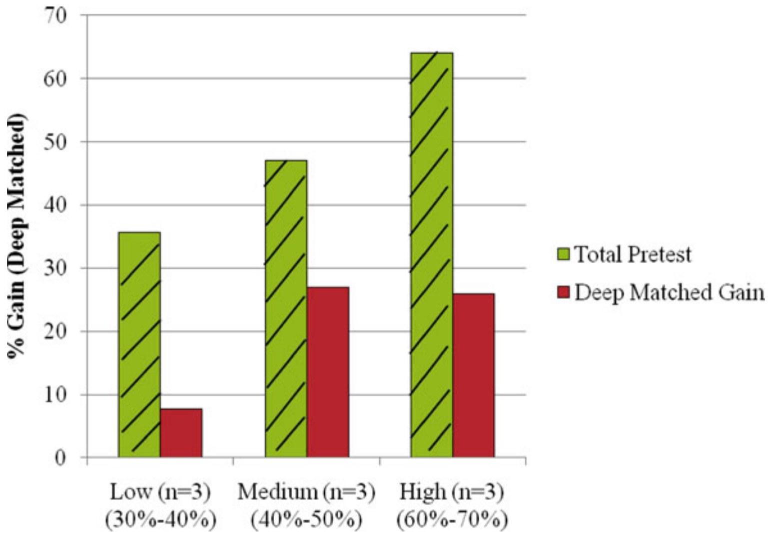

Fig. 5 Hatched bars show the tutees, divided into low (30–40%), medium (40–50%), and high (60–70%) on the basis of their pre-test scores, and the solid bars show how much they improved after tutoring

allows all the students to gain maximally. This suggests that individual differences in learning and/or differences in one's success in achieving elite expertise may not be caused by deliberate practice necessarily (although no doubt engaging in deliberate practice can help), but by a myriad of other factors, such as a desire to excel, persistence, ability to learn, and so forth. Thus, the basic question about achieving elite status is not answered by the finding that the elite experts undertake deliberate practice, because this finding basically regresses the basic question to another question of understanding why some individuals engage in deliberate practice while others do not.

Adaptive Expertise

The second construct that is currently intriguing scholars of expertise is the notion of an adaptive expert. The construct of an adaptive expert was introduced prominently by Hatano and Inagaki in 1986, as a contrast to a routine expert. Routine experts, according to Hatano and Inagaki (1986, p. 266) are experts who are efficient and are outstanding "in speed, accuracy, and automaticity of performance but lack flexibility and adaptability to new problems." Thus, routine experts are "able to complete school exercises quickly and accurately without understanding," whereas adaptive experts have "the ability to apply meaningfully learned procedures flexibly and creatively." (Hatano, 2003, p. xi).

Adaptive experts, in short, are ones who "understand" the procedure or skill, in the sense of understanding the principles and conceptual knowledge guiding the execution of the procedures or skills. With such deeper understanding, adaptive experts obviously can "generalize" their skills to other non-routine problems. Of course this definition of adaptive expertise requires further elaboration in defining what is meant by "understanding" and "generalization." Suppose we simply operationalize the meaning of "generalization" in an objective way without defining it, such as by measuring in some graded way a learner's ability to solve more and more distantly related problems. With such an operational definition of what "generalization" is, we can provide two senses of the term "adaptive expertise" that have been used in the literature. In so doing, we add our elaborations of what we think "understanding" means in each sense.

The first and most common idea of adaptive expertise is the notion of knowing not only how to execute or apply a procedural skill, but an adaptive expert is one who also has conceptual understanding of that skill (Schwartz, Lin, Brophy, & Bransford, 1999). This dichotomy of knowing a procedural skill versus having conceptual understanding of it exists at all stages of skill acquisition, not necessarily only at the expert level. Here is one way of thinking about it. Suppose we have a skill of solving a mathematical problem. The solution can be decomposed into a set of If-Then rules as follows:

If A, Then do Y. [after doing Y, the resulting pattern is C];
If C, Then do Z.

For example, if the unknown variables of Xs are on both sides of the equal sign (condition A), then use legitimate operators to move all the Xs onto one side of the equal sign (execute action Y). Now the resulting equation has changed the condition from A to C. Now If C is true, then action Z can be executed.

One can learn these two rules so well that one can solve all kinds of problems efficiently and accurately in applying these two rules when the problems are similar to the conditions of each rule (i.e., the A's and the C's), as in the case for routine experts. However, if the conditions presented change from A to A + B, then a routine expert would not know what to do. In order to know what to do when conditions A + B show up, one must have reflected on the If A, Then Y rule when one is acquiring it. Reflection can include numerous processes, such as self-explaining why action Y works when condition A is true, seeking what is the characteristics of A for Y to apply. For example, if A is the number 5, one can reflect on whether Y follows because A is a prime number, or because A is less than 10, and so forth. The idea is basically to construct knowledge about A, in a way that generalizes beyond the specific instance of A, thereby allowing the learner to have greater conceptual understanding of A. Thus, we can say that procedural knowledge is simply knowing the two rules, If A, Then Y and If C, Then Z; whereas having conceptual understanding can include understanding the nature of the conditions A and C, their characteristics, the principles that explain their categorical structure, and so forth. Thus, this first idea of adaptive versus routine expertise can be conceptualized as being very much related to the distinctions between conceptual and procedural understanding, a contrast and dilemma that have been around for decades.

However, a more intriguing second idea of adaptive expertise is the notion of a propensity or predisposition to learn while performing. That is, the idea is that while practicing or executing a skill, adaptive experts are ones who seek to learn more from the experience, seek help from others, experiment with new ideas, as if they are not satisfied with what they already know and can do (Bransford & Schwartz, 2009). Thus, this definition of adaptive experts is similar to the characterization of elite experts who intentionally seek challenges in their deliberate practice. In effect, adaptive experts as defined here resemble all "effective learners," and not just adaptive experts. Perhaps only effective learners can become adaptive experts.

Even though this "effective learner" definition of adaptive expertise emphasizes the learning aspect whereas the first definition proposed above emphasizes the conceptual understanding aspect, the two definitions are related in that they have a common component, namely that in order to acquire conceptual understanding, one must reflect and self-explain the concepts or conditions of a rule, much like one must reflect and self-explain while solving a problem or practicing a skill in order to maximize learning. Both definitions can be said to require a constructive component, where new knowledge is constructed while trying to understand the concepts and conditions of rules or while performing the rules.

Not only are the two definitions of adaptive expertise described here similar to the idea of the elite experts engaging in deliberate practice, but moreover, deliberate practice seems to have the components of engaging in reflective practice. That is, in deliberate practice, one can be either reflecting on the conditions of the rule, or reflecting on the outcome of the procedural execution, in order to seek more challenging practice. In short, one could say that to achieve adaptive expertise is to engage in constructive reflection during practice and performance, and such constructive reflection allows one to further learn, generalize, and acquire deeper conceptual understanding. The real question though, is why some learners engage in such constructive reflection and others do not. We have alluded earlier to the notion that motivation and other social and personal factors might be mitigating reasons, but no evidence addresses these issues directly.

Team or Group Expertise

The third construct that has not been pursued very much in the literature is the idea of group expertise. The idea of group expertise has many related and intriguing issues and questions. For example, we know that groups most often perform better than individuals, whether the group is a size of two (dyads), or three (triads), or more (e.g., Barron, 2000; Pfister & Oehl, 2009; Schwartz, 1995; Webb, Nemer, Chizhik, & Sugrue, 1998). But what we don't understand is why. The most mundane reason is to say that groups perform better because different individuals within the group know different aspects of the to-be-solved problem, so that the combined knowledge of the individuals allows more problems to be solved (Ploetzner, Fehse, Kneser, & Spada, 1999). This is a "complementarity" idea. But more intriguing is the notion that even if the individuals in the group have the same knowledge, it seems that they too, can solve more problems correctly (Hausmann, Chi, & Roy, 2004). This is the "co-construction" idea, that two or more people, together, can create some new understanding that neither of them could create alone. Several additional questions arise with respect to group expertise such as: What is the best combination of group members in terms of levels of expertise to optimize the co-construction of new ideas? What is collective knowledge? How can it be measured?

More recently, the challenge involves understanding group and team learning, and not just team or group performance. A team is a pre-determined group in which each member might have a pre-defined role. The question is how to create an expert team that can not only perform effectively but also learn effectively, since groups and teams often have to learn new innovations? That is, a team that performs and learns expertly is not necessarily a team of individual experts, nor necessarily a team led by an expert (Edmondson, Bohmer, & Pisano, 2001). There are other potent factors such as coordination among team members. What is the nature and characteristics of expert coordination (such as timing) is an issue that is being actively explored currently (Cooke, Salas, Cannon-Bowers, & Stout, 2000).

In summary, the three constructs that are being explored in the expertise research currently – deliberate practice, adaptive expertise, and group learning and performance – are silent on the issue of how we can help learners become adaptive experts. Besides the relatively new area of group learning, the first two constructs seem to be mediated by some other unknown factor, such as motivation. There are also many other social (family values, parental guidance) and cultural factors that seem difficult to reproduce for specific learners in order to make them more adaptive. In other words, there are no obvious solutions for how we can train learners to become adaptive and elite experts. Two of the five catalysts mentioned by Martin and Schwartz (2009) seem feasible to implement in training. One is the idea of providing variability instead of reducing variability as usually done in formal instruction. That is, by intentionally introducing variability (as for example, in the condition of rules), then students can see the variability more directly and easily, rather than having to reflect on potential variability, as we postulated above. A second idea is what Martin and Schwartz called "fault-driven adaption". The idea is that if a situation contains either new crisis or chronic bothersome snags, then an individual or a group might decide to adapt. Fault-driven adaption is essentially an effective change caused by an altered situation, in much the same way as conflict-driven conceptual change. And we can imagine a training regime that can include faults such as new crisis, chronic errors, or bothersome tedious repetitive actions. Both of these ideas can be readily implemented in training so as to produce more adaptive experts.

Expertise as Perspective Shift

Besides the question of how to produce elite and adaptive experts, the more fundamental question of how we can accelerate the acquisition of expertise without a decade of practice, is not a question that has a ready answer. One of the reasons is that many of the results from contrastive studies on expertise (i.e., contrasting experts versus novices) do not translate easily into instructional intervention about training for the acquisition of expertise. For example, if we find that experts can see more patterns in an X-ray that novices cannot see (Lesgold et al., 1988), what can we do to accelerate training other than going through what training already is doing, which is to have experts point out x-ray flaws to novices? Similarly, if we find experts to categorize and sort physics problems (Chi et al., 1981) or trees (Medin, Lynch, Coley, & Atran, 1997) or birds (Tanaka & Taylor, 1991) differently from novices, it is not clear how we can teach the categories to novices in a way that can accelerate their learning. That is, they still have to learn the relationships between the features in the objects that are relevant to the categories that the objects belong.

Occasionally, there are more mundane reasons for the length of time it takes to acquire expertise, such as the need to encounter unusual situations. In that case, simulations built to mimic the rare incidents would help accelerate the training of novices, since they can encounter those incidents more often in a simulator (Gott, Lesgold, & Kane, 1996). Lack of access also occurs in other scenarios, such as in apprenticeship. In some workplace apprenticeships, the apprentices do not have good access to the master, therefore they cannot acquire their skills readily and quickly. These kinds of access issues (either accessing rare incidents or accessing an expert) require solutions that can be more easily implemented, if feasible.

Aside from these access issues, no novel approaches have been taken to see if expertise acquisition can be accelerated. One idea to be explored here is perspective shift. Although perspective can be interpreted in many ways, such as spatial perspective, the idea proposed here is a perspective shift across ontological categories (Chi, 1997). For example, a shift between objects and processes can be considered a shift across ontological categories, or a shift between seeing the parts versus seeing the whole might be a second example, or a shift between individual entities versus a system might be a third example. Let us consider two examples. In the old data of experts and novices solving physics problems (Chi et al., 1981), there were some protocols reported in which we asked experts and novices what kind of cues in the problem statement allowed them to decide what kind of a problem it is or how it should be solved. In analyzing two expert and two novices' citations of cues, gross differences emerged (Chi et al., 1981, Table 11, 1982, Table 14). The cues could be either a specific object or concept in the problem statements, such as a spring, an inclined plane or friction, or the cues could be more system level processes, such as that the problem is a "before-and-after" situation, or there are "interacting objects". Table 2 below shows the difference of a single expert and a single novice in the cues they cited as important for determining how a problem is to be solved. The expert cited 21 concrete cues, whereas she cited 74 process cues. The novice did just the opposite: he cited 39 object cues and 2 process cues. Thus, the

Table 2 Physics problem cues (data taken from Chi et al., 1981)

	Object	Process
Expert	21	74
Novice	39	2

Table 3 Ratings of four swimmers (data taken from Leas & Chi, 1993)

	Time	Experts (N=2)	Novices (N=2)
Swimmer 1	51.7	8.00	8.50
Swimmer 2	53.1	6.50	7.50
Swimmer 3	60.2	4.75	6.50
Swimmer 4	61.0	4.75	7.50

experts focused on the processes occurring among the elements within the problem statement, whereas the novices focused primarily on the elements themselves. This constitutes a concrete-object to process shift.

Another example comes from our work on examining expert swimming coaches (Leas & Chi, 1993). In this study, expert swimming coaches (as recognized by the US Swim Association, and with 12 years of coaching experiences) and novice coaches (with 2 years of coaching experience) were asked to view underwater tapes of four swimmers. Their task was to rate each swimmer on a scale ranging from 1 (bad) to 10 (good) and to diagnose what might be wrong with each swimmer's stroke. Table 3 shows the mean ratings of the two expert and two novice coaches, compared with the actual swim times of each swimmer. As one can see from Table 3, novices and experts had the same ranking of ratings, and moreover, these rankings corresponded to the ranking of the swimmers' times. This means that with a minimum of 2 years of coaching experiences, coaches can adequately pick out the good swimmers and differentiate them from the poorer swimmers. The accuracy of the novice coaches makes sense because even a naïve spectator can often tell who is a better swimmer (or dancer, or any other physical performer), and so forth, based on qualitative overall features.

However, we further asked the coaches to give us the cues that they had used to decide on their ratings of the swimmers. Here we found little overlap in the cues cited by the expert and novices coaches. Moreover, there are characteristic differences between the types of cues the novices cited versus the type that experts cited. (Table 4 gives some examples of the cues they had used.) The

Table 4 Swimming diagnoses (data taken from Leas & Chi, 1993)

	Object	Process
Expert		Unequal body roll Rotates to right Wide pull Stroke unbalanced Breathes to one side
Novice	Elbow bent Elbow lock out Right arm not underneath Left arm not extended	Nice body roll

differences can be characterized again as either an object-process difference, or a part-whole difference, or a static-dynamic difference. For example, novices tend to cite a single body part ("elbow bent" or "right arm not underneath") as the flaw in a specific swimmer's stroke, whereas experts tend to refer to the entire holistic movement ("unequal body roll" or "stroke unbalanced") as a flaw in a swimmer's stroke.

These characteristic differences are not incremental, but rather, represent significant shift in perspectives. For example, if we view the difference as one between objects and processes, these two perspectives are distinct ontological categories (Chi, 1997). The difference is similar to the difference in physics cues cited earlier, between citing an explicit concrete object (inclined plane or pulley) as the cue for the kind of problem it is, versus citing cues referring to the entire system, such as a before-and-after situation, meaning that the forces acting on the entire system are equal before some interactions and after some interactions. The question of interest is whether this perspective shift is trainable. For example, in solving simple mechanics problems, would it be feasible to teach students to look for concepts such as a balance-of-forces for the whole system, rather than to teach them to seek individual forces acting on each mass? Similarly, for swimming coaches, can instruction for diagnosis focus on movement of the entire body, rather than individual body parts? Other related areas might be the difference between a focus on individual agents or objects in a dynamic system (such as an eco system), versus teaching students to focus on the entire population (Chi, 2008). This type of instructional approach has not been tried, to our knowledge, to see if the acquisition of expertise can be accelerated.

Conclusion

This chapter is not a review of the expertise literature. Instead, this chapter first outlined the major shifts in the literature in terms of understanding what makes experts excel. Of course, more knowledge is assumed, by definition. But the first approach to the study of expertise had assumed that what differentiated experts from novices were the experts' superior search strategies. However, in light of new empirical evidence, this idea was replaced by another assumption, that what differed between experts and novices was the structure of their greater knowledge. Finally, it was shown that differences in the structure of knowledge led to differences in the way a problem is represented by experts and novices. And the way a problem is represented led naturally to more efficient and more correct solutions. Although this difference in representations offers many insights (for example, in understanding the discrepancy between a teacher's representation of a problem and a student's representation, thereby students will inherently misunderstand a teacher's explanations), how one can teach learners to construct better structured knowledge so that they can construct a better representation remains a challenge. This instructional challenge can be couched as how can we accelerate a learner's understanding or how can we create a more adaptive expert.

The second part of this chapter highlights the three constructs that are being emphasized in the current literature – deliberate practice, adaptive expertise, and team expertise. On the surface, the first two constructs appear to refer to different aspects of expertise: deliberate practice refers to how experts practice in order to achieve elite status, and adaptive expertise refers to some experts who can generalize their understanding to new situations. But in some ways, these two constructs are quite similar: they are both concerned with the production of some exceptional experts, those who have deeper understanding and can generalize and transfer their understanding to non-routine problems. The third construct is concerned with a more concrete practical problem: how to create expert groups or teams, given the nature of collaborative and team work that is required in the real world. Many questions remain unexplored so far about group and team expertise, such as what is the best composition of an expert team, how to optimize a group's learning, and so forth.

The last section of this chapter proposes a new way of thinking about differences between experts and novices. Instead of thinking about experts or more elite and adaptive experts as ones who have conceptual understanding in addition to procedural understanding, or as ones who can generalize their knowledge to non-routine problems, or as ones who practice deliberately, we might want to explore the source of this greater conceptual understanding or greater generalized understanding. One source might be the achievement of an ontological perspective shift. That is, to achieve a certain level of eliteness and adaptive expertise means that one has acquired another perspective. Viewed this way, it makes sense to consider adaptive expertise at all levels of expertise. To enable the acquisition of adaptive expertise then means that we have to understand what is the perspective of the experts, and develop instruction from this perspective. Whether this approach will be more successful at producing adaptive experts remains an empirical question for now.

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Images of Expertise in Mathematics Teaching

Rosemary S. Russ, Bruce Sherin, and Miriam Gamoran Sherin

Abstract In this chapter we present a brief portrait of how researchers engaged in the study of mathematics teaching have understood teaching expertise, a portrait that is attentive to the diversity that has existed and continues to exist in the field. To do so we first adopt a historical perspective and attempt to capture some of the trends in how teaching expertise has been conceptualized, with an emphasis on how these trends were driven by broader changes in educational research. In particular, we trace the study of mathematics teaching through the traditions of process-product research, cognitive research, subject-specific cognitive research, situated cognition research, and design research. We then provide some sense for the diversity of perspectives and approaches to mathematics teaching that are currently prominent by presenting four images of mathematics teaching practice. We describe how researchers have tacitly conceived of mathematics teachers as either diagnosticians of students' thinking, conductors of classroom discourse, architects of curriculum, or river guides who are flexible in the moments of teaching. An awareness of these images of expertise will help the field both recognize and situate new images, allowing us to use them in productive ways to further understand the work of mathematics teaching.

Keywords Mathematics · Teaching · Expertise

Our charge in this chapter is to discuss how teaching expertise has been conceptualized by researchers engaged in the study of mathematics teaching. Although we accept that charge, we must note that there is no possibility of providing anything approaching a unitary account of mathematics teaching expertise, or of the research that seeks to understand that expertise. The problem is that teaching, as a profession, requires its practitioners to engage in a diverse constellation of

R.S. Russ (✉)
School of Education and Social Policy, Northwestern University, Evanston, IL, USA
e-mail: r-russ@northwestern.edu

The authors Rosemary S. Russ, Bruce Sherin, Miriam Gamoran Sherin contributed equally to the writing of this chapter.

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tasks. Mathematics teachers must plan lessons, work with students individually and as a whole class, and they must present explanations, examples, and definitions. Similarly, mathematics teachers develop assessments, grade student work, and keep track of student progress. Complicating the situation still further is the problem that each of these tasks can, in practice, exhibit enormous variability.

This complexity requires that researchers studying mathematics teaching expertise, working as a field, adopt a divide-and-conquer approach. One way in which the field may divide up the undertaking is for individual researchers to work on different subsets of the "diverse constellation of tasks" faced by teachers. So, some researchers might choose to look at how teachers create lesson plans, while others might look at how they lead classroom discussions.

But the situation is a bit more complicated than this divide-and-conquer story suggests. The fact is that individual researchers may look at the problem of understanding teaching expertise from very different angles. Moreover, new perspectives percolate through the field, changing with time, and spreading from one researcher to another. As they do, the problem of understanding teaching expertise is divided and re-divided in such a way that the work of multiple researchers does not fit together cleanly.

The particular way that an individual researcher chooses to conceptualize and study mathematics teaching is likely influenced by a number of factors. First, there are the current trends in the broader landscape of education research – the perspectives that percolate through the field. A second and related influence is that researchers each have their own particular commitments to and assumptions about what aspects of the practice are important for successful teaching and learning. Third, researchers must choose components of teaching practices that are tractable and feasible to study.

In this chapter, we seek to present a brief portrait of how the field has understood mathematics teaching expertise, a portrait that is attentive to the variability and diversity that existed and continue to exist. We will do this in two ways. First, we adopt a historical perspective and attempt to capture some of the broad trends in how teaching expertise was conceptualized, with an emphasis on how these trends were driven by broader changes in the landscape of educational research. In describing these historical trends we treat the field as largely monolithic in its approach and emphases. Our focus then shifts to the present and, in doing so, we attempt to provide some sense for the diversity of perspectives and approaches to mathematics teaching expertise that are currently prominent. To paint a picture of this diversity, we present four images of mathematics teaching practice. In describing these images, we will also attempt to show how our current conceptions of teaching expertise continue to be influenced by perspectives that were prominent in the past. To do so, we first present a short teaching vignette from an eighth-grade mathematics classroom that we will use to ground the discussion throughout the chapter. To be clear, our goal is not to characterize expert mathematics teachers as a class of teachers distinct from novice teachers. Instead, we seek to describe several key aspects of the expertise involved in teaching mathematics today.

A Vignette of Mathematics Teaching: The Crowd Estimation Problem

It was December, and Mr. Louis' 4th period class was nearing the end of a unit on comparing and scaling (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997b). At the beginning of class, students were given a picture of a 14 cm × 9 cm rectangle densely filled with dots (Fig. 1). Students were told to imagine that the picture was an aerial photograph of a crowd at a rally and that each dot represented a person. Their task was to estimate how many people attended the rally. Students began by working on the problem in small groups as the teacher circulated throughout the class. Mr. Louis then invited Tina's group to the front of the room to share their approach. Using the overhead projector to demonstrate, Tina explained that they divided the original rectangle into 126 small squares that were 1 cm × 1 cm. Then they counted 17 dots in one of the small squares. To estimate the total population, they multiplied 17 by 126. Tina concluded, "and we got 2,142. That was our approximate answer."

Following the presentation, Mr. Louis turned to the class and asked, "What do people think about this group's method?" Among several comments from students, Robert responded that Tina's group would have gotten a more accurate estimate for the total population if they had used bigger squares. When prompted to elaborate, Robert explained that "with smaller squares there may be a bunch of dots packed into a small area. In just that particular area or something. Or there might have been not a lot of dots." Robert's point was that the number of dots in a larger square might be more representative of the density of the picture than the number of dots in a smaller square (see Fig. 2).

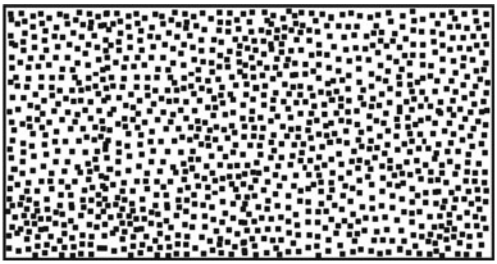

Fig. 1 Estimate the population of the crowd shown in the picture

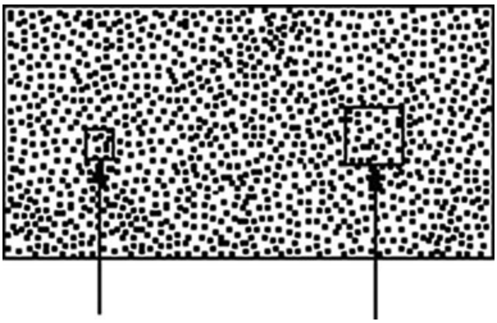

Fig. 2 Two proposed solutions to the crowd estimation problem

Mr. Louis again turned to the class for comments: “What do you think about what Robert just said?” Several students said that they agreed with Robert, including Amy, Jin, and Sal. In contrast, Jeff suggested an alternative method that involved finding the average number of dots in 10 small squares. “It would have been better if instead of ...one small square... they took ten squares from all random spots that were small size and divided the total of all the groups by 10.” After a few minutes, Mr. Louis drew the class’ attention specifically to Robert’s and Jeff’s ideas. “We have two competing ideas here.” He drew a diagram to illustrate the different approaches and encouraged the students to compare and contrast the two methods. “Which way do you think would produce the most accurate estimate of the population?”

As the class discussed Robert’s and Jeff’s methods, students raised a number of issues including the role of averaging (“[For] a better estimate you have to have an average.”), the context in which the sample was drawn (“Robert’s methods would be better if...the big squares had the same number of dots each time.”) and the relationship between the samples (“Is Jeff’s method just... making the square ten times larger?”) While aware of the productive discussion taking place, Mr. Louis also realized that the bell would soon ring. He encouraged students to continue thinking about the lesson: “There’s still a lot of really rich math in here, so let’s try to think about what we think here.” He then assigned students their homework, which included describing a way to estimate the number of blades of grass on a football field and selecting an effective sample to use to determine the favorite rock band of students at their school.

Research Paradigms in the Study of Mathematics Teaching

The prevailing paradigms of research in a given field at any given time heavily influence and serve to organize the particulars of research carried out in that field. Research on teaching in general and mathematics teaching in particular is no exception. Here we review some of the major trends and traditions in the history of research on teaching to provide a background against which we can better understand current research on mathematics teaching expertise.

Process-Product Research

Early research on teaching was driven by a desire to identify relationships and find connections between classroom teaching and students’ learning. Described as “process-product research,” these studies sought to answer questions that took the general form of “What characteristics of teachers and teaching are linked, in some causally relevant way, to desired student outcomes?” (Floden, 2001, p. 7). To answer these questions this work focused largely on observable behaviors of teachers and students in classrooms. Researchers would choose particular behaviors or attributes of teachers to examine (e.g. “experience”) and find ways to quantify those variables (e.g. number of years of teaching) while simultaneously observing and measuring outcome variables in students (e.g. scores on achievement tests). This work then aimed to discover effective teaching strategies by documenting large numbers of classrooms and identifying correlations and covariations between what the teachers did and what the students learned (Rex, Steadman, & Graciano, 2006).

One assumption of this work was that effective teaching strategies were domain general; researchers could look across teaching in different domains and make generalizations about what teaching expertise looked like overall. Thus data from mathematics classrooms was combined with data from science and history classrooms in order to perform these large-scale correlational studies. For example, one productive line of research grounded in the process-product tradition revolved around “wait time” (Rowe, 1974). In this work, the amount of time teachers wait after asking a question and before evaluating a student response was examined in relation to the frequency and complexity of students’ responses. Other research in this tradition explored the influence of various classroom management techniques as well as the influence of different types of teacher questions on student responses (Dunkin & Biddle, 1974). In this work researchers observed teaching practice and compared it to measurable student outcomes using domain general variables such as “wait time,” management techniques, or types of questioning.

Let us now consider our vignette from the perspective of a researcher working within the process-product paradigm. What slice of Mr. Louis’ practice would be of interest to this researcher? Likely he would seek to isolate, observe, and quantify individual features of the instruction or of Mr. Louis himself that contributed to his students’ success or failure in the classroom. For example, the data might be used to explore questions such as: Does the number of times a teacher asks his students to explain their ideas – as Mr. Louis does with Tina, Robert, and Jeff – impact the students’ achievement on a test of the same material? When teachers have students work in small groups to solve problem – as Mr. Louis does with this problem – are students more likely to turn in a correct problem solution? Furthermore, in seeking to answer these questions, it would be assumed that the answers are not domain-specific. So data from Mr. Louis’ mathematics class might be pooled with data from science and social studies classrooms.

A First Look into the Mind of the Teacher

By the 1980s a new paradigm grounded in the intellectual traditions of cognitive science and psychology began to drive research on teaching. Rather than observing and describing teacher behaviors, cognitivist researchers sought to generate accounts of teacher knowledge and thinking. This “approach to the study of teaching assume[d] that what teachers do is affected by what they think” (Clark & Yinger, 1987, p. 231). What might now seem relatively obvious was, against the backdrop of behaviorism, revelatory. Indeed, the promise seemed to be great. It was hoped that researchers might gain more traction in understanding teacher practice if instead of just directly describing behaviors, as research in the process-product tradition had done, research first tried to understand the thinking of the teacher that gave rise to that behavior. This work looked in particular at "three fundamental types of cognitive processes" of teachers including "studies of [teachers'] judgment and policy, of problem solving, and of decision making" (Shulman, 1986, pp. 23–24).

An example of cognitivist research on teaching is the study of teacher planning. For example, Peterson and Clark (1978) interviewed teachers following instruction as a way to explore the relationship between the teachers' goals for a lesson and their decisions about adapting the lesson during instruction. In related work, Clark and Yinger (1979) identified different goals that teachers had in mind as they planned for instruction (e.g., planning in order to structure a lesson versus planning in order to develop an appropriate assessment activity). For researchers within this paradigm, providing detailed accounts of teachers' cognition was essential to understanding and making sense of their classroom teaching. However, these accounts of teacher cognition were still domain general; differences in domains were not considered relevant for examining teacher thinking.

We return again to our vignette to demonstrate how early cognitivist researchers might have attempted to make sense of Mr. Louis' teaching. What slice of our vignette might they have chosen to focus on? To start, such researchers may have sought to uncover Mr. Louis' plans for instruction, his "lesson image" (Morine-Dershimer, 1978–79), and points in the lesson where Mr. Louis expected to shift from one activity to the next. They may have shown Mr. Louis portions of the lesson after instruction with the goal of having Mr. Louis reconstruct his thinking at particular points in time. How did he decide when to move from small group work to the group presentation? Did he have in mind a particular "wrap-up" for the lesson that he then abandoned given time constraints? As in the process-product tradition, these analyses of Mr. Louis' teaching would not be substantively affected by the fact that he teaches mathematics.

A Focus on Subject-Specific Teaching

As the cognitive revolution unfolded over the middle and latter twentieth century, one lesson was clear: Looking across multiple populations and diverse fields, it was repeatedly established that expertise is profoundly domain-specific (Glaser & Chi, 1988). To exhibit expertise in a domain, an expert must acquire a body of knowledge that is specific to that domain. As Shulman summarized it, "the thrust of the cognitive science research program in learning is subject matter specific rather than generic" (Shulman, 1986, p. 25). The implication of this work for teaching was doubly significant. First, it implied that we must expect teaching expertise to exhibit the same kind of domain-specificity as any other discipline. Second, and more subtly, teaching is a discipline that is concerned with helping others – students – to acquire expertise. If student reasoning depends on domains then what teachers must do to influence that reasoning will likely also depend on the domain. Thus, research into effectiveness in teaching, in addition to focusing on cognition, needed to focus in particular on domain-specific cognition.

Shulman (1986) led the field's advance into domain-specific cognitive research on teaching. In particular, he contrasted teachers' subject matter knowledge with what he called their "pedagogical content knowledge." Subject matter knowledge, according to Shulman, concerned one's understanding of the facts and concepts within a domain, while pedagogical content knowledge, on the other hand, had to do with an understanding of how to teach those facts and concepts. A wealth of researchers elaborated on Shulman's claims in the area of mathematics instruction, identifying pedagogical content knowledge in varied domains such as elementary fractions (Marks, 1989) and functions (Even, 1993). Others looked closely at the role of pedagogical content knowledge during instruction, making claims that the depth of one's pedagogical content knowledge is what characterizes the accomplished mathematics teacher (Borko et al., 1992; Putnam, 1992; Sherin, 2002). The assumption behind all of this work is that pedagogical content knowledge is inherently domain specific and crucial for successful teaching practice.

Let us now return again to the case of Mr. Louis. What slice of his teaching would cognitivists committed to domain specificity examine? Researchers from this tradition would be particularly interested in the thinking that Mr. Louis does that is mathematical in nature. For example, they might ask: What knowledge did Mr. Louis use that allowed him to see Jeff and Robert's ideas as competing alternatives? What did Mr. Louis know about students' common misconceptions in mathematics that caused him to select this particular problem to help them understand sample size? For these researchers, answering such questions would likely involve examining the classroom activity in detail and interviewing Mr. Louis about his thinking both in the moment of instruction and during his planning.

A Situative Perspective on Teaching

Like all successful research paradigms, the cognitive perspective engendered a backlash of sorts. At the heart of this backlash was the sense that, in the cognitive tradition, too much explanatory emphasis was located on the in-the-moment cogitations of individual actors. Instead, it was argued, a perspective is needed in which the individual is understood as embedded in physical and social systems, spread over space and time. This perspective has been known by many names; in its more recent incarnations, the names *situated cognition* and *situative perspective* are common. Though the situated perspective surged to prominence in the 1980s and 1990s, it traced its lineage to older traditions, including the instrumental psychology of the Soviet psychologists, in which thinking was thought to arise first on an interpsychological plane (e.g., Vygotsky, 1978).

Adopting a situative perspective has led researchers to see the mathematics classroom as a place and community with a history, and to focus on interactions among teachers, students, and artifacts. Studying mathematics teaching expertise then involves studying, for example, the roles of participants in the classroom discourse (e.g., Moschkovich, 2007; Sfard, 2007), how artifacts and ideas are taken up among community members (e.g., O'Connor, 2001), and how the teacher establishes an environment in which responsibility for learning is shared among participants (e.g., Silver & Smith, 1996).

We return to our vignette once more to demonstrate the focus of attention of situative researchers. These researchers would be interested in questions such as: How does the interaction between Mr. Louis and his students give rise to the various approaches to the problem that are voiced? How do the artifacts and representations used in the classroom mediate or afford the learning that occurs? When and how is new knowledge and language appropriated by Mr. Louis' students? Close examination of students' work in the small groups and their discourse during the large group discussion would be crucial to this analysis, including studying issues of power and agency, the identities and roles the students and Mr. Louis develop or adopt during the course of the lesson, and the negotiation of norms of participation and representation in the classroom. In addition to analyses of this particular moment from Mr. Louis' teaching, those with a situative perspective would also be interested in the history of the class and the students themselves, and how that history impacts what occurs in that moment.

Design Research: Teaching as Curriculum Adaptation

Among the more recent trends to influence research on teaching expertise is what has been referred to as *design research.* Unlike the shifts described above, the design research perspective does not constitute a fundamental change in the way that human reasoning or social systems are understood. Rather, it represents a change in how we conceptualize the relationship between research and practice. In some respects, the relationship between research and practice is seen as *more* intimate. In design research, design and theory development are carried out in tandem, and the boundary between research and design is essentially eliminated (Edelson, 2002). In other respects, the relationship between theory and practice is understood to be loosened. It is explicitly recognized that designs are just that – designs – and that theories of learning do not come close to determining all aspects of an instructional design (Brown & Campione, 1996) Additionally, the design research perspective emphasizes that educational theories and designs must be portable in the sense that they can survive diffusion into the world.

The design research perspective can be seen as having a variety of impacts on the way we understand the nature of teacher expertise. Unlike the paradigms described in the preceding sections, this perspective does not draw our attention primarily to the reasoning and acting of the teacher. Instead, we are led to view the teacher through the lens of the larger instructional system in which the intentions of a curricular designer are brought to bear on students. More specifically, the teacher is understood as playing a particular role within this larger system, as the interpreter and applier of curriculum materials. Within this perspective, research on mathematics teaching focuses on patterns in teachers' use of curriculum materials. For example, Remillard (2005) examines the cognitive resources teachers bring to the work of enacting curricula in their classrooms. In other work, Sherin and Drake (2009) document the different ways that elementary mathematics teachers read, evaluate, and adapt a new mathematics curriculum.

We can return once more to our vignette with the design research perspective in mind. Design researchers entering Mr. Louis' class would likely not be content to observe and analyze only what occurred during his lesson enactment. Instead they would examine how the lesson was enacted as compared to how it was designed and seek to understand Mr. Louis' reasons for adapting the lesson as he did. The design researcher would be interested both in the adaptations Mr. Louis made while planning before the class and those he made in the moments of instruction. For example, Mr. Louis had students discuss their ideas with the entire group. A design researcher would examine the curriculum documents to identify whether this was a change from the original design. If so, why did Mr. Louis change this aspect of the design? When did he decide to change the lesson? Did this change maintain the original goals of the curriculum designers? Design researchers would seek to understand Mr. Louis' teaching as part of a system that includes not only the teacher and the students but also the curriculum designers and the curriculum itself.

Current Research on Mathematics Teaching: Four Images of Expertise

In the preceding sections, our perspective was historical; we attempted only to capture the trends in the field – changes to the broad landscape of education theory – and the new understandings of expertise in teaching that grew out of these changes. In doing so, we essentially treated the field as monolithic. Of course, at any point in time, there is variability among researchers. In this section, we turn to the present day, and we attempt to paint a picture of the variety that exists.

Capturing this diversity in a meaningful way is challenging. The perspectives adopted by researchers are changeable, and boundaries are never clear. To paint our picture we present four *images* of mathematics teaching expertise. Each of these images encapsulates an orientation toward mathematics teaching expertise, and each highlights some facets of expertise and ignores others. To help clarify the differences among these images, we will highlight the kinds of questions that each image might pursue relative to the crowd estimation lesson.

Mathematics Teacher as Diagnostician

One way to conceive of mathematics teaching today is that the central role of the teacher is as a diagnostician. The teacher, like a doctor or mechanic, must examine the mathematical thinking of students, look for symptoms (e.g., wrong or surprising answers), and diagnose their underlying cause (e.g., a faulty conceptualization). Thus, the emphasis here is on the need for teachers to be able to discern the meaning of the mathematical ideas and methods that students raise in class. This aspect of teaching practice has been referred to in a number of ways including “sizing up students’ ideas” (Ball, Lubienski, & Mewborn, 2001), “observing student reasoning” (Kazemi & Franke, 2004), and “drawing inferences about student talk” (Hammer & Schifter, 2001). The skill needed to interpret students’ mathematical ideas should not be underestimated (Even & Wallach, 2004). Students’ ideas can be quite complex, and students do not always articulate their thinking clearly. Furthermore, teachers are often expected to make sense of a student’s idea quite quickly and with little in the way of resources that might offer potential interpretations for the teacher to consider. Wallach and Even (2005), for example, warn of the potential for teachers to *under-hear* or *over-hear* as they work to make sense of the methods students share in class.

Looking at Data

Researchers who adopt the stance of mathematics teacher as diagnostician tend to look closely at interactions between teachers and students around specific mathematical content. They might focus, for example, on the questions teachers ask students about their ideas or on the explanations teachers provide about students’ methods. This approach is strongly connected to the cognitivist’s commitment to subject-specific cognition. The assumption is that the process of diagnosis involves looking at mathematical content in a very detailed and up-close manner.

Studying Teacher Expertise

A focus on the teacher as diagnostician leads researchers to several related lines of inquiry. One area of study examines what teachers understand about student thinking in particular mathematical domains. For example, Even and Tirosh (2002) discuss the extent to which seventh-grade teachers recognize students’ tendency to simplify algebraic expressions without regard to “like terms.” Similarly, Son and Crespo (2009) examine how elementary and secondary teachers reason about a novel student method for dividing fractions. Closely related to such research are investigations of what teachers’ themselves understand about various mathematics topics (see, for example, Borko et al., 1992; Stephens, 2008). The idea here is that the ways teachers diagnose students’ ideas rest heavily on the teachers’ own understanding of the mathematical content.

In other work, researchers aligned with the teacher-as-diagnostician perspective delineate categories of knowledge that support teachers’ ability to interpret students’ thinking. This line of inquiry builds directly on Shulman’s (1986) introduction of pedagogical content knowledge. For instance, Ball, Thames, and Phelps (2008) define “specialized content knowledge” – a “kind of unpacking of mathematics” (p. 402) that allows teachers to, for example, identify common student misconceptions and decide whether or not a novel student method is generalizeable. In other work, Ma (1999) explains that teachers who possess “knowledge packages” (p. 118) – collections of mathematical concepts that a teacher views as strongly connected – are able to provide in-depth, conceptually-based responses to scenarios describing student misconceptions. For these researchers, what is of interest is the kinds of mathematical knowledge that teachers draw on to successfully diagnose students' thinking.

A third line of inquiry revolves around researchers' efforts to help teachers become more effective diagnosticians. For example, the Cognitively Guided Instruction project organized professional development for elementary school teachers around students' understanding of addition and subtraction word problems (Carpenter, Fennema, Peterson, & Carey, 1988). Franke, Carpenter, Levi, and Fenemma (2001) report that, as a result, most participants learned to listen carefully to their students' ideas and that, in some cases, knowledge of students' thinking became generative for the teachers. In other words, teachers' ability to analyze students' strategies influenced the teachers' own learning of mathematics and informed their instructional decisions.

The Crowd Estimation Lesson

We now consider how a researcher focused on studying the ways mathematics teachers diagnose student thinking might examine the Crowd Estimation lesson. Of particular interest to the researcher would be ongoing evidence of Mr. Louis working to understand the ideas that students share in class. For example, during the initial presentation, Mr. Louis requested clarification of the group's approach, asking if Jen "would write some of this down for us" and explain, "What did you do after that?" Shortly after, when Robert suggested an alternative, Mr. Louis probed, "That's interesting. Why do you say that?" Similarly when Amy commented that Robert's method was good because of the bigger squares, Mr. Louis asked her to expand, "Why would that make a difference?" One way to understand Mr. Louis' frequent elaboration requests to students is that he is seeking more information from which to diagnose their thinking. Furthermore, the researcher might also be drawn to particular moments in the lesson where Mr. Louis appears to be drawing on his knowledge of mathematics to diagnose students' ideas. For example, Mr. Louis' understanding of ratio and proportion was likely an important resource in understanding the difference between Robert's and Jeff's methods. Similarly, his pedagogical content knowledge likely played a role in how he chose to represent Robert's and Jeff's methods visually for the class.

Mathematics Teacher as Conductor

A second way to conceive of mathematics teaching expertise is to imagine the teacher as a conductor, directing and shaping the classroom discourse. By many accounts, discourse is an essential component of mathematics instruction today (e.g., National Council of Teachers of Mathematics, 2000). Classroom discourse communities support student participation in important mathematical practices including explanation, argumentation, and justification. Furthermore, research has demonstrated that classrooms in which students regularly talk about mathematics provide valuable access to multiple ways of thinking about and solving problems.

At the same time, managing classroom discourse effectively is not a simple task. As Stein, Engle, Smith, and Hughes (2008) explain, "A key challenge that mathematics teacher's face. . .is to orchestrate whole-class discussions. . .in ways that advance the mathematical learning of the whole class" (p. 314).

Looking at Data

Researchers who draw on the perspective of mathematics teacher as conductor typically focus their investigations on the conversations that take place during class. They often look closely, for example, at who speaks and when, how teachers elicit comments from students, the kinds of questions teachers (and students) ask, and what counts as a valid explanation in a given discussion. This perspective draws heavily on both the cognitive and situative paradigms for teaching. Discourse is thought to involve thinking and meaning making on the part of the teacher; at the same time discourse arises from communities and marks membership in that community (Moschkovich, 2007).

Studying Teacher Expertise

Despite a common focus on the teachers' role in classroom discourse, researchers adopting this stance explore several different lines of inquiry. First, a number of studies investigate stages through which teachers move as they develop their abilities to effectively facilitate mathematical discourse. For instance, Smith (2000) describes key phases in the development of a middle-school teachers' questioning techniques. In other work, Hufferd-Ackles, Fuson, and Sherin (2004) introduce a four-step process of developing a "math-talk-learning community" (p. 4) in which discourse shifted from teacher-directed to student-directed, and from a focus on answers to a focus on mathematical thinking. The emphasis in all this work is on the development of the teacher's expertise as conductor of classroom discourse.

A second, related, approach concerns the teacher's use of classroom norms for communicating about mathematical ideas. Emphasis is on what Yackel and Cobb (1996) define as "sociomathematical norms," shared understandings of what "counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant" (p. 461). Of interest then, is uncovering how teachers lay the groundwork for establishing such norms. For example, early in the year Lampert (2001) explicitly encouraged students to add to one another's ideas in order to establish among the class appropriate ways to respond to, and even challenge, a person's ideas.

Third, some researchers who align with the teacher-as-conductor perspective primarily investigate patterns in the ways that teachers engage in classroom discourse. For example, Forman, Larreamendy-Joerns, Stein, and Brown (1998) describe the use of *revoicing* and *filtering* in order to highlight and clarify students' contributions. In other work, Williams and Baxter (1996) and Nathan and Knuth (2003) illustrate teachers' use of *analytic* and *social* scaffolding design to support worthwhile discussions of student mathematical thinking during class discussions. This approach to research emphasizes the different strategies used repeatedly by teachers in their role as conductors.

The Crowd Estimation Lesson

So how would researchers aligned with the teacher-as-conductor view of teaching expertise analyze the Crowd Estimation lesson? To start, a single lesson would not provide sufficient evidence to allow for an investigation of the development of whole-class discourse in Mr. Louis' classroom. While the researcher might be able to draw a few related conclusions from the data (e.g., that this type of discourse was familiar to students), without access to a series of discussions facilitated by Mr. Louis such analysis would be difficult.

At the same time, the lesson does provide a rich context for examining other aspects of the teacher-as-conductor perspective. First, there is evidence of several sociomathematical norms in place, norms that are mediated by the teacher. For example, Mr. Louis elicited multiple solutions to the estimation task from students, and each strategy was allotted time for discussion. Researchers might also explore patterns in Mr. Louis' discourse with the class. For example, Mr. Louis consistently encouraged students to comment on each other's ideas. He did this by following up a student's comment with a general question to the class: "What you guys think about Robert's idea?" "What do other people think?" He also regularly asked students to explain each other's ideas and strategies: "Can someone summarize what John said?" "What is Jared trying to say?" In exploring these patterns, researchers would try to characterize the nature of the expertise needed to effectively take on the role of teacher as conductor.

Mathematics Teacher as Architect

A third way to conceive of mathematics teaching expertise is that of the teacher as architect. Of central concern in this perspective is the teacher's role in selecting and implementing curriculum materials. Curricula are viewed as the primary vehicle through which policy and reform recommendations reach students (Sykes, 1990). Yet at the same time, a wealth of research emphasizes that curricula are not teacher-proof, and that instead, as teachers use curricula they necessarily interpret and adapt the materials for their own use (Lappan, 1997a). The perspective of teacher-as-architect emphasizes that effectively supporting student learning of mathematics requires expertise on the part of the teacher both in choosing tasks to use with students as well as deciding how those tasks should be carried out.

Looking at Data

Researchers who adopt the perspective of mathematics teachers as architects tend to look closely at one or more of several different activities in which teachers engage around the use of curriculum materials. For instance, researchers may focus on the process through which teachers plan for instruction, or reflect on lessons post-instruction. Alternatively, they may investigate particular components of curriculum implementation. What is of interest is the reasoning that teachers engage in as they design instruction. The teacher-as-architect stance draws on the perspectives of both situated cognition and design research. In line with situated cognition, this approach recognizes that curriculum materials are mediating tools used by teachers to accomplish their goals (Brown, 2009). In addition, in line with design research is the idea that teachers are consumers and adapters of designs, as well as designers of classroom activity themselves. Even when using published curriculum materials, the process through which teachers take the *page as written* and move to the *lesson as enacted* can be thought of as a process of design (Silver, Ghousseini, Charalambous, & Mills, 2009) As Brown (2009) explains, “Teaching by design is not so much a conscious choice as an inevitable reality” (p. 19).

Studying Teacher Expertise

The focus on mathematics teacher as architect has increased in popularity over the last 15 years and has resulted in several related lines of inquiry. One approach examines the extent to which the mathematics activities selected by teachers represent *cognitively demanding tasks* – “problems that promote conceptual understanding and the development of thinking, reasoning, and problem-solving skills” (Stein et al., 2008, p. 315). Along the same lines are studies that examine whether teachers maintain a high level of cognitive demand as a task is carried out (e.g. Smith, 2000). Such research seeks to understand the expertise needed to carry out a mathematics lesson in ways that maintain the integrity of the planned lesson.

In other work, researchers characterize teachers’ typical approaches to using mathematics curriculum materials. For example, Remillard and Bryans (2004) define one group of teachers as “thorough piloters” who allowed the published materials to generally guide the structure of lessons in contrast to another group’s “intermittent and narrow” use of the same curriculum (p. 375). Similarly, Nicol and Crespo (2006) identified different ways that teachers adapted a traditional mathematics curriculum: by extending activities suggested by the text or by creating new problems and questions to insert in lessons. By looking at the impact of these different approaches on instruction, researchers attempt to uncover some of the expertise involved in designing and implementing effective mathematics lessons.

The Crowd Estimation Lesson

A researcher drawing on the teacher-as-architect perspective would likely find several aspects of the Crowd Estimation lesson of interest. One issue might be how Mr. Louis organized the lesson – with students initially working in groups, then a student presentation followed by a whole class discussion, and finally with related homework problems assigned. How did this structure serve to meet Mr. Louis’ goals for the lesson? Did Mr. Louis consider brainstorming strategies as a whole class first, and then having students work in groups to pursue some of the strategies in more detail? How did the mathematical content of the lesson as well as students' experience with similar tasks influence his decisions? The researcher might also want to explore Mr. Louis' choice to have Jen's group present their method to the class. Earlier, Mr. Louis circulated throughout the room as students worked on the task in groups. Were there certain features of Jen's group's method that Mr. Louis wanted the class to see, and wanted the class to see first? How did his choice of Jen's group enable or constrain Mr. Louis to move forward with his planned goals for the lesson? While the lesson itself might provide some evidence related to these issues, the researcher would likely want to interview Mr. Louis to examine both of these issues in depth. In doing so, the researcher would endeavor to uncover ways in which Mr. Louis' expertise enabled him to serve in the role of lesson architect – designing and carrying out the lesson in ways intended to support student learning.

Mathematics Teachers as River Guide

A fourth way to conceive of mathematics teaching expertise is that of the teacher as a river guide, as one whose job it is to be flexible in the moment. Like a river guide, a teacher has a carefully crafted plan; the "river" in this case is a lesson that has been carefully reviewed and whose contingencies have been considered. Yet the river guide's true expertise comes to light during the ride, when the rapids change, or a paddler makes an unexpected move. It is the river guide who must respond quickly and effectively. In the same way, teaching expertise can be viewed as being responsive to the context, to students, and to what occurs in the moment (Berliner, 1994). Our use of the river guide metaphor is intended to emphasize that teachers are on the river with the students. We think of them not just as leading students down the river but also as actively engaged with students in the journey.

Looking at Data

Researchers who adopt the perspective of mathematics teacher as river guide typically focus their investigations on the interactions in the classroom. In particular, they try to identify moments of instruction in which teachers make on-the-fly decisions about how to proceed. Through videotapes of instruction and/or interviews with teachers, the researcher will explore, for example, moments in which teachers deviate from their planned lessons, respond to unexpected student ideas, or adapt an activity in the midst of instruction. This perspective draws on both the cognitive and situated paradigms for teaching. From the cognitive perspective, the teacher's expertise as river guide is reflected in the teachers' understanding of subject matter, students, and so on. From the situated perspective, it is reflected in the way expert teachers react to and fluidly operate within changes in the setting and context.

Studying Teacher Expertise

A focus on the teacher as river guide leads researchers to engage in several related lines of inquiry. One approach involves exploring the nature of improvisation as it is exhibited in the act of teaching. For instance, Sawyer (2004) defines teaching as "improvisational performance" and examines the knowledge teachers draw on as they "think quickly and creatively" during instruction (p. 15). In other work, Heaton (2000) studied the process through which her own mathematics teaching was transformed as she came to "appreciate teaching as an improvisational activity" (p. 60).

> [Today] I moved away from the scripted lesson and made a move that went beyond asking children to explain their thinking. I was connected to the work of teaching in ways that I had not experienced before in mathematics. . .For a moment I was no longer in role of silent bystander. I took control. I knew what I was doing. For a moment, I was teaching. (p. 59)

The emphasis here is the idea that teaching expertise necessarily involves improvisation, deciding in the moment how to respond to the unfolding lesson.

Another approach that draws on the notion of teacher as river guide involves trying to model the on-the-fly decision-making process in which teachers engage. For the example, Schoenfeld (1998) illustrates that a mathematics teacher's actions can be modeled as a reaction to existing beliefs, knowledge, and goals. In particular, he demonstrates how these resources come into play when something unexpected happens in the classroom. Relatedly, Artzt and Armour-Thomas (2002) suggest that teachers engage in cycles of active monitoring and regulating during instruction that are mediated by their beliefs, knowledge, and goals. This line of work emphasizes the role of cognitive resources in enabling teachers to quickly and effectively respond to classroom activity.

Third, some researchers who align with the teacher-as-river-guide perspective focus specifically on the *noticing* that mathematics teachers engage in during instruction (Jacobs, Lamb, & Philipp, 2010; Mason, 1998; van Es & Sherin, 2008). The idea is that because the classroom is a complex environment with multiple events happening at the same time, the teacher cannot pay equal attention to all that is taking place. Instead, a key component of teacher expertise involves deciding where to focus one's attention and, according to Mason (2002) preparing oneself to attend to particular kinds of events. Building on Goodwin (1994), Sherin (2007) refers to this as "teacher's professional vision" – the ability of teachers to identify significant events in the classroom. In this strand of work researchers examine teachers' abilities to parse and make sense of classroom activity, which in turn allow teachers to be responsive to issues as they arise.

The Crowd Estimation Lesson

Returning to the Crowd Estimation Lesson, how might researchers aligned with the teacher-as-river-guide perspective examine the lesson? One event that would likely capture their attention is Mr. Louis' decision to put Robert's and Jeff's ideas before the class for comparison and further elaboration. "We have two competing ideas here." This is certainly a decision made by the teacher in the moment of instruction; Mr. Louis could not have known beforehand precisely what ideas would be raised in class, and in what ways. Instead, in the midst of instruction, with all that is taking place, Mr. Louis likely recognized some features of Robert's and Jeff's strategies that he believed would be worthwhile for the class to investigate. "We have Robert who says this. . .take a larger sample.. . . Jeff said something a little different. 'Take 10 squares like this and average them together.' What is Jeff saying. . .that we do?" In exploring this episode from the lesson the researcher would try to uncover what about Robert and Jeff's ideas peaked the teacher's attention in that moment and how the teacher quickly made the decision to juxtapose those ideas against one another.

Discussion

"It was December, and Mr. Louis' 4th period class was nearing the end of a unit . . .." Thus began our summary of a single episode from a mathematics classroom. Throughout this chapter, we only used this one vignette as a reference point. But even this short vignette was enough to support numerous perspectives on the mathematics teaching expertise possessed by Mr. Louis.

In some respects, this chapter may be understood as a "review of the literature." But the expansive nature of our subject matter (how the field has conceptualized mathematics teaching expertise) and the limits of space (the usual chapter in an edited volume) required that our "review" take a somewhat non-traditional form. This was particularly true of our portrait of research on mathematics teaching expertise as it exists in the present day. There, our review centered around four "images" of the mathematics teacher: *diagnostician, conductor, architect,* and *river guide.* Looking back at these images, we realize that the need to be concise has led us to undertake a productive exercise. We have come to believe that it is productive to see researchers as adopting one or more of a moderately small number of images of mathematics teaching. This recognition helps us to understand some of the diversity in the field, as well as why research has clustered in some areas. To be clear, while we have described these images as independent, they are certainly related. Moving forward, we can imagine it would also be productive for the field to explore the ways that these four images are related.

Another way to understand the ideas presented in this chapter is as a "meta" analysis of mathematics education research on teaching expertise. Just as it is useful for our students of mathematics to be aware of their own thinking, we believe that it is useful for us as researchers to be aware of the perspectives that we adopt in our work, whether explicit or implicit. We expect that this will be particularly important as our field continues to move forward. As new paradigms for understanding the complex environment of the classroom emerge, so also will new images of expertise. An awareness of those images of expertise that currently exist will help us both recognize and situate new images, allowing us to use them in productive ways to further understand the work of mathematics teaching.

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Part II
Expertise in Mathematics Instruction in a Western Setting

Coordinating Characterizations of High Quality Mathematics Teaching: Probing the Intersection

Edward A. Silver and Vilma Mesa

Abstract We present an analysis that probed empirically the relationship among three different views of exceptional mathematics teaching: (a) the operational definition of "highly accomplished teaching" of mathematics used by the National Board for Professional Teaching Standards (NBPTS) in the United States, (b) the effective use of cognitively demanding tasks in the mathematics classroom, and (c) the use of innovative pedagogical strategies. We analyzed samples of instructional practice—lesson artifacts and teachers' commentaries on lessons—submitted by candidates seeking NBPTS certification in the area of Early Adolescence/Mathematics. The instructional samples were systematically probed for evidence of mathematical and pedagogical features associated with the views of cognitive demand and innovative pedagogy, and the features found in the submissions of applicants who were awarded NBPTS certification are contrasted with those who were not awarded certification. Our analyses detected a fairly strong interaction between the NBPTS view of accomplished teaching and the view of effective mathematics instruction associated with cognitively demanding tasks. Nevertheless, even in these lessons that teachers selected for display as "best practice" examples of their mathematics teaching, innovative pedagogical approaches were not systematically used in ways that supported students' engagement with cognitively demanding mathematical tasks.

Keywords Mathematics teaching · Teaching quality · Cognitively demanding tasks · Pedagogical innovation

In the United States at this time several characterizations of high quality mathematics teaching are receiving attention from mathematics educators and public policy professionals. Each has at its core one or more important facets of teaching proficiency. Typically these different characterizations are treated in isolation from each other, emphasizing the distinctions between and among them rather than the ways

E.A. Silver (✉)
School of Education, University of Michigan, Ann Arbor, MI 48109, USA
e-mail: easilver@umich.edu

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in which they might interact with each other. In this chapter we focus on one view of high quality mathematics teaching that has garnered considerable attention from the education policy community in the United States. We report the results of an analysis in which we probed empirically the extent to which samples of teaching practice associated with that view of highly accomplished mathematics teaching also exhibited characteristic features associated with two alternative views of high quality mathematics teaching: (a) the effective use of cognitively demanding mathematics tasks and (b) the use of progressive pedagogical practices. We performed our empirical analysis on samples of instructional activity drawn from actual mathematics classrooms—samples that were selected by teachers as examples of their "best practice."

Characterizations of High Quality Mathematics Teaching

In this section we discuss the three views of high quality mathematics teachers and teaching that we consider in the study reported here. We begin with the notion of highly accomplished (mathematics) teachers and teaching proposed by the National Board for Professional Teaching Standards (NBPTS). The approach taken by the NBPTS was intended to characterize high quality teachers and teaching in a generic way, and then to develop specific characterizations for several school subjects, including mathematics. In the study reported here we begin with the NBPTS view of highly accomplished teaching and teachers and examine the extent to which this view is consistent with two other characterizations of high quality mathematics teaching that derive from research in the field of mathematics education. In contrast to the NBPTS approach, which considers first the features of high quality teaching in general and then tries to specify particular versions for subject matter teachers and teaching, the latter views we consider are derived from research that specifically examined teachers and teaching in mathematics classrooms. In this section we also describe these two alternative perspectives on high quality mathematics teaching.

Highly Accomplished Teaching: NBPTS Certification

As one means of improving the teaching profession in the United States, the NBPTS was established in 1987 to recognize highly accomplished teachers by delineating what high quality practice looks like and then devising a way to identify those who exhibit it. To accomplish its goal, the NBPTS used professional consensus to establish standards for what accomplished teachers should know and be able to do, after which it developed a national voluntary system to assess and certify teachers who meet the standards. Thus, in this view, high quality teaching is what NBPTS certified teachers do in their classrooms.

The NBPTS recognizes accomplished practice in a number of fields. Except for generalist certifications, each field is defined by content area (e.g., mathematics) and students' development level (e.g., Middle Childhood-Early Adolescence, ages 7–16). The NBPTS certification system began with the specification of standards for professional practice, initially at a very broad general level, and then for each content-area/age-level certification field. Figure 1 displays the 12 standards, distributed across four broad areas of competence, along with some sample

Area of Competence	Standard	Sample Elaborations
Commitment to all students	I. Commitment to equity and access	I. Accomplished mathematics teachers value and acknowledge the individuality and worth of each student; they believe that all students can learn and should have access to the full mathematics curriculum; and they demonstrate these beliefs in their practice by systematically providing all students equitable and complete access to mathematics.
Knowledge of Students, Mathematics & Teaching	II. Knowledge of students III. Knowledge of mathematics IV. Knowledge of teaching practice	III. Accomplished mathematics teachers draw on their broad knowledge of mathematics to shape their teaching and set curricular goals. They understand significant connections among mathematical ideas and the application of those ideas not only within mathematics but also to other disciplines and the world outside of school. IV. Accomplished mathematics teachers rely on their extensive pedagogical knowledge to make curricular decisions, select instructional strategies, develop instructional plans, and formulate assessment plans.
The Teaching of Mathematics	V. The art of teaching VI. Learning environment VII. Using mathematics VIII. Technology & instructional resources IX. Assessment	VI. Accomplished mathematics teachers create stimulating, caring, and inclusive environments. They develop communities of involved learners in which students accept responsibility for learning, take intellectual risks, develop confidence and self-esteem, work independently and collaboratively, and value mathematics. IX. Accomplished mathematics teachers integrate assessment into their instruction to promote the learning of all students. They design, select, and employ a range of formal and informal assessment tools to match their educational purposes. They help students develop self-assessment skills, encouraging them to reflect on their performance.
Professional Development & Outreach	X. Reflection & growth XI. Families & communities XII. Professional community	X. Accomplished mathematics teachers regularly reflect on teaching and learning. They keep abreast of changes in mathematics and in mathematical pedagogy, continually increasing their knowledge and improving their practice.

Fig. 1 NBPTS standards for early adolescence/mathematics (adapted from NBPTS, 1998, pp. 11–12)

elaborations, for Early Adolescence/Mathematics (EA/M), which is the certification field we studied (see http://www.nbpts.org/for_candidates/certificate_areas1?ID=8&x=42&y=8 for more information regarding current NBPTS certification areas).

Applicants for NBPTS certification complete a series of assessment tasks in which they are asked to demonstrate knowledge and professional practice of many kinds, and their overall performance determines whether they receive NBPTS recognition. Each component of the assessment is linked to one or more of the standards for the certification area. The EA/M assessment consists of two parts: in one, teachers complete an on-demand, test-center-administered set of exercises to evaluate certain aspects of their content and pedagogical content knowledge; in the other, candidates submit a portfolio that includes contextualized samples of their teaching practice and reflections on their work. For applicants in 1998–1999, which is the data set examined in this study, the portfolio component of the EA/M assessment consisted of six entries, of which four were classroom–based entries. The two portfolio entries (Developing Mathematical Understanding and Assessing Mathematical Understanding) examined in this chapter captured teaching practice via classroom artifacts, samples of student work, and teachers' reflective narratives.

The NBPTS assessment process has been extensively evaluated. Technical analyses of the reliability and validity of the assessment have been conducted (e.g., Bond, Smith, Baker, & Hattie, 2000), and there have been a number of studies investigating the relationship between NBPTS certification and measures of teaching practice and teacher effectiveness, especially in regard to student achievement (e.g., Hakel, Koenig, & Elliott, 2008). In general, the research points to a strong positive relationship between NBPTS certification and student achievement; that is, students of teachers who have attained NBPTS certification tend to perform well on standardized achievement measures.

Effective Use of Cognitively Demanding Mathematics Tasks

An alternative view of high quality mathematics teaching considered in this study is one derived from research on classroom mathematics instruction in the United States and elsewhere. International surveys of the mathematics achievement of students around the world regularly indicate that the average performance of students in the US is mediocre when compared to that of students in many other countries, especially countries in Asia (e.g., Lemke et al., 2004; Mullis, Martin, Gonzalez, & Chrostowski, 2004). A recent analysis of the performance of students in 12 countries who participated in both TIMSS and PISA found that students in the United States have specific weakness in using high-level cognitive processes, such as reasoning and problem solving (Ginsburg, Cooke, Leinwand, & Pollock, 2005).

It is quite likely that the student deficiencies in using high-level cognitive processes on mathematics test items are largely a consequence of the limited opportunities they have to learn mathematics in classroom lessons. Mathematics classroom instruction is generally organized around and delivered through mathematical tasks, activities, and problems. According to Doyle (1983, p. 161), “tasks influence learners by directing their attention to particular aspects of content and by specifying ways of processing information.” In fact, tasks with which students engage constitute, to a great extent, the domain of students’ opportunities to learn mathematics. Students in all seven countries analyzed in the TIMSS Video Study (NCES, 2003) spent over 80% of their time in mathematics class working on mathematical tasks.

Tasks can vary not only with respect to the mathematics content but also with respect to the cognitive processes that they entail. Tasks that require students to analyze mathematics concepts or to solve complex problems offer opportunities for students to sharpen their thinking and reasoning in mathematics. In contrast, tasks that require little more than memorization and repetition offer less opportunity to develop proficiency with high-level cognitive processes. Moreover, the cognitive demands of mathematical tasks can change as tasks are introduced to students and/or as tasks are enacted during instruction (Stein, Grover, & Henningsen, 1996). The Mathematical Tasks Framework (MTF) [see Fig. 2], models the progression of mathematical tasks from their original form to the tasks that teachers actually provide to students and then to the tasks as they are enacted by the teacher and students in classroom lessons.

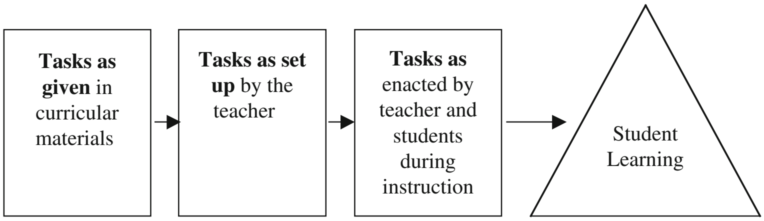

Fig. 2 Mathematical tasks framework (adapted from Stein, Smith, Henningsen, & Silver, 2009, p. xviii)

The tasks, especially *as enacted*, have consequences for student learning of mathematics. The leftmost two arrows in Fig. 2 identify critical phases in the instructional life of tasks at which cognitive demands are susceptible to being altered.

In the TIMSS 1999 video study, the ability to maintain the high-level demands of cognitively challenging tasks during instruction was the central feature that distinguished classroom teaching in countries where students exhibited high levels of mathematics performance when compared with countries like the United States, where performance was lower and teachers rarely maintained the cognitive demands of tasks during instruction (NCES, 2003; Stigler & Hiebert, 2004; Hiebert et al., 2005). In that study, a random sample of 100 eighth-grade mathematics classes in each of seven countries was videotaped during the 1999–2000 school year. Although 17% of the tasks used by teachers in the United States were coded as high level, *none* was implemented as intended. Instead, most “making-connections” problems were transformed into procedural exercises. The authors concluded that 8th grade students in the United States spent most of their time in mathematics classrooms practicing procedures regardless of the nature of the tasks they were given. This claim is consistent with an analysis of mathematics instruction conducted by the Horizon Research Institute, in which only 15% of observed mathematics lessons were classified as providing opportunities for complex thinking, or for mathematical reasoning or sense-making (Weiss & Pasley, 2004; Weiss, Pasley, Smith, Banilower, & Heck, 2003).

Beyond the research documenting modal practice in US. classrooms some other research recently conducted in a variety of American classroom contexts has found that student learning does occur if cognitively demanding mathematical tasks are used regularly and if the high-level cognitive demands are consistently maintained in classroom lessons (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stigler & Hiebert, 2004; Stein & Lane, 1996; Tarr et al., 2008). For example, in a longitudinal comparison of three high schools over a 5-year period, Boaler and Staples (2008) determined that the highest student achievement occurred at the school in which students were supported to engage in high-level thinking and reasoning. Boaler and Staples attribute students' success to the teachers' ability to maintain high-level cognitive demands during instruction, especially the teachers' use of pre-planned questions that elicited and supported students' thinking. Studies by Tarr and colleagues (2008) and by Stein et al. (1996) both found that classrooms in which teachers consistently encourage students to use multiple strategies to solve problems and support students to make conjectures and explain their reasoning were associated with higher student performance on measures of thinking, reasoning, and problem solving.

Emerging from this array of theoretical and empirical work in and on mathematics classrooms is a view of high quality mathematics teaching in which teachers regularly provide students with worthwhile and challenging tasks and generally maintain the level of cognitive demand as students engage with the tasks in a lesson. Thus, this view of high quality mathematics teaching is different from the NBPTS characterization both in kind and in origin. Next we describe a third view that also derives from research in mathematic classrooms, but that is different in kind from both the NBPTS and cognitive demand characterizations.

Innovative Pedagogy

Another alternative view of high quality mathematics teaching encompasses a set of instructional practices that are generally thought to represent progressive ideas about mathematics teaching and that have been associated in various ways with teaching mathematics for understanding. As noted earlier, research in mathematics classrooms in the United States in the upper elementary and middle school grades has found that classroom instruction typically eschews the use of technological tools or concrete models for abstract ideas, tends to focus tasks that make little or no connection to the world outside of school, and pays little or no attention to the development of meaning (e.g., Stigler & Hiebert, 1999; Stodolsky, 1988). Such pedagogy is at odds with current conceptualizations of how people learn best when the goal is developing understanding (Bransford, Brown, & Cocking, 1999). Certain innovative pedagogical practices are often associated with the phrase, *teaching mathematics for understanding*. Over at least the past 60 years a solid body of research evidence has amassed pointing to the benefits of teaching for understanding (sometimes called by various other names, including authentic instruction, ambitious instruction, higher-order instruction, problem-solving instruction, and sense-making instruction) in mathematics (e.g., Brownell & Moser, 1949; Brownell & Sims, 1946; Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Cohen, McLaughlin, & Talbert, 1993; Fuson & Briars, 1990; Hiebert & Wearne, 1993; Hiebert et al., 1996; Newmann & Associates, 1996).

Although there are many unanswered questions about precisely how teaching practices are linked to students' learning with understanding (see Hiebert & Grouws, 2007), there has been increasing emphasis in the mathematics education community in teaching practices that deviate from the canonical version of classroom mathematics instruction noted above and that appear to be more oriented toward the development of students' conceptual understanding. Among the hallmarks of this conceptually oriented version of instruction are teaching practices that are suitable to support multi-person collaboration and communication among students, and to engage students with real-world applications or the use of technological tools or physical models (e.g., Fennema & Romberg, 1999; Hiebert & Carpenter, 1992).

Advocates for conceptually oriented teaching in school mathematics (e.g., NCTM, 1989, 2000) have suggested the potential value of fostering communication and interaction among students in mathematics classrooms through the use of complex tasks that are suitable for cooperative group work and that provide settings in which students need to explain and justify their solutions. Moreover, to increase students' engagement with mathematical tasks and their understanding of concepts, instructional reform efforts have also encouraged the use of hands-on learning activities and technological tools, as well as connecting work done in the mathematics classroom to other subjects and to the world outside school. Beyond exhortations, there is also some research evidence to support these hypotheses about pedagogy that might support students' development of mathematical understanding (e.g., Boaler, 1998; Fawcett, 1938; Fuson & Briars, 1990; Good, Grouws, & Ebmeier, 1983; Hiebert & Wearne, 1993; Stein et al., 1996). Moreover, there is evidence in some studies that these, and other innovative pedagogical strategies, can be applied in superficial ways that emphasize non-mathematical aspects of the activities and sacrifice the complexity of mathematics content (e.g., Cohen, 1990; Ferrini-Mundy & Schram, 1997; Romagnano, 1994; Schoenfeld, 1988; Weiss et al., 2003; Wilson & Floden, 2001).

Emerging from this array of theoretical and empirical work in and on mathematics classrooms is a view of high quality mathematics teaching in which teachers regularly engage in innovative pedagogical practices; that is, pedagogy that deviates from the canonical portrayal found in research on typical classroom teaching. This view is different from both the NBPTS characterization and the cognitive demand view presented above, and it is the third one considered in the study reported here.

Study Methods

In this study, we examined samples of instructional practice—lesson artifacts and teachers' commentaries on lessons—submitted by applicants seeking NBPTS certification. The instructional samples were systematically probed for evidence of mathematical and pedagogical features associated with the views of cognitive demand and innovative pedagogy noted above.[1]

Sample

With the cooperation of the NBPTS, we obtained test center and portfolio exercise score data for all candidates ($N = 250$) who applied for NBPTS EA/M certification in 1998–1999. From this set of 250 applicants we selected a random sample of candidates ($n = 32$; nearly 13% of the population). Our sample was demographically similar to the entire population of EA/M applicants in 1998–1999 and contained a comparable ratio of successful to unsuccessful applicants to that of the full applicant pool; our sample included 13 individuals who obtained NBPTS certification and 19 who did not.[2] The awarding of NBPTS certification is based on a composite of weighted scores on 10 performance indicators (six portfolio entries and four test center exercises), each with an independent, though not equal, contribution to an applicant's overall score.

Data

For each of the 32 individuals in our sample, we obtained copies of the two artifact-based portfolio entries—Developing Mathematical Understanding (DU) and Assessing Mathematical Understanding (AU). These artifact-based entries contained extensive textual portrayals of instructional practice related to developing and assessing student understanding of mathematical ideas, along with supporting artifacts (e.g., students' work, tests, photographs). The DU entry required *two* instructional activities, both focused on the same mathematical idea, which could come from consecutive lessons or from nonconsecutive lessons. In contrast, the AU entry required only *one* activity, and it was required to be different from the idea that was the focus of the DU entry.

[1]This chapter extends another analysis of the same data set that has been reported in Silver, Mesa, Morris, Star, and Benken (2009). In that chapter we reported an analysis of mathematical and pedagogical features of submitted portfolio entries, but we did not distinguish between teachers on the basis of NBPTS certification status. In addition, the purpose of the earlier analysis was different from the intent in this chapter.

[2]Further details regarding the characteristics of our sample with respect to the total population of applicants seeking NBPTS certification in 1998–1999 are given in Silver et al. (2009).

Candidates were instructed to provide all of the following information in each portfolio entry: a written description of the *instructional context* (e.g., grade, subject, class characteristics); a written description of *teacher planning* (e.g., substantive math idea, goals for instructional sequence, challenges inherent in teaching these activities); *analysis of student responses* (actual student work samples for these specific students were appended to the entry); and *candidate's reflections* on the outcomes of each lesson. For both entries, candidates were instructed to select activities in which students were engaged in thinking and reasoning mathematically (e.g. interactive demonstrations, long term projects, journal assignments, problem solving); they were instructed *not* to select activities that focused on rote learning (e.g., students' memorizing procedures).

Data Analysis

Our examination of the NBPTS data consisted of quantitative and qualitative analyses of the two portfolio entries submitted by our sample of 32 applicants. Trained coders examined each entry for evidence of cognitively demanding mathematics tasks and the presence of innovative pedagogical features, and they did so without knowledge of the NBPTS certification status of the applicant whose portfolio entry they were judging.[3] Following the coding of all portfolio entries with respect to cognitive demand and pedagogical features, we conducted further analyses using these codes to compare the portfolio submissions of applicants who were awarded NBPTS certification with those who were not.

Cognitive Demand of Mathematical Tasks in NBPTS Portfolio Submissions

To assess the cognitive demand character of the mathematical tasks in the portfolio entries we developed coding criteria for high-demand and low-demand activities. Low-demand tasks were those that exclusively involved low-level cognitive processes, such as recalling, remembering, implementing, or applying facts and procedures. In contrast, high-demand tasks were those that required students to use high-level cognitive processes, such as analyzing, creating, evaluating, or engaging in metacognitive activity. The framework used to code the cognitive demand of instructional activities is provided in Table 1.

Two independent raters coded each task (64 in DU entries and 32 in AU entries); the overall agreement was acceptably high (80 and 70% respectively); any instances of disagreement were discussed, and a consensus rating was derived. In Table 2 we

[3] We provide here a summary of key points regarding our data analysis methods. Additional information can be found in Silver et al. (2009).

Table 1 Criteria for coding the cognitive demand of mathematics tasks

High cognitive demand
- Tasks require students to explain, describe, justify, compare, or assess
- Tasks require students to make decisions and choices, to plan, or to formulate questions or problems
- Tasks require students to be creative in some way (e.g., to apply a known procedure in a novel way)
- Tasks require students to work with more than one form of representation in a meaningful way (e.g., to translate from one representation to another, interpreting meaning across two or more representations)

Low cognitive demand
- Tasks require students to make exclusively routine applications of known procedures
- Tasks that are potentially demanding are made routine because of a highly guided or constrained task structure (e.g., a complex task is subdivided into non-demanding subtasks; a potentially challenging task is made routine because a particular solution method is imposed by the teacher)
- Task complexity or demand is targeted at non-challenging or non-mathematical issues (e.g., explaining, assessing and describing work is targeted at procedures rather than justification; required explanations are about non-mathematical aspects of a plan or solution)

provide examples of tasks classified as high-demand or low-demand, along with a brief rationale for our decision in each case.[4]

Pedagogical Features of NBPTS Portfolio Submissions

We focused on four pedagogical features identified in the mathematics education reform literature as being innovative and having the potential to cultivate the development of students' mathematical understanding: tasks that involved multi-person collaboration and communication, considered applications in contexts other than mathematics itself, employed technology, or used physical (hands-on) materials. Because a teacher's explanation of instructional context was generally not task-specific for each of the two tasks in a DU entry, we treated the entire DU entry, rather than each activity, as the unit of analysis for the coding of pedagogical features. Thus, 64 items (rather than 96) were coded in this analysis—32 AU entries and 32 DU entries. Agreement was nearly unanimous in the classification. Table 3 displays the judgment criteria we used in the coding and a portfolio entry excerpt providing evidence of the presence of that feature.

[4]Our usage agreement governing the NBPTS materials does not allow us to provide verbatim reproductions. The narrative summaries provide the essential aspects of the task that pertain to decisions regarding cognitive demand.

Table 2 Examples of tasks coded as high-demand and low-demand

Task summary	Coding rationale
High cognitive demand	
• Miniature Golf Course Task. Students had to design a miniature golf course, using at least four solids; they had to produce nets for each shape – showing dimensions, and an isometric drawing of the station. Students had to pass a teacher and peer-inspection that looked for description of the station, nets, isometric drawing, and overall appearance of the course. Comments were expected to be addressed after the inspection (DU)	Students had the liberty to choose the solids, and had to come up with a sensible course; they had many constraints to consider and the net production involved considering reasonable measures for each of the shapes considered. There are also many extracurricular activities involved, which make the task even more complex. This would not be a straightforward activity
• Assessment is based on textbook companion materials; there are 3 questions. Q1 has 7 items, asking about conditions under which systems of equations have one, none, or multiple solutions (*tell how you know that a system of two equations has no solutions*). Students have to provide examples; in the case of one solution, students must provide at least two different ways to solve the system. One item asks the students to write a word problem that can be solved using a system of 2 equations. Q2 has three items to be solved using a graphing calculator. Q3 has three items, all related to a diagram of a shaded region between two lines in the same plane. Students are expected to write a system of inequalities that correspond to the diagram; give a point that is a solution, and a point that is not a solution (AU)	The questions are interesting in that they are "flipped". They are not asking for a solution, but for the conditions to get one or another solution. The demands are higher than when the standard problem/solution is asked for. Students have to create problems that will satisfy a given solution
Low cognitive demand	
• Find Sale Price. Worksheet illustrating how to calculate the price of an item on sale (DU)	Students have to repeat step-by-step procedures modeled in the example provided
• Two-part assessment activity: "geometry walk" and "who am I." In the *geometry walk* students are given a list of 12 shapes and students have to sketch an object found in the real world that has the shape; then they pick 3 objects and explain *why the example has that shape*. In the *who am I* part students are given 14 statements (e.g., my angle degree is 63°, who am I?) (AU)	The assessment confuses 2 dimensional shapes and 3 dimensional objects. Although the task is nonstandard the performance demanded from students is largely based on recalling memorized information; scoring was tilted toward reproduction rather than creativity

Table 3 Criteria and sample excerpts used in coding pedagogical features

Pedagogical feature	Description of criterion	Sample excerpt
Use context outside mathematics	Tasks that involve real-world contexts encountered outside of school, including those related to students' neighborhoods, interests, and cultures	"The assessment is based on a single situation – choosing a car to rent"
Use hands-on materials	Tasks that involve materials used to create some object (e.g., a poster, a physical model) or to make or serve as concrete models of abstract notions (e.g., colored chips to illustrate operations with negative numbers)	"I gave each pair of students a ball, a cylindrical tube, a ruler, and a recording sheet. Students built ramps"
Use multi-person collaboration	Tasks that require that work be done with a partner or in a larger group of students	"They were heterogeneously arranged in carefully selected learning groups of four to five students within that homogeneously grouped class"
Use technology	Tasks in which technological tools—such as calculators, computers, software (e.g., electronic sheets or word processors), and the Internet—are used	"Nineteen students used computer-generated graphs to illustrate their data, while five used pencil and paper"

Relating Mathematical and Pedagogical Features of the Portfolio Entries

We examined the extent to which teachers in our sample used the pedagogical strategies in association with high-demand and low-demand tasks. For the 32 AU and 32 DU portfolio entries, we created 2-by-2 contingency tables, crossing cognitive demand (high or low) with pedagogical feature (present or absent). For each pedagogical feature, each contingency table displayed the number of teachers in our sample who submitted entries that were coded with the corresponding pair of characteristics. For the DU entries, we collapsed the cognitive demand coding for the two submitted activities, and we considered an entry to be high-demand if it contained at least one task that was coded as high-demand. We analyzed the data in these tables using chi-squared tests.

Relating NBPTS Certification Status to Mathematical and Pedagogical Features

To ascertain the interaction between the NBPTS view of high quality teaching and each of the other two views considered in this chapter—the cognitive demand view and innovative practice view—we examined the extent to which teachers in our sample who were awarded (or not awarded) NBPTS certification included (or did not include) high-demand tasks and reflected the presence (or absence) of each pedagogical feature.

As with the other similar analyses, we created 2-by-2 contingency tables, crossing NBPTS certification status (awarded or not awarded) with cognitive demand (high or low) and also with pedagogical feature (present or absent). Each contingency table displayed the number of teachers in our sample who submitted entries that were coded with the corresponding pair of characteristics. We analyzed the data in these tables using Chi-square tests.

Findings

Without knowledge of the NBPTS certification status of applicants, trained coders examined portfolio entries with respect to cognitive demand of the mathematics tasks and presence of innovative pedagogical features. After the portfolio entries were completely coded, we used these judgments to contrast the portfolio entries submitted by the 13 applicants who were awarded NBPTS certification with those submitted by the 19 applicants who were not awarded certification. We report our findings in this section: first with respect to the cognitive demand of the mathematical tasks, next with respect to innovative pedagogical features, and finally with respect to the interaction between cognitive demand and innovative pedagogy in the two sets of portfolio entries.

NBPTS Status and Cognitive Demand

Overall, 17 teachers (slightly more than half of the sample) submitted at least one high-demand task – 6 teachers submitted exactly one such task, 8 submitted exactly two such tasks, and 3 teachers submitted all three tasks that were judged to be cognitively demanding. Thus, 15 teachers submitted only low-demand tasks.

Figure 3 shows the percent of NBPTS certified (and non-certified) teachers who submitted (or did not submit) at least one high-demand activity. These data suggest a strong association between NBPTS certification and the submission of cognitively demanding tasks. In particular, four of every five teachers who submitted exclusively low-demand tasks in these two portfolio entries were not awarded NBPTS certification. Similarly, only one in four teachers who obtained NBPTS certification submitted exclusively low-demand tasks in the two portfolio entries we examined; that is, three-fourths of the teachers who obtained NBPTS certification submitted at least one high-demand task. A chi-square analysis indicated a statistically significant association (χ^2 (32, 1) = 4.98; $p < 0.05$) between a teacher's NBPTS certification status and the inclusion of at least one cognitively demanding task in his or her portfolio.

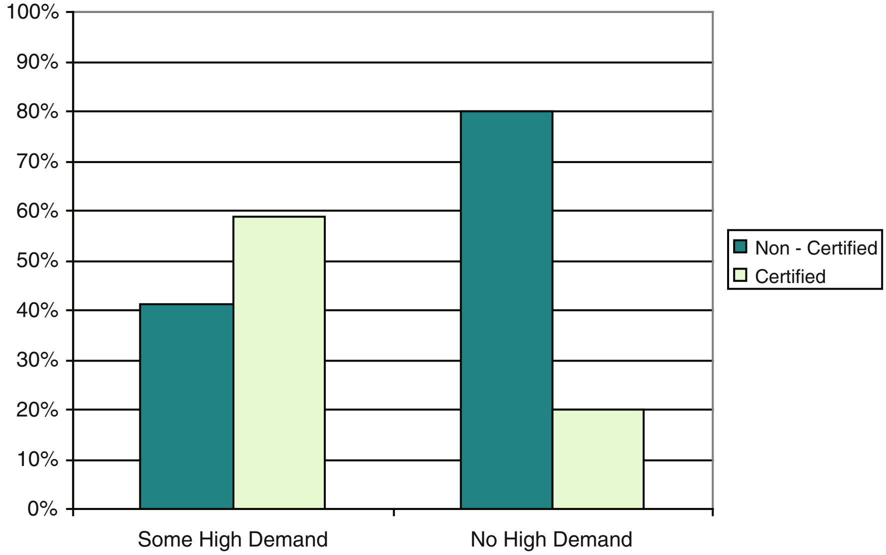

Fig. 3 Percent of teachers submitting (or not) high demand tasks by NBPTS certification status

NBPTS Status and Innovative Pedagogy

Across the portfolio entries we observed much more frequent use of innovative pedagogical approaches than we found cognitively demanding tasks. Overall, the percent of teachers submitting at least one portfolio entry exhibiting each of the innovative features ranged from 100% for the use of contexts outside mathematics to about 60% for the use of technology, with 84% using hands-on activity and 66% including a task that called for collaborative activity. Table 4 shows the distribution of NBPTS certified (and non-certified) teachers who submitted (or did not submit) at least one activity that contained each of the pedagogical features we considered in the portfolio entries we examined.

Because innovative pedagogy was so prevalent in the portfolio entries, these data suggest no more than a weak association between NBPTS certification status and use of the pedagogical features we examined. In fact, certified and non-certified teachers used three of the four pedagogical practices—the use of hands-on activities, contexts outside mathematics, and collaboration—in roughly the same proportion. Only in the case of technology usage was there some difference, with teachers who were awarded NBPTS certification employing this pedagogical feature more frequently. About three of every four teachers who obtained NBPTS certification employed technology in at least one of the two portfolio entries; non-certified teachers were about as likely to submit as to not submit an entry that used technology. Nevertheless, even in the case of technology use, the chi-squared analyses we

Table 4 Number of NBPTS certified and Non-certified teachers giving evidence of using pedagogical features in at least one portfolio entry

	NBPTS certification status			
	Awarded (n=13)		Not awarded (n=19)	
	Feature present	Feature not present	Feature present	Feature not present
Use contexts outside mathematics	13	0	19	0
Use hands-on activities	11	2	16	3
Use multi-person collaboration	9	4	12	7
Use technology	10	3	9	10

performed did not indicate that any of these relationships or trends was statistically significant.

Cognitive Demand and Innovative Pedagogy

We also examined the interaction between cognitive demand and innovative pedagogy. This is an analysis of the extent to which teachers appeared to use innovative pedagogy in support of, or at in close association with, cognitively demanding mathematics tasks. Table 5 shows the frequency of each innovative pedagogical feature in the portfolio entries of teachers who submitted (or did not submit) at least one cognitively demanding task.

Table 5 Number of teachers submitting activities with pedagogical feature by level of cognitive demand of the portfolio entries

	High cognitive demand in the portfolio	
	Present	Not present
Pedagogical feature		
Use contexts outside mathematics	17	15
Use hands-on activities	16	11
Use multi-person collaboration	12	9
Use technology	10	9

From the data displayed in Table 5, we can see that there appears to be no overall relationship between cognitive demand and pedagogical innovation. That is, pedagogical features were detected about as frequently in portfolios in which high-demand activities were included and in portfolios in which no high-demand activities were present. Although the teachers in our sample used innovative pedagogy, these results suggest that they were not using these teaching practices in any systematic way to support students' engagement with cognitively demanding mathematics tasks.

NBPTS Status, Cognitive Demand and Innovative Pedagogy

Although the overall data do not indicate a relationship, the picture might change if we also included NBPTS certification status in the analysis. Table 6 shows the frequency of each innovative pedagogical feature in the portfolio entries of NBPTS certified (or non-certified) teachers who submitted (or did not submit) at least one cognitively demanding task.

Table 6 Number of teachers by NBPTS certification status submitting activities with pedagogical feature by level of cognitive demand of the portfolio entries

	Certified teachers (n = 13)		Non-certified teachers (n = 19)	
	High cognitive demand		High cognitive demand	
	Present	Not present	Present	Not present
Pedagogical feature				
Use outside mathematical contexts	10	3	7	12
Use hands-on activities	9	2	7	9
Use multi-person collaboration	6	3	6	6
Use technology	7	3	3	6

Similar to Table 5, the display of data in Table 6 suggests that there is no clear relationship between cognitive demand and pedagogical innovation when NBPTS certification status is considered. For example, the three NBPTS certified teachers whose portfolio entries did not contain any cognitively demanding tasks submitted activities that used all of pedagogical features (with the exception of one teacher who omitted hands-on activities). The pattern of usage was less uniform for the NBPTS certified teachers whose portfolios contained cognitively demanding tasks, and also for the teachers who were not awarded NBPTS certification, but no clear pattern emerges from the data. Also, because the numbers are so small in the subgroups when all three dimensions are considered simultaneously, we were unable to detect any statistically significant trend for any individual pedagogical feature in relation to cognitive demand and NBPTS certification status simultaneously. We also used cluster analysis considering the total number of pedagogical features present in a portfolio entry in relation to the presence/absence of cognitive demand and the NBPTS certification status of the teacher who submitted the entry. This analysis did not detect statistically significant differences, but it did suggest that the teachers awarded NBPTS certification tended to be more consistent (i.e., had less variance) than their counterparts who did not receive NBPTS certification in the use of pedagogical features in association with cognitively demanding tasks.

Discussion

Our goal in this study was to probe empirically the extent to which samples of teaching practice associated with a view of highly accomplished mathematics teaching as defined by the National Board for Professional Teaching Standards also exhibited characteristic features associated with two alternative views of high quality mathematics teaching: (a) the effective use of cognitively demanding mathematics tasks and (b) the use of progressive pedagogical practices. Toward this end, we examined samples of classroom instruction—lesson artifacts and teachers' commentaries on lessons—submitted by 32 applicants seeking NBPTS certification. The instructional samples were systematically coded with respect to evidence of cognitively demanding mathematical tasks and innovative pedagogy. Finally, we examined the coded data to detect interactions between and among the different views of high quality mathematics teaching.

Our analyses detected a fairly strong interaction between the NBPTS view of accomplished teaching and the view of effective mathematics instruction associated with cognitively demanding tasks. In particular, we found that the teachers who were awarded NBPTS certification were far more likely than their colleagues who were not awarded certification to include high-demand mathematics tasks in the portfolio submissions we examined. Although these two views appear to be related in samples of actual instructional practice we examined in this study, they are clearly not identical. Recall that the decision to award certification is made on the basis of a composite judgment involving ten independent performance indicators and that the judgment of these performances did not explicitly attend to the issue of cognitively demanding mathematics tasks. In fact, when we examined the scores assigned by NBPTS raters to the portfolio entries in our study in relation to our coding of those same entries, we found that the two rating approaches were judging different aspects of the submissions. For example, 17 DU portfolio entries contained two low-demand activities, yet 65% of these entries received "accomplished" scores (a score 3 or greater) from the NBPTS assessors. Thus, the presence of low-demand tasks did not reliably predict a low assessor score on a particular entry, even though they appear to be related more generally to a low total score for the entire NBPTS process. Thus, our findings suggest that these two views of high quality mathematics teaching are related in the practice of teaching, but the relationship is complex.

The picture that emerges from our data analyses regarding innovative pedagogy suggests a different story. The innovative pedagogical features we examined—applications in contexts other than mathematics, multi-person collaboration, technology, or physical (hands-on) materials—were heavily used by the teachers in our sample, regardless of either their NBPTS certification status or their use of cognitively demanding tasks. Although we found that teachers used innovative pedagogical strategies in their classrooms, they did not do so in a way that was closely linked to supporting students' encounters with challenging tasks. Even in our highly select sample of teachers who applied for NBPTS certification—thereby indicating that they thought of themselves as potentially highly accomplished teachers—we found little evidence that innovative pedagogy was used to support students' engagement with cognitively demanding tasks. Such findings are consistent with some other research studies (e.g., Cohen, 1990; Ferrini-Mundy & Schram, 1997), and many anecdotes, suggesting that teachers may implement reform pedagogy in a superficial manner that does not realize its potential.

These findings appear to suggest that there is essentially no connection between pedagogical innovation, as defined here, and either the NBPTS view of highly accomplished mathematics teaching or the use of cognitively demanding mathematics tasks in instruction. Yet, we did find an interesting interaction. The teachers in our sample who not only were awarded NBPTS certification but also submitted at least one cognitively demanding mathematics task appeared to be more consistent than were other teachers in our sample in the use of innovative pedagogy. Though we did not find statistically significant differences, the suggestion of a difference regarding consistency of usage is worth pursuing in follow-up studies with larger samples.

Our investigation of the portfolio entries was not intended to be a validation study of the NBPTS certification process, and a replication involving a larger sample would be needed to make strong claims. Nevertheless, some of our findings do offer some validation of that process. In particular, the lack of correspondence between the awarding of NBPTS certification and the use of pedagogical features can be taken as evidence that the portfolio evaluation process is not heavily influenced by possibly superficial implementation of pedagogical innovation. And the positive association of low-demand mathematics tasks with non-certified teachers and high-demand mathematics tasks with certified teachers suggests that there is some reason to think that the instructional practice of those teachers awarded NBPTS certification is in fact "highly accomplished" in one mathematically important way that is not an explicit part of the NBPTS certification process. Moreover, the finding that at least some of the innovative pedagogy was used in connection with high-demand tasks by NBPTS certified teachers and not by those who were not awarded certification provides yet another indicator that the NBPTS certification process is reasonably well aligned with some other views of high quality mathematics teaching.

Given research evidence indicating both that teachers in the middle grades find it difficult to enact cognitively demanding tasks in mathematics instruction (Stein et al., 1996) and that the consistent, effective use of cognitively demanding tasks in the mathematics classroom increases student achievement (Stein et al., 1996), our findings suggest that there may be something to learn from NBPTS certified teachers about how to utilize such tasks effectively in the mathematics classroom. According to our analysis of the data examined in this study—teacher-selected samples of practice chosen by individuals seeking special recognition—the teachers who were awarded NBPTS certification appeared to deploy cognitively demanding tasks more proficiently than did their counterparts who were not awarded NBPTS certification. One caveat worth noting, however, is that we used a generous criterion when coding for cognitive demand—if some part of an activity exhibited high demand characteristics, it was classified as highly demanding, even if other parts of the activity did not. If we had applied a more stringent criterion—such as requiring that more than one half of an activity was judged to be cognitively demanding—the number of portfolio entries containing high-demand tasks would have been considerably smaller. Nevertheless, even if we applied a more stringent criterion, some of the activities submitted by the teachers awarded NBPTS certification were quite demanding and would likely have been so judged. Thus, it is left to future research to determine how robust the relationship detected in this study would be if more samples of instructional practice were examined and if different criteria were applied. But our findings clearly suggest a strong interaction between these two different views of highly accomplished mathematics teaching.

In the interest of supporting other research inquiry, we wish to underscore two special aspects of the data analyzed in this study that we think merit attention from researchers seeking to understand high quality mathematics teaching. First, the lesson materials and artifacts analyzed in this study were selected by teachers and submitted for evaluation in a process intended to identify highly accomplished teaching. Thus, it is reasonable to assume that the samples represented lessons that the teachers considered to be their best practice. In large-scale observational studies of teaching and in surveys, it is common to request samples of or information about typical teaching practice. Some scholars (e.g., Silver, 2003) have suggested the potential value of also examining instruction that is atypical in some way to detect, for example, what teachers might be capable of doing or inclined to do when they try to exhibit their very best work. The NBPTS portfolio entries offer one example of what such atypical data might look like, and our analysis of these data offers one example of what might be learned.

Second, the data examined were of a hybrid form that combines some features of the data collected via direct observation and data collected via survey responses. Like direct observation, the portfolio entries displayed important details of classroom lessons; similar to survey data, the portfolio entries permitted access to the teacher's perspective. Although the NBPTS portfolio data might appear to overly limited as a source of information about teaching practice because the records do not include direct observation of actual teaching, the data in the NBPTS portfolio submissions are in many ways quite similar to those that have been used and validated by other researchers to study classroom practice using alternatives to direct observation and survey methods, such as "scoop" sampling of instructional artifacts (e.g., lesson plans, student work) to characterize instructional activity (Borko, Stecher, & Kuffner, 2007) and using classroom assignments to judge instructional quality (Clare & Aschbacher, 2001; Matsumura, Garnier, Pascal, & Valdés, 2002). Researchers interested in alternatives to direct observation methods (which are invasive, labor intensive, expensive, and impractical on a large scale) and survey methods (which involve questions susceptible to multiple interpretations, have questionable validity, and provide little information about the details of instructional lessons) might be wise to consider data like those collected in the NBPTS portfolio process to open another window on classroom instructional practice.

At the outset we noted that different views of high quality mathematics teaching are typically treated in isolation from each other, emphasizing the distinctions between and among them rather than the ways in which they might interact with or complement each other. In this chapter we examined the interactions among three ways of characterizing high quality mathematics teaching, and we identified some patterns observed in the interactions detected in samples drawn form actual classroom instruction. We hope that our report will stimulate further research that probes characterizations of high quality mathematics teaching to generate additional insights.

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Expertise in Swiss Mathematics Instruction

Christine Pauli and Kurt Reusser

Abstract This chapter draws on data and findings from several video studies to describe the quality of mathematics teaching in Switzerland. The focus is on features of instructional practice and quality as core components of classroom behavior that reflect the teacher's expertise in creating optimal learning opportunities. The didactic triangle is used as the basis for describing the profile of expertise in Swiss mathematics instruction in terms of three interdependent dimensions of instructional quality. A core element of this profile can be identified in Swiss mathematics teachers' particular strengths in the culture of communication, support and relationships. Findings also paint a generally positive picture of the culture of teaching, learning and understanding (e.g., methods and choreography of teaching) in Swiss mathematics classrooms. However, the culture of objectives, materials and tasks proves to be rather average in international comparison in several respects (level of mathematical content, characteristics of the problems set and the way they are worked on in lessons). In particular, there seems to be room for improvement in the specific context of the didactics of mathematics (e.g., the level of cognitive and mathematical challenge).

Keywords Quality of instruction · Mathematics education · Switzerland · Video studies · Instructional reform

This chapter on expertise in Swiss mathematics instruction was prompted by the findings of the Trends in International Mathematics and Science Study (TIMSS) and the OECD's Programme for International Student Assessment (PISA), which paint a thoroughly positive picture of mathematics instruction in Switzerland (Moser & Notter, 2000; Moser, Ramseier, Keller, & Huber, 1997; Zahner Rossier et al., 2004; Zahner Rossier & Holzer, 2007). Swiss students at both lower and upper secondary level have performed very well in international assessments of mathematics achievement to date. Moreover, Swiss mathematics teachers seem able to achieve

C. Pauli (✉)
Institute of Education, University of Zurich, Zurich, Switzerland
e-mail: cpauli@ife.uzh.ch

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good educational outcomes in a relatively positive and anxiety-free learning environment: Swiss students' self-confidence and interest scores have been found to be average or slightly above average; their anxiety scores, well below the OECD average (Zahner Rossier, 2005). Although there are doubtless numerous reasons for these encouraging findings, it seems reasonable to infer that mathematics instruction and teacher expertise play at least some role. This raises the question of what it is that characterizes the expertise of Swiss mathematics teachers.

The main focus of the current literature on teacher expertise is often on measuring and describing teachers' professional competence in terms of components of teacher knowledge and skills (Baumert & Kunter, 2006; Besser & Krauss, 2009; Blömeke, Kaiser, & Lehmann, 2008; Kunter, Klusmann, & Baumert, 2009). Teacher competence is seen as the first link in a chain of cause and effect running from teaching practice or quality of instruction via student learning to academic achievement and other cognitive and noncognitive outcomes (Pauli & Reusser, 2009; Reinisch, 2009). In this chapter, in contrast, we focus on aspects of instructional quality. This approach is based on the idea that teacher expertise is manifested in the quality of classroom teaching practice. According to current theoretical models of learning and instruction, teaching offers a range of learning opportunities for student learning in the classroom (Fend, 1998; Helmke, 2009; Reusser & Pauli, 1999). Whereas the provision of learning opportunities reflects the professional competence of the teacher (knowledge, skills, beliefs, motivation), students' actual learning outcomes also depend on the extent to which they recognize and are able to take advantage of these learning opportunities.

In this chapter, we therefore draw on expert ratings of videotaped lessons as well as on student ratings of aspects of instructional quality to describe instructional practice and quality in Swiss mathematics lessons. Furthermore, we make a theoretical distinction that has proved helpful in the assessment and description of instruction (Aebli, 1983; Messner & Reusser, 2006; Oser & Baeriswyl, 2001), distinguishing the surface level of lesson organization (e.g., methods, instructional scripts, lesson choreography) from the deeper level of quality of instruction (quality of teaching and learning processes, quality of teacher–student interaction). Drawing on video and questionnaire data obtained from both teachers and students in the context of several video studies, we describe Swiss mathematics instruction at both levels and thus develop a profile of the expertise of Swiss mathematics teachers (sections "Characteristics of Swiss Mathematics Instruction in International Comparison" and "Student and Expert Judgments of the Instructional Quality of Swiss Mathematics Teaching"). First, however, we describe the data sources available and provide some background information on the context in which the participating teachers work.

In section "The Role of Instructional Reform", we investigate the impact of innovations and reform initiatives on the thinking and practice of Swiss mathematics teachers. As research on teacher training has shown, teachers' beliefs about the processes of learning and instruction and about the role that students play in these processes are of fundamental importance in the implementation of reforms (Baumert & Kunter, 2006; Reusser, Pauli, & Elmer, 2011; Turner, Christensen,

& Meyer, 2009). We therefore take Hans Aebli's (1983) model of psychological didactics, which has been widely adopted in teacher training programs in the German-speaking part of Switzerland, as the starting point for our investigation of how a specific reform initiative – based on the ideas of progressive education (individualized learning, student orientation, and learner autonomy) – has influenced teaching practice. Finally, we summarize and systematize the findings presented in the chapter. By considering the strengths and weaknesses identified in terms of three dimensions of instructional quality, we develop a profile of teacher expertise from the didactic perspective.

Data Sources

Our main data source in this chapter is the *TIMSS 1999 Video Study* (Hiebert et al., 2003) and the *Swiss Video Study* that was embedded in it. Designed as a video survey (Stigler, 1998), the TIMSS 1999 Video Study aimed to document everyday classroom instruction in a variety of countries and, on this basis, to describe patterns of teaching practices within each country. There was a particular focus on comparing mathematics teaching in the United States and in those countries that showed comparatively high achievement on TIMSS assessments. To this end, representative samples of approximately 100 (Switzerland: 140) mathematics lessons each were videotaped in Australia, the Czech Republic, Hong Kong, the Netherlands, Switzerland, and the United States. Numerous features of both the structure of lessons (e.g., forms of interaction, activities) and the mathematical content covered (e.g., characteristics of the problems set and the way these problems were worked on in the lesson) were analyzed. Japanese mathematics lessons collected for the TIMSS 1995 Video Study were re-analyzed as part of the TIMSS 1999 Video Study (Hiebert et al., 2003).

In Switzerland, the TIMSS 1999 Video Study was extended within the context of the Swiss Video Study (Reusser, Pauli, & Waldis, 2010).[1] In addition to the video recordings obtained within the TIMSS 1999 Video Study, the Swiss database includes extensive survey data (teacher and student questionnaires), a cognitive abilities test, and mathematics assessments, embedded in a longitudinal design. In the Swiss Video Study, the video data were reanalyzed in a number of respects. In particular, the instructional quality of the lessons was assessed. The same quality evaluations were conducted in a subsample of the German mathematics lessons videotaped for the TIMSS 1995 Video Study (Clausen, Reusser, & Klieme, 2003), thus making it possible to compare the quality of German and Swiss mathematics lessons.

[1]This research was funded by the Swiss National Foundation (SNF grant 4033-054871), the Ecoscientia Foundation (Zurich, Switzerland), and CORECHED (Swiss Conference for the Coordination of Educational Research).

This chapter also draws on the findings of a video study investigating mathematics instruction in Germany and Switzerland (Klieme, Pauli, & Reusser, 2009) that was conducted in collaboration between a German research group (principal investigator: Eckhard Klieme) and our Swiss research group (principal investigators: Kurt Reusser, Christine Pauli). One of the features that distinguishes this German–Swiss video study, which was based on a sample of 20 classes in each country, from the Swiss Video Study described above is that more than one lesson delivered by each teacher was recorded (two lesson units; five lessons in total), and that the content taught was standardized. The study also included a teacher survey; approximately 150 mathematics teachers in each country reported on aspects such as their self-perceptions and experience of teaching.

The data provided by the TIMSS Video Study and the German–Swiss Video Study make it possible to determine where Swiss mathematics teachers stand in international comparison on various indicators of teaching expertise.

Characteristics of Swiss Mathematics Instruction in International Comparison

Before drawing on selected findings from the TIMSS 1999 Video Study and the Swiss Video Study to describe key characteristics of Swiss mathematics instruction, we outline some particularities of the Swiss teaching context.

One defining characteristic of the Swiss context is that Switzerland has three main language regions.[2] These regions differ not only in the language of instruction (German, French, Italian), but also in certain aspects of their education systems, including pre- and in-service teacher training. As a federation of 26 cantons, Switzerland does not have a centralized education system. Most cantons in the German- and French-speaking areas (but not in the Italian-speaking areas) implement a three-track secondary system based on academic ability. Until a few years ago, many cantons ran different training programs for candidate teachers aspiring to teach in the different tracks (i.e., school types). Working conditions also differ across the three tracks, as reflected in the data from the representative sample of the TIMSS 1999 Video Study. For example, under 5% of the teachers at the least academically demanding school type in the German-speaking part of Switzerland (most of whom were all-rounders responsible for teaching several subjects) had at least 2 years' university-level training in mathematics (compared with 67% of teachers in the intermediate school type and 100% of teachers in the most academically demanding school type). A similar picture emerges for the French-speaking part of the country. In contrast, the great majority (at least 85%) of the teachers in the Italian-speaking part of the country studied mathematics at university.

[2]The three language regions and all school types were adequately represented in the Swiss sample of the TIMSS 1999 Video Study ($N = 140$). It was not possible to include the country's fourth language (Romansh), which is spoken only in a small area.

What then are the surface-level characteristics of mathematics instruction in Switzerland? Analyses of data from the TIMSS 1999 Video Study make it possible to examine how Switzerland compares with other high-achieving countries and the United States in terms of lesson organization, instructional practices, and the mathematical content of lessons. With the exception of Japan, however, these analyses have revealed many similarities in mathematics teaching across all countries examined, including the United States. In other words, Swiss mathematics lessons did not deviate from the general international pattern of teaching practices that emerged from these analyses. For example, the findings of the TIMSS 1999 Video Study indicated that mathematics teaching in Switzerland is (also) dominated by problems with a low level of complexity that can be solved within a few minutes by repeating known procedures; most of these problems have no relevance to practical applications or to students' everyday lives. Challenging mathematical activities such as constructing mathematical proofs or exploring, presenting, and discussing multiple solution methods were something of a rarity in the Swiss sample.

Analyses examining not only the frequency or duration, but the sequencing or choreography of activities over the course of a lesson (Oser & Baeriswyl, 2001) revealed a somewhat higher level of variability in the Swiss sample relative to the other countries (Givvin, Hiebert, Jacobs, Hollingsworth, & Gallimore, 2005). In international comparison, Swiss mathematics instruction thus seems to be characterized less by the presence of certain lesson features or by a clearly identifiable pattern of teaching practices, but by greater variability across lessons. It seemed reasonable to hypothesize that this diversity might be attributable to systematic differences across the country's three main language regions. The empirical data did not support this hypothesis, however. With few exceptions, the international video coding system revealed no systematic differences across the language regions in the features of teaching investigated (Pauli & Reusser, 2010b).

Instead, the function of the lesson in the learning process (introductory vs. follow-up lesson) proved to be relevant. Surveys of the participating teachers (Pauli & Reusser, 2010a) and of experts in pre- and in-service teacher training revealed that – from both the teacher and the expert perspective – the arrangement of mathematics lessons depends on whether new material is introduced (introductory lesson) or known material is consolidated and practiced (follow-up lesson). Data obtained through the international teacher questionnaire made it possible to categorize the videotaped lessons as either introductory or practice/follow-up lessons. As expected, the two lesson types differed in some aspects of teaching; for example, there was more independent student work in practice and follow-up lessons than in introductory lessons. The distinction between introductory and follow-up lessons – which was, incidentally, also observed to a certain extent in sequences of mathematics lessons from the United States, Japan, and Germany, although in smaller samples (Clarke et al., 2007) – can be interpreted as indicating that Swiss teachers plan their lessons with a view to the different stages of the learning cycle, not all of which can generally be covered in a single lesson (Aebli, 1983, p. 276). We return to this "learning process orientation" as a defining characteristic of Swiss mathematics teaching below.

A certain diversity of instructional choreographies was also found within the two lesson types, as an analysis of lessons in the German-speaking part of the country showed (Hugener & Krammer, 2010). This finding is reflected in the data of the teacher survey included in the Swiss Video Study. Based on the findings of the TIMSS 1995 Video Study, Stigler and Hiebert (1999) developed the idea of culture-specific instructional scripts. The Swiss Video Study sought to assess these scripts through an open-ended question in the teacher questionnaire asking participants to describe the typical structure of an everyday mathematics lesson (or of an introductory and a follow-up lesson). Content analysis of these descriptions (Pauli & Reusser, 2010a) revealed that, for *introductory lessons*, most teachers in all three language regions described a pattern of instruction that corresponds to the typical structure of *fragend-entwickelnder Unterricht*, a kind of instructional dialogue between teacher and students. However, some 27% of descriptions in all three language regions revealed a second pattern, which can be labeled "exploratory/discursive". The descriptions of a typical *follow-up lesson* also revealed two main lesson types. Whereas most teachers described a mix of individual and teacher-directed collective work on problems and teacher-guided discussion of solutions, 12% of teachers described an alternative approach, namely individualized instruction with personal learning plans.

Overall then, the data indicate that there is no single teaching script dominating Swiss mathematics lessons. Rather, Swiss teachers seem to draw on different instructional scripts, as further survey data confirm (Pauli & Reusser, 2003; Stebler & Reusser, 2000). These data indicate that it is less the language region or the country in which a teacher works that determines the didactic approach taken, than the degree to which innovations and reform initiatives are implemented in lessons (see also Blömeke & Müller, 2008) in correspondence with the teacher's personal beliefs about teaching and learning (see section "The Role of Instructional Reform"). As a federal state, moreover, Switzerland does not have a national curriculum or centralized teaching strategies. Rather, teachers have considerable freedom in their choice of methods and approaches. Apart from the broad educational goals laid out in cantonal curricula and a certain amount of compulsory teaching material, it is left largely to individual teachers to decide whether and how to integrate new teaching methods and reforms into their classroom practice.

This considerable freedom of discretion was not only apparent in the teacher survey data and in the videotaped mathematics lessons, but also reflected in what Swiss experts in mathematics teaching and teacher training expected to observe in everyday mathematics classrooms in Switzerland. In group interviews conducted in the run-up to the TIMSS 1999 Video Study, experts were unable to compile a single "hypothesized country model" describing mathematics instruction in Switzerland (Hiebert et al., 2003, pp. 209–211). Instead, they expected most lower secondary mathematics teachers in the country to have a rather traditional, teacher-directed instructional style, but another pattern of lessons to be seen in reform-oriented classrooms. Both approaches were seen to have a place. One advantage of this tolerance of different methods is doubtless the pragmatic approach to instructional reform that can be observed throughout Switzerland, which prevents the premature, overly radical, or flawed implementation of proposed reform models (see section "The Role of Instructional Reform"). However, one potential disadvantage is the associated lack of commitment to the systematic development of instruction. For example, it is currently left largely to teachers to decide whether and to what extent to implement individualized forms of teaching in their lessons. Given that the heterogeneity of school classes is set to increase in the coming years, this model is bound to reach its limits. New forms of and commitments to training and instructional development will thus be required.

Student and Expert Judgments of the Instructional Quality of Swiss Mathematics Teaching

Instructional research has repeatedly shown that it is less the surface features of instruction that determine students' learning outcomes than the deeper level aspects of instructional quality (see, e.g., Brophy, 2006; Helmke, 2009; Helmke & Weinert, 1997; Klieme & Rakoczy, 2008; Seidel & Shavelson, 2007). The cognitive activation of students, a supportive learning environment or student-oriented teaching style, clarity and structure of presentation, and efficient classroom management have been identified as particularly important quality dimensions (Helmke, 2009; Klieme et al., 2009; Kunter et al., 2006; Lipowsky et al., 2009). In this section, we examine the extent to which these aspects of instructional quality are apparent in Swiss mathematics classrooms. To this end, we draw on the expert and student ratings obtained for a representative sample of 140 mathematics lessons in the context of the Swiss Video Study.

Using 4-point rating scales, the participating students assessed various aspects of instructional quality; their responses were collated to form six scales: clarity and structure, classroom management, individual support, cognitive activation, social climate and student autonomy. As the analyses show, lower secondary students' evaluations of their mathematics instruction were generally positive across the three language regions of Switzerland examined (Fig. 1). The only instructional feature to receive less favorable ratings was "scope for student autonomy" (Waldis, Grob, Pauli, & Reusser, 2010b).

The question arises whether these positive student ratings are attributable to the quality of instruction – that is, to the expertise of Swiss mathematics teachers – or whether they are more a reflection of Swiss students' fundamentally positive attitudes toward learning mathematics. Various findings indicate that most Swiss students have a positive approach to learning mathematics. In the Swiss Video Study, for example, open-ended items in the student questionnaire tapping quality of motivation in mathematics lessons painted a thoroughly positive picture: Both in grade 8 and at the second assessment in grade 9, Swiss students tended to show a self-determined motivational orientation. In particular, they emphasized the practical value of mathematics (Buff, Reusser, & Pauli, 2010). The student questionnaire further assessed mathematics interest on an 8-item scale and revealed positive ratings overall, although the gender differences known from the literature were apparent (with girls showing less interest), as were differences across school types depending on the language region (Waldis, Grob, Pauli, & Reusser, 2010a). In

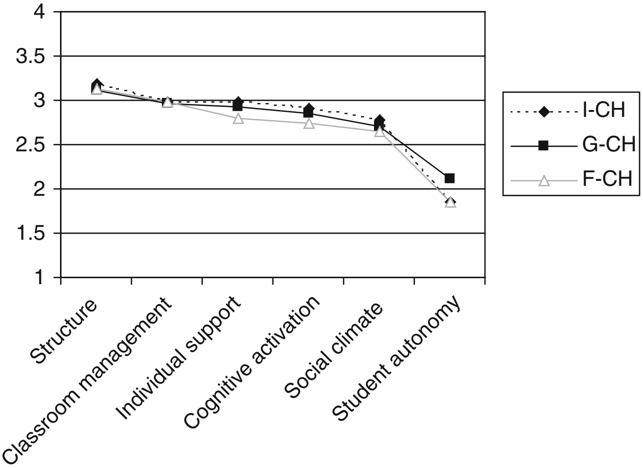

Fig. 1 Mean student ratings of instructional quality. I-CH: Italian-speaking Switzerland ($n = 27$), G-CH: German-speaking Switzerland ($n = 74$); F-CH: French-speaking Switzerland ($n = 39$). Data base: Representative sample of 140 Swiss lessons (and classes) from TIMSS 1999 Video Study (see also Waldis et al., 2010b, p. 189)

the German-speaking part of Switzerland, for example, students attending the least academically demanding school type reported higher interest than did their peers in the most academically demanding school type.

There is thus much evidence to suggest that Swiss students' positive perceptions of their instruction *also* reflect a generally positive attitude to learning mathematics. However, this does not exclude the possibility that "objectively" measurable aspects of the quality of instruction and of teacher expertise also contribute to the favorable student ratings. Rather, a reciprocal relationship can be assumed.

As Fig. 2 shows, *observer ratings* of instructional quality in the videotaped mathematics lessons were good to very good (clarity/structure) for all features assessed: clarity/structure, classroom management, cognitive activation, and student orientation (see Waldis et al., 2010b).

Overall, both students and observers evaluated the quality of Swiss mathematics instruction favorably. The different school types revealed specific learning cultures with respect to the "scope for student autonomy" and "cognitive activation," with the least academically demanding schools granting greater scope for student autonomy within the German- and Italian-speaking areas and the most academically demanding schools offering a higher level of cognitive activation within the German- and French-speaking areas (not shown in Figs. 1 or 2). In the Italian-speaking region, where students are not tracked to separate school types, but streamed within schools according to their ability in certain subjects, no such pattern emerged.[3]

[3]It would go beyond the scope of this chapter to report the results for different school types; for details, see Waldis et al. (2010b).

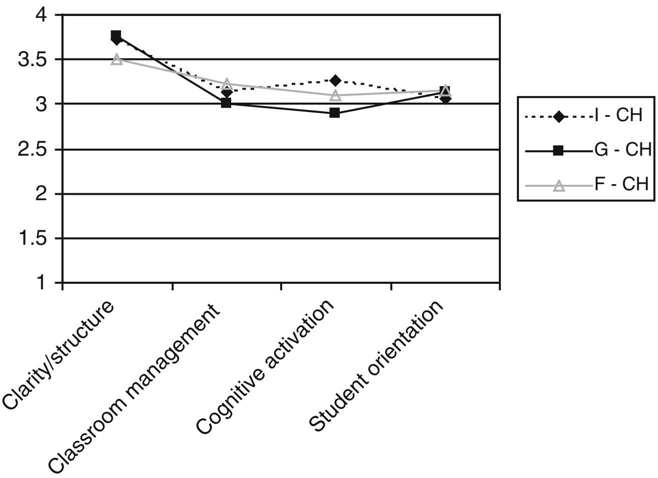

Fig. 2 Mean expert ratings of instructional quality. I-CH: Italian-speaking Switzerland ($n = 27$), G-CH: German-speaking Switzerland (n = 74); F-CH: French-speaking Switzerland ($n = 39$). Data base: Representative sample of 140 Swiss lessons from TIMSS 1999 Video Study (see also Waldis et al., 2010b, p. 196)

The profile of Swiss mathematics instruction can be further defined by comparing observer ratings of the quality of instruction in Swiss and German mathematics classrooms. Data from a subsample of 30 Swiss lessons (from the TIMSS 1999 Video Study) and 30 German mathematics lessons derived from the TIMSS 1995 Video Study (Clausen et al., 2003) showed that the Swiss lessons were rated higher than the German lessons in terms of classroom management, individualization, and student-oriented teaching, but ratings of cognitive activation and clarity/structure did not differ. This general pattern of results was echoed in the later binational video study of mathematics instruction in 20 German and 20 Swiss classes (Klieme et al., 2009). Here again, the Swiss lessons scored higher on some features indicative of motivationally supportive instruction, namely student perceptions of social relatedness and autonomy support and observer ratings of room for autonomy (Rakoczy, 2008).[4] In contrast, the German students perceived a higher level of competence support and, to some extent, the observers evaluated the cognitive demands of the German lessons to be higher (p. 187 ff.).

Interestingly, these findings are very much in line with an evaluation of instructional quality in three Swiss mathematics lessons selected by the project management of the TIMSS 1999 Video Study as "typical." In group interviews conducted in four countries (Australia, Czech Republic, Hong Kong, United States), international expert groups agreed that these three lessons showed a high level of teacher

[4]Note that the same instruments (student questionnaire, rating inventory) were not used in this study.

direction, a high level of student involvement, and a generally positive atmosphere. However, the expert groups did not agree on the quality of the mathematical content (Givvin, Jacobs, Hollingsworth, & Hiebert, 2009; Petko, Krammer, Pauli, & Reusser, 2010).

Overall, the findings summarized here suggest that Swiss mathematics teachers have particular strengths in the domain of *student-oriented teaching* and creating a *supportive learning atmosphere*, especially where granting students autonomy in the learning process, individualization, and good classroom relations are concerned. However, the *cognitive demands* of Swiss mathematics instruction and the level of *cognitive activation* achieved are no higher than average.

The Role of Instructional Reform

One important component of teachers' professional competence is the ability to develop their instructional practice and to implement reforms successfully. Beliefs about teaching and learning play an important role here (Philipp, 2007; Richardson & Placier, 2001; Turner et al., 2009). In this section, we investigate the impact of a specific reform initiative on mathematics instruction in the German-speaking part of Switzerland.[5] This initiative is characterized by a revised understanding of the role that students play in the learning process.

Teacher training in the German-speaking part of Switzerland has traditionally been strongly influenced by the model of "psychological didactics" proposed by Hans Aebli, a student of Jean Piaget (Aebli, 1951). Aebli's standard work on the "basic forms of teaching" (Aebli, 1961, 1983) has been a core component of many teacher training programs for decades. One key element of Aebli's approach is its focus on students' learning processes (Messner & Reusser, 2006, pp. 67–68). From Aebli's perspective, it is less the surface features of instruction that are decisive for lesson planning and hence the quality of instruction, than the deeper level structures – that is, the extent to which instruction succeeds in enabling the intended learning processes. The relatively large variation in teaching methods observed in Swiss mathematics classrooms, and the distinction of introductory and follow-up lessons (see section "Characteristics of Swiss Mathematics Instruction in International Comparison"), can be seen as evidence that the thinking and practice of many Swiss mathematics teachers is shaped by this approach.

Aebli's approach was founded on a constructivist understanding of student learning, based on Piaget's constructivist epistemology and theory of cognitive development. Like Piaget, Aebli maintained that learners actively construct and transform knowledge by integrating new information and experience into what they have previously come to understand, and by revising and reinterpreting old

[5] Because the reform situation differs across the three language regions in certain respects (e.g., differing models of reform, use of terminology, strategies of instructional development and in-service training, etc.), we restrict our analyses to data from the German-speaking part of the country.

knowledge in order to reconcile it with the new. But whereas Aebli agreed with Piaget on the active role of the learner in the learning processes, unlike Piaget he attributed a central role to the guidance and mediation of learning through interaction with the teacher. This is expressed in his model of "problem-based knowledge construction," a teacher-guided problem-solving approach with the goal of achieving deep and flexible understanding (see also Pauli, Reusser, & Grob, 2007).

However, models of student-oriented instruction rooted in progressive education in the German tradition (*Reformpädagogik*) also have a long tradition in Switzerland. These models emphasize student autonomy and co-determination of learning arrangements. In the 1990s, these ideas saw a significant renaissance in a teaching reform initiative that emerged essentially from classroom practice and became known as "Extended Forms of Teaching and Learning" (ETL). This reform model strives to extend the repertoire of teaching methods, focusing primarily on the organization of learning activities, and aiming to give students more opportunities for co-determination and individualized learning. Typical learning arrangements are individualized weekly learning plans, project teaching, and workstations (Croci, Imgrüth, Landwehr, & Spring, 1995; Pauli et al., 2007). Whereas these forms of teaching and learning primarily aim to provide *organizational* and *procedural* autonomy support (Stefanou, Perencevich, DiCintio, & Turner, 2004), the aspect of *cognitive* autonomy support has attracted increasing attention in recent years, especially in the context of mathematics teaching. Against this background, the ETL model also calls for more opportunities for students to engage in independent problem solving and higher order thinking (see also Affolter et al., 2006).

The question arises of how this reform model has been received and implemented by mathematics teachers in the German-speaking part of Switzerland. To assess how familiar these teachers are with didactic principles and reform initiatives, the teacher questionnaire administered to the teachers of the TIMSS 1999 Video Study sample in the context of the Swiss Video Study included a question asking how often they organized their lessons according to three didactic principles: (1) *operative didactics* ([guided]) problem-based knowledge construction; e.g., Aebli, 1983; Wittmann, 1981); (2) the *genetic–socratic exemplary approach* (e.g., Wagenschein, 2008) and (3) the model of *extended forms of teaching and learning* (ETL). As Fig. 3 shows, the ETL model proved to be well known and frequently implemented in the German-speaking mathematics classrooms of the TIMSS 1999 Video Study sample; only 2% of teachers stated that they were unfamiliar with the model. More than 41% stated that they "frequently" or "almost always," and 52% that they "occasionally," taught according to the principles of ETL. There was somewhat less awareness and implementation of operative didactics; nevertheless, 39% of teachers stated that they "frequently," and 22% that they "occasionally," taught according to this principle. Most teachers were also familiar with the genetic-Socratic approach, but its implementation in the classroom was much more limited.

Given that a large group (41%) of teachers stated that they frequently or almost always taught according to the *principles* of ETL, it was interesting to examine what distinguishes the instruction of these *reform-oriented* teachers from that of

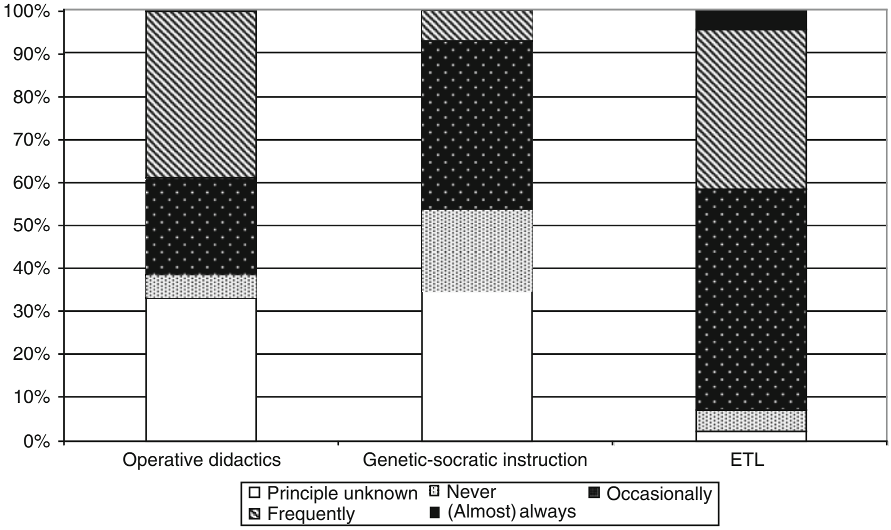

Fig. 3 Teacher awareness and implementation of didactic principles ($N = 66$; teachers of German-speaking areas of Switzerland, based on the TIMSS 1999 Video Study sample). Operative didactics: (guided) problem-based knowledge construction (Aebli, Wittmann); genetic–socratic exemplary approach (Wagenschein); ETL model: reform model of extended forms of learning and teaching

their more *traditionally oriented* colleagues. This question was addressed in a series of steps, drawing on data from both teacher self-reports and student and observer ratings.

Based on the teacher *self-reports*, the classroom practice of reform-oriented and traditionally oriented teachers was first compared in terms of aspects of instruction and social interaction. Specifically, teachers were given a list of 32 forms of teaching and learning and asked to state the frequency with which each featured in their own practice (Pauli & Reusser, 2010a; Pauli, Reusser, Waldis, & Grob, 2003). For those forms of teaching and learning characteristic of the ETL model (e.g., weekly learning plans, workstations, individual guidance), a significant difference in the expected direction was found between reform-oriented and traditionally oriented teachers, with higher levels of implementation in reform-oriented classrooms (Pauli et al., 2003). However, few differences emerged in the forms of teaching and learning typical of traditional instruction. Figure 4 compares selected forms of instruction. In addition to the form of the teacher-led instructional dialogue that is typical of traditional instructional practice, the figure presents findings for two forms of instruction representing the aspect of granting students scope for self-directed learning that is central to the ETL model (weekly learning plan, workstations), and two forms of instruction that are characteristic of reform efforts in the specific context of mathematics (discovering solution methods, discussing solution methods).

Although the distribution of the teaching practices "weekly learning plan" and "workstations" differed significantly in the expected direction, there were no

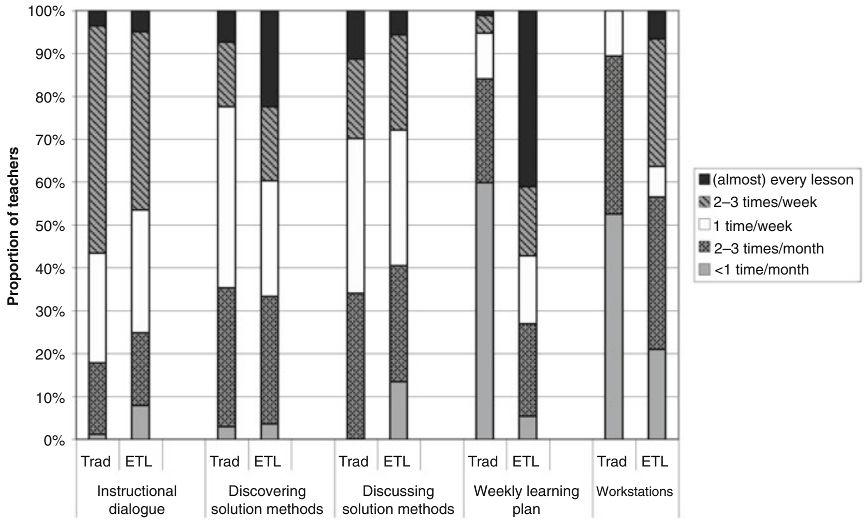

Fig. 4 Comparison of the frequency of implementation of instructional methods by traditionally oriented ("Trad.", $n = 38$) and reform-oriented ("ETL", $n = 28$) teachers in the German-speaking areas of Switzerland, based on the TIMSS 1999 Video Study sample. Significant differences emerged for weekly learning plan: χ^2 (4, $N = 66$) = 25.49 ($p = 0.000$) and workstations: χ^2 (4, $N= 66$) = 12.23 ($p = 0.016$). No significant differences were found for the other methods

statistically significant differences between the reform-oriented and the more traditional teachers in the other forms of teaching and learning. These findings indicate that, in the teachers' self-conceptions, the practice of ETL is characterized primarily by creating more opportunities for self-regulated learning, and that there is less of an emphasis on discursive approaches to mathematical problems. Indeed, further analyses showed that there was no systematic relationship between the teacher-reported frequency of these two reform-oriented teaching practices (Pauli et al., 2007) or between reform-oriented instructional practice in terms of the organization of learning activities (opportunities for self-directed learning) and constructivist-oriented beliefs about teaching and learning. Interestingly, the reform-oriented teachers did not differ significantly from their more traditional colleagues in the frequency of teacher-led instructional dialogue, which plays a major role in Aebli's approach. The proportion of reform-oriented teachers who facilitated an instructional dialogue less than once a week is also relatively small at 25% (traditional teachers: 18%; see Fig. 4). These findings indicate that the ETL reform model does not mean a radical *switch* to open forms of instruction, but rather a *broader spectrum of teaching practices at the organizational level*, creating more opportunities for self-directed and individualized learning. This does not necessarily include exploratory and discursive approaches to mathematical problems.

Another interesting question is whether reform-oriented teaching practices influence *student perceptions* and *expert ratings* of instructional quality. Analyses indicate that this is indeed the case. Students and experts rated reform-oriented teachers significantly higher on various dimensions of instructional quality (cognitive activation, student orientation, clarity and structure), and students reported higher levels of positive emotional experience (Pauli et al., 2007, 2003). However, these findings were not reflected in the development of students' interest and achievement over the course of a school year, where neither a positive nor a negative effect was observed (Pauli et al., 2007, 2003).

Based on the international video analyses and on further analyses conducted in Switzerland, it was also possible to examine how the quality differences detected were reflected in observable features of teaching practice. The international analyses revealed few, rather weak relations between teacher orientation and classroom practice. Less whole-class work was observed in the reform-oriented teachers' lessons, but there were no significant differences in the *culture of tasks* (i.e., the quality of mathematical content or the characteristics of the problems set and the way they were worked on in the lesson). In other words, the reform-oriented teachers' lessons were also dominated by repetitive, low-complexity tasks that could be solved by applying known procedures. More challenging mathematical activities were rarely observed (Pauli, Reusser, & Grob, 2010).

However, differences between the two teacher groups were found for *classroom interaction* and *learning support*, as a further analysis of the Swiss sample showed (Krammer, 2009). A detailed analysis of the activities and interactions occurring in phases of independent student work showed that reform-oriented teachers invested significantly more time in cognitively activating forms of individual learning support (i.e., feedback that encouraged students to continue thinking independently) and less time in evaluative feedback (feedback on the accuracy of task completion). In addition, their students had more opportunity to cooperate during phases of independent student work. In sum, Krammer's (2009) findings indicate that, relative to their more traditionally oriented colleagues, reform-oriented teachers dedicate more lesson time to independent student work (either individually or in pairs/groups), and that this time is used productively to guide and support individual learning processes.

In summary, the analyses presented indicate that instructional reform initiatives play a notable and generally positive role in lower secondary mathematics instruction in Switzerland. In particular, the ETL model, which is informed by the tradition of progressive education, is widely known and its impact on instructional practice and quality is perceptible to both students and observers. What characterizes this reform model is that (in contrast to some concepts of open education) it does not prescribe a radical transformation of traditional, teacher-directed instruction, but strives to *extend* the repertoire of teaching methods in terms of both the organization of learning activities (e.g., weekly learning plans) and ways of encouraging and supporting student learning processes. The present analyses of data from several video studies suggest that the ETL model is currently being implemented primarily at the *organizational* level, with students being given greater scope for autonomy through weekly learning plans and cognitively activating individualized learning support. There is room for improvement in lesson *content* in terms of the level of cognitive and mathematical challenge, the quality of the problems set, and the way they are worked on in lessons – that is, in the deep structure of teaching and learning processes.

Conclusion

This chapter drew on data and findings from several video studies to describe the quality of mathematics teaching in Switzerland. The chapter focused on features of instructional practice and quality as core components of classroom behavior that reflect the teacher's expertise in creating optimal learning opportunities. Despite this focus on instruction, it is important to remember that observable teacher behavior and instructional quality are influenced by multiple factors at different levels of the education system, as well as by the characteristics of those on the "uptake" end – that is, the students and their parents. These relationships, which are articulated in the model of the provision and uptake of learning opportunities (Fend, 2002, 2008; Helmke, 2009; Reusser & Pauli, 2003), need to be taken into consideration when interpreting the present findings on expertise in Swiss mathematics instruction.

In terms of the students and parents on the "uptake" end of learning opportunities, for example, Swiss mathematics teachers have to date benefited from relatively favorable conditions. A survey of German and Swiss mathematics teachers embedded in the German–Swiss video study showed that Swiss teachers seem to be aware of this fact (Lipowsky, Thussbas, Klieme, Reusser, & Pauli, 2003). Their ratings of student and parental interest were fairly high; their ratings of student and parent appreciation of their work, very high. Interestingly, the teacher ratings mirrored the difference in student interest ratings found across school types in the German-speaking part of Switzerland (see section "Student and Expert Judgments of the Instructional Quality of Swiss Mathematics Teaching"): Overall, the teachers rated students at the least academic school type to show much higher interest in mathematics than students in the most academically demanding school type. The opposite pattern of results was found in German teachers, whose overall ratings were also lower than those of their Swiss colleagues. A similar pattern emerged for teacher self-perceptions. For example, Swiss mathematics teachers had higher self-efficacy than their German colleagues, and German teachers reported more stress than their Swiss colleagues, with the least favorable constellation again being found in German teachers working in the least academic track of the [three-tiered] German secondary system, the Hauptschule. These findings indicate that teachers in Switzerland, even at the least academically demanding schools, feel able to practice their profession to the desired level. In the terms of the model of the provision and uptake of learning opportunities, this too can be seen as a reciprocal relationship.

In the following, we use the model of the didactic triangle as the basis for presenting our conclusions on expertise in Swiss mathematics instruction. In this model, the teacher, students, and content correspond to the points of the triangle describing the

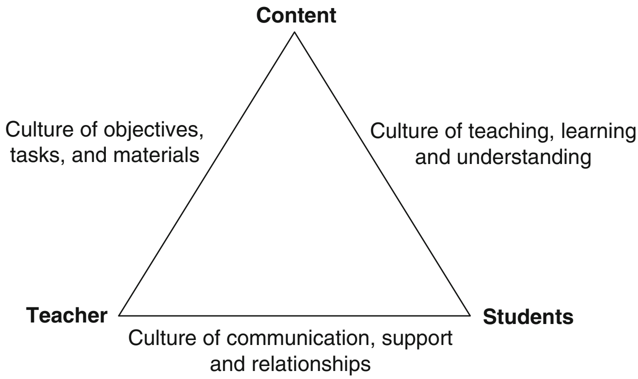

Fig. 5 Didactic triangle (Reusser, 2008, 2009)

teacher's scope of practice in the classroom (Reusser, 2008, 2009). The three sides of the triangle define three interdependent dimensions of instructional quality, namely the culture of objectives and materials; the culture of teaching, learning, and understanding; and the culture of communication, support, and relationships (see Fig. 5). Considering the empirical findings presented above in terms of these three quality dimensions can help to build up a detailed profile of expertise in Swiss mathematics instruction.

A core element of this profile can be identified in Swiss mathematics teachers' particular strengths in the *culture of communication, support, and relationships*, as reflected by the positive ratings of instructional quality given by both students and independent observers in the video studies and as confirmed by the comparison of observer ratings of German and Swiss mathematics lessons (Clausen et al., 2003). Data obtained through the PISA 2000 student questionnaire point in the same direction: Swiss students rated their teachers to be supportive and, in particular, the quality of teacher–student relations to be high, but perceived levels of pressure to achieve to be relatively low (Klieme & Rakoczy, 2003, p. 344). The findings that Swiss students reported anxiety levels below the OECD average in mathematics (see Introduction) and higher wellbeing in school than, for example, German students (Fend, 1998) can be attributed to this generally positive culture of communication, support, and relationships. However, these findings should not be interpreted as indicating that there is no scope for teacher improvement in this dimension of instructional quality. For example, Krammer's (2009) analyses identify a need for enhancement of adaptive individual learning support. Overall, a rather low proportion of the individual learning support provided was evaluated to be cognitively activating (i.e., to stimulate further thought). This proportion was significantly higher in the lessons of reform-oriented teachers than in those of their traditionally oriented colleagues, highlighting the potential of the ETL model as a platform for adaptive instruction.

The findings presented also paint a generally positive picture of the *culture of teaching, learning and understanding* – in other words, the actual process of learning and its choreography – in Swiss mathematics classrooms. The various video analyses give the impression of well-managed lessons with few disruptions, guided primarily by the teacher, but also involving a fairly high proportion of independent student work in international comparison. Although a form of instructional dialogue also plays a key role in Swiss mathematics instruction, the video analyses reveal a notable variety of teaching practices and methods. One reason for this is that Swiss teachers deliberately include phases of consolidation and practice in their lessons, as reflected in the distinction between introductory and follow-up lessons observed in both the instructional scripts described by teachers and the video recordings. Another reason is the relatively widespread implementation of the ETL reform model. This model is characterized by its combination of traditional, teacher-directed forms of instruction and highly individualized forms of instruction such as weekly learning plans, which has doubtless contributed to its high acceptance among teachers. The model seems practicable because it can be implemented to differing degrees and with differing focuses; it is flexible enough to be adapted to different contexts and conditions (e.g., different schools, classes, etc.). From the perspective of instructional research, it is also worth noting that the risk of negative effects on academic outcomes – as have been found for some radical concepts of "open education" (Giaconia & Hedges, 1982; Gruehn, 2000; Lüders & Rauin, 2004) – is limited: unlike radical reform concepts, the ETL model includes forms of direct instruction, which numerous empirical studies have shown to play an important role in student learning (see, e.e., Brophy, 1999; Helmke, 2009).

Given the ETL model's positive effects on student perceptions and experiences of instruction – and in view of findings from international empirical studies demonstrating positive effects of student-oriented instruction and good teacher–student relations (Cornelius-White, 2007) – one potential point of intervention for developing teacher expertise in the culture of teaching and learning would therefore be to encourage the more widespread implementation of the ETL reform model. The model can also be regarded as offering useful strategies for dealing with heterogeneous classes. In view of the planned or already realized move away from special classes and toward the inclusion of special needs students in mainstream education and continuing immigration (in Switzerland and elsewhere), this heterogeneity is bound to increase in the future. Whereas the literature criticizes the unreasonably high expectations and "euphoric hopes" associated with didactic concepts of within-class differentiation (Trautmann & Wischer, 2008), ETL seems to offer a practicable approach that stands the empirical test, having demonstrably positive effects on students' experience of instruction and wellbeing, without negative effects on educational outcomes. However, the data suggest that although teachers' implementation of the ETL model has to date (positively) influenced the *culture of communication and support* and extended their *repertoire of teaching methods*, there has been little change in *aspects of mathematics-specific instruction* in the narrow sense (e.g., the level of cognitive and mathematical challenge; provision for independent and discursive approaches to challenging problems). There thus seems to be room for improvement in the specific context of the didactics of mathematics.

Consequently, the *culture of objectives, materials and tasks* in Swiss mathematics classrooms is rather average in international comparison in several respects (level of mathematical content, characteristics of the problems set and the way they are worked on in lessons). Analyses of the Swiss Video Study data revealed considerable differences across school types in the German- and French-speaking parts of the country. Data from the Italian-speaking part of the country, where students are not tracked to separate secondary school types, but streamed within schools according to their ability in certain subjects, show that these differences are only partly the result of teachers adapting the level of challenge to better meet their students' cognitive needs: In the Italian-speaking region, observer ratings of cognitive activation did not differ significantly across lessons at the two achievement levels (Waldis et al., 2010b). It thus seems reasonable to surmise that the differences are also partly attributable to the teachers' training and responsibilities (see section "Characteristics of Swiss Mathematics Instruction in International Comparison"): In contrast to the Italian-speaking part of Switzerland, where the teachers at both levels receive the same training (Pauli & Reusser, 2010a), almost all of the lessons videotaped in the least academically demanding schools in the German-speaking part of the country were taught by "all-rounders" with no university-level training in mathematics, whose teaching commitments included various subjects beside mathematics. Given these teachers' rather modest mathematical knowledge base and the scarce time they have to prepare lessons, the implementation of didactically more demanding models, as is called for in the current literature, does not seem practicable. Since the data were collected, teacher training at tertiary level has been restructured to place a much stronger focus on content knowledge and pedagogical content knowledge, meaning more of a subject focus in teaching responsibilities. It remains to be seen how these changes will influence the *culture of materials and tasks* in Swiss mathematics classrooms.

In terms of instructional development, Swiss teachers evidently have a pragmatic approach to instructional reform initiatives. As the principles of Hans Aebli's psychological didactics can be assumed to have an important influence on teaching practice at least in the German-speaking region of Switzerland, this should not come as any surprise. In Aebli's approach, which is rooted in cognitive psychology (Baer, Fuchs, Füglister, Reusser, & Wyss, 2006), didactic decisions are based on not the surface-level characteristics of instruction, but on the deeper level of the quality of student learning processes (see also Oser & Baeriswyl, 2001). Teachers in the German-speaking part of Switzerland can thus be expected to have a *learning process orientation* that is manifested, for example, in a pragmatic approach to different forms of instruction and social interaction in the classroom. In contrast to what is sometimes suggested in the current discussion of "constructivist learning environments" (see Tobias & Duffy, 2009), Aebli did not consider teacher guidance and support of learning activities to be at odds with a constructivist understanding of learning. For Aebli, teacher-guided instructional dialogue was a key element, if not *the* key element, of cognitively guided instruction based on a constructivist understanding of teaching and learning. Extending the repertoire of teaching methods to include forms that grant students more self-direction and autonomy is, however, certainly compatible with Aebli's focus on student learning processes, although Aebli himself paid little attention to these aspects in his own work (Pauli, 2006).

One clear indication that teachers in German-speaking Switzerland have a learning process orientation is that the Swiss Video Study found no systematic relationship between reform-oriented instructional practice in terms of the organization of learning activities (opportunities for self-directed learning) and constructivist-oriented beliefs about teaching and learning. In contrast, a constructivist orientation correlated positively with the reported frequency of opportunities for independent problem solving (Pauli et al., 2007) – in other words, with an instructional feature that focuses more on intended student learning processes than on the surface structure of instruction. This learning process orientation offers a good basis for instructional development, both from the perspective of general didactics and in the context of mathematics-specific conceptualizations of challenging, cognitively activating, and adaptive instruction. It is important to capitalize on Swiss mathematics teachers' learning process orientation through innovative forms of in-service training and instructional development that are congruent with their subjective theories of teaching and learning.

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Responding to Students: Enabling a Significant Role for Students in the Class Discourse

Ruhama Even and Orly Gottlib

Abstract This is a case study of a highly regarded high-school mathematics teacher in Israel. It examines the kinds of responses to students' talk used repeatedly by the teacher, directing and shaping the classroom discourse, during different parts of the lesson. The main data source included 21 h of observations in two of this teacher's classrooms. Analysis of the video-taped lessons showed that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. This was mainly due to the teacher's responsiveness to students. The most common teacher response was elaborating. Accompanying talk occurred considerably less, and the teacher rarely expressed puzzlement or opposition when responding to students' talk. The chapter demonstrates how the teacher combined her attention to students' talk, with the goal of making progress on the main topic.

Keywords Teacher responsiveness · Classroom discourse · Instructional decisions · Expertise in math teaching · Elaborating talk · Accompanying talk

Introduction

Expertise in mathematics teaching is frequently associated in the literature with devoting considerable class time to solving problems, proposing and justifying alternative solutions, critically evaluating alternative courses of action, leading to different methods of solving problems, not necessarily anticipated by the teacher ahead of time (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2001; Even & Lappan, 1994; National Council of Teachers of Mathematics, 2000). Expertise in teaching mathematics is often linked to encouraging students to make conjectures, explain their reasoning, validate their assertions, discuss and question their own thinking and the thinking of others, and argue about what is mathematically true (Collins, Brown,

R. Even (✉)
Department of Science Teaching, Weizmann Institute of Science, Rehovot, Israel
e-mail: ruhama.even@weizmann.ac.il

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& Newman, 1990; Even & Tirosh, 2002; Wood, Williams, & McNeal, 2006). Hence, expertise in mathematics teaching implies, among other things, a significant and influential role for students in the class discourse.

To enable a significant and influential role for students in the class discourse the mathematics teacher needs to play the role of diagnostician ("Images of Expertise in Mathematics Teaching" in the chapter by Russ, Sherin, & Sherin, this book). Research and professional rhetoric suggest that awareness to, and understanding of, students' mathematics learning and thinking are central to good teaching (e.g., Barnett, 1991; Even, 1999; Even & Markovits, 1993; Fennema et al., 1996; Llinares & Krainer, 2006; National Council of Teachers of Mathematics, 1991; Scherer & Steinbring, 2006). Consequently, the development of such awareness and understanding has become part of the curriculum of teacher education for both prospective and practicing teachers in recent years (e.g., Even, 1999, 2005a; Markovits & Even, 1999; Fennema et al., 1996; Tirosh, 2000).

Yet, improving teachers' understanding of what their students say, write or do still leaves the problem of how teachers may use this understanding to make better instructional decisions. How they may encourage and enable a significant and influential role for students in the class mathematics discourse, while, as river guides ("Images of Expertise in Mathematics Teaching" in the chapter by Russ et al., this book), respond to the students, to the context, and to what occurs in the moment (Berliner, 1994). This is not an easy task, as research suggests (Chazzan & Ball, 1999; O'Connor, 2001; Simon, 1997; Wood, 1994). For example, Even (2005b) illustrates the difficulties teachers encounter when facing the need to address students' mistakes, even after the teachers developed rich and profound understandings of the nature and sources of these mistakes. Ball (1993) describes the challenge of responding to students who present novel ideas that are not in line with standard mathematics, even in the case of an expert teacher with deep disciplinary understandings. Research suggests that expert teachers are better than novice teachers at productively altering the direction of their lesson in response to students' questions or comments (Brown & Borko, 1992). Yet, as Ball's study shows, responding to students' talk and action is problematic even for expert teachers.

A review of the literature provides limited information on the ways teachers attend and respond to students during mathematics lessons. Most studies have been conducted as part of intervention programs, involving a small number of lessons. Moreover, information on the ways teachers respond to students' talk and action during mathematics lessons is often derived from studies that do not specifically focus on that, but rather on class discourse (Even & Schwarz, 2003; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Resnick, Salmon, Zeitz, Wathen, & Holowchak, 1993; Sherin, 2002), patterns of interaction (Bauersfeld, 1988; Wood, 1994; Voigt, 1995; Lobato, Clarke, & Ellis, 2005), and teaching strategies (Fraivilling, Murphy, & Fuson, 1999). Missing are studies that focus purposely on teachers' responses to students' talk and action during a relatively long period of regular mathematics lessons. Our study focuses on this.

The chapter examines how a teacher who has a reputation of encouraging a significant and influential role for students in the class discourse, responds to students' mathematical talk in class. The chapter examines the kinds of responses used repeatedly by the teacher, directing and shaping the classroom discourse, during different parts of the lesson.

Methodology

This is a case study of an experienced high-school mathematics teacher, highly regarded by her colleagues and other members of the mathematics education community in Israel. In addition to teaching high school mathematics, she has been a central member of several curriculum development teams, was a member of the national syllabus committee for junior-high mathematics, and has served as educator for prospective and practicing mathematics teachers. In her various roles she regularly sought for innovations in content and ways of teaching, and systematically reflected on her own teaching and the learning processes of her students. In numerous formal and informal conversations she often expressed the importance she attributed to being attentive and responsive to students and to encouraging students to take a significant role in the lesson.

The main data sources include observations of the teaching of mathematics in two of this teacher's classes. One of the classes the teacher taught was a 9th grade class and the other a 10th grade class; both in the high-school where she regularly taught mathematics – an academic oriented Jewish religious girl school. The 9th grade class was composed of lower-achieving students whereas the 10th grade class was composed of high-achieving students.

The second author observed 9 lessons in the 9th grade class and 8 lessons in the 10th grade class (the length of each lesson ranged between 36 and 88 min). Total time of observation was 21 h: about 10.5 h in each class. About one-half of the observed lessons in the 9th grade class were on functions; the rest were on geometry. Similarly, about one half of the observed lessons in the 10th grade class were on analysis; and the rest were on geometry. This research design enabled us to examine the nature of the teacher's ways of attending to students' talk and action during a rather long period of regular mathematics lessons, in a variety of settings: different classes, and when teaching different mathematical topics.

All 17 observed lessons were videotaped; notes were taken during and after each observation, and informal conversations were often held with the teacher. At the end of the data collection period, an 80-min long semi-structured interview was held with the teacher. The interview focused on her way of teaching, students' participation in the lessons, her response to students' talk, and differences in response in different settings. Later on, two additional semi-structured interviews were held with the teacher, focusing on her way of teaching, on the structure of a typical lesson of hers, and on the teaching sequence in each observed lesson.

Of the 17 observed lessons, 16 lessons consisted of whole-class work, small group/individual work, and class organization; one lesson consisted of small group/individual work and class organization only. Detailed data analysis of the lessons included only the talk during whole-class work, which comprised more than one-half of all lesson time – close to 12 h. The interviews and observations of the small group/individual work were used to support or downplay interpretations and to provide additional information about the teacher's responsiveness to students.

Following Even and Schwarz (2003), analysis of teacher responsiveness included an examination of the occurrence of four kinds of teacher responsiveness: *Accompanying* talk refers to talk in which the teacher attended to a student's talk without elaboration, typically acknowledging that she follows the student's talk. *Elaborating* talk refers to talk in which the teacher elaborated utterances and expressed deeper cognitive involvement. *Opposition* refers to talk in which the teacher explicitly expressed disagreement and objection. *Puzzlement* points to talk expressing confusion, perplexity or bewilderment.

Teacher responsiveness to students may be related to the purpose of the lesson segment. Thus, data analysis focused also on identifying the purposes of different components of the teaching sequence in each lesson. The coding we used for this is based in part on the coding system developed in the TIMSS-Video Study (Hiebert et al., 2003), but was modified to fit with the teacher's view, as indicated in interviews and conversations with her, and with the observational data. Thus, we combined two categories from the TIMSS-Video Study's coding system (Hiebert et al., 2003) – "Introducing new content" and "Practicing new content" – into one category, "Work on the main topic", because this category fits better with the class practice and with the teacher's description of the structure of her lessons. We also added to the TIMSS-Video Study's coding system the category "Extending beyond the main topic" because the teacher explicitly stated in a conversation that she often does that intentionally. The resulting coding system for this study includes four main categories. The first three categories center on mathematical work; the last one on class organization:

- *Work on the main topic*: focuses on introducing, investigating, extending, and deepening the main topic of the lesson.
- *Reviewing content introduced previously*: focuses on reminding students of, and clarifying, content learned earlier in the lesson, in previous lessons, or in lower grades.
- *Extending beyond the main topic*: focuses on extending and enriching students' knowledge and understanding of mathematics.
- *Class organization*: focuses on mathematical organization (e.g., distributing materials or homework assignments) or on non-mathematical work (e.g., disciplining students).

Finally, we examined what kinds of responses characterized each of the first three lesson components.

Responsiveness to Students in the Lessons

Analysis of the data suggested that teacher responsiveness to students characterized the mathematical work during whole-class work sessions. Almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. The teacher was attentive and responsive to different kinds of students' talk, including students' questions, answers, hypotheses, claims, remarks, mistakes, etc. The nature of the mathematical activity triggered by, built or followed on, students' utterances varied, and included, for example, discussing students' answers, investigating students' hypotheses, clarifying concepts critical for work on assigned tasks, strengthening previously learnt materials, answering students' queries, explaining the nature of mathematics and the work of mathematicians, etc.

Overall, two kinds of teacher response – elaborating and accompanying – were used repeatedly by the teacher, whereas opposition and puzzlement seldom occurred. The most common response was elaborating; accompanying occurred less frequently. Nonetheless, both elaborating and accompanying talk occurred during every lesson that included whole-class work.

Analysis of the data shows several similarities and some differences in the use of the four kinds of teacher responsiveness among the three lesson components. Below we describe and exemplify the kinds of responses practiced by the teacher during each lesson component: work on the main topic, reviewing content introduced previously, and extending beyond the main topic.

Work on the Main Topic

Work on the main topic comprised of introducing, investigating, extending, and deepening the main topic of the lesson. This kind of activity occurred during every lesson, and most of the total lesson time was devoted to it. Usually, the teacher initiated this kind of mathematical work. Typically, it involved collaborative whole-class work built on students' small group/individual problem solving.

All four kinds of responses were enacted by the teacher when working on the main topic. The most common response was elaborating; accompanying occurred less frequently. Teacher opposition and puzzlement occurred only a small number of times. Below are illustrations of the different kinds of teacher responsiveness to students when working on the main topic.

Opposition During Work on the Main Topic

The teacher seldom expressed disagreement or objection to students' ideas when working on the main topic. When she did, it was when students' suggestions severely deviated from the main point. One of these rare events occurred when she introduced the topic of similarity of polygons. The teacher started the lesson by asking the 10th grade students to explain the meaning of similarity in everyday life. The first students' suggestions were all closely tied to the mathematical notion of similarity: "Same angles but not the same sides" or "The ratios between the sides are equal". The teacher repeatedly rejected these suggestions, emphasizing that she was looking for something not in the mathematical world: "You explain it from a mathematical point of view. I'd like a description from everyday life."

Elaborating Talk During Work on the Main Topic

A common teacher behavior was to elaborate students' talk and express deep cognitive involvement in students' suggestions. For example, as part of the work on the topic of similarity of polygons, following the previous exchange on what similarity might mean in everyday life, yet before being given the formal definition of similarity of polygons, the 10th grade class students were assigned the task to imagine that they were using a camera or a photocopy machine. They were asked then to draw polygons that would be similar to the ones in Fig. 1 (drawn on triangular lattice), and to find the angle measures of the original and the new polygons.

After small group/individual work, a whole class work began, focusing first on the angle measures of the given shapes. The angles of the triangle in Fig. 1a were easily found, based on the fact that it is an equilateral triangle. But the triangle in Fig. 1c was a challenge. One student suggested that the top angle is a right angle based on its appearance. As a result, a discussion on whether one can be sure of that arose, eventually rejecting this method. This discussion was characterized by the teacher elaborating students' ideas and expressing profound cognitive involvement in their suggestions:

S: There is an angle of 90°.
T: How do you know? You see. Is it allowed?
S: I don't know.
S: Is seeing allowed?
T: Seeing is allowed but you cannot decide based on seeing.

Work on finding the angle measures continued, led by the teacher who kept using elaborating talk throughout this discussion. A student suggested that the left base-angle is 60°, based on what they found regarding the equilateral triangle in Fig. 1a and the problem of whether "seeing" is allowed in mathematics (i.e., is a valid

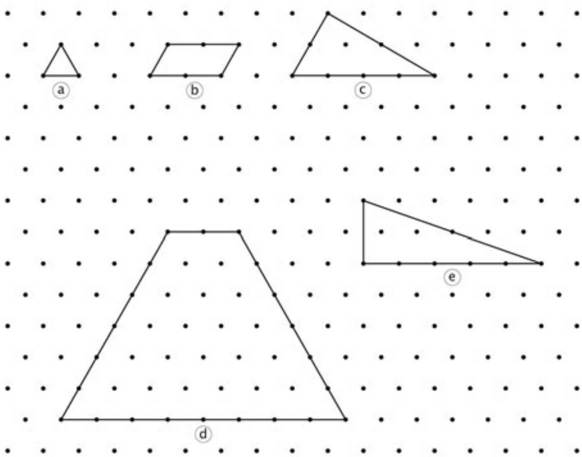

Fig. 1 Shapes used in the similarity activity

tool for determining mathematical truths) emerged again, distinguishing between the nature of “seeing” in each case:

S: There is also a 60° angle. It’s like the angle of triangle 1a.
T: Right. There is a 60° angle here. I agree. It’s like the angle of triangle 1a.
S: Can we do that? Do you allow us to do that?
T: What? What did we do?
S: According to the dots.
T: According to the dots we determined that
S: Ah, then everything will be much easier.
T: Sure… This is allowed, to “see” that there is an equilateral triangle here. What is the difference between “seeing” that this is a 90° angle and between “seeing” that there is an equilateral triangle?
S: Here it is 90° [incomprehensible] as if you see it. And here you can base it. You know that their distance is equal [distances between dots on the triangular lattice].
T: It is given to us that the distances are equal, so actually it is not based on “seeing”. We decided that this is an equilateral triangle based on what is given. It is given to us that the distances are equal, and it is given to us, and it means that the triangle is an equilateral triangle. In contrast, here when I look at the angle and it looks like 90°. But maybe it is 91° So, can someone continue and show…

A student then added the following construction (see Fig. 2a) and claimed that the right base-angle is 30°. She argued that the side AC is an angle bisector because it is both a height and a median in an isosceles triangle. The teacher requested a justification: “You claim that this is in the middle. Who wants to say, again it is a bit ‘seeing’ and a bit, I’d like a clear strong explanation, why is it in the middle?” Attempting to prove this, another student suggested to complete the triangle into a rhombus, and added the following construction (see Fig. 2b). From here it was straightforward for other students to point out that the diagonals of a rhombus form right angles at their intersection and bisect each other. Thus, they concluded that the side AC is indeed an angle bisector and the right base-angle is 30°. Trying in vain to use the Pythagorean theorem in order to prove that the top angle is 90°, students eventually suggested using the angle sum of a triangle property.

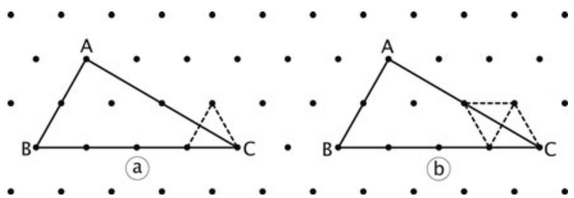

Fig. 2 Finding angle measures of the triangle in Fig. 1c

This episode of finding the angle measures of the triangle in Fig. 1c exemplifies how work on the main topic that was led by the teacher, depended on, and was responsive to, students' talk. It illustrates how work on the main topic comprised of the teacher elaborating students' suggestions and taking part in developing ideas students suggested. The teacher was attentive to students' ideas, and embraced their suggestions as a starting point for mathematical examinations. Thus, problems were mostly solved according to students' proposals and suggestions. The teacher adopted students' suggestions even when they were unproductive or mistaken, spending the time needed to examine their potential and adequacy.

In addition to making students' proposals and suggestions part of the content dealt with in the lesson, the teacher was also attentive to students' requests regarding the issues and topics to be dealt with, and often accepted their requests. For example, after finding the angle measures of the polygons in Fig. 1a–c the teacher shifted the focus of the discussion in the 10th grade class to the formal definition of similarity of polygons and to properties of similarity of different polygons. Then, just before the end of the lesson, the teacher started to explain the homework assignment when a student interrupted her, requesting to complete the task of finding the angle measures of the triangle in Fig. 1e. The teacher accepted this request and the rest of the lesson time was devoted to discussing this. Even though it was the end of the lesson, the teacher responded to students' ideas using elaborating talk. For example, following a student's remark that the triangle was not a 30-60-90° triangle, the teacher asked the students whether they were certain of that. After students responded positively she asked them to explain this claim. Finally the class discussed the two explanations suggested by the students and another one suggested by the teacher.

The discussions outlined above about similarity evolved after the teacher formally opened up a whole-class discussion, asking the class to suggest how to find the angle measures of the polygons in Fig. 1, orchestrating a collaborative whole-class problem solving session. Yet, there were quite a few instances when work on the main topic that comprised teacher's elaborating students' suggestions occurred rather spontaneously. An example for that is an episode taken from a series of lessons on the quadrilateral family in the 9th grade class. During one of the lessons, the class worked on determining which quadrilaterals have reflective symmetry. The students were asked to fold a paper in half, and cut out different quadrilaterals (parallelogram, trapezoid, kite, rectangle, rhombus, square) using the fold line as the line of symmetry. After several unsuccessful attempts to cut a parallelogram that is not a rectangle, according to these instructions, one student noticed that another student "succeeded" to cut such a parallelogram (in fact that student did not use the fold line as the line of symmetry). Astonished, the student inquired, "How did you do it?" The teacher overheard the conversation. She picked up the cut paper, presented it to the whole class, declared that one student succeeded in the task, and asked the class how the student managed to cut out the parallelogram. Eventually, the class discovered that the student did not use the fold line as the line of symmetry.

Accompanying Talk During Work on the Main Topic

Another common teacher behavior was to attend to a student's talk without elaboration, typically acknowledging that she followed the student's talk. There were a few times when the teacher used only accompanying talk; yet, more often she combined accompanying with elaborating talk.

Occasionally, when students gave correct answers, or when gathering students' thoughts as a starting point for work on the main topic, the teacher attended to students' ideas without elaboration. The following illustration of using this type of accompanying talk is taken from an episode that occurred after the teacher expressed disagreement with the suggestions that the 10th grade students proposed for the meaning of similarity in everyday life because they were all closely tied to the mathematical notion of similarity. A student then proposed something different:

S: The same shape but smaller.
T: The same shape but smaller.
The teacher then asked the students to give her examples from everyday life for similar shapes:
S: Perhaps Babushka [a Russian nested doll – Matryoshka doll]?
T: Ah, Babushka, Babushka dolls.

As can be seen, the teacher used in these short excerpts accompanying talk, basically repeating the student's words: "The same shape but smaller", "Ah, Babushka, Babushka dolls."

Another example for accompanying talk that is not embedded in elaborating talk is taken from an activity that followed the activity described above of cutting out different shapes using the fold line of a paper as the line of symmetry. The teacher asked the 9th grade students to report which shapes they succeeded to cut out. The list on the board included the following shapes: circle, square, rectangle that is not square, rhombus that is not square, parallelogram that is not rectangle or rhombus, trapezoid, and kite.

T: Okay, then out of all these – which ones did you succeed at [cutting out]?
S's: Circle, square, rectangle that is not square,
T: [marks on the board each shape the students mention, holding her marker by the next shape on the list: rhombus that is not square].
S: Parallelogram, no, parallelogram I didn't succeed.
S's: Rhombus, trapezoid, rhombus that is not square.
T: [continues to mark each shape the students mention. Eventually all shapes are marked but the parallelogram]. Okay.

Later in the lesson, a student raised again the case of the parallelogram, and the teacher responded by opening up a discussion regarding whether a parallelogram has a line of symmetry. This time, as she often did, the teacher used accompanying talk combined with elaborating talk. The following excerpt illustrates this. It occurred when the teacher drew a parallelogram with a straight line parallel to one pair of its sides, and asked the students to prove that the two adjacent angles of the parallelogram on opposite sides of the line are not equal to each other.

S: I think that the adjacent angles need to be 180°.
T: Very good.
S: [incomprehensible] Never mind.
T: We said that this and this it's 180°. If they are equal then what?
S: 90°.
T: And what will the parallelogram be then?
S: A rectangle.
T: If one is acute then what happens to the other one?
S: The other one is obtuse so that there is 180°.

As can be seen in this excerpt, the teacher first used accompanying talk: "Very good" and "We said that this and this it's 180°" which basically repeats a student's idea. But then she began to use elaborating talk, and actively participated in the construction of the proof.

Puzzlement During Work on the Main Topic

Even though whole-class work on the main topic comprised of immense students' participation, and teacher attention and responsiveness to students characterized by-and-large the mathematical work during whole-class work sessions, the teacher rarely expressed confusion when responding to students' talk. One of these unusual episodes where the teacher's response reflected puzzlement occurred during a lesson in the 9th grade class that centered on exploring relationships among rectangles that have a fixed perimeter or a fixed area. When examining whether a fixed perimeter implies a fixed area the teacher phrased the problem as: "If the perimeters of two rectangles are equal then the areas are equal: Is this claim correct?" A student interpreted this as if the problem was whether there exists a rectangle whose perimeter equals its area. For a few seconds the student and the teacher expressed puzzlement until another student pointed out the reason for confusion. The teacher responded rather astonished:

T: No! No, no, this is not what I meant!
S: Then what did you mean?
T: Not that the perimeter equals the area.
S: Then?
T: Rather that I have two rectangles [draws two rectangles on the board]... Does the fact that the perimeter of this one equals the perimeter of that one mean that the area of this one equals the area of that one? Not that the perimeters equal the areas.

The teacher, who sensed that she misunderstood what a student had said, insisted in the episode described above on clarifying the confusion. However, there was one time when the teacher acknowledged that she was puzzled by a student's talk, yet she chose not to clarify this confusion. It happened when the students' talk was not at the heart of the main point: When collecting students' suggestions for rectangles with a fixed area, a student provided a long complicated explanation on how she found, without using a calculator, that 6 is the other dimension of a rectangle whose area is 15 squared units and one of its dimensions is 2.5. The teacher acknowledged that she was attentive, but confusedly concluded: "Okay, I didn't really understand what you said" and continued with the lesson.

Reviewing Content Introduced Previously

Reviewing content introduced previously comprised of reminding students of, and clarifying, content learned earlier in the lesson, in previous lessons, or in lower grades. This kind of mathematical activity occurred during most of the lessons. It tended to be rather short and only a small part of the total lesson time was devoted to it. Reviewing content introduced previously rarely occurred as a teacher initiative during the observed lessons. In those few times that it did, it occurred at the beginning of a lesson, and served as a means for the teacher to collect information regarding students' readiness for the planned work on the main topic. Nonetheless, reviewing content introduced previously occurred almost always as a teacher's response to students' queries or requests that emerged during work on the main topic.

Two out of the four kinds of responses examined – elaborating and accompanying – were performed by the teacher when reviewing content introduced previously. The most common response was again elaborating; accompanying occurred less frequently. Like in the case of work on the main topic, the teacher led the review – triggered by students' queries and requests – building on students' active participation. Thus, the activity depended on, and was responsive to, not only the student's initial talk that initiated the review, but often also to on-going students' talk. Below are illustrations for elaborating talk and accompanying query used by the teacher when reviewing content introduced previously.

Elaborating Talk During Reviewing Content Introduced Previously

To signal the end of the small group/individual work in the 10th grade class, regarding the polygons in Fig. 1, as a transition to a whole class discussion, the teacher said: "Girls, start to talk about, about the relationships between sides and angles. Is there any connection between this and similarity?" The first student's response was: "What the heck is similarity anyhow?" The teacher responded by restating the idea she presented before the small group/individual work: "We didn't define it yet. But we understand it as some kind of enlargement or reduction by a photocopy machine." Expressing deep involvement in the student's query, the teacher aimed to make sure that they found common ground. She drew on the board two parallelograms, one of which was derived from the other by reducing only one pair of opposite sides, asking the students to determine whether the two are similar to each other. After a collaborative examination the class concluded that reduction (or enlargement) by a photocopy machine (i.e., similarity) reduces (or enlarges) all sides of a shape in the same proportion.

Another example for the teacher's use of elaborating talk when reviewing content introduced earlier in response to students' queries or requests is taken from a concluding lesson on the quadrilateral family in the 9th grade class. In a previous lesson the teacher defined a rhombus as a parallelogram with one pair of equal adjacent sides, and the class worked on the rhombus properties and relationships with other members of the quadrilateral family. In the concluding lesson, after some work on finding characteristics of a rhombus based on the definition and previous work on parallelograms, one of the students pointed to one of the rhombus characteristics found by the class – that all sides are equal – and questioned why the teacher said previously that a rhombus is a parallelogram with one pair of equal adjacent sides, whereas all the sides are equal. The teacher promised to address it later. The class finished the planned work on finding rhombus characteristics, and the teacher returned to the student's query regarding what a rhombus is: a parallelogram with one pair of equal adjacent sides (as defined in a previous lesson) or a parallelogram in which all the sides are equal (as concluded in the current lesson). Attending to the student's confusion, the teacher responded by reviewing the definition of a rhombus, clarifying the distinction between the definition that was introduced in a previous lesson and the rhombus attributes found in the current lesson:

> [The student] said that if we know that a rhombus has four equal sides, then why did we begin by saying such a thing [points to the definition of a rhombus on the board: a parallelogram with two equal adjacent sides]? Does anyone have an idea?
> [Pause]
> Okay, let me tell you. We could have said that a rhombus is a parallelogram with four equal sides, right? [But] in definitions we try to say as little as possible. That means, I don't want to say everything I know about a rhombus as its definition. I say as little as possible in the definition, and all the rest I can prove by myself. In other words, we managed to prove, based on the fact that we knew that this pair is equal, we managed to prove that all sides are equal. This we managed to prove.

The teacher used elaborating talk also when reviewing content introduced in previous school years in response to students' queries or requests. For example, during an analysis lesson the teacher asked her 10th grade class to find the dimensions of a square box (i.e., a right square prism) made from a 60 cm long string with the maximum volume. After the presentation of the problem, a student questioned whether a square box could also be a cube. Even though the students have already studied these shapes in lower grades, the teacher assessed that other students may also be confused regarding the distinction and relationship between the two, and decided to clarify it. She did not answer succinctly, but rather elaborated the distinction:

> Okay. Let us first clarify some concepts. What is a square box? I am sketching and sketch with me. . . A square box is a box with a square base. It doesn't have to be a cube. . . Make it high enough so that it doesn't look like a cube.

And the teacher continued to explain that square box is not synonym to cube. Rather, it denotes a whole family of boxes in which a cube is only one special case.

Accompanying Talk During Reviewing Content Introduced Previously

Occasionally when reviewing content introduced previously the teacher used accompanying talk, by and large combining it with elaborating talk. When the teacher initiated the review it typically served as a means for collecting information regarding students' readiness for the planned work on the main topic. In such cases the teacher often first attended to students' ideas without elaboration, typically acknowledging that she followed the student's suggestions. For example, when starting the topic of the quadrilateral family the teacher started the work by reviewing what a parallelogram is:

> T: Who remembers what a parallelogram is? Raise your hands. Who remembers what a parallelogram is?
> S: A quadrilateral with two opposite parallel sides.
> T: A quadrilateral with two opposite parallel sides.

As can be seen, the teacher used in this short excerpt accompanying talk, basically repeating the student's words: "A quadrilateral with two opposite parallel sides."

However, reviewing content introduced previously occurred almost always as a teacher's response to students' queries or requests that emerged during work on the main topic. In such cases when the teacher used accompanying talk she often combined it with elaborating talk. For example, a short time after the teacher clarified the distinction between square box and cube, a student announced that volume was difficult for them. The teacher attended to this statement of difficulty and reviewed the relevant content, which again had been already studied in a lower grade. She started by unpacking the sources of difficulty, "Do you know how to calculate the volume of a box?" After students responded saying "No", the teacher reminded them of a problem on which they worked in the previous school year, when they were in the 9th grade. That problem dealt with finding the dimensions of an open box constructed from a square cardboard sheet, which can hold the largest amount of chocolate. The teacher drew a square box on the board, and together with the class calculated its volume by making reference to filling the box with chocolate:

> T: What is the volume? The number of 1 cm $\times$ 1 cm $\times$ 1 cm chocolate cubes that fill. . . How many would fill the first layer? . . .How many would fill the whole box?... Who wants to tell me how one calculates the volume of a square box or a non-square?
> S: The area of the base times the height.

T: The area of the base times the height. Is it clear why?... Is there anyone who doesn't understand? The area of the base gives the number of cubes in the first layer.

As can be seen in this excerpt, there was a time when the teacher used accompanying talk, basically repeating the student's words: "The area of the base times the height". But before and after this she used elaborating talk to remind the students of the formula for calculating volume of boxes, which was needed in order to solve the problem of maximum volume. While doing that, the teacher focused on explicating the meaning of volume of a box by making connections to a problem on which the class worked in the previous school year. She exhibited a systematic way of filling the box with one-unit chocolate cubes – layer after layer – reflecting the structure of the formula, emphasizing the meaning of a volume of a box as the number of one-unit chocolate cubes that would completely fill it.

Extending Beyond the Main Topic

Extending beyond the main topic comprised of extending and enriching students' knowledge and understanding of mathematics. Extending beyond the main topic occurred during most lessons, tended to be rather short, and a rather small part of the total lesson time was devoted to it. This kind of mathematical work was usually triggered by students' talk.

Three kinds of responses were enacted by the teacher when extending beyond the main topic. The most common response was elaborating; accompanying occurred less frequently, and teacher puzzlement occurred once. Below are illustrations of the different kinds of teacher responsiveness to students when extending beyond the main topic.

Elaborating Talk During Extending Beyond the Main Topic

The teacher often expressed deep cognitive involvement in students' suggestions, even when it meant deviating from the main topic of the lesson. Sometimes it implied working on new mathematical content. For example, as part of the work on the angle measures of the triangle in Fig. 1c, a student suggested to check whether it is a right triangle, inquiring whether when one side of a triangle is one-half of a second side, it implies that it is a "pretty" triangle: the name used in this class for a right triangle with angle measures of 30-60-90°. As a response, the teacher deviated from the main topic and made this query an object of examination for the whole class. Led by the teacher, the class unpacked the student's query, clarifying what the givens are in the implied conjecture. The teacher called students' attention to the fact that the conjecture is close to be the converse of a theorem they had proven in a previous lesson: In a right triangle, the side opposite the 30° angle is one-half of the hypotenuse. The class continued to work on rephrasing the conjecture, using more formal terms. Finally, the teacher assigned as homework checking whether the conjecture was correct.

In the above episode, extending beyond the main topic comprised of work on new mathematical content: phrasing, and proving or refuting a conjecture regarding right triangles. The episode below had a similar nature. This episode occurred during one of the lessons in the 9th grade class, which centered on finding all rectangles that have a fixed perimeter or a fixed area. When working on the case of a fixed perimeter of 16 units, one of the students found the 4×4 rectangle, which is also a square. She then noticed that the perimeter of this square and its area are equal to each other, and raised the question whether this is true for all squares. Later on, when collecting all students' suggestions for rectangles with a fixed perimeter of 16 units on the board, the teacher pointed to the 4×4 rectangle, and repeated the student's question:

> [The student] asked this question, and I want you to examine this question: I have a square. I saw that the perimeter and the area result in the same numbers. Is it true for all squares in the world that their area and perimeter are the same?

Another student pointed to the 2×2 square, showing that its area does not equal its perimeter. The teacher then explicated that this was a counter example, because it showed that the claim is not true for all squares. By doing that, the class not only worked on new mathematical content: proving or refuting a student's conjecture regarding the equality between a square's area and perimeter, but the teacher also explained an important general mathematical principle, of refutation by a counter example.

Thus, as this episode illustrates, in addition to work on new mathematical content, there were times when extending beyond the main topic comprised of developing students' understanding about general norms and conventions in the discipline of mathematics. The episode described earlier regarding the definition of a rhombus in the 9th grade class also exemplifies this. In that episode, the teacher was attentive to a student's confusion regarding what a rhombus is: either a parallelogram with one pair of equal adjacent sides or a parallelogram in which all the sides are equal. Attending to the student's confusion, the teacher responded by reviewing the definition of a rhombus. Yet, she used the student's question also as a vehicle for explaining the minimalism principle of mathematical definitions, deepening students' understanding of mathematical norms and conventions beyond the main topic:

> In definitions we try to say as little as possible. That means, I don't want to say everything I know about a rhombus as its definition. I say as little as possible in the definition, and all the rest I can prove by myself.

The teacher continued to explain that mathematical definitions are not like dictionary definitions, which include as many characteristics as possible about the defined words (concepts).

Accompanying Talk During Extending Beyond the Main Topic

Occasionally, the teacher used only accompanying talk; but more often she combined accompanying with elaborating talk. This happened, for example, in the above illustration of elaborating talk, when a student suggested checking whether the triangle in Fig. 1c is a “pretty” triangle (i.e., a right triangle with angle measures of 30-60-90°):

S: If one side is one-half of the second, can we say that this is a “pretty” triangle?
T: If a triangle has one side that is one-half of the second, I am repeating the question, if a triangle has one side that is one-half of the second, does it imply that the triangle is a right triangle?
S: And one angle is 60°
T: And one angle is 60°
. . .
T: If in a right triangle. . . there is an angle of 60° [incomprehensible] and the ratio between the two sides that are not opposite the 60° angle. . .
S: She [another student] said: Can we say, the sides that include the angle?
T: The sides that include the angle. Great phrasing. And the sides that include the angle: What about them?
S: Their ratio
T: And their ratio is 1–2. We succeeded to phrase it better. Earlier we said that one [side] is one-half of the other, and now that the ratio is 1–2. Then
S: The triangle is a right triangle.
T: Very true. Then the triangle is a right triangle.
As can be seen, the teacher used in these short excerpts accompanying talk. She basically repeated the students’ words, leading the class to unpack the student’s question, clarifying what the givens are in the implied conjecture. Yet, this accompanying talk was integrated with elaborating talk, situating the conjecture as “almost” the converse of a theorem they had proven in a previous lesson:
T: I repeat the theorem. It’s a converse of a theorem. . . It’s somewhat converse, it’s not really converse, but it’s almost converse.

Puzzlement During Extending Beyond the Main Topic

The teacher response reflected puzzlement only once during whole-class work that extended the main topic. It occurred when the teacher asked the 10th grade class whether the fact that a theorem in mathematics is true implies that the converse of that theorem is true as well. A student interpreted this question as if the teacher was referring to a specific theorem with which the class dealt a few minutes earlier. For a few seconds the student and the teacher expressed puzzlement until another student pointed out the reason for confusion. The teacher explained to the whole class the source of confusion and called students' attention to the potential problematic use of language:

> See how problematic language can be. I think of one thing and [the student] thinks of another. And we try to communicate. It's really a deaf persons dialog.

Conclusion

This chapter examined how an experienced high-school mathematics teacher, who had a reputation of encouraging a significant and influential role for students in the class discourse, responded to students' mathematical talk in class. The chapter examined the kinds of responses used repeatedly by the teacher, directing and shaping the classroom discourse.

Analysis of the lessons showed that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk. This was true in general, and also during each lesson component (work on the main topic, reviewing content introduced previously, and extending beyond the main topic). Moreover, the teacher was attentive and responsive to different kinds of students' talk.

For example, the teacher made a student's mistake regarding a parallelogram's line of symmetry an object for mathematical exploration for the whole class, and made use of it to discuss an important mathematical topic. She incorporated an examination of the student's mistaken cut parallelogram into a public discussion, revealing what the student did wrong, concluding eventually that a parallelogram does not have a line of symmetry. The teacher attended to the student's work, and acknowledged its value, even though it was wrong, by asking the whole class to examine its validity, and by showing how work on mistakes can advance understanding.

On another occasion, the teacher answered a student's specific question about what a rhombus is, reviewing content introduced in a previous lesson. Yet, she did not deviate from the main teaching sequence at a point that could have been confusing for the class. Instead, she acknowledged the importance of a student's question by promising to respond to it later.

Still in a different occurrence, the teacher attended to a student's hypothesis, and acknowledged its value by asking the whole class to examine its validity. By attending to the student's hypothesis, the teacher deviated from the main topic and made the student's conjecture an object of examination, asking the class to prove or refute the conjecture regarding the equality between a square's area and perimeter. She then exploited the opportunity and made use of a student's answer not only to respond to the student's original hypothesis, but also to extend students' knowledge beyond the topic at stake, and explained an important general mathematical principle, of refutation by a counter example. She signaled that raising hypotheses is a valued activity, and used the opportunity to extend the problem solving activity the class has been already doing. By expecting the other students to participate in the problem solving process, and by using a solution suggested by a student, the teacher indicated also that students' input counts.

The finding that almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk was mainly due to the teacher's responsiveness to students. The most common teacher response was elaborating. The teacher constantly elaborated students' utterances and expressed profound cognitive involvement in what students said. In addition to being the most common teacher response to students in general, elaborating was also the most common teacher response during each of three different lesson components. When working on the main topic the teacher embraced and elaborated students' ideas both as a starting point for mathematical examinations and throughout the work. She embraced students' suggestions for whole-class examination when they were productive and correct and also when they were unproductive or mistaken, and she took an active part in developing students' ideas. Although reviewing content introduced previously and extending beyond the main topic rarely occurred as a teacher initiative, but rather were comprised mainly of the teacher's response to students' queries, remarks or requests that emerged during work on the main topic, the teacher's elaborating talk then was similar in nature to that during work on the main topic. Here too the teacher seldom responded succinctly, but rather provided elaborated reviews or extensions (triggered by students' queries and remarks), building on students' active participation.

Responding to students using accompanying talk occurred considerably less than elaborating talk. Yet, the teacher often acknowledged that she followed the student's talk by attending to a student's talk without elaboration. Typically the teacher combined brief accompanying talks in much longer elaborating response; in general, and also during each of three different lesson components. Occasionally, though, accompanying talk was used not as a component of elaborating talk. Sometimes this happened when students provided a correct answer, and the teacher then repeated the student's answer without elaboration and quickly returned to work on the main topic. A few other times it occurred as a teacher initiative when she gathered students' thoughts, hypotheses or solutions, as a starting point for whole-class work.

The teacher rarely expressed puzzlement or confusion when responding to students' talk. Because almost the entire whole-class work comprised of mathematical activity that was triggered by, built or followed on, students' talk, this reflects an utter sensitivity, awareness and knowledge about students, and about their thinking and ways of talking. Teacher puzzlement regarding students' talk occurred a small number of times during work on the main topic and once when extending the main topic, but not when reviewing content introduced earlier. Yet, there seems to be no relationships between the occurrence of teacher puzzlement and the nature of the lesson component. It appears that the teacher expressed confusion whenever she could not follow students' talk regardless of the part of the lesson in which it occurred. Yet, she consistently insisted on clarifying the confusion unless it was extremely remote from the main issue.

Teacher responsiveness to students in the form of opposition also seldom occurred. The teacher expressed disagreement or objection to students' ideas a few times during work on the main topic. In contrast with her practice during most of the observed time, in those few events, when students' ideas deviated considerably from the main point, the teacher did not embrace or follow on students' suggestions, queries and remarks. Instead, she pointed out what they should focus on. Yet, this response is quite different from the more common teacher practice of objection to students' wrong answer (Resnick et al., 1993).

This chapter examined the kinds of responses used repeatedly by an experienced high-school mathematics teacher, directing and shaping the classroom discourse, during different parts of the lesson. The chapter presents the ways in which the teacher encouraged and enabled a significant and influential role for students in the class mathematics discourse, while as river guide ("Images of expertise in mathematics teaching" in the chapter by Russ et al., this volume), responded to the students, to the context, and to what occurred in the moment (Berliner, 1994). The chapter demonstrates how the teacher combined her attention to students' talk, with the goal of making progress on the main topic. She was sensitive to students' difficulties in regard to content learned previously, but devoted only a short time to reviewing content introduced previously, using these episodes to enhance understanding. She also exploited opportunities to extend beyond the main topic, developing understanding of the nature of work in mathematics and the nature of the discipline.

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Effects of a Research-Based Learning Approach in Teacher Professional Development

Florian H. Müller, Irina Andreitz, Konrad Krainer, and Johannes Mayr

Abstract The article examines the effects of teacher professional development, which follows a research-based learning approach focused on "action research" (Altrichter, Feldman, Posch & Somekh 2007). Using integrated research methods, the study examines the extent to which the four-semester university programme, "Pedagogy and Subject Didactics for Teachers" (PFL), has an impact on its participants. The study follows a longitudinal design, which focuses on input factors, processes, and outcomes. Its core component consists of testing for teaching-related analysis components using a video task (Krammer et al., 2006) conducted before and after the course. Based on an instructional video sequence on the topic of geometry, the study assesses the extent to which participants of the PFL mathematics course differ from those of other PFL courses.

Keywords Teacher professional development · Action research · Video analysis · Competence in analyzing · Teacher interest · Learning strategies · Mathematics teachers

Introduction

Teachers often participate in traditional professional development events which are of short duration and communicate abstract knowledge; however, these have not only been criticized frequently by participants, but they have also demonstrated little overall effect (see Altrichter, 2010; Lipowsky, 2010; OECD, 2009; Scheerens, 2009).

F.H. Müller (✉)
Institute of Instructional and School Development, University of Klagenfurt, Klagenfurt, Austria
e-mail: florian.mueller@uni-klu.ac.at

This article is an extended version of a previous article. It integrates new findings of a mathematics course and compares these with other courses.

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It appears that the possibility to examine one's own profession or teaching in a self-directed manner and during exchanges with colleagues, is a key to professional development. This research-based learning approach has been reflected for some time in theoretical and conceptual considerations, as well as in practical applications for a variety of professionalization measures (e.g., Altrichter, 2002; Dirks & Hansmann, 2002; Feindt, 2009; Hollenbach & Tillmann, 2009; Horstkemper, 2003; Roters, Schneider, Koch-Priewe, Thiele, & Wildt, 2009). In particular in the last two decades, a variety of innovative models of teacher professional development have been designed, implemented and evaluated all over the world. In the field of mathematics teacher education, for example, such models are discussed not only in research papers (like in the *Journal of Mathematics Teacher Education*, launched 1998) but also in the International Handbook of *Mathematics Teacher Education* (Wood, Jaworski, Krainer, Sullivan, & Tirosh, 2008) and in several specific books (e.g. Even & Loewenberg Ball, 2009). Studies analyzing research on mathematics teachers' professional growth (see e.g. Llinares & Krainer, 2006) show that teachers' learning is not only promoted by meaningful activities, but also by teachers' (oral and written) reflections on these activities, in many cases related to students' or teachers' own mathematical learning. Sustained and intensive professional development, often designed as teachers' participation in a "community of practice" (more and more also virtual communities using new technological tools such as videopapers, blogs, etc.) and integrated into the daily life of the school (see e.g. Krainer & Wood, 2008; Sowder, 2007 or Wood et al., 2008) is more likely to be effective than short-term- and practice-distant professional development activities that address teachers mainly as "single fighters".

Concurrently, the degree of attention given to these innovative approaches is not reflected in the amount of empirical research available. There are only few large-scale findings relative to the conditions, processes, and effects of research-based learning approaches for professional development (see e.g. Adler, Ball, Krainer, Lin, & Novotná, 2005).

There are some indications regarding the effects of approaches similar to action research. As part of a broadly based meta-analysis, Cordingley, Bell, Thomason, and Evans (2003) were able to show that a collaborative Continuing Professional Development approach (CPD) had positive effects on confidence, feelings of self-efficacy, motivation to work in a team, and willingness to change one's own actions in a teaching setting (see also Cordingley, Bell, Thomason, Rundell, & Firth, 2005). As far as the students are concerned, the research overview revealed indications about the differences between collaborative and individual professional development. These differences refer to outcomes, such as learning motivation, performance, and attitude towards subjects, as well as active participation during the course. It is also evident that the research overview offered by Cordingley and colleagues does not refer exclusively to action research as a method of professional development, but to professional development and developmental measures that consider several aspects of action research. Other overviews and individual studies report similar findings, although these can vary significantly depending on the study and type of professional development. Nevertheless, recent research shows that collaborative and reflective teacher professional training impacts teacher cognition and partly students' characteristics (e.g., Gärtner, 2007; Gough, Kiwan, Sutcliffe, Simpson, & Houghton, 2003; Gow, Kember, & McKay, 1996; for a summary, see e.g., Benke, Hospesová, & Tichá, 2008). On a global basis, however, research efforts still remain too incomplete to allow valid statements to be made.

This article introduces findings from monitoring research conducted for the university courses "Pedagogy and Subject Didactics for Teachers" (PFL) at the University of Klagenfurt (Austria), which generally follow the action-research approach. Initially, the theoretical background and concept behind the PFL courses will be described, followed by a report and discussion of the research design and selected results.

The starting point for learning-based research approaches, which extend beyond teacher education and professional development, is based on the scientific insight into and practical experience with professional development, which is not only an intellectual and academic process, but is also an active practical, emotional, and social process (Altrichter, 2002). A central consideration in this context is that the simple transfer of scientific concepts and innovations is very difficult and at times, impossible. There are many reasons for this; for example, theoretical knowledge is often inert and was not obtained in authentic, complex and team orientated learning situations.

Research-based learning aims at reducing the gap between knowledge and action by focusing on one's own actions. In this context, one of the most prominent approaches taken to teacher education is so-called "Teacher Research" or action research (see e.g., Burns, 2007; Altrichter, Feldman, Posch & Somekh, 2008; Elliott, 1991; Hollingsworth, 1997; Wittwer, Salzgeber, Neuhauser, & Altrichter, 2004; Posch, Hart, Kyburz-Graber, & Robottom, 2006), which finds its theoretical basis in the action theories of Schön (1987) and Stenhouse (1975), among others. In this vein, professional development should be conceptualized through a repeated cycle of action and reflection. Here, teachers systematically investigate their own teaching practices, interpret the insights they gain, and create new action ideas (reflection), which are then implemented (action) and evaluated. Relative to this, Schön (1987) notes the "reflective adoption of practical solutions to problems," which is based on having an experimental attitude regarding real life practices. Reflection is viewed as one of the main competencies of those in the teaching profession, and not just in action research (see e.g., Bromme, 1994). This ability is of particular consequence relative to the implementation and objectives of teacher professional development.

Another core concept of action research that is in line with Elliott (1991) is that individual research and the further development of one's own teaching practices or those of the school, are embedded into a professional community. The significance of professional communities in action research has been pointed out by Altrichter (2002), who also established a systematic relationship between the concept of "situated learning" (see Lave & Wenger, 1991), and teacher education.

To overcome the risk of excessive self-referencing[1] of teacher groups (because collegial cooperation can also prevent learning; see e.g. Corcoran, Fuhrman, & Belcher, 2001), the support and intervention offered by outside colleagues or experts can be integrated into action research projects. These act as a corrective factor or "critical friends".

The Philosophy of the PFL Programme

The University of Klagenfurt has been offering the 2-year university programme "Pedagogy and Subject Didactics for Teachers" (PFL) since the early 1980s. The PFL programme consists of several courses, each dedicated to one or more subjects (Krainer, 1999). The courses are based primarily on the concept of action research (Posch, Rauch, & Mayr, 2009). One maxim for these courses is the close linking of pedagogical knowledge and pedagogical content knowledge (Shulman, 1987), and identifying the teacher's own actions as the starting point for teacher professional development. In this context, academic issues are, at least initially, of secondary importance. PFL courses initiate personal teaching-related development projects, which the participants typically implement at their own school. The projects are supported by experts from research and practice, and the intensive exchange of information among teachers in terms of collegial advisory services forms a part of the PFL course. During the course, participants prepare on average two reflective papers that document the development process of teaching projects, the objective of which is to evaluate one's own teaching actions. The courses also integrate the school environment by focusing not only on researching one's own teaching actions, but also observing the projects of colleagues and school development initiatives (see e.g. Krainer, 2001). Beyond the project phases, as part of three, 1-week module workshops held during the course, participants also receive information on subjects, such as evaluation methods or new trends in pedagogical knowledge and pedagogical content knowledge, which they can link with their teaching projects. Additional work group meetings offer more opportunities for exchange with colleagues.

The following box offers some competences the PFL courses aim at.

[1] It must be assumed that an experimental and reflective attitude towards one's own teaching practice is not a given, nor can it be assumed that it exists for all teachers (cf. Copeland, Birmingham, De la Cruz, & Lewin, 1993; Ferry & Ross-Gordon, 1998). To develop such an attitude must also be seen as an objective of teacher professional development.

Examples of Competences the PFL Courses Aim at

The PFL programme rather focuses on the development of pedagogical content knowledge, pedagogical knowledge, motivational orientation, attitudes and beliefs, critical reflection or networking competences than on content knowledge (for example content knowledge in mathematics or science).

Pedagogical Content Knowledge

- about students' preconceptions
- about methods for cognitive activation
- about the use of different instructional methods

Pedagogical Knowledge

- about classroom management
- about social learning arrangements
- about evaluation strategies

Attitudes and Beliefs

- constructivist view of learning and teaching
- life-long learning

Self-Related Cognitions

- development or stabilization of teachers' interest and motivation
- teachers' self-esteem/self-efficacy

Reflection and Networking Competences

- critical reflection on classroom practice
- how to work in teams

At present, four parallel courses for secondary teachers are offered which include mathematics, sciences, English, a cross-subject course for art, history and German (ArtHist), as well as a course designed specifically for the primary level including the issue of integration.

The following section outlines the concept of monitoring the research for PFL courses. It will present and discuss the results of this research for the four teaching courses held from 2006 to 2008 (respectively 2007–2009 for the PFL mathematics course).

Research Design

Theoretical Background

The monitoring study follows a longitudinal design which focuses on the present courses with regard to input, ongoing processes, and outcome. The theoretical background is a concept which is based on the association between the experiences of teachers, the opportunities to learn in the courses and the uses of the learning environment (see Helmke & Weinert, 1997), which can be presented in a simplified form as follows:

Participants enter the course with specific input conditions (expectations, interests, competencies, etc.), and encounter specific learning opportunities (in the form of individuals, information, etc.). They employ these depending on their input conditions and the quality of the learning environments (such as by applying specific learning strategies). The learning benefits drawn from each course at the individual level (expanding knowledge, changes in thinking, etc.) are viewed as being dependent on the aforementioned input conditions, learning opportunities, and learning strategies. The competencies developed as part of the course (in the widest sense, see Allemann-Ghionda & Terhart, 2006) should have an impact on teaching actions, so that the course contributes to a further development of teachers' practice.

Research Plan and Questions

The research design (see Fig. 1) identifies four dates during which data collection took place. (1) Prior to each PFL course, participants were surveyed using an online questionnaire that covered previous professional experience, interests, self-assessed occupational competencies and existing knowledge, and the reasons for participating in the course. (2) At the beginning of each course a video test was conducted in order to measure teachers' competences in analyzing lecture units. (3) At the end of the first year, participants were asked to complete a questionnaire regarding their assessment of the learning environments and their utilization (learning strategies). (4) All measurements were repeated at the end of the course and supplemented with several additional questions regarding satisfaction with the PFL and scales measuring the learning motivation of the participants. This article reports on the findings of the initial survey, the two video tests, and the final survey.

With regard to this article, the following questions are considered leading components of the research:

- What are the motivating factors behind a decision to participate in the PFL-programme?
- What learning strategies do participants use during the course?
- Do the participants' interests in the professional activities, and their own self-assessment of competences and knowledge, change during the course?
- Is there a change in the ability to analyze lecture units with regard to the learning opportunities of students (teaching video on the topic of geometry)?
- Are any differences noted between mathematics and other courses?

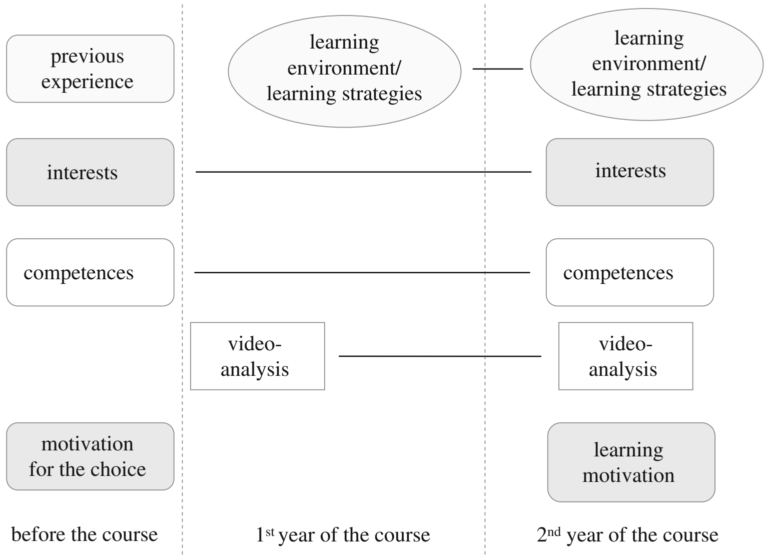

Fig. 1 Research design

Instruments

The following section outlines briefly the instruments used for collecting the survey data, and it introduces the instruments used in the video tasks. Detailed statistical information about the scales and items are described elsewhere (Müller, Andreitz, & Mayr, 2010).

Questionnaires

Motivation to participate in a course (six scales)

The first two scales that focus on motives for participating in the course (see also Fig. 1) were constructed on the basis of the self-determination theory by Ryan and Deci (2002, 2008). Notably, a distinction was made between the aspects "self-determined" motives (item example: I am taking the course because I enjoy learning something new) and "controlled" motives (because of the high prestige associated with university courses) (see also Müller, Palekčić, Beck, & Wanninger, 2006). Further, scales were added that capture the interest in "Development of the school system" and "Development of classroom teaching" as a motivation to take the course. In addition, scales regarding the participation motivation of "Maintain and promote professional motivation" and "Making social contacts" were prepared.

Job-related interests and competencies (eight scales each)

Slightly adapted versions of the six dimensions of teacher interest scales (LIST; Mayr, 1998) were used prior to and at the end of the course to assess the interests and competencies for a teaching career (e.g., "Teaching" or "Address specific needs”). In addition, two areas that refer specifically to the content of PFL courses were included: “Reflect on own actions” and “School development”. Each item referred to the teachers’ interest in the relevant activity and the competence for conducting that activity.

PFL-specific knowledge areas (four scales)

The teachers were asked to assess the state of their knowledge before and after professional development with regard to aspects that are considered to be essential in the PFL courses. These are: (1) methods of promoting teaching (measures of inside differentiation and individualization in the classroom); (2) pedagogical content knowledge and pedagogical knowledge (newest concepts); (3) knowledge about performance standards (educational standards); and (4) management and evaluation (evaluation methods).

Learning strategies (five scales)

The three items with the highest factor loading for the scales “Create associations,” “Critical review,” and “Repeat,” and the scale “Effort,” were used from the surveys on learning strategies (LIST; Wild, 2000). In addition, the scale “Reflection” was created (item example: I think about how I can improve my actions), since there were some indications that this learning strategy is initiated particularly through the conception of PFL courses.

Satisfaction with the course

One item was formulated with regard to overall course satisfaction, two other items referred to the satisfaction with the 1-week modules, and the working groups.

Video Task

At the beginning and the end of the course, participants were asked to analyze videographed sections of problem- and action-oriented mathematics lessons on the topic of the “Pythagorean theorem” (from the DVD “Introductory Sequences” by Reusser, Pauli, & Krammer, 2004). They were asked to identify learning opportunities that activate the students at a cognitive level and then substantiate their answers (see Table 1). Exactly the same video scenes were used at the beginning and at the end of the course.

Table 1 Tasks related to the video sequences

Questions on the video
① **Learning opportunities/cognitive activation in the classroom** Identify events or moments from this lesson which activated the students’ learning and thinking processes. Briefly describe these and provide reasons why you view them as learning opportunities for the students.
② **Optimization opportunities** Please describe options which the teacher could use to further increase learning opportunities for students in this lesson.

The research design was adopted from the work group "Bi-national video-supported professional development for teachers in Germany and Switzerland"[2] (Krammer et al., 2006, 2008, 2009). In this study, however, the instructional video was used only as a diagnostic instrument to measure the participants' analytical competence. The objective of the video task was to evaluate whether participants were using the pedagogical knowledge and pedagogical content knowledge addressed in the course as they analyzed a video sequence, in addition to self-assessing their knowledge and competence.

The open answers provided by participants were subjected to a content analysis (Mayring, 2000) in which the category system by Krammer and colleagues (2009) was adopted. This category system contains seven main categories: (A) Characteristics of instruction design with regard to content; (B) Characteristics of instruction relative to interaction and social forms; (C) Characteristics of instruction with the objective of achieving an active examination of the content by the learner; (D) Comprehension orientation; (E) Behavior characteristics/Characteristics of the teacher; (F) Observation of student behavior; and (G) Learning atmosphere. In addition, the categories Direct Instruction by Teacher (as the main category I) and Reciprocal Teaching (as subcategory A1) are also formed, since information was frequently provided for these aspects, particularly in t_2, but no provision was made for these categories in the German-Swiss research group.

Sample of the Study

Participants in the four PFL courses were surveyed between 2006 and 2009 (questionnaire: $Nt_1 = 131$; Nt_1 and $Nt_2 = 84$; video task: Nt_1 and $Nt_2 = 54$). The average age of teachers is 46 years ($SD = 8.6$); they teach at university entrance secondary institutions (*Gymnasien*) (30%), vocational middle and secondary schools (28%), general secondary schools (*Hauptschulen*) (27%) and other types of schools (15%). Thirty-three teachers took part in the PFL mathematics course. This group will be analyzed consistently relative to teachers who chose another PFL course (ArtHist, English, Sciences) covering the same school types.

Results

Questionnaire Analysis

Table 2 provides an overview of the motivation for participation and illustrates that the self-determined motivation towards taking the course is significantly higher than controlled motivation. In this vein, the course is selected due to intrinsic motivation,

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[2]The authors would like to express their thanks for being granted access to the required work materials.

Table 2 Motives for participating in the course: non mathematics, mathematics

Scales	Items	Cronbach's alpha	M_{t1} (SD_{t1}) (non math)	M_{t1} (SD_{t1}) (math)
Reasons for selection (self-determination theory)				
Self-directed	7	0.80	4.1 (0.6)	3.9 (0.8)
Controlled (extrinsic)	7	0.68	2.3 (0.6)	2.1 (0.6)
Reasons for selection (specific)				
System development	2	0.72	3.4 (1.2)	3.4 (1.2)
Teaching development	2	0.75	4.7 (0.5)	4.7 (0.5)
Improve own motivation	2	0.65	3.7 (0.9)	3.5 (1.2)
Social contacts	1	–	3.1 (1.1)	3.2 (1.0)

N = 128; Scale: 1 = Does not apply, 5 = Applies (Question: "Why did you choose this course?")

an interest in the topic, curiosity, and the desire to develop one's own competence in the area. Extrinsic motivation, such as obtaining a certificate, the prestige of the course, or "feelings of guilt" if one does not participate in professional development played a subordinate role.

However, with regard to the controlled motivation (extrinsic motivation), 20% of participants demonstrated decidedly stronger values (means of greater than 3.5). These participants had already attributed more importance to the significance of tests, certificates, and associated career opportunities at the beginning of the course. In this context, Posch et al. (2009, p. 212) point out that the courses can also represent the "starting point for an exit from the teaching profession," and that they can stimulate new career ideas and open new career options (see also Benke et al., 2008, p. 289).

The primary motivation (relative to content) for selection of the course is the development of one's own teaching. All other kinds of content-related motivation are ranked below.

No significant differences were found between PFL mathematics participants and those of other PFL courses.

The high degree of self-determined motivation and the content's focus on teaching development are good motivating criteria for sustained learning processes in the course (see also Smith & Gillespie, 2007). These concur as well with the alignment of the PFL courses relative to content. It has been shown elsewhere that the participants in such courses remain highly self-determined over the 2-year period (Müller et al., 2010).

Overall, the participants are very satisfied with the course (see Table 3). This applies particularly to regional groups in which the participants work intensively on their individual instructional project in cooperation with their colleagues and support staff.

An observation of the learning strategies used by the participants shows that different approaches are used to various extents (see Table 4). In particular, the in-depth learning strategies of reflection, creating associations, and critical review

Table 3 Assessment of satisfaction with course

Scales	M_{t2} (SD_{t2}) (non math)	M_{t2} (SD_{t2}) (math)	T-test p
Satisfaction			
With 1-week seminars	4.1 (0.8)	4.2 (0.9)	n.s.
With regional groups	4.5 (0.8)	4.2 (0.8)	0.05
With the course overall	4.3 (0.8)	4.1 (0.7)	n.s.

N = 84; Scale: 1 = Does not apply, 5 = Applies

Table 4 Learning strategies

Scales	Items	Cronbach's alpha	M_{t2} (SD_{t2}) (non math)	M_{t2} (SD_{t2}) (math)	T-test p
Scales from LIST					
Create associations	3	0.70	4.3 (0.6)	4.5 (0.4)	n.s.
Critical review	3	0.78	3.9 (0.7)	3.7 (0.9)	n.s.
Effort	3	0.70	3.5 (0.7)	4.2 (0.5)	0.01
Repetition	3	0.68	3.6 (0.7)	4.4 (0.5)	0.01
Additionally constructed scale					
Reflection	3	0.74	4.5 (0.5)	4.8 (0.3)	0.04

N = 84; Scale: 1 = Does not apply, 5 = Applies

are utilized. Relative to the research-based learning conception of professional development, these represent results that correspond with expectations. Notably, repetition strategies are used less frequently. Similarly, the Effort scale also features a significantly lower mean.

In the PFL mathematics course, the two latter learning strategies of repetition and effort are used significantly more often than in the other courses. However, it remains to be seen whether these differences can be traced back to the learning environment in the course or to the differences in the culture relative to the various subjects.

The interest shown for different activities of teachers, and the competencies and knowledge assessed by the teachers themselves were investigated in a longitudinal study (see Table 5). That fact that interests changed little during the 2 years was to be expected, even though a significant increase is noted regarding participant interest in the "Development of school" scale, which was created specifically for the PFL courses. This applies as well to the non-mathematical PFL courses regarding the aspect "Reflect on one's own actions." These two aspects, school development and development of own actions, are essential objectives of the PFL courses.

With regard to the competencies experienced, three scales indicate substantial changes over the 2 year period of the study. For example, participants assess their teaching competence somewhat higher after 2 years, along with the competence to reflect on their own actions and conduct school development. With regard to PPL mathematics, these changes are visible only for the "Development of the school system" scale. There is no change for the scale 'adress specific needs' of students.

Table 5 Change in interest, competence and knowledge

Scales	Items	Cronbach's alpha	PFL: Non mathematics				PFL: Mathematics			
			M_{t1} (SD_{t1})	M_{t2} (SD_{t2})	p	d	M_{t1} (SD_{t1})	M_{t2} (SD_{t2})	p	d
Interests										
Teaching	6	.68	4.0 (0.6)	4.1 (0.6)	0.50	0.1	4.5 (0.4)	4.5 (0.4)	0.85	0.0
Address specific needs	5	.74	3.6 (0.7)	3.7 (0.6)	0.25	0.1	3.6 (0.5)	3.6 (0.4)	0.75	0.0
Reflect on own actions	4	.80	3.8 (0.7)	4.0 (0.7)	0.07	0.3	4.1 (0.7)	4.2 (0.4)	0.42	0.2
Develop school	4	.79	3.2 (0.9)	3.5 (0.9)	0.02	0.4	3.2 (0.8)	3.7 (0.7)	0.05	0.7
Competencies										
Teaching	6	.78	3.8 (0.6)	4.0 (0.6)	0.01	0.5	4.2 (0.6)	4.2 (0.6)	0.84	0.0
Address specific needs	5	.76	3.1 (0.8)	3.2 (0.6)	0.47	0.1	3.1 (0.8)	3.3 (0.9)	0.28	0.2
Reflect on own actions	4	.74	3.4 (0.6)	3.6 (0.6)	0.05	0.5	3.7 (0.5)	3.8 (0.6)	0.69	0.2
Develop school	4	.79	3.1 (1.0)	3.4 (1.0)	0.03	0.4	3.1 (0.5)	3.7 (0.5)	0.02	1.2
Knowledge										
Methods to improve learning processes	4	.79	3.1 (0.8)	3.6 (0.7)	0.00	0.6	3.0 (0.5)	3.7 (0.6)	0.00	1.4
Pedagogical content knowledge and pedagogical knowledge	2	.77	2.6 (0.8)	3.3 (0.9)	0.00	0.7	2.4 (0.7)	3.4 (0.9)	0.01	1.3
Performance standards	3	.66	3.0 (0.8)	3.4 (0.8)	0.00	0.5	3.3 (0.8)	4.1 (0.6)	0.00	1.1
Management and evaluation	3	.73	2.6 (0.9)	3.4 (0.8)	0.00	1.0	2.7 (0.7)	3.5 (0.9)	0.01	1.0

$N = 84$; t_1: 2006 (prior to course); t_2: 2008/09 (at the end of the course); $p < 0.01$ (t-test, double-sided, between t_1 and t_2); d = effect size.
Scale: Interest: "How do you carry out the following activities?" 1 = don't like to; 5 = really enjoy doing it.
Scale: Competence: "How good are you at doing the following tasks?" 1 = not at all; 5 = very good.
Scale: Knowledge: "In your opinion, what is your level of knowledge?" 1 = very low; 5 = very high.

All self-assessments of the knowledge areas increased significantly. In particular, the scales “Pedagogical content knowledge and pedagogical knowledge” and “Management and Evaluation” achieved higher self-assessments at the end of the course than at the beginning. These knowledge areas also featured the lowest values at the beginning of the course, which indicates significant development potential. Similarly, knowledge of methods for promoting learning and performance standards also increased.

With regard to the scale “Performance standards,” a noticeable and highly significant difference between PFL mathematics (mean 4.1) and other PFL courses (mean 3.4) is observed for t_2. This development stems primarily from the fact that the issue of performance standard established a focus of the PFL mathematics course.

Video Task

The verbal responses for the video task were categorized initially by three people independently, based on the category system of the research group “Bi-national video-supported professional development for teachers in Germany and Switzerland” (see section “Instruments”). Subsequently, the category assignments were validated within the research group as part of a discussion. This article reports on the results of the partial task “Identification of learning opportunities/cognitive activation in the classroom”.

Figures 2 and 3 shows the average number of entries for some categories (‘G: Learning atmosphere’ and ‘I: Direct instruction’) and subcategories (like ‘A4: Situating’ or ‘A4: Reciprocal teaching’) by way of example. Because of the large number of categories, the average number of entries for observation units in the individual categories is low.

The sum of all entries for cognitive activation does not reflect any significant increase for both groups in the longitudinal study ($Mt_1 = 5.08$; $Mt_2 = 5.93$; t-Test: $t = 1.2$, $p = 0.11$). Hence, the changes refer to individual aspects:

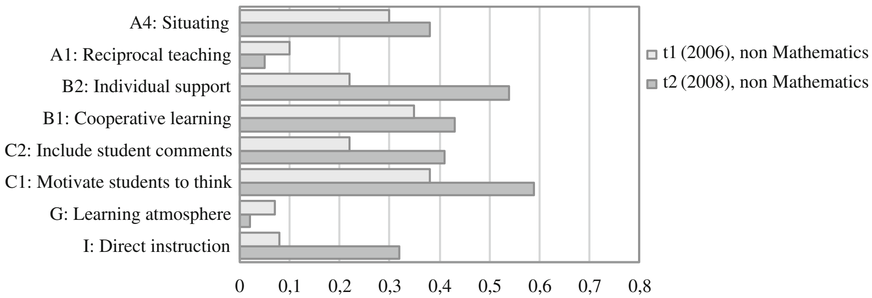

Fig. 2 Average number of entries for the categories of the video task (non mathematics)

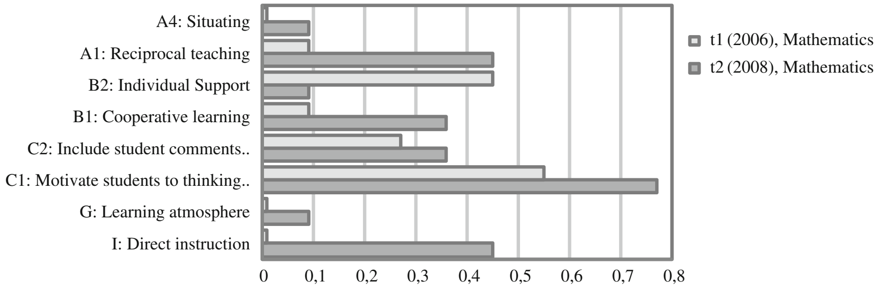

Fig. 3 Average number of entries for the categories of the video task (mathematics)

PFL Non-Mathematics

Significant increases (t-test, double sided; N = 34) are observed for the aspects of "Individual support" ($Mt_1 = 0.22$, SD = 0.77; $Mt_2 = 0.54$, SD = 0.63; p = 0.05), "Motivate students to think" ($Mt_1 = 0.38$, SD = 0.99; $Mt_2 = 0.60$, SD = 0.95; p = 0.04), and "Direct teacher instructions" ($Mt_1 = 0.08$, SD = 0.40; $Mt_2 = 0.32$, SD = 0.51; p = 0.02). No significant changes were observed for most of the other categories; e.g., "Reciprocal teaching" for "Situating," "Inclusion of student comments" for "Cooperative learning," or "Learning atmosphere."

PFL-Mathematics

Only the "Direct Instruction" category has significantly more entries at the end of the course ($Mt_1 = 0.00$; $Mt_2 = 0.45$, SD = 0.54; N = 20). The category of individual support is rarely mentioned for t_2 and decreases significantly compared to t_1 ($Mt_1 = 0.45$, SD = 0.60; $Mt_2 = 0.09$, SD = 0.30). None of the other categories undergo a significant change. This lack of significant differences can be traced back to the small sample.

In addition to the number of individual entries, the "quality" of analyses was also analyzed. In this case, quality has been defined as the reasoning for each entry including an indication of associations to other passages of the teaching sequence. Figure 4 denotes that the number of reasons provided increased significantly (t-test, double sided, p = 0.02, d = 0.33). The correlation of the quality of analysis between t_1 and t_2 is r = 0.57 ($p < 0.01$). Overall, however, teachers gave few explanatory statements at both points in time. The number of reasons provided is lower for t1 with PFL mathematics than for other courses.

A differentiated look at the quality variance in the analyses shows that the number of explantions provided and the associations only increase to a significant level, from t1 to t2, if the participants analyzed and evaluated some in-house or external teaching videos as part of the 2-year course (see Fig. 5). Nine of all 54 participants in the video study were involved in this process as part of or external to the course. The correlation between the quality of analysis and experience with video analyses is r = 0.41 ($p < 0.01$).

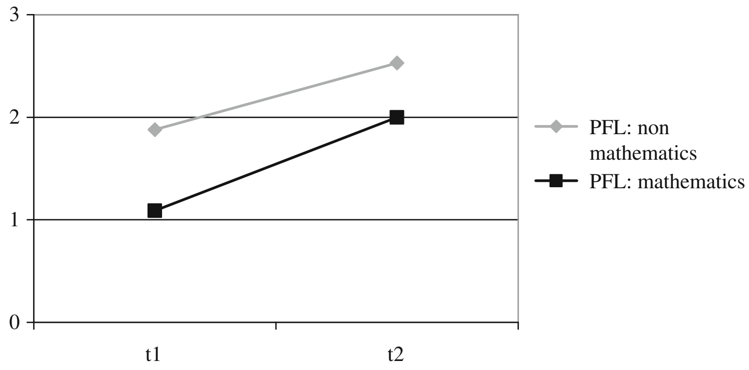

Fig. 4 Change in analytical competence: quality of analyses

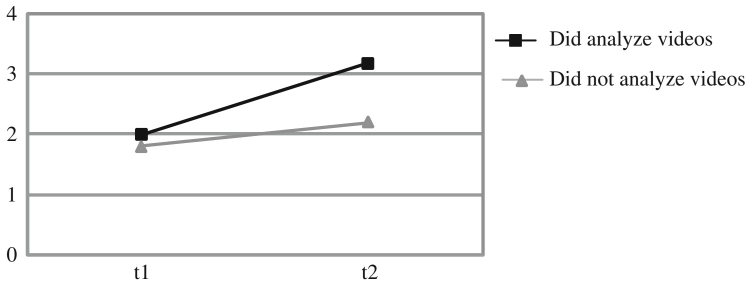

Fig. 5 Change in analytical competence: quality of analyses in partial groups N = 54 (math and non math PFL courses)

No differences were found between mathematics and other courses when experience with video analyses was considered.

Predicting changes in analytical competence

In addition to the experiences acquired by participants with regard to the analysis of teaching sequences during the course, two other aspects can also be used to explain the increase in reasons provided. On the one hand, analytical competence develops more strongly if the participants receive explicit instructions from PFL-trainers regarding how these teaching issues are analyzed (even if no teaching videos are used). The correlation between this assessment and the quality of video analysis is $r = 0.39$ ($p < 0.01$). On the other hand, a strong interest in course content as a motive for selecting the course is a good prerequisite to expand one's teaching-related analytical competence. No other indications for predicting differences for analytical competence were observed.

Summary and Discussion of Findings

Professional development opportunities such as the PFL course, which focuses on a research-based learning approach and on a close link between theory and individual teaching approaches as well as supports teachers' learning processes over a long time period, are assessed postively by teachers. This applies particularly to working in small groups (regional group meetings), which allows for in-depth discussion of the development of one's own teaching as part of a collegial exchange along with the intensive support provided by experts. Of course, the high degree of self-directed motivation, the strong interest in linking pedagogy and subject didactics, and an interest in personal development of competences at the beginning of the course also play roles in the high level of satisfaction and acceptance of these courses.

The participants primarily apply elaborative learning strategies, such as "critical review" and "reflection." However, it is not only the learning strategies employed that correspond with the leading theme of "reflecting practitioner" in the PFL courses, since the longitudinal study also shows that interest and self-assessed competence regarding reflection on one's own practice increase as well. Yet it is also the periodic self-assessment of competence for teaching and a desire to participate in school development processes that increase at the end of the course. The same applies to course-specific knowledge areas such as pedagogical content knowledge and pedagogical knowledge, management and evaluation, methods for supporting learning processes, and knowledge of learning standards.

Given a careful interpretation, the change in interests and competencies can be viewed as evidence of the effectiveness of the courses. However, it is not known whether these subjective assessments also correspond with changes in behavior in the classroom setting. To look more closely at this item, the research design must be expanded to include the corresponding activities (e.g. teaching observations in the PFL participants' classrooms before and after the course).

In this study, the video task serves as a diagnostic instrument to measure the teaching-related analytical competencies of the participants and to validate their self assessed competencies. The categorization and number of entries indicate that at the end of the course, teachers have become more sensitive regarding individual aspects of learning situations. Overall, however, the difference in aggregated entries in the video task is not significant between the two test dates.

An analysis of the quality of answers for the video task, which considers the reasons provided and the associations made in the responses, results in increases from t1 to t2. In this vein, mathematics teachers are not superior to other PFL course participants. On the contrary, it is evident that the number of contributions (explanatory statements and associations) for both t1 and t2 is less than the quality of contributions by PFL participants from other subjects. Finally, additional clarification to this result can only be obtained through studies that involve larger samples. In the course of such a study, factors such as the motivation of participants *during* their work on the video task can also be taken into account, since they may influence the result.

The teachers who increased in their analytical competence were mainly those who worked with teaching videos as part of the course, as well as participants who indicated that they were given explicit information on how experts analyze teaching-related issues. The finding that an examination of teaching videos will increase competence for video-based teaching analyses seems almost trivial, yet it shows again that active application and the idea of practice are indeed significant factors in professional development (see also Neuweg, 2010).

Outlook

Until now, externaly conducted research on university courses has been limited to assessments by course participants with regard to their interests, competencies, motivation, or learning strategies, as well as the illustrated video task that records the competence found in analyzing one's teaching. The effects on teachers' actions; i.e. on teaching practice as well as on students' learning attitude and performance have been only refined to teachers' own investigations and some analyses by PFL staff. Similarly, the effects of courses on individual schools and other colleagues have not been investigated on a large scale.

Therefore, it is still an open question to what extent the results of this study are really relevant to the teaching practice. Irrespective of this uncertainty, the findings suggest that practice sequences should increasingly be built into research-based learning approaches for teacher professional development.

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Teacher Expertise Explored as Mathematics for Teaching

Elaine Simmt

Abstract The nature of the teacher's encounter with mathematics has taken prominence in the last decade as teacher educators research the mathematics of teaching. In this chapter teacher's expertise is articulated as mathematics for teaching (MFT). A model, theorized from complex learning systems, is discussed in this chapter. It posits MFT as multi-layered and nested knowledge involving subjective understanding at the core, enveloped by an understanding of the collective knowing that emerges from the interaction among individuals, which in turn is nested in knowledge of evolving and emergent curriculum structures, and further nested in a knowledge of the broader culture of mathematics. In this chapter the MFT model is read against the actions and interactions of a group of mathematics teachers in a professional development session to reciprocally explore the teachers' encounter with mathematics while explicating the model.

Keywords Complexity theory · Teacher education · Binary operations · Commutativity · Mathematics for teaching

Over the past decade there has been a great deal of discussion in the mathematics education community about the nature of the teacher's encounter with mathematics.[1] There is growing consensus that teachers engage in mathematics differently than do research mathematicians and others who create and use mathematics as part of their work (Ball, Hill, & Bass, 2005; Adler & Davis, 2006; Proulx & Bednarz, 2008). Broadly speaking, the scientist and economist use mathematics as a tool for understanding particular kinds of phenomena; the mathematician creates mathematics through the study of structure, change, invariance, etc. However, the mathematics the teacher enacts is quite different. She begins with this body of knowledge (systems, objects, processes. . .) we call mathematics (see Davis, 1996)

E. Simmt (✉)
University of Alberta, Edmonton, AB, Canada
e-mail: esimmt@ualberta.ca

[1]Take for example, Working Group 30 at ICME-11 on Mathematics for Teaching and the November 2009 special issue of *For the Learning of Mathematics*.

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and is charged with the responsibility of working with mathematics from this body of knowledge with the children and youth she teaches. Different from the scientist, economist or mathematician's work the teacher's work is an encounter with mathematics as both a body of knowledge already established and as knowledge that will emerge through an individual's cognitive activity. For the teacher, mathematics is at once a product of a culture and a new production of the learner. The teacher's expertise then involves the ability to appropriately interpret the mathematics of the learner as meaning-making acts by the individual and as acts within the cultural domain of mathematics. The teacher's expertise enables her to negotiate the middle way between individual meaning making and cultural knowledge.

Let us take the case of dividing fractions and speculate about the nature of the teacher's expertise and the potential impact of that expertise. She has access to mathematical objects such as the algorithm, "invert the divisor and multiply" and a variety of understandings of division and of rational numbers. Additionally, she understands the student as a learner who makes meaning out of his experiences, she has knowledge of the curriculum sequence and she understands the class dynamics. Thus the teacher's pedagogical moves (Powell & Hanna, 2006) take into account the mathematical objects that exist in and for the community and individual meaning making. It might be argued that in the case of dividing fractions that all too often the teacher sacrifices individual meaning making while privileging the algorithm (invert and multiply). This can result in students who are able to compute a quotient but cannot explain why the algorithm works or what sort of situations involves the division of fractions. This is not to say that there is not also the possibility that the teacher privileges individual meaning making by encouraging students to generate their own algorithms (for example). In this case one consequence is that it is possible that the students do not get access to a well-established cultural object (algorithm noted above) that is efficient for working in an algebraic context. In an attempt to understand the expertise needed for such mathematical lessons I ask: What is the nature of the knowledge teachers "need" for teaching children and youth mathematics? And, how might teachers develop expertise for teaching mathematics?

Such questions have received a great deal of attention especially since Shulman (1986) discussed teacher knowledge as general pedagogical knowledge, knowledge of learners, knowledge of educational contexts, knowledge of educational purposes, content knowledge, curriculum knowledge, and pedagogical content knowledge. Ball and her colleagues (e.g., Ball, Thames, & Phelps, 2008) have been working to further refine Shulman's categories of content and pedagogical content knowledge. Ball and her colleagues' elaboration within subject matter knowledge distinguish among common content knowledge, knowledge at the mathematical horizon and specialized content knowledge. They make distinctions within pedagogical content knowledge pointing to knowledge of content and students, knowledge of content and teaching and knowledge of curriculum. In the research with my colleague

Brent Davis[2] (Davis & Simmt, 2006) we too have explored the nature of teachers' mathematics. Although we find Ball et al.'s (2008) categories useful we frame our work somewhat differently. For example, we understand *mathematics for teaching* (MFT) not as an object or a set of skills stored in ones head but rather as an emergent phenomenon that is enacted in the context of teaching mathematics (see also Powell & Hanna, 2006). Also important to our work with teachers are two premises. Firstly, knowledge is a process not a thing. Von Foerster's (2003) definition of knowledge is useful here. He explains that knowledge is persistently taken as a commodity or a substance when it is the "processes which integrate past and present experience to form new activities, either as nervous activity internally perceived as thought and will or externally perceivable a speech and movement" (p. 200). Secondly we presume that teachers are *knowers*. In other words, we work from a surplus rather than deficit model. We do not begin with what the teachers do not know but rather what they do know. Teachers know mathematics; they know how to teach; and they know how to learn. Further, we assume that this knowing emerges in the context of their interactions with others (whom are most often but not always students).[3]

Mathematics for Teaching

In work already reported Davis and I used the case of the teachers exploring multiplication to develop a model for observing and interpreting MFT (Davis & Simmt, 2006). The model suggests MFT as nested knowledge (Fig. 1), with knowledge of subjective understanding at the core, enveloped by an understanding of the collective knowing that emerges from the interaction among the individuals, which in turn is nested in knowledge of evolving and emergent curriculum structures, and further nested in a knowledge of the broader culture of mathematics which is also growing (mathematical objects).

Some elaboration on the various kinds of knowledge that comprise this model for observing MFT is useful. Subjective understanding (Pirie & Kieren, 1989) and classroom collectivity (Bowers & Nickerson, 2001) happen in "lived time" and hence are commonly understood as dynamic and ever transforming. A teacher observes the growth of these phenomena in the minutes, hours, days and weeks when working with learners and classes of learners. However, it is not only subjective understanding and classroom collectivity that are dynamic. The outer-layered

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[2]In this chapter I make extensive use of work done in collaboration with Brent Davis. Throughout the chapter I use the "we" pronoun whenever I am talking about our shared research site or referring back to ideas explored in earlier papers or discussions we have had. However the interpretations of our work that you read here are my interpretations and Davis should not be held responsible for errors or weaknesses in the arguments I present.

[3]This is a key assumption in our work and based on an enactivist theory of knowing (Varela, Thompson, & Rosch, 1993).

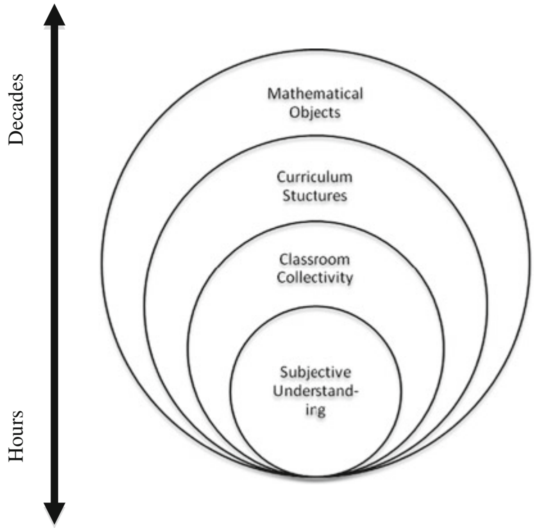

Fig. 1 Davis–Simmt model with relative time scale for observing MFT (adapted from Davis & Simmt, 2006)

phenomena that are commonly thought of as fixed are better interpreted as mutable (indeed dynamic) when re-thought as learning systems and observed over longer periods of time. Curriculum structures are commonly thought of as either curriculum resource materials or mandated programs of study. In the work of curriculum theorist Ted Aoki (see Pinar & Irwin, 2005) we find an elaboration of the notion of curriculum, one in which thought, action and relationships among the teacher, learners and objects take shape as curriculum-as-lived. With this elaboration and the recognition of the temporality of curriculum-as-planned (mandated) curriculum is understood as dynamic, evolving, and transforming over days, weeks, months and years. The fourth layer, mathematical objects, can be observed as dynamic when one examines the history of mathematics over years, decades and centuries (Swetz, 1994). Indeed there are many accounts of the histories of mathematics that illustrate the changing nature of mathematics. Those accounts point to the emergent collective learning systems we identify as bodies of knowledge or disciplines.

Investigating Teachers' Mathematics

In our work we offer teachers tasks and place them in situations that can be explored mathematically with others and that have the potential to trigger their MFT. Our challenge has been two-fold: to create learning environments for teachers that at once enable their pedagogical awareness and their mathematics to be brought forward into the situation so as to expand their professional knowing to engage in new teaching situations; and to theorize the teacher's mathematics knowing that emerges within those research environments. One might recognize the former challenge as offering professional development and the latter as a creating a research site. It is worth making a distinction here between this approach to exploring teachers' mathematics and how other colleagues might explore teachers' mathematics. Because we begin with the assumption that teachers' knowledge occurs in interaction with others we do not observe for their activity with individual activities such as paper and pencil questionnaires or one-on-one interviews. It is also assumed that the task itself need not specifically call for a pedagogical question (that is questions about the knowledge of students and teaching). The pedagogical considerations, as demonstrated by Proulx (2009a) naturally arise for the teachers with the context of the activity, as the teachers engage with each other in the tasks. My claim is that those are precisely the specialized *knowings* that teachers enact as part of their MFT. I believe another difference is in how we position ourselves in our research settings. As professional development facilitators and as educational researchers *we too* are working on our MFT. Our knowing and the environments we create evolve with the actions and interactions of all the participants. It is not uncommon for us to come to new mathematical and pedagogical insights as we work with the teachers as they work on the tasks. Like other researchers and teacher educators we set general topics for exploration based on our understanding of the needs and desires of the teachers but we do not prescribe a set of learning outcomes to be achieved; rather we create learning spaces to *work on mathematics* collectively (Proulx, 2009b).

Research Design

The data used for illustrative purposes in this chapter comes from research conducted with a group of K-12 teachers from a school district in Western Canada. A core group of 8–10 teachers met every 3–4 months in professional development workshops over the course of 2 school years. Two of the teachers from this group were observed in their classrooms a year after the in-service sessions. The teachers' in the group varied from a first year teacher to a number of teachers with more than 10 years experience. The teachers in our study had a minimum of a 4-year university degree. They would have taken at least one university mathematics course and one curriculum and instruction course specific to mathematics. All of the teachers taught mathematics as part of their load, although most of the K-6 grade teachers were generalists and responsible for teaching a number of subjects. In spite of their pre-service education and the years of experience they brought to our professional development sessions, the teachers further developed their expertise in those sessions.

The sessions the teachers participated in were structured as workshops around a particular area or concept taken from the school mathematics program of studies. Because the teachers represented the K-12 curriculum we made a point of selecting topics that provided numerous opportunities to make connections across as many grades as possible. For example, multiplication turned out to be a particularly rich site for the emergence of the teachers' knowledge likely because of the significance of that topic in the teachers' histories. The teachers could connect with the topic and found value in learning about what the students did in the grades prior to coming to their classes and what the students would do with multiplication when they moved on to higher grade levels.

Each session was facilitated by two researcher/teacher educators (henceforth referred to as the facilitators) and observed by one to three graduate student assistants. The graduate students video taped the sessions and took field notes based on their own particular interests. By asking the students to observe based on their own interests we found we had a wider range of interpretations of the events brought into our research discussions. Each session was de-briefed by the researchers. When de-briefing we discuss the mathematics that emerged in the session, the mathematics and pedagogical insights that were new for us, the curricular connections that were raised and other things that were notable for anyone of us. Then we return to the videotapes to examine the things we marked for further analysis. We do not transcribe the tapes in full but only those sections one of the research team selects for further analysis/interpretation. Once those sections are transcribed a mathematical activity trace is done (Reid, 1995). This trace highlights the mathematical actions, objects, metaphors, analogies and explanations that emerge in the interactions of the participants. The model for observing MFT was developed through holistic work with the data informed by enactivism and complexity and each new session was used to recursively inform and elaborate the theory.

An Illustration of the Emergence of MFT

Binary operations is a fundamental topic in elementary mathematics and foundational to mathematical understanding. It goes without saying that a teacher's expertise involves much more than being able to carry out the binary operations or being able to explain algorithms for binary operations. What follows is an illustration of teachers' mathematics as it emerges in interaction among a group of teachers (and two researchers) working on a prompt involving division.

In this particular session the teachers explored the question: What is division? The first responses came quickly: "fair sharing" or "equal distribution." But few other suggestions followed. The question was followed up with a problem for the teachers to work on taken from Silver and Cai (2005).

> Pose a problem that can be solved using the division statement $540 \div 40 =$?
> How many different problems can you pose and solve?
> Two problems will only be different if they give different answers [1].[4]

The teachers in the group individually and together came up with a number of different problems that would require the operation $540 \div 40$. [2]

- *There are 540 kids in the school and 40 kids in a classroom. How many teachers will be hired?* [an example of grouping/sharing]

[4] I use numbers in square brackets to highlight key utterances, actions and interpretations in order to refer back to them.

- *There are 40 streetlights along a street. How far apart will they be?* [an example of partitioning]
- *540 students. Put 40 kids on each bus. We need 14 buses. Just like you need 14 teachers!* [grouping]
- *Brandon has 40 girlfriends. He has $540. How much will he spend on each girlfriend at Christmas?* [partitioning]

Then, after some discussion of the particular problems the following interpretations for division were added to the list.[5] [3]

- Factors (find missing factors)
- Missing dimensions for area
- Repeated subtraction
- Subset making
- n groups?
- How many groups of n?
- Undoing multiplication

As the participants worked through these problems and discussed them with each other and the facilitators the focus of the teachers' collective thinking moved from identifying some problems that would satisfy the constraints of the problem posed (see examples noted above), to examining the different interpretations of division that emerged (sharing or distributing/grouping, partitioning/making equal intervals), to exploring the nature of the solution space (whole number, whole number with remainders, rational, money and measures). The teachers' collective expertise involved all of those understandings of division. As the conversation was slowing down the following exchange occurred.

Teacher1[6]: The other thing I thought about [was] division and subtraction. [4] I was thinking of my class [5th grade] [5]. They really have difficulty in understanding that the big number comes first [6]. And they will tell you that two divided by six is three, for sure. And I'd say, "do it on your calculators, what do you get?" And they'd just go, "like there must be something wrong." And same thing [happens] with subtraction. They don't understand. And somehow, saying that putting the big number first isn't enough. [7]

Teacher2: And when you say, "How many groups of so are in such and such?" [8] And then—

Facilitator1: That would be carried on from addition and multiplication where order doesn't matter— [9]

[5]This list is a composite and does not reflect a particular order or the comments of a single teacher or group.

[6]A comment about the notation used in the transcripts – an m-dash is used to indicate an interruption in the speaker's speech; ellipses are used to indicate missing speech.

Teacher3: Yah! Suddenly, order matters and the learners—

Facilitator2: Addition and multiplication are commutative; order doesn't matter. Now [in the case of] subtraction and division, it's not that the big number comes first. It's that order matters. . . [10]

Teacher4: It's interesting you made a point about order mattering. . . most of what I do is teach the Learning Assistance Class and children struggle with reading. . . [11] There are a number of them who struggle with attending to the order of the print. That's a challenge. I wonder if the two are connected?

This illustration is useful to us for discussing teachers' expertise in terms of MFT and for noting the gaps in their expertise that are revealed and subsequently filled while working on their own mathematics. Using the Davis–Simmt MFT model (Fig. 2) I classify the utterances and activity of the participants in the group and of the group itself. An elaboration of the classification follows.

The session began with the prompt to find some problems that fit a particular set of constraints; selecting the task is observed as an act of curriculum knowing by the facilitators. With the selection and posing of this task, we observe the space of the curriculum-as-planned and the curriculum-as-lived. This knowing is mapped on to the Davis–Simmt model for observing MFT in the curriculum structures layer [1]. As the participants contemplate the task and begin to suggest problems which fit the constraints their knowledge of mathematics emerges and so too does their

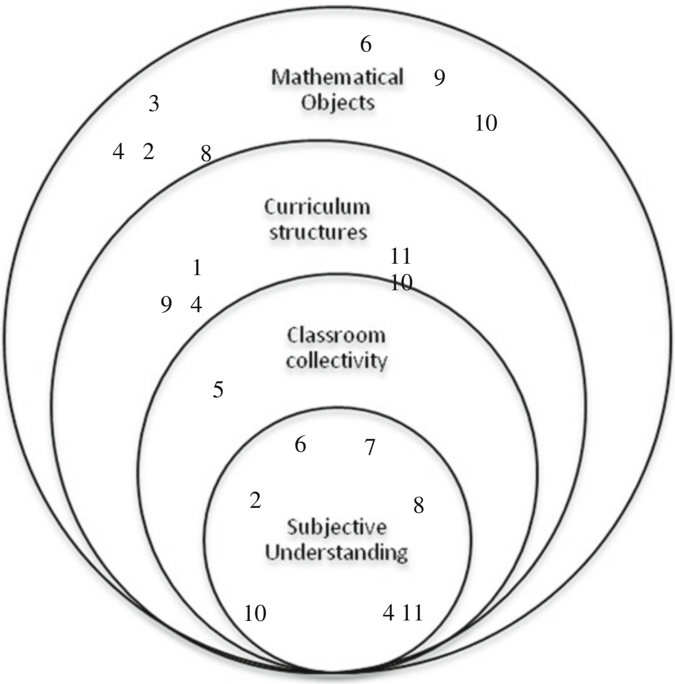

Fig. 2 Classification of utterances and activity as elements of MFT

awareness of the participants within the group. The participants' past experiences as teachers are integrated with their present experience in this group of teachers. Consider for example how two of the problems are drawn from the school context and the fourth problem included the name of a young male teacher in the workshop (the only male in the group). Such actions reflect the teachers' understandings of learning and learners and how to make meaning out of the shared experiences. The teachers "know" that one way of making a problem relevant to the learner is to set it in familiar contexts and include the learners themselves in the context; their knowledge of subjective understanding is brought forth in these interactions [2].

After a few of the examples were shared among the group their attention was drawn to the underlying understandings of division that were represented in the examples provided as they shifted from talking about specific examples to discussing categories: division as fair shares, repeated subtraction, finding a factor, finding a missing dimension in a measure and so on demonstrating the multiple interpretations these teachers have of division. In discussion we observe the emergence of the groups' understanding of the mathematical object of division [3].

It was late in the workshop when the fifth grade teacher raised the issue of students who confound the order an expression is written in. She expressed her concern that students do not know to "put the big number first." As already noted, the facilitators did not intentionally build in to the tasks pedagogical concerns. But the teacher's knowing in this context included her experiences with learners hence it was completely complicit with her MFT. This teacher connected her experiences with learners who make a similar error in division and subtraction and in doing so reveals knowledge of learners, curriculum and mathematics all at once [4].[7] Of particular note is the teacher's comment, "And somehow, saying that putting the big number first isn't enough." This is an interesting comment to unpack. On one hand the teacher understands the learner's need for more than simply being told something which is a demonstration of her understanding of subjective understanding [6] and on the other hand she reveals a gap in her mathematics. She doesn't seem to have a mathematical explanation to offer the learners for ordering the mathematical expressions they write from particular contexts. This seems to be a gap in the teacher's understanding of commutativity (and non-commutativity) as it relates to subtraction and division. This is particularly interesting since this teacher is very unlikely to make a mistake writing an expression from a context, computing differences or quotients, or even discussing what subtraction or division is. Note that a second teacher wondered if the student's difficulty is in the meaning making. She asked about how the learner understood division. By pointing out how the learner might make sense of the operation as grouping revealed this second teacher's understanding of both mathematics and subjective understanding [8], and at the same time contributed meaning to the group of teachers working through these ideas at that very moment.

[7] It is useful to note that students would have been subtracting and dividing prior to the fifth grade but always with whole number solutions.

One of the facilitators connected the problem the teacher encountered with how the mathematics content was sequenced within the curriculum; that is the non-commutative operation of subtraction follows addition and division follows multiplication [9]. In doing so he demonstrated how he made meaning of mathematical objects as elements in the curriculum-as-planned. The second facilitator conjectured that the problem for students whom make such mistakes is based on an over-generalization of order not mattering in the cases of addition and multiplication, operations students learn prior to subtraction and division. This reveals her understanding of the curriculum-as-planned and curriculum-as-lived structure, the subjective understanding of the learner, and mathematical objects [10]. Finally, a third teacher entered the conversation making a connection to her experiences teaching children to read. With this utterance the conversation moved from the specifics of teaching children binary operations to a more general conversation of the difficulty some students experience learning to read, once again demonstrating the teacher's knowledge of subjective understanding [11].

Opportunities Missed and Made for Enhancing MFT: Developing Expertise

In this discussion I address the opportunities missed and made for enhancing teacher's expertise (or her MFT). In particular I point to how the in-service session provided opportunities for the teachers to further develop their MFT and speculate how it came about that such fundamental gaps exist.

The task given to the teachers in the workshop (to find problems for the division expression 540÷40) provided a particularly interesting space to explore the teachers' expertise in terms of their MFT. As a facilitator of the session, I thought the problem would be very useful for exploring the explanations, models, metaphors and strategies teachers use for division. However, I did not anticipate the gap in the teacher's understanding of commutativity. Because I am a teacher educator I am ultimately faced with the question of what are the formative experiences for teachers (both pre-service and in-service) that are most worthwhile. It is worth unpacking how that gap may have been have been left/created in the teacher's MFT in spite of her formal tertiary education and experiences in the classroom.

The problem of order in binary operations (or reading) requires a teacher's attention as she mediates the knowing of the individual and of the culture. Imagine a child responding to the question, "If three birds fly away from a wire and there were five to begin with, how many birds remain on the wire?" The child communicates his meaning making with the response, "Three minus five equal two" or writes, 3–5=2. The teacher (Teacher1) interacting with the child is confronted with the difference between the way the child has expressed his knowing and the conventional from of communicating this expression used in the mathematical community. We know the teacher's mathematics for teaching emerges in interaction with the learner. It is not known how the mathematics teacher who posed this dilemma came to use "the big number first" as a strategy with her learners but it is useful to speculate. She knows subtraction is not commutative, or maybe she knows that when communicating subtraction in an expression you first write down the minuend and take from it the subtrahend (mathematical objects). It is possible that because her class of learners are working strictly within the whole numbers she knows the children will not study integers for a few more years (curriculum structure). Hence she anticipates that the problems these particular learners will encounter at this point in time (classroom collective) have been designed to only have whole number answers, so she thinks of the situation as the big number coming first (subjective understanding).

The teacher's expertise is demonstrated by how her actions integrate all four aspects of MFT in the interactions she has with others. As is evident in the illustration, at once MFT involves the mathematical objects, the curriculum structures, the classroom collective, and the subjective understandings. Difficulties arise if the teacher ignores any one of those nested dimensions of MFT. Ignore the mathematical objects as they exist in the culture and the child grows up with idiosyncratic understanding; ignore the order of presentation of the curriculum and there is too much to teach; ignore the child's understanding and the child resorts to memory and rote learning. Teacher1's comment about the difficulties students experience when beginning subtraction and division points to the need for a teacher to have expertise across all four layers of MFT because she lives at the intersection of the body of mathematics as a cultural artefact and mathematics as individual meaning making in a curricular and classroom context.

It follows that MFT is a concern for the teacher educator whose task it is to identify and create experiences for the novice (and experienced) teacher to develop appropriate mathematics. Let us return to our example of commutativity to consider pre-service education. In the undergraduate teacher education program at the University of Alberta, Canada secondary mathematics majors are required to take 12 half-year mathematics courses and elementary mathematics minors are required to take up to 4 half-year mathematics courses. In university mathematics the study of commutativity usually occurs as a topic in the contexts of linear algebra and abstract algebra. In those cases it is discussed as a property of a set and is generally used for classification purposes; that is for distinguishing a set as commutative group or a field. Knowing a set is commutative then allows one to use the set in particular ways because you know how it behaves. Such knowledge is useful for the mathematician but the teacher needs a further understanding of commutativity. She needs to know what the implications of commutativity are for the teaching of binary operations on whole numbers, integers, rationals and reals because her task is to offer appropriate experiences for learners coming to understand number systems, binary operations and elementary algebra.

In the work of the teacher commutativity is a significant concept as students learn to operate on whole numbers. Indeed teachers place much emphasis on the fact that addition is commutative, $2 + 3 = 3 + 2$. Their experience with children as learners suggests the value of this kind of emphasis. On a number of occasions in our study it was clear to us that the teachers take advantage of addition and multiplication being commutative to help children develop their sense of number and quantity and to become more familiar with addition and multiplication "facts." However, a pupil's experience with addition comes first in arithmetic and is prior to the introduction of negative numbers. Thus when the pupil moves on to subtraction there is no reason for him to anticipate that the expression 5–3 does not equal 3–5; this is a simple generalization of the child's experience with addition. In contrast, when teaching commutativity in a university algebra course the instructor has the advantage of teaching students whom have an understanding of real numbers and already understand that there are distinct outcomes to these two computations. As university students are introduced to new groups they use existing understandings of commutativity to make sense of objects (sets) that are not commutative. But for children learning binary operations for the first time such understanding is not available to them to work from. Hence a school teacher's mathematics needs to include explicit understanding that encountering the non-commutative situation of subtraction over the whole numbers calls for a new kind of object, a new number system and for more deliberately ordered expressions for communication purposes.

Although there exist many researched-based models for developing mathematical expertise for teaching in the context of in-service education and in pre-service education there is little information about the extent to which these models are having an impact on the broader programmatic decisions in pre-service teacher education, at least in Canada.[8] At the University of Alberta, the programmatic elements with respect to disciplinary knowledge have remained relatively stable over the last six decades. Secondary mathematics majors study most of their mathematics content with students from the Faculty of Science. Less than 20% of their instruction in mathematics is focused on mathematics for teaching. In the case of the elementary generalist they study 50% of their mathematics in the Faculty of Education where professors are using the research in MFT to inform their instruction however 50% is a single half-year course. They also take 1 half-year course offered by the Department of Mathematics. That course is specifically designed for pre-service teachers. The research in MFT speaks loudly about the need for rethinking the mathematics pre-service teachers study in their undergraduate programs. Fortunately, in Alberta in-service education does not have the same constraints as does pre-service education. Hence it is in this environment that I have been able to study MFT and to design educational opportunities grounded in the research on MFT.

Concluding Remarks

With this chapter I join others in the mathematics education community to assert that a teacher's expertise involves a specific form of mathematics. That mathematics is marked by a different kind of encounter than we find in the work of the research mathematician and others who create and use mathematics. In other words, it is not simply that the mathematics is different but there is also a difference in what

[8]One notable exception is the programming at the Université du Québec à Montréal See Bednarz and Proulx (2005) for a description.

teachers are expected to do with the mathematics and there is a difference in the contexts in which they do their mathematics. Teachers' expertise lies in that fact that their mathematics is done in interaction with others. Their expertise lies in the fact that they must understand mathematics as at once well-established knowledge and as enacted knowing. Their expertise lies in the fact that they engage in mathematics as both a cultural product and as personal constructing. Their expertise includes the collective we call a class and within a structure we understand as the curriculum.

For teachers to be able to work effectively with children and youth there is a need for teacher educators to understand the kinds of mathematics teachers will encounter in their work life and provide courses that offer appropriate mathematical experiences and content. As a community it is important for teacher educators to continue to conduct research into teachers' mathematics and to create teacher education programs that reflect the research we do. Our collective research is beginning to suggest that university mathematics courses designed for the future scientist, economist and research mathematician are inappropriate for future teachers (Proulx & Bednarz, 2008). As a community we have a need to examine our programs and offer new possibilities grounded in research rather than tradition. Further research and creative program planning is needed to create courses within programs which focus specifically on mathematics for teaching so that teacher education has the potential to develop the mathematics teacher's expertise.

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Part III
Expertise in Mathematics Instruction in an Eastern Setting

Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics

Yeping Li, Rongjin Huang, and Yudong Yang

Abstract This study aimed to characterize Chinese teachers' expertise in mathematics instruction through analyzing five selected expert teachers' videotaped lessons, their lesson designs and reflections. A prototype view of teaching expertise was adopted and used to identify similarity-based central tendencies that are shared among these expert teachers. Data analyses revealed six central tendencies of these teachers' lesson instruction and thinking. The case analysis of one expert teacher's lesson instruction was also used to provide rich descriptions and illustrations of the prototype of these teachers' teaching expertise. The findings help us to develop a better understanding of the complexity of teaching expertise valued in China, and are important to teacher educators in their efforts to improve professional development for teachers.

Keywords Expert teacher · Teachers' expertise · Chinese teacher · Prototype · Teacher thinking · Lesson instruction

Introduction

To improve the quality of classroom instruction, one popular approach used in practices is to learn directly from expert teachers. This is especially the case in China (Huang, Peng, Wang, & Li, 2010), where expert teachers are officially conferred and socially recognized with teaching taken as a professional practice that is open to public scrutiny and discussion (Li & Li, 2009). A school-based teaching research system has long been established in China since 1952 and various teaching research activities commonly exist in individual schools and beyond (Yang, 2009). Exemplary teaching behavior is identified both formally and informally through frequent classroom observations and exchanges of ideas among teachers in and outside

Y. Li (✉)
Department of Teaching, Learning and Culture, College of Education and Human Development, Texas A&M University, College Station, TX 77843-4232, USA
e-mail: yepingli@tamu.edu

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of their own schools. Teachers' work accomplishment is formally acknowledged by rank and title according to not merely their years of experience, but more on their teaching performance, teaching research outcomes, and level of instructional leadership (Li, Huang, Bao, & Fan, 2011). Expert teachers in China are not just experienced teachers; they are part of the teaching culture in China and also play an important role in nurturing that culture. The policy and practices in ranking and promoting teachers in China make it practically feasible for others to model after expert teachers' classroom instruction.

While the approach of learning from expert teachers is commonly practiced in China, how Chinese expert teachers perform in classroom instruction has not been well understood especially to outsiders. Although existing studies have revealed that Chinese mathematics teachers have a profound understanding of the mathematics that they teach (An, Kulm, & Wu, 2004; Li & Huang, 2008; Ma, 1999), much remains unknown about expert teachers and their teaching expertise that is valued in China. The identification and examination of expert teachers' expertise in teaching also becomes necessary for teacher educators in China in their efforts to help other teachers develop expert-like performance in classroom instruction. In this study we thus aimed to explore teaching expertise valued in China through examining expert teachers' classroom instruction.

The importance of identifying and examining teachers' expertise has long been recognized in educational research. In the United States, many researchers identified unique features in expert teachers' knowledge and their teaching practices through comparing their behaviors and performances to those of novice teachers (e.g., Borko & Livingston, 1989; Leinhardt, 1989; Leinhardt & Smith, 1985; Livingston & Borko, 1990; Swanson, O'Connor, & Cooney, 1990). While the comparative approach has been effective in helping identify unique features that expert teachers have (in comparison with novice teachers), possible findings are often restricted to exclude those features that are not unique to one or two expert teachers focused in individual studies. This limitation makes it difficult to justify whether the identified features are shared by other expert teachers. Rather than contrasting two diverse groups of teachers, we tended to focus on a group of expert teachers in China and their classroom instruction in this study. It is an approach that utilizes a prototype view of expertise in teaching to identify and examine possible resemblance in teaching performances among expert teachers (e.g., Smith & Strahan, 2004; Sternberg & Horvath, 1995). Without comparing with novice teachers in this study, we were less restricted in identifying and examining features that are salient in expert teachers' classroom instruction.

Existing studies on the issue of expertise often faced a dilemma in identifying expert teachers and examining their expertise. On one hand, some researchers tended to focus on expertise through studying experts. They faced the difficulty of identifying those who can be taken as experts in their study. In fact, researchers often used different criteria to identify and select "expert" teachers across studies (e.g., Berliner, 1986, 2001). Some may rely on the measure of years of teachers' teaching experience, whereas others may use students' testing scores and/or local educators and administrators' recommendations. The inconsistency in identifying and selecting expert teachers makes it very difficult to compare and summarize findings across different studies. On the other hand, others developed and utilized a list of specified features to identify experts. But this method has also been criticized, because there is not such a well-defined standard that can absolutely separate all experts from non-experts (Sternberg & Horvath, 1995). The complexity in identifying experts and examining expertise becomes acute in studying teachers' expertise in teaching in the West where teaching is not subject to public scrutiny (e.g., Kaiser & Vollstedt, 2008). Expert teachers, who are not necessarily experts in teaching, may be selected to examine their expertise in classroom instruction. However, it presents less of a concern when identifying and studying expert teachers in China, where the policy and practices in ranking teachers mainly in terms of their teaching is widely used. Thus, we planned to use case study methodology to select a group of expert teachers and then examine their shared similarities in teaching.

Chinese Expert Teaching in School Mathematics: Background and Theoretical Considerations

Mathematics Teacher as a Profession in China

As a system with a long history in education, China has a deeply-rooted cultural value orientation about teachers and teaching. Teaching is taken as a professional, not private and personal, practice in China. There are common beliefs and values of what teachers need to do for their students (*Jiao Shu Yue Ren*). Chinese teachers are respected for having in-depth knowledge and being moral examples (e.g., Beishuizen, Hof, van Putten, Bouwmeester, & Asscher, 2001). In particular, expert teachers in China are often identified as the ones who bear most of the culturally-valued moral characters and expertise for others to follow. What teachers can learn from others is the type of knowledge and skills that are publically valued and often locally proven as effective. Individual teachers realize the need to improve their professional knowledge, and they know what they can learn from others through their daily teaching activities. Teachers' expertise is practical, and teachers acknowledge expertise differences from one teacher to another.

According to the regulation of secondary teacher's position promotion in China, the positions of secondary teacher include senior-rank teacher, secondary teacher level 1 (intermediate), and secondary teacher level 2 (primary). For each level of teacher position, there is a specification in terms of political, moral and academic aspects. For example, the requirements for being a teacher at the senior rank include (1) 5 or more years' experience as a secondary school teacher at level 1 or with a Ph.D. degree, and (2) demonstrating the ability to take the responsibility as a "senior" secondary teacher. In particular, the second condition is further specified as that the candidate should (a) have a systematic and sound academic training and professional knowledge, have plentiful teaching experience for being able to teach effectively, win prizes in teaching contests at the municipal level or beyond, or specialize in political and moral education and classroom management and have rich experiences; (b) engage in secondary school education research and teaching and be able to write teaching experience summaries, scientific reports and research papers that integrate theory and practice at certain academic level, or make remarkable contributions to the improvement of other teachers' academic level and teaching ability.

Teachers' ranking and promotion policy functions as an important mechanism in specifying aspects of teachers' professional expertise that is valued for promotion and supporting teachers' professional development in China (Li et al., 2011). From an academic perspective, it spells out in detail what the system values as teachers' professionalism. In addition to the general regulation provided by the Ministry of Education in China, the provincial regularity (e.g., Educational Bureau of Jiangsu Province, 2004) provides more detailed requirements and methods, including even quantitative measures. For example, in specifying the teaching quality of teachers of an exceptional class (特级教师, an honorary title for some senior-ranking teachers), it is required that the teacher has developed his/her unique teaching style. The teacher is a leader of the teaching subject at the municipal or county level, has shown the quality of his/her teaching with some public and exemplary lessons, and has won a prize of a teaching contest at the national level. In specifying the expected contribution in education reform and research (esp. in teaching method research), it is required that the teacher has a monograph or more than three research papers published in journals at the provincial level or beyond. In this study, we refer expert teachers as those with a senior rank including an honorary rank of exceptional class.

Theoretical Considerations

Three models of expertise helped structure the framework of this study. The first is the national teacher ranking and promotion system (NTRPS) developed and used in China. As mentioned above, this system provides a ready model of expertise for this study to identify and select expert teachers that are officially recognized in China. Experts, or teachers with at least a senior rank based on NTRPS, are those who demonstrate an accomplished practice in classroom instruction. The system used to judge teacher practice is content specific and based on a consensus among practitioners starting from the school level. Teachers who are evaluated as accomplished in terms of NTRPS are awarded higher ranks that come with a monetary increase and professional status change. All teachers who are selected in this study had an advanced ranking (i.e., the senior rank) in the NTRPS model.

Although expert teachers in China are evaluated and promoted through the NTRPS, expert teachers' classroom instruction is often recognized locally among teachers but has not been examined systematically to document their expertise in teaching. To examine teaching expertise of Chinese expert teachers, we adopt a prototype view of teaching expertise as a theoretical framework (i.e., the second model of expertise in this study) to analyze, interpret, and describe the classroom instruction practices of several expert teachers (Smith & Strahan, 2004; Sternberg

& Horvath, 1995). This theoretical framework is originally developed by Sternberg and Horvath (1995). According to Sternberg and Horvath, teaching is a complex and holistic practice that can exhibit various features across classrooms. Expert teaching can better be described in terms of a "*prototype* that represents the central tendency of all the exemplars in the category" (p. 9, emphasis in original). They called for a "reconceptualization of teaching expertise" as a natural, similarity-based, family resemblance category of expertise that is shared by expert teachers. A prototype of teaching expertise is a summary representation of the central tendencies of teachers' classroom instruction in this category, and its content is evolving along with the identification and assembly of such central tendencies from different studies. Thus, a prototype view of teaching expertise should help us develop a more inclusive understanding of teaching expertise than a pre-defined standard, and "provide a basis for understanding apparent 'general factors' in teaching expertise" (p. 9).

Sternberg and Horvath (1995) developed this prototype view of teaching based on Rosch's (1973, 1978) notion of prototype in cognitive psychology research on natural language concepts, and other psychological studies on expert performance in various domains. In particular, they derived from psychological research a list of prototypical features of expert teaching in knowledge (content knowledge, pedagogical knowledge, practical knowledge), efficiency (automatization, executive control, reinvestment of cognitive resources), and insight (selective encoding, selective combination, selective comparison). For example, for the executive control, Sternberg and Horvath listed three sub-category features including planning, monitoring and evaluating. For the feature of planning, they specified that "expert anticipates difficulties in the execution of a lesson plan" (p. 15). However, they did not carry out specific studies of expert teaching by themselves. Instead, they called for studies to validate their list of prototypical features and examine teaching expertise as a similarity-based category. Smith and Strahan (2004) used Sternberg and Horvath's framework to study possible similarities among three expert teachers with diverse profiles. Based on their analyses of a variety of data collected from classrooms and professions (e.g., lesson observations, lesson transcripts, participant surveys, structured interviews), Smith and Strahan derived six central tendencies in broad categories (e.g., "these teachers maximize the importance of developing relationships with students", "these teachers show evidence that they are masters of their content areas", p. 365). As indicated by Smith and Strahan, their study complements Sternberg and Horvath's examination to gain insights on teaching expertise from a different angle. While Sternberg and Horvath examined teaching expertise mainly on cognitive mechanism and/or ability, they focused on teachers' practical (or tacit) knowledge of teaching practices. Although Smith and Strahan's study is limited with only three expert teachers, their study illustrated the value of examining teaching expertise with a prototype view.

Moreover, Lin (1999) used the prototype view of expertise in teaching to differentiate elementary mathematics teachers' expertise between novices and experts through structured interviews about classroom events. As this study also showed the value of this framework for capturing mathematics teachers' expertise through interviews, it should be pertinent to use the prototype view to guide the examination of teachers' performance in classroom instruction directly.

By focusing on mathematics expert teachers in China, the current study was designed to develop a prototype of mathematics teaching expertise through examining commonly recognized expert teachers' classroom instruction. With the prototype view of teaching expertise, we aimed to provide a rich description of similarities of what mathematics expert teachers do and say that will contribute to our understanding of the teaching expertise valued in China. The rich description and summary representations should provide the prototypical features, as shared by expert teachers, to inform mathematics teacher educators in their efforts to improve teachers' professional practices.

The third model of expertise used in this study is the cognitive analysis of teachers' classroom instruction that helps us to understand what expert teachers do and say in teaching. Classroom instruction is a dynamic and complex process that can be analyzed using different lens with various details. Different from the cognitive modeling of classroom instruction process that was commonly used in previous studies on expertise (e.g., Leinhardt, 1989; Leinhardt & Greeno, 1986; Schoenfeld, Minstrell, & van Zee, 2000), we intended to generate both rich descriptions and summary representations of expert teachers' instruction. In particular, we took a similar lens as the 1999 TIMSS video study to focus on the three important aspects in classroom instruction: content, students, and instruction (Hiebert et al., 2003). These three aspects are further specified as below:

(a) Content aspects: the lesson's content treatment, tasks used and connections made;
(b) Student aspects: students' learning and engagement in lesson activity;
(c) Instruction aspects: the teacher's use of instructional methods and discourse in content introduction and activity arrangement, lesson coherence, and activity variations.

As this approach was feasibly used in analyzing Chinese teacher's classroom instruction in a previous study (Li & Li, 2009), we expected that instructional analyses in these three important aspects would allow us to identify and examine similarities in mathematics teaching shared by expert teachers in China.

The Current Study

This study aimed to investigate teaching expertise in school mathematics that is valued in China. In particular, we took a case study approach to examine five expert teachers' video-taped exemplary lessons, their lesson designs and reflections on their video-taped lessons. By taking a prototype view of expertise in teaching, the study was designed to examine the extent of similarities in these expert teachers' thinking and instructional practices. In particular, the following three questions guided collection and analysis of data:

(1) What similar components can be identified from these expert teachers' classroom instruction?

(2) How similar are these expert teachers in their knowledge, thinking, organizing and presenting content in teaching?
(3) To what extent are these expert teachers similar in engaging and guiding students with different strategies?

Method

Data Sources

Five middle school mathematics teachers with a senior rank (hereafter, these teachers are denoted as T1∼T5) were identified by local mathematics educators and teachers as experts who demonstrated exemplary teaching (see Table 1 for their background information). They were invited (and thus agreed) to participate in this study. All participants were informed that the data collection was for research purposes only.

Table 1 Background information of the five participating teachers

Teacher	Education degree	Teaching experience	Teaching grade	Award	Participations of teacher PDPs*
T1	Bachelor	28 years	6–9	First prize of teaching contest of young teachers (D*, C levels)	Five years' teacher PDPs
T2	Master	14 years	6–12	First prize of teaching contest of young teachers (D, C levels)	Key teacher training programs (D, C levels)
T3	Bachelor	15 years	6–9	First prize of teaching contest of young teachers (D, C, N levels)	Senior teacher study programs, master teacher candidate training program
T4	Bachelor	16 years	7–8	First prize (D level) and second prize (C level) of teaching contest	Master teacher workshop at district level.
T5	Bachelor	12 years	6–9	First prize (D level) and second prize (C level) of teaching contest	Master teacher workshop and math gifted students coach.

Note: "*" PDP denotes Professional Development Program; D, C, and N mean District, City and National levels respectively.

Table 1 shows that all five teachers held a bachelor's degree in mathematics, one also with a master's degree in mathematics education. These teachers had an average of 17 years' teaching experience, ranging from 12 to 28. All of them won various awards in teaching contests at the municipal and/or national levels (for more information about different teaching contests in China, see Li & Li, 2009). They also participated in a variety of professional development programs (PDPs).

Although these teachers all needed to teach different content topics in middle schools, they were asked to provide a video-taped lesson in algebra that can represent his/her excellence and expertise in teaching. With the consent from all five teachers, further data collection was carried out to get relevant information about themselves and their teaching. In particular, the following two types of data were also collected in this study:

(1) Participating teachers' professional background. A questionnaire was designed to collect background information about teachers themselves, their beliefs about effective teaching, and their design and reflections of their own video-taped lessons.
(2) Participating teachers' views of others' classroom instruction and their views on effective mathematics instruction. Two video-taped lessons taught by two other expert teachers were provided to these five participating teachers. In these two video-taped lessons, the same topic, *a system of linear equations*, was taught using different instructional approaches. The participants were asked to watch the video-taped lessons and then share their views about the lessons by filling out a specifically designed questionnaire. We intended to use open-ended questions in the questionnaire so that the respondents could comment on the lessons based on what they value. In this way, the respondents' comments can help reveal not only their lesson evaluations but also the focal aspects in their evaluation. In addition, they were asked to answer two open-ended questions about good mathematics teaching and learning, and ways of evaluating the effectiveness of a mathematics lesson.

Data Analysis

These expert teachers' own video-taped lessons, their responses to the questionnaire related to lesson design and their reflections were taken as the main data for analyses in this study. These experts' comments on two video-taped lessons and also their views on effective mathematics teaching and learning were used as supplementary materials. All the data for this study was analyzed in the original language of Chinese. Selected data was translated into English to provide evidence in the later sections of this chapter. In particular, these teachers' videotaped lessons are transcribed verbatim, along with some contextual information and time recording for all the activities that happened in these lessons.

To address our three research questions directly, we analyzed these experts' video-taped lessons both holistically and analytically (Li & Li, 2009). The holistic approach aimed to capture overall features of the five expert teachers' lesson instruction in order to identify the main aspects of a possible prototype of expert teaching. Our further analyses included the following stages: (1) identifying and describing main segments of each lesson; (2) comparing these lesson segments and developing a common list of segments capturing the lesson structures across teachers (Merriam, 1998); and (3) a special video analysis software, Studio-code, was used to code all five lessons according to the developed segment list. This analysis helped generate the baseline data to derive and describe similar features of these experts' teaching.

With regard to these teachers' lesson designs, reflections and their comments on others' lessons, we developed a coding system through constant comparison within and across cases (Corbin & Strauss, 2008). Three categories of codes were developed and used: teacher knowledge, instructional process, and teaching skills and teacher characteristics. *Teacher knowledge* includes subject content knowledge (including treatment of important content points), treatment of difficult content points, development of mathematical thinking and abilities, and mathematical applications (problem solving and problem posing). *Instructional process* includes student-centered activities (i.e., self exploratory learning, collaborative and exchange), teacher-directed activities (e.g., presenting problems, explaining concepts and summarizing key points), contextual learning, and learning motivation. *Teaching skills and teacher characteristics* include basic teaching skills (i.e., blackboard writing, teaching language, and use of multi media), and improvisational ability. A list of (sub-) categories with explanatory examples is provided in Appendix 1.

Although the majority of these categories are self-explanatory, here we illustrated several selected categories. For example, T2 anticipated that "the majority of students are used to memorize the meaning of proportion. It is very difficult for students to understand the concept and its applications in real contexts". This statement is coded as "difficult points and treatment" under "teacher knowledge". As another example, T1 appreciated that in the video-taped lesson he watched, "the teacher used a problem with real-world context (chickens and rabbits stay in the same cage) to introduce the concept of system of linear equations". Then, the statement is coded as "contextual learning".

Except the analyses of the five experts' video-taped lessons as a whole, we also identified an expert's lesson as typical from the perspectives of content treatment, instructional strategies, and students' engagement. Then we provided some more detailed descriptions and analyses of the selected lesson to provide a rich picture of expert teaching.

Characterizing Expert Teachers' Instruction: Results and Discussion

This section is organized into two parts. First, the central tendencies of these expert teachers' teaching in terms of their instructional design, lesson instruction, and reflection were presented and summarized. Secondly, an expert teacher's classroom instruction was described and analyzed to further illustrate the prototypical features of experts' teaching.

Central Tendencies of These Experts' Mathematics Teaching

The video-taped lessons provided by these five teachers include different content topics in algebra: percentage (T1), proportion properties (T2), the formula method for solving quadratic equation (T3), a review of inverse proportion function (T4), and a review of linear and inverse proportion function (T5). The nature of these lessons also has some variations, with three lessons on introducing new content topics (Lesson 1–3 as taught by T1–T3, respectively) and two review lessons (Lesson 4–5 as taught by T4–T5, respectively). Being aware of these variations, the analyses and reports of video-taped lesson instruction focus on the identification of prototypical features and family resemblance across these experts' lesson instruction.

Based on the results from coding the five teachers' comments and reflections (see Appendix 2), we found seven commonly mentioned categories. Combined with a consideration of the interconnection among these categories and some similarities among these video-taped lessons, we identified six central tendencies of these experts' teaching in school mathematics. They include: (1) having sound subject content knowledge of teaching topics; (2) appropriately identifying and dealing difficult content points in students' learning; (3) emphasizing the development of students' mathematical thinking and ability; (4) using mathematics problem solving and posing for developing effective classroom instruction; (5) emphasizing and practicing student-centered instruction; and (6) motivating students.

These central tendencies will be explained and illustrated below with information derived from lesson instruction, teachers' explanation about their lesson designs, and their reflections.

Central tendency 1: Having sound subject content knowledge of teaching topics

Subject content knowledge refers to school mathematic topics that these teachers taught and the advanced mathematics related to teaching topics. All these expert teachers had a sound subject content knowledge, which was evidenced with the following facts: their bachelors' degree in mathematics, advanced professional rank, wining various teaching contests, their performances in video-taped lessons, and their opinions expressed in the questionnaires. First of all, teacher preparation program in China is dominated with the perceived need of providing a sound mathematical content training for prospective teachers (e.g., Li, Huang, & Shin, 2008; Li, Ma, & Pang, 2008). In particular, it intends to equip prospective mathematics teachers with: (1) a foundation for having profound mathematics knowledge and highly advanced mathematics literacy, and (2) an extensive review and study of school mathematics, with a focus on developing prospective teachers' ability to solve problems in school mathematics.

Secondly, as one of the prerequisite conditions of being promoted to the senior ranking, teachers are expected to have systematic and sound acquisitions of fundamental mathematics content knowledge, plentiful teaching experience and good teaching effectiveness, or to specialize in political and moral education, and classroom management and achieve a high performance with a rich experience (Huang et al., 2010). These five expert teachers' senior ranking, as awarded through NTRPS in China, provides another strong indication of their in-depth content knowledge in school mathematics they teach.

Thirdly, these expert teachers were all winners of various teaching contests organized in China. One of the critical aspects in evaluating contest lessons is about the content being presented and organized in lessons (Li & Li, 2009). These expert teachers' winning in various teaching contests suggested that they need not only to have a sound mathematics content knowledge by themselves but also to be able to demonstrate their understanding and skills related to the subject content.

In these video-taped lessons provided by these expert teachers, they demonstrated their in-depth understanding of the content topic being taught and their thoughtful considerations in making pedagogical treatment of the content. In fact, we did not find any mistakes in terms of content treatment (e.g., mathematical accuracy of concepts and formula, dealing with students' mistakes or misconceptions in lesson instruction). In the following paragraphs, we gave some examples of how these expert teachers thought about and dealt with important content points.

In their answers to the questionnaires, these expert teachers clearly expressed their thinking about content treatment in terms of connections between previous knowledge, knowledge being taught, and the knowledge to be learned in the future. For example, T1 identified the instructional objectives of his video-taped lesson as follows:

> Understanding the connections and differences between percentages and fractions, and mastering the conversion among percentages, fractions, and decimals.

This is a public lesson given to a sixth grade class in another school. The purpose of this lesson is to demonstrate how this newly added topic (moved from grade 8 to grade 6) can be taught innovatively. The video-taped lesson was prepared and used for participating in a teaching contest. The teacher was highly satisfied with the lesson, because "I introduced new knowledge through exploring a real life problem and consolidated the learned knowledge through interactions between the teacher and students, and extended students' learning through homework."

T1 further explained how to help students understand percent concept and master the conversion among percentages, fractions, and decimals through discussing contextual problems and using visual diagrams. In his teaching, he directly introduced an environmental prevention problem about how serious sand storms can harm human life (by showing photos on screen), and invited students to find out possible ways to prevent or ease the impact of a sand storm. When some students suggested planting trees, the teacher tabulated four types of trees, the total of planted trees, and the total of survived trees (as shown in Fig. 1).

Then, he asked students which type of tree has the highest survival rate. One student answered type D because it has the largest number of survived trees, while another student explained it should be type C because 84 out of 100 planted trees survived. Building on students' responses, the teacher highlighted that it is necessary to consider the rate of two numbers in order to decide which type of tree has the highest survival rate.

After that, the teacher listed all the survival rates (fraction) of the four types of trees. Immediately, the teacher asked students how to compare these fractions.

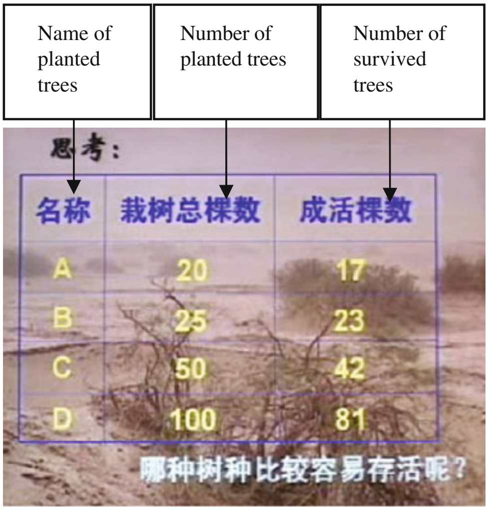

Fig. 1 Situated problems of environmental protection

Then one student said to transform fractions to have the same denominator. Another student said to convert them into decimals. Following students' suggestions, the teacher listed all the fractions with the same denominator 100 correspondingly, and students are asked to list all the decimals with their oral responses (See Table 2). Finally, the teacher stated that the new special fractions (with the same denominator of 100) were called percentages. It is denoted as %, and read as percentage *Pai Fen Hao*. Then, the teacher presented the formal definition of percentage on the screen and read the percentage of 85%. Students were asked to read three other percentages.

Table 2 Different presentations of fractions

Fraction	Fraction with the dominator of 100	Percentage	Decimal
$\frac{17}{20}$	$\frac{85}{100}$	85%	0.85
$\frac{23}{25}$	$\frac{92}{100}$	92%	0.92
$\frac{42}{50}$	$\frac{84}{100}$	84%	0.84
$\frac{81}{100}$	$\frac{81}{100}$	81%	0.81

After that, two sets of problems focusing on conversions of percentages, decimals, and fractions were provided for practices. As described above, the teacher used a situated problem-solving approach to strategically construct and consolidate the new concept and procedure operations. The important content points were emphasized and the treatment of important content seemed quite effective.

As another example, T5 pointed out that "the salient features of review lessons are to sum up, systematize and optimally re-organize learned knowledge, and illustrate the development, extension and internal connections of leaned knowledge". He aimed to achieve these goals through a problem-driven and technology-enriched instructional approach. For example, with the appropriate use of technology (e.g., *Sketchpad*), the teacher successfully organized students to solve a series of interconnected problems (See Table 3).

Table 3 Types of problems used in one review lesson

Types of problems	Sample problems
(A) Judging whether an expression is an inverse proportion function	$y = \frac{1}{2x},\ y = -\frac{2}{x}$
(B) Finding the expression of an inverse proportion function	Given an x-coordinate –1 of the intersection point of an inverse proportion function and a straight line $y = -2x$
(C) Comparing the value of the y-coordinate of points on an inverse proportion function (three methods: computation, graph, and using properties)	1. Given A$(-2, y_1)$, B$(-1, y_2)$ which are the points of an inverse proportion function $y = \frac{-4}{x}$? 2. Given three points A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) of the function of $y = \frac{k}{x}$ $(k < 0)$. If $x_1 < 0 < x_2 < x_3$, what are the relationships among y_1, y_2, y_3 in terms of their values?
(D) Comprehensive application	Given a point P (x, y) on hyperbola $y = \frac{k}{x}$ $(k > 0)$, passing the point to draw perpendicular lines toward x-axis and y-axis with the intersections A and B. Then, what is the area of the rectangle of OAPB?

The difficulties of these problems are increased: from definition judgment to comprehensive application. These problems address different aspects of the definition, graph and prosperities of inverse proportion functions. For problem type (A), it is aimed to clarify the definition of inverse proportion functions ($y = \frac{k}{x}$ $(k \neq 0)$); the problem type (B) is aimed to build the connection between functional expressions (linear function and inverse proportion function) and functional graphs, which includes the mathematical thinking of integrating numerical and pictorial representations; the problem type (C) is aimed to develop students' ability in applying the properties and graph of inverse proportion function flexibly; problem type (D) is intended to develop students' ability in comprehensive application of different strands of knowledge (e.g., graph of function, and area of rectangle) by integrating numerical and pictorial representations. Through solving these problems, not only was students' learned knowledge connected and extended, the structure of their learned knowledge optimally organized, but also the underlying mathematical thinking method of integrating numerical and pictorial representations was highlighted.

In summary, these efforts to enhance students' conceptual understanding and developing mathematics thinking methods and problem solving ability reflect that these teachers not only had a sound understanding of the concepts they teach, but also had a deep understanding of the mathematical thinking methods underlying these concepts.

Central tendency 2: Appropriately identifying and dealing difficult content points in students' learning

Apart from identifying important content knowledge and designing instructional strategies to help students master them, anticipating difficult content points and designing possible ways to help students overcome these learning difficulties was also the common feature of these experts' teaching. Many different strategies were developed and used to tackle students' learning difficulties, including exploring contextual problems, playing games, solving a series of interconnected and varying mathematical problems. For instance, T1 identified students' learning difficulties of percentage as the conversion between percentages, fractions and decimals, and he thus designed a way to ease the difficulty through observing and discovering the conversion methods (see the information presented in the aforementioned *central tendency* 1 for details).

T2 identified students' learning difficulties in her video-taped lesson as understanding the concept and properties of proportion, and she attempted to resolve these difficulties through organizing a series of number games played as explained below:

> The learning difficulty is embedded in self-exploring the properties of proportion. Thus, I designed a series of games, and helped students understand the properties from numerical presentations to symbolic representations through playing these games.

T3 indicated that creating problems by students themselves is difficult, and then the teacher used the following strategies to help overcome the difficulty:

> First I explained a worked-out example and summarized the procedures of solving quadratic equations using a formula. Then, some common mistakes with the use of the formula method of solving quadratic equations were provided for students to identify and correct, which aimed to consolidate the use of the formula method. After that, a more difficult problem was provided for students to solve. Finally, the students were asked to create their own equations.

T4 identified students' difficulty in understanding inverse proportion function such as the features of increase and decrease. He thus provided a series of interconnected and varying problems for students to practice as follows:

Prototype Question 1: Given A$(-2, y_1)$, B$(-1, y_2)$ which are the points of inverse proportion function $y = \frac{-4}{x}$, what is the size relationship between y_1 and y_2?

Variation 1: Given that A$(-2, y_1)$ and B$(1, y_2)$ are on the graph of inverse proportion function $y = \frac{-4}{x}$, what is the size relationship between y_1 and y_2?

Variation 2: Given that A(x_1, y_1), B(x_2, y_2), $C(x_3, y_3)$ are the points on the graph of inverse proportion function $y = \frac{k}{x}$ (k $<$ 0), and $x_1 < 0 < x_2 < x_3$, then what is the size relationship among y_1, y_2, y_3?

Paying attention to appropriately identifying difficult content points and designing effective ways to tackle students' learning difficulties is crucial for success in teaching. This is one of basic features in Chinese mathematics classroom instruction (Huang & Li, 2009) and also highly valued by expert teachers in this study.

Central tendency 3: Emphasizing the development of students' mathematical thinking and ability

These experts also paid close attention to developing students' mathematical thinking and ability. It presents another central tendency in Chinese experts' teaching. For example, T1 emphasized the "mathematical thinking of integrating numerical and pictorial representations", and the "classification and transformation thinking", while T2 appreciated the importance of transformation thinking and the development of rigorous mathematics thinking ability. In addition, T3 highlighted the value of developing students' mathematical thinking, and cultivating students' mathematical abilities as stated in her instructional objectives:

> Through applying the formula method in solving quadratic equations and creating problems, the lesson is aimed at advancing students' computation skills and knowledge application ability, and developing a good computation habit.
>
> Through experiencing the process of discovering the formula, it is aimed at developing students' rigorous reasoning ability, with an attention to developing their mathematical thinking such as using symbols to represent numbers, classification, transformation, and integrating numerical and pictorial representations.

T4 gave the following description of how to develop students' mathematical thinking through reviewing direct and inverse proportion functions:

> In teaching, we implicitly introduced mathematical thinking methods such as transformation, integration of numerical and pictorial representations, and function and equation. It was aimed to help students get an experience in solving daily life problems using function and its graphs, develop their mathematical application ability, get an experience in using graphical information to solve problems, and develop students' visual thinking ability.

Likewise, T5 emphasized in students' explorations of flexible and open-ended problems and implicitly introduced the method of integrating numerical and pictorial representations to motivate students to learn.

Central tendency 4: Using mathematics problem solving and posing for developing effective classroom instruction

The use of problem solving and posing is commonly practiced in many classrooms across different educational systems. For example, in all the seven systems that participated in the TIMSS 1999 Video Study, Hiebert et al. (2003) reported that eighth-grade mathematics classrooms were most commonly taught by spending at least 80% of lesson time on working with mathematical problems. Likewise, if taking mathematics problems as including routine and non-routine, symbolic and contextual problems, it is not surprising that we also identified a family-resemblance feature of mathematics problem solving and posing in these Chinese experts' teaching.

In particular, these expert teachers demonstrated several features in how to use problem solving and posing for making an effective mathematics lesson. They emphasized the approaches of learning new knowledge through solving contextual problems (T1) or symbolic problems (T2 and T3), and of consolidating knowledge and developing students' mathematical thinking and ability through investigating deliberately selected problems (T1∼T5).

For example, T1 designed and used a contextual problem for students to explore new concepts and to consolidate new knowledge as stated in his completed questionnaire:

> The concept of percentage was introduced through exploring a contextual problem situation that requires students to compare the survival rate of planted trees. Through discussing this situational problem, the environmental protection awareness was aroused. Meanwhile, through solving practical problems, the students' ability to apply knowledge was enhanced. And students' awareness of applying mathematics was also fostered. Most importantly, this problem situation contains the elements for exploring the conversions among percentages, fractions and decimals.

T2 indicated that she could improve her lesson in the following aspects:

> By asking students to give more daily life examples related to proportion applications and apply the proportion concept in solving contextual problems, it makes learning of the concept more accessible and acceptable to students. Emphasizing proportion application itself is one important element of learning the topic.

T3 was excited about her creative design of asking students to create their own problems as illustrated below:

> Through creating problems by students themselves, their motivation was stimulated, and their computation skills and knowledge application ability were enhanced.

Both T4 and T5 highlighted the importance of selecting and making good problems in designing and organizing review lessons. For instance, T5 explained how to creatively vary textbooks' problems and solutions through modifying problems as follows:

> In review lessons, we should go beyond textbooks' problems and solutions through varying a problem from different aspects. For example, changing a problem's conditions or results can be used to examine if students understand relevant concepts in depth. It can also be used to develop students' reasoning and exploratory ability, and develop their creative thinking and transfer ability. Through posing and solving such varying problems, we can cultivate students' problem exploration ability and help them make mathematical connections among different problems and develop their problem-solving ability.

With regard to the selection of problems, these experts' first priority is that selected problems should focus on certain critical aspects of the concepts learned. Second, the difficulty of the selected problems should be feasible for students and the order of presenting problems should be arranged in terms of their difficulty to increase step by step. For example, in order to apply the property of proportion, T2 presented the following two questions: (1) Find unknown x, so that 2, 3, 4, and x can form a proportion; and (2) if $ad = bc$ (a, b, c, and d are not zero), then how many proportions can be derived from the same equation? Students found different solutions of x (6 and $\frac{8}{3}$) based on different proportions ($2 : 3 = 4 : x$; $3 : 4 = 2 : x$; $2 : 4 = 3 : x$). Through solving this problem, it was found that at least two proportions: $2 : 3 = 4 : x$ and $2 : 4 = 3 : x$ are derived from the same equation of multiplication: $2x = 4 \times 3$. Then, the teacher raised one question: if $ad = bc$ (a, b, c, and d are not zero), then how many proportions can be derived from the same equation? Some students found proportions such as, $a : c = b : d$ and $a : b = c : d$. The teacher concluded that based on the same equation with multiplications (simplicity), we can derive many different proportions (multiplicity). In fact, the second question was finally used as part of homework.

Moreover, these teachers preferred to use different ways to present and organize examples and exercise problems. For example, one teacher wrote and explained the process of solving problems with frequently questioning students. Another teacher asked students to express their solutions orally while the teacher wrote down the students' solutions on the blackboard simultaneously. Another teacher asked students to solve problems individually or in groups, then invited some students to write their solutions on blackboard and organized a whole class discussion.

It is a common effort for these experts to solicit multiple solutions and highlight the flexibility of using appropriate methods. For example, T2 asked students to find all possible proportions consisting of a, b, c and d when $ad = bc$. T3 also highlighted the flexibility of using different methods to solve quadratic equations after introducing four different methods. For example, equation $x^2 - 4x + 4 = 0$ can be solved using the formula method, but it is the most convenient to use the method of making a complete square.

Overall, organizing classroom practice is also a common and important segment of these experts' teaching. Based on the analysis of the instructional activities in the video-taped lessons, we noticed that these experts are thoughtful in their selection and arrangement of problems, the ways of using problems and generating multiple solutions.

Central tendency 5: Emphasizing and practicing student-centered instruction

In contrast to the teacher-led instruction, student-centered instruction refers to the type of instructions that emphasize the following elements: students' self exploration, collaborative exchange, group activities, and students' active participation. These expert teachers appreciated and implemented student-centered instruction through various strategies. For example, T1 indicated that he strives to stimulate students' active exploration and collaborative exchange in his lesson instruction. He also valued lesson summary through interactions between the teacher and students, and among students themselves. He pursued the learning process as "thinking-manipulative-observation-synthesis", and appreciated "students' participation in instructional activities actively, broadly and deeply." T2 intended to develop students' ability of analysis, synthesis and exploration through their self-exploration and comparison. In particular, she led students to discover rules, verify the rules, and prove the rules so that the meaning and properties of proportion were explored through playing various games.

T3 emphasized students' self-learning through organizing students' problem posing activity, namely, creating quadratic equations for their classmates to solve. She also realized that it would also be a good idea to ask students to collect relevant mathematics history on quadratic equation development before class and share them in the class. The teachers (T4 and T5) who taught lessons with a lecture-dominated style still realized that they need to put more efforts to encourage students to actively engage in the process of leaning when reflecting on their lessons. For example, T5 believed that "... hands-on activities, self exploration, and collaborative exchange are important approaches of learning mathematics. The fundamental task of classroom instruction is to implement the principle that students should be the center of lesson instruction. Students' willingness and active participation are important indicators of students being the center of lesson activities."

T1, T2 and T3 spent around 86% of their lesson time on student-centered activities (e.g., individual seatwork, collaborative problem solving in groups, discussions, writing solutions on the blackboard, and students' reflection and sharing) and interactive activities (between the teacher and students for constructing knowledge and solving problems, and discussing important issues). As shown in our case analyses in the next section, the student-centered instruction was obvious (e.g., about 90% of the lesson time was spent on these two types of activities). This finding is supported by Huang and Li's (2009) observation that "Chinese master teachers greatly emphasized student-centered teaching, such as student participation, student mathematical thinking, student self-exploratory learning, student problem-posing and opinion-expressing, and collaborative discussions" (p. 305).

Central tendency 6: Motivating students

All five expert teachers emphasized the importance of motivating students. They developed and used various strategies to stimulate students' learning interests. T1, T3 and T4 believed that "the most effective way of teaching mathematics is to cultivate thier interest in learning mathematics." T2 emphasized stimulating students' learning through organizing numerical games and presenting problems from concrete to abstract ones. T4 also emphasized motivating students through creating their familiar and interesting learning situations, and using inspiring questions. T5 emphasized in motivating students through mathematics application, using multi-media and visual presentations. Meanwhile, through the process of teaching, teachers should appreciate and encourage students so that students enjoy from participating in problem-solving activities and being motivated.

For example, T1 created the environmental protection situation to motivate students' learning and lay a foundation for learning the new content of percentage (See tendency 1 for details). While T2 motivated students through providing a series of games with specific numbers (i.e., find unknown x so that 2, 3, 4, and x can form a proportion) to abstract symbols (i.e., if $ad = bc$ (a, b, c, and d are not zero), then how many proportions can be derived from the same equation) (see tendency 4 for details). Likewise, T3 tried to motivate students through introducing mathematics history and problem posing.

Characterizing Expert Teaching: A Case Analysis

In order to enrich the prototypical view of expert teaching, we deliberately selected one expert teacher (T3, named Ms. Li hereafter) as the representative of this cohort of expert teachers. Making this selection is due to the following considerations. First, the content of the lesson is the formula method of solving quadratic equations which is a classic, difficult, and core content in school algebra. Second, Ms. Li's approach of introducing a new topic through solving mathematical problems is the most common method in mathematics lessons. Third, it contains the complete four phases of a typical Chinese lesson: review, introduction, practicing, and summary and assignment (e.g., Huang & Wong, 2007; Leung, 1995).

Our analysis of the lesson focused on the following dimensions: overall description of the lesson; the features of dealing with the important and difficult contents; mathematics problems used; and students' engagement and instructional strategies (see Li & Li, 2009).

Overall Description of the Lesson

The video-taped lesson was given to a normal class with about 50 eighth graders that Ms. Li had been teaching mathematics for more than 2 years. The whole lesson lasted about 50 min. Ms. Li apparently had a good relationship with her students, as students actively participated in the lesson's instructional activities. According to Ms. Li, the lesson showed her typical teaching style. The video-taped lesson was well prepared for a teaching contest.

The lesson aimed to help students learn the formula method of solving quadratic equations. It consisted of the four segments: (1) reviewing relevant knowledge, (2) introducing the formula method through solving problems and discussion, (3) applying and using the formula method, and (4) summarizing and assigning homework.

Reviewing relevant knowledge. Through questioning, students recalled three methods of solving some special quadratic equations: taking square root (for $ax^2 = b$), factoring, and making a complete square methods. Students were asked to give the detailed procedure of completing square method step by step.

Introducing the formula method through solving problems and discussion. After reviewing, students were asked to solve a specific quadratic equation: $2x^2 - 5x + 1 = 0$. Through solving this equation, the teacher intended to draw students' attention to two issues: (1) the condition of ensuring that equation $x^2 = a$ has real solutions is $a \geq 0$, and (2) no matter how the coefficients are changed, the procedure of solving quadratic equations is the same. Thus, students were motivated to search for a general formula/procedure for solving any quadratic equations.

Immediately, the following equation $ax^2 + bx + c = 0$ ($a \neq 0$) was given to students to solve individually. After a while, a student was asked to write his solution on the blackboard and the teacher organized a whole class discussion on the solution. Several critical questions were discussed:

what is the condition for taking square root of $b^2 - 4ac$, and why there is not absolute sign in the formula although $\sqrt{a^2} = |a|$ (all of them are prerequisite for understanding the formula method to be introduced). Based on the student solution, and relevant discussion, the teacher presented formula method of solving quadratic equation: $ax^2 + bx + c = 0$ ($a \neq 0$). In sum, Ms. Li indicated that there are four methods of solving quadratic equations.

Applying and using the formula method. Students were asked to solve following equation: $2x^2 - 5x = -1$ by using the formula method directly, which is more effective than making a complete square method. After summarizing the procedures of using the formula method, students were asked to discern and correct five solutions of quadratic equations with typically common mistakes. This exercise was aimed to draw students' attention to correctly using the formula method. Then, a little more complicated equation with fractions and decimals coefficients was assigned to students individually and their solutions were shared with the whole class. Finally, an activity of creating quadratic equations was organized in four student groups. Each group needed to create several quadratic equations for classmates to solve using the formula method. For example, one student as representative of that group presented the following equation: $5(x + 1)^2 + 2(x + 1) = 3$ and explained that they anticipate others' use of a substitution method (i.e., $X = x + 1$ substitution method).

Summarizing and assigning homework. By questioning "what are your attainment and experience in this lesson", students were encouraged to express their gains and experience. Finally, some homework both from textbooks and self created ones was assigned.

The Treatment of Important and Difficult Content Points

Ms. Li stated the following instructional objectives: (1) solving quadratic equations using formula method; (2) developing computation skills and knowledge application ability, and nurturing good computation habits through the application of the formula method and problem posing; (3) experiencing the process of inducing the formula of solving quadratic equations, and further developing the rigor and preciseness of mathematical reasoning, and cultivating mathematical thinking methods including using symbols to represent numbers, classifications, and transformations; and (4) introducing relevant mathematics histories related to the development of the formula method for solving quadratic equations and introducing Ancient Chinese mathematician's contributions to the development of the formula method.

The difficult content points of students' learning include (1) discovering the formula for solving quadratic equations, and (2) posing quadratic equations that have solutions. To deal with the difficulty of introducing the formula, Ms. Li presented a simple quadratic question that can be solved with the complete square method first. After solving this problem, Ms. Li intended to help students become familiar with the procedures of solving specific quadratic equations and a classification method for discussing different cases. These discussions should provide a base for the follow-up student-led discovery of the formulas for solving general quadratic equations.

To develop students' computation and application ability, Ms. Li used a problem posing activity which is anticipated as a difficult part of the lesson. She carefully designed the lesson to reduce this difficulty. First, she guided students to complete a worked-out problem and sum up the procedures of solving quadratic equations using the formula method. Then, Ms. Li arranged a set of exercise problems, discerning students' common mistakes in using the formula method in order to deepen students' understanding of the formula application. After that, another quadratic equation with fraction and decimal coefficients was provided to students to solve using the formula method. When students became familiar with the formula, they were ready (knowledge, skills and motivation) to create their own quadratic equations to challenge their classmates.

Problems Used or Posed During the Lesson

All the problems used and proposed during the lesson and their instructional purposes are summarized in Table 4. In segment 2, Ms. Li used problem 1 to help

Table 4 Problems used in different segments of the lesson

Segment	Problem used/posed	Instructional purpose
2	Two problems P1: $2x^2 - 5x + 1 = 0$;	The first problem was used for helping students become familiar with making a complete square method and the classification thinking method, and motivating students to search for a general formula
	P2: $ax^2 + bx + c = 0 (a \neq 0)$	The second problem was used to introduce the formula of solving quadratic equations
3	Four sets of problems: P3: $2x^2 - 5x = -1$;	The first problem (P3) was used for helping students become familiar with the procedure of using the formula method
	P4: One set of problems discerning students' typical mistakes;	The second set of problems (P4) was used to clarify and consolidate the application of formula method
	P5: $\frac{1}{3}x^2 - x - 0.5 = 0$;	The third problem (P5) was used to strengthen the use of the formula method
	P6: One activity: creating quadratic equations – one set of problems posed by students:	The fourth activity (P6) was used to challenge students in using the formula comprehensively and flexibly
	$2x^2 - 4x + 6 = 0$; $3x^2 - 6xy + y^2 = 0$; $5(x + 1)^2 + 2(x + 1) = 3$; $x^2y^2 + 3xy + 1 = 0$	The equations proposed by students require using the formula numerically and symbolically, and using the formula after substituting the unknown (e.g., let $X = x + 1$ or xy)

students become familiar with previously learned procedures of making a complete square method and the classification thinking method, and motivate students to learn the new topic. She then used problem 2 to discover the formula for solving quadratic equations. In the segment 3, a series of deliberately selected problems and activities (P3–P6) were then used to help students become familiar with the formula method, clarify and consolidate possible use of the formula, and develop student's ability in solving and posing problems related to quadratic equations.

Through solving these problems, the teacher aimed to help students develop the formula method by students themselves and apply the formula systematically and progressively. Meanwhile, the underling mathematical methods including generalizing from special cases, the classification thinking method, and the substitution method were explored explicitly.

Student Engagement in Learning and Instructional Strategies

We examined student's engagement through investigating time distribution to different activities and the features of classroom interactions. To examine students' engagement, we identified three types of classroom activities: (1) teacher-led activity, (2) teacher–student interactive activity, and (3) student-centered activity. *Teacher-led activity* refers to presenting problems, explaining concepts, activity transition, and summarizing key points. *Teacher–student interactive activity* includes all interactions between teacher and students when constructing knowledge, solving problems, and discussing important issues. *Student-centered activity* includes individual seatwork, collaborative group problem solving and discussions, writing solutions on the blackboard, and students' reflections and sharing. The time spent in different activities was shown in Table 5. As a whole, the teacher spent about 9% of the lesson time for presenting problems, explaining concept, transiting activities, and summarizing key points. More than half of the lesson time (53%) was used with teacher-student interactions in developing concepts, solving problems, and sharing solutions. And about two-fifths of the lesson time (39%) was spent on students-centered activities.

Table 5 Time spent for different activities in various segments

Segment	Teacher-led activity	Interactive activity	Student-centered activity
1		1:00 (2%)*	
2	2:20 (4.5%)	11:40 (22.6%)	4:30 (8.7%)
3	1:45 (2.8%)	14:30 (28%)	13:30 (26%)
4	1:10 (2%)		2:10 (4.2%)
Total	5:15 (9%)	27:10 (53%)	20:10 (39%)

Note: "*" 1:00 (2%) means 1 min and 0 seconds that is 2% of the lesson time.

Based on the time distribution, Ms. Li was very effective in using class time without any task off period. She was very frugal in direct talk for activity organization and transition (using a total of about 9% of the lesson time). Ms. Li tried very hard to get students engaged in the process of learning through frequent interaction with students, and student-centered activities. On one hand, through questioning and probing, the students were encouraged to construct new knowledge and consolidate knowledge through participating in various classroom activities (53% of the lesson time). On the other hand, through individual or group activities, the students were involved in independent thinking and collaborative exchanges. It is impressive that students were organized in groups to create their own problems, and explain their intentions in public. Moreover, Ms. Li encouraged students to reflect on their learning experience and attainments. These student-centered activities (39% of the lesson time), not only motivated students' learning interests, and got them involved in the learning, but also benefited the development of their ability in problem posing and problem solving, and reflection on their own learning process.

Summary of the Case of Expert Teaching

On the surface, this seems to be a typical lager-size whole class instruction, and the teacher controlled the whole process of teaching. However, in no way, can we conclude that this was a students' passive learning class. Actually, the aforementioned analyses bring us the following observations. It is a well-organized, skillfully delivered lesson. The set of instructional objectives (Central tendency 1) was carefully prepared and successfully implemented through appropriately dealing with difficult content points (Central tendency 2). Students actively discovered the formula method of solving quadratic equations, and applied the formula method flexibly and appropriately (Central tendency 5). Several mathematical thinking methods such as using symbols to represent numbers, the classification thinking method, and substitution thinking were realized and applied (Central tendency 3). The two creative attempts of the lesson: equation posing and integrating mathematics history in lesson instruction were implemented successfully and they resulted in positive effects on motivating students and developing their creative ability (Central tendency 3–6). All of these achievements were evidenced through students' reflective summarization of the lesson (Central tendency 5).

Ms. Li used a variety of strategies to engage students in mathematics learning. These strategies include reading aloud individually and in chorus, solving problems individually and in groups, presenting and explaining solutions, problem posing and justification, reflecting on particular learning activities and the whole lesson (Central tendency 5 & 6). The lesson looks like an art performance, unfolding smoothly. There are several high climates, including students' success in discovering the formula, discerning typical mistakes, posing valuable equations, and sharing their learning experiences.

In addition to Ms. Li's excellent classroom management skills, the presentation and exploration of deliberately selected problems played a crucial role. Essentially, the lesson was unfolded in line with the progress of solving interconnected problems, which were used to discover new knowledge and apply knowledge in various situations from simple case to complex problems (Central tendency 4). The teacher provided these problems one by one to help make a coherent lesson.

Discussion and Conclusion

In this study, we identified six central tendencies of teaching expertise in school mathematics in the context of Chinese classrooms. They include: (1) having sound subject content knowledge of teaching topics; (2) appropriately identifying and dealing with difficult content points in students' learning; (3) emphasizing the development of students' mathematical thinking and ability; (4) using mathematical problem solving and posing for developing effective classroom instruction; (5) emphasizing and practicing student-centered instruction; and (6) motivating students. We can further group these central tendencies into three broad categories: (a) teacher knowledge for teaching (e.g., content knowledge, and knowledge about students' learning and teaching); (b) mathematics-specific instruction (e.g., developing mathematical thinking methods and students' mathematical ability, and mathematics problem solving and posing); and (c) student-oriented approaches (e.g., student-centered instruction and motivating students). A case of expert lesson instruction was used to illustrate how these tendencies were embodied in an expert teaching.

The findings of this study have some implications. On one hand, this study further supports relevant findings on expert teachers' knowledge and teaching strategies (Berliner, 2001; Borko & Livingston, 1989; Smith & Strahan, 2004). These expert teachers did demonstrate their sound knowledge in subject matter, student learning, and relevant teaching strategies. The prototypical features of Chinese experts' teaching expertise are consistent with some others' reporting in terms of: high efficiency (Sternberg & Horvath, 1995) and coherence (Chen & Li, 2011; Li & Li, 2009), and the use of different and rich instructional strategies (Li & Li, 2009; Lin, 1999). Student-centered instruction was in line with Smith and Strahan's findings, echoing with Huang and Li's (2009) reporting about Chinese experts' views of effective mathematics teaching. On the other hand, some central tendencies revealed in this study further enrich our understanding of teaching expertise valued in China. In particular, with a focus on experts' mathematics teaching, this study revealed that Chinese experts tended to pay close attention to identifying students' learning difficulties, developing mathematical thinking methods and students' problem-solving ability (see Zhang, Li, & Tang, 2004), and consolidating knowledge learning through solving deliberatively selected and interconnected problems.

As pointed out by Sternberg and Horvath (1995), different dimensions of expert teaching usually are interrelated. Our study seems to suggest that expert teachers' profound knowledge for teaching led them to appropriately analyze important content points and difficult content points, and to select mathematically worthwhile problems and relevant strategies to construct, develop, consolidate and apply knowledge. As pointed out by Borko and Livingston (1989), expert teachers should have elaborate, interconnected and accessible cognitive schemata.

As discussed at the beginning, the articles written by Sternberg and Horvath (1995) and Smith and Strahan (2004) provided us a great guidance in taking the prototypical view of expert teaching. However, no real classroom instruction was provided and described in either article to showcase expert's teaching if putting together their lists of prototypical features. In a way, readers would think that their list of prototypical features of teaching expertise is ideal. Different from analyses conducted by Sternberg and Horvath (1995) and Smith and Strahan (2004), we took five expert teachers' lesson instruction as main data sources to identify central tendencies of teaching expertise valued in China. In fact, we can put our identified prototypical features back together and use these experts' video-taped lessons to depict how teaching expertise can be manifested in real classrooms. Indeed, all the video-taped lessons were carried out very smoothly and coherently. Not only did these expert teachers have an in-depth knowledge about the content they teach, but also performed as virtuoso (Paine, 1990) guiding students to construct new knowledge and mathematics thinking, consolidate the new knowledge, experience the learning process, and achieve their instructional objectives. These experts are also flexible in using different strategies of introducing new content (e.g., reviewing or problem solving), practicing new content (individual or group, problem solving or problem posing, seat work or public sharing) and summarizing key points of lessons (teacher-directed, or students self reflection). Without doubt, the experts conducted these lessons efficiently (Sternberg & Horvath, 1995) by following certain explicit patterns of mathematics teaching. They are knowledgeable in anticipating students' learning difficulties and developing and using relevant strategies to tackle them.

The findings of this study allowed us to develop a better understanding of Chinese mathematics teachers' expertise in teaching that goes beyond what we already learned about Chinese mathematics teachers' in-depth understanding of school mathematics they teach (e.g., Li & Huang, 2008; Ma, 1999). The use of five expert teachers' lesson instruction further helped us make direct connections between expertise in teaching and experts' teaching performances. It also extends the prototypes of teaching expertise that Smith and Strahan (2004) and Sternberg and Horvath (1995) have shown us. We believe that continued research will help expand further about what we can learn about the nature of teaching expertise from diverse angles.

Finally, this study aimed to focus on examining and understanding teaching expertise that is valued in China. Yet, much remains unclear about how Chinese experts may develop their expertise in teaching during their journeys of pursuing professional development and promotion. As this is beyond the scope of this study, further research will be needed to identify and examine possible effective approaches and practices used in China to develop teachers' expertise in teaching.

Appendix 1: The Categories and Examples of Teachers' Comments and Reflections

Categories	Sub-categories	Examples
Teacher knowledge	Content knowledge	Knowing the connection and differences between percentage and fraction, mastering the conversion between percentage, fraction, decimals and fraction (T1)
	Students' learning difficulties and treatment	The teacher appropriately anticipated students' difficulties when learning system of linear equations, and designed the strategies to overcome the difficulties (T3)
	Developing mathematical thinking methods and abilities	I implicitly introduced the transformation thinking method, the method of integrating numerical and pictorial representations, and the thinking method of using function and equation (T4)
	Developing mathematical application	To help students get an experience in solving contextual problems by using function and its graphs, and develop their mathematical application ability (T4)
Instructional process	Student-centered activities	Through participating in various learning activities, students are motivated to explore, collaborate and develop their affective experience in learning mathematics (T1)
	Teacher-directed activities	I believe that in this lesson it would be much better if the teacher directly explained the concept. Because the definition is rigorous, the teacher should help students understand the rigor of the definition (T3)
	Contextual learning	Knowing how to use mathematics language to express real life situations concisely through exposing various real life examples of using percentage (T1)
	Learning motivation	Motivating students to learn through exploratory activities from concrete to abstract cases (T2)
Teaching skills and teacher characteristics	Basic teaching skills	Board writing is good, teaching language is concise (T3)
	Use of multiple media	Using colorful pictures to present real life situations, using multiple media to teach (T1)
	Improvisational ability	For teacher, the highest teaching ability should be reflected in improvisational ability in classrooms (T1)

Appendix 2: Frequencies of the Codes Appeared in the Five Teachers' Comments and Reflections

Sub-categories	T1	T2	T3	T4	T5
Content knowledge	6	6	5	5	5
Students' learning difficulties and treatment	1	9	2	2	1
Developing mathematics thinking methods and abilities	6	4	4	8	9
Developing mathematics application	12	1	5	3	4
Student-centered activities	6	9	6	7	5
Teacher-directed activities	1	7	0	0	3
Contextual learning	10	2	1	3	1
Learning motivation	2	3	2	5	4
Basic teaching skills	0	0	2	3	3
Use of multiple media	1	0	0	1	1
Improvisational ability	1	0	0	0	1

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The Japanese Approach to Developing Expertise in Using the Textbook to Teach Mathematics

Akihiko Takahashi

Abstract Japanese teachers and educators distinguish between "teaching the textbook" and "using the textbook to teach mathematics." In order to provide a better learning experience for their students, all teachers should be able to use the textbook to teach mathematics effectively. This means that teaching the textbook is not enough. But what is the knowledge and expertise that Japanese teachers are expected to develop, and when and how do Japanese prospective teachers and novice teachers acquire that knowledge and expertise? A study was conducted with selected Japanese prospective and practicing elementary school teachers to reveal the knowledge and expertise they use to design a lesson based on the contents of a textbook page. Using the findings from the study, I will discuss what this distinction between "teaching the textbook" and "using the textbook to teach mathematics" means in terms of teacher knowledge and expertise, and how Japanese teacher education programs help teachers develop that knowledge and expertise.

Keywords Teaching · Problem solving · Professional development · Expertise

Introduction

Japanese public school teachers are required to use one of the government-authorized textbooks as a major resource for teaching mathematics. Textbook companies carefully examine the Course of Study (the national standards) and follow the teaching guide published by the government when they create new materials.

Although most teachers use textbooks as their primary instructional material (McKnight et al., 1987; Tyson & Woodward, 1989), Japanese teachers and educators recognize that there are different ways to use textbooks, and those ways have

A. Takahashi (✉)
School of Education, DePaul University, Chicago, IL, USA
e-mail: atakahas@depaul.edu

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different impacts on student learning. Japanese teachers and educators distinguish between "teaching the textbook" and "using the textbook to teach mathematics." To teach the textbook, teachers need little knowledge about mathematics; they can just tell students what is in the textbook. But to use the textbook to teach mathematics, teachers need a much deeper understanding of mathematics and of how students learn, and expertise for teaching.

In order to provide a better learning experience for their students, all the teachers should be able to use the textbook to teach mathematics effectively. This means that teaching the textbook is not enough. But what is the knowledge and expertise that Japanese teachers are expected to develop, and when and how do Japanese prospective teachers and novice teachers acquire that knowledge and expertise?

In order to answer these questions, a study was conducted with selected Japanese prospective and practicing elementary school teachers to reveal the knowledge and expertise they use to design a lesson based on the contents of a textbook page.

Using the findings from the study, I will discuss what this distinction between "teaching the textbook" and "using the textbook to teach mathematics" means in terms of teacher knowledge and expertise, and how Japanese teacher education programs help teachers develop that knowledge and expertise.

The Japanese View of Good Mathematics Teaching

The Japanese Problem Solving Approach

Although there are several beliefs and views of good mathematics teaching among Japanese mathematics educators and teachers, problem solving has been a major focus in Japanese mathematics curricula for nearly 50 years. Numerous teacher reference books and lesson plans using problem solving have been published since the 1960s. Government-authorized mathematics textbooks for elementary grades, published by six private companies, have had more and more problem solving over the years. As a result, almost every chapter in recent Japanese mathematics textbooks for elementary grades begins with problem solving as a way to introduce students to new concepts, and even to procedures.

A few key publications have greatly influenced how problem solving is used in Japanese mathematics education. Polya's *How to Solve It* (Polya, 1945) was translated and published in Japanese in 1954, and studied by various researchers and educators in Japan. Japanese researchers, teachers, and administrators worked collaboratively through Lesson Study, a professional development approach that is popular in Japan, to develop mathematics instruction based on Polya's four phases of problem solving (A. Takahashi, 2000). One of the results from the studies of problem solving, *the Open-ended Approach*, was published in 1977 (Shimada). The open-ended approach has been widely used in Japanese classrooms since then. The Ministry of Education in Japan, in various documents since the beginning of the 1980s, has emphasized the need for students to develop problem-solving skills to learn and use mathematics. The position statement from the NCTM's *An agenda for* *action: Recommendations for school mathematics of the 1980s* (1980) that "problem solving must be the focus of school mathematics" was referenced in various research articles and resource materials for teachers in Japan during the 1980s. Also, *Teaching Problem Solving: What, Why & How* (Charles & Lester, 1982) was translated into Japanese in 1983.

Stigler and Hiebert (1999) used the phrase "structured problem solving" to describe Japanese mathematics lessons, and similar descriptions were reported in the proceedings of the US-Japan Seminar of Mathematical Problem Solving (Becker & Miwa, 1987; Becker, Silver, Kantowski, Travers, & Wilson, 1990). "Structured problem solving" involves presenting students with challenging problems designed to provoke creative mathematical activity and discussion through which students acquire new knowledge and skills.

Japanese teachers believe that, with appropriate supports, students can successfully solve these challenging problems by themselves. But they also expect that students will use a variety of approaches – incorrect and correct, naïve and sophisticated. For students to learn from this experience, the good teacher must lead students in a whole-class discussion around comparing individual approaches and solutions. Through their extensive study of lessons based on problem solving, Japanese teachers and educators have come to recognize that this whole-class discussion is the heart of teaching through structured problem solving and have named this discussion part *neriage.*

Neriage (Extensive Discussion)

The term *neriage* has been widely used among Japanese teachers and researchers of mathematics education as a technical term since 1980s. *Neriage* is a noun form of a verb *neriageru*, which means to "polish up". Japanese teachers use it to describe the dynamic and collaborative nature of a whole-class discussion in the lesson (Shimizu, 1999). The most important role of the teacher in *neriage* is to orchestrate students' ideas for and approaches to solving the problem and to help them polish their solutions in order to learn new mathematical content. During the process, a teacher highlights the important mathematical ideas and concepts that are the goals of the lesson. This is why Japanese teachers see *neriage* as the heart of teaching mathematics through problem solving: the solving of the problem by each student at the beginning of the lesson is preparation for *neriage*. It is important for students to struggle with the problem and find their own way to solve the problem, because this experience will be the foundation for them to make a connection between their previous learning and the content that they will learn through *neriage*.

The following problem (see Fig. 1) is typical of one found in Japanese mathematics textbooks for 4th grade; it is presented to students who have just learned the formulas for finding the area of rectangles and squares. (The actual texbook page from which this problem comes can be found in the Appendix.) The objective is for students to understand how they might use formulas that they have learned previously, to find the area of shapes that they have not seen before. In order to use their prior knowledge to find the area of this unfamiliar shape, the students should use strategies like area-preserving transformations (cutting and re-arranging) or area-doubling transformation (copying and re-arranging). Thus, the teachers should be able to use this problem to help students learn general strategies for using previously-learned area formulas to find the area of unfamiliar shapes (Watanabe, Takahashi, & Yoshida, 2008).

In order for the students to accomplish the goal of the lesson, Japanese educators expect the teachers to help the students develop the idea for finding the area of this shape (Fig. 1) through *neriage*.

A structured problem solving approach to this lesson would begin by asking the students to find the area by themselves. Students are expected to come up with several different approaches and solutions.[1] Japanese teachers usually monitor students' work during individual or group problem-solving time - using a seating chart, for example, to record how each student approached the problem - and devise a plan for the discussion.

Based on this plan, the teacher then assigns students to share their solutions. A teacher might begin with one of the students who used the most common method to share his/her method, and then ask another student to share a different one. At this time, Japanese teachers usually avoid saying whether the answers are right or wrong in order to provide students with the opportunity to think carefully about each solution method. Teachers make careful use of the blackboard to help students see all the solution methods and to help them understand each method.

Up to now, the class discussion is very similar to a favorite school activity, "Show and Tell." If the goal of the lesson were just to find a solution to the problem, the

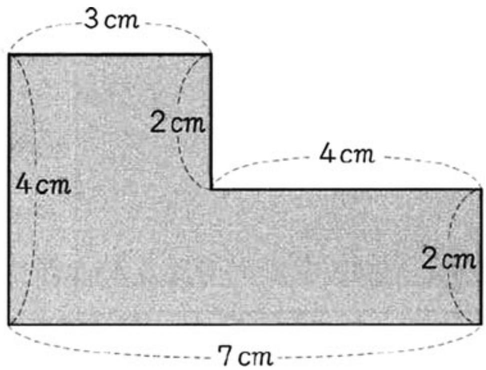

Fig. 1 Reprint from the *Mathematics Textbooks For Elementary Grade 4th grade* p. 58 (Sugiyama, Iitaka, & Ito, 2006). Reprinted with permission from Tokyo Shoseki Publishing Co.

[1]Although the textbook page includes three diagrams implying possible solutions, Japanese educators generally encourage teachers not to show these diagrams unless the students really need it. Experienced teachers typically show the problem on the board, and discourage students from opening the book during the class.

lesson could end here. But because the goal is for students to acquire new mathematical knowledge, a different kind of discussion is now needed. *Neriage* begins after the students have presented their various solution methods, and takes the discussion beyond "Show and Tell" (A. Takahashi, 2008).

Neriage is a critical component of a problem-solving lesson because this is where, using the students' own solution methods, teachers can lead students to acquire new mathematical ideas and concepts. The effectiveness of the lesson as a whole thus hinges on the quality of the *neriage*. But *neriage* is difficult to do well; the expertise required for effective *neriage* is suggested by the existence in Japanese of some technical terms used to describe its component skills, such as *kikanshido*, *hatsumon*, and *bansho* (these terms will be described in the following section). In planning for the *neriage* portion of a lesson, teachers have to use all their knowledge of mathematics, their knowledge about teaching mathematics, their knowledge of students, and their skill at facilitating whole-class discussion.

Because of the extent of knowledge and skill required for effective *neriage*, and the careful planning required for it, Japanese educators believe that *neriage* is the proving ground of a teacher's knowledge and expertise, and that a teacher's knowledge and expertise can be revealed by how he or she plans for it. This connection between a teacher's expertise and how he or she plans for *neriage* is explored in a study described later in this chapter.

Knowledge and Expertise Required for Using the Textbook to Teach Mathematics

It is obvious that teachers cannot teach content beyond their knowledge (National Mathematics Advisory Panel, 2008), but knowledge of content is not nearly enough to teach effectively. For example, knowing the algorithm for multi-digit multiplication is not the same as understanding why the algorithm works with any two multi-digit numbers, which in turn is not the same as knowing how to help students acquire understanding of and proficiency with that algorithm.

The steps for multiplying two multi-digit numbers are typically written out in the textbook. A teacher might attempt to teach students how to multiply numbers by telling them what the textbook says and correcting wrong errors that they make. This is "teaching the textbook"; it doesn't require much knowledge and is rarely, if ever, effective. "Using the textbook to teach mathematics," as will be seen later, involves drawing on the textbook as a resource for designing lessons aimed at developing student understanding, and doing so requires considerable knowledge and expertise.

Japanese mathematics educators and teachers identify three levels of expertise between "teaching the textbook" and "using the textbook to teach mathematics" (Sugiyama, 2008):

Level 1: The teacher can tell students important basic ideas of mathematics such as facts, concepts, and procedures.

Level 2: The teacher can explain the meanings of and reasons behind the important basic ideas of mathematics in order for students to understand them.
Level 3: The teacher can provide students opportunities to understand these basic ideas, and support their learning so that the students become independent learners.

Sugiyama (2008) writes that during the early twentieth century, which is considered an early stage of the Japanese public education system, most elementary school teachers were at Level 1. They told their students the facts and expected them to memorize those facts through practice, and textbooks of the time were designed to support that form of instruction. Although it is very important for teachers to be able to tell students important facts, a teacher at Level 1 is not today considered a professional.

Teachers at Level 2 have to know mathematics beyond what is used in everyday life or what is required to solve problems in elementary school textbooks. For example, knowing the “invert and multiply” rule for division of fractions is enough to be a Level 1 teacher but is not enough for Level 2 teachers. Level 2 teachers should be able to explain how multiplying by the reciprocal of a fraction produces the quotient. This type of knowledge is important for helping students understand mathematics. Japanese mathematics educators regard a teacher at Level 2 as a professional.

Although Level 2 teachers are considered professionals, Japanese mathematics educators believe that all teachers of mathematics should be at Level 3, because Level 2 teachers cannot provide adequate opportunities for most students to develop proficiency with understanding.

The differences between Level 3 teachers and teachers at the other levels can be understood by looking at how they might use a problem in a textbook. A Level 1 teacher would present the problem and show the steps for solving it. A Level 2 teacher would show the steps and explain why those steps are correct and useful. A Level 3 teacher, in contrast, would present students with the same problem, providing structure and guiding the conversation so that that students would arrive at a new understanding as a result of their own efforts to solve it. The philosophy behind Level 3 teaching is that students should have a reasonable amount of independent work, such as problem solving, in order to develop knowledge of, understanding of, and skill with mathematics (National Research Council, 1989; Polya, 1945).

These differences exist between teaching the textbook and using the textbook to teach mathematics. But to make this happen in the classroom clearly requires much greater knowledge and expertise.

What This Distinction Among Level 1, Level 2, and Level 3 Teachers Means in Terms of Teacher Knowledge and Expertise

Japanese mathematics educators can safely assume that most university students have a Level 1 knowledge of mathematics. Their concern, therefore, is to move those students toward Level 2. Sugiyama (2008) argues that one of the major goals of teacher preparation is to develop a good understanding of the teaching materials.

The courses for elementary mathematics teacher preparation in Japan mainly focus on examining the contents of mathematics for elementary grades and developing deeper understanding of those contents. This process is often similar to *kyozaikenkyu*, which means "studying teaching materials for establishing deeper understanding for better teaching" (Watanabe et al., 2008).

For example, there are several formulas for finding the area of basic geometric figures. Most students who come to a teacher preparation program already know those formulas and how to use them to find areas of basic figures. Although some students might forget those formulas, looking back at their elementary school textbooks can help them recall all the facts. The university courses help the teacher candidate see how the formulas are developed, how they are related to each other, how they are related to other areas in mathematics, and potential difficulties students might have with learning the formulas. Investigating a topic in this way is typical of *kyozaikenkyu* and is an essential part of teachers' preparation for everyday teaching; hence these courses also introduce the teacher candidates to *kyozaikenkyu* as a critical skill for becoming a part of the teaching profession.

Because the contents of mathematics for all the elementary grades cannot be covered in short courses, even becoming a Level 2 teacher may not be possible by simply completing university courses. Therefore, preparing teacher candidates to conduct *kyozaikenkyu* is important, as it equips them to continually deepen their knowledge and understanding of mathematics throughout their career.

Moving student teachers to Level 3 is even further beyond the scope of what can be accomplished during the university training. Because teaching is a cultural activity and cannot be learned like the use of a computer, the prospective teachers can not become experts quickly by merely listening to lectures, reading textbooks, and watching videos. Teaching is something that needs to be "learned implicitly, through observation and participation, and not by deliberate study" (Stigler & Hiebert, 1999, p. 86). Therefore, becoming a Level 3 teacher is demanding and time-consuming, requiring continuous learning after the teacher preparation program is completed.

Although moving toward Level 3 requires continuous professional development beyond the teacher preparation program, teacher candidates can learn what is required for Level 3 through coursework. Their preparation should help the candidates understand what it means to be a Level 3 teacher, should aim to convince the student teachers of the importance of striving to become Level 3 teachers, and should show them the pathways by which they might get there.

As part of helping teacher candidates understand what it means to be a Level 3 teacher, the following major technical terms that describe the expertise involved in Level 3 teaching are usually introduced and discussed during the university training.

The term *hatsumon* means "Posing key questions to students." *Hatsumon* is important for teaching through structured problem solving because the way a problem is posed influences students' learning significantly. Good *hatsumon* provokes students to think mathematically, using their prior learning to learn something new (Shimizu, 1999).

Kikanshido and *kikanjyunshi* are both used to refer to the deliberate activity of a teacher moving among the students' desks while they work, monitoring and supporting them in their efforts. During *kikanshido/kikanjyunshi*, the classroom teacher observes how each student is solving the problem, considers the order in which students might be invited to present their solutions, and provides appropriate support to individual students. The teacher does not spend too much time with any one student, however, since a major goal of this phase is for the teacher to know how all students are approaching the problem (Shimizu, 1999). Also, because *kikanshido/kikanjyunshi* is for the teacher to prepare for polishing the students various solutions and helping them learn important mathematics through *neriage*, the Japanese teachers avoid telling students how to find the answer to the problem at this time.

Bansho is the word for blackboard writing. For Japanese teachers, blackboard writing is a critical component of teaching, serving several purposes. One is to organize various students' approaches to the problem to enable comparison and discussion of these approaches. It helps the students follow the *neriage* process. Another is to help the students summarize the lesson when they take notes during the class (Yoshida, 2005).

For teachers to become Level 3, using the textbook to teach mathematics, Japanese mathematics educators believe that the teachers should have clear goals for the lesson and plans for *hatsumon*, *kikanshido*, and *bansho,* none of which come from the textbook page.

How do Level 1, Level 2, and Level 3 Teachers Plan a Lesson Differently?

As a way to uncover the differences in teacher expertise at Level 1, Level 2, and Level 3, an empirical study was conducted with selected prospective and practicing teachers in Japan to see how they would plan a lesson from the same page of a mathematics textbook.

The Subjects of the Study

The subjects of this study were selected from prospective teachers and practicing teachers in one city in Japan. Level 1 teachers and Level 2 teachers were asked to participate in this study voluntarily from the teacher preparation program at the major university in the city. The participants were given questionaires in written format and asked to complete them by themselves. This was not a part of their course work.

Prospective teachers from the first year of the teacher preparation program were selected as Level 1 teachers. All students demonstrated their proficiency in elementary and junior secondary level mathematics by passing a rigorous and comprehensive entrance examination which covered the major academic subjects, including mathematics. They had not, however, completed the teaching methods course for mathematics. For Level 2 teachers, students in the fourth year of the university teacher education program were selected. These students had completed the teaching methods courses as well as the student teaching requirements. Japanese mathematics educators and teachers regard these students as well-prepared to become teachers.

Level 3 teachers were selected from the teacher leaders in the area around the university by one of the leading researchers in mathematics education in the area. These teachers are respected among other teachers as knowleadgeble and experienced classroom teachers in elementary school mathematics. Although they specialize in elementary school mathematics, they are self-contained classroom teachers and teach most subjects everyday in local elementary schools.

Method

Subjects were asked to complete a questionnaire designed to elicit how they use their knowledge of mathematics and pedagogy to plan a lesson based on a textbook. Teachers worked from one textbook page from the mathematics textbook series most widely used in Japanese public elementary schools.[2] All questions were posed in Japanese and the subjects of this study wrote their responses in Japanese. Four Level 1 teachers, four Level 2 teachers, and three Level 3 teachers participated in this study, and all the responses from these teachers were analyzed in Japanese and translated into English after the analysis.

In English, the questions that were posed to the prospective and practicing teachers were these:

1. To teach this content, how would you segment your given 45 minutes. Please explain how you would divide the lesson into segments and how many minutes you would spend for each segment.
2. What is the most important point of the lesson for students to understand? Why do you think it is important?
3. In order for students to understand the main point of this lesson, which part of the lesson do you think should be emphasized? Why do you think that part of the lesson should be emphasized?
4. This textbook page instructs students to find multiple ways to calculate the area of the given shapes. Why do you think the textbook asks students to come up with multiple ways?
5. What methods do you think students will come up with? Besides Naoko's answer, please anticipate students' responses and give them in order from most likely to least likely.
6. During and after the lesson, what point of view and method would you use to evaluate whether (a) students have attained the goal of the lesson, and (b) if the flow of the lesson and teacher's questions/reactions were appropriate?

[2] See Appendix.

Results

As the following summaries will show, there were clear differences in how teachers at different levels would design a lesson based on the given textbook page.

1. To teach this content, how would you segment your given 45 minutes. Please explain how you would divide the lesson into segments and how many minutes you would spend for each segment.

The responses to this question among Level 2 teachers and Level 3 teachers were quite similar, and quite distinct from responses from Level 1 teachers.

Each Level 1 teacher segmented the lesson differently. Three out of four allocated time at the beginning of class to guide students to find a way to calculate the area of the shape. Only one Level 1 teacher planned to ask the students to solve the problem by themselves without any guidance. One of the Level 1 teachers planned to use all the questions and tasks on the page while the other Level 1 teachers modified some of the questions and tasks on the page.

Level 2 and Level 3 teachers segmented the lesson in very similar ways. The lesson in the textbook begins with an introduction to the problem and asks students to solve the problem by themselves without any guidance or hint to calculate the area. One Level 2 teacher allocated time at the beginning of the lesson for reviewing what the students had learned about finding area. After students spent time solving the problem by themselves, all the Level 2 and Level 3 teachers planned to have time for a class discussion to compare and discuss various methods for finding the area of the shape. These responses clearly indicate that all the Level 2 and Level 3 teachers planned based on the form of structured problem solving that Stigler and Hiebert (1999) described as characteristic of the Japanese approach to teaching mathematics.

The major difference between Level 2 and Level 3 teachers was the length of the time they allocated for individual problem solving versus whole class discussion. All Level 3 teachers allocated more time for whole class discussion than for individual problem solving. Among the Level 2 teachers, three of them allocated more time for individual problem solving than for whole class discussion while the fourth one allocated equal time.

Another difference was in the time allocated for practice. Although the textbook page includes two similar problems at the bottom of the page, two of the Level 3 teachers allocated no time for the problems. According to the teacher's manual the textbook page is designed for one class period, 45 minutes. These two Level 3 teachers would apparently choose to spend all the class time on solving one problem, leaving these two problems for homework exercises. Most Level 1 teachers and Level 2 teachers included all the tasks on the textbook page in the 45 minutes lesson.

The following Table 1 shows an example from a teacher at each level.

2. What is the most important point of the lesson for students to understand? Why do you think it is important?

Table 1 Some of the responses to question 1

A Level 1 teacher's response	A Level 2 teacher's response	A Level 3 teacher's response
Think about how to find the area (5 min)	Review prior learning (5 min)	Introduction to the problem: encourage students to use prior learning to find the answer (10 min)
Explain Naoko's idea [which is described in the textbook] (5 min)	Individual problem solving (10 min)	Individual problem solving: let each student explain the solution method by using diagrams, math sentences, or words (10 min)
Ask students to describe other approaches and make sure all the approaches reach to the same area (10 min)	Comparing and discussing (10 min)	Whole class discussion for examining each solution: present own solution and understand other solutions. Find similarities and differences among the solutions (20 min)
Exercises (15 min)	The first exercise (10 min)	Summarizing: reflect on own solution (5 min)
Check the answer for the excises (10 min)	The second exercise (10 min)	

All the responses to this question include either or both of the following two points: that the area of the shape can be found several different ways, and understanding that the area of the shape can be found by using the prior knowledge of how to find the area of basic figures such as rectangles and squares. All the Level 1 and Level 2 teachers point out one of these while all the Level 3 teachers mention both.

Two Level 1 teachers argue that the most important point is to understand that there are multiple approaches for finding the area. Both of them explain that it is important to see a problem with multiple viewpoints. Another Level 1 teacher argues that it is important for students to be aware that they can solve problems using their prior learning because this thinking skill would help students when they grow up.

Three out of four Level 2 teachers highlighted that the most important point of the lesson is for students understand that the area of the composed shape can be found by manipulating the shape, such as by cutting it into two rectangles and rearranging the parts into a rectangle or square without changing the area. Compared to the ideas of the Level 1 teachers, these teachers identified an important generalization of the different approaches. Another Level 2 teacher argues that the most important point is to help students understand the merits of each approach for finding the area in terms of simplicity and reliability.

Each Level 3 teacher provided a more comprehensive answer than the Level 1 and Level 2 teachers. In addition to the Level 1 and Level 2 teachers' answers, Level 3 teachers were explicit about what the students should learn through comparing and discussing multiple approaches for finding the area. For example, students should understand that some of the approaches that they discussed in the class might be useful for finding the area of other shapes. Two of the Level 3 teachers also wrote about helping students foster their ability to express their ideas by using mathematical expressions and diagrams. One of the Level 3 teachers wrote that the discussion should help students develop the ability to foresee, with other geometric shapes, that their area could be calculated using the formulas for area of rectangles and squares.

3. In order for students to understand the main point of this lesson, which part of the lesson do you would think should be emphasized? Why do you think that part of the lesson should be emphasized?

In responding to this question, most Level 1 teachers pointed to the part of the lesson where the students think about how to find the area of the shape, while all the Level 3 teachers emphasized the whole class discussion of comparing and examining the solution methods. The Level 2 teachers were evenly split between the thinking part of the lesson and the discussion (See Fig. 2).

4. This textbook page instructs students to find multiple ways to calculate the area of the given shapes. Why do you think the textbook asks students to come up with multiple ways?

All the Level 1 teachers and two out of four Level 2 teachers responded that finding multiple ways to calculate the area of the shape will help students think more flexibly, but did not explain how this relates to the purpose of mathematics education. On the other hand, two Level 2 teachers and all the Level 3 teachers provided some rationale for how finding multiple ways to calculate the area would contribute to learning mathematics. For example, one rationale was that finding multiple methods would help students understand the important idea, such as rearranging the shape into a rectangle or a square in order to calculate the area of geometric shapes. Others wrote that finding multiple ways would encourage students to seek better ways to find the area.

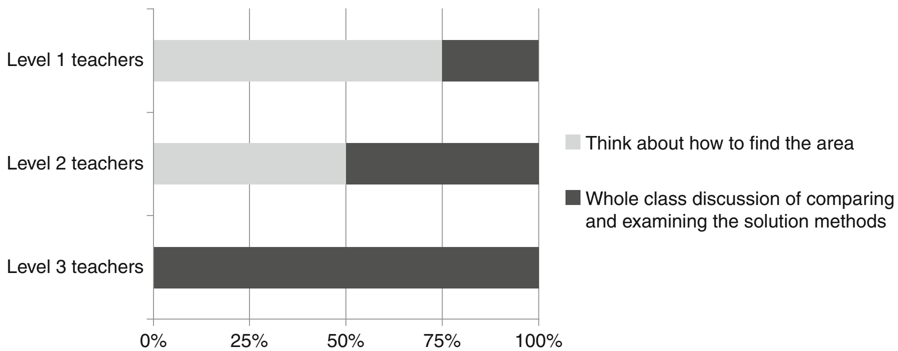

Fig. 2 Responses to question 3 which part of the lesson you would think to be emphasized

5. What methods do you think students will come up with? Besides Naoko's answer, please anticipate students' responses and give them in order from most likely to least likely.

Figure 3 shows the top anticipated solutions. Three out of four Level 1 teachers, three out of four Level 2 teachers, and all three Level 3 teachers chose Solution A as the most likely. One Level 1 teacher and one Level 2 teacher chose Solution C as the most likely. All Level 1, Level 2, and Level 3 teachers included Solutions A, B and C except for one Level 1 teacher who did not include Solution B. These three solutions all involve considering the shape as two or three rectangles and calculating the area of the shape using the formula for the area of rectangle. Only two teachers – one Level 2 teacher and one Level 3 teacher – included Solution F, which divides the shape into the unit squares and counts them to find the area. This is the only one solution that does not require using any formula for finding the area.

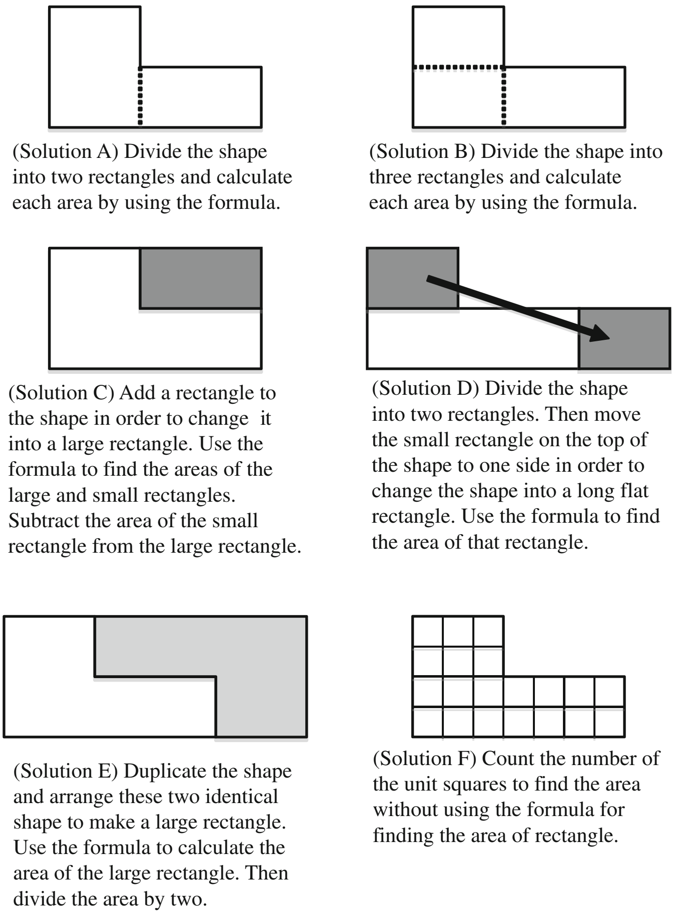

Fig. 3 Anticipated responses

One of the notable differences between the Level 1 and Level 2 teachers on one hand and the Level 3 teachers on the other concerns Solution C. Most Level 1 teachers and Level 2 teachers anticipated that Solution C would be the most likely or the second most likely. No Level 3 teacher thought it was the most or second most likely.

6. During and after the lesson, what point of view and method would you use to evaluate whether (a) students have attained the goal of the lesson, and (b) if the flow of the lesson and teacher's questions/reactions were appropriate?

Level 1 teachers all said that they would evaluate if students attained the goal of the lesson by asking students to solve similar problems. Their responses mainly focus on whether the students can solve similar problems and how they solve the problems at the end of the lesson or afterward. On the other hand, all the Level 2 teachers responded that they would base their evaluation upon the students' work during the class. They would also ask each student to write a journal response at the end of the lesson as a way to evaluate if the flow of the lesson and teacher's questions and reactions were appropriate.

Level 3 teachers also planned to use the students' work during the lesson. Moreover, all three argued that careful observation of how the students respond to the questions and tasks during the lesson is important to evaluate the lesson flow and teacher's questions and reactions.

Discussion

From the results of this study, two areas of expertise can be identified as important for using the textbook effectively in mathematics teaching: expertise in structured problem solving, and expertise in anticipating student responses.

Expertise in Structured Problem Solving

Stigler and Hiebert (1999) note that Japanese mathematics lessons often follow a sequence of the following five activities:

Reviewing the previous lesson
Presenting the problem for the day
Students working individually or in groups
Discussing solution methods
Highlighting and summarizing the major points.

To teach mathematics using structured problem solving, teachers need to understand the purpose of each of the five activities listed above.

Although all the teachers were asked to plan a lesson based on the same page of the textbook, there are notable differences in how they used the contents of that page. The responses to question 1 demonstrate that Level 2 and Level 3 teachers may share a common view of the lesson structure.

First, the Level 2 and Level 3 teachers plan to give the students the opportunity to solve the problem by themselves. Second, the Level 2 and Level 3 teachers include substantial time for the class discussion to compare and discuss various approaches for finding the area of the shape. They did this even though the textbook page includes no suggestions about having a whole-class discussion. Furthermore, they argued that the whole-class discussion on students' solution methods should be the main point of the lesson. The teachers' common responses regarding the structure of the lesson suggest that they share the structured problem solving framework for designing their lessons and see the importance of having *neriage* in this lesson. The Level 1 teachers, on the other hand, designed their lesson based on their own idiosyncratic views of lesson structure.

The responses to question 3 show that all the Level 3 teachers agree that the main point of the lesson is the whole class discussion around comparing and examining the solution methods. In contrast, all the Level 1 teachers see the main point as thinking about to how to find the area.

Although experienced Japanese teachers, like the Level 3 teachers in this study, often emphasize that having a meaningful whole class discussion is more important than letting each student find the area individually, it might not be easy for novice teachers to understand this. The novice Level 2 teachers focus mainly on illuminating the merits of each approach for finding the area of the particular shape in the problem. In contrast, Level 3 teachers see the purpose of the discussion as helping students see how the ideas can be used for any composition of shapes. This is one of the reasons why teachers must learn continuously to attain Level 3. The university courses and student teaching may be able to teach what structured problem solving lessons look like and teach the basic framework of that approach. But pre-service preparation cannot equip teachers to see the most important points in each activity during the lesson.

Another notable difference among three levels of teachers is how they use the questions and tasks of the textbook pages.

Level 1 teachers tended to follow the instruction in the textbook. Level 2 and the Level 3 teachers, on the other hand, tended to follow the flow of structured problem solving. Although both Level 2 and Level 3 teachers would avoid explaining how to solve the problem before individual students attack the problem, there are some differences in segmenting the lesson. Level 2 teachers seem to try to cover most or all the contents of the page. The Level 3 teachers seem less concerned with covering everything on the page. Their lessons use the major task on the page of the guiding questions and activities. This demonstrates that the Level 3 teachers chose to focus on the key question (*hatsumon*) for their students rather than simply using what is on the textbook page. Thus, one might say that Level 1 and Level 2 teachers tend to teach the textbook by following the contents on the page, whereas Level 3 teachers use the major tasks in the textbook to teach mathematics.

Expertise in Anticipating Students' Responses

Although anticipating students' responses is always important when designing a lesson, it is especially essential for planning *neriage*, the whole-class discussion of students' solution methods. Because the discussion will change based on how students solve the problem, anticipating all the solution methods, including possible misunderstandings, helps teachers prepare to handle the discussion flexibly.

The responses to question 5 show that most of the teachers could anticipate a variety of solution methods. The solutions include not only dividing the shape into rectangles but also adding a rectangle to the shape to form a large rectangle, Solution C. But, the Level 1 and 2 teachers anticipated Solution C as the most likely approach whereas the Level 3 teachers considered it neither the first nor second most likely approach. Level 1 and Level 2 teachers may be anticipating solutions based on their own experience solving the problem, while the Level 3 teachers anticipate solutions based on their insight into the students' viewpoints. As with the ability to understand the important points in a discussion, coursework and student teaching are probably insufficient for developing the necessary expertise to see a problem from the students' perspective.

All the teachers in this study were able to find the area of the shape on the page in several different ways, which suggests that they all possess the necessary knowledge of mathematics for teaching this lesson. For designing the flow of the lesson, the Level 2 teachers, like the Level 3 teachers, organized the lesson around structured problem solving. But, the important differences in how Level 2 and Level 3 teachers planned the lesson suggest that the knowledge about teaching mathematics acquired in pre-service preparation is different from the expertise needed for Level 3 teaching. In order to develop that expertise, teachers need to continue learning well after their coursework is complete.

Helping Practicing Teachers Increase their Knowledge and Expertise

Two Major Types of Professional Development

When designing professional development programs for practicing teachers, it is useful to recognize that professional development falls into two categories.

The first category, which I call *phase 1* professional development, focuses on increasing a teacher's *knowledge* for teaching mathematics. This includes content knowledge of mathematics, pedagogical content knowledge for teaching mathematics, curricular knowledge for designing lessons, and general pedagogical knowledge (Shulman, 1986). In order for teachers to develop such knowledge, phase 1 professional development usually provides opportunities to learn through reading books and articles, listening to lectures, and watching videos or demonstration lessons. Most university coursework falls into this category.

The second category, *phase 2* professional development, focuses on developing *expertise* for teaching mathematics. This includes the expertise needed to develop lessons for particular students, to use various questioning techniques, to design and implement formative assessments, to anticipate students' responses to questions and tasks, and to make purposeful observations of students during class. For teachers to develop such expertise, they need opportunities to plan lessons carefully, to teach the lesson based on the plan, and to reflect upon the teaching and learning based on careful observation. Japanese teachers and educators obtain these experiences through *lesson study* (Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Akihiko Takahashi & Yoshida, 2004; Yoshida, 1999).

The Japanese Lesson Study Model

The practice of lesson study originated in Japan. It is the primary form of professional development there, and is credited with dramatically improving classroom practices in the Japanese elementary school system (Fernandez, Chokshi, Cannon, & Yoshida, 2001; Lewis, 2000; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; A. Takahashi, 2000; Yoshida, 1999).

Lesson study embodies many features that researchers have noted are effective in changing teacher practice, such as using concrete practical materials to focus on meaningful problems, taking explicit account of the contexts of teaching and the experiences of teachers, and providing on-site teacher support within a collegial network. It also avoids many shortcomings of typical professional development, which has been criticized as short-term, fragmented, and externally administered (Firestone, 1996; Huberman & Guskey, 1994; Little, 1993; Miller & Lord, 1994; Pennel & Firestone, 1996).

Lesson study promotes and maintains collaborative work among teachers while giving them systematic intervention and support. During lesson study, teachers collaborate to: (1) formulate long-term goals for student learning and development; (2) plan and conduct lessons based on research and observation in order to address these long-terms goals through actual classroom practices for particular academic content; (3) carefully observe the level of students' learning, their engagement, and their behavior during the lesson; and (4) hold debriefing sessions with their collaborative groups to discuss and revise the lesson accordingly (Lewis, 2002b).

One of the key components in these collaborative efforts is the *research lesson* – a single lesson, typically prepared by a group of teachers, which is observed in the classroom by the lesson study group and other practitioners, and is then analyzed during the group's debriefing session.

During the research lesson, the observers carefully note how the lesson unfolds, gathering data based on the lesson plan that the lesson study group has prepared. The research lesson is followed by a debriefing session, in which teachers review the data together in order to: (1) make sense of educational ideas within their practice; (2) challenge their individual and shared perspectives about teaching and learning; (3) learn to see their practice from the student's perspective; and (4) enjoy collaborative support among colleagues (Akihiko Takahashi & Yoshida, 2004).

Through the lesson study process teachers have opportunities to develop the skills for Level 3 teaching. This collaborative participant-centered professional development an effective approach for developing *expertise* based on the *knowledge* that the teachers learned through phase 1 professional development.

A Framework for Designing Programs for Prospective and Practicing Teachers

Providing a variety of effective programs and usable resources for prospective and practicing teachers is an important role for the universties and the school systems. The first step in designing such programs and resources is to develop a framework to identify the purposes and target audiences. Based on the earlier discussion regarding teacher knowledge and expertise, the three levels of teaching, and the phase 1 and phase 2 categories of professional development, I propose the following matrix as a framework for mathematics teacher education (Table 2).

Phase 1 for Level 1

Level 1 is the foundation for becoming a teacher of mathematics, since one cannot teach mathematics if one does not know the content. Usually prospective teachers who come to a university or a teacher-training institute already possess the basic knowledge required for Level 1 teaching. If this is not the case, there should be programs to review content knowledge, such as through online courses or individual tutoring. Although they might be needed for only a small number of prospective teachers, such programs could help more people become teachers. Online courses and resources might be appropriate since the target audience may be small in number but geographically widely spread.

Phase 1 and Phase 2 for Level 2

According to Japanese mathematics educators, developing knowledge and expertise for Level 2 teaching should be the major focus of university teacher training programs for prospective teachers. Since knowing the content of mathematics is not enough, Level 2 teaching requires knowledge beyond being able to solve mathematics problems for elementary and middle school students. For example, to teach the formula for finding the area of a parallelogram, Level 2 teachers must know how the formula was developed, why the formula works for any parallelogram regardless of its size and orientation, and how the formula is related to other formulas for finding the area of basic geometric shapes.

The knowledge required for Level 2 teaching is a special kind of knowledge for mathematics teachers, and is often called pedagogical content knowledge (Shulman, 1986). Since the knowledge is only required for teaching mathematics, universities and teacher-training institutes should design special courses and resources for prospective teachers of mathematics. In other words, providing regular university level mathematics courses is not sufficient and not appropriate for

Table 2 A framework for developing programs and resources for mathematics teacher education

	To become a Level 1 teacher	To become a Level 2 teacher	To become a Level 3 teacher
Phase 1 Professional development	Strengthen knowledge of mathematics. . . . . .through: • Studying textbooks and workbooks • Using online resources and courses	Acquire knowledge of mathematics teaching and learning– • Pedagogical content knowledge • Knowledge of the curriculum • Knowledge of the students • Knowledge of pedagogy. . . . . .through: • University courses • Professional development workshops • Online resources • Classroom videos • Classroom observations, including participating in research lessons	Update knowledge of mathematics teaching and learning. . . . . .through: • Workshops • Evening and summer coursework
Phase 2 Professional development		Understand the process of lesson study . . . . . .through: • Designing mock-up research lessons as part of university coursework • Lesson study during student teaching	Develop expertise for teaching (*neriage* etc.). . . . . .through Lesson Study

prospective teachers. Providing dedicated courses and resources for prospective teachers should be the major focus of Phase 1 professional development in preparing Level 2 teachers.

At the same time, prospective teachers should develop an understanding of what a good lesson looks like and how to design lessons. For example, Japanese teacher candidates learn the basic form of structured problem solving, the sequence of five activities and their roles, the technical terms *hatsumon*, *kikanshido*, and *kansho* and the purpose and importance of the activities they denote.

Phase 2 professional development in Level 2 teaching should focus on introducing the idea and the process of lesson study. Engaging in lesson study offers teacher candidates not only practice in developing lessons and teaching lessons based on a plan, but also practice in observing students' learning processes and reflecting upon a lesson.

Phase 1 and Phase 2 for Level 3

Achieving Level 3 is quite demanding and requires extensive Phase 2 professional development. It is essential to understand the philosophy of teaching and learning mathematics, to develop a vivid image of the ideal mathematics class as a model, and to know key instructional techniques for enabling students to learn mathematics independently. Most knowledge and understanding for Level 3 teaching may be obtained through Phase1 professional development programs such as reading books, listening to lectures, and observing well-designed mathematics classes. However, acquiring the knowledge and understanding is not sufficient to develop the expertise needed for Level 3 teaching. To develop this expertise requires considerable teaching experience, with reflection. Japanese teachers and researchers work collaboratively through lesson study to develop expertise for Level 3 teaching.

Conclusion

Japanese educators say that "good teachers know how to read between the lines of the textbook." We saw that in the results of the study described in this chapter. The textbook makes no explicit mention of whole-class discussion, but the Level 3 teachers in this study said that the most important part of the lesson would be the whole-class discussion on comparing and examining the various approaches to finding the area of the shape.

Level 1 teachers might be able to teach the textbook, but they are not equipped to use the textbook to teach mathematics. Level 2 teachers may be ready to use the textbook to teach mathematics, but their teaching may not be very effective. Because of their deep understanding of the content of school mathematics, their ability to anticipate student responses to a task, and their skill at *hatsumon, kikan-shudo,* and *neriage*, Level 3 teachers are able to use ideas from the textbook to help their students learn new mathematics for themselves.

Japanese educators also say, "You can be a teacher if you complete the teacher preparation program that universities provide. However, becoming a good teacher is not so easy. It requires a life-long learning and collaboration with colleagues." This small-scale research study provides some insight into why this statement is true, by revealing the depth of knowledge and expertise one needs to be a Level 3 teacher. Although a very few teachers might be able to attain Level 3 by themselves, school systems and universities need to establish a system for helping the majority of teachers develop both the knowledge and the expertise to become Level 3 teachers, to be able to use their textbook to teach mathematics effectively.

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Appendix

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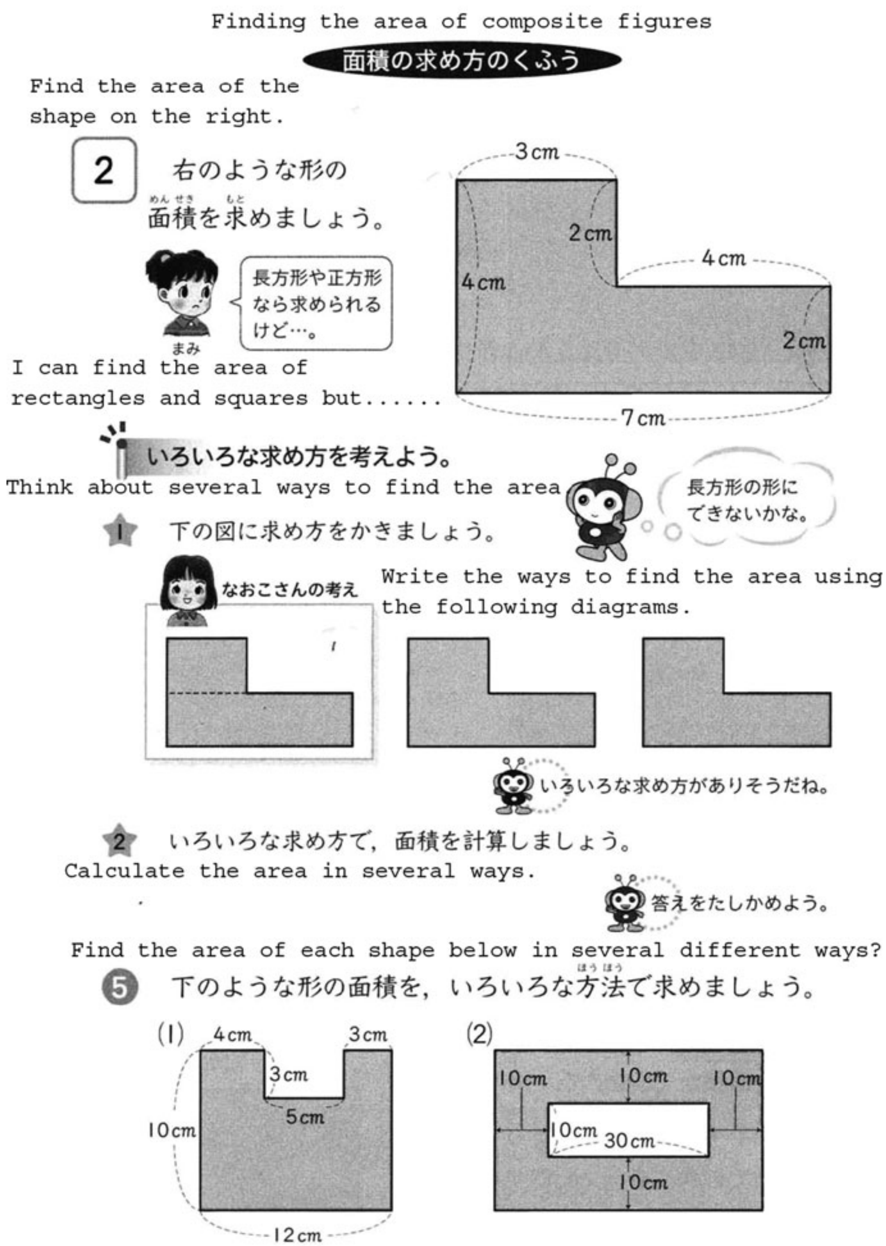

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Perceptions of School Mathematics Department Heads on Effective Practices for Learning Mathematics

Suat Khoh Lim-Teo, Kwee Gek Chua, and Joseph Kai Kow Yeo

Abstract Teachers' knowledge of discipline-specific pedagogy affects classroom practices which in turn affect learning by pupils. While there are many desirable practices which demonstrate strong principles of mathematics-specific pedagogy, teachers' adoption and use of these practices in part depend on the value placed on them by the school management. This chapter presents the quantitative and qualitative findings of a survey as well as insights from interviews of primary schools' mathematics department heads on their perceptions of the importance of various teachers' practices in contributing towards effective mathematics teaching and learning. The study shows that the department heads value practices which contribute towards conceptual learning and pupil motivation to learn which go beyond achievement in performance tests.

Keywords Mathematics teaching · Effective mathematics teaching practices · Mathematics heads of departments · Perceptions of effective teaching

Introduction

One of the most widely accepted axioms in education is that good teachers matter. Good teachers are effective and their students learn. Teachers who make a difference have been the focus of research for a long while. There have been studies looking at generic teacher qualities and characteristics, teacher behaviors and actions in particular subject areas and factors determining these actions, and links between teacher actions and student learning.

While teachers are the central "actors" in the classroom environment, their choice of actions in crafting their lessons is affected by stakeholders including primarily their pupils but also the school leaders, the parents of their pupils and, further removed, the central education authorities determining the curriculum. Teacher

S.K. Lim-Teo (✉)
National Institute of Education, Singapore, Republic of Singapore
e-mail: suatkhoh.teo@nie.edu.sg

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educators who provide teacher preparation programs and professional development courses have also significant inputs at various stages of teachers' professional careers. Teachers' actions and practices are thus the result of making choices in a complex environment which encompass situational factors and demands and expectations of all the stakeholders.

In Singapore, mathematics teacher educators, grounded in educational theories and necessarily strengthened through pedagogical experience, often have firm and generally agreed ideas of what constitutes effective pedagogies which produce rich mathematics learning. However, there appears to be a pervasive perception that teacher educators at universities and experienced practitioners out in schools may not be in agreement over what works in real classrooms and such differences may result in tensions for beginning teachers moving from teacher preparatory programs into the school environment. While the beginning teachers as professionals need to work out their own choices of teaching actions, a deeper understanding by teacher educators of the senior school practitioners' viewpoints could help in resolving tensions and result in easier transitions for beginning teachers from teacher preparatory programs to schools.

The purpose of this chapter is to present a study which seeks to understand the perceptions of school mathematics heads of departments on the effectiveness of various practices carried out in mathematics teaching in terms of pupil learning. This particular group of stakeholders is deemed to be very influential over teachers as practitioners and thus such understanding would be highly informative to teacher education faculty in their teacher education work.

Rationale for Study

The process-product research of the 1970s and 1980s addressed the issue of teacher quality primarily by studying the impact of specific teacher behaviors on student performance (see Brophy & Good, 1986; Doyle, 1986). Later studies on teacher effectiveness have focused on how teacher cognition and decision-making affected the quality of classroom instruction (Ball, 2000; Calderhead, 1996).

In mathematics education, Mewborn (2003) summarized that connections between student achievement and teacher characteristics have not been conclusive and that the relationship is nonlinear and threshold effects seem to occur. Although research has not shown conclusively what these effective teacher qualities are and how they develop or how exactly they result in student learning, there appears to be general consensus about some characteristics of good teachers. According to the National Commission on Teaching and America's Future (2003), good teaching requires that teachers have a deep knowledge of the subjects they teach, a repertoire of instructional skills to teach the content, knowledge about their students, and attitudes that support high levels of learning for all students. The standards for excellence in teaching mathematics in Australian schools (AAMT, 2006), which was a joint effort of a professional body, the Australian Association of Mathematics Teachers and mathematics educators/researchers at Monash University, stated the necessary expertise in three domains for teachers to be deemed as excellent in their profession. The three domains are professional knowledge, professional attributes and professional practice, each of which is shaped by and inter-related to the others.

Research studies can also explore the characteristics of effective mathematics teachers as viewed from different perspectives by different groups of stakeholders. Kaur (2004) studied the qualities of good mathematics teachers from the perspectives of Singapore mathematics teachers and found that the desired teacher qualities belonged to the following categories: Personal qualities, rapport/relationship with students, teaching qualities and expectation of student work. Kaur also found that good mathematics teachers constantly upgrade their knowledge of mathematics and teaching of mathematics through professional development courses conducted by institutions of higher learning or professional bodies. Sanders (2002) explored what schools thought made a good mathematics teacher by looking at criteria listed by 80 secondary schools in the UK to candidates applying for the position of mathematics teachers. She found that all the requirements fell into five broad categories – qualifications, teaching skills, professional qualities, personal qualities and “value-addedness” which included extra skills and knowledge beyond those which focused on the mathematics classroom. It was interesting that the requirements for teaching skills were generic and not specific to mathematics.

Mathematics teacher practices in classroom as a research area has been around for a long time although not in Singapore due to difficulties in gaining access to classrooms. This was changed with the setting up of the Centre for Research in Pedagogy and Practice at the National Institute of Education, Singapore by the Ministry of Education. In its first core study which was a very large scale study to provide a window on Singapore lessons, students in mathematics classes were observed to be mainly engaged in activities focusing on factual/rote knowledge and procedural computation. Yeo and Zhu (2005) analyzed data collected from 59 primary five lessons and noted that answer checking, individual seatwork and monologue teaching occupied 39%, 32% and 13% respectively of all the phases in the mathematics lessons. While the large amount of time spent on teacher monologue and questioning merely to elicit specific pre-determined “correct” answers may seem to go against sound pedagogical principles, especially viewed in the paradigm of constructivism, Luke (2005) who was in overall charge of this study postulated that it could well be an effective approach for establishing the correct concepts and use of terminology in subjects such as mathematics. Thus, while large scale studies provide baseline data, comparative data across subjects are not really informative and truly useful studies into effective teaching must therefore be subject or discipline specific as they yield knowledge which can be applied in context. In summarizing research on whether subject knowledge mattered, Darling-Hammond (2006) wrote:

> Knowing how students understand (or misunderstand) particular subjects and having a repertoire of strategies to help students engage ideas central to the discipline is at the core of pedagogical content knowledge (Shulman, 1987; Grossman & Schoenfeld, 2005). The subject does matter centrally for teachers, not only in its own right as the grist for teaching, but also as the context for developing understanding for teaching that enables learning.

Any mathematics educator observing the TIMSS Video Study would realize that the depth and richness of the discussions carried out in mathematics lessons in Japan were facilitated by teachers who did not just have strong generic skills but who also had deep pedagogical content knowledge for teaching mathematics. Through her many studies of mathematics-specific teacher practices, Ball (2000) has stated that depth of teachers' understanding of Mathematics Pedagogical Content Knowledge (MPCK) is a major determinant of teachers' choice of examples, explanations, exercises, items and reactions to children's work. While mathematics teacher education is a well-established area in educational research history of the United States, it is still relatively new in Singapore.

In seeking to understand mathematics teachers' pedagogical content knowledge and what this expertise means in the Singapore context, a research project called the MPCK project was undertaken from 2003 by a team from the Mathematics and Mathematics Education Academic Group of the National Institute of Education. Studies within the project had generated findings on pre-service primary mathematics teachers' understanding of mathematics and mathematics teaching approaches and strategies as could be determined from written tests (Cheang et al., 2007; Lim-Teo, Chua, Cheang, & Yeo, 2007) and a similar but smaller study was also carried out for practicing teachers from a few participating schools. However, while these studies provided evidence or otherwise of the teachers' knowledge and understanding, it is the ability to amalgamate, consolidate and translate such theoretical understanding into actual practices and activities in mathematics lessons that will result in effective pupil learning. Thus, as teacher educators, the MPCK project team was also keen to find out whether what was deemed desirable practices were actually taking place in schools and whether teachers were encouraged to value such practices as effective for mathematics learning.

In Singapore schools, heads of various subject departments form the middle management and they are the ones who influence and guide teachers under their charge. They are also expected to be pedagogical leaders since the school leaders cannot be curriculum leaders in every discipline area. It is thus reasonable to expect that the pedagogical practices of teachers are strongly influenced by the views of the mathematics heads of department (HODs) of their schools even more than the pedagogical principles and methods encountered during pre-service preparation. These reporting officers of the mathematics teachers would have their own professional perceptions of the relative contributions towards pupils learning engendered by different teaching practices. Such perceptions, which reflect school cultures in terms of what the school management values among various teaching practices, will affect the teachers' use of the different practices in their mathematics lessons. Knowing the practices valued by heads of mathematics departments in schools will provide data from the practitioners' perspectives and also help the MPCK project researchers to understand the rationale behind teachers' choice of practices.

The MPCK project thus designed and implemented a study to examine the perceptions of mathematics HODs in Singapore primary schools with regard to pedagogical practices for effective learning of mathematics at the primary school level. While acknowledging that generic teaching skills and teacher dispositions and characteristics are of great importance, the study focused specifically on practices which applied to mathematics teaching and learning. This chapter will describe the study and its findings.

The Study

Context

In the Singapore education system, the curriculum is determined centrally by the Curriculum Planning and Development Division (CPDD) of the Ministry of Education and implemented across all schools. Other than for very few specialized schools, all the mainstream schools implement this common curriculum due to compulsory national examinations at milestone junctures of schooling. There are regular communication sessions between CPDD and schools to ensure that curriculum objectives are clearly understood and content changes or new emphases are made known to the schools. In fact, all textbooks used by schools have to be certified by the CPDD as complying with the requirements of the curriculum in terms of content and level of difficulty. There is thus a high degree of adherence to the specified curriculum by all schools.

In addition, in this very centralized system, teachers are centrally employed by the Ministry of Education and posted to the schools according to staffing needs. In fact, the teachers are already appointed employees of the Ministry of Education before being sponsored for pre-service teacher preparation at the single teacher education in Singapore, the National Institute of Education (NIE). The NIE offers all teacher preparation programs and a large portion of professional development courses done by teachers in Singapore.

There are around 180 primary schools in Singapore and it must be noted that primary teachers in Singapore are not subject specialists but are generalists who teach a range of subjects, the three main subjects being English, Mathematics and Science. The three primary teacher preparation programs of NIE cover generic knowledge and pedagogy courses in specific subject areas. The curriculum subjects to be taken by pre-service teachers are mainly determined by the employer, the Ministry of Education, based on the needs of the system and hence Mathematics is a subject which is taken by almost all the student teachers. There are no pre-requisite requirements other than at the basic level of a credit pass at the Cambridge GCE "O" level examinations and this can result in issues concerning the mathematics competency level of the teachers.

Pedagogical practices of mathematics teachers are learned and developed in two environments, firstly, during their teacher preparation programs or subsequent professional development courses and secondly, in the school environment. Mathematics pedagogy courses in NIE's teacher preparation programs tend to encourage learner-centered constructivist approaches to teaching mathematics and these practices are also endorsed and encouraged by the CPDD in the syllabus documents and communications with school curriculum leaders. However, as mentioned earlier, the actual practices observed in Singapore's mathematics classrooms tended to emphasize procedural correctness and knowledge of meanings of concepts without discussion of the how these meanings could have been derived. While feedback from student teachers indicates that the methods which they acquire and develop during their teacher preparation courses are valued as meaningful and effective for learning, there is also feedback from practicing teachers that such methods were too time-consuming and thus impractical. This disconnect could be attributed to the "achievement-in-examinations" culture which pervades the society and the school system in Singapore. It appears that instrumental understanding is preferred as the more time-efficient and effective way of achieving high student performance at examinations, especially the high-stakes national examination at the primary school-secondary school interface.

In the highly competitive environment of Singapore, not only are students and parents concerned with individual examination performance, schools also are concerned with the overall performance of their students in the national examination. In such a context, it was of interest to the research team to find out whether mathematics HODs would value more the practices which efficiently generate better examination results as compared to those which foster longer-term understanding since the HODs are the reporting officers for teachers and are also responsible for leading and setting directions for teachers teaching their subjects. Generally, schools use the Ministry of Education's Enhanced Performance Management System (EPMS) forms for staff appraisal and rate their teachers using the performance indicators which place great emphasis on generic pedagogy and class management. The researchers do not know of any school which uses mathematics-specific appraisal tools for assessing the performance of mathematics teachers.

In understanding the sample of the study, the reader should note that while there is a growing proportion of graduate teachers in the primary schools, subject HODs may or may not be graduates or have any specific academic qualifications for the subject. These HODs would usually have started out as generalist teachers but, in addition to recognition of their leadership capabilities, they get appointed to such positions due to their experience and strengths in teaching the particular subject. Once they assume such a management position, their teaching duties will, in general, be limited to that specific subject. Teachers who are selected to be HODs by schools are usually sent for a management program of which a small portion is in the curriculum subject area. Also, while teaching excellence for and inclination towards a particular subject and management potential are the usual factors affecting the appointment of HODs, a requisite experience in terms of number of years is not a necessary condition.

Procedure

The study was carried out through a questionnaire. These questionnaires were mailed to all the primary school mathematics HODs. The questionnaire required the HODs to rate a list of 35 items on practices carried out in mathematics lessons, assessing their importance in contributing to effective mathematics learning. The questionnaires were returned by post.

After the quantitative analysis, a series of one-to-one interviews was conducted. A sample of 10 HODs was chosen from the pool of respondents for the one-one interviews to seek clarifications on their rating of the items and obtaining a fuller picture of HOD's thinking on practices in mathematics teaching. This sample comprised HODs with varying number of years of teaching experience, academic and professional qualifications. Although the sample was small, it included HODs with different demographic characteristics and who belonged to schools with differing pupil abilities. More elaboration will be given in the qualitative analysis section.

The interview started, as an introduction, by eliciting information on how these key personnel appraise their teachers' mathematics teaching performance. Questions were asked on the mathematics practices which they valued, particularly those with very high ratings and those they valued the least. They were then asked to elaborate on how these practices have impact on pupils' learning in terms of performance and attitude. Each interview lasted approximately 1 h.

The Instrument

The instrument used was a questionnaire in which the participants were asked to rate a list of 35 practices of mathematics teachers which may be observable in mathematics teaching. The rates, on a scale of 0–10, were their assessment of the importance of each practice as contributing to effective mathematics learning by pupils. There were also some open-ended questions where the HODs could expand on their views. While the open-ended questions were crafted for the purpose of elaboration on their responses, the development of the items in the questionnaire was built upon earlier work of the MPCK project team, to be explained below.

In the MPCK study, the project team developed an MPCK framework consisting of four constructs which mathematics teachers were expected to have and to apply to their teaching. These were: (i) Content knowledge with deep understanding, (ii) Multiple representations of concepts, (iii) Understanding the cognitive demands of mathematical tasks, and (iv) Ability to identify learner difficulties and misconceptions. The first construct here does not refer to higher level of mathematics content as covered at university levels but rather a deep understanding of content relevant to the primary school mathematics curriculum. Interested readers may refer to Lim et al. (2007) for more details of the project's framework of MPCK.

While the constructs gave a framework to what we understood as MPCK, they were rather abstract and it was necessary to actualize them into a list of teaching approaches, practices or actions which could be observed or deduced from what happened during mathematics lessons. These practices were identified through the:

(a) analysis of the video-recordings of some units of mathematics lessons conducted by beginning teachers,

(b) researchers' experience through years of observing student-teachers in their school practicum, and
(c) teaching methods advocated by the mathematics curriculum document of Singapore's Ministry of Education

The term coined for these practices was *MPCK-in-action practices*. The list of practices was distilled down to 35 practices which could be observed in mathematics lessons. These 35 MPCK-in-action practices were considered by experienced mathematics teacher educators to be important for pupil learning and it was found that they could be grouped under ten categories as given in Table 1. To check for content validity, experienced mathematics teacher educators not in the research team were asked to examine and categorize these practices. In general, there was no clear mapping of practices to the four MPCK constructs given earlier since knowing that a particular construct was present must be imputed from some combination of practices exhibited by a teacher and this combination could vary between different situations and contexts. Nevertheless, there was consensus that all the practices taken holistically would cover the four MPCK constructs.

The titles of these categories, for example *Sequencing of Activities* and *Explanation*, carried neither positive or negative values but each practice within a category carried adjectival or adverbial descriptors for which an observer would need to make a value judgment as to whether such a practice was demonstrated. For example, while *Explanation* is a category, the item *Explains Clearly and Concisely* carries the descriptors of "clearly" and "concisely". While some categories contained more items, there were some which contained only one or two practices, because the category was rather holistic and could not be broken down into different component practices.

In addition to these 35 items and the open-ended questions, the survey questionnaire also asked for demographic data pertaining to the HODs' qualifications, teaching experience and experience as a mathematics HOD. Also, the HODs were given opportunities to clarify and elaborate their views through three open response questions. Specifically, the questions provided opportunities for them (i) to explain their ratings, (ii) to identify two most important practices which could include practices not listed among the 35 items and (iii) to explain how these valued practices would influence pupils' learning outcomes.

Analysis of Data

From the data collected, quantitative analysis included calculating the mean ratings of each item and ranking the items in order of importance. The data was also analyzed to see if the responses were affected by the HODs' experience or qualifications.

For qualitative data, the interviews were transcribed and studied mainly to provide elaboration on why the HODs regarded some practices were considered more

Table 1 Survey items according to categories

General categories	Observable mathematics teachers' practices
Sequencing of activities	1. Sequences learning activities logically
	2. Structures examples/tasks from simple to complex
	3. Adopts the concrete-pictorial-abstract approach where applicable
Choice of activities	4. Identifies and selects the most suitable learning activity to teach a certain topic
	5. Designs/modifies learning activities to match pupils' learning needs
	6. Uses a variety of learning activities to develop the given concept
Connections between topics and between concepts	7. Builds on pupils' prior knowledge to teach new knowledge
	8. Relates/applies concepts to the real world context
	9. Provides opportunity to integrate topics/concepts learnt
	10. Makes links between topics/concepts
Balance between concept development and mathematical procedures	11. Consciously emphasizes the underlying reasons/explanations for the given mathematical procedure
	12. Focuses the pupils on the essential steps and necessary conditions in procedures
	13. Emphasises computational speed and accuracy
	14. Allows pupils to explore alternative procedures in solutions
	15. Provides pupils the opportunity to identify and rectify errors in presentation of solutions
	16. Places due emphasis on conceptual understanding
Explanation	17. Explains mathematical terms accurately
	18. Explanations are appropriate to the learners' level
	19. Explains clearly and concisely
	20. Uses appropriate range of examples
	21. Uses non-examples to enhance pupils' understanding of concepts
	22. Uses multiple modes of representations for developing concepts or establishing procedures
	23. Provides counter-examples for the concept/procedure
Role model in demonstrating mathematical processes	24. Models exemplary mathematical behaviour (e.g., Being logical and systematic in presentation, using mathematical instruments correctly)
Mathematical communication	25. Uses correct mathematical terms and language
	26. Makes provision for pupils' mathematical communication
Mathematics curriculum	27. Displays knowledge of Singapore mathematics curricular emphasis and syllabus requirements
Questioning techniques	28. Monitors pupils' understanding through appropriate questions
	29. Uses structured questioning to facilitate development of concepts
	30. Uses structured questioning to establish mathematical procedures
	31. Asks questions to stimulate mathematical thinking processes e.g., comparing, classifying, generalizing, deducing etc
Responding to students	32. Detects pupils' errors/misconceptions
	33. Analyses the cause of pupils' errors/misconceptions
	34. Takes appropriate actions to rectify errors and/or correct misconceptions
	35. Adopts alternative explanation/representation when pupils encounter difficulty in learning concepts/procedures

important than others. The responses were examined to identify the themes and patterns and whenever possible typical commentaries from the HODs were used to illustrate these themes and patterns.

Findings

Quantitative Analysis of Survey Items

From all the primary schools in Singapore, 81 HODs responded by completing and returning the questionnaire. The demographic data of these HODs showed that about two-thirds of them had more than 10 years of teaching experience. The distribution according to years of mathematics teaching experience is given in Table 2.

Table 2 Distribution of HODs across experience in teaching mathematics

Years of mathematics teaching experience	Percentage of the 81 respondents (%)
Up to 5 years	11.1
6–10 years	22.2
11–20 years	23.5
More than 20 years	43.2

About 25% have a degree as their highest academic qualification though not necessarily a degree in Science or Mathematics and 56% had taken a full-time professional diploma specially tailored for preparing experienced teachers to become department heads.

The Cronbach alpha for the whole instrument is 0.97. This high reliability coefficient of the 35-item instrument coupled with the fact that each item is equally weighted based on the first principal component analysis confirms that each item contributed to an important aspect of the positive impact of good teacher practices.

A one-way ANOVA results on the mean of all items was employed to verify whether the responses were affected by the respondents' number of years of teaching mathematics, number of years as HOD, their highest academic qualification and professional qualifications. None of them yielded any significant differences at 0.05 level, showing that the HODs' profile and background do not play any crucial role in identifying similar good teachers' practices in contributing to effective mathematics learning.

On a scale of 0–10 rating, with 10 being the most important, the mean rating score for each item fell between 7.46 and 9.35 and the standard deviation for each item between 0.59 and 3.52.

There were nine items with very high mean ratings exceeding 9.10. For every one of these items, at least 44% (36 HODs) gave a perfect rating of 10 and 73% or more gave ratings of 9 and 10. These items are given in Table 3 in descending order of mean ratings.

Table 3 Highly rated items of mathematics teacher's practices

Item no.	Mathematics teacher's practices	Mean	S.D.
19	Explains clearly and concisely	9.35	0.79
34	Takes appropriate actions to rectify errors and/or correct misconceptions	9.30	0.59
18	Explanations are appropriate to the learners' level	9.28	0.90
35	Adopts alternative explanations/representation when pupils encounter difficulty in learning concepts/procedures	9.21	0.89
33	Analyses the cause of pupils' errors/misconceptions	9.19	0.90
32	Detects pupils' errors/misconceptions	9.15	1.01
3	Adopts the concrete-pictorial-abstract approach where applicable	9.14	1.32
5	Designs/modifies learning activities to match pupils' learning needs	9.12	1.07
2	Structures examples/tasks from simple to complex	9.10	1.17

The high ratings of items 18 and 19 indicate the importance of clear and precise explanations. Also, all four items belonging to the last category *Responding to students* which concerns detecting, analysing and rectify errors and misconceptions appropriately are also very highly rated. These practices are very strongly regarded by the HODs as essential to promote clear understanding by pupils and strong performance in achievement tests.

Table 4 below shows the three items with the lowest ratings. These three items were the only ones with mean ratings below 8.00. The lowest rated item 21 had a low mean because three respondents gave it a zero rating, a fact which also accounts for the high standard deviation. However, the same three HODs gave high ratings of 8 and above to item 23 which was to provide counter-examples for concept or procedure. Nonetheless, item 23 was also a comparatively low-rated practice. In item 23, the use of counter-examples was for procedures as well as concepts and this item may have been rated more highly due to "procedures". It is quite common for teachers to show how pupils should *not* carry out procedures but uncommon for primary mathematics teachers to show non-examples of concepts or counter-examples of statements. The relatively low rating of item 13 was a little surprising considering

Table 4 Three lowest rated items of mathematics teacher's practices

Item no.	Mathematics teacher's practices	Mean	S.D.
21	Uses non-examples to enhance pupils' understanding of concepts	7.46	3.52
13	Emphasises computational speed and accuracy	7.49	2.68
23	Provides counter examples for the concept/procedure	7.96	2.04

that Singapore pupils' good computational skills seem to indicate a strong emphasis by their teachers.

Results from Open-Ended Questions

The responses to the open-ended questions in the survey showed the practices which they valued contributed towards two overarching goals: (a) conceptual understanding by pupils and (b) the affective aspects of motivating pupils through helping them understand and enjoy mathematics. What came through in the open responses was that HODs were very concerned with true mathematical learning beyond mere computational speed and accuracy which also explained the relatively lower rating of item 13. They valued the practices which contributed towards these goals and these practices can be grouped into five themes as follows.

The first is that effective mathematics teachers should adopt the concrete-pictorial-abstract (C-P-A) approach wherever possible in the teaching of mathematics to ensure pupils have correct conceptual understanding. This is not surprising as the C-P-A approach has been advocated in the Singapore mathematics syllabus since the 1980s. The HODs also saw activity-based lessons and the practice of using manipulatives as effective for engaging learners.

The second theme substantiates the ratings of two of the top three items, i.e., the ability to present materials and explain clearly. Several HODs commented that *The teachers must be strong in mathematics pedagogies and must ensure that their explanation is clear,. . ., presented in a logical and systematic way*. Choice of activities, importance of sequencing the activities with clear explanations were perceived as essential in mathematics pedagogy.

Yet another recurring theme is the recognition that mathematics is a hierarchical subject, therefore there is a need to review prior knowledge and to teach from what is familiar to a new concept which is unfamiliar. One HOD wrote, *Mathematics is a subject which requires pupils to build on prior knowledge and concepts learned* while another HOD shared that teachers must *ensure that pupils have the foundation before proceeding to the current topic*.

The fourth theme is the importance of relating mathematics to the real world context so that pupils can see meaning in the mathematics they are learning and will be motivated to learn. As written by one of the HODs, *I think it is essential that pupils are able to relate what they have learnt to the real world because then they will see meaning in what they are learning* and by another, *I certainly think that relating the mathematical topics to real life context is absolutely necessary to cultivate their interest in the subject.*

The last theme is related to the second highest rated item and the ability to diagnose errors and remediate errors is also another important mathematics teaching practice. As one HOD puts it, *Teachers must have adequate intervention skills when they diagnose a particular learning disability or lack of mastery of a mathematics skill.*

For the last question in the questionnaire, the HODs did not refer to the list of teaching practices to explain in depth how particular practices would influence pupils' mathematics learning outcomes. Instead they explained why the two main goals as given above were important for pupil learning to take place. With regard to conceptual understanding, one HOD wrote: *Understanding concepts is crucial for mathematics learning. No meaningful learning takes place if pupils are not aware of underlying concep*ts *within each topic.* Another wrote: *Experiential learning helps pupils understand and "see" concepts on their own.* From what they have stated, it was clear that the HODs value the conceptual understanding and they expect teachers are able to ensure that pupils gain conceptual understanding through different learning experiences. The HODs saw value in experiential learning because pupils were more likely to take ownership of their learning, have deeper understanding of the concepts and have longer retention of what they have learnt.

Qualitative Analysis

The ten HODs selected for interview had varying years of teaching experience and years as HOD. Table 5 below provides a matrix showing the characteristics of the 10 HODs, who are labelled C, E, G, K, L, N, P, R, T and W. The six HODs who hold a bachelor's degree or higher have their degrees indicated. Except for K and P, all the others had undergone the diploma in departmental management program for preparing them to be HODs.

As mentioned earlier, about two-thirds of the 81 HODs who responded to the survey had more than 10 years of mathematics teaching experience and this was the case for seven out of the ten selected for interview. However, compared to the overall survey data where around a quarter were degree holders, among those HODs with less teaching experience and those who were more newly appointed, such HODs tended to be degree holders. This is also reflective of Singapore's move towards an all graduate teaching force in primary schools, through recruitment and through upgrading of non-graduate teachers. As such, degree holders in primary schools were expected to take on more leadership roles despite having less teaching experience. P, C and G are examples of those who were graduates recruited into teaching while T, K and W were non-graduate teachers who subsequently went on to obtain a degree.

Table 5 Experience of the HODs interviewed

	Number of years of teaching mathematics				
No. of years as HOD	< 3 yrs	3–5 yrs	6–10 yrs	11–20 yrs	> 20 yrs
< 3 yrs	P (B.Sc)		C (B.Sc) G (M.Ed)	T (BA) K (BSc)	W (B.Sc)
3–5 yrs				E	N
> 5 yrs				L	R

In the one-to-one interviews, the HODs shared their experiences in their recent appraisals of mathematics teachers, giving examples and some profile descriptions of their mathematics teachers who were considered “strong” or “weak” teachers. They also provided more information on what they value in pupils’ mathematics learning in relation to Singapore mathematics curriculum framework. The analysis of the HODs’ interviews surfaces some emerging trends which will be discussed below.

Appraisal Procedures

Every school carries out lesson observation as part of the appraisal process. Most teachers are observed once a year while “weaker” teachers were observed twice. However, for some schools, only new teachers are observed. Most HODs regard pre-conference sessions as developmental for the teachers. During their observations, eight of the ten HODs used school-designed observation checklists while the rest used observation reports. These checklists and reports are given to teachers during the post conference. The post conference sessions are generally considered as feedback sessions by the reporting officer (HOD) and teachers are asked to reflect on their practices in the observed lessons either orally or in written form. This is usually done on the very day itself but if not possible, then within the week of observation.

HODs were also careful to include practices outside lesson time in their appraisal. Other supplementary modes of appraisal include checking the quality of worksheets designed by teachers and whether they were appropriately customised for the class level or ability. In assessing their quality of marking workbooks or worksheets, teachers are rated in terms of accuracy, comments and follow-up actions. As mentioned earlier, many schools would use the Ministry of Education’s generic EPMS framework for yearly appraisal of their teachers and the mathematics-specific competencies would be slotted into the appropriate generic competencies.

Practices that Are Valued Highly by the HODs

The HODs gave certain important general competencies and attributes which were not specific to mathematics teaching. Three HODs, E, R and N mentioned passion as well as flexibility. They believed that without passion, teachers will not have the motivation to continue to improve their practices and the commitment to exercise desirable practices on a sustained basis. By flexibility, they meant the teacher’s ability to change their planned lesson as the lesson was carried out. Two HODs, R and N, believed that without good class management, literally no teaching could be carried out, much less effective teaching leading to effective learning. These general aspects were not covered in our questionnaire because the project was focusing on particular practices applicable to mathematics only. However, when the attribute of “flexibility” was further explained in the context of monitoring pupil understanding and taking appropriate teaching actions to correct misconceptions or foster understanding, this quality of an effective teacher could then be observed through a combination of several of the MPCK-in-action practices.

It was interesting that two HODs considered strong content to be a necessary condition for strong PCK. While K considered *a strong grasp of the subject* as part of a mathematics teacher's "capability", E stated that ... *without content knowledge, I think the rest cannot be achieved. Even though the teacher may be able to deliver, have very good rapport, but there is no communication. There's no transfer of knowledge...*

Almost all HODs stressed that pedagogy was important although they tended to be vague in what they meant by effective pedagogy before being prompted to consider the practices given in the survey. They generally reiterated good mathematics teaching practices which were not very different from those mentioned in the open-ended response part of the survey. The interviews thus confirmed the valued practices under the five themes as given earlier but added elaborations on their reasons for placing importance on these practices.

In general, the sequencing of activities, the C-P-A approach and practices which built up conceptual understanding and making connections to real world were seen as very effective for learning. For the C-P-A approach or using real-world examples, reasons given by three HODs, T, L and W, were that, due to the pupils' developmental level, there was need to scaffold from concrete materials, pictures or real life experience to the "abstract" mathematical concepts. Another HOD, K, further added motivation for the children to learn the concept(s) as a reason for using real life examples in difficult topics such as rates. One of the HODs, N, mentioned that the C-P-A approach or using manipulatives may not be necessary for upper primary level classes as she felt teachers could use IT or other aids to establish concepts. She also pointed out that teachers needed to build upon pupils' experience. This practice of building on prior knowledge was also cited by L who also saw logical sequencing of activities as very crucial to developing concepts for "otherwise, the children will get very confused".

From the quantitative analysis of the survey, the cluster of practices to respond to pupils' misconceptions, errors were very highly rated to be very important. HODs P, W, K and E viewed detecting pupils' misconceptions and errors and analysing the causes as the way to help teachers realize why children do not understand their lessons. P and W further elaborated that unless the teacher probes into the thinking of the pupils, it is futile for the teacher to conduct remedial lessons, teaching the same thing over and over again. P also mentioned questioning techniques as one of the methods for teachers to monitor pupils' learning.

Beyond learning concepts, a few of the HODs moved to discuss problem solving as a priority area. In this area, the profile of pupils in the school was an influencing factor. C, who was HOD in a school which constantly produced top performers, mentioned that "it is important that a mathematics teacher models alternative ways of solving a problem" since this would teach the pupils the need to be flexible in their mathematical problem solving. This view was also expressed by G who thought questioning could be used to encourage pupils to look for different solutions. L, who was the HOD of a school with a higher proportion of weaker pupils viewed *structuring simple to complex* as very important because progression from simpler tasks and questions to harder questions will "help and lead them to the solution." In addition, she also regarded *explains mathematical terms accurately* as vital because it helps pupils to understand word problems. *Knowing how to use the mathematical tools* was also considered very important as her pupils need a lot of guidance in this area. N, who was concerned that her pupils were not very strong in mathematics, thought that teachers should ask structured questions with precise vocabulary to guide pupils and that it was important for teachers to be accurate in mathematical language.

During the interviews, the HODs sometimes referred to their own school situations and the above discussion suggests that the different reasons for HODs valuing the good practices are very much dependent on two main factors: (a) their pupils' characteristics and abilities and how these practices can best assist and motivate pupils in their learning, and (b) the HODs' own views of the nature of mathematics.

There are also external influences on HODs' perceptions of good practices such as cluster initiatives and professional development of the HODs themselves. For instance, G mentioned that she saw questioning techniques as important to develop in pupils a habit of looking for alternative solutions. This was in line with the project *Habits of Mind* which her school had implemented after a workshop conducted at cluster level. T stated that takeaways from the National Institute of Education's course for preparation of HODs and other in-service courses have impacted her perceptions of what good MPCK-in-action meant.

Practices that Are Less Valued by HODs

Some anecdotal explanations of their lower ratings provided the researchers new insights into their perceptions. While some of the practices are generally highly valued, they were considered not appropriate for all pupils, especially the weaker ones.

Confirming the quantitative findings, the use of counter examples, non-examples and multiple representations as part of the teaching pedagogy was less valued. From the interviews, W felt that "teachers were not ready for such a methodology" and that if one was not careful in giving counter examples, "it could serve to confuse the pupils". This view was shared by P who noted that there were many ways of representing a concept but children may be confused if too many representations were used. It was noted by L that "it is difficult to find counter examples" which accounted for her not rating that practice very highly.

Also, the researchers were a little surprised at the relatively low rating given to the item *emphasising computational speed and accuracy* since it was expected that Singapore pupils were generally strong in their computation skills and the calculator was only allowed to be used in the national primary school leaving examination in 2009. The interviews threw some light on this with two main reasons. T and N explained that with changing times and use of technology such as calculators and computers, computation skills were no longer deemed as important. As a very experienced teacher, T showed her concern for pupil development. She explained that lower ability pupils needed time to carry out their computations correctly and speed should take lower priority. Being against teachers being over rigid, she had also rated the practice *focuses the pupils on the essential steps and necessary conditions in procedures* rather low. Another HOD, E, whose pupils were of higher mathematical ability, provided the reason that, in the climate of developing pupils to be innovative and enterprising, over focus on essential steps in problem solution may be too prescriptive and stifle thinking.

The information from the interviews suggests some practices were regarded as less important by HODS for a few reasons: the nature or characteristics of their pupils, the relevance of the practices at the different grade levels, and the readiness of the teachers in their schools.

Practices that Are Considered as Having an Impact on Pupils' Mathematical Development

The interviewed HODs have a broad view and a diverse blend of beliefs on what constitutes pupil mathematical development. A number of them identified mathematical development to encompass areas like (i) test performance, (ii) ability to appreciate how mathematics is related to daily life, (iii) stretching pupils cognitively and (iv) having a positive attitude towards the subject.

K was of the view that mathematics is a hierarchical subject and therefore, a spiral approach that builds on pupils' existing knowledge and skills would be effective in enhancing mathematical development. Two HODs, L and N, whose pupils' mathematical ability were below-average also emphasised building on prior knowledge for the same reason stated above. Two other HODs, R and W advocate a hands-on approach through learning activities to help pupils enjoy mathematics. Though the approach may not translate into better mathematics performance, they are confident that it will have an impact on these pupils' performance in the long run or positively affect their dispositions towards mathematics.

One theme that emerged was that teachers are important role models in the mathematical development of pupils. Teachers' modelling of how to overcome errors and misconceptions is also found to be important as it will help pupils to persevere in the event that they make mistakes or are stuck in problem solving. Teachers' modelling of exemplary mathematical behavior like being logical and systematic is also important because this will help pupils to be systematic and logical in their presentation.

Another theme was the need to go beyond mathematical concepts and skills specified in the syllabus or textbooks. The HODs would want teachers to teach thinking skills and processes, as well as make mathematics meaningful by moving beyond textbook questions. They felt that this would serve to make learning holistic and also improve pupils' view of mathematics learning.

Generally, HODs' evaluation of the practices that impact on pupils' mathematical development depended to a large extent on the pupils' characteristics, how the pupils under their charge learn best and also how the HODs themselves defined mathematical development.

Implications and Conclusion

The study showed that in general, school mathematics heads of departments value all the pedagogical practices given in the questionnaire and this finding gives assurance that the curriculum leaders in schools would be emphasising such practices to their teachers. The survey and interview findings also showed that the HODs value more those practices which promote understanding of concepts and practices which enhance their pupils' attitude towards mathematics rather than those which only concentrate on developing skills in procedures. The HODs' evaluation of the effectiveness of any particular practice is primarily determined by the characteristics of pupils in their schools, their knowledge of their mathematics teachers' readiness and the HODs' personal pedagogical practices, actions and experiences.

The findings of the study thus seem to contradict the view that, in general, mathematics teaching in Singapore tends to be didactical and instrumental, with great emphasis on correct answers and correct procedures since these approaches were seen to be effective in achieving high examination performance levels. There are several reasons to explain this apparent contradiction. First, as is quite within cultural norms, HODs in a perception survey could well be giving politically correct responses as they clearly understand that teacher educators and the Ministry of Education's Curriculum Department are inclined towards deep conceptual learning and nurturing desirable attitudes towards mathematics. Second, while HODs may encourage teachers to provide time and energy towards understanding of mathematics concepts in lower levels, the examination-driven teaching strategies of having intensive practice by pupils get increasingly emphasised in the last 2 years of the primary school, leading to the very high stakes national primary school leaving examination. These teaching strategies at this stage would emphasize speed, accuracy and exposure to difficult examination-type word problems. Exploration of alternative methods/conceptions or spending time on mathematical ideas become unaffordable luxuries at this stage.

While the HODs strongly felt that the pedagogical practices listed in the questionnaire were desirable for effective teaching and learning of mathematics, there were some concerns with the ability or readiness of their teachers to carry out some of these practices well and the suitability of some of the practices for their pupils. This was particularly the case for those practices which require teachers to use counter-examples, non-examples and alternative methods. Effective use of such practices requires the teachers to have deep understanding of mathematical concepts and procedures far beyond the level at which they are teaching and the concerns reflected by the HODs also imply some lack of deep mathematical understanding in their teachers.

In our study, we have only sought to understand the HODs' perceptions of what teacher practices are important for effective learning by their pupils. We have not gone further to understand the links between how these perceptions by HODs affect or determine teachers' choice of actions. The underlying assumption is that teachers will be guided by their HODs' expectations since the HODS are their reporting officers' holding positional authority over them and have considerable inputs in the teachers' annual performance assessment. Due to the fact that many primary school teachers are generalist teachers and not degree graduates, their sense of professionalism and self confidence may not be well established. Thus, in a culture where respect for authority is strongly encouraged, the teachers would be strongly influenced by their HODs' expectations especially where there is a lack of conviction that they can make better pedagogical decisions. Nevertheless, such influences may still not result in actual practice as they could lack the capability of deeper understanding both of content structures or pedagogical principles to carry out these practices valued by their HODs or the Curriculum authorities. Teachers are also faced with dilemmas of choosing between various courses of actions and lack of time for lesson preparation given the multi-faceted demands made on them from school leaders, pupils and parents. Thus the interface between what HODs' expectations and final actions taken by teachers is affected by a whole range of factors and is an area for further research.

While there is no shortage of professional development courses for Singapore teachers, it has been the experience of NIE that teachers tend to select courses which interest them rather than courses which they need to improve their practice. Moreover, generalist teachers may also have some mathematics phobia themselves which make them shy away from courses which demand deeper understanding of mathematics concepts from themselves. The findings from other studies in the MPCK project show that teachers are not particularly strong in articulating their principles and providing pedagogically sound rationale for making decisions about their teaching. They tended to use very general statements and are vague in their explanations of what they would do in a particular teaching incident. In particular, some were unable to produce simple counter-examples to illustrate a misconception to pupils. It was shown in one of the studies (Cheang et al., 2007) that student teachers who had taken a specially designed subject knowledge course for deepening their own mathematical understanding performed significantly better than those who did not in the questions designed to test their own understanding of concepts. This finding indicates that it is important for teachers who need to deepen their own mathematical understanding to be helped to do so provided time and effort are committed to such courses.

In Singapore, it is quite normal for schools to promote the latest initiative of the Ministry of Education and to organize workshops or encourage their teachers to take up courses in general areas such as Understanding by Design or Multiple Intelligences. However, while these professional development efforts may provide foundational understanding as a starting point, the project team is of the opinion that, with the help of their HODs, mathematics teachers should analyze their own practices more carefully, identify particular areas for improvement and seek means to address these areas. It is important for school leaders and teachers to understand that pedagogical content knowledge is to a large extent subject-specific and that generic appraisal may not lead to improvement in their practices.

In their annual appraisal of mathematics teachers, the HODs used generic observation checklists/reports rather than a mathematics-specific instrument. The list of 35 items which this study has used could be a starting point for discussions within particular schools to determine the relevant and desirable mathematics teaching practices for their teachers. With a more subject-specific checklist, HODs can then help their teachers to do profiling with respect to their practices in teaching mathematics, thereby guiding and motivating them to be more focused and targeted in professional development and improvement. HODs can also reflect on their own responses to help them understand their own perceptions, leading them to develop those under their charge more effectively.

The findings of this study and the other studies in the MPCK project provide data and tools which mathematics curriculum leaders and teacher educators can use for better understanding of teacher practices and capabilities in the teaching of mathematics. While it is easy to share findings with teacher educators in Singapore and to improve pre-service teacher education which is within the purview of the NIE, communication to school practitioners needs to be carefully and sensitively carried out so as to have better partnership in planning, designing, and implementing teacher professional development.

Further research studies could be carried out on the actual effectiveness in pupil learning which result from enhancing teachers' practices in the classroom. Such intervention studies would likely be resource intensive since they will need researchers working together with teachers for extended periods at school level. As Singapore's education system progresses in becoming more research-guided, the culture should evolve into a non-threatening and non-judgmental relationship where researchers and teachers work together to improve the practice of teaching for the benefit of our students.

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Exploring Korean Teacher Classroom Expertise in Sociomathematical Norms

JeongSuk Pang

Abstract The teacher is expected to help students be engaged in meaningful discourse to support their mathematical development. Although there is widespread awareness of such an expectation, the concern exists that many teachers do not quite grasp the vision of the idea. This study compared and contrasted more successful and less successful teachers in playing such a role in order to get a better understanding of teacher expertise as practiced in actual mathematics classrooms. As such, this study probed in what ways teachers contribute to creating unequally successful mathematics classrooms and what kinds of learning opportunities would emerge for the students. By identifying the differences and similarities between the teachers' instructional behavior, the possibility is explored that the subtle but vital differences of teacher expertise have implications for the sociomathematical norms that become established in the classrooms. Given that the two classes established similar social participation patterns but a different quality of mathematical discourse, this study highlights the importance of the teacher's role in sustaining sociomathematical norms and discusses implications for the elements and development of teacher expertise.

Keywords Teacher classroom expertise · Sociomathematical norms · Korean mathematics instruction · Effective mathematical discourse · Changing teaching practice

Introduction

With a vision of high quality teaching and learning, educational leaders have sought to change not only what to teach but also how to teach mathematics (National Council of Teachers of Mathematics [NCTM], 2000, 2007). Both content and

J. Pang (✉)
Department of Elementary Education (Mathematics Education), Korea National University of Education, San 7, Darak-ri, Gangnae-myeon, Cheongwon-gun, Chungbuk 363-791, South Korea
e-mail: jeongsuk@knue.ac.kr

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process in school mathematics are emphasized. For this reason, the teacher is requested to change her traditional instructional behavior which was focused mainly on delivering well-packed information. She is rather expected to foster students' sense-making about mathematics on the basis of sound knowledge of their mathematical learning as well as that of mathematics and pedagogy (Hill, Sleep, Lewis, & Ball, 2007). The teacher is supposed to provide students with mathematical tasks that stimulate their intellect and orchestrate mathematical discourse by making effective pedagogical decisions (Smith, Silver, & Stein, 2005). The teacher is also expected to have the habit of analyzing her instruction by consistent reflection on student learning (Artzt & Armour-Thomas, 2002; NCTM, 2007).

In the same vein, recently the Korea Institute of Curriculum and Evaluation (KICE) established standards of mathematics teaching evaluation as a way to improve mathematics teachers' professionalism (Im & Choe, 2006). The standards require the teacher to have solid knowledge of students as well as that of mathematics and pedagogy, to plan lessons aligned with the curriculum and tailored to students' ability, to implement the designs on the basis of establishing a good learning environment, and to improve her own expertise by reflecting on her daily teaching practice and collaborating with others.

However, such expectations of teacher expertise are not easily achieved. The extent to which significant change occurs in the classroom depends largely on how the teacher comes to make sense of high-quality instructional behavior and responds to it. Despite the widespread endorsement of curricular and pedagogical changes, many teachers have not grasped the full implications of such innovation (Kirshner, 2002). Teachers too easily adopt new teaching techniques such as the use of real-world problems or manipulative materials, but without re-conceptualizing how such an instructional change relates to fostering students' conceptual understanding or mathematical dispositions. More precisely, teachers encourage students to present their solution methods but do not utilize such contributions as the catalyst for exploring the underlying mathematics (Choppin, 2007). This is the case even for teachers who are committed to implementing new recommendations (Fennema & Nelson, 1997; Smith et al., 2005).

Korean teachers are not exceptional in this matter. For instance, the teacher often emphasizes the completion of each activity in the textbook rather than focusing on the mathematical content embedded in the activity or mathematical connections among such activities (Pang, 2002). Students are often busy *doing* activities without necessarily *thinking* about mathematics. The real issue is then to understand not the form but the quality of the teacher's instructional behavior. What kinds of mathematical and social exchanges occur and in what ways does the teacher play a significant role in making such changes mathematically meaningful?

This study is intended to better understand teacher expertise as situated in actual Korean mathematics classrooms. However, this study makes a significant departure from previous research trends where a single classroom is extensively studied. Moreover, in previous studies the classroom is taught by a researcher rather than a regular teacher or is supported by researchers who play an important role in shaping the teacher's instructional methods (e.g., Lampert & Ball, 1998; McClain, 2002;

Stephan & Whitenack, 2003). Comparing and contrasting more successful and less successful teachers can provide a unique opportunity to reflect on the subtle but important issue of exploring teacher expertise at the classroom level. The plans and implementation of mathematics instruction of teachers, as compared to researchers' direct influence on such actions, can portray more realistically the challenges they may encounter. Given this, this study probes in what ways teachers play a leading role in creating more successful or less successful mathematics classrooms and what kinds of learning opportunities are created for the students in these classrooms. This study then identifies the similarities and differences between the teachers' instructional behaviors in order to gain insights into the issues and challenges both for teachers to pursue developing their expertise and for teacher educators to support such efforts.

Theoretical Background

Social and Sociomathematical Norms

This study explores teacher expertise as it is activated in mathematics teaching. A general guideline for the understanding of mathematics teaching practice is an "emergent" theoretical framework (e.g., Cobb & Bauersfeld, 1995). In this perspective, mathematical meanings are neither decided by the teacher in advance, nor discovered by students. Rather, they *emerge* in a continuous process of negotiation through social interaction.

Along with the emergent perspective, two constructs of *social norms* and *sociomathematical norms* are mainly used to characterize each mathematics classroom (Yackel & Cobb, 1996). General social norms are the characteristics that constitute the classroom participation structure. They include expectations, obligations, and roles adapted by classroom participants as well as overall patterns of classroom activity. For example, the general social norms in an inquiry-oriented classroom include the expectation that students invent, present, and justify their own solution methods and the role that a teacher listens carefully to students' contributions and rephrases them for further discussion.

Sociomathematical norms are the more fine-grained aspects of these general social norms that relate specifically to mathematical discourse and activity. The differentiation of sociomathematical norms from general social norms is of great significance because emphasis is given to the ways of explicating and acting out mathematical practices that are embedded in classroom social structure. Some examples of sociomathematical norms include norms that determine what counts as an acceptable, justifiable, easy, clear, different, efficient, elegant, and sophisticated explanation (McClain & Cobb, 2001; Stephan & Whitenack, 2003; Yackel & Cobb, 1996). For instance, the sociomathematical norms in an inquiry-oriented classroom may include the expectation that students should present their solution methods by describing actions on mathematical objects rather than simply accounting for computational manipulations.

While participating in establishing social and sociomathematical norms, students can develop the capability to make mathematical judgments and, more generally, acquire mathematical beliefs and values which ultimately lead them to become intellectually autonomous in mathematics (Yackel & Cobb, 1996). As such, the psychological correlates of the sociomathematical norms are taken to be mathematical beliefs and values.

However, several studies showed that sociomathematical norms have a positive influence not only on students' mathematical dispositions but also on their concept development. As students learn how to engage in the establishment and maintenance of sociomathematical norms, they begin to develop mathematically oriented explanations and justifications which, in turn, enrich their conceptual learning. Specifically, Kazemi and Stipek (2001) attributed students' conceptual thinking to four specific sociomathematical norms: (a) an explanation based on a mathematical argument, (b) understanding of mathematical relations among multiple strategies, (c) errors as meaningful sources to re-conceptualize a problem, explore contradictions, and pursue different strategies, and (d) collaborative work stemming from individual accountability and consensus through mathematical argumentation. In the same vein, Tatsis and Koleza (2008) identified the mathematical justification norm, differentiation norm, validation norm, and relevance norm.

Recent studies also tend to broaden the meanings of sociomathematical norms. Pang (2001) claims that previous studies tend to document briefly sociomathematical norms (and also social norms) mainly as a precursor to the detailed analysis of the students' conceptual learning established in the classroom community. She instead explores the possibility of positioning the sociomathematical norms construct as more centrally reflecting the quality of students' mathematical engagement in collective classroom processes and predicting their conceptual learning opportunities. Lopez and Allal (2007) use the term sociomathematical norms in a broad sense by claiming that any norm of social interaction negotiated and interpreted through mathematical meaning is a sociomathematical norm. Similarly, Clark, Moore, and Carlson (2008) introduce the sociomathematical norm of "speaking with meaning" in order to include meaning about any mathematical ideas, teaching, and even student learning in a professional learning community beyond being limited strictly to mathematical justifications or arguments.

Teacher Expertise in Establishing Social and Sociomathematical Norms

Teacher expertise can be examined through various means including her knowledge and belief of mathematics and mathematics teaching or teaching processes such as lesson plans, implementation, and reflection. Regardless of different approaches, teacher expertise is ultimately related to carrying out effective mathematics instruction. As such, this chapter investigates teacher expertise as situated in classroom teaching, specifically with regard to orchestrating mathematical discourse. The quality of classroom discourse is the essential factor of meaningful mathematics instruction (NCTM, 2007). Similarly, the ability to sustain quality mathematical discourse is the critical element of teacher expertise. Given this, many studies have recently focused on what and how a teacher needs to promote classroom discourse in order to develop students' mathematical ability (Walshaw & Anthony, 2008). For instance, Hufferd-Ackles, Fuson, and Sherin (2004) describe a developmental trajectory for the teacher to establish a reform-oriented classroom community in terms of questioning, explaining mathematical thinking, sources of mathematical ideas, and responsibility for learning. Stein and her colleagues (2008) specify five key practices for the teacher to facilitate productive mathematical discussions: (a) anticipation of students' mathematical responses, (b) monitoring of their responses, (c) purposeful selection of the responses for public discussion, (d) purposeful sequence of the responses, and (e) connection of the responses.

Building on the prior studies on the teacher's role in mathematical discourse, this chapter intends to explore in what ways teacher expertise is enacted in actual classrooms with regard to sociomathematical norms. The construct of sociomathematical norms is expected to be a meaningful lens by which we can probe teacher expertise in managing classroom discourse for students' mathematical development. To be clear, sociomathematical norms (and also social norms) are understood to be established by social interaction between the teacher and the students (Stephan & Whitenack, 2003; Yackel & Cobb, 1996). The development of sociomathematical norms is based not only on the teacher's guidance but also on students' contributions in terms of their explanations, justifications, and argumentations of solution methods. In this sense, students are regarded as playing an important role in developing and maintaining such norms. For instance, students sometimes stick to their personally preferred mode of explanation even in the case that a specific sociomathematical norm has been endorsed and implemented by the teacher (Levenson, Tirosh, & Tsamir, 2006).

However, the teacher plays a leading role in establishing normative behavior in the classroom. Levenson et al. (2006) report that certain social norms such as explaining their own thinking were well established in the classroom whereas others such as the challenging of peer ideas were still in need of the teacher's prompting. This happens more likely in the elementary school classroom in which students are beginning to learn what it means to do and know mathematics. The teacher often makes the decisive contribution to the classroom discourse (Smith et al., 2005). Such decision-making process is not always easy for the teacher who needs both to respect students' own contributions and to consider the mathematical objective (Ball, 1993; Hufferd-Ackles et al., 2004; McClain, 2002). The teacher is required to judge consistently the essence of students' activities and explanations against her own pedagogical goals in order to make them mathematically meaningful.

The evolution of sociomathematical norms provides the teacher with an opportunity to have more direct influence on students' mathematical development than is true of the teacher who merely attends to general social norms. In fact, the teacher can actively promote students' mathematical development by taking a proactive role in filtering students' explanations and justifications through a mathematical lens, monitoring the path of discourse, and providing any necessary guidance as a representative of the mathematical community (McClain, 2002; Stephan & Whitenack, 2003).

There have been many studies which identify various sociomathematical norms established from elementary school to undergraduate mathematics class and illustrate the process by which such norms are constructed and negotiated in the particular setting such as an inquiry-based mathematics classroom or teaching experiments (e.g., Dixon, Egendoerfer, & Clements, 2009; Levenson et al., 2006; Stephan & Whitenack, 2003; Yackel, Rasmussen, & King, 2001). Some studies mention the role of the teacher in initiating and guiding the establishment of sociomathematical norms. Specifically, Yackel (2002) identifies different functions a teacher might serve in the development of collective argumentation while analyzing both productive and unproductive classroom episodes. As such, she elaborates on the essence of the proactive role of the teacher in inquiry-based mathematics classrooms. Similarly, Choppin (2007) suggests teachers' specific actions that support the development of collaborative discussions: (a) seek warrants for students' explanation to develop norms for mathematically acceptable explanations, (b) maintain a non-evaluative stance and seek comments from other students, (c) slow down and clarify the discussion, and (d) attempt a synthesis or summary of a discussion.

Specifying the teacher's role in mathematics discourse is a recent research endeavor (Walshaw & Anthony, 2008). As a result, an understanding of what a teacher expertise looks like in classroom teaching, specifically in relation to sociomathematical norms, is still in its formative stages. Therefore, the teacher's role needs to be closely examined with regard to how the teacher pursues her pedagogical goals based on students' participation and contributions. This chapter intends to explicate the teacher's role with regard to establishing and sustaining sociomathematical norms and to trace vividly in what ways such a role is closely tied to support students' conceptual understanding. This is based on Kazemi and Stipek (2001)'s proposal that "the concept of sociomathematical norms provides a useful framework for thinking about what teachers need to do to promote the development of students' mathematical ideas" (p. 78). The examples in this chapter can clarify to what extent teachers can be successful in paying attention to students' understanding of mathematical concepts. By comparing and contrasting two teachers' instructional behavior, I pursue the possibility that the subtle but vital differences in terms of teacher expertise as situated in classroom practice implicate the sociomathematical norms that become established in the classrooms.

Method

This study is an exploratory, qualitative, comparative case study (Yin, 2002) for which the primary data sources are classroom video recordings and transcripts. The data used in this chapter are from a 1 year project of transforming teaching practices in Korean elementary schools. As a kind of purposeful sampling, the classroom teaching practices of 15 elementary school teachers eager to improve their teaching practices were preliminarily observed and analyzed. An open-ended interview with each teacher was conducted to investigate his or her beliefs in mathematics and its teaching.

Five classes from different schools that aspired to be compatible with current curricular and pedagogical recommendations were selected. Two mathematics lessons per month in each of these classes were videotaped and transcribed. As noted, most research on sociomathematical norms was conducted in a specific setting in which the lessons were inquiry-based and carefully designed by the collaboration between a research team and the classroom teacher in a way to promote students' understanding of specific mathematical contents. In contrast, this study took place in a more naturalistic way in which the teacher decided the topics of the videotaped lessons on the basis of her preference and implemented her own lesson plans.

Individual interviews with the teachers were conducted three times to trace the construction of their teaching approaches. These interviews were audiotaped and transcribed. Additional data included videotaped inquiry group meetings in which the participant teachers met once per month and watched the videotaped lessons together. The main role of the researcher was to set the stage for the monthly group meetings and to provide the teachers with opportunities to analyze their own teaching practices as well as those of others. The teachers were encouraged to discuss whatever seemed meaningful in relation to the videotaped lessons. In this way, they talked about mathematics, curriculum, and pedagogy with the minor help of the researcher. This was possible because the teachers participated in this study voluntarily without enrolling in any specifically programmed workshops or seminars. This design was a part of an attempt to examine teacher expertise in a more naturalistic setting while helping them develop a keen sense of how to foster students' mathematical proficiency and understanding. The interview and inquiry group data were used to understand the successes and difficulties that might occur in the process of changing the culture of primary mathematics classrooms, as well as the complex relationship among the teachers' learning, beliefs, and classroom teaching.

For the purpose of this chapter, two of the five teachers (Ms. K and Ms. Y) were compared and contrasted with relation to social and sociomathematical norms established in their classrooms. Both Ms. K and Ms. Y earned a master's degree in mathematics education and they had about 15 and 10 years of teaching experience, respectively. Both teachers happened to teach the same grade during the project period. While analyzing individual classes using general classroom flow, teacher's approaches, students' approaches, and students' learning opportunities, Ms. K was particularly outstanding in that the students' ideas and mathematical thinking were consistently respected and meaningfully discussed. In addition, the other participant teachers designated Ms. K as the most powerful and influential teacher both in the monthly meetings and in their interviews. Ms. Y was selected because she demonstrated dramatic changes in her instructional approaches. The transformation was noticeable not only by the researcher but by the participant teachers as well.

Whereas she was successful in transitioning toward desirable teaching practices, Ms. Y experienced some difficulties in the process.

Note that this chapter intentionally chose the most successful teacher and less successful teacher instead of the least successful teacher. Such comparison between expert and novice teacher may be too obvious to provide practical implications for teachers who are eager to improve their mathematics instruction but often meet some challenges in the process. Instead, comparison between unequally successful teachers may provide us with subtle but significant issues and challenges in the improvement of teacher classroom expertise in mathematics instruction.

Data analyses had two stages: individual analysis of each classroom and comparative analysis. Because case study should be based on the understanding of the case itself before addressing an issue or developing a theory (Stake, 1998), each teacher's teaching practice was very carefully scrutinized in a bottom-up fashion. The central feature of the analysis was to compare and to contrast preliminary inferences with new incidents in subsequent data in order to determine if the initial conjectures were sustained throughout the data set. In addition, the interview and monthly meeting data were used to explore in what ways the teachers would reflect on their own teaching as well as peers' instruction. The data were also included to analyze whether there had been considerable changes in the teachers' perceptions of their teaching and what would be the reasons for such transformation. In this way, the data provided useful background information in relation to the teacher's decision-making process in the classroom.

Next, the data from the individual classes were employed for comparison according to the social norms and the sociomathematical norms of the two classrooms. The social norms concern the classroom participation structures, whereas sociomathematical norms concern the collective engagement patterns specific to mathematical activity and discourse. The discussion of sociomathematical norms informed the dynamics of understanding the teacher expertise by focusing on how the teacher and the students struggled together to make sense of their mathematical activities. The relationship between sociomathematical norms and teacher expertise was examined in this part of the analysis. This was intended to explore the possibility of promoting sociomathematical norms as a significant analytic tool through which we can understand teacher expertise in terms of different qualities of classroom mathematical discourse.

As described above, the topics of the videotaped lessons were selected by individual teachers in order to place them in a more naturalistic setting. If the lessons with the same topics were analyzed, however, any subtle difference noted in teachers' approaches would be more salient than those with different contents. For this reason this chapter includes a keen analysis in relation to specific lessons with the same topics by social norms and sociomathematical norms. For the purpose of this chapter, the role of the teacher in initiating and sustaining such norms was highlighted and the quality of classroom discourse was analyzed in relation to students' understanding of mathematical contents and processes.

Results

Initial Difference in the Structure of Lessons Between Teachers

Ms. K's teaching practice was consistent across lessons during the year. Each of her lessons consisted of a brief review of the previous lesson, her introduction of new activities, students' individual or small-group work, and a whole-class discussion and summary. The general characteristics of her teaching included reconstructing the learning sequence or the activities in the textbook on the basis of mathematical significance, providing detailed guidance for the new main tasks before students' own activities, focusing on students' thinking and contributions, and emphasizing mathematically important contents during the whole-class discussion (see Pang, 2009 for details). These characteristics were coherent regardless of the mathematics contents to be covered in the instruction. Students were actively involved in classroom activity and appropriately played their roles such as explaining their own solution methods.

Ms. Y's teaching practice evolved during the year. At first, Ms. Y taught mathematical contents step by step and provided appropriate praise and encouragement within a permissive but very calm classroom atmosphere. Students complied with the teacher and provided a short answer or choral response as requested. To be clear, Ms. Y posed questions such as "What shall we do to solve this problem?", "Who will explain?", or "Do you all agree?" However, students' answers in most cases were limited to short or pre-determined responses. Another noticeable observation was that Ms. Y seemed to be very concerned about going through all the activities and problems in the textbook. In fact, the teacher faithfully followed the sequence of activities in the textbook, not necessarily recognizing the interrelations among them.

Similarity Between Teachers: Similar Social Norms

A noticeable change in Ms. Y's instructional behavior occurred after the first inquiry group meeting in which she looked at Ms. K's videotaped lessons. Since both teachers were teaching sixth graders, Ms. Y could see more directly how a teacher's different approach even with the same contents and materials would result in different mathematical development on the part of students. In the meeting, Ms. Y expressed her excitement about the variety and the depth of students' mathematical ideas in the Ms. K classroom, while contrasting them with her own instruction.

Ms. Y started to develop a worksheet intended for students to explore important mathematical ideas for themselves. Note that the current Korean textbook includes detailed guidance on how to solve a given problem with systematic questions. Since the worksheet includes mainly the problem, students were expected to figure out solution methods for themselves. Ms. Y often emphasized the process of problem solving and encouraged multiple ways to approach a problem. These changes made the structure of her lessons remarkably similar to Ms. K's by creating a desirable environment in which students were more fully engaged in mathematical activity and could communicate their ideas.

Consequently, both teachers were quite successful in establishing social norms that could be consistent with the pedagogical approaches advocated in the new curriculum (Ministry of Education, 1997; Ministry of Education and Human Resources Development [MEHRD], 2007). Specifically, two norms need to be emphasized. One was that students are expected to figure out the solution methods of given problems for themselves. The other was that they are supposed to explain or justify their solution methods during the whole-class discussion period and to make sense of explanations given by their peers. These norms proved to be highly consistent regardless of the contents to be covered throughout the year. However, a social norm reported frequently in the related previous studies – students are expected to ask clarifying questions directly to classmates while indicating agreement or disagreement with others' reasoning – was not observed (e.g., Stephan & Whitenack, 2003). This may be related to Korean classroom microculture in which the teacher plays the major role in managing classroom interaction and thus the turn-taking of teacher and students rather than student and student is evident.

Critical Difference Between Teachers: Sociomathematical Norm of Difference

Two teachers' success in sustaining specific social norms described above facilitated the emergence of normative mathematical meanings. The nature and quality of classroom discussion, however, were somewhat different to the extent of how the teacher used students' contributions and encouraged them to articulate mathematical reasons behind the correct answer. Within this analysis, I pursue the possibility that the subtle but critical breakdown between teachers expertise has implications for the sociomathematical norms that become established in their classrooms. In this section, the lessons about a fundamental idea of permutations from sixth-grade classrooms were analyzed to illustrate how the teachers initiated and promoted a sociomathematical norm of difference which, in turn, had a different impact on the depth of students' mathematical understanding. The lessons were taught in the midst of the project period.

As Koreans use only one elementary mathematics textbook series, we need to look at how the idea of permutations is introduced in the textbook. The main activity in the textbook is to figure out how to choose two representatives out of three candidates, followed by making 3-digit numbers with 3 or 4 number cards including 0. Whereas the textbook deals only with the case of permutations, its concomitant workbook includes the case of selecting 2 representatives and that of choosing 1 president and 1 vice-president out of 3 candidates. It must be pointed out that the workbook is mainly for students' self-practice and that the textbook with the teacher's manual is the major instructional resource for teachers. Given that Ms. K and Ms. Y used the problem in the workbook, both teachers regarded the difference between permutation and combination as an important content point in this lesson. However, it is very different how they helped students appreciate such an essential difference.

Many students in Ms. Y's classroom produced the same answer, 6 possibilities, both for the case of electing 2 representatives and for that of electing a president and a vice-president. Some of them interpreted 2 representatives as 1 president and 1 vice-president. Ms. Y began with whole-class discussion by encouraging two students to present their methods via an overhead projector. She intentionally chose these students because they produced 6 and 3 possibilities respectively for the case of electing 2 representatives from 3 people (Young-Dae, Hyung-Ju, and Bo-Mi). Ms. Y then asked students how they derived such different answers.

Ms. Y: Where did the difference come from? Who will present? Yun-Jeong?
Yun-Jeong: One included the same choices but the other didn't.
Ms. Y: That's right. One included the same choices. Let's look at this. (With regard to the 6 possibilities shown on the projector, she pointed out one by one where the repeated cases were.) So, what do you have to do to solve the first case? Da-Hae?
Da-Hae: I think that it would be better to exclude the same choices.
Ms. Y: Anyone else? Kwon-Min?
Kwon-Min: I think it is correct to include the same choices.
Ms. Y: Include the same choices? Why do you think so?
Kwon-Min: Because, if Hyung-Ju were elected as the president, then Bo-Mi couldn't be a president.

Building on Yun-Jeong's idea, Ms. Y expected students to agree to exclude the repeated cases by pointing out each one of them. However, Kwon-Min disagreed with the idea. Confronted with this difficulty, Ms. Y asked students to read the problems again and to find out the difference between the problems (i.e., electing 2 representatives and electing 1 president with 1 vice-president). She emphasized that many students had the same answer to the two problems, indicating that the problems would be the same. Ms. Y asked Hae-Jin to explain the difference as indicated in the problems.

Hae-Jin: I think the two [problems] are different. The first case is to elect representatives, but the second is to elect a president and a vice-president.
Ms. Y: Uh, that's right. Anyone else? Yun-Seok?
Yun-Seok: In the first problem, you have to elect two representatives among three people. Electing Young-Dae and Hyung-Ju are the same as electing Hyung-Ju and Young-Dae. In the second [problem], if Young-Dae were a president, then Hyung-Ju could be a vice-president, and vice versa. Therefore, the two problems are different.

Ms. Y: The two problems are different! But many of you answered six possibilities. What resulted in six possibilities? What resulted in three possibilities? What was the factor?

Hyung-Ju: One included the same choices whereas the other didn't.

Ms. Y: Yes, then what do you have to do? How can you conclude this problem (electing two representatives)?

Seong-Gyun: For this problem, you have to exclude the same choices

Ms. Y: Um. Do you all agree? (Many students say "Yes", but Ji-Sun raised her hand)

Ji-Sun: I think differently from my friends because there are 6 possibilities.

Ms. Y: The reason is?

Ji-Sun: The reason is because a president and a vice-president are different so you should include the repeated choices.

Ms. Y: Then, what does this problem ask for? Selecting representatives, or selecting a president and a vice-president?

Students: Representatives.

Ms. Y: Then where can we apply what Ji-Sun said?

Students: The next [problem].

Ms. Y: Right. Now can you understand how you made a mistake? Correct it.

Whereas Hae-Jin explained the problems per se with little mathematical thinking, Yun-Seok came up with a clear idea of what made the two problems different and justified his claim with specific examples. However, Ms. Y did not probe his mathematical thinking. She rather tended to reinforce what students had to do to get the right answer. Note that Yun-Seok's idea was presented by Kwon-Min in the previous dialogue but their conclusions were different. Kwon-Min used the example to include the repeated choices, thus supporting 6 possibilities, whereas Yun-Seok used it to exclude the ones supporting 3 possibilities. For this reason, this illustrative example could have been explored further to differentiate permutations from combinations. Ms. Y instead focused on whether or not the repeated choices need to be included while regarding students' confusion as a mistake. However, students' confusion was not a trivial mistake as evidenced by Ji-Sun's refusal even after the long discussion. During the rest of the class Ms. Y gave students only two more problems with permutations and checked the answers at the end. Consequently, students were initially exposed to investigating the critical difference between permutations and combinations but had limited opportunity to pursue it.

In contrast, Ms. K carefully orchestrated the path of classroom discourse towards the mathematical distinction. Before students were engaged in solving the same problems (i.e., selecting 2 representatives and selecting a president with a vice-president), Ms. K first asked students to find out the difference between the problems and to predict whether or not the answers would be the same. Within this pre-activity, the teacher encouraged students to think about what it meant to consider the order of an arrangement of objects. This made students solve successfully the first set of problems. Ms. K then gave students a similar set of problems – electing 2 representatives and electing a president with a vice-president out of 5 people – while encouraging them to find out a pattern. After students answered 10 and 20 possibilities respectively, Ms. K asked which had more possibilities and why. Students found that the number of combinations is smaller than that of permutations because it excludes the repeated choices.

Ms. K gave students the third problem: "There are 4 points, A, B, C, and D in a circle. How many line segments can you make by connecting two points in the circle?" Ms. K encouraged students to think about this problem in their heads, saying that she posed this problem on the basis of what they had studied. In fact, this problem was not included in the teacher's original lesson plan. Since students had successfully solved the previous two sets of problems, Ms. K expected students to solve this problem without any difficulties. When asked whether or not they needed to consider the order of points, however, students were confused. With the teacher's clarifying question of whether the line segment connecting A and B and that of B and A would be the same, students agreed that the problem would not require the order of points and came up with the 6 answer possibilities. Ms. K then posed a concomitant problem on the spot: "There are 4 people, A, B, C, and D. How many ways can you line two of them up?" While formulating this problem, Ms. K drew students' attention to deciding whether or not the order of people would matter, leading them to compare this with the previous circle problem. Students easily noticed the difference and found the answer.

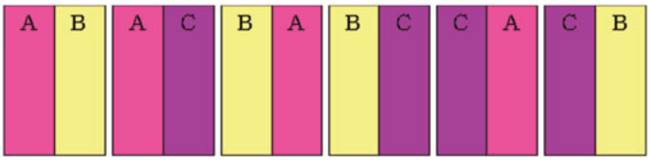
Ms. K then asked students to remember the three sets of problems they had solved and to determine what they had in common with the present problem. After students had time to think about this problem with their partner, Ms. K initiated the whole-class discussion.

Ms. K:	It seems that there is something in common. Who will present–Jeong-Hoon?
Jeong-Hoon:	The numbers are always even.
Ms. K:	The numbers are always even! Yes, they look like they are. Anything else? What is different when you consider the order and when you don't? Su-Hyang?
Su-Hyang:	In the case of considering the order there is the same number per A, B, C, D respectively [interrupted by the teacher].
Ms. K:	She said something important! What Su-Hyang said was, when there is the order, the possibilities after choosing A, those after choosing B, C, or D are all the same. Right? What's the next?
Su-Hyang:	In case of not considering the order, if there were 3 possibilities in A, there would be 2 in B and, so on, decreased by one.
Ms. K:	She found a really important thing! Let's look at it together. [With the problem of selecting 2 out of A, B, C, and D, the teacher explains in detail what Su-Hyang said.] Why is it decreasing?
Students:	Because there is something repeated.

In this way, the topic of discussion changed from solving a problem to searching for the similarities and the differences among the problems. During the discussion a student said that each pair of orders should be added in case that the order matters, whereas such orders should be subtracted if order does not matter. Given this, students agreed that the number of combinations is smaller than that of permutations. Building on this agreement, Ms. K further probed the relationship between combinations and permutations. A student answered that the number of permutations divided by 2 is the number of combinations. Note that the three sets of problems solved in this lesson dealt with selecting 2 out of 3, 4, and 5 objects respectively. To build on the excitement of this idea, Ms. K even encouraged students to explore whether or not this idea would work for the case of electing 3 in place of 2 people as homework. In this way, Ms. K's students had a chance to explore the essential difference between permutation and combination throughout the lesson, while experiencing challenging problems built on their understanding and conjectures.

Subtle Difference Between Teachers: Sociomathematical Norm of Notation

Another difference between the two teachers is related to the sociomathematical norm of effective notation. From the beginning of the lesson, Ms. K was directive in guiding how students were expected to represent their solution methods. With regard to the first set of problems Ms. K asked students to use small rectangular-shaped post-its with which they could easily paste and change the order (see below). Ms. K encouraged students to use different colors or letters such as A, B, C to represent people. The following was a representative example students used to solve the problem of electing a president and a vice-president among 3 people.



When asked to write down the possibilities, students easily wrote pairs of orders such as AB. Throughout the lesson students used the concrete manipulatives and efficient notation initiated by the teacher. Note that the third set of problems included A, B, C, and D to represent points in a circle as well as people. Regardless of what A, B, C, and D signified, however, students could focus on the important decision of whether or not they should consider the order of objects.

In contrast, Ms. Y did not provide any specific manipulatives for students. Rather, students were expected to come up with ideas on how to represent the solution process on the worksheet. As the first set of problems entailed finding out the possibilities of selecting two representatives and a president with a vice-president among Young-Dae, Hyung-Ju, and Bo-Mi, respectively, students wrote easily the names in order. However, they had a considerable amount of difficulty with regard to the second problem of arranging 4 runners among their small group members. As there were 24 possibilities, some students wrote repeated arrangements and others omitted several arrangements. Generally speaking, students spent a lot of time writing the exact names of the runners.

During the whole-class discussion, two students presented their solution methods, which happened to be very similar. The differences included writing surnames of runners and using "-" or "→" between names. Both students applied a systematic order to their arrangements: (a) arrange 4 runners randomly, (b) fix the first and the second runners followed by trading the third runner by the fourth, and (c) change the second runner and repeat the process. Being satisfied with the fact that a correct and clear solution was presented, Ms. Y drew all students' attention to figuring out how to arrange 4 runners in a systematic way. Ms. Y then pushed students to agree that there would be 24 possibilities because there were 4 runners and 6 possibilities after the first runner was decided. With regard to the third problem of making 3 digit numbers with 1, 2, and 3, students solved it easily by using a table or a tree diagram. Ms. Y praised the students for the notations by which they could figure out visually the solution methods at a glance.

In short, Ms. K explicitly communicated which notation she expected students to use in solving a series of problems whereas Ms. Y gave students an opportunity to devise their methods for each problem and approved them one by one. Writing others' names as well as one's own might be fun for elementary school students. However, arranging them in 24 ways requires a mathematically systematic method. After most students spent a considerable amount of time in writing names per se, Ms. Y indicated how and when particular notation might be used to illustrate their mathematical ideas building on the presenting students' contributions. In contrast, Ms. K did not give students the opportunity to develop various types of notations which might be mathematically rich and meaningful. She instead supported students by a visual aid of post-its and short letters in place of long names as needed. While preferring the pedagogically efficient method, Ms. K helped students appreciate the essential difference between combinations and permutations behind the given problems.

Discussion

Teacher's Role in Sustaining Sociomathematical Norms

The results of this study lead us to probe the role of the teacher in promoting students' mathematical development. Both teachers implemented high-quality mathematics instruction. The teachers gave students challenging tasks and encouraged them to solve by themselves and present their solution methods during the whole-class discussion. They restated what the presenting students said or drew all students' attention to the mathematically important contents. More precisely, both teachers even used the same task – electing 2 representatives or a president with a vice-president among 3 people – and initiated a discussion which brought students' attention to the mathematical concepts of permutations and combinations.

Despite the similar, reasonable discourse structure, a closer analysis revealed subtle but important differences in terms of the two teachers' expertise. Ms. Y listened to how students solved the given problems and noticed their difficulty of differentiating combinations from permutations. After the right answers were presented, however, the teacher directed students to move on to the next problem and followed the sequence of activities as planned. She did not address students' difficulty in understanding the underlying mathematical differentiation for the rest of the lesson. In contrast, Ms. K used students' contributions and difficulties as a stepping stone for delving into the conceptual issues between permutations and combinations. Moreover, Ms. K posed a related problem on the spot to address students' conceptual difficulties. It is a common feature of expert teachers that progressively support students' understanding of mathematical concepts to provide systematic and interconnected problems (Smith et al., 2005). It was noticeable that Ms. K further asked students to examine the mathematical similarities and differences among the problems they had solved. The teacher's interactions with the students regarding what would make the problems mathematically different or similar allowed the emergence of the relationship between permutations and combinations.

Another teacher expertise is related to the norm of mathematical notation. In Ms. Y's classroom, various notations were used to solve each problem, and, in fact, multiple representations should be encouraged. Some of them such as writing the names of runners may even reflect real-life situations but they are mathematically inefficient. Any notation was accepted as long as it served to obtain the right answer. In contrast, only one type of notation suggested by the teacher was used to solve all the problems in Ms. K's classroom. The teacher's explicit role in establishing the norm of mathematically effective notation drew students' attention more to the mathematical essence of the concept rather than to a resemblance to real-life situations, visual appearance, or correctness.

The results show that students' learning opportunities are very much constrained by the mathematically significant distinctions embedded within the classroom discourse. The similarities and differences between Ms. Y's and Ms. K's teaching practices clearly show that students' learning opportunities do arise not from general social norms but from sociomathematical norms of a classroom community. Recall that sociomathematical norms concern the quality of students' collective engagement in mathematical practices of a classroom community (McClain & Cobb, 2001; Stephan & Whitenack, 2003). Although both teachers in this study frequently used an enjoyable activity format, how the teacher handled the activity was directly related to the content and quality of students' experiences. In this respect, the construct of sociomathematical norms, not general social norms, should be the focus for discussing teacher expertise in initiating and pursing mathematically meaningful discourse (Kazemi & Stipek, 2001).

Elements and Development of Teacher Expertise

Recent pedagogical suggestions require teachers to monitor classroom discourse carefully and to provide necessary support to ensure the quality of discourse as well as students' mathematical development (MEHRD, 2007; NCTM, 2007). Playing such a role requires a great deal of expertise on the part of the teacher. The challenge for the teacher is to use the social structure of the classrooms to nurture students' development of mathematical thinking as well as their understanding of specific mathematical concepts and processes. For this reason, as stated by Choppin (2007), "learning to develop and direct collaborative discussions effectively is a form of expertise that is new to most teachers." (p. 137)

The successes and challenges that Ms. Y experienced in the process of improving her instruction lead us to consider what matters regarding teacher expertise at the classroom level. Recall that Ms. Y, with great enthusiasm for improving her teaching practice, was quite successful in changing the classroom activity structure in a way that encouraged students' participation and elicited students' solutions and elaboration of their solution methods. Such a promising change, however, did not ensure mathematically powerful discourse. Certainly, the whole-class discussion in Ms. Y's classroom included the important content of permutations and combinations. Yet the teacher was not directive in presenting the correct answer and explaining the underlying mathematics. She instead minimized such a role and gave students an opportunity to present their own methods, often followed by applause and praise. But the string of presentations was used mainly to complete each problem rather than to engage students in genuine mathematical inquiry, which was relatively evident in Ms. K's classroom.

In fact, many teachers find it easy to help students explain their thinking but not to reflect on and build from students' explanations to develop mathematical ideas. It is indeed more challenging pedagogically to support the consequent development of discourse beyond what might be apparent for students (Choppin, 2007; Hufferd-Ackles et al., 2004; Kazemi & Stipek, 2001; Stein et al., 2008; Stephan & Whitenack, 2003). The teacher needs to decide what to pursue in depth among various presentations, when to provide additional information or questions that further challenge students' ideas, and how to modify her original lesson plan based on students' contributions in keeping with the mathematical agenda (Leikin & Dinur, 2007; NCTM, 2007). The teacher also needs to be sensitive to how her decisions influence the opportunities for students' understanding of mathematics (McClain, 2002; Smith et al., 2005). Such a decision-making process requires new ways of thinking about the teaching and learning dynamic. This study clearly addresses the need for a clear distinction between attending to the social practices of the classroom and attending to students' conceptual development within those social practices. Reconceptualizing teaching and learning in this way can pose great difficulty even if the teachers are eager and willing to teach differently, as demonstrated in Ms.Y's case. But these challenges must be met by teachers and teacher educators if teaching expertise is ever to be realized in mathematics instruction.

Another topic to be discussed is what constitutes teacher expertise. Implementing effective discourse requires a great deal of knowledge of students as well as mathematics and an accumulated set of experience on the part of the teacher. The two teachers in this study showed their mathematical sense of what should be included in the lesson as evident in selecting the main task (i.e., selecting 2 representatives or a president with a vice-president out of 3 people) not from the textbook but from the workbook, which was rare in Korean contexts. Both teachers concentrated on mathematical content throughout the lesson. This content-oriented instruction is somewhat different from a process-oriented one in which students' participation is often emphasized at the expense of the development of mathematical ideas.

The difference between Ms. Y and Ms. K seems to result from their knowledge of students rather than from their mathematical knowledge per se. In comparison with Ms. K, Ms. Y was not sensitive to the difficulties her students experienced, which could have been the catalyst to sharpen the understanding of the fundamental difference between permutations and combinations. Most of the teacher's comments served to facilitate the flow of the discussion and to complete each problem. Being sensitive to students' constraints as well as their current capabilities requires a deep understanding of students' mathematical learning processes. Although Korean teachers have relatively strong and sound knowledge of mathematics (Li, Ma, & Pang, 2008), they often do not have equivalent knowledge of how students learn mathematics. However, this knowledge becomes critical as long as the new role of the teacher evolves beyond just explaining mathematical knowledge in a coherent, progressive, and systematic way, which has been the common practice in Korea (Grow-Maienza, Hahn, & Joo, 1999).

Note that Ms. Y was in the process of changing her instructional approaches by participating in the project. As such, she needed to initiate social norms as well as sociomathematical norms by which students' ideas and contributions could be the focus of the instruction. Because the sixth-grade students had been familiar with participating in traditional mathematics instruction, the teacher might need to help students understand her new expectations for their explanations beyond simple answers. As students become more comfortable with their new roles, the teacher may shift from soliciting students' various ideas to communicating and reasoning about mathematically important ones. During this transition process, as illustrated by Ms. Y, the teacher may experience the tension between completing tasks with correct solution methods and creating meaningful pedagogy tailored to students' ideas as well as other challenges. This inherent tension seems inevitable for the teacher who attempts to promote her teaching expertise (Ball, 1993; Dixon et al., 2009; Simon, 1997).

The final remark is concerned with how to develop teacher expertise. The new role requires more than adding a few new teaching techniques. Individual teacher's own commitment and enthusiasm may not be enough. Given that Ms. K's expertise as revealed through her instruction had a positive impact on Ms. Y's teaching practice, it is important that teachers need to form a sustained community of practice to develop their teaching expertise. As the subtle but pivotal difference in maintaining mathematically significant norms may be evident only through the comparison and contrast of teaching approaches, teachers need to accumulate such experience in a community of practice in which participants discuss various practices and support one another's efforts for the purpose of improving their expertise. This chapter is expected to contribute to our understanding of what teacher expertise means and how it may be realized in mathematics instruction, in particular with regard to sustaining mathematically powerful discourse in Korean contexts.

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Expertise of Mathematics Teaching Valued in Taiwanese Classrooms

Pi-Jen Lin and Yeping Li

Abstract This study was designed to explore aspects of teaching expertise displayed in expert teachers' mathematical instruction valued in Taiwanese classrooms. Three expert teachers were identified and selected in this study. A prototypical view of teaching expertise was used to guide our analyses and identification of similarity-based, family resemblance of expert teachers' instruction. These expert teachers' mathematics instruction was examined in light of some common aspects of good mathematics instruction, including problems or tasks being selected and sequenced in classroom instruction, students' solutions then being selected and sequenced for the whole-class discussion, questions being asked and responses to students during the class discussion, and the transition from one activity to another. As a result, mathematics teachers' expertise in teaching was revealed as prototypical features in five aspects that are shared among these three expert teachers. The rich description and summary features provide great details and insight to the teaching expertise that is important for developing good mathematics classroom instruction valued in Taiwan.

Keywords Expert teacher · Expertise · Mathematics teaching · Prototypical features · Taiwanese classrooms

Introduction

Ma's (1999) study suggests that sampled Chinese teachers, but not their counterparts in the US, perceived mathematics concepts as interconnected and had a profound understanding of school mathematics they teach. The result provides a contrasting picture in teachers' knowledge that is consistent with students' achievement gap in school mathematics between China and the United States documented in other cross-national studies. Such results indicate that Chinese teachers tend to have a

P.-J. Lin (✉)
Graduate Institute of Mathematics and Science Education, National Hsinchu University of Education, Hsinchu, Taiwan
e-mail: linpj@mail.nhcue.edu.tw

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solid understanding of school mathematics they teach, which can help contribute to effective mathematics instruction in China. However, if going beyond teachers' knowing and understanding of mathematics knowledge, much remains unclear about the nature of expertise in mathematics instruction that contribute to effective classroom instruction valued in China. As world-wide efforts to improve students' learning of mathematics have led to ever-increasing interest in learning more about educational policy and practices in East Asia, including the Chinese mainland and Taiwan (e.g., Li & Kulm, 2009; Li & Shimizu, 2009), examining and understanding Asia teachers' expertise in mathematics teaching should be important to those who strive to find ways to improve teachers' quality and their classroom instruction in many education systems. In particular, in this chapter we aimed to identify and examine the elements of expertise in mathematics teaching valued in Taiwanese classrooms.

To examine the elements of expertise in mathematics teaching valued in Taiwan, it becomes necessary for us to identify and examine expert teachers. However, there is not a ready answer to the question of what counts as an "expert" specialized in mathematics instruction in Taiwan. The question of identifying an expert teacher bears a close and direct connection with the question of what counts as high-quality classroom instruction. In a recent special issue of ZDM (Li & Shimizu, 2009), high-quality classroom instruction in several education systems in East Asia was identified through either public evaluation, locally defined informal criteria, or as in line with recommended instruction in a system. It is a common characteristic for those teachers, whose classroom instruction was identified as high quality, to have more than 10 years of teaching experiences in several studies (e.g., Huang & Li, 2009; Kaur, 2009; Lin & Li, 2009; Pang, 2009; Shimizu, 2009). The development of high-quality teaching highly relies on teachers' years of teaching experience, but an experienced teacher is not guaranteed to be an expert teacher (Berliner, 1994). This suggests that teaching experience is an important factor, but to be an expert teacher one must go beyond being an experienced teacher. Wade (1998) distinguishes an expert from an experienced teacher in that an expert teacher has more efficient processing of information than an experienced teacher during the planning and interactive phases of teaching. An expert teacher also has strong mathematics knowledge for teaching and enacts high quality mathematics instruction (Hill et al., 2008). While few might disagree that expert teachers can develop and carry out high-quality mathematics instruction, much remains to be understood about the expertise of mathematics instruction that expert teachers may have. The purpose of this article is to examine and describe the features of expertise displayed in *expert teachers*' classroom instruction valued in Taiwan.

The following sections are organized into four parts. In the first part (section "Identifying and Examining Expert Teachers' Classroom Instruction in Taiwan: Research Background and Theoretical Perspectives"), the research background and theoretical perspectives are presented and discussed. In particular, ways of selecting expert teachers from Taiwanese's perspectives will be identified and summarized on the basis of a literature review. The theoretical perspectives for examining experts' teaching in this article will then be presented and discussed. In the second part (section "Method"), we focus on several selected experts' teaching and examine the prototypical features of expertise displayed in their mathematics instruction. The results obtained from data analyses are summarized in part 3 (section "Prototypical Features of Expertise Displayed in Experts' Mathematics Teaching"), and the prototypical features of experts' teaching are illustrated with one expert teacher's mathematics instruction in six consecutive lessons. In the last part (section "Concluding Remarks"), we summarize the prototypical features of teaching expertise displayed in the three experts' teaching and discuss some important factors relevant to the development of these teachers' expertise.

Identifying and Examining Expert Teachers' Classroom Instruction in Taiwan: Research Background and Theoretical Perspectives

Identifying Expert Teachers from Taiwanese's Perspectives

In this section, we aim to identify the criteria of an expert teacher. We will summarize the findings from existing studies on the definition or the criteria of an expert teacher. The literature includes Chinese journals, theses or dissertations, and refereed conference proceedings in Chinese obtained online. Based on the literature review, we try to identify and summarize the commonly acceptable criteria for identifying expert teachers.

Similar to the case of evaluating and identifying what counts as high quality classroom instruction of mathematics, there is no universal agreement on the definition of expert teachers. To generate acceptable criteria for identifying an expert teacher involved in the study, searching for the key word from the database of published papers via website was taken as the first step. There were 25 papers in Chinese that emerged from the database. The expert teachers in published papers taught in various subject areas. Of the 25 papers, six papers focus on mathematics teachers, six on science teachers, and 13 on social science teachers. Generally speaking, the studies on the identification of an expert teacher relied on the criteria of teachers' years of teaching experience, their professional performance, and professional service rather than on measuring their knowledge or skills. Table 1 summarizes relevant information taken from six selected papers.

Table 1 indicates that among the six studies, the teaching experience of at least 10 years is a common feature for being an expert teacher in these studies (Chen, 2007; Huang, 2002; Kao & Chen, 2002; Lee, 2004a, 2004b; Shiao, 1995). The experienced teacher winning an award of the teaching contest at either the national or local level is a commonly used criterion to identify and select an expert (Chen, 2007; Kao & Chen, 2002; Lee, 2004a, 2004b). The award at the national level is highly reputable among all the contests of classroom instruction in Taiwan. Likewise, to win an award at the local level, the teacher's instruction should be inspected with extremely rigorous criteria and approved by a committee consisting of reputed school teachers, principals, and the education bureau at the local level.

Table 1 Criteria of identifying and selecting an expert teacher from Taiwanese's perspectives

Criteria of being an expert teacher	Studies
More than 10 years of teaching experience	Chen (2007), Huang (2002), Lee (2004a, 2004b), Kao and Chen (2002), Shiao (1995)
Winning an award of teaching contest(s)	Chen (2007), Lee (2004a, 2004b), Kao and Chen (2002)
Being a mathematics master teacher at the national level	Chen (2007), Lee (2004a), Shiao (1995)
Recommended by school principals	Lee (2004a, 2004b)
Other professional contributions: writing mathematics textbooks; a member of curriculum reviewing committees	Lee (2004a, 2004b), Huang (2002)

In addition, most expert teachers are master teachers enlisted at the national level (Chen, 2007; Lee, 2004a; Shiao, 1995). The master teachers are required to participate in a master-teacher training program hosted by the Ministry of Education. Master teacher's instruction needs to be observed in public periodically. A master teacher is also obligated to demonstrate a good model of instruction for other teachers in a school or school district. Thus, exemplary lesson development led by master teachers has been taken as an important approach to develop high quality classroom instruction valued in various innovative curricula. Finally, a teacher identified as an expert could also be recommended by the school principal according to the teacher's experience and his/her professional contributions (Lee, 2004a, 2004b).

In summary, with more than 10 years of teaching experience, being a master teacher, and winning an award of the mathematics teaching contest at the national or local level have been commonly considered and used as basic requirements in identifying and selecting an expert teacher participated in previous studies. Such common requirements used in previous studies provide us a reference in identifying expert teachers in this study.

Theoretical Perspectives

Two models of teaching expertise are used to structure our perspective of analyzing experts' mathematics teaching in this study. The first is a theoretical framework that takes a prototype view of expertise in teaching (Smith & Strahan, 2004; Sternberg & Horvath, 1995). The second is a model of teaching expertise that focuses on commonly recognized features of exemplary mathematics instruction valued in East Asia. The combination of these two models allows us to develop a perspective of conceptualizing and identifying prototypical features of experts' teaching valued in Taiwanese classrooms.

A Prototype View of Expertise in Mathematics Teaching

Examining expertise based on experts' teaching is not a new idea in educational research. In the United States, researchers tended to explore the nature of expert teachers' knowledge and instructional performance through comparing expert and novice teachers' classroom instruction (e.g., Borko & Livingston, 1989; Leinhardt, 1989; Livingston & Borko, 1990; Swanson, O'Connor, & Cooney, 1990). The comparative approach has been effective in identifying some features unique to expert teachers. At the same time, it poses a possible restriction by looking for those features that are not shared by novice teachers. In fact, it is questionable whether expertise should refer to only those features that expert teachers have and novice teachers do not. Rather than comparing two different groups of teachers, in this study we aim to focus on a group of expert teachers in Taiwan.

To examine teaching expertise of expert teachers in Taiwan, we adopt a prototype view of teaching expertise as a theoretical framework (Smith & Strahan, 2004; Sternberg & Horvath, 1995). This theoretical framework is originally developed by Sternberg and Horvath, who viewed teaching as a complex and holistic practice that can exhibit various features across classrooms. Other than developing a list of necessary and sufficient features, Sternberg and Horvath proposed to describe and examine experts' teaching in terms of a "*prototype* that represents the central tendency of all the exemplars in the category" (p. 9, emphasis in original). Teaching expertise is conceptualized as a natural, similarity-based, family resemblance that is shared by expert teachers. A prototype of teacher expertise is a summary representation of the central tendencies of teachers' classroom instruction in this category.

Although Sternberg and Horvath (1995) did not carry out specific studies of expert teaching by themselves, they derived from psychological research a list of prototypical features of expert teaching in knowledge (content knowledge, pedagogical knowledge, practical knowledge), efficiency (automatization, executive control, reinvestment of cognitive resources), and insight (selective encoding, selective combination, selective comparison). For example, for the executive control, Sternberg and Horvath listed three sub-category features including planning, monitoring and evaluating. For the feature of monitoring, they specified "expert detects students' failures of comprehension or interest during the execution of a lesson plan" (p. 15). Although they did not carry out specific studies of expert teaching by themselves, they called for studies to validate their list of prototypical features and examine teaching expertise as a similarity-based category. Later, some other researchers used this framework in examining teachers' expertise in teaching. For example, Smith and Strahan (2004) used Sternberg and Horvath's framework to study possible similarities among three expert teachers with diverse profiles (one certified in the Early Adolescence/English Language Arts area, and two certified in the Middle Childhood/Generalist area). With the analyses of a variety of data collected through lesson instruction and interviews, Smith and Strahan derived six central tendencies in broad categories, such as these teachers have a sense of confidence in themselves and in their profession. Likewise, Lin (1999) adopted the prototype view of expertise in teaching to differentiate elementary mathematics teachers' expertise between novices and experts through structured interviews about classroom events. Although none of these studies focused on the analysis of mathematics teachers' classroom instruction, these studies showed the feasibility and value of using this framework to examine teachers' expertise in different subject areas.

In our study, we plan to use a case study approach to identify several expert teachers in Taiwan and examine their expertise in teaching. The prototype view of teaching expertise becomes feasible as we intend to identify similarity-based prototypical features of experts' teaching valued in Taiwan. Different from previous studies that used the prototype view of teaching expertise, we will adopt the prototype view to analyze, interpret, and describe classroom instruction practices of these identified expert teachers.

Examining Experts' Teaching in Light of Features of Good Mathematics Instruction Valued in Various Countries

Classroom instruction is a dynamic and complex process that can be analyzed using different lens with various details. In order to examine experts' teaching in this study, it becomes necessary for us to take a specific lens. Because classroom instruction of these expert teachers selected in the current study is exemplary in Taiwan, it occurs to us to describe and examine experts' teaching in light of features/characteristics of good mathematics instruction.

Recently, a special issue of ZDM (Li & Shimizu, 2009) was published with a focus on the characteristics and development of exemplary mathematics instruction valued in several selected high-achieving education systems in East Asia (e.g., Kaur, 2009; Lin & Li, 2009; Mok, 2009; Pang, 2009; Shimizu, 2009). In each article, the features of exemplary classroom instruction are identified through either analyzing classroom instruction, from master teachers' perspective, professionally active and experienced teachers' perspective, or from students' views. Kaur (2009) reviewed the studies on effective teaching in Australia, New Zealand, the United Kingdom, and the USA and summarized the features of effective instruction as follows: student motivation and participation, skills in communicating mathematics, the provision of cognitive scaffolding, the use of tasks and tools that afford challenging opportunities for learning and rich assessment, linking to learners' knowledge and interests, the use of higher-order questioning, and coherence of curriculum.

Huang and Li (2009) also reviewed existing studies on effective teaching and summarized some common features of Chinese mathematics instruction as follows: (1) setting and achieving comprehensive and feasible teaching objective; (2) having a detailed and well designed lesson plan that not only covers sufficient content to teach but also offers alternatives to develop the content coherently; (3) emphasizing the formation and development of knowledge and mathematics reasoning; (4) emphasizing knowledge connection and instruction coherence; (5) practicing new knowledge with systematic variation problems; and (6) making a balance between the teacher's guidance and students' self explorations; (7) summarizing main ideas and providing proper homework. Table 2 summarizes the features of exemplary

Table 2 Features of exemplary mathematics instruction documented in different studies

Features of good mathematics instruction	Studies
Before teaching: developing a detailed and well designed lesson plan	
• Setting feasible instructional objectives	Huang and Li (2009)
• Designing the tasks with high-level cognitive demands	Hill et al. (2008)
• Designing contextual problems based on students' experience	Lin and Li (2009), Pang (2009)
During teaching: having rich mathematics	
• Achieving instructional objectives	Lin and Li (2009), Pang (2009)
• Exploring mathematics concept(s) based on students' activities	Pang (2009)
• Absence of mathematics errors	Hill et al. (2008)
• Discourse focused on mathematical thinking (explanation, justification, and reasoning)	Huang and Li (2009), Hill et al. (2008), Lin and Li (2009), Pang (2009)
• Identifying, selecting, and discussing various solutions with a focus on its process	Lin and Li (2009), Pang (2009)
• Asking students various questions to promote thinking and discussion during the process of discussing students' solutions	Lin and Li (2009), Pang (2009), Huang and Li (2009)
• Appreciation in response to students	Hill et al. (2008)
• Summarizing key points in due course and assigning homework	Huang and Li (2009)

mathematics instruction displayed in the classroom that are valued in Korea, China, and Taiwan.

Recent research also suggested that teachers' mathematical knowledge for teaching (MKT) is essential to effective classroom instruction (e.g., Hill et al., 2008; Sowder, Phillip, Armstrong, & Shappelle, 1998). In particular, Hill et al. (2008) indicated that the teachers who have strong mathematics knowledge for teaching tend to enact high quality mathematics instruction in classrooms. They identified the elements of high-quality mathematics instruction from relevant literature and through their own analyses of classroom instruction, as also summarized in Table 2. They indicated that the teachers with strong MKT provide students constant opportunities to think mathematically, to report on their thinking, and to politely agree or disagree with one another. On the contrary, the teachers with weak MKT floundered with the mathematical content.

The elements featured in exemplary mathematics instruction across several education systems in East Asia include: Teachers make good preparations of a lesson; teachers are used to construct a lesson based on their students' learning; the lesson is started with presenting students contextual problems; and the mathematical concepts are often introduced through students' exploration of mathematical activities. The richness of mathematics embedded in instructional activities and discourses constitutes the focus of classroom instruction (Huang & Li, 2009; Kaur, 2009; Lin & Li, 2009; Mok, 2009; Pang, 2009; Shimizu, 2009). The mathematics richness includes: carefully identifying, selecting, and sequencing students' various solution methods before the whole-class discussion; explaining and justifying what they discovered; comparing and contrasting various solutions in terms of mathematically significant ideas; and frequently asking various questions to further clarify, compare, diagnose, and extend students' mathematical thinking.

Approaching to the end of a lesson, teachers summarize important mathematical ideas and put them together with students. These elements of high-quality mathematical instruction are to be used in helping guide our analyses of expert teachers' classroom instruction in Taiwan in this study.

Method

Identification and Selection of Expert Teachers

The identification and selection of expert teachers in the current study were based on their professional accomplishment, performance, contribution, and experience of teaching. In particular, identifying a teacher as an expert in the current study was completed through three steps. The first step was to recruit 25 experienced teachers with more than 7 years of teaching who have participated in various teacher professional development programs, based on recommendations from teacher educators. These programs help teachers to conceptualize the ideas of innovative effective teaching emphasised in the standards-oriented curriculum. The second step was to select candidates from the 25 experienced teachers based on (1) whether they received awards at the national level or local level (e.g. "Shiu-Duau Award", "Chu-Chian Award", etc), (2) whether they won a prize in mathematical teaching contests at the national or local level, (3) whether being a master teacher of mathematics at the national level, and (4) their experience in reviewing or writing textbooks. The final step to identify an expert was that the candidate must be accredited by his/her school principal as exhibiting the best quality in mathematics instruction. As a result, one teacher (hereafter, named T1) successfully passed the three steps of identification process, while two teachers (hereafter, named T2 and T3) passed all three steps except the criterion of experience in reviewing or writing textbooks as part of the second step.

Participants

The selection process resulted in three fifth-grade teachers, one male (T2) and two female (T1 and T3), identified as expert teachers to participate in this study. They all had at least 12 years of teaching experience and earned a master degree in mathematics education. T1 had been a master teacher of mathematics for 6 years; T2 and T3 had been master teachers for 6 and 3 years, respectively. Their classroom instruction was frequently authorized and recommended by their school principal for other teachers to observe. They were awarded as the "Power Teacher" and also won several prizes for mathematical activity design at the local level. T1 did not teach at upper elementary grade levels until she started to be involved in a teacher professional program. She has been a member of a textbook reviewing committee. T2 had been teaching at the middle grade level and T3 at the high school level during most of their years of teaching.

T1, T2, and T3 have participated in successive 3-year professional development programs for 10, 8 and 6 years. The goals of the successive 3-year professional development programs are to (1) help teachers identify accurately and critically instructional objectives and construct logical sequences of the objectives; and (2) carry out in-depth analyses of students' difficulties in understating mathematical concepts. The participants involved in the program were routinely engaged in designing and developing a lesson plan by comparing various textbooks on the same content to be taught. After implementing the lesson plan in a classroom, they were asked to reflect on the lesson instruction collaboratively, and plan for the follow-up lesson.

T1 stayed in a school with 780 students that is located in a city area, while T2's and T3's schools with about 260 students on average are located in a suburban area. These three teachers' classrooms shared some similarities in term of classroom arrangements. Students' desks were all arranged in groups of 5 or 6 to facilitate students' group work and collaboration that occurred during many of their lessons. The aisles between the groups provided extra space for groups of students working together on mathematical tasks and for visitors to sit beside students to observe.

Data Collection and Analysis

After identifying and selecting these expert teachers, extensive data were collected for the study. Because the focus of the study was on the expertise in mathematics instruction, the data collected for the study includes two units with 6 and 5 lessons from each teacher throughout an entire year. Their lesson plans, videotaped lessons, and lesson observations were taken as the main data sources. In addition, the interviews with teachers about their lesson planning and their reflections on program participations conducted at the end of the year of the study were also used as the data for analyses in this study.

All these collected data were transcribed and analysed in the original language of Chinese. Selected data were translated into English to provide evidence in the later sections of this chapter. All the transcriptions were assembled, and read repeatedly. We used a grounded theory methodology and open coding (Strauss & Corbin, 1998) to develop a better understanding of the aspects of expertise exhibited. A line-by-line analysis was carried out to identify the categories of how different aspects of expertise were developed. The similarities across the video-taped lessons were identified in reference to the aforementioned elements of high-quality mathematical instruction. Prototypical features of teaching expertise across mathematics content topics that emerged through contrast and comparison are categorized in the following five aspects: (1) developing and sequencing problems for and in classroom instruction, (2) selecting and sequencing students' solutions for the whole-class discussion, (3) creating more opportunities for students to engage in discussions and interact with more students, (4) responding to students during the class discussion, and (5) transiting from one activity to another. The first aspect is emerged from a lesson preparation and the second aspect emerged after students solved a given problem individually or in groups. Other aspects are relevant to the process of classroom discussion. The coding schema of these teachers' video-taped lessons was developed as summarized in Table 3. The frequencies and the levels (on a scale from 1 to

Table 3 Coding schema used for video-taped lessons

		Levels					
Code	Description of the code	5	4	3	2	1	Frequencies
1. Developing and sequencing problems for and in classroom instruction							
1.1	Creating and using tasks with high-level demands and realistic context for evoking multiple solutions and eliciting the anticipated solutions	√					
1.2	Sequencing the problems to be posed	√					
2. Selecting and sequencing students' solutions							
2.1	Predicting the anticipated multiple solution methods	√					
2.2	Identifying the similarities and differences among various solutions						√
2.3	Sequencing students' various solutions for class discussion on the basis of multiple representations and conceptual development	√					
3. Creating more opportunities for students to engage in discussions and interact with more students							
3.1	Asking various questions for different purposes						√
3.2	Asking key questions in time and asking follow-up questions for various purposes	√					
4. Responding to students during the class discussion							
4.1	Interpreting students' productions						√
4.2	Highlighting and summarizing the main point at the end of the discussion						√
5. Transiting from one activity to another							
5.1	Transiting from one activity to another corresponding to students' learning	√					
5.2	Creating specific problems/tasks for assessing students' understanding and as a part of preparation for the next lesson						√

Note: 1. "√" in the level column means that the coding is done by level 5, 4, 3, 2, and 1.
2. "√" in the frequencies column means that the code is counted by frequencies.

5) of each sub-aspect were encoded. A score of 5 means very good, while 1 means very poor.

Four graduate students who are in-service teachers served as the raters. They were trained by the first author to identify the analysis unit and understand the meaning of each category. There are $6 \times 3 + 5 \times 3 = 33$ lessons to be coded, where 6 lessons in unit 1 and 5 lessons in unit 2 for each teacher. The coding was an intensive work. The four graduate students were divided into 2 pairs. Initially, they read lesson transcripts separately accompanied with watching lesson videos together and then tried out their coding. Once completing the coding for a lesson, the raters' reliability was checked as the agreement between two raters within each pair. The reliability of two pairs for lesson one was 0.65 and 0.68, respectively. Two raters of each pair discussed to resolve possible differences in order to reach a consensus. Two raters of each pair then coded the second lesson again. Two pairs of rater's reliability became 0.83 and 0.85. Two raters in each pair discussed their different codes altogether again. Then, the reliability of each pair from coding the third lesson was 0.95 and 0.96, respectively. And the reliability of cross-pair was 0.93. It means that the rater's reliability within or across pairs were high enough. After that, four raters coded the lessons individually.

To enrich our understanding of the characteristics of the teaching expertise as displayed in mathematics instruction, one expert teacher's classroom instruction was used for illustrations as needed. In particular, we use T1's lessons of exploring the area of various shapes including triangles, parallelograms, and trapezoids that span six consecutive days (six 40-min lessons).

Before this sequence of six lessons, the fifth graders who were also in T1's class at grade 4 had learned about the area of rectangles. The instructional objectives of the six consecutive days as described in the textbook include: (1) determining how many square units are in given cardboard parallelograms; (2) exploring the area of a parallelogram and developing students' conceptual understanding of the formula of a parallelogram; (3) determining the number of square units in a given cardboard triangle; (4) developing the area formula of a triangle; (5) exploring the area formula of trapezoid; and (6) calculating the area of given complicated figures composed of various simple shapes.

Prototypical Features of Expertise Displayed in Experts' Mathematics Teaching

The teaching expertise displayed in the three experts' teaching has some common features. In general, they mastered in designing and using tasks that support rich mathematics thinking, in carefully selecting and sequencing students' solutions for whole-class discussion, questioning and using students' errors or misconceptions for instruction, responding to students' questions adequately, and being able to summarize main ideas for students. Each prototypical feature will be discussed below in detail.

Developing and Sequencing Problems or Tasks for and in Classroom Instruction

Two units of lesson instruction for each teacher were coded in accordance with the coding schema described in previously. The participating teachers have participated in several terms of teacher professional programs, so that they were used to providing 2–4 problems for students to resolve in most of their lessons. The six lessons on the trapezoid area are in the unit that is the same for the three teachers, and the other unit for T1, T2 and T3 is proportion, ratio, and ratio respectively. Even though the three teachers created new tasks for students, they still kept the same hours in a unit as outlined in the teachers' guide. As scheduled in the teachers' guide, the lesson hours for trapezoid area, proportion, and ratio are 6, 5, and 5, respectively. The number of problems proposed by each teacher distributed in unit 1 with 6 lessons and unit 2 with 5 lessons is summarized in Table 4.

Table 4 The number of problems proposed by each teacher distributed in unit 1 and unit 2

	T1		T2		T3	
	U1	U2	U1	U2	U1	U2
Lesson 1	4	2	2	2	2	2
Lesson 2	3	3	2	3	2	2
Lesson 3	3	3	3	3	2	3
Lesson 4	3	3	4	3	4	4
Lesson 5	1	4	3	4	4	4
Lesson 6	4	–	4	–	4	–
Total	18	15	18	15	18	15

The levels of the three teachers' developing and sequencing these problems for and in lesson instruction are displayed in Table 5.

Skilled in Creating and Using Tasks with High-Level Demands and Realistic Context for Evoking Multiple Solutions and Eliciting the Anticipated Solutions

Table 5 shows that the frequencies of the highest level with scale 5 of the code 1.1 displayed in unit 1 for T1, T2, and T3 are 15, 16, and 14 out of 18, respectively, where 18 is the number of problems used by each of these three teachers in unit 1. Likewise, there are high frequencies at the highest level with scale 5 displayed in the three teachers' teaching of the unit 2 that has 15 problems proposed. This indicates that the three teachers skilled in creating and using tasks with a high-level cognitive demand in classrooms. They often used such tasks that require students to explore and understand the nature of mathematical concepts or relationships. These teachers can intervene to ensure the mathematics in the real-world examples reach students and evoke multiple solutions and elicit anticipated solutions.

Table 5 Levels of developing and sequencing problems for and in classroom instruction

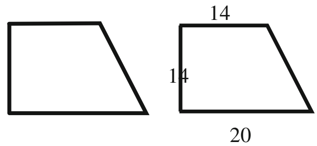
			Teachers					
			T1		T2		T3	
Code	Description of the code	Level	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)
1.1	Creating and using tasks with high-level demands and realistic context for evoking multiple solutions and eliciting the anticipated solutions	5	15	14	16	14	14	15
		4	3	1	2	1	4	0
		3	–	–	–	–	–	–
		2	–	–	–	–	–	–
		1	–	–	–	–	–	–
1.2	Sequencing the problems to be posed	5	6	5	5	5	5	5
		4	–	–	1	–	1	–
		3	–	–	–	–	–	–
		2	–	–	–	–	–	–
		1	–	–	–	–	–	–

Note: U1(6,18) means Unit 1 includes 6 lessons with 18 problems proposed. The same for U2(5,15).

For instance, T1 would not directly use the problems in the textbook which present students the area formula for the trapezoid, but rather would create a high-level demand task to explore the meaning of the area for the case of trapezoid.

In particular, after wrapping up a review of the area formulas for a rectangle, parallelogram, and triangle, T1 presented the class with the following task (as Fig. 1) with a high-level cognitive demand that she created for developing the area formula for a trapezoid on the fifth day of the lesson sequence:

Fig. 1 Two congruent cardboard trapezoids provided by T1 with task 1



Task 1: Each student was given two congruent cardboard trapezoids as Fig.1 to figure out the area of a trapezoid (altitude=14, upper-base=14, bottom-base=20):

The task was at the high cognitive demand level because it required students to reflect on their previous experience and understand the relationships between trapezoid, parallelogram, triangle, and rectangle. As a result, students presented 12 different solution methods. They tried to either place one adjacent to the other to arrange them into a parallelogram or a rectangle or cut one of the two trapezoids to rearrange them into a parallelogram or a rectangle. On the basis of students' multiple solutions, this task evidently showed T1's success in making mathematics challenging and accessible to students. Meanwhile, T1's approach exhibited her respect and value to students' experiences. The task itself illustrates T1's expertise in mathematics and teaching.

Skilled in Sequencing the Problems to be Posed on the Basis of Students' Learning

Table 5 shows that the proportions of the highest level with scale 5 of the code 1.2 displayed in unit 1 for T1, T2, and T3 are 6, 5, and 5 out of 6, respectively, where 6 is the number of sequences problems in unit 1. The number of ordering problems used in a lesson was counted as 1, so that 6 frequencies were in total for the 6 lessons of unit 1. Likewise, there is a high proportion at the highest level with the scale of 5 displayed in the three teachers' teaching of unit 2. This indicates that these teachers were skilled in identifying what problem types or mathematical activities are more difficult for students at a particular age. They were used to utilizing the textbook with a critical perspective and being very skillful in logically restructuring the learning sequence on the basis of mathematical significance and students' prior knowledge.

For instance, T1 realized that the area formula for a rectangle is often the first formula students learn. Therefore, she clarified students' understanding of the area formula for a rectangle by providing the following sequence of activities before exposing students to other activities that are geared toward developing the area formula for the parallelograms, triangles, and trapezoids. The sequence of activities consisted of: (1) covering a given rectangle with arbitrary units and finding its area by counting or multiplying the number of rows by the number of units in each row; (2) covering the given rectangle with a standard unit such as square-centimeter units and finding the area by multiplying the number of rows by the number of units in each row; (3) developing a shortcut to cover the entire rectangle by showing that it is only necessary to see how many rows by the number of units in each row; (4) marking how many squares could fit across and down, continues to multiply to find the area; (5) identifying and measuring the base and altitude of the rectangle; (6) developing the area formula of a rectangle $A = (\text{length} \times \text{width})$ that is multiplying the number of rows by the number of units in each row as corresponding to the number squares across the base (length) and down the altitude (width).

T1 realized that once students have worked on the area formula for a rectangle, the area of a parallelogram or a triangle can be developed. Students were given the opportunity of exploring the relationship between a parallelogram and rectangle by cutting a given cardboard parallelogram to rearrange the pieces into a rectangle. The next stage in developing the formula is to "discover" the relationship between the area of a parallelogram and that of the corresponding rectangle with the same length and width. Likewise, students' understanding of the area of a triangle can be developed from the case of parallelogram. She also realized that the formula of trapezoid can be developed from a parallelogram or triangle.

Selecting and Sequencing Students' Solutions for the Whole-Class Discussion

Predicating students' possible solutions and ordering students' various solutions that emerge during a lesson reflects the level of a teacher's knowledge and competence, so the coding was given as the level code. We also looked for how often the three teachers identified possible similarities and differences among students' various solutions, so the frequency counts were used for differentiation, as shown in Table 6.

Skilled in Predicting the Anticipated Multiple Solution Methods

The level of predicting students' various solutions including students' misconceptions shown on the lesson plan was coded from each problem. Each predication of solutions for each problem was a unit of analysis. If there is no more solutions in the real teaching than those given by the instructor in the lesson plan, then the code of prediction is at level 5. The code at level 4 means that there was one solution that was not anticipated in the lesson plan. The coding of the levels 3, 2, and 1 means that there were 2, 3, and 4 or more solutions given by the students in the lesson but they were not anticipated by the teacher.

Table 6 Levels and frequencies of selecting and sequencing students' solutions for the whole-class discussion

			Teachers					
			T1		T2		T3	
Code	Description of the code	Level/ Frequency	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)
2.1	Predicting the anticipated multiple solution methods	5	12	13	14	14	11	14
		4	4	1	2	1	5	1
		3	2	1	2	–	2	–
		2	–	–	–	–	–	–
		1	–	–	–	–	–	–
2.2	Identifying the similarities and differences among various solutions	Frequencies	18	15	18	15	18	15
2.3	Sequencing students' various solutions for class discussion	5	6	5	5	4	5	4
		4	–	–	–	1	–	1
		3	–	–	–	–	–	–
		2	–	–	–	–	–	–
		1	–	–	–	–	–	–

Table 6 shows that of the 18 problems given in the unit 1 by T1, T2 and T3, they had 12, 11, and 11 problems respectively at the highest level of 5 of predicting students' possible solutions. No more than 3 solutions given by students in each unit was not included in each teacher's anticipation. Likewise, the three teachers had good predictions of students' various solutions for the problems given in the unit 2. This indicates that the three teachers were knowledgeable about multiple solution methods. Their prediction of multiple solution methods commonly used by students showed that they understood the students. As we observed, students in the three experts' classes also appeared comfortable with having multiple methods to solve a problem.

For instance, T1 anticipated that many methods of finding the area of a trapezoid would be provided by students before developing the area formula. In fact, for Task 1 presented in section "Skilled in Creating and Using Tasks with High-Level Demands and Realistic Context for Evoking Multiple Solutions and Eliciting the Anticipated Solutions", students did provide many different solutions (see Fig. 2) as T1 anticipated.

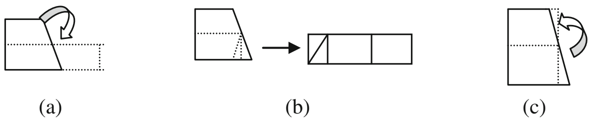


Fig. 2 Students' different solutions to a given task in T1's lesson

Many teachers tend to draw a parallelogram placed in a horizontal position, as shown in Fig. 3(1). T1 anticipated students' difficulty in identifying the altitude of parallelogram if placed in a different position. In her lesson, she provided a parallelogram as shown in Fig. 3(2).

As T1 anticipated, Fig. 3(3) sketches a student's error in drawing the altitude of the base in Fig. 3(2). Apparently, T1 skilled in identifying the similarities and differences among various solutions.

Table 6 shows that the frequencies of comparing the similarities and differences among multiple solutions are the same across the three teachers. This indicates that after students presented various solutions, teachers always asked students to identify possible similarities and differences among different methods from students. It reveals teachers' perceived differences among various methods. For instance, after students presented various solutions for Task 1, T1 asked students to categorize their solutions. Students sorted the solutions (see Fig. 4) into two categories: One was using two given trapezoids to form a parallelogram or rectangle, such as solutions (1) and (2) and the other was cutting a trapezoid to rearrange different geometric shapes, such as the solutions (3)–(11). Moreover, T1 distinguished solution #11

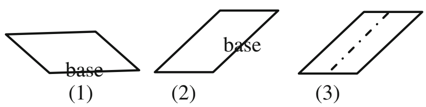


Fig. 3 Students' errors in drawing the altitude corresponding to the base

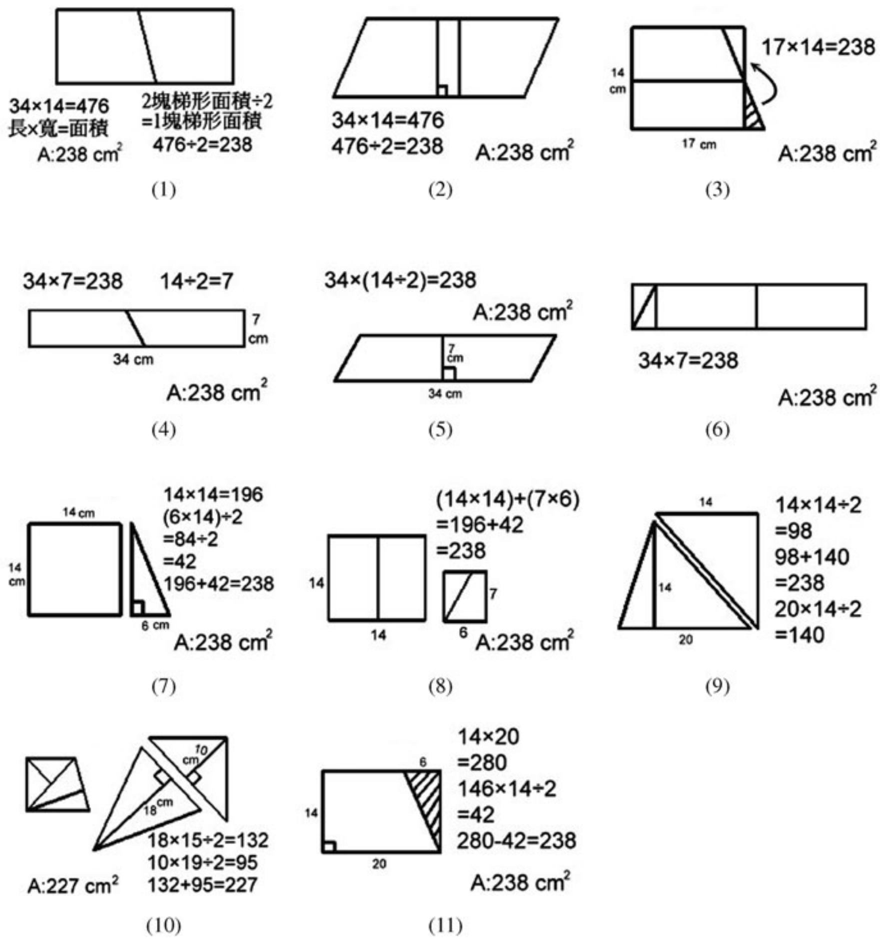


Fig. 4 Students' ways of rearranging a trapezoid into a rectangle

from the second category, since solution #11 transformed the trapezoid into a rectangle by adding a triangle and then deducted the area of the added triangle from that of the rectangle.

Skilled in Sequencing Students' Various Solutions for Class Discussion on the Basis of Multiple Representations and Conceptual Development

Sequencing students' various solutions in a logical order plays an essential role in leading a successful classroom discourse. Table 6 shows that this is another element of the teaching expertise displayed by these expert teachers. In order to getting students' attention to the whole class discussion in a short instructional time, these teachers selected and sequenced the order carefully. The solutions being selected as used by most students could have a wrong answer, incomplete answer, or correct answer. For those solutions with the correct answer, the quality of those solutions including multiple representations and conceptual development was also attended to in making a selection.

For instance, solutions (3)–(5), and (6) in Fig. 4 selected for the whole class discussion are commonly found by folding the altitude into half from a trapezoid only, and T1 distinguished them by cutting them in various ways. Figure 4-(4), (5), (6) were cut along the folded line and the upper part was placed adjacent to the bottom part. For solutions of (5) and (6) in Fig. 4, the trapezoid was rearranged into a rectangle.

Creating More Opportunities for Students to Engage in Discussions and Interact with More Students

The three teachers tried hard to create more opportunities for students to engage in discussion via initiating various questions for different purposes and asking key and follow-up questions. We evaluated how often the teachers asked students questions and what for, so that the types of questions characterized by the different purposes were coded. Moreover, of the questions, we were also concerned with whether the questions are good enough for stimulating students' thinking mathematically or if the questions would potentially ignite follow-up discussion. Thus, asking key questions on time was coded as frequency count, as seen in Table 7.

Skilled in Asking Various Questions for Different Purposes

Table 7 shows that the total numbers of questions asked by T1 are 529 and 395 in 6 lessons of unit 1 and 5 lessons of unit 2, respectively. It means that T1 asked one question in every half-minute on average. Likewise, it is similar with T2 and T3. The data indicates that in these expert teachers' classrooms, students had frequent opportunities to think mathematically and to report their thinking. In particular, these teachers asked students to clarify how they get their answers, encouraged students to explain their reasoning, asked students to distinguish one solution from another, diagnosed students' misconceptions, and helped correct students' misconceptions.

In addition, Table 7 shows that making comparison among different solution methods is ranked as the second highest frequency with 153 in these lessons. This kind of questions asked by each teacher was about 8 times per lesson on average (i.e., 153 times in 18 lessons, 116 times in 15 lessons) in one lesson. This indicates that these teachers encouraged students to compare, and contrast multiple solution methods. When a student proposed an incomplete solution or a difficult question, these expert teachers frequently put the question back to the whole class and invited help from other students to continue to deal with the difficulty.

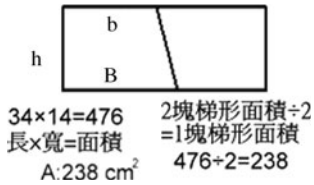
Table 7 Levels and frequencies of creating more opportunities for students to engage in discussions and interact with more students

				Teachers					
				T1		T2		T3	
Code	Description of the code		Level/ Frequency	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)
3.1	Asking various questions for	Explanation	Frequencies	245	177	235	184	253	186
		Clarification		49	25	45	45	42	42
		Completion		20	15	23	13	18	14
		Comparison		153	110	150	115	148	116
		Diagnosis		25	15	20	18	24	15
		Summary		18	15	16	15	18	15
		Correction		19	18	15	15	18	15
	Total			529	395	504	405	521	403
3.2	Asking key questions in time and asking follow-up questions		5	510	377	488	388	510	386
			4	10	9	8	7	6	7
			3	2	3	2	3	2	5
			2	2	2	3	3	1	2
			1	5	4	3	4	2	3

Consequently, more and more students jumped into the discussion to resolve the difficulty.

For instance, T1 recognized that there were many ways to develop the area formula of a trapezoid as corresponding to various solutions presented in Fig. 4. One of the easiest ways was to rely on the area of rectangle. She took advantage of solution (1) in Fig. 4 as an example of finding the area of a trapezoid (b = upper-base, B = bottom-base, h = altitude)

Fig. 4(1)



She asked students a series of questions as follows:

T1: What figure have you formed?
S_{12}: A rectangle.
T1: What is its length?
S_{12}: (14+20) or (b+B)
T1: What is its altitude?
S_{12}: h
T1: What is its area?
S_{12}: h × (b+B).
T1: How does the area of the trapezoid compare to the area of the rectangle?
S_{12}: Half as much.
T1: How might we write a formula for the area of a trapezoid?
S_{12}: $\frac{h(b+B)}{2}$

Skilled in Asking Key Questions in Time and Asking Follow-Up Questions

The results presented in Table 7 show that the three teachers' questioning skills were frequently ranked at the level of 5. The proportions of T1's, T2's and T3's questioning skill ranked at the highest level were 510/529, 488/504, and 510/521 respectively. This indicates that these three teachers asked most questions properly. After students explained their solution methods, the follow-up conversations were devoted to asking students many questions with various purposes. These teachers did not begin the lesson on the area formula of the trapezoid by reciting other formulas from previous learning and ended the lesson by asking students to memorize the new area formula. Rather, they displayed frequently the mathematical richness evidenced in the questions they prompted or asked students.

For instance, on the second day of the lesson sequence, the questions T1 asked were geared toward the key concept for developing the area formula of a parallelogram. The key concept was that the cutting line must be perpendicular to the base when you cut the parallelogram and make it a rectangle. The critical questions asked by T1, after her students' seatwork, include: *How did you decide your cutting line? Why did you cut it and rearrange it into a rectangle? How did the length and width of a rectangle correspond to the base and altitude of the parallelogram?*

On the fifth day of the lesson sequence, the key idea behind finding the area formula of a trapezoid was transforming the trapezoid into a parallelogram or a rectangle either by placing it adjacent to the original or cutting the trapezoid. For this purpose, T1 asked some critical questions for clarifying students' thinking, such as *Can only a rectangle be made from the two congruent trapezoids? Can it be a parallelogram?* The question *Should two trapezoids be used to make a parallelogram to find the area of the trapezoid*? was used to prompt students' multiple solution strategies. To help students find the area and length relationships between the two geometric shapes, T1 asked the following questions *Is the area of the rectangle the same as that of the trapezoid being transformed?*; *What are the length and width of the rectangle in relation to the original two trapezoids?* and *What are the length and width of the rectangle corresponding to the original trapezoid?* To help students connect their concrete experience with the corresponding mathematics expression, T1 asked the question: *Is the meaning of the "* $\div 2$*" in* $(34 \times 14) \div 2$ *the same as* $34 \times (14 \div 2)$*?* To help students generalize the area formula, T1 asked *Can the formula (upper-base + bottom-base)* $\times h \div 2$ *be used for computing the area of all trapezoids?*

Responding to Students During the Class Discussion

How the three teachers responded to students is another aspect of the expertise displayed in mathematical instruction. These teachers sometimes responded to students' questions by interpreting students' solutions or students' comments. Thus the frequencies of the teachers' interpreting of students' productions were coded. How often the teacher helped students make a summary at the end of a discussion or at the end of a lesson was counted as well. The results of teachers' responding to students are summarized in Table 8.

Skilled in Interpreting Students' Productions

Table 8 shows that the three teachers tried to understand and appropriately interpret students' comments, questions, and solutions. Of all the interpretation of students' responses, their interpretation of students' solutions was the most frequent in the classroom. This indicates that there were many occasions during the class discussion for the expert teachers to respond to students. When a student offered the correct answer, they did not take that answer outright, as many other inexperienced or novice teachers often do. Instead, they pressed for a mathematical explanation. They

Table 8 Frequencies of responding to students during the class discussion

Code	Description of the code		Frequency	Teachers					
				T1		T2		T3	
				U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)
4.1	Interpreting students'	Questions	Frequencies	72	62	77	57	80	55
		Solutions		216	182	221	184	207	175
		Comments		54	30	47	32	55	37
4.2	Highlighting and summarizing the main point at the end of the discussion		Frequencies	18	15	18	15	18	15

appeared to be processing what students were saying. They were able to identify if there was evidence indicating that students' explanations were reasonable.

For instance, after students presented the answer (34×14)÷2 and 34×(14÷2) in Fig. 4-(1), (5), respectively, T1 asked them to distinguish the meaning of the "÷2" in the two expressions. The "÷2" in (34×14) ÷2 is half of the parallelogram composed by two congruent trapezoids. The "÷2" in 34×(14÷2) is half of the altitude of the given trapezoid.

Skilled in Highlighting and Summarizing the Main Point at the End of the Discussion

Table 8 also shows that the frequencies of highlighting and summarizing the main point at the end of the discussion occurring in the classroom is the same as the number of the problems proposed in these two units. This indicates that the three expert teachers always summarized the main point that often took place at the end of the discussion for a problem or at the end of the lesson. One way of highlighting or summarizing the main point at the end of the discussion for a problem was to compare the similarity and difference among various solutions. The efficiency was not the focus of comparing various solutions. Instead, the comparison was used to identify which of the methods is successful in achieving the instructional objectives.

Another way of highlighting the main point occurring at the end of a lesson was to summarize the instructional objectives for students, make a comparison among different activities, or review the lesson. If time permitted, these teachers would pose an extended task as an assignment that needs students to apply what they have just learned in the current lesson.

Transiting from One Activity to Another

The smooth transition from one activity to another can be counted with a degree of quality measure. For T1, the activities proposed within 6 lessons in unit 1 were 4, 3, 3, 3, 1, and 4. The total number of transition was 3 + 2 + 2 + 2 + 0 + 3 = 12, where the first 3 is counted in lesson 1 as from activity 1 to activity 2, from activity 2 to activity 3, and from activity 3 to activity 4. The total of 12 transitions was observed in unit 1 with 18 lessons, shown in Table 9. Approaching the end of a lesson, the three teachers sometimes created assessment tasks for students as their homework. These tasks were initiated from their classroom discussion.

Skilled in Transition from One Activity to Another Corresponding to Students' Learning

Table 9 shows that the expertise displayed in these expert teachers' lessons includes their smooth transition from one activity to another as corresponding to students' learning. For instance, before area formulas were introduced, T1 first provided students with the opportunity to compare areas of different regions with and without units. When introducing area units, T1 then let students experience covering a region

Table 9 Levels and frequencies of transiting from one activity to another

			Teachers					
			T1		T2		T3	
Code	Description of the code	Level/ Frequency	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)	U1(6,18)	U2(5,15)
5.1	Transition from one activity to another corresponding to students' learning	5	11	10	12	8	11	10
		4	1	–	–	2	1	–
		3	–	–	–	–	–	–
		2	–	–	–	–	–	–
		1	–	–	–	–	–	–
5.2	Creating specific problems/tasks for assessing students' understanding	Frequencies	4	3	4	3	3	4

with nonstandard and then standard units. At this stage, she helped students to understand that a square unit is the standard unit used in area measurement. When students were thoroughly familiar with counting the number of units covering the rectangles, it was time for the teacher to introduce the area formula for a rectangle. After learning how to use the formulas, T1 then let students find the areas of regions in which they have to combine formulas.

Skilled in Creating Specific Problems/Tasks for Assessing Students' Understanding and as a Part of Preparation for the Next Lesson

Table 9 also shows that the three teachers would not create assessment tasks for students at the end of each lesson, but rather they provided the assessment tasks at the first three or four lessons when students' conceptual knowledge was developed. During the classroom instruction, asking students to solve specifically created problems to assess their learning is a pattern of teaching in the expert teachers' lessons. Students' responses to given tasks helped the expert teachers understand the effect of instruction and improved their awareness of where students may need extra help. The assessment tasks can also be used for the purpose of extending the current lesson instruction as part of the preparation for the next lesson. The classroom discourse on mathematical ideas was the major source of information for the expert teachers to examine if individual students truly understand what he or she learned in the lesson.

For instance, at the end of the fifth day's lesson, T1 generated an assessment task based on the classroom discourse as an assignment as follows.

Assignment task: During today's instruction, you were asked to find the area of a trapezoid (its bases are 14, 20 cm, its altitude is 14 cm). Ming's answer was presented as (34×14)÷2 and Mei's was 34×(14÷2). What do Ming and Mei's answers mean by "÷2"? Write down your explanation.

The assessment tasks as an assignment were also commonly used as a medium for communicating about students' learning with their parents.

In addition, these expert teachers exhibited versatile skills in mathematics instruction. They made connections among mathematical ideas and created smooth transitions between topics. To motivate students to learn, they used various methods, such as doing group work, hands-on activities, and using information technologies. To engage students in activities, they devoted to design tasks to meet students' mathematical level. To promote high-quality student–teacher interactions, various questions were asked to serve different instructional purposes. To inspire students to move to an advanced level, these experts dared to face the challenge to deal with students' questions, misconceptions and difficulties.

Concluding Remarks

The above section presents rich descriptions of similarity-based features of teaching expertise exhibited in these three expert teachers' classroom instruction. Taken together, the summative aspects of teaching expertise displayed in these expert teachers' classroom instruction include designing and using good tasks for evoking and eliciting students' thinking mathematically, predicting students' possible methods of solutions, selecting and sequencing students' solutions in a good order, asking questions purposely at a right time, and responding timely to students. Each aspect heavily relies on the expert teachers' knowledge, in particular, knowledge of mathematics and students' learning. Although we did not measure teachers' knowledge, this study, in some ways, helps to demonstrate what strong knowledge may be needed in order to exhibit such performance in mathematics instruction. This study's findings implicitly suggest these teachers' strong content knowledge, pedagogical content knowledge, and knowledge of students' learning that underpin the various aspects of expertise exposed in the expert teachers' performance in teaching.

Findings of this study provide a strong support for the three aspects of teaching expertise (i.e., knowledge, efficiency, and insight) outlined by Sternberg and Horvath (1995). At the same time, the study's focus on expert mathematics teachers' classroom instruction helps provide the rich content of the three aspects of teaching expertise that is valued in Taiwan. In particular, these expert teachers carried out their instruction with efficiency. They were willing and capable of restructuring the instructional activities from the textbook, flexible in the transition from one activity to another when needed, skillful in asking questions to serve various purpose and addressing students' misconceptions along the ongoing process of classroom instruction. In addition, these expert teachers were insightful about students' solution methods and proficient in selecting and sequencing them in a good order for the class discussions. They were also knowledgeable about students' learning of specific content topics that helped them anticipate students' possible solutions and difficulties.

As we indicated at the beginning of the chapter, the study of teachers' expertise in teaching is important not only for the improvement of classroom instruction but also for the practices of teacher education. The rich description and prototypical features obtained from this study reveal the type of insights that we can gain. Because this study is still restricted to three expert teachers, further research is needed to enrich our understanding of teacher expertise in mathematics teaching. Moreover, as the development of teaching expertise is not the focus of this study, further research is also needed to identify and examine possible factors and practices used in Taiwan to mediate the development of expert teachers' expertise in mathematics instruction.

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Part IV
Cross-National Comparison and Reflections

Cross-Nationally Comparative Results on Teachers' Qualification, Beliefs, and Practices

Svenja Vieluf and Eckhard Klieme

Abstract A growing body of research compares educational processes and outcomes cross-nationally, but up to now there are only few studies on teachers and their expertise involving more than a handful of countries. Drawing on data from the OECD-Teaching and Learning International Survey the present chapter aims at filling this research gap. It compares different aspects of teacher quality – namely mathematics teachers' qualification, beliefs about the nature of teaching and learning and classroom teaching practices – across 23 countries. Results of descriptive and multivariate analyses show the three facets and their subscales to be distinct but interrelated across countries. At the same time significant differences in profiles are observed cross-nationally. The findings suggest both, global and country-specific effects on teacher quality.

Keywords Mathematics teachers · Teacher quality · Teacher beliefs · Conceptions of teaching · Teaching practices · International comparisons · Cross-cultural

Introduction

Comparative research in education has been following different paradigms. Qualitative approaches characterise educational regimes in two or more countries or regions juxtaposing local findings and subsequently drawing conclusions about similarities and country specifics (as an example, see Döbert, Klieme, & Sroka, 2004). In a second paradigm, direct empirical comparisons are made between select countries; for example, there are multiple studies comparing mathematics education in the USA and Japan (Becker, 1992; Stigler & Hiebert, 1999). A third approach – cross-national large scale surveys involving representative samples from larger numbers of countries – additionally facilitates analysis of cross-cultural generalizability and country level effects.

S. Vieluf (✉)
German Institute for International Educational Research, Frankfurt, Germany
e-mail: vieluf@dipf.de

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The latter paradigm has become most influential in student assessment. In research on teacher expertise on the other hand, there are only few attempts to examine a larger sample of countries. Even though mathematics has been a prominent subject in international large scale surveys run both by the IEA (FIMS, SIMS, and TIMSS) and OECD (PISA), they do not provide rich data on mathematics teachers.[1] PISA does not survey teachers at all, and the IEA studies have focused on professional background variables such as a teacher's level of training, the amount and quality of teaching experience, and status as a professional worker.

More recently, the IEA has begun to cover cognitive and affective aspects of teacher expertise as well. TIMSS 2011 will incorporate scales measuring teacher motivation and self efficacy (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009, p. 108). In 2007–2009, the Teacher Education and Development Study (TEDS-M) examined knowledge and beliefs of future teachers from 20 countries. In parallel to these IEA initiatives, the OECD launched its Teaching and Learning International Survey (TALIS) in 2008 (see OECD, 2009 for the initial report) which covered – besides other aspects of school quality and teachers' work places – several scales related to teacher quality.

The present chapter builds on the TALIS data base[2] to study three aspects of teacher quality in cross-national comparison: mathematics teachers' qualification, their beliefs about the nature of teaching and learning, and profiles of classroom teaching practices in mathematics lessons. In addition to comparing means and profiles the chapter also examines the generalizability of relations between these three indicators. The next section will introduce the constructs used in our comparative study and relate them to the overarching concepts of teacher quality and teacher expertise.

Theoretical Background

Teacher Expertise and Teacher Quality in Mathematics

Within educational psychology, the constructs of expertise and professionalism, and knowledge and competence can hardly be discriminated when the quality of teachers

[1] IEA is the International Association for the Evaluation of Educational Achievement, which launched the First and the Second International Mathematics Study (FIMS 1964; SIMS 1977) as well as the Third International Mathematics and Science Study (TIMSS 1995) which later became the Trends in Mathematics and Science Study 1999, 2003, and 2007. The Programme for International Student Assessment (PISA) was launched by the Organisation for Economic Co-operation and Development (OECD).

[2] The authors of the present chapter have been affiliated with TALIS as research fellow and members of the international TALIS expert group, respectively. They authored the chapter on *Teacher Beliefs and Teaching Practices* in the initial report edited by OECD (Klieme & Vieluf, 2009). The authors would like to thank Michael Davidson and Ben Jensen (project leaders, OECD), Ralph Carstensen and Steffen Knoll (project managers at IEA-DPC, the international study contractor), as well as David Baker, Aletta Grisay and Jaap Scheerens (members of the TALIS Questionnaire Expert Group) for excellent collaboration.

and/or teaching is discussed. Those who tend to use the notion of *expertise* (like Bromme, 2008) understand teaching to depend on a combination of knowledge structures, including schemata for perception and action, skills, and routines that are developed through extended practice while moving from the status of a novice to the status of an expert. When Bromme (2008; see also Bromme, 1997) equates teacher expertise with teachers' professional knowledge and skills related to teaching and learning in school, however, he refers to the seminal work on *professional teacher knowledge* done by Shulman (e.g., Shulman, 1987). In this tradition, three forms of professional knowledge are frequently discussed: (1) content knowledge, (2) pedagogical content knowledge, and (3) pedagogical knowledge (e.g., Borko & Putnam, 1996; Helmke, 2003; Lipowsky, 2006). There is evidence that pedagogical content knowledge – that is knowing how a specific content area is taught and learned – is most important in predicting the quality of teaching and learning processes, and finally the outcomes of student learning. Ball and Hill (2008) as well as Baumert et al. (2009) developed tests of pedagogical content knowledge in the area of elementary and secondary-level mathematics respectively, and were able to predict student achievement growth from teachers' test scores. However, neither Shulman nor other authors have drawn a clear distinction between knowledge and beliefs. (See section *Teachers' Beliefs About Teaching and Learning* below for a discussion of these notions). Therefore, Baumert and Kunter (2006) came up with a rather comprehensive definition of *professional teacher competence* as the interplay of the three knowledge dimensions with teachers' beliefs, motivation, and self-regulation competencies.

In their recent overview of *teacher quality* in mathematics, Ball and Hill (2008) take an even broader perspective when discussing different approaches to measuring the quality of teachers. They set out defining high-quality teachers as those who "consistently and effectively foster students' learning" (p. 95). However, they do not establish student achievement growth as *the* measure of teacher quality, as econometricians have done (e.g., Hanushek, 2002). Student achievement can hardy be accounted to one teacher. Moreover, the effectiveness approach lacks the pedagogical substance needed to guide teacher education. Therefore, Ball and Hill worry about the "many problems with using direct measures of student learning to gauge teacher quality" (2008, p. 95).

Ball and Hill also discuss teacher qualification – that is teacher education, certification, and experience – as another approach for measuring teacher quality. Advanced academic degrees, a major in the subject being taught, and professional experience have been described as desired qualifications or as indicators of teacher quality. However, results regarding their association with student achievement are inconsistent (for a summary of research see Zuzovsky, 2009; for teacher certification see Libman, 2009). For the case of mathematics, Ball and Hill (2008, p. 85) conclude: "Overall, course taking and certification are relatively imprecise discriminators of teacher quality". This is also in line with results from economics of education (Hanushek & Rivkin, 2007) and from international studies (Mullis & Martin, 2007). Nevertheless, the professional background may have an impact on teacher competence (as defined above) and teaching practices and thus a more indirect effect on student learning.

Instead of effectiveness and qualification measures, Ball and Hill prefer direct measures of instructional practice, identifying "teachers who provide students with error-free, substantial mathematics and who can manage with mathematical adeptness the range of students' mathematical productions. There may also be other dimensions of instructional quality, such as the cognitive challenge of students' classroom work or the pedagogical aspects of classroom practice, that we would want to include" (2008, p. 95).

To sum up, empirical research on teacher quality has been discussing a number of different, though related constructs. Expertise is just one out of many notions used in this context. No single study like the TALIS, which the present chapter is based on, can cover all relevant aspects. Rather, following the broader view expressed by Ball and Hill (2008), teacher qualification, teacher beliefs about the nature of teaching and learning (chosen as a core element of professional competence), and instructional practices are covered here. The TALIS framework for *Teacher beliefs and teaching practices* (Klieme & Vieluf, 2009) assumed (1) teacher qualification, including teacher education and professional development, to impact (2) teacher beliefs about the nature of teaching and learning, which in turn would have an influence on (3) classroom teaching practices. This line of argument will be taken up in the present chapter. Although the present study may well be considered a study on teacher expertise, the more neutral term *teacher quality* will be used. Also, it should be noted that TALIS was a domain-general survey, sampling teachers from all kinds of subject areas, which did not allow subject-specific knowledge or beliefs to be addressed. However, the present chapter exclusively studies the TALIS sub-sample of mathematics teachers.

Teachers' Beliefs About the Nature of Teaching and Learning

Teachers' beliefs can be defined as "psychologically held understanding, premises, or propositions about the world that are felt to be true" (Richardson, 2003, p. 2). Within mathematics education, there has been a long history of research into teachers' as well as students' beliefs (Leder, Pehkonen, & Törner, 2002). In his overview of the state-of-the-art, Pehkonen (2004, p. 2) sees beliefs "situated in the 'twilight zone' between the cognitive and the affective domain". Mathematics educators have focused on beliefs about the nature of mathematics (e.g., Grigutsch, Raatz, & Törner, 1998; Hannula, Kaasila, Laine, & Pehkonen, 2005; Törner & Grigutsch, 1994), but Pehkonen (2004) also mentions beliefs on mathematics learning and teaching, self-related beliefs (such as self efficacy), and beliefs about the social context of mathematics education.

Following the seminal work by Peterson, Fennema, Carpenter, and Loef (1989; see also Fennema, Carpenter, & Loef, 1990), a reception/direct transmission view on teaching and learning is often contrasted with a constructivist view. Although these views were originally introduced as *pedagogical content beliefs* in the area of mathematics, they may be applied to teaching and learning in general.

- As a traditional strand of professional beliefs, the direct transmission approach – according to Staub and Stern (2002) – is rooted in behaviorism, which proposes a teacher directed approach to learning and instruction. Teachers should explicitly communicate concrete knowledge and exemplary approaches to specific assignments in a clear and structured way. Also, attentiveness and discipline in the classroom are considered to be highly important. Teachers who support this approach tend to view their students as recipients of knowledge that is passed on to them from their teachers.
- Constructivist beliefs assign students a more active role in the process of acquiring knowledge. Constructivism – which many scholars regard as the more modern, reform oriented kind of pedagogy – assumes that learning is embedded in its settings and conditions, and that learners actively construct their knowledge based on previous experiences. Many different instructional approaches are based on constructivist theories. Central to these approaches is that teachers are not seen as transmitters of information, but rather as facilitators of students' self-regulated learning processes. Thus, teachers holding this view emphasize facilitating student inquiry, prefer to give students the chance to develop solutions to problems on their own, and allow students to play an active role in instructional activities (Staub & Stern, 2002).

As exemplified by Kirschner, Sweller, and Clark's (2006) critique of constructivist (*minimal guidance*) instruction and the scholarly debate it triggered (Tobias & Duffy, 2009), the discussion about success and failure of constructivist vs. direct instruction is still unsettled from a researchers' perspective (see section *Classroom Teaching Practices* on this issue). In the present context, however, it is important to note that *constructivist* vs. *direct transmission teacher beliefs* still represent two distinct ways of professional thinking which are quite popular among teachers, and which in the case of mathematics may even be predictive of their students' achievement trajectories (Staub & Stern, 2002). Therefore, TALIS attempted to study these beliefs in an international comparison.

Classroom Teaching Practices

Classroom teaching practices have been shown to be related to effective classroom learning and student outcomes (Brophy, 2000; Brophy & Good, 1986; Seidel & Shavelson, 2007; Wang, Haertel, & Walberg, 1993). Existing evidence suggests there is no single best way of optimizing instruction. Well-structured lessons with close monitoring, adequate pacing and classroom management, clarity of presentation, informative and encouraging feedback – which are known as key aspects of *direct instruction* – bear a positive impact on student achievement. However, researchers inspired by reform pedagogy and humanistic psychology, e.g., Deci and Ryan (1985), argue that student motivation and non-cognitive outcomes require additional facets of quality, such as a classroom climate and teacher-student relations which support autonomy, competence and social relatedness. Finally, in order to foster *cognitive activity* (Mayer, 2004) – rather than *activity per se* – and conceptual understanding, instruction has to use *deep*, challenging content (Brown, 1994), which in the case of mathematics means making connections between mathematical facts, procedures, ideas, and representations (Hiebert & Grouws, 2007); argumentation and non-routine problem solving should be promoted. Thus, teachers have to orchestrate learning activities in a way that serves the needs of their specific class.

Klieme, Pauli, and Reusser (2009) condensed this knowledge into a framework of three *basic dimensions of instructional quality*: (a) clear, well-structured classroom management, (b) supportive, student-oriented classroom climate, and (c) cognitive activation with challenging content. Empirical support for the separation of these dimensions and their impact on student learning comes from the German extension to the TIMSS 1995 video study (Klieme, Schümer, & Knoll, 2001), from a German large scale study on mathematics teachers (Baumert et al., 2009), from a Swiss-German video study in math instruction (Lipowsky et al., 2009), but also from international work in educational effectiveness (e.g., Creemers & Kyriakides, 2008). By incorporating both (socio-)constructivist thinking and classical process-product-research, the framework may help to build a bridge between constructivism and direct instruction (Tobias & Duffy, 2009). Lipowsky et al. (2009) consider the basic dimensions as *latent* factors which are related to, but not identical with specific instructional practices.

We assume classroom practice to be influenced by teachers' beliefs. Generally teachers with direct transmission beliefs are expected to focus more on structure and discipline and to use more lecturing, while on the other hand we anticipate a correlation between constructivist beliefs and more student-centred practices as well as a focus on self-regulated learning, collaboration, problem-solving and cognitive challenge. However, the results of studies examining these relationships are inconsistent. While some studies showed beliefs to be related with classroom teaching practices in Western countries (e.g., Dubberke, Kunter, McElvany, Brunner, & Baumert, 2008; Peterson et al., 1989; Staub & Stern, 2002), but also in Asia (Kember & Kwan, 2000), other authors find no such link (e.g., Wilcox-Herzog, 2002). The inconsistency of findings may be partly due to differences in the operationalization of the constructs.

Cross-Cultural Comparison of Teacher Beliefs About the Nature of Teaching and Learning and Classroom Teaching Practices

Cross-cultural studies examining teachers' knowledge and beliefs mainly focus on comparisons of the USA with East Asia and examine two or three countries only (e.g., An, Kulm, & Wu, 2004; An, Kulm, Wu, Ma, & Wang, 2006; Cai, 2006; Correa, Perry, Sims, Miller, & Fang, 2008; Ma, 1999; Zhou, Peverly, & Xin, 2006). These studies highlight specific differences between countries, but they do not inform about differences and similarities of beliefs on an overarching level. Some research comparing teachers or future teachers from a larger variety of countries comes from IEA studies such as TIMSS and MT21. The results are mixed: Incremental vs. entity beliefs about student abilities, epistemological beliefs about mathematics, and instructional goals sometimes are shared and sometimes vary between countries (see LeTendre, Baker, Akiba, Goesling, & Wiseman, 2001; Mullis et al., 2008; Schmidt et al., 2007).

With regard to teaching practices in mathematics, SIMS already identified a surprising level of similarity among systems. Teachers were using whole-class instructional techniques, relying heavily on prescribed textbooks, and rarely giving differentiated instruction or assignments (Burstein, 1992). Later, TIMSS – including the 1995 and 1999 video studies – found global patterns regarding the general repertoire of practices. Thus, a high degree of convergence was found across countries when the presence of certain features of lessons was examined (LeTendre et al., 2001; Mullis & Martin, 2007). However, analysing the sequencing of lessons, Stigler and Hiebert (1999) identified scripts that seemed to be country specific. For example, teachers across most (industrialized) countries employ whole class work, seat work and lecturing, but the sequence of these practices and the frequency of shifts between them significantly vary (Givvin, Hiebert, Jacobs, Hollingsworth, & Gallimore, 2005). When the TIMSS 1995 video study was published, many – including Stigler and Hiebert – believed the instructional script found in Japanese classrooms to be the cause for high level mathematics achievement in Japan. Later, the 1999 TIMSS video study, which included another five high achieving countries (i.e. Hong Kong, the Czech Republic, the Netherlands, Switzerland, and Australia), revealed that those countries had quite different profiles in teaching practices, thus devaluating any attempt at directly linking student achievement to teaching practices on a national level (Hiebert et al., 2003). Some early conclusions drawn from the TIMSS video studies may be flawed due to ecological fallacy.

Aims and Hypotheses

As the previous sections have shown, cross-cultural research is still left with open questions about cross-national differences and similarities of teacher quality. The present chapter will shed light on this question by examining three indicators of teacher quality across a large sample of 23 countries. More specifically, it aims to answer the following research questions: (1) How similar or different are countries with regards to the quality of their teacher population, considering (a) the composition of their mathematics teacher force in terms of their professional qualification and experience, (b) profiles of beliefs about the nature of teaching and learning, and (c) profiles of classroom teaching practices? (2) Are these three aspects of teacher quality related, and are the relations similar across countries?

Based on previous research, especially the TIMSS study, we expect to find characteristic differences between countries regarding the qualification of teachers (Mullis et al., 2008). We further expect both, *direct transmission* and *constructivist* ideas, to be present across countries. However, influences of national cultures and policies suggest differences in the magnitude and pattern of endorsement of the two views. Regarding classroom teaching practices, comparative research, especially the TIMSS video studies, has proven that mathematics teachers possess a similar repertoire, and that more traditional activities dominate in almost all countries (Hiebert et al., 2003). Thus, *structureing* practices would likely be more frequent than *student orientation* and *enhanced activities* in every country. However, according to previous research in comparative education (including TIMSS, PIRLS and PISA), countries have quite different profiles in terms of alternative or enhanced teaching practices, which we also expect for the present study.

Based on theoretical considerations and previous research (e.g., Dubberke et al., 2008; Peterson et al., 1989; Staub & Stern, 2002) we further expect to find direct transmission beliefs to be related to structuring and constructivist beliefs to correlate with student orientation and enhanced activities.

Method

The research questions described are examined with data from the Teaching and Learning International Survey (TALIS). TALIS uses a teacher and a principal questionnaire to gather information on teachers' beliefs, attitudes and practices and their conditions. The data collection for the first cycle took place in fall 2007 in the Southern Hemisphere and in spring 2008 in the Northern Hemisphere. The target population is all teachers who, as part of their regular duties, provide instruction in programs at the lower secondary level (ISCED level 2[3]) in one of the 23 participating countries. A two-stage stratified sample design was used. Firstly a representative sample of schools providing lower secondary education was drawn, and secondly a representative sample of teachers within these schools was selected. Therefore the data has a multilevel structure with teachers nested within schools (for more information see OECD, 2009, 2010).

Sample and Description of Population Characteristics

The analyses for this chapter are based on a subsample of the TALIS participants who were randomly selected to represent the ISCED level 2 teaching force in the 23 participating countries. Within the questionnaire, teachers were asked to identify the first ISCED level 2 class they typically teach after 11 a.m. on Tuesdays. Those teachers who reported to teach a mathematics class at this specific slot in the timetable will be labelled mathematics teachers in the following. Altogether 73,100

[3]For a detailed description of ISCED levels see United Nations Educational, Scientific and Cultural Organization (2006).

teachers completed the TALIS questionnaire in 2008 and 2009. The sizes of the samples drawn vary by country, with Malta having the smallest teacher sample (1,143 teachers), and Brazil the largest (5,843 teachers; for a more detailed description of sampling procedures see OECD, 2010). The subsample used for this article consists of 9,259 mathematics teachers, which equals 13% of the total sample and 10–19% of each of the total country samples. Between 132 mathematics teachers in Malta and 957 mathematics teachers in Brazil are included.

Altogether 62% of the mathematics teachers are female and 38% male. Also within 19 of the 23 participating countries the percentage of female mathematics teachers is higher than that of male mathematics teachers.[4] A majority of the mathematics teachers is between 30 and 50 years old, both in the total sample (58%) and in most of the country-subsamples (44–77%). Only 18% of the mathematics teachers are 30 years or younger, and 24% are 50 years or older.[5]

Measures

Individual background characteristics – gender, experience, level of education, participation in professional development – are measured with single items. To collect data on mathematics teachers' beliefs, attitudes and practices items were summarized to form scales.

Confirmatory factor analysis (CFA) was used to confirm the expected dimensional structure of the scales. In accordance with scientific conventions (Hu & Bentler, 1999; Schermelleh-Engel & Moosbrugger, 2002), the following values for fit indexes were seen as indicative of an acceptable model fit: CFI > 0.90, RMSEA < 0.08 and SRMR < 0.08. In addition to the general model fit across and within each of the countries, the cross-cultural invariance of the factor loadings, intercepts and residual variances was tested using multiple group confirmatory factor analysis (MGCFA) and different restrictions on the parameters. Such analysis of cross-cultural equivalence informs about the generalizability of constructs (Van de Vijver & Poortinga, 1982), but it can also be interpreted as a multi-method approach to construct validation (Marsh, Martin, & Hau, 2006). The analysis was carried out with the software Mplus, version 5.1 (Muthén & Muthén, 1997–2008). Additionally the internal consistence (Cronbach's Alpha) of the scales was also examined, both across countries and for each country separately. (For detailed results see OECD, 2010).

For the assessment of mathematics teachers' beliefs about the nature of teaching and learning TALIS draws on scales developed by Fennema et al. (1990) and adapted by Staub and Stern (2002). The original questionnaires are designed to measure mathematics teachers' agreement with a cognitive

[4]The five countries with a larger percentage of male mathematics teachers are Australia, Denmark, Mexico, Norway, and Turkey.

[5]Noticeable exceptions are Italy where 60% of the teachers are 50 years or older, and Turkey where 56% are 30 years or younger.

constructivist perspective vs. a direct transmission orientation as two poles of one dimension. Many items are worded in a mathematics specific way, for example referring to word arithmetic. Because TALIS examines teachers teaching different subjects the items were revised to measure domain general beliefs. Moreover the scales were shortened to fit in the time frame of the TALIS study. With the eight items used in TALIS two scales were built: direct transmission beliefs and constructivist beliefs. They were assessed on a four-point Likert scale, ranging from 1 = *strongly disagree* to 4 = *strongly agree*. The two indices for teachers' beliefs about the nature of teaching and learning comprise the items shown in Table 1.

The fit of a confirmatory factor analysis (CFA) model is good for the total sample: CFI = 0.94, TLI = 0.91, RMSEA = 0.04 and SRMR = 0.03. Reliabilities for the two scales measuring mathematics teachers' beliefs about the nature of teaching and learning tended to be rather poor ($\alpha = 0.47$ for direct transmission beliefs and $\alpha = 0.61$ for constructivist beliefs for the total sample). Furthermore, the scales are not fully invariant across countries; the general structure and the factor loadings are relatively similar, but intercepts and residual variances differ noticeably between countries.

Classroom teaching practices were examined by teachers' frequency estimations on a 5-point scale, ranging from *never or hardly ever* to *in almost every lesson*. Based on the triarchic model by Klieme, Lipowsky, Rakoczy, and Ratzka (2006) three indices were established: structuring, student-orientation and enhanced activities. The items measuring classroom teaching practices are detailed in Table 2.

The model fit for the whole model including all three scales is acceptable for the total sample (CFI = 0.90, TLI = 0.87, RMSEA = 0.06, and SRMR = 0.04). Reliabilities for the three scales measuring classroom teaching practices are mostly satisfactory, both for the whole sample ($\alpha = 0.73$ for structuring, $\alpha = 0.70$ for student orientation and $\alpha = 0.72$ for enhanced activities) and for single countries. Across countries, the three scales measuring classroom teaching practice have a similar structure and also relatively similar factor loadings, but they are also not completely cross-culturally invariant.

Table 1 Items wording for beliefs about the nature of teaching and learning

Direct transmission beliefs	Constructivist beliefs
Effective/good teachers demonstrate the correct way to solve a problem.	My role as a teacher is to facilitate students' own inquiry.
Instruction should be built around problems with clear, correct answers, and around ideas that most students can grasp quickly.	Students learn best by finding solutions to problems on their own.
How much students learn depends on how much background knowledge they have; that is why teaching facts is so necessary.	Students should be allowed to think of solutions to practical problems themselves before the teacher shows them how they are solved.
A quiet classroom is generally needed for effective learning.	Thinking and reasoning processes are more important than specific curriculum content.

Table 2 Item wording for classroom teaching practices

Structuring	Student orientation	Enhanced activities
I explicitly state learning goals.	Students work in small groups to come up with a joint solution to a problem or task.	Students work on projects that require at least 1 week to complete.
I review with the students the homework they have prepared.	I give different work to the students that have difficulties learning and/or to those who can advance faster.	Students make a product that will be used by someone else.
At the beginning of the lesson I present a short summary of the previous lesson.	I ask my students to suggest or to help plan classroom activities or topics.	I ask my students to write an essay in which they are expected to explain their thinking or reasoning at some length.
I check my students' exercise books.	Students work in groups based upon their abilities.	Students hold a debate and argue for a particular point of view which may not be their own.
I check, by asking questions, whether or not the subject matter has been understood.		

Model fit and reliability are unsatisfactory in some cases, especially for teacher beliefs about the nature of teaching and learning. However, scales have been shown to work well for the total sample in most countries (OECD, 2010). Therefore, we believe that we can trust in the psychometric quality of these scales – as long as we restrict ourselves to correlation and regression models, without comparing country means when scales are not equivalent across countries.

Statistical Modelling

The TALIS data have a hierarchical structure with teachers nested within schools. Since the school samples of mathematics teachers are very small, no multilevel analyses are carried out, but standard errors are corrected for possible cluster effects. For all analyses Mplus factor scores were used as indicators for latent constructs (for details regarding their computation also see OECD, 2010). Descriptive analyses and correlations are computed with population weights and Balanced Repeated Replicates (BRR) methodology with Fay's adjustment for variance estimation. The Software WesVar was used for the former and a special SPSS macro developed for TALIS for the latter kind of analysis (for a more detailed description see OECD, 2010). To deal with missing data listwise deletion was used for all analyses.

To examine associations between the different indicators of teacher quality multiple group regression analysis with the program Mplus was used. Two models were analysed respectively, one in which all beta weights are allowed to vary and one in which all beta weights are fixed to be equal across countries. Fit indexes are used to judge the cross-national invariance of regression coefficients. Comparing the two models ΔCFI > –0.01, ΔRMSEA > 0.01 and ΔSRMR > 0.01 are seen as indicative of differences between countries (Chen, 2007; Cheung & Rensvold, 2002).

Standardized net effects (beta weights) are reported for the model with equal regression weights and controlling for teacher's gender, years of experience as a teacher, and level of education (a Master's degree or higher versus a lower level of qualification). For standardization the standard deviations of the predicted variable and those of continuous predictor variables are used. An effect is considered statistically significant if the *p*-value is below 0.05.

Results

Teacher Qualification

Across all countries, one third of the mathematics teachers report more than 20 years of professional experience, while 39% have been working in their job less than 10 years. Between 10 and 20 years of work experience are reported by 28% of the mathematics teachers. Country differences are significant (Chi-Square = 1,095.87; $df = 132$; $p < 0.01$). A comparatively large proportion of mathematics teachers with more than 20 years of professional experience can be found in Austria, Italy, and in the Eastern European countries (except Poland). Turkey, Malaysia, and Malta, on the other hand, have comparatively less experienced teaching staff in mathematics. Here more than 50% report less than 10 years of professional experience. All of the other countries lie in between these extremes.

In most of the TALIS countries the initial training of mathematics teachers takes place in colleges and universities and at least a Bachelor's degree is required for employment. Accordingly about 90% of the mathematics teachers across countries report at least this level of educational attainment. Three exceptions are Austria, Belgium and Slovenia, where more than 50% of mathematics teachers report to have completed ISCED level 5B only. Continuing education until a Master's degree is common in Italy, Spain, the Eastern European countries (except Hungary) and, to a lesser extent, Korea and Austria. In all of the other countries less than a third of the mathematics teachers report this level of attainment. Finally a PhD is generally very rare (1%). Differences between countries are significant (Chi-Square = 33563.46; $df = 88$; $p < 0.01$).

The vast majority of mathematics teachers – 89% across countries – have taken mathematics as a field of study during their academic training. Significant differences between countries are found (Chi-Square = 1205.45; $df = 22$; $p < 0.01$). All European countries except Italy (86%) score at or above the average, while Australia (86%), Brazil (84%), Iceland (73%), and Malaysia (85%) score below.

Professional development in TALIS refers to all "activities that develop an individual's skills, knowledge, expertise and other characteristics as a teacher" (OECD, 2009). Therewith TALIS adopts a broad definition, including both, traditional workshops and courses and more modern practices, that is observation visits to other schools, participation in networks for professional development, individual or collaborative research on a topic of professional interest, education conferences or seminars, mentoring and/or peer observation, and coaching as part of a formal school arrangement. Finally, extra occupational qualification programs (e.g., a degree program) are included as well.

Across countries, most of the mathematics teachers report they regularly participate in at least one of these professional development activities. On average teachers report to have spent 19 work days on professional development during the preceding 18 months. However large variation is found regarding the total number of days for the total sample ($SD = 32$) and for all country subsamples ($SD = 6$ to $SD = 67$). Moreover, the average reported days of attendance also vary between countries ($R^2 = 0.09$; $F = 46.92$; $df = 20$, $p < 0.01$). Belgium Fl., Ireland, and Malta have the lowest means (6 days). Mexico has the most active teachers with regards to their professional development (36 days on average) followed by Bulgaria, Poland, Italy, and Spain (more than 20 days on average). The high mean scores can partly be explained by the fact that many of the mathematics teachers in the countries concerned report to attend qualification programs (Mexico, Bulgaria, Poland) and/or individual and collaborative research activities (Mexico, Italy, Poland, Spain), which are significantly more time consuming.

Across all countries workshops and courses are the most common forms of professional development. In most countries, at least three out of four teachers have participated in this kind of professional education, with the Slovak Republic and Turkey as exceptions. Modern forms of professional development which involve more cooperation and reflection are also present across all countries, but less common. For all programs significant differences between countries are found ($p < 0.01$; $df = 22$, and Chi-Square = 409.30, Chi-Square = 365.90, Chi-Square = 656.57, Chi-Square = 515.93, and Chi-Square = 463.12 for each of the variables respectively). A comparatively large percentage (> 60 %) of teachers (a) participates in networks for professional development in Iceland, Slovenia, and Poland, (b) observes other teachers' instruction in Estonia, Korea, and Iceland, (c) participates in mentoring arrangements in Korea, Poland, and the Slovak Republic, and (d) reports research visits in Mexico, respectively. Thus in summary, the highest percentages of teachers involved in these more modern activities are found in Iceland, Korea, and Poland.

Teachers' Beliefs About the Nature of Teaching and Learning

Teachers' beliefs about the nature of teaching and learning form two scales across all participating countries, which are sufficiently invariant to compare correlations across countries (see OECD, 2010 for an in-depth discussion on scale invariance). These scales capture constructivist beliefs and direct transmission beliefs, as expected. Thus, the two aspects can be identified within all countries. However, multiple group confirmatory factor analysis (MGCFA) shows the item intercepts to vary significantly, which questions the validity of mean score comparisons (see section Measures). Therefore, in the following mean score comparisons are reported for single items only.

Figures 1, 2, 3 and 4 show that mathematics teachers' agreement with all items measuring teachers' beliefs about the nature of teaching and learning is generally high: In a majority of countries the mean scores for all items are higher than the theoretical average of the response scale (> 2.50). The items measuring direct transmission beliefs receive slightly less support than those measuring constructivist beliefs, but the differences are small. However, not all teachers agree with the items to a similar extent. The standard deviations equal between 0.60 and 0.80 respectively.

Table 3 shows the variance within countries to be considerably larger than the variance between countries: Country indicators (so called dummy variables) explain 2% to 19% of the total variance in each of the items measuring teachers' beliefs about the nature of teaching and learning. But even though the differences between countries are small as compared to within country differences, they are still significant for all of the items. The largest cross-country-differences can be found for the importance of a quiet classroom for efficient instruction. Teachers in Mexico, Iceland, the Slovak Republic, and Ireland have a low mean score for this item, while teachers in Austria, Bulgaria, Portugal, Brazil, Turkey, and Italy put more emphasis on quietness in the classroom. Comparatively small country effects are found for the statement that teachers' main role is to *facilitate students' own inquiry*, and that *thinking and reasoning processes are more important than specific curriculum content*.

Response patterns further seem to be related to geographical regions. Based on the profiles four groups were built: Group A consists of the Northern European

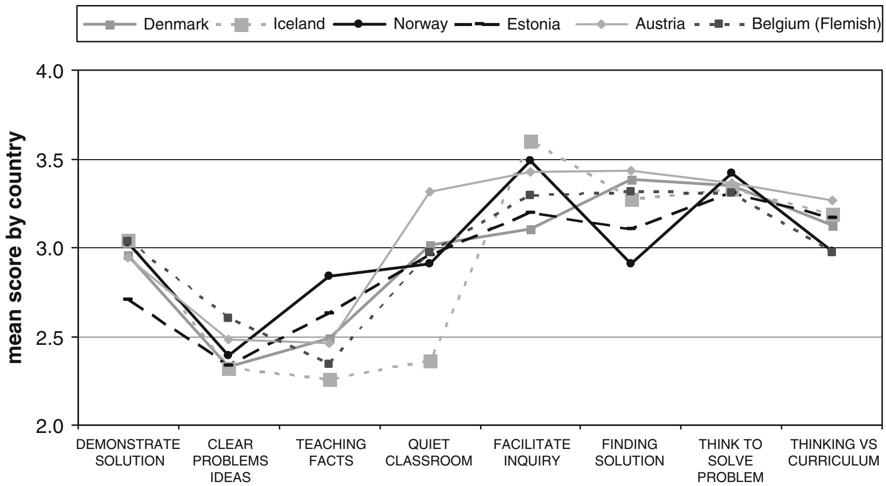

Fig. 1 Mean scores for all items measuring teachers' beliefs about the nature of teaching and learning by country (only Northern and Central European countries)

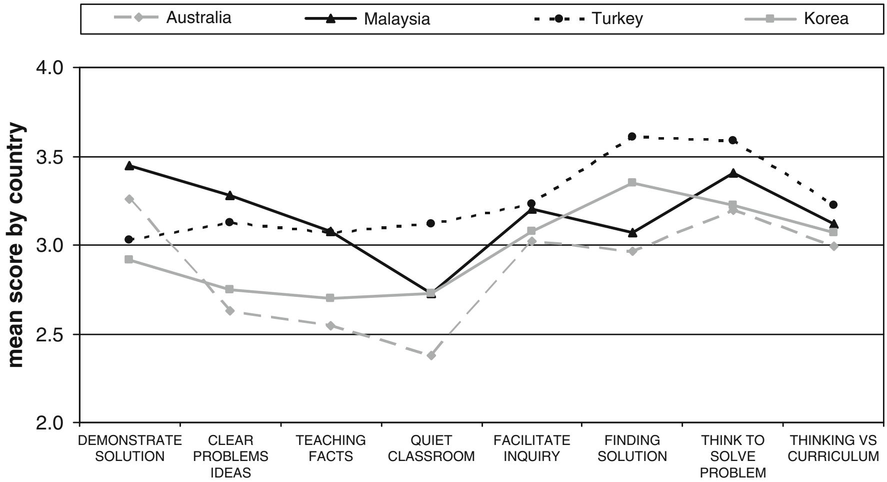

Fig. 2 Mean scores for all items measuring teachers' beliefs about the nature of teaching and learning by country (only Asian countries and Australia)

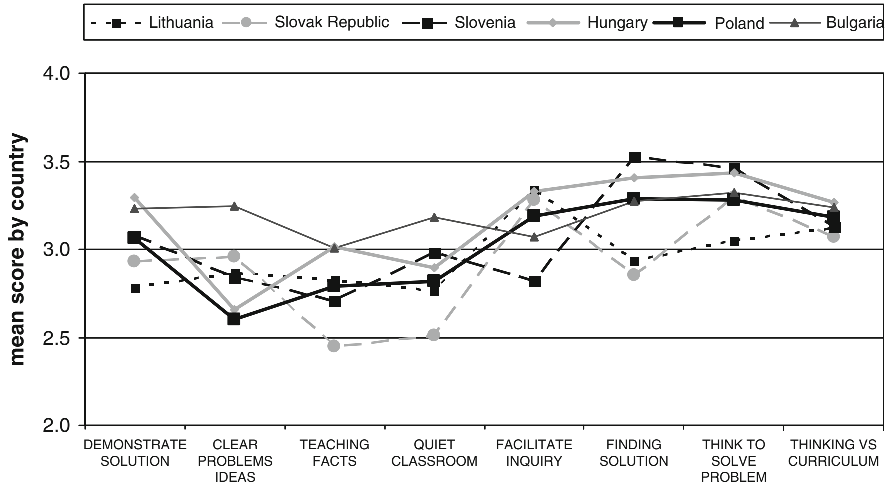

Fig. 3 Mean scores for all items measuring teachers' beliefs about the nature of teaching and learning by country (only Eastern European countries)

countries, but also Estonia, Austria and the Flemish Part of Belgium. Asian countries and Australia form group B, and the former communist European countries (except for Estonia) group C. Group D unites all Southern European and South American countries plus Ireland. In group A teachers agree with items measuring constructivist beliefs more strongly than with those measuring direct transmission beliefs (Fig. 1). This tendency is also apparent, but less clear in group B, except for Malaysia (Fig. 2). By contrast the average agreement with all items is relatively similar in group C (Fig. 3) and especially in group D (Fig. 4).

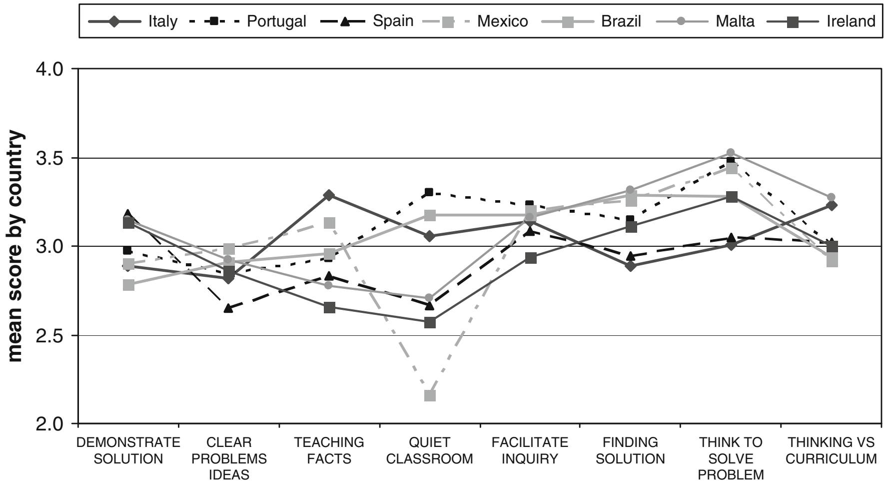

Fig. 4 Mean scores for all items measuring teacher beliefs about the nature of teaching and learning by country (only Southern European and South American Romanic countries and Ireland)

Table 3 Country effects on items measuring teacher's beliefs about the nature of teaching and learning

	Demon-strate solution	Clear problems/ ideas	Teaching facts	Quiet classroom	Facilitate inquiry	Finding solution	Think to solve problem	Thinking vs. cur-riculum
R^2	0.06	0.07	0.09	0.19	0.02	0.08	0.06	0.03
F-value	30.16	44.42	39.64	41.11	23.28	33.69	14.84	11.33
df	22	22	22	22	22	22	22	22
p	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Classroom Teaching Practices and Their Relationship with Teachers' Beliefs About the Nature of Teaching and Learning

As for teacher beliefs about the nature of teaching and learning, our theoretical expectation about the structure of classroom teaching practices was supported across all countries. Three dimensions of classroom teaching practice – namely, structuring student orientation, and enhanced activities – could be identified within all countries. However, once again, the intercepts vary significantly, so that mean score comparisons are reported for single items only.

The results show that, around the globe, most structuring and student orientation are regularly employed by teachers. The country means are mainly above 2.00, indicating that teachers use these practices at least in one out of four lessons. *Checking understanding* is among the most frequently reported classroom teaching practices in a large majority of countries (mean scores > 3.50). Relatively low mean scores are found on the other hand for *student classroom planning*, *ability grouping*, and *small group work* (mean scores < 2.00 in a majority of countries). In mathematics classrooms teachers across all countries also report an infrequent use of enhanced activities (projects, students making products, debates/arguments, and written reasoning/essay). These practices are more common in science and the humanities (see Klieme & Vieluf, 2009).

Again the within country variance is larger than the variance between countries, which explains 5% to 16% respectively. But for all items country effects are significant (see Table 4). Country dummies explain a comparatively large proportion of variance for working in small groups, checking the exercise books, reasoning/essay writing and student classroom planning. Comparatively small country effects are found for giving different work to the students that have difficulties learning and/or to those who can advance faster, holding debates/arguments and for checking understanding.

Regarding general patterns of classroom teaching practices, one basic difference between countries is illustrated by Figs. 5 and 6. They show that structuring is reported to be considerably more frequent than student orientation in Southern Europe. In contrast, Northern European teachers report lower frequencies for most of the classroom teaching practices covered in TALIS, and especially for those that aim at structuring the lesson. Some of the student oriented teaching practices on the other hand are more common in Northern than in Southern Europe.

Thus, teachers in Northern countries do not only show strong support for constructivist compared to direct transmission beliefs, as discussed in section *Teachers' Beliefs About the Nature of Teaching and Learning*, but they also use student oriented teaching practices quite often as compared to their colleagues in Southern Europe. This observation indicates a parallelism of beliefs and practices. With a case number of 23 and a non-random selection of these countries, correlations between both aspects cannot be statistically tested on the country level but the associations can be examined within countries (see Table 5).

The results of regression analyses of classoom teaching practices on beliefs about the nature of teaching and learning – controlling for gender, experience, and highest level of education – in fact show that structuring rather associated with direct transmission beliefs, while student orientation rather goes along with constructivist beliefs (Table 5). However, significant differences between countries exist: The model fit drops substantially when the beta-weights are fixed to be equal across countries (ΔCFI = 0.29–0.60 and ΔRMSEA = 0.04–0.05). A closer look at within country regressions suggests that these differences mainly concern the strength of associations, not the direction of coefficients. For structuring significant and positive effects of direct transmission beliefs are found in seven countries, and significant effects of constructivist beliefs in six. Constructivist beliefs have a positive effect on student orientation in eight countries and on enhanced activities in nine. Relations between direct transmission beliefs and the latter two practices are significant in only three and five countries respectively. Also within countries the associations are rather weak.

Table 4 Country effects on items measuring classroom teaching practices

	State learning goals	Review home-work	Previous lesson summary	Check exercise	Check under-standing	Small groups	Different work	Student classroom planning	Ability groups	1 week projects	Student make product	Write reasoning/ essay	Debate/ argument
R^2	0.08	0.13	0.08	0.16	0.06	0.20	0.05	0.15	0.12	0.07	0.09	0.17	0.05
Z-value	48.90	74.51	80.05	76.07	35.67	55.98	45.74	55.61	50.61	38.19	39.44	72.44	82.26
df	22	22	22	22	22	22	22	22	22	22	22	22	22
p	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

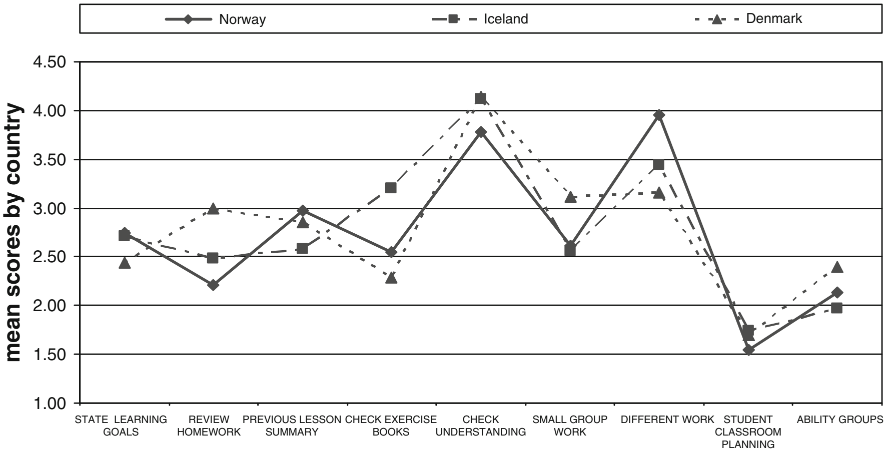

Fig. 5 Mean scores for items measuring classroom teaching practices by country (Northern European countries)

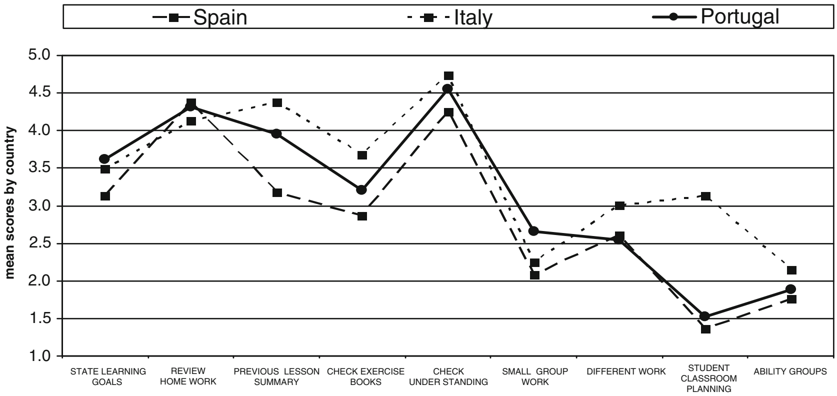

Fig. 6 Mean scores for items measuring classroom teaching practices by country (Southern European countries)

Relationships Between Teachers' Professional Background and Their Beliefs About the Nature of Teaching and Learning and Classroom Teaching Practices

Table 6 shows the results of multiple group regression analyses predicting teachers' beliefs about the nature of teaching and learning with indicators of professional qualification. When all coefficients are restricted to be equal across countries, direct transmission beliefs are positively related with professional experience and negatively with participation in workshops and courses. None of the other effects is significant.

Table 5 Results of multiple group regression analyses explaining classroom teaching practices with beliefs about the nature of teaching and learning. (Three regression analyses are reported, each with two independent variables and teachers' gender, experience, and level of education as control variables)

	Classroom teaching practices		
	Structuring	Student orientation	Enhanced activities
Direct transmission beliefs	0.11**	0.05**	0.05**
Constructivist beliefs	0.06**	0.09**	0.04**

Notes: $^{*}p \leq .05$; $^{**}\ p \leq .01$

Table 6 Results of multiple group regression analyses explaining beliefs about the nature of teaching and learning with teacher qualification (Two regression analyses are reported, each with seven independent variables and teachers' gender as control variable)

	Beliefs about the nature of teaching and learning	
	Direct transmission	Constructivist
Professional experience	0.04**	−0.01
Highest level of education (Bachelor or below vs. Master/PhD)	0.01	0.03
Studied mathematics	0.05	−0.06
Days of professional development	−0.00	0.00
Workshops/courses	−0.10**	0.01
Networks for professional development	−0.01	0.04
Mentoring	0.01	0.03

Notes: $^{*}p \leq .05$; $^{**}\ p \leq .01$

Classroom teaching practices are more closely related with teacher qualification: First of all, the level of education has a negative, but weak effect on student orientation and enhanced activities. Moreover teachers who have studied mathematics report to use more structuring than out-of-field-teachers. Finally, attendance of workshops and courses is positively related with student orientation, and teachers participating in networks or mentoring programs report to use all three practices more often, especially student orientation (Table 7).

Analysis of invariance shows that the correlations of indicators of teacher qualification with beliefs about the nature of teaching and learning as well as correlations with classroom teaching practices are not equivalent across countries. For all regression models the fit drops noticeably when regression coefficients are restricted to be equal (ΔCFI = 0.30–0.44 and ΔRMSEA = 0.03–0.04). However, more detailed analyses of within country effects show that differences between countries mainly concern the strength of the associations, not their direction.

Table 7 Results of multiple group regression analyses explaining classroom teaching practices with teacher qualification (Three regression analyses are reported, each with seven independent variables, and teachers' gender as control variable)

	Classroom teaching practices		
	Structuring	Student orientation	Enhanced activities
Professional experience	0.02	0.02	0.01
Highest level of education (Bachelor or below vs. Master/PhD)	0.00	−0.07*	−0.04*
Studied mathematics	0.12**	−0.00	−0.00
Days of professional development	0.00	0.00	0.00
Workshops/courses	0.05	0.06*	0.03
Networks for professional development	0.07**	0.12**	0.08**
Mentoring	0.14**	0.17**	0.11**

Notes: *$p \leq .05$; ** $p \leq .01$

Discussion

While there are many studies comparing student achievement cross-nationally, most empirical research on teachers focuses on single countries only. In the present contribution we drew on a large international database to explore cross-cultural differences and similarities regarding three aspects of teacher quality. The results show both, similarities and differences across the 23 countries participating in TALIS.

Cross-National Differences and Similarities in Levels and Patterns of Teacher Quality

Similarities Between Countries

First of all, the findings show that basic features of teacher qualification systems are similar across participating countries. Almost all of the secondary mathematics teachers have attained a university degree, and most (> 70%) have studied mathematics. Most common is a Bachelor's degree, but about a third has also attained a Master's degree. A PhD is generally rare. To expand their teaching skills and to stay up-to-date with instructional methods, teachers in all participating countries attend professional development, especially courses and workshops. Arrangements demanding a higher level of cooperation and active reflection – like networks for professional development and mentoring – are also familiar cross-nationally, but less widespread.

These results are consistent with findings from the TIMS-study. However, one difference becomes apparent: In TIMSS only 78% of 8th grade students have teachers with a university degree as compared to 90% of the TALIS teachers. A close look at the data shows that this is mainly due to the fact that more developing countries (e.g., Tunisia, Algeria, Morocco, Ghana, Lebanon) participated in TIMSS with higher rates of teachers who had completed secondary school only (Mullis et al., 2008). Hence, differences in teacher qualification may be larger when less affluent countries are also included in the sample.

Remarkably, TALIS shows that basic dimensions of teachers' beliefs about the nature of teaching and learning (namely constructivist vs. direct transmission views) can be cross-nationally identified. The agreement with all items measuring teachers' beliefs about the nature of teaching and learning is high – a result that was also found in MT21 (Schmidt et al., 2007). Hence, the instruments seem to cover well what teaching and learning means to teachers in different countries. Moreover, it is impressive to see that constructivist views are supported by a majority of mathematics teachers in all countries. This shows constructivist ideas to be present in different philosophical traditions and educational discourses.

Dimensions of classroom teaching practices (namely structuring, student orientation, and enhanced activities) could also be measured cross-nationally. Like TIMSS we found a similar repertoire in different regions of the world (Mullis & Martin, 2007; LeTendre et al., 2001): Across countries most mathematics teachers report to regularly state learning goals, review homework, check exercise books, check student understanding, use group work, and summarize the previous lesson.

Altogether these findings show that at more general levels of abstraction mathematics teachers in different countries are quite similar regarding their qualification, beliefs about the nature of teaching and learning and classroom teaching practices. However, going into more detail, significant differences regarding all three indicators of teacher quality become apparent.

Differences Between Countries in Terms of Teacher Qualification

The most striking difference between countries regarding teachers' level of education is the high percentage of teachers without a Bachelor or Master degree in Austria, Belgium and Slovenia. This can be explained by a peculiarity of the education systems in these countries: the training of mathematics teachers used to take place in special institutions – at least for some tracks or educational levels. However, recently – in the course of the European Bologna process – equalization to other systems is taking place in these European countries.

Aside from the level of education, differences are also found for the proportion of out-of-field-teaching: This is comparatively low in Eastern Europe and higher in many Southern, Northern and non-European countries. Mathematics teachers who have studied mathematics are likely to have more content knowledge and pedagogical content knowledge than out-of-field-teachers, and research suggests a positive (but non-linear) relation of subject specific training with student achievement (Darling-Hammond, 1999; Monk, 1994). Hence, teachers in countries with a large percentage of out-of-field-teaching may be on average less well prepared for their job. Moreover – as out-of-field teaching often concerns schools with a socially disadvantaged student population (Ingersoll, 2003) – the cross-national differences may be relevant for explaining system level variation in equity.

While the attendance rates in workshops and courses for professional development are relatively similar across countries, considerably more variance is found regarding networks for professional development, observation visits, research visits, and mentoring. Research suggests that professional development that involves teachers in professional learning communities may be more effective in changing classroom teaching practices, promoting student-centred approaches and enhancing student achievement than traditional programs (e.g., Bolam et al., 2005; Supovitz, 2002; Supovitz & Turner, 2000; Vescio, Ross, & Adams, 2008). Thus, in countries where this is common (e.g., Iceland, Korea and Poland) teachers are better supported with becoming a *reflective practitioner* (Schön, 1983).

Differences Between Countries in Terms of Teacher Beliefs About the Nature of Teaching and Learning

Significant country effects are further found for the level of endorsement of each of the items measuring teachers' beliefs about the nature of teaching and learning. Such differences were expected, as teachers' professional beliefs are considered to be influenced by *folk pedagogies* (Bruner, 1996) or *personal history-based lay theories* (Holt-Reynolds, 1992), and bearing in mind that previous research found distinct patterns of teacher beliefs and practices even for countries that are very close with regards to their cultural background and their education systems (e.g., Germany and Switzerland; Leuchter, Pauli, Reusser, & Lipowsky, 2006).

In the TALIS sample, the preference for constructivist beliefs is especially pronounced in Northern and Central Europe, reflecting the long-standing tradition of reform pedagogy in this region. However, a comparatively strong relative endorsement of constructivist views was also found in Korea, despite its different philosophical traditions. Similar results have been reported for other Confucian countries (Cheng, Chan, Tang, & Cheng, 2009; Lingbiao & Watkins, 2001; Tang, 2008), and in fact Lee (1996) and Shim (2008) pointed to some intersection of Confucian philosophy with European constructivist ideas. In Southern Europe and South America the pattern is less clear. Here, the relative agreement with a direct transmission view as compared to a constructivist view is higher than in other countries. Interestingly, these regions are also characterized by comparatively traditional general values (Inglehart, Basàñez, Díez-Medrano, Halman, & Luijkx, 2004). This suggests that in addition to country specific pedagogic traditions there may also be an influence of more general values on beliefs about the nature of teaching and learning.

Differences Between Countries in Terms of Classroom Teaching Practices

Finally, just like TIMSS (Givvin et al., 2005), we also found characteristic differences in profiles of classroom teaching practices. Most noticeable is the comparatively frequent self-reported use of student oriented teaching practice in the Northern European countries. It is especially group work and adaptive practices which are more common in this region than in other parts of the world. At the same time structuring teaching practices are reported to be common, but less frequently used than in Southern Europe. This may reflect the concern of the *Nordic Model* for promoting weak and socially disadvantaged students in comprehensive school systems (e.g., Lie, Linnakylä, & Roe, 2003).

Associations Between Different Indicators of Teacher Quality

The present study uses indicators of teacher quality from three different research traditions, namely teacher qualification, teachers' beliefs about the nature of teaching and learning, and classroom teaching practices. Results show that across countries these different aspects are indeed associated with each other, but they still represent quite distinct facets of teacher quality.

No significant correlation is found for teachers' professional experience and their level of initial education with beliefs about the nature of teaching and learning or classroom teaching practices. This reflects the large body of research in economics of education – mostly within countries, especially in the USA – that finally led to the conclusion that teacher experience is a weak indicator for teacher quality (for a more detailed discussion, see Ball & Hill, 2008). The finding is further consistent with the observation that beliefs about the nature of teaching and learning are often acquired prior to professional education and can be quite stable over the life span (e.g., Borko & Putnam, 1996; Pajares, 1992; Wilson, 1990). However, it should be noted that TALIS only asks for the level of educational attainment, while the curricula, the specific content, and the quality of initial education programs may also be relevant for the acquirement and differentiation of beliefs and a repertoire of practices.

In contrast to initial teacher education, professional development is shown to be associated with beliefs and practices in TALIS. The relationships are rather weak, but significant for the total sample as well as the country subsamples. Networks and mentoring have stronger effects than workshops and courses. Furthermore the former kinds of professional development – which regularly go along with an intensive professional exchange and a high level of teacher commitment – are rather related with student orientation and enhanced activities than with structuring. However, as the study is cross-sectional, the causal chain behind this correlational pattern could be twofold: Teachers with more diverse and/or more intensive didactical practices may be more willing to participate in professional development, or professional development may inspire teachers to use classroom teaching practices in a more explicit way. Results of previous research on effects of professional development on teacher behaviour and student achievement are rather inconsistent (for a discussion see e.g., Buczynski & Hansen, 2010). To establish causality experimental settings may be used in the future, comparing the effects of different kinds of professional development programs in a variety of countries.

Correlations between beliefs and practices are in accordance with theoretical expectations and previous research. Teachers who have a rather constructivist view on the nature of teaching and learning also use more student orientation, while structuring is less closely related to teachers' beliefs is about the nature of teaching and learning. The associations are rather weak. This is consistent with previous research (e.g., Levitt, 2001; Seidel, Schwindt, Rimmele, & Prenzel, 2008; Wilcox-Herzog, 2002), and in TALIS it can also be explained with the abstract nature of the beliefs examined, which generally implies less relevance for actual behaviour (see e.g., Alisch, 1981). However, TALIS is the first study to show that the magnitude of these associations also varies between cultures.

More generally, in comparing 23 countries it was found that the associations between all different indicators for teacher quality differ between education systems. It is mainly the strength of the association not the prefix that is different across countries. Nevertheless these results suggest that it may be necessary to define and examine teacher quality in a country-specific way.

Conclusions and Implications for Research and Practice

In summary, the results regarding cross-country differences in teacher qualification, beliefs and practices neither support the theory of national cultures, which assumes education to be largely culture specific (e.g., Bennett, 1987; Bracey, 1997), nor the theory of *institutional isomorphism* which holds the influence of international institutions responsible for a general harmonization of education systems (e.g., Spindler & Spindler, 1987). They are – if anything – consistent with the *global culture dynamics* approach suggested by LeTendre et al. (2001). The authors argue that organizational characteristics of schooling, but also instructional practices, are similar around the world because "the modern institution of school has penetrated most nations" (p. 5). At the same time their approach also assumes effects of national or regional laws as well as "national, regional, or local systems, customs and expectations on schooling" (p. 12). Accordingly, we found similarities, but also significant and characteristic differences between countries.

The finding of differences in profiles and structure of teacher quality emphasizes the importance of a careful analysis of cross-national equivalence in any study aiming at level oriented comparisons, but also whenever results and practices from one country are transferred to another. The same conclusion may hold for related constructs such as teacher expertise, professional knowledge, and teacher competence. Theoretical paradigms like the expert-novice-distinction, Shulman's taxonomy of professional knowledge, or the notion of competence (most often being defined as a mixture of cognitive and attitudinal dispositions) have been used and empirically applied in educational research world wide. However, the present study may induce a more careful approach to these paradigms in cross-cultural contexts. Previous research, being based on these globally accepted theoretical paradigms, seems to have neglected the role of culture in defining, understanding, and measuring teacher expertise and teacher quality. Especially, conceptions of constructivism have been used without reflecting its cultural foundations. More cross-cultural research on teacher expertise and teacher quality, both qualitative and quantitative, is needed.

Putting constructivism into perspective is another important message to mathematics education practitioners. The triarchic model of instruction, which has been supported in the present study, assumes structure, support, and cognitive activation to be basic dimensions of high-quality teaching. Pure constructivists tend to neglect the dimension of structure, which is indispensable for cognitive learning as well as for student motivation.

From a teacher education point of view it should be noted that structuring teaching practices are implemented more often by teachers who had studied mathematics, while all three dimensions seem to be correlated with professional networking and mentoring.

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Reflections on Teacher Expertise

Alan H. Schoenfeld

Abstract This chapter offers some broad reflections on issues of mathematics teacher expertise. The first main section stresses the importance of teachers' and researchers' beliefs and values – more generally, their conceptual models regarding "what counts" in the act of teaching. This raises the question of how one might frame explorations of teacher expertise in general, an issue I explore in the context of a general model of the research process. The second main section explores the very nature of teaching itself. If expertise in teaching is the culmination of a developmental process, then one should ask, "what develops?" That is: what does a teacher draw upon, in the moment, as he or she teaches? I argue that a teacher's actions are a function of that teacher's *resources* (including knowledge), *goals, and orientations.* Hence the study of the development of expertise should focus on the growth and change of teachers' resources, goals, and orientations. The concluding discussion reflects on these issues and considers some next steps in research the field might undertake.

Keywords Teaching · Decision making · Resources · Goals · Beliefs · Orientations

Conceptualizing Research on Teaching Expertise

In this chapter I reflect on themes raised in this book and, more broadly, the literature on (mathematics) teacher expertise. I begin with two stories. The first concerns an experience I had more than 20 years ago, and which remains vividly with me today. In the mid-1980s my research group was constructing computer software that was intended to help students develop deep understandings of functions and graphs. My programmer told me that his wife was interested in exploring our software. She had previously had trouble with algebra – she had taken the course four times, and forgotten the contents each time – and she was interested in seeing what she could learn

A.H. Schoenfeld (✉)
School of Education EMST, University of California, Berkeley, CA, USA
e-mail: alans@berkeley.edu

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by using the software. I came into our laboratory as she was trying (unsuccessfully, of course) to create a vertical line using the equation y = mx + b.

If the graph of the equation $y = x$ is at 45° from the horizontal axis, she reasoned, the graph of $y = 2x$ should be at 90° – that is, vertical. After all, one is doubling the coefficient of x, so shouldn't the angle be doubled? She tried it. It didn't work, so she doubled the coefficient again, and then again; she was baffled. What followed was an exchange in which I gave her varied opportunities to recall what she remembered about slope ("rise over run") and to rebuild her understandings from the ground up. After almost an hour, she had an epiphany: on a vertical line the "run" would be zero, and it is impossible to divide by zero. She shouted with joy – it made sense! I was confident that this time the meaning of slope and what its computation meant would "stick."

We had taped this interaction, and I was proud of the tape. A few months later a colleague came by, and I showed her the tape. She fidgeted through it, and then, when it was over, said, "You know, Alan, when we build our instruction we work with master teachers. A number of the teachers I work with could have explained the content to this student in much more straightforward ways." From her point of view, my teaching was terrible.

The second story concerns a conversation with David Clarke about expert teaching. As part of his research, Clarke has collected tapes of competent teaching from around the world.[1] The teachers whose tapes he collects have been selected by procedures that resemble many of those in this book: the teachers have been teaching for some time, are often award winners, and are nominated either by researchers or administrators for their expertise. Clarke, who is interested in the prioritization of rich mathematical discourse in instruction, had been looking at a high school classroom tape from a particular country. What the tape showed was, in essence, pure lecture. At that time Clarke was being visited by a researcher from a country that neighbors the one where the tape had been made. Trying to be neutral, Clarke said, "I have this fascinating tape of a teacher who has been nominated as competent. In the entire lesson, the students seldom speak, and then only to say the word: yes, and this word is almost always a choral response by the whole class." His colleague's response was, "So?" This scholar, like the one who had nominated the tape, saw nothing problematic in a situation where high school students were the object of a well constructed lecture.

The point of these stories is that beliefs and values, which I will subsume under the more general category *orientations*, are of great importance not just when one is teaching but when one is reflecting on or evaluating teaching. I thought that my exchange with the student was an instance of exemplary teaching, because it got to the heart of the student's (mis)understanding and helped her make sense of something that had been deeply problematic for her; my colleague thought my teaching

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[1] Because he recognizes that the label "expert" is value laden, Clarke prefers to use to term "competent teaching" instead of "expert teaching." My purpose here is to problematize the term "expert teaching" – so I use it here as many people do, and then show that judgments of expertise are culture-bound.

was terrible because I wasted time and allowed her to get “lost” when I could have laid out the content in a much more direct manner. Clarke found the teaching in the videotape a challenge, because it was missing a dimension of classroom interaction that he – like many of the researchers represented in this volume – would consider essential in the (Australian) classrooms with which he is most familiar. In contrast the particular teacher had been recruited because of local acclamation as a competent teacher. Nor did the other expert researcher with whom Clarke discussed the tape see student spoken mathematics as an important aspect of mathematics instruction, either as process or as product.

In short, our orientations *as researchers* to what counts in teaching are every bit as consequential in shaping what we study and how we study it as teachers’ orientations are consequential in shaping the ways they run their classrooms. They shape our conceptual models of the teaching process, and of the process of conducting research on teaching.

As noted in at least one chapter in this volume, there exists very little research on teacher expertise that examines the ways in which teacher expertise has an impact on student performance. That is because the field is young, and tools and techniques for the robust characterization of teachers’ actions have yet to be developed and used widely. It is probably safe to assume that once such tools and techniques have been developed, the field will turn its attention to student outcomes. Yet, there is still a problem: what will be used as outcome measures? Experimental or correlational results might be very different if the outcome measures used focus on skills and procedures, or if they focus on conceptual understanding and problem solving.

This is not a hypothetical issue. Consider the following, for example. Ridgway et al. (2000) compared students’ performance at Grades 3, 5, and 7 on two examinations. One examination was a standardized high-stakes, skills-oriented test (the State of California’s STAR test). The other was an examination produced by the Mathematics Assessment Resource Service (MARS). The MARS tests are designed to cover a broad range of skills, concepts, and problem solving. Both tests were administered to more than 16,000 students at grades 3, 5, and 7. Table 1 provides

Table 1 Comparison of students’ performance on two examinations (reproduced with permission from Schoenfeld, 2007a)

	SAT-9	
MARS	Not proficient	Proficient
	Grade 3 ($N = 6136$)	
Not proficient	27%	21%
Proficient	6%	46%
	Grade 5 ($N = 5247$)	
Not proficient	28%	18%
Proficient	5%	49%
	Grade 7 ($N = 5037$)	
Not proficient	32%	28%
Proficient	2%	38%

the distribution of scores at the three grades. Student scores are recorded as being either “proficient” or “not proficient.”

What one sees from the score distributions is the following. At every grade level, if a student scores “proficient” on the MARS test, there is a very good chance that the student will also score “proficient” on the SAT-9 (For example, 46/52 – about 89% – of those judged proficient on the MARS test at grade 3 were also judged proficient on the SAT-9). However, the converse is not true. Only 46/67 – about 69% – of the third graders who scored proficient on the SAT-9 also scored proficient on the MARS tests.

This study demonstrates how the choice of measures can make a big difference in the ways one interprets the results of research. Suppose one were to define “teaching expertise” as “demonstrating clearly how to perform mathematical procedures and providing students with ample opportunities to practice those procedures.” If the SAT-9 were used as an outcome measure of student performance, this kind of expertise would, most likely, correlate with high student scores. If, however, the outcome measure used was the MARS tests, then it is quite possible that this kind of teaching “expertise” would result in relatively low test scores. In sum, a researcher’s conceptual model of what represents “students’ mathematical proficiency” will shape the choice of measures the researcher employs – and thus the interpretations of findings.

Specifically, if one is oriented toward “direct instruction” and one picks teachers who are good at it, then one can find measures that will “prove” that those teachers’ students do extremely well. This is not a hypothetical example: At the height of the “math wars” in California, testimony before the California Board of education attempted to prove that direct instruction is the superior teaching method, using precisely this kind of approach (Schoenfeld, 2004).

In sum, researchers’ conceptualization of what counts as expertise is very much a function of those researchers’ own orientations toward teaching. Their choice of measures to “capture” what counts in classroom – whether those measures focus on student scores, on the frequency of teacher behaviors (e.g., as in the process-product paradigm), or on the documentation of student-student interactions – is consequential. The researcher’s conceptual model of what is important shapes what the researcher observes and privileges. The researcher’s conceptual models of instruction and of instructional outcomes shape the researcher’s choice of measures, the ways those measures are implemented, and the ways that the results of those measures are interpreted.

To be sure, this is not just the case in the study of teacher expertise. Research on teacher expertise is no different than any other research: one’s conceptual models, one’s measures, the ways one combines and reports data, the ways one “maps back” from the data to interpret the results, can all be problematic. The complexities are represented in Fig. 1, reproduced with permission from Schoenfeld (2007a), see that chapter for a broad discussion of issues in conducting empirical studies such as those in this volume.

Here I will simply point to some of the complexities involved in studies of teaching expertise. The “real world situation” in Fig. 1 can be taken to be an instance, or a series of instances, of expert teaching.

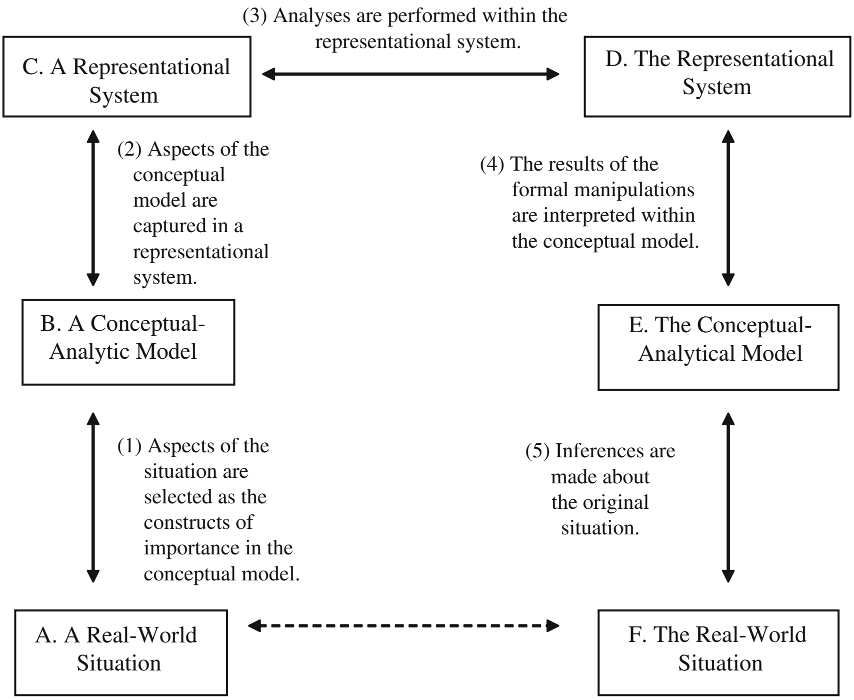

Fig. 1 A schematic representation of the process of conducting empirical research (reproduced with permission from Schoenfeld, 2007a)

Along arrow (1), what is the researcher's model of teaching expertise? For example, does it include giving a polished lecture, with clear explanations; is it focused on relational teaching; is student-to-student interaction considered valuable (and under what circumstances)? What the researcher values will shape the very data that are collected, observed, and described or quantified.

Arrow (2) represents the investigator's choice of representations of teaching practice and outcomes (These may vary from "rich, thick descriptions" of teaching to counts of certain teacher behaviors to a choice of tests of student understanding). Note that these choices, which reflect the researcher's conceptual model, are consequential. For example, will the researcher tally or otherwise indicate the frequency of teacher questioning? Will the research indicate the *nature* of teacher questioning? One teacher may ask a large number of questions, but they may all be in the form of IRE sequences and be aimed purely at procedural mastery. Another teacher may ask fewer questions, but they may be conceptual in nature. Does the researcher's coding scheme capture the differences? Will the complexity of the teacher's questions be examined? Some teachers, when students are having difficulty, ask questions that simplify the task and reduce the level of challenge; other teachers maintain the level of challenge. Does this matter to the researcher? If the outcomes are measures of student proficiency, what tests, or interviews, or other measures are used? As the discussion of the MARS tests above indicates, one can draw very different inferences about student proficiency (and thus about teacher expertise) depending on the measures used.

Arrow (3) represents a more methodological concern: are the methods used to analyze the data used appropriately? This concern applies to both quantitative and qualitative studies. If statistics are used, are the entities that are captured in numbers coded in reliable and replicable ways? (Recall the issue of teacher questions – is a question simply “a question,” or does the nature of the question make a difference, and if so, how reliable is the categorization scheme?) If qualitative descriptions are offered, how much of the original data (videos, transcripts, or other source material) must be offered before the reader can be confident of the representativeness and accuracy of those descriptions?

With Arrow (4) we return to issues of interpretation related to conceptual models. It may well be that the statistics employed in “direct instruction” studies referred to earlier were correctly used – but the inference that direct instruction led to improved student learning was highly questionable, given that the measures of student learning used in those studies were limited to the performance of rote skills. Of course, for the investigators, the ability of students to perform rote operations may have represented “mathematical competency.” That is precisely why it is important to “unpack” the conceptual models that researchers employ.

Finally, Arrow (5) represents the inferences drawn about the original situation – and quite often the generalizations thereof. In the previous example, the conclusions of individual experimental studies were “direct instruction is most effective for student learning.” In the present context (that is, a book on teaching expertise), one issue that needs to be problematized is the very notion of “expertise.” A useful distinction is one introduced by Hatano and Inagaki (1986), the contrast between “routine expertise” and “adaptive expertise.” A routine expert is someone whose performance in a domain is highly competent, as long as the issues the individual deals with fall within the realm of the familiar. An adaptive expert is someone who is much more flexible – someone who, in addition to coping with familiar situations, can also cope with the unexpected. On a personal note, this contrast between routine expertise and adaptive expertise is an issue that bedeviled me early in my career. The definition of a problem solving “expert” in the cognitive science literature when I began doing my problem solving research was of a routine expert – a person who knows the domain inside-out and can answer textbook problems without needing to work hard. That definition ruled out of existence the very kinds of problem solving experts I cared about, those who were able to make progress on challenging problems that they didn’t know how to solve at first glance! Similarly, is an expert teacher the kind of teacher who presents material clearly and has an engaging classroom, perhaps making good use of group work or other activity structures (routine expertise)? Or, is a teaching expert in addition someone who can react to an unusual comment from a student and use it as the starting point for productive classroom interactions (adaptive expertise)?

We as a research community are navigating through very complex territory, and recognition of the challenges inherent in Fig. 1 places significant burdens on us. As researchers it behooves us to: be as explicit as possible about our conceptual models, so that others may better understand what we do and do not take into account; to be comparably explicit about out methods, so that others can understand, replicate, and apply them; and be cautious about drawing conclusions that are warranted by assumptions, models, and data.

A Theory of Teaching; Deconstructing One Example of Expertise

Presumably a major reason for studying teacher expertise is that expertise is the "target" for professional development: if one knows what comprises expert teaching, one would hope to find ways to help teachers develop such competencies. I take that to be a fundamental rationale for this volume.

But if one hopes to enhance teacher development, it helps to have a theory of what develops. That is: if you hope to shape teacher's decision-making in the classroom, it helps to have a theory of how and why teachers make the decisions they do.

In the next few pages I shall try to suggest the dimensions of such a theory. The full theory, exemplified in detail, requires a book-length exposition (Schoenfeld, 2010); here I tell one story briefly to suggest how things work, and what the foci of attempts to develop (adaptive) expertise must be.

In view of the preceding discussion, I hasten to say that the lesson described below is representative of a teaching style considered to be "expert" by some researchers in the United States. However, the point of the example is not to elaborate on that style of teaching as representing "expertise." Rather, the point is to show that this teacher's rather complex decision making can be modeled in very fine detail using a small number of theoretical constructs. My claim is that the analytical approach illustrated below can be used to examine and elaborate all teachers' decision-making, no matter what style of teaching is examined. Hence this kind of approach can be used as a mechanism for developing deeper understandings of teachers' actions in the classroom, no matter which pedagogical goals are privileged in that particular cultural context.

The lesson I discuss was taught by Jim Minstrell, a high school physics teacher-researcher who, by any measure, meets the definition of "adaptive expert." Minstrell was a recipient of the US Presidential Award For Excellence in Science Teaching, and he has written and reflected extensively on how and why he teaches the way he does, and on the impact of his teaching on student learning (see, e.g., Minstrell, 1992; Minstrell & Stimpson, 1996; van Zee & Minstrell, 1997a, 1997b). Here I discuss an introductory lesson, for which Minstrell has a number of major goals. Minstrell wants his students to come to understand that physics is not a discipline that consists of the mindless application of formulas, but rather that it is a discipline of sense-making, where there is discretion in selecting and applying formulas; that students can figure things out for themselves; that his classroom will function as a community of inquiry, and that the content of the course should be grounded in reason, not authority. These goals are driven by Minstrell's orientations – his beliefs and values regarding his perception of physics as a discipline and how students should interact with it. Minstrell has a substantial knowledge base, which he employs in the service of his goals and orientations. This knowledge base includes disciplinary (content) knowledge, of course; it includes pedagogical content knowledge, for example conceptions and misconceptions his students are likely to reveal, and how to build on the former and deconstruct the latter; and it involves a number of pedagogical routines that, by virtue of their interactive form, solicit and build upon ideas from the students. One such routine, which is shared by other accomplished teachers who also share Minstrell's orientation toward encouraging and building on student ideas and sense-making, is given in Fig. 2.[2] I will illustrate the use of the routine by describing how Minstrell employed it early in the lesson, and then show how his orientations toward teaching (in concert with his knowledge base) played a major role in shaping his decision-making.

To make the point that human discretion is involved even in the application of formulas, he had asked eight students to measure the width of a table. The values they obtained were:

106.8; 107.0; 107.0; 107.5; 107.0; 107.0; 106.5; 106.0.

Minstrell's major question for the day was, "What is the best number for the width of the table?" The first stage of this discussion consisted of having the class address the question of whether they should combine some or all of the numbers to get the best value; and if it was just some of them, which ones they should be. This question, posed repeatedly, led to discussions of why one might trust some numbers more than others (e.g., if they were obtained by acknowledged experts), what one should do with outliers, and whether and when one might exclude extreme values from a computation (as, for example, e.g., in Olympic gymnastics competitions).

Once the class determined that all the numbers listed above would be used to determine the width of the table, Minstrell asked how to combine those eight numbers to get a "best number." In terms of the routine in Fig. 2, he had completed steps A1 and A2. In response to Minstrell's question about "best number" in step A2 a student said "average them." This was on target and did not raise other issues (in the flow chart decision D1 = "no"), but it did call for elaboration (D3 = "yes").

[2] I hasten to add that Minstrell does not "follow" a flow chart or decision procedure when he teaches. The flow chart *replicates* his decision-making, which in practice is totally fluid and natural. That is part of the story about expertise: over time, an individual develops skills that are flexible and look so casual and practiced that the structure underlying them is hard to see; indeed, the decisions to do particular things may be automatic and not conscious. If any of the readers of this chapter are practiced cooks, they can reflect on how easy it appears when they prepare a favorite dish – but how it took a long time to develop the fluency that makes the preparation *look* easy. For example, as he or she is preparing a dish an experienced cook might see that the temperature in a sauté pan is too low or too high, and adjust accordingly. That cook certainly does not say to him or herself, "I must monitor the temperature and look for certain signs. If they indicate that the pan is not hot enough, I raise the heat; if they indicate that the pan is too hot, I lower the heat." The point, however, is that the cook's actions can be *modeled* by such decision procedures.

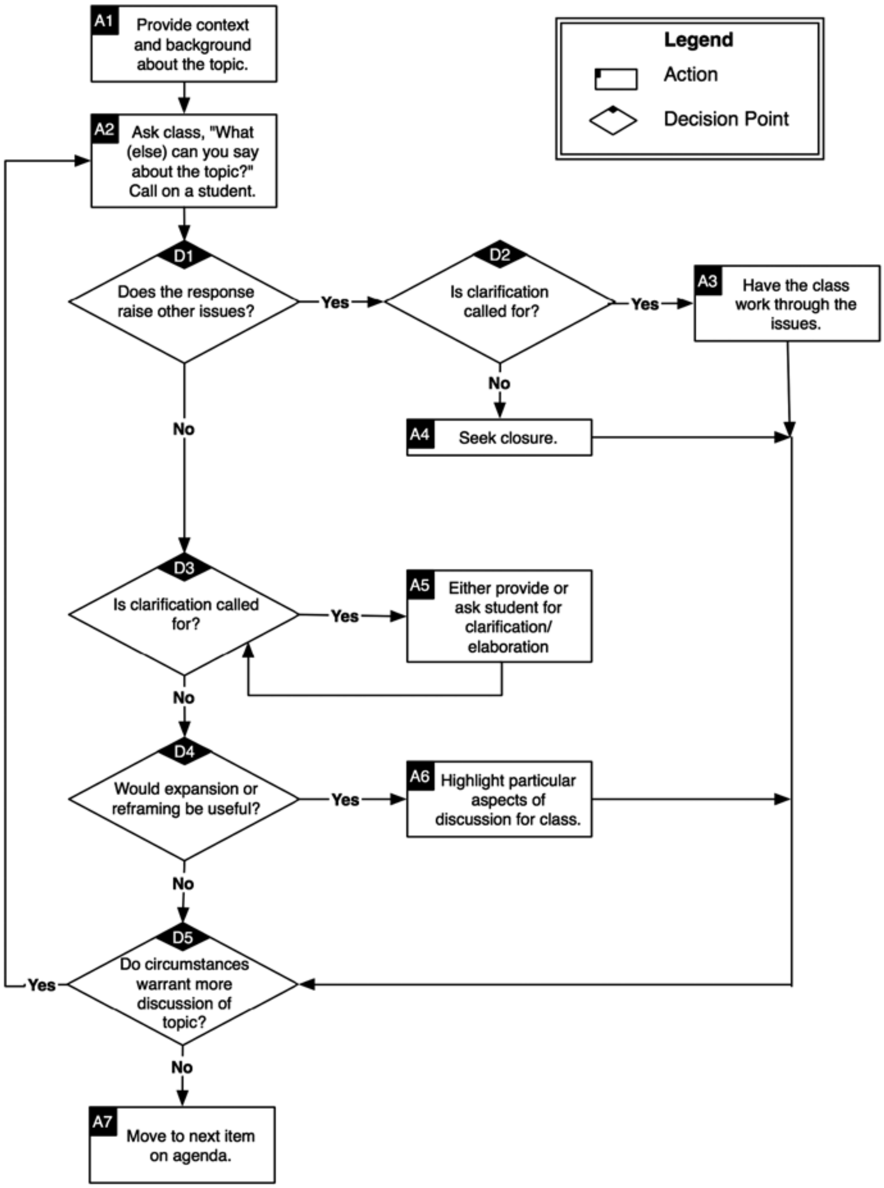

Fig. 2 A highly interactive routine for soliciting and working with student ideas (with permission from Schoenfeld, 2002)

Thus Minstrell asked the student to flesh out his answer: "We might average them. Now what do you mean by 'average' here?" The student's response, "Add up all the numbers and then divide by whatever amount of numbers you added up," provided the explanation that Minstrell needed, so no further clarification was necessary

(D3 = "no"). Minstrell then decided that some expansion would be useful (D4 = "yes"). He continued by mentioning that the student's specification of how to compute the average is known as an "operational definition." That discussion completed one "tour" through the flow chart.

Having obtained and discussed one potential method for obtaining the "best number," Minstrell returned to step A2 and opened the next round of conversation: "Any other suggestions there for what we might do to get a best value?" A student comment, "You've got a bunch of numbers that are the same number," led to a discussion of the fact that the number 107 appeared more frequently than any other number, and that the number that appears most frequently, the mode, is also a good candidate for "best number." With the second round of conversation completed (in a manner entirely consistent with the flow chart in Fig. 2), Minstrell returned to step A2: "Anybody think of another way of giving a best value?" He expected a discussion concerning the median. Either a student would suggest the median and he would have the class clarify its meaning or, if no student generated the concept, he would inject it into the conversation and ask what they thought. (This is what happened later in the lesson.)

Instead, something unusual happened. In response to Minstrell's request for "another way," a student said:

> This is a little complicated but I mean it might work. If you see that 107 shows up 4 times, you give it a coefficient of 4, and then 107.5 only shows up one time, you give it a coefficient of one, you add all those up and then you divide by the number of coefficients you have.

This comment is decidedly out of the ordinary. It is oral, so it "vanishes" rapidly. It is ambiguous – is the student referring to the total number of coefficients, eight, or the number of different coefficients, five? In the former case the student is suggesting a formula for weighted average; in the latter case the suggestion is not mathematically productive.

Before proceeding I note that there is a wide range of possible responses to the student's comment. Some teachers might not know the formula for weighted average or might not recognize the ambiguity in the student's statement; some might wish to stick to the curriculum; some might wish to explore the full ramifications of the student's suggestion, in its productive and unproductive versions.

That is, there are teachers whose response to the statement would be "That's a very interesting idea. I'll talk to you about it after class." From a teacher's perspective there are advantages and disadvantages to this response. On the one hand, it keeps the lesson on track and, if the teacher is uncertain about what the student has said, may avoid a discussion that the teacher, because of uncertainty about the content, might find uncomfortable.

Other teachers might recognize one interpretation of what the student said as being the weighted average, and present the weighted average to the class. Doing so does not take more than a few minutes, so the disruption to the teacher's lesson plan is not that great; more content is covered; and, the student is (tacitly) rewarded for a making a suggestion that led to a productive classroom discussion.

Yet other teachers might pursue the student's comment in its full complexity, exploring both interpretations of "divide by the number of coefficients you have" and seeing where they lead. Such a discussion has the potential to take a long time, especially if the teacher has the students work through the ideas. Thus, it could derail the teacher's intended agenda. However, it also makes very clear the fact that the teacher takes student ideas seriously, and it provides tangible evidence of the teacher's commitment to having the class be a community of sense making.

Minstrell chose this last path. He skillfully navigated through both interpretive paths, leading the class to a deeper understanding of the issues entailed by the student's comment. This exploration did take a significant amount of time, and the rest of the lesson was more rushed than it normally would have been (See Schoenfeld, 2010, for extensive detail).

I tell this story for two reasons. Minstrell is a case in point both of teaching in general and of teaching expertise. In what follows I describe the architecture of teachers' decision making, using Minstrell as an example. Then I point to relevant issues in the study of expertise.

The fundamental argument I make in Schoenfeld (2010) is that teachers' in-the-moment decision making is a function of their orientations (their beliefs, values, preferences, etc.), their goals (which are established in the light of their orientations), and their resources (especially their knowledge). I argue that if enough is known about each of these for a specific teacher in a specific context, one can explain that teacher's actions on a line-by-line basis. Moreover, teachers' actions are typically of two kinds: the application of well-established routines to deal with familiar and expected circumstances, and consequential in-the-moment decision making (and possibly the choice of new routines) when unexpected circumstances occur.

Minstrell's hour-long lesson, described in Schoenfeld (2010) along with a number of other cases, provides perfect exemplification of these ideas. Minstrell entered the class with a lesson plan that had been constructed in the light of his orientations toward physics. Having taught this lesson numerous times in the past, Minstrell had a well-articulated lesson image. Indeed, through the first parts of the lesson, Minstrell's resources, goals, and orientations play out as planned. Then comes a consequential moment, when the student makes the unexpected comment.

As noted above, there is a wide range of possible responses to the student's comment. Minstrell's choice of response – *every* teacher's choice of response – is fundamentally shaped by his orientations. For Minstrell, having his classroom function as a community of inquiry and honoring sense-making comments from his students are very high priorities. Thus, Minstrell will pursue the student's question, even though there is a cost in terms of time. In terms of formal modeling, Minstrell establishes a new top-level goal: pursue the implications of the student's comment. The next (actually concurrent) question is, how will he do so? There, Minstrell's resource (knowledge) base is critically important; he must choose among the alternatives he knows to be available. In his case, the routine described in Fig. 2 is a preferred modus operandi; it too is consistent with his orientations, in that it solicits student ideas and engages them actively in the act of sense-making. Thus, Minstrell's highest priority goals for the time being become (a) to clarify the student's statement and (b) to pursue the implications of the clarifications; he pursues them using the interactive routine in Fig. 2. Once he has done so, he has achieved these new (emergent) goals. With the unexpected topic taken care of, the highest priority goal is then the continuation of the discussion of "best number." The class has completed its discussions of average (mean) and mode, so his next goal is to pursue the discussion of median. He is back on familiar territory, and uses a familiar routine to pursue it.

My argument *in general* is that teacher's in-the-moment classroom decision-making can be characterized, on a moment-by-moment basis, using the theoretical constructs illustrated above. That is, any teacher's decision making in any particular instructional context can be modeled as a function of that teacher's orientations, goals, and resources.[3] That being the case, the questions pertaining to (any context-specific definition of) teaching expertise are (a) what orientations, goals and resources represent that conception of teaching expertise, and (b) how to catalyze their growth.

Let me turn to (a), the question of expertise, and use Minstrell as a starting point for discussion. First, in line with the comments in the first part of this chapter, I note that my characterization of Minstrell as an expert is a reflection of my values as a researcher (and the values of others, notably the panel that awarded Minstrel the Presidential Award for Excellence in Science Teaching). There are certainly observers who would find Minstrell's extended "detour" to address the student's comments to be time poorly spent; and there are some who would find his questioning strategies to be a poor use of time (Recall the colleague who told me that "telling" is much more efficient when it comes to giving students information). Be that as it may, Minstrell meets my definition of expertise. So, let us examine (a small subset of) his orientations, goals, and beliefs.

Minstrell's orientations are fundamentally concerned with the idea that physics is a sense-making domain, and that students should experience it as such. His classroom actions (and his goals for what his students will experience) are predicated on these orientations. Such orientations come at many levels. For example, he actively engages his students in sorting through complex issues. Other teachers wouldn't (One memory that has stuck with me for more than 25 years is of a discussion with a teacher who took a very procedural approach to the class I was observing, giving student step-by-step instructions on how to solve problems. I asked him, "Have you ever thought of giving the students a challenging problem without instructions on how to solve it, and seeing what they would do with it?" His response was, "Not these students. It would just confuse them." That teacher's orientations resulted in his students being deprived of sense making experiences). In short, Minstrell has a set of orientations that are consistent with my view, as a teacher and researcher,

[3] Of course, it is unreasonable to expect the reader to accept this argument on the basis of the brief discussion given in this chapter. See Schoenfeld (2010) for extended documentation of this claim.

of what an expert teacher's orientations should be. These orientations lead to the establishment of certain goals – to have the class discuss complex issues, to sort through them, etc. But, goals are not enough. In Minstrell's case, his resource base is what allows him to meet those goals. Where other teachers might have to work hard to understand and disambiguate the student's unexpected comment, Minstrell's content knowledge enabled him to recognize (and then pursue) the entailments of that comment without difficulty. He then had access to a number of productive teaching routines. He could have lectured on the weighted average, for example; but he preferred to have the content emerge from classroom discussion, and he had the resources (the routine in Fig. 2, combined with his content knowledge, which informed the choices he made while enacting the routine in Fig. 2). In short, Minstrell's expertise is a function of his expert orientations, goals, and resources.

It is worth noting that these expert orientations, goals, and resources are all tightly linked, and they develop slowly (The literature on expertise indicates that in every domain, expert-level proficiency takes between 5,000 and 10,000 h to develop). A teacher might aspire to conducting an interactive sense-making classroom of the type Minstrell conducts, but if the teacher lacks the content knowledge or the fluency in using routines such as that described in Fig. 2, the teacher's classroom actions will fall short of his or her aspirations. And, developing that resource base takes time. Thus, a program of moving teachers toward (one's vision of) expertise must, of necessity, focus on knowledge, goals and orientations simultaneously; and it must expect to take some time to succeed.

Discussion

The two main sections of this chapter offer two views, at very different grain sizes, of the processes of conducting research on teacher expertise. My purpose in the first section was to problematize the enterprise – to show that the ways that researchers frame and execute their studies of expertise depends on the researchers values and orientations, and that there are many places in the conduct of such research where tacit assumptions relating to the researchers' conceptual frameworks, or their measures, are highly consequential. In particular, any assumptions about teaching expertise that are not grounded in proof of enhanced student performance are just that – assumptions. I share the assumptions of many of the authors in this book, but I also recognize that until I can "connect the dots" and provide an evidential link between particular teacher actions and that teacher's students' enhanced learning, I am acting on the basis of (well-founded) assumptions.

There is a substantial body of evidence to show that curricula that focus on sense making in mathematics result in enhanced student performance: students who study from such curricula tend to do as well on tests of skills as students who study from skills-oriented curricula, and tend to outperform those students on tests of conceptual understanding and problem solving (ARC Center, 2003; Schoenfeld, 2007b; Senk & Thompson, 2003). Thus, there is good reason to believe that the characterizations of expertise that typify this volume do indeed lead to enhanced student learning. But, as a field, we need to be explicit about our assumptions and the chains of evidence that justify our conclusions. Along those lines, I stress that the nature of the measures used to characterize both teacher performance and student learning is critically important: the inferences one can draw depend critically on the character of those measures and on the generalizations one can legitimately make on the basis of them (I am currently engaged in a project with Robert Floden in which we are trying to develop a coding scheme to capture what we believe is proficient teaching, and link it to student performance on student interviews and tasks that provide students with the opportunity to demonstrate robust mathematical understandings. Given the complexities of the enterprise, I have substantial admiration for the achievements of the authors of the chapters in this volume).

The second main part of this chapter offered the barest outline of a case study (backed by numerous publications, e.g., Schoenfeld, 1998, 2002, 2008, 2010) suggesting the main aspects of teachers' in-the-moment decision making – and thus, the things that need to grow (in the "right" directions) if teachers are to evolve into experts. Attention to all these dimensions is necessary. Having the "right" orientations is critical, for a teacher's orientations shape that teachers' goals; but, teachers need to employ resources (knowledge, routines, etc.) in the service of those goals, so goals in the absence of such resources are unfulfillable (This is the reason for the consistently large difference between the number of teachers who profess "student-centered" teaching beliefs and who actually teach in a student-centered manner). Thus, a major next step in research on helping teachers develop the kinds of expertise described in this volume will be to chart the growth and change of teachers' orientations, goals, and knowledge as they have the kinds of experiences intended to help them develop expertise. Needless to say, the cautions discussed in the first part of this chapter will apply to that research as well.

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Reflections and Future Prospects

Gabriele Kaiser and Yeping Li

Abstract This concluding chapter summarizes what can be learnt from this book concerning the concept and nature of expertise, and how expertise is theoretically conceptualized and empirically measured. The chapter discusses differences between the Eastern and Western perspectives on expertise, and exemplifies their different orientations of teaching towards the subject of mathematics and the individual students. Furthermore, we discuss and analyze the current state-of-art research on expertise and possible research directions for the future.

Keywords Professional knowledge of teachers · Expertise · Novice-expert-model · Cultural differences

It is now common knowledge that teacher expertise in mathematics instruction varies individually and affects teaching performance. However, there is still very limited understanding of the nature of teacher expertise in mathematics instruction. As teachers and teaching have become recognized as a vital part for enhancing students' academic achievement, understanding the nature of teachers' expertise is an unavoidable issue. In fact, with ever-increasing emphasis in current worldwide educational efforts to improve students' mathematics learning, those who care about finding ways of improving mathematics classroom instruction and teacher education have stressed the importance of knowing and understanding what is needed for making and developing expert-like mathematics. Towards this end, this book makes a much-needed and important contribution to the international community of mathematics education and teacher education.

Understanding and evaluating teacher expertise has been a perplexing issue in many education systems for years. Taking an international perspective to examine teacher expertise that is appreciated in the East and the West should help advance our understanding of the issue. For example, existing cross-national studies have

G. Kaiser (✉)
Faculty of Education, Psychology, Human Movement, Didactics of Mathematics, University of Hamburg, Von-Melle-Park 8, Hamburg 20146, Germany
e-mail: gabriele.kaiser@uni-hamburg.de

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revealed remarkable differences between Japan, Hong Kong, and the United States in mathematics classroom instruction (Hiebert et al., 2003; Stigler & Hiebert, 1999), between Mainland China and the United States in teachers' knowledge of mathematics for teaching (e.g., Ma, 1999). In particular, Ma's study (1999) revealed that Chinese elementary teachers had a profound understanding of the fundamental mathematics they teach, whereas US teacher participants lacked a strong knowledge base in mathematics. It is in the same spirit of expanding possibilities for learning from many more education systems that this book succeeds in helping readers develop a better understanding of teacher expertise that is valued in different systems.

In this concluding chapter, we will discuss what we can learn from this book and provide our prospects for research development in the future. In particular, consistent with several issues being highlighted in the introduction chapter, we will focus on (1) some similarities and differences between Eastern and Western perspectives on teachers' expertise or its specifications and (2) cultural differences in viewing expert teachers and teaching. Based on these summaries and discussions, we will then suggest directions for future research and the development of teaching practice with regard to teacher expertise in mathematics instruction.

Conceptualizing and Specifying Teacher Expertise

In her fundamental chapter on theoretical perspectives and trends in research on expertise, chapter "Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise" by Chi (this book) develops an overall framework for describing and conceptualizing the nature of expertise from a general perspective. She emphasizes that in contrary to what had been presumed so far, the amount of knowledge is not an important aspect for discriminating experts from novices in solving knowledge-lean tasks: the domain-relevant knowledge and its structure are actually very important for successfully solving complex and knowledge-rich tasks. Especially the representation of the domain-relevant knowledge seems to be a distinctive feature of expertise. Chi points out that teacher experts will generally have a normatively correct and deeper representation of the topics they teach in comparison to novice teachers, who will have a more shallow or incomplete representation of the teaching topics and other important aspects.

A more concrete approach to elaborate on the construct of teacher expertise is the approach by Blömeke (2002), who describes the importance and impacts of (subject-related) knowledge on teacher expertise and its development. She describes teachers with expertise as experts, can develop mental models of the situation in the classroom by referring to previous knowledge especially under enormous time pressure. These models allow them to select the relevant information out of a huge amount of information, to process it, and to come to decisions of different kinds, to recognize problems and to react sensitively and successfully from a pedagogical point of view. On the level of the cognitive processes involved, this means the inclusion of components of expertise such as rapid judgment in the situation, combining and structuring of events observed in the classroom intercourse into a few categories, and the willingness to change the course of classroom interaction when necessary. This description has the following consequences on the teaching profession: it is consensus that teachers with expertise have a more adequate level of knowledge; that they are able to structure the teaching-and-learning-processes more adequately and in a goal oriented manner; that they see the teaching-and-learning-process in a holistic way, combining the subject's teaching requirement, the organization of the teaching-and-learning-process, and the students' needs as a whole using more abstract concepts. Additionally, the knowledge of teachers with expertise is more coherent and organized according to the teaching situation, not according to single students (Bromme, 1992). Teaching expertise includes the transfer of learned rule-knowledge to more complex if-then-rules, taking information on the teaching situation into consideration.

Consistent with the above conceptualization of teacher expertise by Chi (this book) and Blömeke (2002), teachers' sound subject knowledge has been taken as an important part of their expertise in mathematics instruction in many chapters of this book. In fact, it becomes a common feature, which is highlighted by contributions from the East and the West in one way or another. Yet, there are also notable differences both across and within the East and West in terms of the ways of specifying knowledge as part of teacher expertise. In general, contributions from the East tend to focus on teachers' instructional practices to discuss and specify teachers' expertise in a more holistic way. Thus, teachers' knowledge as part of their expertise is identified by analyzing teachers' instructional practices, and is not taken as a stand-alone facet but rather as an integrated aspect of what teachers are capable of doing. Possible differences across the contributions from the East can be found in the nature of instructional practices that have been analyzed, varying from classroom teaching (chapter "Characterizing Expert Teaching in School Mathematics in China – A Prototype of Expertise in Teaching Mathematics" by Li, Huang, & Yang, this book; chapter "Expertise of Mathematics Teaching Valued in Taiwanese Classrooms" by Lin & Li, this book; chapter "Exploring Korean Teacher Classroom Expertise in Sociomathematical Norms" by Pang, this book), textbook use (chapter "The Japanese Approach to Developing Expertise in Using the Textbook to Teach Mathematics" by Takahashi, this book), to mathematics department heads' perceptions (chapter "Perceptions of School Mathematics Department Heads on Effective Practices for Learning Mathematics" by Lim-Teo, Chua, & Yeo, this book). In contrast, contributions from the West likely examine and analyze teachers' knowledge as an important, yet stand-alone aspect of teacher expertise. Teacher expertise is regarded in an analytical way as containing different components, including knowledge, beliefs, and teaching performance. Even further, chapter "Teacher Expertise Explored as Mathematics for Teaching" by Simmt (this book) conceptualizes and specifies teacher expertise as mathematics for teaching, which differs from the type of mathematics that mathematicians use and talk about. The differences in conceptualizing teacher expertise and the nature of mathematics knowledge in teacher expertise suggest different perspectives and approaches that can possibly be used to examine teacher expertise.

Apart from the central importance of structured knowledge as a key aspect of expertise, Chi describes three constructs emphasized in current research on expertise in her chapter. In particular, the first construct is *deliberate practice*, which is defined as “expanding intentional efforts to achieve further improvement through focused, concentrated, well-structured, programmatic, and goal-oriented practice. Moreover, the goals of practice are set to go beyond one’s current level of achievement, and evaluated by identification of errors, and so on.” (chapter “Theoretical Perspectives, Methodological Approaches, and Trends in the Study of Expertise” by Chi, this book) This construct is specifically crafted to explain why some teachers can become real experts but others cannot. This question is of great interest to cognitive psychologists and educational researchers. Yet, the construct itself does not provide a detailed account for the development of structured knowledge in expertise growth.

There is a general consensus that the development of expertise is a tedious learning process, which may last for ten years. The learning theories which serve as a basis to account for expertise development are more and more orientated towards constructivism and emphasizing the domain-specificity of the knowledge. For example, Blömeke adapts the model of Neuweg (1999) in order to describe the development from novice to expert teachers (Blömeke, 2002, p. 81; own translation). This model describes the development of the teachers’ expertise concerning various aspects identified as being important for teachers’ expertise in various approaches. For example, how the teaching situation is perceived or how the behavior of a teacher is determined. With this model a detailed description of a possible development from novices to experts is provided referring to the different theoretical models on the development of expertise (Fig. 1).

Concerning the development of expertise, Blömeke (2002) describes teachers at the beginning of their teaching practice as being in transition from competence level to mastery level, which means in detail to broaden the achieved competencies through the development of everyday routines. These everyday routines allow the experts to perceive the classroom situation as a whole and not only as consisting of

	level	level	competence	level	level
Considered elements	Context-free	Context-free and *situational*	Context-free and situational	Context-free and situational	Context-free and situational
Sense for the essentials	No	No	Worked out	*Immediate*	Immediate
Perception of the whole situation	Analytical	Analytical	Analytical	*Holistic*	Holistic
Determination of the behavior	By rules	By rules and guidelines	Through extensive planning	Through limited planning	*Intuitive*

Fig. 1 Model on the development of expert teachers (Blömeke, 2002)

single students. This enables the experts to recognize the essential aspects of a situation immediately without long analyses and it allows experts to teach competently without having planned each detail of a lesson carefully in advance. This aspect is emphasized in the study by Shimizu (2008) where novice teachers were compared to experienced teachers.

In her framework, Blömeke (2002) emphasizes the necessity of content-related knowledge and refers to the classification developed by Shulman (1985), which comprises subject matter content knowledge, pedagogical content knowledge, pedagogical, and curricular knowledge. As discussed above, the necessity of high levels of knowledge in these areas as a prerequisite for expertise is emphasized in the various chapters of this book.

Likewise, Schoenfeld and Kilpatrick (2008) developed a framework for proficiency in teaching mathematics, consisting of a set of dimensions, which they consider indispensable for expertise in mathematics teaching and which are strongly related to teaching and learning processes. They name broad and in-depth, sound knowledge of school mathematics, different heuristic strategies and meta-cognitive control strategies as well as a growing competence to reflect teaching-and-learning processes as dimensions of expertise. Knowledge of school mathematics, which should be both deep and broad, plays a central role in their theoretical approach. The breadth of knowledge covers the multiple ways of conceptualizing the relevant mathematics as well as of representing it in various ways, of understanding the key aspects of each topic, and of seeing connections to other topics at the same level. The deepness refers to knowledge on the curricular origin and further conceptual development of the content. Schoenfeld and Kilpatrick (2008) describe the outcome of this kind of knowledge: it allows proficient teachers to prioritize and organize content in such a way that students are introduced to basic/important ideas and not lost in an abundance of details. Furthermore, this knowledge allows teachers to respond flexibly to questions posed by students. This kind of knowledge is called "knowledge of mathematics for teaching" by Ball, Thames, and Phelps (2007) and according to Schoenfeld and Kilpatrick (2008) it involves "more than 'just' knowing the mathematics in the curriculum" (p. 322). Proficient teachers or expert teachers can respond more flexibly to the students' questions than novices. Furthermore, expert teachers are able to craft and manage learning environments, are able to develop classroom norms, and support classroom discourse in the sense of teaching for understanding. To summarize, teachers with expertise show a greater consciousness towards mathematical learning processes and their content as well as towards the development of the students' thinking processes than novices, which is emphasized by Llinares and Krainer (2006) as well.

Cultural Differences

The aforementioned differences in conceptualizing teacher expertise between the East and the West may also be linked to the unspoken difficulty of identifying expert teachers between these two cultures. After having taken a careful read of the corresponding chapters in this book, readers can notice that not every chapter focuses on expert teachers. In fact, identifying expert teachers can pose a bigger challenge to researchers in the West than in the East, as teaching is regarded a private practice in the West but not in the East (Kaiser & Vollstedt, 2007; Li & Li, 2009). Thus, it is more understandable that researchers in the West take a more hypothetical approach to conceptualize teacher expertise, which is regarded as being necessary to be an expert teacher. In contrast, it is relatively easier for researchers in the East to first identify those teachers who are expected to have expertise and then analyze their expertise in a holistic or analytical way. It presents a procedure similar to many studies on expertise in psychology. Cross-cultural differences in teaching practice and people's views of teaching practices suggest an important dimension when examining and understanding teacher expertise in different cultural contexts.

In different chapters describing an Eastern or Western background, another apparent remarkable difference is the description on the various roles of expert teachers. Chapter "Images of Expertise in Mathematics Teaching" by Russ, Sherin, and Sherin (this book) develop four metaphors of expertise in their chapter:

- the role of teachers as diagnosticians, which refers mainly to the teacher's ability to interpret students' thinking and students' strategies;
- the role of teachers as conductors, shaping the classroom discourse and using classroom norms for communicating about mathematical ideas;
- the role of teachers as architects selecting cognitively demanding tasks;
- the role of teachers as river guides, which involves improvisation, deciding on the spot how to unfold the lesson;

This description of expertise clearly focuses on the learning process and the individual student, his or her learning and the organization of learning processes in order to promote the students' learning.

A comparison to the different aspects of expertise from an Eastern perspective shows clear differences. Yang (2010) differentiates in his study on expert teachers in China multiple roles, which have to be played by an expert teacher:

- expert in teaching, i.e., organizing good teaching processes;
- researcher, i.e., conduct teaching research and publish papers in professional and academic journals;
- teacher educator, i.e., mentor non-expert teachers and facilitate non-expert teachers' professional development;
- scholar, i.e., an expert teacher should have profound knowledge base in mathematics and other areas;
- expert in examination, i.e., have the ability to pose examination problems;
- exemplary model for students and colleagues.

Similar descriptions are developed by Li, Huang, and Yang in their study on expert teachers in this book, in which they describe that expert teachers should serve as moral role models who stand for culturally valued moral characteristics and expertise for others to follow. They continue with the function of expert teachers as researchers and elaborate that it is important to engage in research and write scientific papers in order to be identified as expert teachers. Being an expert teacher needs to contribute to the improvement of other teachers' academic level and teaching ability, which includes the teacher educator role. From a Western perspective, it is surprising that scientific research is widely required, especially as this means that expert teachers must have written a monograph or more than three research papers published in journals at the provincial level or beyond. They add that an expert teacher should be the leader of the teaching subject at the municipal or county level, who has shown high quality teaching with public and exemplary lessons, and who should have won a prize at a teaching contest at the national level. According to Huang and Li (2009) the teacher promotion system, commonly practiced in China, provides a platform for teachers to value and pursue mathematics classroom instruction excellence. Yang (2009) emphasizes that in contrast to Western culture, where the policy of closed classroom doors is followed (Kaiser & Vollstedt, 2007), the classroom teaching of Chinese mathematics teachers is open for colleagues' observation, studies and discussions, mainly based on the Teaching Research Groups.

Comparing Eastern and Western perspectives on expertise as related to expert teachers' roles, one can describe the Eastern perspective on teacher expertise as more holistic, aiming for a systemic change of the teaching-and-learning processes in school by strengthening teachers as researchers and developing expertise in scientific work. Furthermore the holistic view in the East is accompanied by the public recognition of expert teachers, who are responsible for the development of mathematics education on a broader basis including not only teachers, but curricular aspects as well. The Western perspective is clearly focused on the teaching-and-learning process within the classroom, where experienced teachers shall display their expertise especially in interactions with the students. Characteristic for the Western approach to expertise is the focus on the individual student, who is put into the centre of reflections and actions; the promotion of learning processes of individual students is a major goal of the classroom activities.

Therefore, the differences between the Eastern and the Western approaches concern the different foci on levels of change: while in the Eastern conception a change on a systemic level is desirable, the Western conception refers to changes on the local level. These differences with systemic change focusing on groups of actors in the Eastern conception and the local change with a focus on the individual student relate to strong cultural differences, which are described by cultural psychology as orientation towards collectivistic oriented countries in contrast to individualistic oriented countries. In collectivistic oriented countries, societal actions are seen as commitment against social networks, whereas in individualistic oriented countries the conviction that societal action is a result of freely negotiated contracts is dominant (Hofstede, 1980, 2001; Mascolo & Li, 2004). Transferring this differentiation to individualistic and collectivistic orientation towards education implies (cf. Triandis, 1995) that in collectivistic oriented countries the role of social relations is more strongly emphasized in the learning process: according to this theoretical approach students rather learn due to their commitment towards their teachers, their family, and the social group, who conversely have the responsibility to provide every necessary support. Failing in school is in this social paradigm attributed to a lack of effort and the required changes rather aim for higher efforts of the students and not for a change of the schooling framework to the benefit of the individual student. In individualistic oriented countries, students are more strongly seen as autonomous subjects, who learn on an individual basis, mainly independent from other individuals. Lacking learning success is explained by referring to inadequate social conditions such as too difficult tasks, poor explanation skills of the teachers or, in general, poor lessons and faults by the teacher. The changes required refer to a change in these social conditions such as changing the teaching styles or the lesson structure, the tasks, or even the school system, but do only seldom refer to the individual student.

These differences in the cultural and psychological paradigms underlying Eastern and Western educational approaches seem to be adequate to explain at least partly the strong holistic focus on expertise in Eastern countries by embracing the professional development of whole teacher groups and in general the educational system, including work on curricular aspects. In addition, the strong individualistic orientation of Western cultures, which expects teachers to provide effective learning environments, good classroom management and so on, leads to a conceptualization of expertise which focuses on the individual student's teaching-learning-processes. Cai, Wang, Wang, and Garber (2009) confirm these results from another perspective: namely effective mathematics teaching from the teachers' perspective. They report that most Asian teachers are more mathematics content-oriented, they emphasize that an effective teacher should understand the content thoroughly and organize teaching in well-structured lessons. This is in strong contrast to teachers from America and Europe, who tend to be more person-oriented and emphasize that an effective teacher should be passionate about mathematics, he or she should have good listening skills and provide enough room and time for students to learn for understanding.

Although there are these strong culturally-based differences between Eastern and Western countries, the cross-national comparative results of the OECD-study TALIS (Teaching and Learning International Survey) point out significant commonalities as well as differences in the teachers' beliefs about the nature of teaching and learning between the participating countries that reveal a few unexpected results (Schmidt et al., 2007). For example, Western countries emphasize the individual student, on the other hand – rather unexpectedly – countries sharing Confucian traditions follow the same constructivist ideas.

To summarize, there are apparently strong cultural differences concerning the description of expertise in mathematics education. These might explain the different ways of implementing expertise in mathematics education.

Areas of Future Research Directions

The above discussion highlights several aspects to be learned from this book. While we can learn much from reading the book, many more questions can actually be raised about teacher expertise. Here we would like to share three areas of further research with the readers.

The first area relates to teacher expertise itself. The aforementioned similarities and differences in the conceptualization of expertise between the East and West suggest some important aspects that need further examination. In particular, although sound subject knowledge is commonly regarded as being an important part of teacher expertise, it remains unclear what exactly expert or experienced teachers know about mathematics. Further research is needed to examine the expert teachers' level of knowledge about school mathematics, and to find out whether it is important for them to also know advanced mathematics. Different from Ma's study that compares Chinese and US teachers' mathematics knowledge, we suggest to examine subject knowledge in expertise as a consistently changing and dynamic body of knowledge. The nature of mathematics subject knowledge in teacher expertise may vary dramatically between novice and expert teachers, and it is important for us to know and understand what kind and level of structured knowledge expert teachers need and how it is developed.

In Chi's description of three current constructs on expertise research, the second construct is *adaptive expertise*. Chi (this book) describes adaptive expertise as "the notion of knowing not only how to execute or apply a procedural skill, but an adaptive expert is one who also has conceptual understanding of that skill". She emphasizes that adaptive experts understand the procedures or the skills in a profound way, so that they are able to generalize their skills to other non-routine problems. In order to acquire a conceptual understanding, it seems to be necessary to reflect and self-explain the solution of the problem during the problem solving process, which leads to a deeper understanding. This emphasis and high importance of reflection and metacognition is in accordance with new trends in mathematics education, which stress the necessity of metacognition for higher-order thinking processes. Chi's discussion suggests adaptive expertise in mathematics instruction as an important area of research studies. In mathematics education research, a growing number of studies investigates how to help students develop adaptive expertise in problem solving (e.g., Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009), which might help to develop similar studies for teachers. However, much remains unclear about the nature of adaptive expertise in mathematics instruction. Research on adaptive expertise becomes especially important as it can help to understand what is needed for being a real expert teacher and not a routine expert or just an experienced teacher.

Looking into areas in which further research is needed, we take the development of expertise and its wider promotion as one more area of research. As a continuation of this book, Yeping Li and Ruhama Even will edit a special issue of ZDM – The International Journal on Mathematics Education at the end of 2011. The issue on "Approaches and Practices in Developing Teachers' Expertise in Mathematics Instruction" begins with the observation that while educational research has dramatically increased its emphasis on teachers and teaching practice over the past few decades (e.g., Sikula, 1996; Townsend & Bates, 2007), the need for improving teachers' expertise has emerged ever-increasingly in various ways. This includes the need for practicing teachers' continuous knowledge and practice development in mathematics and pedagogy, teachers' training for undertaking and implementing changes in the curriculum and instruction, and teachers' professional promotion. While various approaches and practices (e.g., lesson study in Japan, teaching research group and apprenticeship practice in China, and video case based learning in the US) have been generated and implemented to address different needs across educational systems, much remains to be learned about specific approaches and practices that have been developed and used effectively. Knowing and understanding effective approaches and practices for developing practicing teachers' expertise in mathematics instruction have become especially important to those who care about the ways of improving mathematics classroom instruction and broad teacher professional education. Li and Even emphasize, that until today, researchers have not come to a consensus on how to define and assess teachers' expertise. In contrast, as described in this chapter, distinct differences between the various approaches common in the Eastern and the Western debate exist (Lappan & Li, 2002). This themed issue is proposed as a sequel to this collaborative book publication on teachers' expertise in mathematics instruction, for the international mathematics education community to develop and share relevant research in the much-needed topic area of approaches and practices utilized to develop such expertise. Hopefully, this book will serve as a starter for rich and extensive debates on the definition and development of expertise, how to promote it, and will lead to reflective ideas on its further embedding in joint cross-cultural endeavors on expertise in mathematics teaching.

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