

# Pedestrian, Crowd and Evacuation Dynamics

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## Glossary

**Collective intelligence** Emergent functional behavior of a large number of people that results from *interactions* of individuals rather than from individual reasoning or global optimization.

**Crowd** Agglomeration of many people in the same area at the same time. The density of the crowd is assumed to be high enough to cause continuous interactions with or reactions to other individuals.

**Crowd turbulence** Unanticipated and unintended irregular motion of individuals into different directions due to strong and rapidly changing forces in crowds of extreme density.

**Emergence** Spontaneous establishment of a qualitatively new behavior through non-linear interactions of many objects or subjects.

**Evolutionary optimization** Gradual optimization based on the effect of frequently repeated random mutations and selection processes based on some success function (“fitness”).

**Faster-is-slower effect** This term reflects the observation that certain processes (in evacuation situations, production, traffic dynamics, or logistics) take more time if performed at high speed. In other words, waiting can often help to coordinate the activities of several competing units and to speed up the average progress.

**Freezing-by-heating effect** Noise-induced blockage effect caused by the breakdown of direction-segregated walking patterns (typically two or more “lanes” characterized by a uniform direction of motion). “Noise”

means frequent variations of the walking direction due to nervousness or impatience in the crowd, e. g. also frequent overtaking maneuvers in dense, slowly moving crowds.

**Panic** Breakdown of ordered, cooperative behavior of individuals due to anxious reactions to a certain event. Often, panic is characterized by attempted escape of many individuals from a real or perceived threat in situations of a perceived struggle for survival, which may end up in trampling or crushing of people in a crowd.

**Self-organization** Spontaneous organization (i. e. formation of ordered patterns) not induced by initial or boundary conditions, by regulations or constraints. Self-organization is a result of non-linear interactions between many objects or subjects, and it often causes different kinds of spatio-temporal patterns of motion.

**Social force** Vector describing acceleration or deceleration effects that are caused by social interactions rather than by physical interactions or fields.

## Definition of the Subject

The modeling of pedestrian motion is of great theoretical and practical interest. Recent experimental efforts have revealed quantitative details of pedestrian interactions, which have been successfully cast into mathematical equations. Furthermore, corresponding computer simulations of large numbers of pedestrians have been compared with the empirically observed dynamics of crowds. Such studies have led to a deeper understanding of how collective behavior on a macroscopic scale emerges from individual human interactions. Interestingly enough, the non-linear interactions of pedestrians lead to various complex, spatio-temporal pattern-formation phenomena. This includes the emergence of lanes of uniform walking direction, oscillations of the pedestrian flow at bottlenecks, and the formation of stripes in two intersecting flows. Such self-organized patterns of motion demonstrate that efficient, “intelligent” collective dynamics can be based on simple, local interactions. Under extreme conditions, however, coordination may break down, giving rise to critical crowd conditions. Examples are “freezing-by-heating” and “faster-is-slower” effects, but also the transition to “turbulent” crowd dynamics. These observations have important implications for the optimization of pedestrian facilities, in particular for evacuation situations.

## Introduction

The emergence of new, functional or complex collective behaviors in social systems has fascinated many scientists. One of the primary questions in this field is how cooper-

ation or coordination patterns originate based on elementary individual interactions. While one could think that these are a result of intelligent human actions, it turns out that much simpler models assuming automatic responses can reproduce the observations very well. This suggests that humans are using their intelligence primarily for more complicated tasks, but also that simple interactions can lead to intelligent patterns of motion. Of course, it is reasonable to assume that these interactions are the result of a previous learning process that has optimized the automatic response in terms of minimizing collisions and delays. This, however, seems to be sufficient to explain most observations.

In this contribution, we will start with a short history of pedestrian modeling and, then, introduce a simplified model of pedestrian interactions, the “social force model”. Furthermore, we will discuss its calibration using video tracking data. Next, we will turn to the subject of crowd dynamics, as one typically finds the formation of large-scale spatio-temporal patterns of motion, when many pedestrians interact with each other. These patterns will be discussed in some detail before we will turn to evacuation situations and cases of extreme densities, where one can sometimes observe the breakdown of coordination. Finally, we will address possibilities to design improved pedestrian facilities, using special evolutionary algorithms.

## Pedestrian Dynamics

### Short History of Pedestrian Modeling

Pedestrians have been empirically studied for more than four decades [1,2,3]. The evaluation methods initially applied were based on direct observation, photographs, and time-lapse films. For a long time, the main goal of these studies was to develop a *level-of-service concept* [4], *design elements* of pedestrian facilities [5,6,7,8], or *planning guidelines* [9,10]. The latter have usually the form of *regression relations*, which are, however, not very well suited for the prediction of pedestrian flows in pedestrian zones and buildings with an exceptional architecture, or in challenging evacuation situations. Therefore, a number of simulation models have been proposed, e.g. *queuing models* [11], *transition matrix models* [12], and *stochastic models* [13], which are partly related to each other. In addition, there are models for the *route choice behavior* of pedestrians [14,15].

None of these concepts adequately takes into account the self-organization effects occurring in pedestrian crowds. These are the subject of recent experimental studies [8,16,17,18,19,20]. Most pedestrian models, however, were formulated before. A first modeling approach that

appears to be suited to reproduce spatio-temporal patterns of motion was proposed by Henderson [21], who conjectured that pedestrian crowds behave similar to gases or fluids (see also [22]). This could be partially confirmed, but a realistic gas-kinetic or fluid-dynamic theory for pedestrians must contain corrections due to their particular interactions (i. e. avoidance and deceleration maneuvers) which, of course, do not obey momentum and energy conservation. Although such a theory can be actually formulated [23,24], for practical applications a direct simulation of *individual* pedestrian motion is favorable, since this is more flexible. As a consequence, pedestrian research mainly focuses on *agent-based models* of pedestrian crowds, which also allow one to consider local coordination problems. The “social force model” [25,26] is maybe the most well-known of these models, but we also like to mention *cellular automata* of pedestrian dynamics [27,28,29,30,31,32,33] and *AI-based models* [34,35].

### The Social Force Concept

In the following, we shall shortly introduce the social force concept, which reproduces most empirical observations in a simple and natural way. Human behavior often seems to be “chaotic”, irregular, and unpredictable. So, why and under what conditions can we model it by means of forces? First of all, we need to be confronted with a phenomenon of motion in some (quasi-)continuous space, which may be also an abstract behavioral space such as an opinion scale [36]. Moreover, it is favorable to have a system where the fluctuations due to unknown influences are not large compared to the systematic, deterministic part of motion. This is usually the case in pedestrian traffic, where people are confronted with standard situations and react “automatically” rather than taking complicated decisions, e. g. if they have to evade others.

This “automatic” behavior can be interpreted as the result of a *learning process* based on trial and error [37], which can be simulated with *evolutionary algorithms* [38]. For example, pedestrians have a preferred side of walking, since an asymmetrical avoidance behavior turns out to be profitable [25,37]. The related *formation of a behavioral convention* can be described by means of *evolutionary game theory* [25,39].

Another requirement is the vectorial additivity of the separate force terms reflecting different environmental influences. This is probably an approximation, but there is some experimental evidence for it. Based on quantitative measurements for animals and test persons subject to separately or simultaneously applied stimuli of different nature and strength, one could show that the behavior in

conflict situations can be described by a superposition of forces [40,41]. This fits well into a concept by Lewin [42], according to which behavioral changes are guided by so-called *social fields* or *social forces*, which has later on been put into mathematical terms [25,43]. In some cases, social forces, which determine the amount and direction of systematic behavioral changes, can be expressed as gradients of dynamically varying potentials, which reflect the social or behavioral fields resulting from the interactions of individuals. Such a social force concept was applied to opinion formation and migration [43], and it was particularly successful in the description of collective pedestrian behavior [8,25,26,37].

For reliable simulations of pedestrian crowds, we do not need to know whether a certain pedestrian, say, turns to the right at the next intersection. It is sufficient to have a good estimate what percentage of pedestrians turns to the right. This can be either empirically measured or estimated by means of route choice models [14]. In some sense, the uncertainty about the individual behaviors is averaged out at the macroscopic level of description. Nevertheless, we will use the more flexible microscopic simulation approach based on the social force concept. According to this, the temporal change of the location  $\mathbf{r}_\alpha(t)$  of pedestrian  $\alpha$  obeys the equation of motion

$$\frac{d\mathbf{r}_\alpha(t)}{dt} = \mathbf{v}_\alpha(t). \quad (1)$$

Moreover, if  $\mathbf{f}_\alpha(t)$  denotes the sum of social forces influencing pedestrian  $\alpha$  and if  $\boldsymbol{\xi}_\alpha(t)$  are individual fluctuations reflecting unsystematic behavioral variations, the velocity changes are given by the *acceleration equation*

$$\frac{d\mathbf{v}_\alpha}{dt} = \mathbf{f}_\alpha(t) + \boldsymbol{\xi}_\alpha(t). \quad (2)$$

A particular advantage of this approach is that we can take into account the flexible usage of space by pedestrians, requiring a continuous treatment of motion. It turns out that this point is essential to reproduce the empirical observations in a natural and robust way, i. e. without having to adjust the model to each single situation and measurement site. Furthermore, it is interesting to note that, if the fluctuation term is neglected, the social force model can be interpreted as a particular *differential game*, i. e. its dynamics can be derived from the minimization of a special utility function [44].

### Specification of the Social Force Model

The social force model for pedestrians assumes that each individual  $\alpha$  is trying to move in a desired direction  $\mathbf{e}_\alpha^0$

with a desired speed  $v_\alpha^0$ , and that it adapts the actual velocity  $\mathbf{v}_\alpha$  to the desired one,  $\mathbf{v}_\alpha^0 = v_\alpha^0 \mathbf{e}_\alpha^0$ , within a certain relaxation time  $\tau_\alpha$ . The systematic part  $\mathbf{f}_\alpha(t)$  of the acceleration force of pedestrian  $\alpha$  is then given by

$$\mathbf{f}_\alpha(t) = \frac{1}{\tau_\alpha} (v_\alpha^0 \mathbf{e}_\alpha^0 - \mathbf{v}_\alpha) + \sum_{\beta (\neq \alpha)} \mathbf{f}_{\alpha\beta}(t) + \sum_i \mathbf{f}_{\alpha i}(t), \quad (3)$$

where the terms  $\mathbf{f}_{\alpha\beta}(t)$  and  $\mathbf{f}_{\alpha i}(t)$  denote the repulsive forces describing attempts to keep a certain safety distance to other pedestrians  $\beta$  and obstacles  $i$ . In very crowded situations, additional physical contact forces come into play (see Subsect. “**Force Model for Panicking Pedestrians**”). Further forces may be added to reflect attraction effects between members of a group or other influences. For details see [37].

First, we will assume a simplified interaction force of the form

$$\mathbf{f}_{\alpha\beta}(t) = \mathbf{f}(\mathbf{d}_{\alpha\beta}(t)), \quad (4)$$

where  $\mathbf{d}_{\alpha\beta} = \mathbf{r}_\alpha - \mathbf{r}_\beta$  is the distance vector pointing from pedestrian  $\beta$  to  $\alpha$ . Angular-dependent shielding effects may be furthermore taken into account by a prefactor describing the anisotropic reaction to situations in front of as compared to behind a pedestrian [26,45], see Subsect. “**Angular Dependence**”. However, we will start with a **circular specification** of the distance-dependent interaction force,

$$\mathbf{f}(\mathbf{d}_{\alpha\beta}) = A_\alpha e^{-d_{\alpha\beta}/B_\alpha} \frac{\mathbf{d}_{\alpha\beta}}{\|\mathbf{d}_{\alpha\beta}\|}, \quad (5)$$

where  $d_{\alpha\beta} = \|\mathbf{d}_{\alpha\beta}\|$  is the distance. The parameter  $A_\alpha$  reflects the *interaction strength*, and  $B_\alpha$  corresponds to the *interaction range*. While the dependence on  $\alpha$  explicitly allows for a dependence of these parameters on the single individual, we will assume a homogeneous population, i. e.  $A_\alpha = A$  and  $B_\alpha = B$  in the following. Otherwise, it would be hard to collect enough data for parameter calibration.

**Elliptical Specification** Note that it is possible to express Eq. (5) as gradient of an exponentially decaying potential  $V_{\alpha\beta}$ . This circumstance can be used to formulate a generalized, elliptical interaction force via the potential

$$V_{\alpha\beta}(b_{\alpha\beta}) = AB e^{-b_{\alpha\beta}/B}, \quad (6)$$

where the variable  $b_{\alpha\beta}$  denotes the semi-minor axis  $b_{\alpha\beta}$  of the elliptical equipotential lines. This has been specified according to

$$2b_{\alpha\beta} = \sqrt{\frac{(\|\mathbf{d}_{\alpha\beta}\| + \|\mathbf{d}_{\alpha\beta} - (\mathbf{v}_\beta - \mathbf{v}_\alpha)\Delta t\|)^2}{-\|(\mathbf{v}_\beta - \mathbf{v}_\alpha)\Delta t\|^2}}, \quad (7)$$

so that both pedestrians  $\alpha$  and  $\beta$  are treated symmetrically. The repulsive force is related to the above potential via

$$\begin{aligned} f_{\alpha\beta}(\mathbf{d}_{\alpha\beta}) &= -\nabla_{\mathbf{d}_{\alpha\beta}} V_{\alpha\beta}(b_{\alpha\beta}) \\ &= -\frac{dV_{\alpha\beta}(b_{\alpha\beta})}{db_{\alpha\beta}} \nabla_{\mathbf{d}_{\alpha\beta}} b_{\alpha\beta}(\mathbf{d}_{\alpha\beta}), \end{aligned} \quad (8)$$

where  $\nabla_{\mathbf{d}_{\alpha\beta}}$  represents the gradient with respect to  $\mathbf{d}_{\alpha\beta}$ . Considering the chain rule,  $\|z\| = \sqrt{z^2}$ , and  $\nabla_z \|z\| = z/\sqrt{z^2} = z/\|z\|$ , this leads to the explicit formula

$$\begin{aligned} f_{\alpha\beta}(\mathbf{d}_{\alpha\beta}) &= Ae^{-b_{\alpha\beta}/B} \cdot \frac{\|\mathbf{d}_{\alpha\beta}\| + \|\mathbf{d}_{\alpha\beta} - \boldsymbol{\gamma}_{\alpha\beta}\|}{2b_{\alpha\beta}} \\ &\quad \cdot \frac{1}{2} \left( \frac{\mathbf{d}_{\alpha\beta}}{\|\mathbf{d}_{\alpha\beta}\|} + \frac{\mathbf{d}_{\alpha\beta} - \boldsymbol{\gamma}_{\alpha\beta}}{\|\mathbf{d}_{\alpha\beta} - \boldsymbol{\gamma}_{\alpha\beta}\|} \right) \end{aligned} \quad (9)$$

with  $\boldsymbol{\gamma}_{\alpha\beta} = (\mathbf{v}_\beta - \mathbf{v}_\alpha)\Delta t$ . We used  $\Delta t = 0.5$  s. For  $\Delta t = 0$ , we regain the expression of Eq. (5).

The elliptical specification has two major advantages compared to the circular one: First, the interactions depend not only on the distance, but also on the relative velocity. Second, the repulsive force is not strictly directed from pedestrian  $\beta$  to pedestrian  $\alpha$ , but has a lateral component. As a consequence, this leads to less confrontative, smoother (“sliding”) evading maneuvers. Note that further velocity-dependent specifications of pedestrian interaction forces have been proposed [7,26], but we will restrict to the above specifications, as these are sufficient to demonstrate the method of evolutionary model calibration.

### Evolutionary Calibration with Video Tracking Data

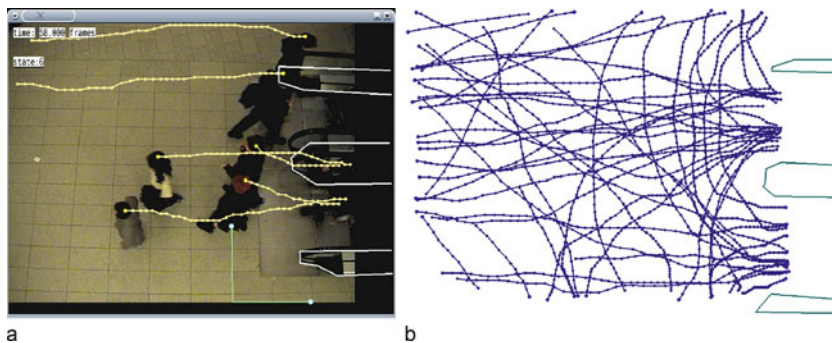
For parameter calibration, several video recordings of pedestrian crowds in different natural environments have been used. The dimensions of the recorded areas were

known, and the floor tiling or environment provided something like a “coordinate system”. The heads were automatically determined by searching for round moving structures, and the accuracy of tracking was improved by comparing actual with linearly extrapolated positions (so it would not happen so easily that the algorithm interchanged or “lost” close by pedestrians). The trajectories of the heads were then projected on two-dimensional space in a way correcting for distortion by the camera perspective. A representative plot of the resulting trajectories is shown in Fig. 1. Note that trajectory data have been obtained with infra-red sensors [47] or video cameras [48,49] for several years now, but algorithms that can simultaneously handle more than one thousand pedestrians have become available only recently [87].

For model calibration, it is recommended to use a hybrid method fusing empirical trajectory data and microscopic simulation data of pedestrian movement in space. In corresponding algorithms, a virtual pedestrian is assigned to each tracked pedestrian in the simulation domain. One then starts a simulation for a time period  $T$  (e.g. 1.5 s), in which one pedestrian  $\alpha$  is moved according to a simulation of the social force model, while the others are moved exactly according to the trajectories extracted from the videos. This procedure is performed for all pedestrians  $\alpha$  and for several different starting times  $t$ , using a fixed parameter set for the social force model.

Each simulation run is performed according to the following scheme:

1. Define a starting point and calculate the state (position  $\mathbf{r}_\alpha$ , velocity  $\mathbf{v}_\alpha$ , and acceleration  $\mathbf{a}_\alpha = d\mathbf{v}_\alpha/dt$ ) for each pedestrian  $\alpha$ .
2. Assign a desired speed  $v_\alpha^0$  to each pedestrian, e.g. the maximum speed during the pedestrian tracking time. This is sufficiently accurate, if the overall pedestrian



Pedestrian, Crowd and Evacuation Dynamics, Figure 1

Video tracking used to extract the trajectories of pedestrians from video recordings close to two escalators (after [45]). **a** Illustration of the tracking of pedestrian heads. **b** Resulting trajectories after being transformed onto the two-dimensional plane

density is not too high and the desired speed is constant in time.

3. Assign a desired goal point for each pedestrian, e. g. the end point of the trajectory.
4. Given the tracked motion of the surrounding pedestrians  $\beta$ , simulate the trajectory of pedestrian  $\alpha$  over a time period  $T$  based on the social force model, starting at the actual location  $\mathbf{r}_\alpha(t)$ .

After each simulation run, one determines the relative distance error

$$\frac{\|\mathbf{r}_\alpha^{\text{simulated}}(t+T) - \mathbf{r}_\alpha^{\text{tracked}}(t+T)\|}{\|\mathbf{r}_\alpha^{\text{tracked}}(t+T) - \mathbf{r}_\alpha^{\text{tracked}}(t)\|}. \quad (10)$$

After averaging the relative distance errors over the pedestrians  $\alpha$  and starting times  $t$ , 1 minus the result can be taken as measure of the goodness of fit (the “fitness”) of the parameter set used in the pedestrian simulation. Hence, the best possible value of the “fitness” is 1, but any deviation from the real pedestrian trajectories implies lower values.

One result of such a parameter optimization is that, for each video, there is a broad range of parameter combinations of  $A$  and  $B$  which perform almost equally well [45]. This allows one to apply additional goal functions in the parameter optimization, e. g. to determine among the best performing parameter values such parameter combinations, which perform well for *several* video recordings, using a fitness function which equally weights the fitness reached in each single video. This is how the parameter values listed in Table 1 were determined. It turns out that, in order to reach a good model performance, the pedestrian interaction force must be specified velocity dependent, as in the elliptical model.

Note that our evolutionary fitting method can be also used to determine interaction laws without prespecified

Pedestrian, Crowd and Evacuation Dynamics, Table 1  
Interaction strength  $A$  and interaction range  $B$  resulting from our evolutionary parameter calibration for the circular and elliptical specification of the interaction forces between pedestrians (see main text). The calibration was based on three different video recordings, one for low crowd density, one for medium, and one for high density. The parameter values are specified as mean value  $\pm$  standard deviation. The best fitness value obtained with the elliptical specification for the video with the lowest crowd density was as high as 0.9

Model	A	B	“Fitness”
Extrapolation	0	–	0.34
Circular	0.11 $\pm$ 0.06	0.84 $\pm$ 0.63	0.35
Elliptical	4.30 $\pm$ 3.91	1.07 $\pm$ 1.35	0.53

interaction functions. For example, one can obtain the distance dependence of pedestrian interactions without a prespecified function. For this, one adjusts the values of the force at given distances  $d_k = kd_1$  (with  $k \in \{1, 2, 3, \dots\}$ ) in an evolutionary way. To get some smoothness, linear interpolation is applied. The resulting fit curve is presented in Fig. 2 (left). It turns out that the empirical dependence of the force with distance can be well fitted by an exponential decay.

### Angular Dependence

A closer study of pedestrian interactions reveals that these are not isotropic, but dependent on the angle  $\varphi_{\alpha\beta}$  of the encounter, which is given by the formula

$$\cos(\varphi_{\alpha\beta}) = \frac{\mathbf{v}_\alpha}{\|\mathbf{v}_\alpha\|} \cdot \frac{-\mathbf{d}_{\alpha\beta}}{\|\mathbf{d}_{\alpha\beta}\|}. \quad (11)$$

Generally, pedestrians show little response to pedestrians behind them. This can be reflected by an angular-dependent prefactor  $w(\varphi_{\alpha\beta})$  of the interaction force [45]. Empirical results are represented in Fig. 2 (right). Reasonable results are obtained for the following specification of the prefactor:

$$w(\varphi_{\alpha\beta}(t)) = \left( \lambda_\alpha + (1 - \lambda_\alpha) \frac{1 + \cos(\varphi_{\alpha\beta})}{2} \right), \quad (12)$$

where  $\lambda_\alpha$  with  $0 \leq \lambda_\alpha \leq 1$  is a parameter which grows with the strength of interactions from behind. An evolutionary parameter optimization gives values  $\lambda \approx 0.1$  [45], i. e. a strong anisotropy. With such an angle-dependent prefactor, the “fitness” of the elliptical force increases from 0.53 to 0.61, when calibrated to the same set of videos. Other angular-dependent specifications split up the interaction force between pedestrians into a component against the direction of motion and another one perpendicular to it. Such a description allows for even smoother avoidance maneuvers.

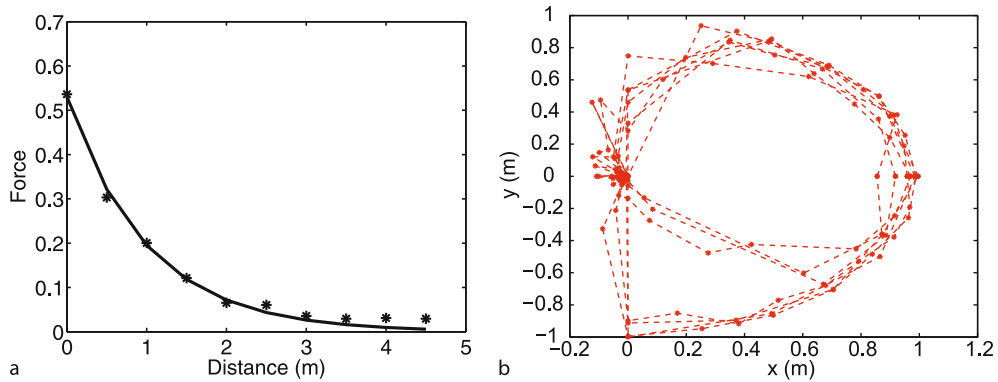
### Crowd Dynamics

#### Analogies with Gases, Fluids, and Granular Media

When the density is low, pedestrians can move freely, and the observed crowd dynamics can be partially compared with the behavior of gases. At medium and high densities, however, the motion of pedestrian crowds shows some striking analogies with the motion of fluids:

1. Footprints of pedestrians in snow look similar to streamlines of fluids [15].





Pedestrian, Crowd and Evacuation Dynamics, Figure 2

Results of an evolutionary fitting of pedestrian interactions. **a** Empirically determined distance dependence of the interaction force between pedestrians (after [45]). An exponential decay fits the empirical data quite well. The dashed fit curve corresponds to Eq. (5) with the parameters  $A = 0.53$  and  $B = 1.0$ . **b** Angular dependence of the influence of other pedestrians. The direction along the positive  $x$ -axis corresponds to the walking direction of pedestrians,  $y$  to the perpendicular direction

2. At borderlines between opposite directions of walking one can observe “viscous fingering” [50,51].
3. The emergence of pedestrian streams through standing crowds [7,37,52] appears analogous to the formation of river beds [53,54].

At high densities, however, the observations have rather analogies with driven granular flows. This will be elaborated in more detail in Sects. “Force Model for Panicking Pedestrians” and “Collective Phenomena in Panic Situations”. In summary, one could say that fluid-dynamic analogies work reasonably well in normal situations, while granular aspects dominate at extreme densities. Nevertheless, the analogy is limited, since the self-driven motion and the violation of momentum conservation imply special properties of pedestrian flows. For example, one usually does not observe eddies, which typically occur in regular fluids at high enough Reynolds numbers.

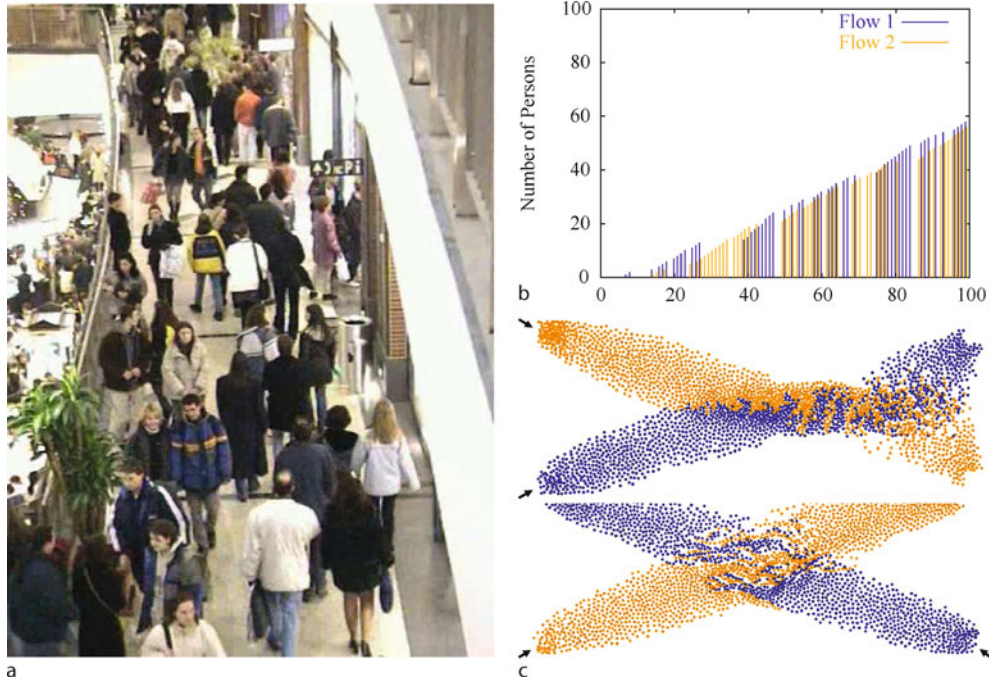
### Self-Organization of Pedestrian Crowds

Despite its simplifications, the social force model of pedestrian dynamics describes a lot of observed phenomena quite realistically. Especially, it allows one to explain various self-organized spatio-temporal patterns that are not externally planned, prescribed, or organized, e. g. by traffic signs, laws, or behavioral conventions. Instead, the spatio-temporal patterns discussed below emerge due to the non-linear interactions of pedestrians even without assuming strategical considerations, communication, or imitative behavior of pedestrians. Despite this, we may still interpret the forming cooperation patterns as phenomena that establish social order on short time scales. It is actu-

ally surprising that strangers coordinate with each other within seconds, if they have grown up in a similar environment. People from different countries, however, are sometimes irritated about local walking habits, which indicates that learning effects and cultural backgrounds still play a role in social interactions as simple as random pedestrian encounters. Rather than on particular features, however, in the following we will focus on the common, internationally reproducible observations.

**Lane Formation** In pedestrian flows one can often observe that oppositely moving pedestrians are forming lanes of uniform walking direction (see Fig. 3). This phenomenon even occurs when there is not a large distance to separate each other, e. g. on zebra crossings. However, the width of lanes increases (and their number decreases), if the interaction continues over longer distances (and if perturbations, e. g. by flows entering or leaving on the sides, are low; otherwise the phenomenon of lane formation may break down [55]).

Lane formation may be viewed as *segregation phenomenon* [56,57]. Although there is a weak preference for one side (with the corresponding behavioral convention depending on the country), the observations can only be well reproduced when repulsive pedestrian interactions are taken into account. The most relevant factor for the lane formation phenomenon is the higher relative velocity of pedestrians walking in opposite directions. Compared to people following each other, oppositely moving pedestrians have more frequent interactions until they have segregated into separate lanes by stepping aside whenever another pedestrian is encountered. The most long-lived pat-



Pedestrian, Crowd and Evacuation Dynamics, Figure 3

**Self-organization of pedestrian crowds.** **a** Photograph of lanes formed in a shopping center. Computer simulations reproduce the self-organization of such lanes very well. **b** Evaluation of the cumulative number of pedestrians passing a bottleneck from different sides. One can clearly see that the narrowing is often passed by groups of people in an oscillatory way rather than one by one. **c** Multi-agent simulation of two crossing pedestrian streams, showing the phenomenon of stripe formation. This self-organized pattern allows pedestrians to pass the other stream without having to stop, namely by moving sideways in a forwardly moving stripe. (After [8])

terns of motion are the ones which change the least. It is obvious that such patterns correspond to lanes, as they minimize the frequency and strength of avoidance maneuvers. Interestingly enough, as computer simulations show, lane formation occurs also when there is no preference for any side.

Lanes minimize frictional effects, accelerations, energy consumption, and delays in oppositely moving crowds. Therefore, one could say that they are a pattern reflecting “collective intelligence”. In fact, it is not possible for a single pedestrian to reach such a collective pattern of motion. Lane formation is a self-organized collaborative pattern of motion originating from simple pedestrian interactions. Particularly in cases of no side preference, the system behavior cannot be understood by adding up the behavior of the single individuals. This is a typical feature of complex, self-organizing systems and, in fact, a widespread characteristics of social systems. It is worth noting, however, that it does not require a conscious behavior to reach forms of social organization like the segregation of oppositely moving pedestrians into lanes. This organiza-

tion occurs automatically, although most people are not even aware of the existence of this phenomenon.

**Oscillatory Flows at Bottlenecks** At bottlenecks, bidirectional flows of moderate density are often characterized by oscillatory changes in the flow direction (see Fig. 3). For example, one can sometimes observe this at entrances of museums during crowded art exhibitions or at entrances of staff canteens during lunch time. While these oscillatory flows may be interpreted as an effect of friendly behavior (“you go first, please”), computer simulations of the social force model indicate that the collective behavior may again be understood by simple pedestrian interactions. That is, oscillatory flows occur even in the absence of communication. Therefore, they may be viewed as another self-organization phenomenon, which again reduces frictional effects and delays. That is, oscillatory flows have features of “collective intelligence”.

While this may be interpreted as result of a learning effect in a large number of similar situations (a “repeated game”), our simulations suggest an even simpler, “many-

particle” interpretation: Once a pedestrian is able to pass the narrowing, pedestrians with the same walking direction can easily follow. Hence, the number and “pressure” of waiting, “pushy” pedestrians on one side of the bottleneck becomes less than on the other side. This eventually increases their chance to occupy the passage. Finally, the “pressure difference” is large enough to stop the flow and turn the passing direction at the bottleneck. This reverses the situation, and eventually the flow direction changes again, giving rise to oscillatory flows.

**Stripe Formation in Intersecting Flows** In intersection areas, the flow of people often appears to be irregular or “chaotic”. In fact, it can be shown that there are several possible collective patterns of motion, among them rotary and oscillating flows. However, these patterns continuously compete with each other, and a temporarily dominating pattern is destroyed by another one after a short time. Obviously, there has not evolved any social convention that would establish and stabilize an ordered and efficient flow at intersections.

Self-organized patterns of motion, however, are found in situations where pedestrian flows cross each other only in two directions. In such situations, the phenomenon of

stripe formation is observed [58]. Stripe formation allows two flows to penetrate each other without requiring the pedestrians to stop. For an illustration see Fig. 3. Like lanes, stripes are a segregation phenomenon, but not a stationary one. Instead, the stripes are density waves moving into the direction of the sum of the directional vectors of both intersecting flows. Naturally, the stripes extend sideways into the direction which is perpendicular to their direction of motion. Therefore, the pedestrians move forward *with* the stripes and sideways *within* the stripes. Lane formation corresponds to the particular case of stripe formation where both directions are exactly opposite. In this case, no intersection takes place, and the stripes do not move systematically. As in lane formation, stripe formation allows to minimize obstructing interactions and to maximize the average pedestrian speeds, i. e. simple, repulsive pedestrian interactions again lead to an “intelligent” collective behavior.

### Evacuation Dynamics

While the previous section has focused on the dynamics of pedestrian crowds in normal situations, we will now turn to the description of situations in which extreme crowd

Pedestrian, Crowd and Evacuation Dynamics, Table 2

Incomplete list of major crowd disasters since 1970 after J. F. Dickie in [59], <http://www.crowddynamics.com/Main/Crowddisasters.html>, [http://SportsIllustrated.CNN.com/soccer/world/news/2000/07/09/stadium\\_disasters\\_ap/](http://SportsIllustrated.CNN.com/soccer/world/news/2000/07/09/stadium_disasters_ap/), and other internet sources, excluding fires, bomb attacks, and train or plane accidents. The number of injured people was usually a multiple of the fatalities

Date	Place	Venue	Deaths	Reason
1971	Ibrox, UK	Stadium	66	Collapse of barriers
1974	Cairo, Egypt	Stadium	48	Crowds break barriers
1982	Moscow, USSR	Stadium	340	Re-entering fans after last minute goal
1988	Katmandu, Nepal	Stadium	93	Stampede due to hailstorm
1989	Hillsborough, Sheffield, UK	Stadium	96	Fans trying to force their way into the stadium
1990	New York City	Bronx	87	Illegal happy land social club
1990	Mena, Saudi Arabia	Pedestrian Tunnel	1426	Overcrowding
1994	Mena, Saudi Arabia	Jamarat Bridge	266	Overcrowding
1996	Guatemala City, Guatemala	Stadium	83	Fans trying to force their way into the stadium
1998	Mena, Saudi Arabia		118	Overcrowding
1999	Kerala, India	Hindu Shrine	51	Collapse of parts of the shrine
1999	Minsk, Belarus	Subway Station	53	Heavy rain at rock concert
2001	Ghana, West Africa	Stadium	> 100	Panic triggered by tear gas
2004	Mena, Saudi Arabia	Jamarat Bridge	251	Overcrowding
2005	Wai, India	Religious Procession	150	Overcrowding (and fire)
2005	Bagdad, Iraque	Religious Procession	> 640	Rumors regarding suicide bomber
2005	Chennai, India	Disaster Area	42	Rush for flood relief supplies
2006	Mena, Saudi Arabia	Jamarat Bridge	363	Overcrowding
2006	Pilippines	Stadium	79	Rush for game show tickets
2006	Ibb, Yemen	Stadium	51	Rally for Yemeni president



densities occur. Such situations may arise at mass events, particularly in cases of urgent egress. While most evacuations run relatively smoothly and orderly, the situation may also get out of control and end up in terrible crowd disasters (see Table 2). In such situations, one often speaks of “panic”, although, from a scientific standpoint, the use of this term is rather controversial. Here, however, we will not be interested in the question whether “panic” actually occurs or not. We will rather focus on the issue of crowd dynamics at high densities and under psychological stress.

### Evacuation and Panic Research

Computer models have been also developed for emergency and evacuation situations [32,60,61,62,63,64,65,66,67,68]. Most research into panic, however, has been of empirical nature (see, e. g. [69,70,71,72]), carried out by social psychologists and others.

With some exceptions, panic is observed in cases of scarce or dwindling resources [69,73], which are either required for survival or anxiously desired. They are usually distinguished into escape panic (“stampedes”, bank or stock market panic) and acquisitive panic (“crazes”, speculative manias) [74,75], but in some cases this classification is questionable [76].

It is often stated that panicking people are obsessed by short-term personal interests uncontrolled by social and cultural constraints [69,74]. This is possibly a result of the reduced attention in situations of fear [69], which also causes that options like side exits are mostly ignored [70]. It is, however, mostly attributed to social contagion [69,71,73,74,75,76,77,78,79,80,81], i. e., a transition from individual to mass psychology, in which individuals transfer control over their actions to others [75], leading to conformity [82]. This “herding behavior” is in some sense irrational, as it often leads to bad overall results like dangerous overcrowding and slower escape [70,75,76]. In this way, herding behavior can increase the fatalities or, more generally, the damage in the crisis faced.

The various socio-psychological theories for this contagion assume hypnotic effects, rapport, mutual excitation of a primordial instinct, circular reactions, social facilitation (see the summary by Brown [80]), or the emergence of normative support for selfish behavior [81]. Brown [80] and Coleman [75] add another explanation related to the prisoner’s dilemma [83,84] or common goods dilemma [85], showing that it is reasonable to make one’s subsequent actions contingent upon those of others. However, the socially favorable behavior of walking orderly is unstable, which normally gives rise to rushing by everyone. These thoughtful considerations are well compatible

with many aspects discussed above and with the classical experiments by Mintz [73], which showed that jamming in escape situations depends on the reward structure (“payoff matrix”).

Nevertheless and despite of the frequent reports in the media and many published investigations of crowd disasters (see Table 2), a quantitative understanding of the observed phenomena in panic stampedes was lacking for a long time. In the following, we will close this gap.

### Situations of “Panic”

Panic stampede is one of the most tragic collective behaviors [71,72,73,74,75,77,78,79,80,81], as it often leads to the death of people who are either crushed or trampled down by others. While this behavior may be comprehensible in life-threatening situations like fires in crowded buildings [69,70], it is hard to understand in cases of a rush for good seats at a pop concert [76] or without any obvious reasons. Unfortunately, the frequency of such disasters is increasing (see Table 2), as growing population densities combined with easier transportation lead to greater mass events like pop concerts, sport events, and demonstrations. Nevertheless, systematic empirical studies of panic [73,86] are rare [69,74,76], and there is a scarcity of quantitative theories capable of predicting crowd dynamics at extreme densities [32,60,61,64,65,68]. The following features appear to be typical [46,55]:

1. In situations of escape panic, individuals are getting nervous, i. e. they tend to develop blind actionism.
2. People try to move considerably faster than normal [9].
3. Individuals start pushing, and interactions among people become physical in nature.
4. Moving and, in particular, passing of a bottleneck frequently becomes incoordinated [73].
5. At exits, jams are building up [73]. Sometimes, intermittent flows or arching and clogging are observed [9], see Fig. 4.
6. The physical interactions in jammed crowds add up and can cause dangerous pressures up to 4,500 Newtons per meter [59,70], which can bend steel barriers or tear down brick walls.
7. The strength and direction of the forces acting in large crowds can suddenly change [87], pushing people around in an uncontrollable way. This may cause people to fall.
8. Escape is slowed down by fallen or injured people turning into “obstacles”.
9. People tend to show herding behavior, i. e., to do what other people do [69,78].



Pedestrian, Crowd and Evacuation Dynamics, Figure 4  
Panicking football fans trying to escape the football stadium in Sheffield. Because of a clogging effect, it is difficult to pass the open door

10. Alternative exits are often overlooked or not efficiently used in escape situations [69,70].

The following quotations give a more personal impression of the conditions during crowd panic:

1. “They just kept pushin’ forward and they would just walk right on top of you, just trample over ya like you were a piece of the ground.” (After the panic at “The Who Concert Stampede” in Cincinnati.)
2. “People were climbin’ over people ta get in ... an’ at one point I almost started hittin’ ’em, because I could not believe the animal, animalistic ways of the people, you know, nobody cared.” (After the panic at “The Who Concert Stampede”.)
3. “Smaller people began passing out. I attempted to lift one girl up and above to be passed back ... After several tries I was unsuccessful and near exhaustion.” (After the panic at “The Who Concert Stampede”.)
4. “I couldn’t see the floor because of the thickness of the smoke.” (After the “Hilton Hotel Fire” in Las Vegas.)
5. “The club had two exits, but the young people had access to only one”, said Narend Singh, provincial minister for agriculture and environmental affairs. However, the club’s owner, Rajan Naidoo, said the club had four exits, and that all were open. “I think the children panicked and headed for the main entrance where they initially came in,” he said.” (After the “Durban Disco Stampede”.)
6. “At occupancies of about 7 persons per square meter the crowd becomes almost a fluid mass. Shock waves can be propagated through the mass, sufficient to ... propel them distances of 3 meters or more. ... People

may be literally lifted out of their shoes, and have clothing torn off. Intense crowd pressures, exacerbated by anxiety, make it difficult to breathe, which may finally cause compressive asphyxia. The heat and the thermal insulation of surrounding bodies cause some to be weakened and faint. Access to those who fall is impossible. Removal of those in distress can only be accomplished by lifting them up and passing them overhead to the exterior of the crowd.” (J. Fruin in [88].)

7. “It was like a huge wave of sea gushing down on the pilgrims” (P. K. Abdul Ghafour, Arab News, after the sad crowd disaster in Mena on January 12, 2006).

### Force Model for Panicking Pedestrians

Additional, physical interaction forces  $f_{\alpha\beta}^{\text{ph}}$  come into play when pedestrians get so close to each other that they have physical contact (i. e.  $d_{\alpha\beta} < r_{\alpha\beta} = r_{\alpha} + r_{\beta}$ , where  $r_{\alpha}$  means the “radius” of pedestrian  $\alpha$ ). In this case, which is mainly relevant to panic situations, we assume also a “body force”  $k(r_{\alpha\beta} - d_{\alpha\beta})\mathbf{n}_{\alpha\beta}$  counteracting body compression and a “sliding friction force”  $\kappa(r_{\alpha\beta} - d_{\alpha\beta})\Delta v_{\beta\alpha}^t \mathbf{t}_{\alpha\beta}$  impeding relative tangential motion. Inspired by the formulas for granular interactions [89,90], we assume

$$\mathbf{f}_{\alpha\beta}^{\text{ph}}(t) = k\Theta(r_{\alpha\beta} - d_{\alpha\beta})\mathbf{n}_{\alpha\beta} + \kappa\Theta(r_{\alpha\beta} - d_{\alpha\beta})\Delta v_{\beta\alpha}^t \mathbf{t}_{\alpha\beta}, \quad (13)$$

where the function  $\Theta(z)$  is equal to its argument  $z$ , if  $z \geq 0$ , otherwise 0. Moreover,  $\mathbf{t}_{\alpha\beta} = (-n_{\alpha\beta}^2, n_{\alpha\beta}^1)$  means the tangential direction and  $\Delta v_{\beta\alpha}^t = (\mathbf{v}_{\beta} - \mathbf{v}_{\alpha}) \cdot \mathbf{t}_{\alpha\beta}$  the tangential velocity difference, while  $k$  and  $\kappa$  represent large constants. (Strictly speaking, friction effects already set in before pedestrians touch each other, because of the psychological tendency not to pass other individuals with a high relative velocity, when the distance is small.)

The interactions with the boundaries of walls and other obstacles are treated analogously to pedestrian interactions, i. e., if  $d_{\alpha i}(t)$  means the distance to obstacle or boundary  $i$ ,  $\mathbf{n}_{\alpha i}(t)$  denotes the direction perpendicular to it, and  $\mathbf{t}_{\alpha i}(t)$  the direction tangential to it, the corresponding interaction force with the boundary reads

$$\mathbf{f}_{\alpha i} = \{A_{\alpha} \exp[(r_{\alpha} - d_{\alpha i})/B_{\alpha}] + k\Theta(r_{\alpha} - d_{\alpha i})\} \times \mathbf{n}_{\alpha i} - \kappa\Theta(r_{\alpha} - d_{\alpha i})(\mathbf{v}_{\alpha} \cdot \mathbf{t}_{\alpha i})\mathbf{t}_{\alpha i}. \quad (14)$$

Finally, fire fronts are reflected by repulsive social forces similar those describing walls, but they are much stronger. The physical interactions, however, are qualitatively different, as people reached by the fire front become injured and immobile ( $\mathbf{v}_{\alpha} = \mathbf{0}$ ).

### Collective Phenomena in Panic Situations

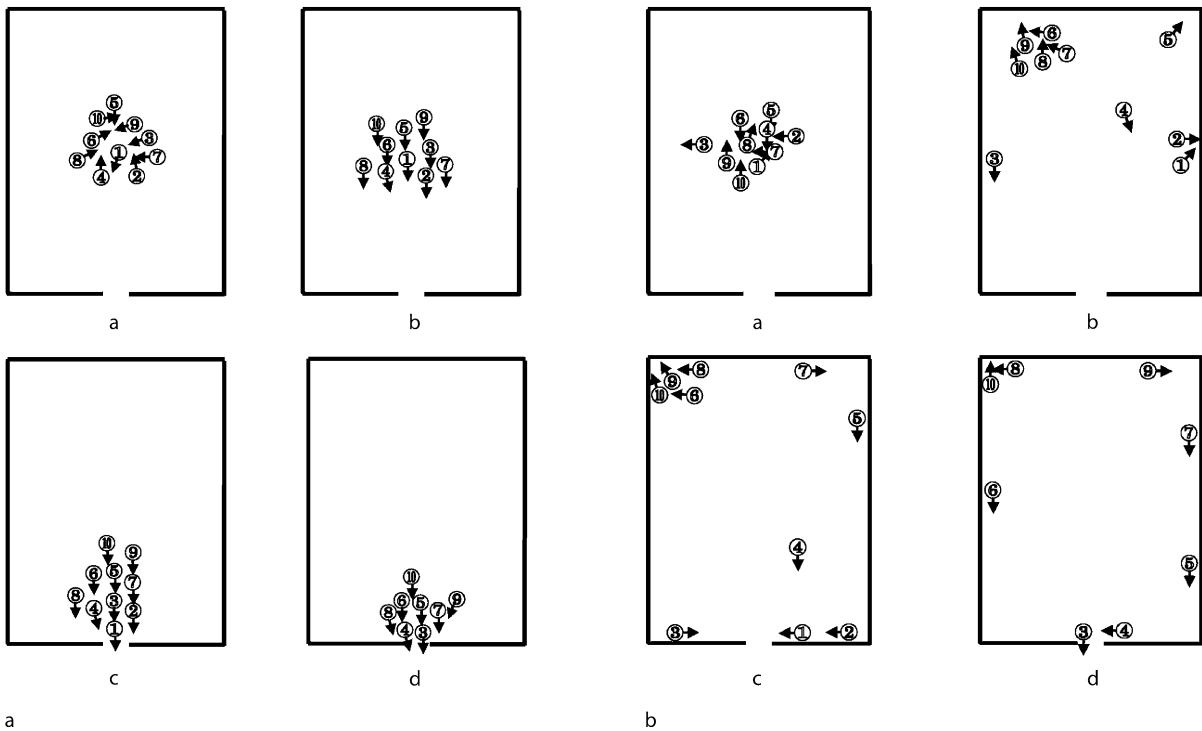
In panic situations (e. g. in some cases of emergency evacuation) the following characteristic features of pedestrian behavior are often observed:

1. People are getting nervous, resulting in a higher level of fluctuations.
2. They are trying to escape from the source of panic, which can be reflected by a significantly higher desired velocity  $v_{\alpha}^0$ .
3. Individuals in complex situations, who do not know what is the right thing to do, orient at the actions of their neighbors, i. e. they tend to do what other people do. We will describe this by an additional herding interaction.

We will now discuss the fundamental collective effects which fluctuations, increased desired velocities, and herding behavior can have. In contrast to other approaches, we do not assume or imply that individuals in panic or emergency situations would behave relentless and asocial, although they sometimes do.

**Herding and Ignorance of Available Exits** If people are not sure what is the best thing to do, there is a tendency to show a “herding behavior”, i. e. to imitate the behavior of others. Fashion, hypes and trends are examples for this. The phenomenon is also known from stock markets, and particularly pronounced when people are anxious. Such a situation is, for example, given if people need to escape from a smoky room. There, the evacuation dynamics is very different from normal leaving (see Fig. 5).

Under normal visibility, everybody easily finds an exit and uses more or less the shortest path. However, when the exit cannot be seen, evacuation is much less efficient and may take a long time. Most people tend to walk relatively straight into the direction in which they suspect an exit, but in most cases, they end up at a wall. Then, they usually move along it in one of the two possible directions, until they finally find an exit [18]. If they encounter others, there is a tendency to take a decision for one direction and move collectively. Also in case of acoustic signals, people may be attracted into the same direction. This can lead to over-crowded exits, while other exits are ignored. The same can happen even for normal visibility, when people



Pedestrian, Crowd and Evacuation Dynamics, Figure 5

**a** Normal leaving of a room, when the exit is well visible. **b** Escape from a room with no visibility, e. g. due to dense smoke or a power blackout. (After [18])

are not well familiar with their environment and are not aware of the directions of the emergency exits.

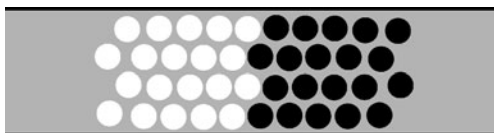
Computer simulations suggest that neither individualistic nor herding behavior performs well [46]. Pure individualistic behavior means that each pedestrian finds an exit only accidentally, while pure herding behavior implies that the complete crowd is eventually moving into the same and probably congested direction, so that available emergency exits are not efficiently used. Optimal chances of survival are expected for a certain mixture of individualistic and herding behavior, where individualism allows *some* people to detect the exits and herding guarantees that successful solutions are imitated by small groups of others [46].

**“Freezing by Heating”** Another effect of getting nervous has been investigated in [55]. Let us assume the individual fluctuation strength is given by

$$\eta_\alpha = (1 - n_\alpha)\eta_0 + n_\alpha\eta_{\max}, \quad (15)$$

where  $n_\alpha$  with  $0 \leq n_\alpha \leq 1$  measures the nervousness of pedestrian  $\alpha$ . The parameter  $\eta_0$  means the normal and  $\eta_{\max}$  the maximum fluctuation strength. It turns out that, at sufficiently high pedestrian densities, lanes are destroyed by increasing the fluctuation strength (which is analogous to the temperature). However, instead of the expected transition from the “fluid” lane state to a disordered, “gaseous” state, a solid state is formed. It is characterized by an at least temporarily blocked, “frozen” situation so that one calls this paradoxical transition “*freezing by heating*” (see Fig. 6). Notably enough, the blocked state has a *higher* degree of order, although the internal energy is *increased* [55].

The preconditions for this unusual freezing-by-heating transition are the driving term  $v_\alpha^0 e_\alpha^0 / \tau_\alpha$  and the dissipative friction  $-v_\alpha / \tau_\alpha$ , while the sliding friction force is not required. Inhomogeneities in the channel diameter or other impurities which temporarily slow down pedestrians can further this transition at the respective places. Finally note that a transition from fluid to blocked pedes-



Pedestrian, Crowd and Evacuation Dynamics, Figure 6  
Result of the noise-induced formation of a “frozen” state in a (periodic) corridor used by oppositely moving pedestrians (after [55])

trian counter flows is also observed, when a critical density is exceeded [31,55].

**Intermittent Flows, Faster-Is-Slower Effect, and “Phantom Panic”** If the overall flow towards a bottleneck is higher than the overall outflow from it, a pedestrian queue emerges [91]. In other words, a waiting crowd is formed upstream of the bottleneck. High densities can result, if people keep heading forward, as this eventually leads to higher and higher compressions. Particularly critical situations may occur if the arrival flow is much higher than the departure flow, especially if people are trying to get towards a strongly desired goal (“acquisitive panic”) or away from a perceived source of danger (“escape panic”) with an increased driving force  $v_\alpha^0 e_\alpha^0 / \tau$ . In such situations, the high density causes coordination problems, as several people compete for the same few gaps. This typically causes body interactions and frictional effects, which can slow down crowd motion or evacuation (“*faster is slower effect*”).

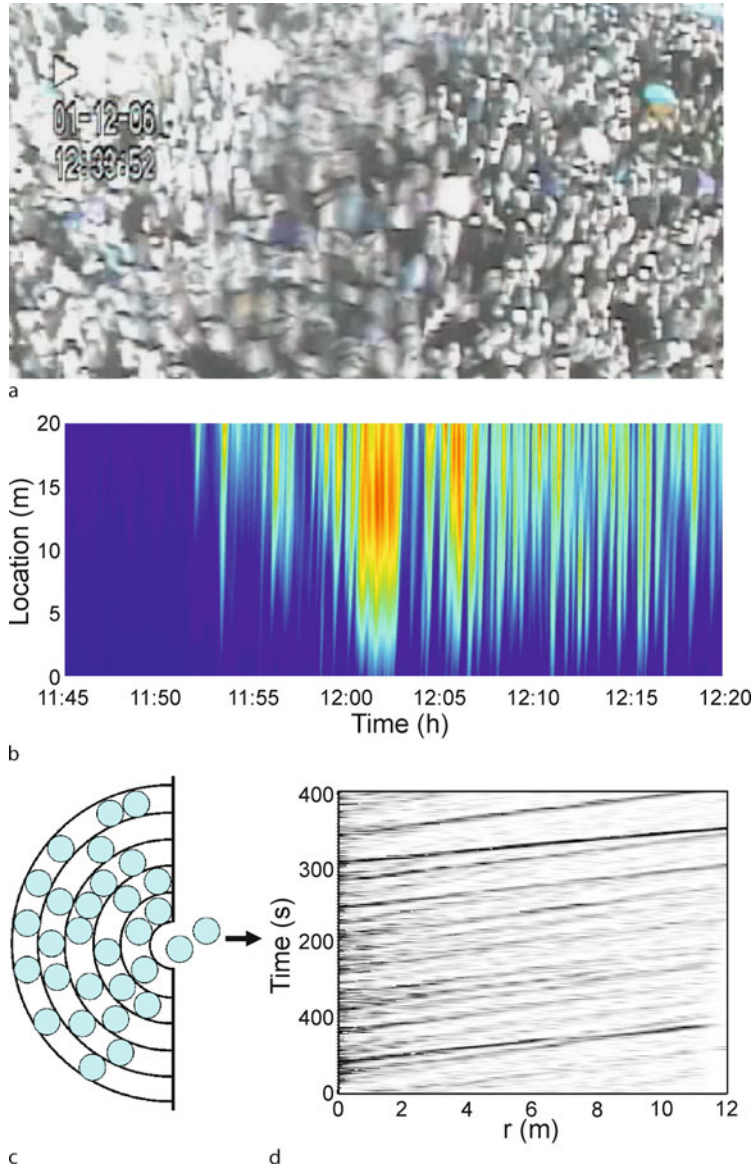
A possible consequence of these coordination problems are intermittent flows. In such cases, the outflow from the bottleneck is not constant, but it is typically interrupted. While one possible origin of the intermittent flows are clogging and arching effects as known from granular flows through funnels or hoppers [89,90], stop-and-go waves have also been observed in more than 10 meter wide streets and in the 44 meters wide entrance area to the Jamarat Bridge during the pilgrimage in January 12, 2006 [87], see Fig. 7. Therefore, it seems to be important that people do not move continuously, but have minimum strides [25]. That is, once a person is stopped, he or she will not move until some space opens up in front. However, increasing impatience will eventually reduce the minimum stride, so that people eventually start moving again, even if the outflow through the bottleneck is stopped. This will lead to a further compression of the crowd.

In the worst case, such behavior can trigger a “*phantom panic*”, i. e. a crowd disaster *without* any serious reasons (e. g., in Moscow, 1982). For example, due to the “faster-is-slower effect” panic can be triggered by small pedestrian counterflows [70], which cause delays to the crowd intending to leave. Consequently, stopped pedestrians in the back, who do not see the reason for the temporary slowdown, are getting impatient and pushy. In accordance with observations [7,25], one may model this by increasing the desired velocity, for example, by the formula

$$v_\alpha^0(t) = [1 - n_\alpha(t)]v_\alpha^0(0) + n_\alpha(t)v_\alpha^{\max}. \quad (16)$$

Herein,  $v_\alpha^{\max}$  is the maximum desired velocity and  $v_\alpha^0(0)$  the initial one, corresponding to the expected velocity of





Pedestrian, Crowd and Evacuation Dynamics, Figure 7

**a** Long-term photograph showing stop-and-go waves in a densely packed street. While stopped people appear relatively sharp, people moving from right to left have a fuzzy appearance. Note that gaps propagate from right to left. **b** Empirically observed stop-and-go waves in front of the entrance to the Jamarat Bridge on January 12, 2006 (after [87]), where pilgrims moved from left to right. Dark areas correspond to phases of motion, light colors to stop phases. **c** Illustration of the “shell model”, in particular of situations where several pedestrians compete for the same gap, which causes coordination problems. **d** Stop-and-go waves resulting from the alternation of forward pedestrian motion and backward gap propagation

leaving. The time-dependent parameter

$$n_\alpha(t) = 1 - \frac{\bar{v}_\alpha(t)}{v_\alpha^0(0)} \quad (17)$$

reflects the nervousness, where  $\bar{v}_\alpha(t)$  denotes the average speed into the desired direction of motion. Altogether,

long waiting times increase the desired speed  $v_\alpha^0$  or driving force  $v_\alpha^0(t)e_\alpha^0/\tau$ , which can produce high densities and inefficient motion. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger so high pressures that people are crushed or falling and trampled. It is, therefore, imperative, to have sufficiently

wide exits and to prevent counterflows, when big crowds want to leave [46].

**Transition to Stop-and-Go Waves** Recent empirical studies of pilgrim flows in the area of Makkah, Saudi Arabia, have shown that intermittent flows occur not only when bottlenecks are obvious. On January 12, 2006, pronounced stop-and-go waves have been even observed upstream of the 44 m wide entrance to the Jamarat Bridge [87]. While the pilgrim flows were smooth and continuous (“laminar”) over many hours, at 11:53 am stop-and-go waves suddenly appeared and propagated over distances of more than 30 m (see Fig. 7). The sudden transition was related to a significant drop of the flow, i. e. with the onset of congestion [87]. Once the stop-and-go waves set in, they persisted over more than 20 min.

This phenomenon can be reproduced by a recent model based on two continuity equations, one for forward pedestrian motion and another one for backward gap propagation [91]. The model was derived from a “shell model” (see Fig. 7) and describes very well the observed alternation between backward gap propagation and forward pedestrian motion.

**Transition to “Crowd Turbulence”** On the same day, around 12:19, the density reached even higher values and the video recordings showed a sudden transition from stop-and-go waves to *irregular* flows (see Fig. 8). These irregular flows were characterized by random, unintended displacements into all possible directions, which pushed

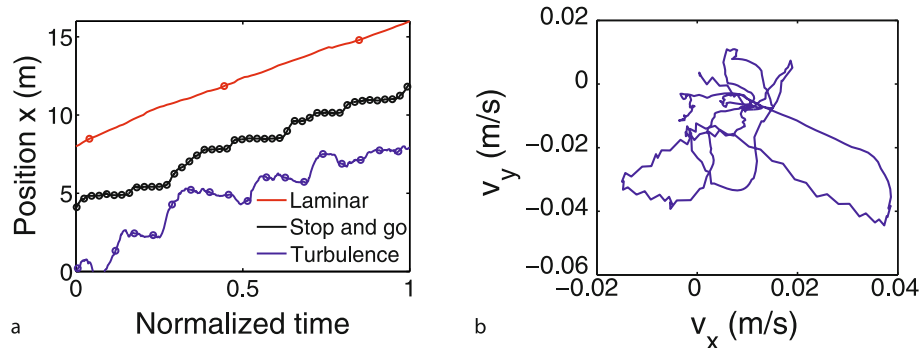
people around. With a certain likelihood, this caused them to stumble. As the people behind were moved by the crowd as well and could not stop, fallen individuals were trampled, if they did not get back on their feet quickly enough. Tragically, the area of trampled people grew more and more in the course of time, as the fallen pilgrims became obstacles for others [87]. The result was one of the biggest crowd disasters in the history of pilgrimage.

How can we understand this transition to irregular crowd motion? A closer look at video recordings of the crowd reveals that, at this time, people were so densely packed that they were moved involuntarily by the crowd. This is reflected by random displacements into all possible directions. To distinguish these irregular flows from laminar and stop-and-go flows and due to their visual appearance, we will refer to them as “*crowd turbulence*”.

As in certain kinds of fluid flows, “turbulence” in crowds results from a sequence of instabilities in the flow pattern. Additionally, one finds a sharply peaked probability density function of velocity increments

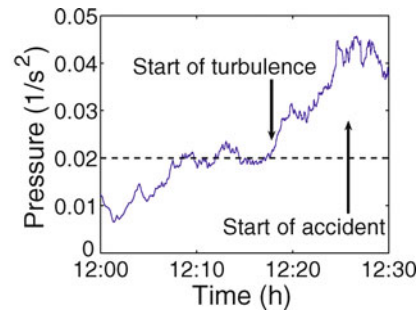
$$V_x^\tau = V_x(\mathbf{r}, t + \tau) - V_x(\mathbf{r}, t), \quad (18)$$

which is typical for turbulence [92], if the time shift  $\tau$  is small enough [87]. One also observes a power-law scaling of the displacements indicating self-similar behavior [87]. As large eddies are not detected, however, the similarity with *fluid* turbulence is limited, but there is still an analogy to turbulence at currency exchange markets [92]. Instead of vortex cascades like in turbulent fluids, one rather finds a hierarchical fragmentation dynamics: At extreme



Pedestrian, Crowd and Evacuation Dynamics, Figure 8

Pedestrian dynamics at different densities. **a** Representative trajectories (space-time plots) of pedestrians during the laminar, stop-and-go, and turbulent flow regime. Each trajectory extends over a range of 8 meters, while the time required for this stretch is normalized to 1. To indicate the different speeds, symbols are included in the curves every 5 seconds. While the laminar flow (*top line*) is fast and smooth, motion is temporarily interrupted in stop-and-go flow (*medium line*), and backward motion can occur in “turbulent” flows (*bottom line*). **b** Example of the temporal evolution of the velocity components  $v_x(t)$  into the average direction of motion and  $v_y(t)$  perpendicular to it in “turbulent flow”, which occurs when the crowd density is extreme. One can clearly see the irregular motion into all possible directions characterizing “crowd turbulence”. For details see [87]



Pedestrian, Crowd and Evacuation Dynamics, Figure 9

**Left:** Snapshot of the on-line visualization of “crowd pressure”. Red colors (see the lower ellipses) indicate areas of critical crowd conditions. In fact, the sad crowd disaster during the Muslim pilgrimage on January 12, 2006, started in this area. **Right:** The “crowd pressure” is a quantitative measure of the onset of “crowd turbulence”. The crowd disaster started when the “crowd pressure” reached particularly high values

densities, individual motion is replaced by mass motion, but there is a stick-slip instability which leads to “rupture” when the stress in the crowd becomes too large. That is, the mass splits up into clusters of different sizes with strong velocity correlations *inside* and distance-dependent correlations *between* the clusters.

“Crowd turbulence” has further specific features [87]. Due to the physical contacts among people in extremely dense crowds, we expect commonalities with granular media. In fact, dense driven granular media may form density waves, while moving forward [93], and can display turbulent-like states [94,95]. Moreover, under quasi-static conditions [94], force chains [96] are building up, causing strong variations in the strengths and directions of local forces. As in earthquakes [97,98] this can lead to events of sudden, uncontrollable stress release with power-law distributed displacements. Such a power-law has also been discovered by video-based crowd analysis [87].

### Some Warning Signs of Critical Crowd Conditions

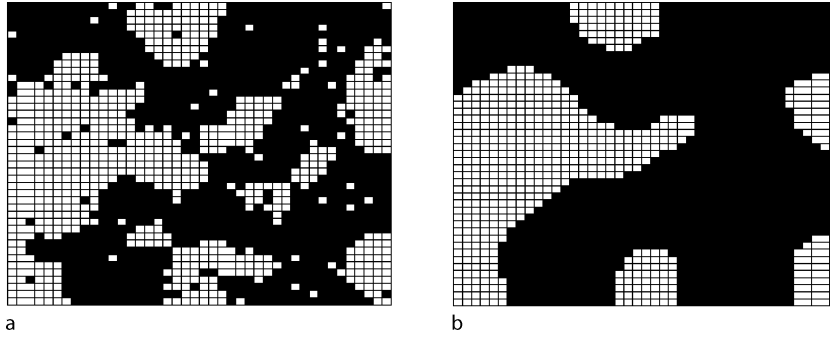
Turbulent waves are experienced in dozens of crowd-intensive events each year all over the world [88]. Therefore, it is necessary to understand why, where and when potentially critical situations occur. Viewing real-time video recordings is not very suited to identify critical crowd conditions: While the average density rarely exceeds values of 6 persons per square meter, the local densities can reach almost twice as large values [87]. It has been found, however, that even evaluating the local densities is not enough to identify the critical times and locations precisely, which also applies to an analysis of the velocity field [87]. The decisive quantity is rather the “crowd pressure”, i. e. the density, multiplied with the variance of speeds. It allows one to identify critical locations and times (see Fig. 9).

There are even advance warning signs of critical crowd conditions: The crowd accident on January 12, 2006 started about 10 minutes after “turbulent” crowd motion set in, i. e. after the “pressure” exceeded a value of  $0.02/s^2$  (see Fig. 9). Moreover, it occurred more than 30 min after stop-and-go waves set in, which can be easily detected in accelerated surveillance videos. Such advance warning signs of critical crowd conditions can be evaluated on-line by an automated video analysis system. In many cases, this can help one to gain time for corrective measures like flow control, pressure-relief strategies, or the separation of crowds into blocks to stop the propagation of shock-waves [87]. Such anticipative crowd control could increase the level of safety during future mass events.

### Evolutionary Optimization of Pedestrian Facilities

Having understood some of the main factors causing crowd disasters, it is interesting to ask how pedestrian facilities can be designed in a way that maximizes the efficiency of pedestrian flows and the level of safety. One of the major goals during mass events must be to avoid extreme densities. These often result from the onset of congestion at bottlenecks, which is a consequence of the breakdown of free flow and causes an increasing degree of compression. When a certain critical density is increased (which depends on the size distribution of people), this potentially implies high pressures in the crowd, particularly if people are impatient due to long delays or panic.

The danger of an onset of congestion can be minimized by avoiding bottlenecks. Notice, however, that jamming can also occur at widenings of escape routes [46]. This surprising fact results from disturbances due to pedestrians, who try to overtake each other and expand in the wider area because of their repulsive interactions.



Pedestrian, Crowd and Evacuation Dynamics, Figure 10

The evolutionary optimization based on Boolean grids [99] uses a two-stage algorithm. **a** In the randomization stage, obstacles are distributed over the grid with some randomness, thereby allowing for the generation of new topologies. **b** In the agglomeration stage, small nearby obstacles are clustered to form larger objects with smooth boundaries

These squeeze into the main stream again at the end of the widening, which acts like a bottleneck and leads to jamming. The corresponding drop of efficiency  $E$  is more pronounced,

1. if the corridor is narrow,
2. if the pedestrians have different or high desired velocities, and
3. if the pedestrian density in the corridor is high.

Obviously, the emerging pedestrian flows decisively depend on the geometry of the boundaries. They can be simulated on a computer already in the planning phase of pedestrian facilities. Their configuration and shape can be systematically varied, e. g. by means of evolutionary algorithms [28,100] and evaluated on the basis of particular mathematical performance measures [7]. Apart from the *efficiency*

$$E = \frac{1}{N} \sum_{\alpha} \frac{\mathbf{v}_{\alpha} \cdot \mathbf{e}_{\alpha}^0}{v_{\alpha}^0} \quad (19)$$

we can, for example, define the *measure of comfort*  $C = (1 - D)$  via the discomfort

$$D = \frac{1}{N} \sum_{\alpha} \frac{(\mathbf{v}_{\alpha} - \overline{\mathbf{v}_{\alpha}})^2}{(v_{\alpha}^0)^2} = \frac{1}{N} \sum_{\alpha} \left( 1 - \frac{\overline{v_{\alpha}^2}}{(v_{\alpha}^0)^2} \right). \quad (20)$$

The latter is again between 0 and 1 and reflects the frequency and degree of sudden velocity changes, i. e. the level of discontinuity of walking due to necessary avoidance maneuvers. Hence, the optimal configuration regarding the pedestrian requirements is the one with the highest values of efficiency and comfort.

During the optimization procedure, some or all of the following can be varied:

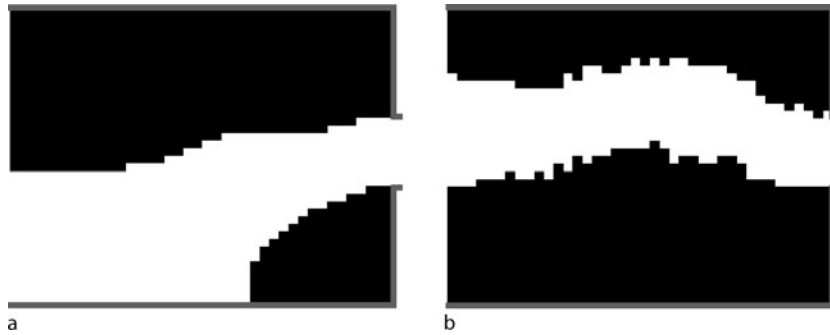
1. the location and form of planned buildings,
2. the arrangement of walkways, entrances, exits, staircases, elevators, escalators, and corridors,
3. the shape of rooms, corridors, entrances, and exits,
4. the function and time schedule. (Recreation rooms or restaurants are often continuously frequented, rooms for conferences or special events are mainly visited and left at peak periods, exhibition rooms or rooms for festivities require additional space for people standing around, and some areas are claimed by queues or through traffic.)

In contrast to early evolutionary optimization methods, recent approaches allow to change not only the dimensions of the different elements of pedestrian facilities, but also to vary their topology. The procedure of such algorithms is illustrated in Fig. 10. Highly performing designs are illustrated in Fig. 11. It turns out that, for an emergency evacuation route, it is favorable if the crowd does not move completely straight towards a bottleneck. For example, a zigzag design of the evacuation route can reduce the pressure on the crowd upstream of a bottleneck (see Fig. 12). The proposed evolutionary optimization procedure can, of course, not only be applied to the design of new pedestrian facilities, but also to a reduction of existing bottlenecks, when suitable modifications are implemented.

### Future Directions

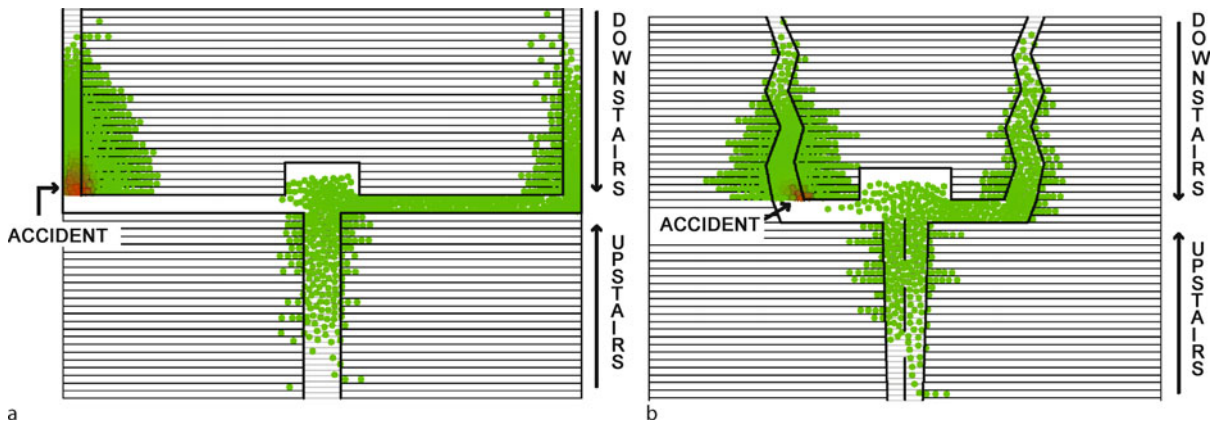
In this contribution, we have presented a multi-agent approach to pedestrian and crowd dynamics. Despite the great effort required, pedestrian interactions can be well quantified by video tracking. Compared to other social interactions they turn out to be quite simple. Neverthe-





Pedestrian, Crowd and Evacuation Dynamics, Figure 11

Two examples of improved designs for cases with a bottleneck along the escape route of a large crowd, obtained with an evolutionary algorithm based on Boolean grids. People were assumed to move from left to right only. **a** Funnel-shaped escape route. **b** Zig-zag design



Pedestrian, Crowd and Evacuation Dynamics, Figure 12

**a** Conventional design of a stadium exit in an emergency scenario, where we assume that some pedestrians have fallen at the end of the downwards staircase to the left. The dark color indicates high pressures, since pedestrians are impatient and pushing from behind. **b** In the improved design, the increasing diameter of corridors can reduce waiting times and impatience (even with the same number of seats), thereby accelerating evacuation. Moreover, the zigzag design of the downwards staircases changes the pushing direction in the crowd. (After [8])

less, they cause a surprisingly large variety of self-organized patterns and short-lived social phenomena, where coordination or cooperation emerges spontaneously. For this reason, they are interesting to study, particularly as one can expect new insights into coordination mechanisms of social beings beyond the scope of classical game theory. Examples for observed self-organization phenomena in normal situations are lane formation, stripe formation, oscillations and intermittent clogging effects at bottlenecks, and the evolution of behavioral conventions (such as the preference of the right-hand side in continental Europe). Under extreme conditions (high densities or panic), however, coordination may break down, giving rise to “freezing-by-heating” or “faster-is-slower effects”, stop-and-go waves or “crowd turbulence”.

Similar observations as in pedestrian crowds are made in other social systems and settings. Therefore, we expect that realistic models of pedestrian dynamics will also promote the understanding of opinion formation and other kinds of collective behaviors. The hope is that, based on the discovered elementary mechanisms of emergence and self-organization, one can eventually also obtain a better understanding of the constituting principles of more complex social systems. At least the same underlying factors are found in many social systems: Non-linear interactions of individuals, time-dependence, heterogeneity, stochasticity, competition for scarce resources (here: Space and time), decision-making, and learning. Future work will certainly also address issues of perception, anticipation, and communication.

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