CHAPTER 2

ENERGY IN PHYSICAL PROCESSES

So far we have not made use of an important aspect of physical phenomena. Whenever something happens in the physical world, processes are accompanied by an additional quantity—*energy*. We will see that energy plays a unique role, unlike the roles of quantities which are often mistaken for it such as electricity, motion, or heat.[1]

First, we will investigate chains of processes which teach us that a description in terms of amounts of fluids, electricity, or motion alone does not suffice: we need a property which quantifies the coupling of processes—namely energy. After this qualitative introduction, we will discuss quantitative measures for this new quantity by studying waterfalls, and hydraulic and electric processes. Then we shall take a closer look at energy transfer and energy storage. Finally, the description will be extended to rotational and magnetic phenomena.

2.1 ENERGY AND COUPLING IN CHAINS OF PROCESSES

Processes usually occur in chains. One process drives another, sometimes creating long chains. This phenomenon teaches us that there must be a physical quantity which relates one process to the next. We introduce energy to quantify the coupling of processes.

2.1.1 Processes Driving Other Processes

Examples of processes driving other processes are easy to find. Even limiting our view to hydraulic and electric phenomena we can identify coupling. In an electric water pump, we make use of an electric process to drive a hydraulic one (Fig. 2.1), and a

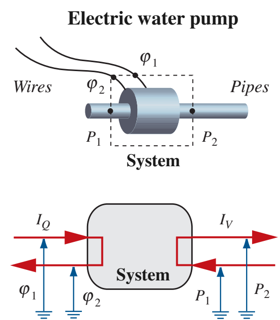

Figure 2.1: A process diagram of an electric pump shows an electric process driving the flow of a fluid. The electric process runs downhill, while the hydraulic process runs uphill. The pump couples the processes.

1. There are good reasons why we mistake other physical quantities for energy—at least in common sense reasoning. See Section I.2 in the Introduction for a brief discussion of this issue.

H.U. Fuchs, *The Dynamics of Heat: A Unified Approach to Thermodynamics and Heat Transfer*, Graduate Texts in Physics, DOI 10.1007/978-1-4419-7604-8_3,

combination of turbine and generator couples a hydraulic process to the flow of electricity. If we extend our view and allow for additional processes such as rotation and heat, we find even more instances of coupling (Table 2.1): a rotational process drives the production of heat in the grinding of one mill stone against another; in a turbine, the flow of water drives a rotational process; and in the flow of a viscous fluid through a pipe the hydraulic process drives the production of heat (Fig. 2.2).

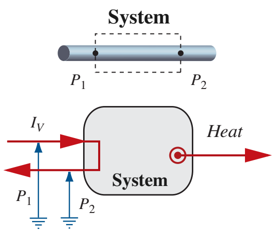

Figure 2.2: In viscous flow, the fluid flows from higher to lower pressure, driving the production of heat (notice the symbol of a source of heat).

Table 2.1: Examples of coupling of processes

	Hydraulics	Electricity	Rotation	Heat
Hydraulics	Hydraulic ram	Turbine plus generator	Turbine	Resistive flow
Electricity	Electric pump	Transformer	Electric motor	Electric heater Peltier device
Rotation	Hand pump	Generator	Gearbox	Friction
Heat	Solar water pump	Thermoelectric generator	Heat engine	Absorption refrigerator

Coupling of processes. A single process is quantified in terms of the two fundamental quantities used to conceptualize it: the fluidlike quantity and its associated potential. In hydraulic and electric processes, these pairs are volume and pressure, and electric charge and electric potential, respectively. These pairs of quantities are different for different processes—they are basically unrelated. Therefore, the question arises how different processes can be coupled. How can one process drive another in a determined manner? Two processes—such as an electric process driving a hydraulic one—must always be related or coupled in the same way. It never happens that the electric process drives the hydraulic process differently at different times. We expect a well defined relation between the two.

In other words, the same process should always accomplish the same result, assuming that conditions do not change. Therefore, we need a measure of how much a process driving another is accomplishing. We may also say that a process is *working* to accomplish a result. The measure introduced for "work" and "accomplishment" has to do with *energy*.

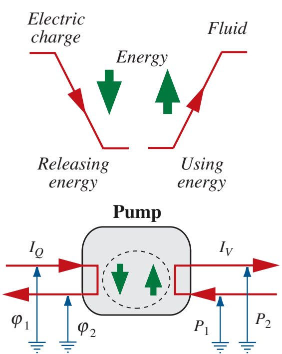

Figure 2.3: An electric pump couples electric and hydraulic processes. The driving process releases energy which is used (bound) in the follow-up process. The amount of energy released or used (bound) is the measure of how much processes "work" or "accomplish." Release and binding of energy are depicted by fat vertical arrows.

Releasing and using (binding) energy. A voluntary process driving another process consists of water or electricity flowing through a potential difference from higher to lower levels. The reverse—involuntary—process consists of a fluidlike quantity being "pumped uphill" through a potential difference (Fig. 2.3, top).

Now we introduce the measure of how a process is working. We say that the driving process *releases energy* in the fall of the fluidlike quantity (Fig. 2.3, top) which is used to drive the follow up process, i.e., the pumping of another fluidlike quantity. In the latter case, we speak of the *binding of energy* to the quantity flowing "uphill." Therefore, we can use the amount of *energy released* as the measure of how much a process works, and the amount of *energy used (bound)* for how much has been accomplished. Releasing and using energy is now introduced as an additional graphical element in the system diagrams depicting physical processes (Fig. 2.3, bottom).

2.1.2 Chains of Processes: Transferring Energy

Consider a chain of processes as in Fig. 2.4. Processes coupled in a device are like single links which can be connected to form long chains. Consider a turbine driven by a current of water, which drives a generator, which in turn can be used to drive the production of heat in an immersion heater (Fig. 2.4). Energy is not only the measure relating two otherwise unrelated processes in a device. The coupling of processes must work through long chains. While the coupling of two consecutive processes inside a device is the result of releasing and binding of energy when the first processes drives the second, two devices are coupled by the flow of quantities such as water or charge from one device to another (Fig. 2.4). Consider the coupling of the generator and the immersion heater in Fig. 2.4. In the generator, energy is bound to the current of charge flowing from lower to higher electric potential. In the immersion heater, energy is released in the fall of charge from higher to lower potential.

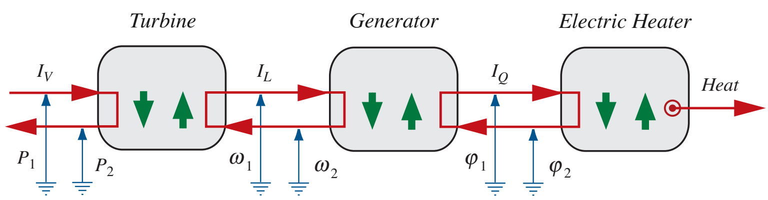

Figure 2.4: Processes can be joined in a chain. Flow processes provide for the coupling between devices or systems. (*L* is the symbol for angular momentum, the fluidlike quantity transferred in rotation; see Section 2.5.)

It seems to be reasonable to assume that the same amount of energy that was bound to the current of charge in the generator is released in the immersion heater. The second process perfectly reverses the first; electric current and voltage (potential difference) are the same. Therefore, we assume that the energy bound in the first process is transferred from the generator to the system following it (Fig. 2.5). In fact, whenever devices are coupled in processes, energy is transferred.

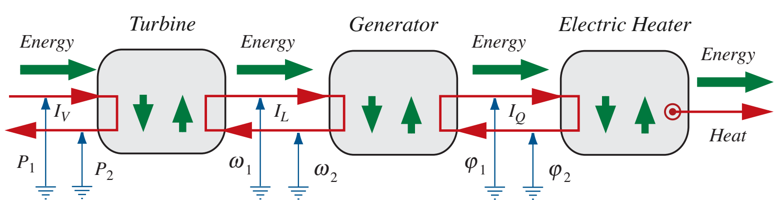

Figure 2.5: Energy is transferred from system to system together with the quantities exchanged in processes—such as fluids and electric charge. The transfer is depicted by fat arrows pointing from one system to the next.

2.1.3 Interrupting and Resuming Processes: The Storage of Energy

Chains of processes need not work continuously. It is possible to interrupt them, and resume them later or at some other place. Therefore, it should be possible to store the energy transferred through a chain so it can be used again for other processes.

Consider a pressure vessel which is being filled with a fluid with the help of a pump (Fig. 2.6). We need energy to operate the pump, i.e., we deliver it to the pump. It is released and used there and then transferred with the fluid to the pressure vessel. The vessel, therefore, is not only a storage device for fluids, but it also stores energy. We can use a pressure vessel filled with a fluid—and therefore with energy—to drive the operation of a turbine and generator, thus emitting the energy which was absorbed before by the vessel.

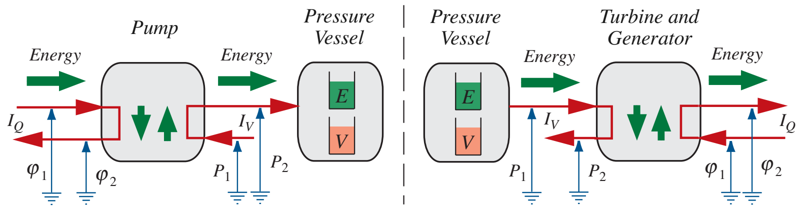

Figure 2.6: Chains of processes can be interrupted and then resumed later. This can be explained in terms of energy storage. The symbolic containers with letters V or E represent the storage of volume and energy, respectively.

2.1.4 Conservation of Energy: Can Energy Be Lost or Created?

Consider different electric pumps. If we drive them in an identical manner all the time, we expect the same result, which may be measured in terms of amount of water pumped to a given height. It is found that different pumps operate differently; some will pump less water than others.

Most processes in nature seem not to run at perfect efficiency—where the efficiency is measured in terms of the energy used in the desired process compared to the energy released by the driving process. Perfect efficiency would correspond to the case when the energy bound is 100% of the energy released. This is what we have assumed for all the processes depicted in the diagrams of Figs.2.3 to 2.6.

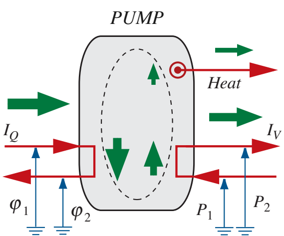

Figure 2.7: Energy is not "lost" in a process which does not run perfectly. Rather, the amount of energy which seems to be missing is driving an unwanted process—the production of heat.

Does this mean that the lower quality pumps lose energy compared to the better ones? Actually, this is not the case. It is found that the engine drives two processes at once—the desired one, and an undesired production of heat—where each process uses part of the energy released. Together they use 100% of the energy available. What we have found to be true for pumps also holds for other processes. The apparent "loss" of energy is associated with an undesired production of heat which accompanies the process the engine was designed for (Fig. 2.7).

There appears to be another reason for loss of energy. Energy storage devices usually lose some of their energy in the course of time. Again, we can explain this not as an actual loss but as the result of "leaking away" of energy. The energy which is not available any longer can always be detected in nature—at least in principle.

In summary, there is no reason to believe that energy can simply disappear. Neither can it be created. If we wish to set in motion a chain of processes, we always need an energy storage device which has to supply the energy running through the chain. Today we take this as one of the fundamental principles of nature: energy cannot be created, nor can it be destroyed: *energy is a conserved quantity*.

2.1.5 The Properties of Energy

Energy is a buzz word for much of what we read about in science and technology. Our usage of the term is often fuzzy which leads to imprecise images of what energy is all about. We usually speak of generating and losing energy, even though energy is conserved. We talk about converting energy, and we give it myriad names, even though there is only one type of energy: we speak of electrical, hydraulic, and mechanical energy, kinetic and potential energy, work and heat, and so forth. Most disturbingly, we mix up energy with the fundamental quantities flowing in physical processes, namely, electricity, heat, and motion.

Actually we have to learn very little about energy and what we learn repeats itself again and again in every field of physics. From what we have discussed so far, we recognize that there is just a single quantity called *energy* which accompanies all processes. This quantity has the following properties:

- Energy is released and used in processes.
- Energy can be transported from system to system.
- Energy can be stored in systems.
- Energy is conserved; it can neither be created nor destroyed.

The second and third items in the list make energy a quantity to which the laws of accounting can be applied; in other words, energy satisfies a *law of balance* (Section 2.4). The properties of energy will now be investigated more carefully and with quantitative means.

2.2 Power: The Rate at Which Energy is Released in a Process

Nature presents us with a perfectly simple process which can serve as the archetype of physical processes—a *waterfall*. Other processes are interpreted analogously: a process consists of the flow a fluidlike quantity from a higher to a lower level (Fig. 2.3). We introduce *energy* as the measure of how much the fluidlike quantity is working, i.e., how much it can achieve, when it falls down a gradient of its potential. *Power* is the rate at which the fluidlike quantity is working. We will say that energy is released in a process, and power—the rate of working—is the rate at which energy is released. Common sense reasoning indicates that the power of a process will depend on the flow of the fluidlike quantity and the height of its fall.[2]

2.2.1 Power of an Electric Process

A simple experiment which can be used to quantify the measure of power is the electric heating of water. The rate of heating of water may be measured in terms of the rate of change of its temperature. If we always take the same amount of water at the same temperature, and observe the same rate of change of temperature, we can be sure that the electric process is "working at the same rate." In terms of energy we may say that this quantity has been released at the same rate in the immersion heater every time we repeat the experiment. On the other hand, if twice as much water can be heated at this rate, the electric processes must run at twice the rate.

Different runs of this experiment show that the rate of change of temperature is the same for identical bodies of water whenever the product of electric current through the

2. This is how Sadi Carnot expressed his idea of the power of heat. See the Introduction for a short discussion of his idea and the roots of common sense conceptualizations of phenomena and processes.

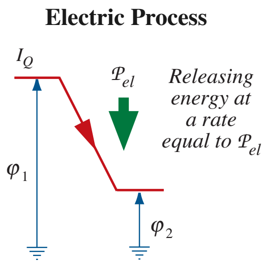

Figure 2.8: Energy released in an electric process. The rate at which energy is released (the power of the electric process) depends upon the flow of charge and the potential difference.

immersion heater and the electric potential difference across the device is the same. In other words, the rate of working of the electric process can be measured in terms of the product of electric current and voltage:

$$\mathcal{P}_{el} = -\Delta\varphi_{el} I_Q \quad \text{or} \quad \mathcal{P}_{el} = U I_Q \qquad \textbf{(2.1)}$$

The symbol $\mathcal{P}$ is used for the rate at which energy is released; from now on we will call this quantity *power*. Therefore, we speak of the *electric power of a flow of electric charge*. The SI unit of power is the Watt (W). The minus sign in the first form of the equation is arbitrary. It means that the power of a voluntary process is counted as a positive number, while the power of an involuntary process is taken to be negative.

The equation can be interpreted graphically using the waterfall image of a process (Fig. 2.8). In an electric process that drives another process, electric charge flows "downhill" through a potential difference and in turn releases energy at a rate that depends upon the flow of charge and the potential difference in the simple manner indicated by Equ.(2.1).

2.2.2 Hydroelectric Power Plants and the Power of a Gravitational Process

We need a measure of the *power of a fall of water*, i.e., the rate at which energy is released in a gravitational process. By allowing water to accomplish a measurable result at a certain rate, we can define the power of a fall of water. Data on hydroelectric power plants yields the information we need (Table 2.2). If we take the product of electric current and voltage at the terminal of the generator as the measure of the rate of working of the water rushing down from the artificial lake to the turbine, we can see which factors determine the rate at which a waterfall releases energy.

Table 2.2: Examples of hydraulic power plants [a]

Hydraulic power plant	Current of Mass I_m / kg/s	Vertical fall of water Δh / m	Voltage and current [b] UI_Q / V · A	UI_Q / $I_m \Delta h$
Bavona	18,000	890	$137 \cdot 10^6$	8.6
Nendaz	45,000	1014	$384 \cdot 10^6$	8.4
Handeck III	12,500	445	$48 \cdot 10^6$	8.6
Chatelard	16,000	814	$107 \cdot 10^6$	8.2
Tiefencastel	16,700	374	$50 \cdot 10^6$	8.0

a. Hydraulic power plants with artificial lakes in Switzerland.
b. Product of voltage and electric current measured for the generator.

The results in Table 2.2 demonstrate that—except for an almost constant factor—the current of mass of water (measured in cubic meters per second) and the vertical drop of the water from the artificial lake to the turbine and generator station (measured in meters) determine the rate at which energy is released by the falling water. In fact, this quantity depends linearly on both factors. Doubling the current of water, or doubling the drop, will each lead to a doubling of the rate of release of energy.

Power of a waterfall. Specifying a waterfall first of all means quantifying the flow of water falling down. This is done with the help of the *current of (gravitational) mass* I_m (measured in kilograms per second). The second obvious quantity determining the properties of a waterfall is the vertical drop Δh (measured in meters).

The power of a waterfall, i.e., gravitational power, depends upon another parameter which is suggested by the fact that the strength of the gravitational field g must play a role. We expect the drop of water through a certain height to accomplish much less on the surface of the Moon than on the surface of our planet. Now we are ready to calculate the rate at which energy is released:

$$\left|\mathcal{P}_{grav}\right| = \left|g \Delta h I_m\right| \tag{2.2}$$

Gravitational Process

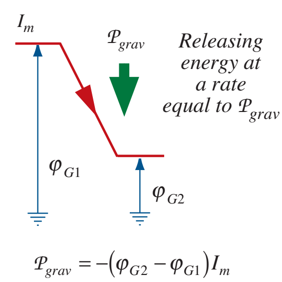

Figure 2.9: Energy released in a gravitational process. The rate at which energy is released (the power of the gravitational process) depends upon the flow of mass and the gravitational potential difference.

Potential. There is a simple graphical interpretation of the formula for the power of a waterfall (Fig. 2.9). We combine the first two factors on the right side of Equ.(2.2) into a new quantity which we call the *level* or *potential of gravitational processes*:

$$\varphi_G = g h \tag{2.3}$$

According to the results in Table 2.2, g should be somewhat larger than 8 N/kg. We know from independent measurements that it is closer to 9.8 N/kg (Section 1.4.4). The discrepancy is a result of the imperfection of the processes in power plants.

We may now write the power of the process as the product of the difference of the gravitational potential and the current of mass falling through this difference of levels:

$$\mathcal{P}_{grav} = -\Delta \varphi_G I_m \tag{2.4}$$

Note the analogy between this result and the one for electricity (Equ.(2.1)). The expression introduced for the gravitational potential is analogous to the one found in Chapter 1 (see Fig. 1.26).

2.2.3 The Efficiency of Processes

Note that the experimental determination of the factor in the last column of Table 2.2 leads to values that are a little bit smaller than g. This is due to the fact that the processes leading from the waterfall to the generator are not ideal: some of the energy released by the water is used for other purposes—mostly for the production of heat as a result of friction.

Ideally, all the energy released in a process would be used for the desired follow up process. Realistically, this does not happen, since parallel processes such as friction bind part of the energy released (Fig. 2.10). To measure the efficiency of the transfer of energy to the desired process, the ratio of the powers involved is used:

$$efficiency = \frac{\mathcal{P}_{desired\ process}}{\mathcal{P}_{driving\ process}} \tag{2.5}$$

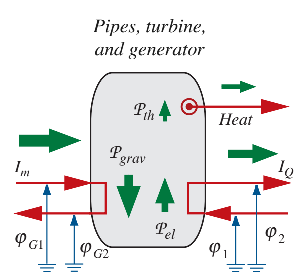

Figure 2.10: In a non-ideal coupling of processes, less than 100% of the energy released is used in the desired process.

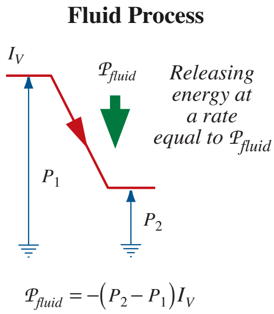

Figure 2.11: Energy released in a fluid process (hydraulic process). The rate at which energy is released (the power of the fluid process) depends upon the flow of volume of fluid and the pressure difference.

The power of an electric process is measured as the product of voltage and electric current (Equ.(2.1)). Applying this rule to the values presented in Table 2.2 we see that the overall efficiencies of modern hydroelectric power plants are quite high, of the order of 80% to 90%.

2.2.4 Power of a Hydraulic Process

Analogical reasoning suggests, and experiments confirm, that the type of relation found for the power of electric and gravitational potentials also holds for hydraulic processes:

$$\mathcal{P}_{hyd} = -\Delta P I_V \tag{2.6}$$

Just consider a turbine driving an electric generator. The electric process is found to be identical as long as the product of the pressure difference and the flux of volume is kept constant. In summary, all types of processes investigated demonstrate the same basic structure (see Section 2.2.6 and Table 2.3): knowing one field of nature helps us to understand other subjects.

2.2.5 Power in Inductive Processes

So far, we have studied devices such as pumps, turbines and generators, artificial lakes and pipes, resistors, electric engines, etc. They all demonstrate that the release of energy is followed by its use when processes are coupled.

Inductive elements (Section 1.6) seem to confront us with a somewhat different case. First, the other devices work strictly in one way—in resistors, volume or charge always flow "downhill"—while processes in inductors run both ways. Second, most of the systems mentioned before can run in steady state without involving the storage of energy; inductive devices, however, work dynamically only, and they also serve as energy storage devices.

Third and most important, it is not readily apparent if there are two processes coupled in such devices, one running "downhill", driving the second one "uphill." Closer inspection shows, however, that there are processes coupled to the obviously visible electric or hydraulic ones. Let us see what they are in the case of electromagnetic induction.

The phenomenon of electromagnetic induction is coupled to the growth or decay of magnetic fields due to electric currents. The magnetic field acts as the storage system for the energy (Section 2.4.5) which is released or bound by the electric current—depending on whether the current is increasing in time, or decreasing. If the electric current through an inductive device is increasing with time, i.e. if $dI_Q/dt > 0$, it runs "downhill" through the inductive potential difference $\Delta\varphi_L$ (Section 1.6). We have just learned that this process is associated with the release of energy at the rate

$$\mathcal{P}_{el} = -\Delta\varphi_L I_Q \tag{2.7}$$

There should be a process running "uphill" on the energy made available. This process exists: it is the building up of a magnetic field which at the same time acts as the storage device for the energy released in the electric process (Fig. 2.12a).

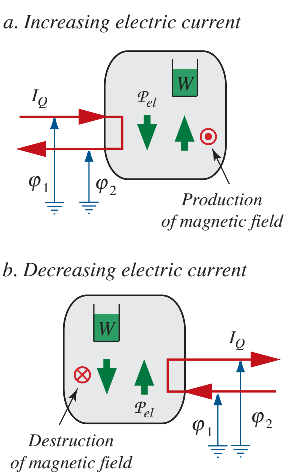

Figure 2.12: In an inductive electric process, energy is released or bound. The process is coupled to the creation or destruction of a magnetic field which acts as the storage device for energy in the inductive element.

If the electric current through the inductive element decreases with time, i.e. if $dI_Q/dt < 0$, the magnetic field decreases as well, releasing energy which is picked up by the electric current. As a result, this current is driven "uphill" through the induced potential difference $\Delta\varphi_L$ (Fig. 2.12b).

The case of hydraulic induction is quite analogous. However, here we do not have a magnetic field associated with the current. Rather it is the quantity of motion of the flowing fluid which is built up or reduced in the device which acts similarly to the magnetic field.

2.2.6 Processes and Power in General

If a fluidlike quantity falls "downhill" it releases energy at a certain rate. This rate we call the *power* of the process. The energy that is released drives a follow-up process "uphill", and it is said to be used by or bound to the flowing quantity (Fig. 2.13). The law for the energy released or used is this:

The power of a process always depends on two factors—the potential difference and the current flowing through this potential difference:

$$\mathcal{P}_X = -\Delta\varphi_X I_X \qquad \textbf{(2.8)}$$

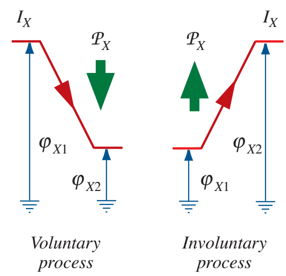

Figure 2.13: Processes and the power of processes. The same fundamental structure is discovered in all physical processes.

The letter X stands for the flowing fluidlike quantity which determines the type of process: mass, volume, and electric charge for gravitational, hydraulic, and electric processes, respectively (Table 2.3). For a given process, we have to use the proper fluidlike quantity and its associated potential. Thus, for a hydraulic process, X corresponds to V, and $\Delta\varphi_X$ corresponds to ΔP.

Table 2.3: Comparison of different processes

	Flowing quantity	Current	Potential	Potential difference	Power
Gravity	Gravitational mass	Current of gravitational mass	Gravitational potential	$\Delta\varphi_G$	$-\Delta\varphi_G I_m$
Hydraulics	Volume of fluid	Current of volume	Pressure	ΔP	$-\Delta P I_V$
Electricity	Electric charge	Current of electric charge	Electric potential	$\Delta\varphi_{el} = -U$	$-\Delta\varphi_{el} I_Q = U I_Q$

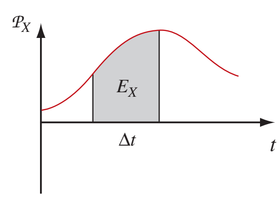

Figure 2.14: The integral over time of the power of a process yields the energy released or used in that process.

Amounts of energy released or used in a process. Sometimes, we want to be able to say "how much has happened" in a process. In other words, we want to know how much energy has been released or bound as the result of a process lasting for a certain period. The amount of energy released in a process—which is sometimes called *work*[3]—can be obtained by integrating the power over time (Fig. 2.14). In general, this quantity can also be calculated as the product of the amount X_e of the fluidlike quantity flowing through a potential difference, and the potential difference $\Delta\varphi$:

$$W_X = -\Delta\varphi_X X_e \quad \text{(2.9)}$$

This expression is correct only if the potential difference stays constant during the process. The relation is particularly simple to prove for a process running at a constant rate. The unit of energy (released) is the Joule (1 J = 1 W·s).

2.2.7 Electric and Hydraulic Circuits: The Balance of Power

An indication of the balance of energy comes from the consideration of energy released or bound in closed electric and hydraulic circuits: the sum of all terms of electric or hydraulic power add up to zero.

This is a consequence of Kirchhoff's second rule which we encountered in hydraulics and electricity (Chapter 1). Consider a simple electric circuit containing a battery, a resistor, and an electric motor (Fig. 2.15). The current of charge flowing through all three elements is the same, and the voltages across them add up to zero:

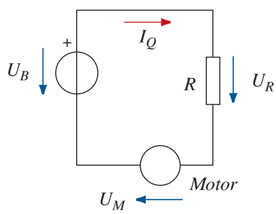

Figure 2.15: The sum of the potential differences in a closed circuit is always zero. Therefore, the sum of the electric power terms of all the elements combined must be zero as well.

$$U_B + U_R + U_M = 0 \quad \text{(2.10)}$$

The current is flowing through each of the elements leading to the release or binding of energy. If we multiply Equ.(2.10) by the current I_Q, we obtain $U_B I_Q + U_R I_Q + U_M I_Q = 0$. Since the terms represent the electric power in the elements, this is equivalent to

$$\mathcal{P}_{el,B} + \mathcal{P}_{el,R} + \mathcal{P}_{el,M} = 0 \quad \text{(2.11)}$$

This means that the energy bound in the electric process in the battery is equal to the energy released in the resistor and the motor combined as the consequence of the fall of the electric charge. In everyday language we say that the energy delivered by the battery is used by the resistor and the motor.

2.3 ENERGY TRANSFER AND ENERGY CARRIERS

Energy released in a process does not come out of the blue, and energy that is bound does not disappear. Either it is transferred into or out of the system or it comes from storage or will be stored (Section 2.4). Here we shall investigate the transfer of energy. There is a simple form of coupling of the flow of the fluidlike quantities with the flow of energy into and out of systems. It is as if mass, volume, and charge acted as *carriers of energy* in the processes they are responsible for.

2.3.1 Energy Carriers, Potentials, and Energy Currents

A simple example demonstrates how nature works. Consider the steady-state flow of a viscous fluid through a straight pipe as in Fig. 2.2. So far we have introduced the concept of energy in the following manner: since the fluid flows from a point of high

3. The words *power* and *work* are used inconsistently in different fields of physics. In mechanics, for example, work means a quantity of energy *transferred*, not released.

pressure to a point of lower pressure, energy is released in the hydraulic process at a certain rate (Fig. 2.16). The energy released is bound in the following thermal process. Remember that the production of heat due to friction is all that happens in the pipe; therefore, we assume that 100% of the energy released is bound in the follow up process.

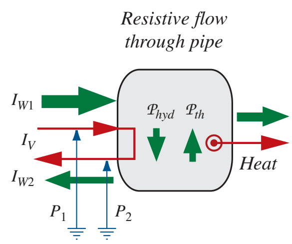

Figure 2.16: The energy released in the "fall" of fluid from high to low pressure must be supplied to the system. Energy is flowing into—and out of—the device with the fluid under pressure. The amount released and used flows out together with heat.

To be specific, let us introduce concrete numbers. Assume there is a fluid current of 0.10 m^3/s, and a pressure drop of 0.50 bar. According to Equ.(2.6), energy must be released at a rate of 5.0 kW. In other words, 5000 J energy are released each second and made available for the production of heat. We believe that the energy must be supplied to the system. Since the only possibility for this to happen is through the flow of fluid into—and out of—the pipe, we say that the fluid flowing under pressure carries with it some energy: we associate an *energy current* with the fluid (Fig. 2.16). In this sense we can call the fluid the *energy carrier* with respect to the system.

Naturally, we should expect the energy current to depend upon two factors. First, it must be proportional to the current of fluid; two equal currents under identical conditions will have twice the effect of a single one. Second, the pressure of the fluid must play a role. Let us see how energy and carrier currents are related.

If a fluid flowing into the system at a certain level (pressure) carries energy, so must the fluid flowing out of the system. Therefore, we assume that the rate at which energy is released is the difference between the currents of energy into and out of the pipe due to fluid flow. Since this makes the difference of the energy flows equal to the product of the pressure difference and the volume current, i.e.,

$$|I_{E1}| - |I_{E2}| = |(P_2 - P_1)I_V|$$

the simplest expression for a single current of energy I_E is the product of the flux of volume and the pressure of the fluid as it enters—or leaves—the system:

$$I_{E,fluid} = P I_V \tag{2.12}$$

There is a simple image which can be used to remember this relation. We may look upon the pressure as the "load factor" of the "carrier current." The current of volume is "loaded" with energy according to the value of the pressure. The flux of energy therefore is the product of a carrier current and its load factor.

Again, this is the structure of energy flow in all fields of physics. Consider the different devices and processes studied so far—gravitational and electric ones in addition to hydraulic: we always arrive at exactly the same relation for the expected energy currents.

> *The flux of a current of energy entering—or leaving—a system is the product of the flux of the carrier current and its associated potential (Fig. 2.17):*

$$I_{E,X} = \varphi_X I_X \tag{2.13}$$

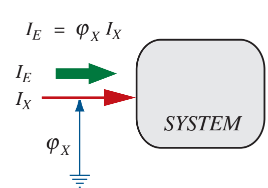

Figure 2.17: The relation of flux of energy, "carrier" flux, and the "load factor," represented in a process diagram.

As we have seen in Chapter 1, the electric potential is not an absolute quantity. Values of electric potentials must always be measured with respect to a chosen level, i.e., the "ground." The same is true for the gravitational potential; here on our planet we commonly measure levels or heights relative to sea level. Of the levels we know so far, only the hydraulic one is absolute. Fluxes of energy in electric and gravitational processes therefore do not have quite the same independent meaning as in fluid flow. Only the difference of two energy currents flowing into and out of a system together with a single current of a fluidlike quantity is independent of arbitrarily chosen levels. This difference is equal to the power of the associated process (Fig. 2.16).

This already tells us that the notion of energy being "carried" by the current of a fluidlike quantity should not bee taken too literally. In particular, as we shall see below, "carried" does not mean that the carrier current "contains" the energy being supplied. We should look upon Equ.(2.13) as meaning that energy always flows *at the same time* as the fluidlike quantity—rather than together with or directly bound to the carrier. It is certainly correct to state that *energy never flows alone*: at the same time, there must always be one or more flows of other physical quantities.[4]

2.3.2 Energy Transfer in Compression

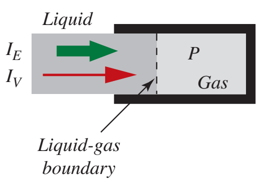

Figure 2.18: A gas in a vessel is compressed by a liquid flowing in. The gas has a pressure P.

There is an example of energy transfer that will play a particularly important role in our study of thermodynamics: energy flows associated with compression or expansion of a (compressible) fluid.

Imagine an imaginary wall separating a gas inside a container from a liquid that flows in or out so the gas is either compressed or expanded (Fig. 2.18). At the liquid-gas boundary we have a flux of volume of liquid at pressure P (which is the pressure of the gas enclosed by the liquid and the walls of the vessel). The current of volume of the liquid is I_V, so there is an energy flux $I_E = PI_V$ entering the gas. At the same time, the gas is compressed at a rate that equals the flow of volume of liquid. Since the volume of the gas is decreasing—we might say, volume of gas is "disappearing"—we describe the effect by a (negative) *production rate* of volume Π_V. In summary, a gas at pressure P being compressed at rate Π_V receives energy at the rate equal to

$$I_{E,comp} = -P\Pi_V \tag{2.14}$$

2.4 ENERGY STORAGE AND THE BALANCE OF ENERGY

In some sense energy is like amounts of water: we can account for it. We have seen this principle applied in the steady state processes investigated in the previous sections. Energy flows through chains of processes, and since we believe that it is a conserved quantity, we know that the flow does not change in magnitude.

4. There is another point that needs to be taken into consideration. When we get into details of transport processes in later chapters, we shall see that there are three fundamentally different types of flows: conductive (flow through matter, caused by a potential gradient), convective (transport of a quantity stored in a fluid, as a consequence of fluid flow), and radiative (transport of a physical quantity with radiation). The relation between energy fluxes and fluxes of fluidlike quantities only holds for conductive transports. Conductive currents are those that are associated with their (own) potentials, so Equ.(2.13) (or Equ.(2.8), for that matter) make sense in this respect. A conductive current I_X is a current associated with or driven by the potential difference φ_X. As we shall see in Chapters 7 and 8, energy transfers in convection and radiation take different forms.

Changes in the flows in the course of time are possible, however, if energy is stored in systems. Only if we take into account storage of this quantity do we arrive at a general law of balance.

2.4.1 The General Law of Balance of Energy

Unless we believe that energy is either generated or disappears if chains of processes are interrupted, we must accept the idea that energy can be stored (Section 2.1). Bodies—and physical systems in general—can contain energy, and they can absorb it and emit it, thus changing the amounts stored.

As in the case of amounts of water—or amounts of electric charge—a law of balance relates what happens to the quantity stored as the consequence of flow into and out of the system. Because energy can neither disappear nor appear out of the blue, we know that amounts stored can only be changed as the result of flows. This is what we call the *law of balance of energy* for a system:

> *Energy can be stored and it can flow. The sum of all fluxes of energy $I_{E,net}$ with respect to a system tell us how fast the amount of energy stored will change:*
>
> $$\frac{dE}{dt} = I_{E,net} \tag{2.15}$$
>
> *(Fig. 2.19). This form holds for every moment. For a process lasting for a certain period, we may also say that the change of the amount of energy stored is determined by the total amount of energy $E_{e,net}$ transferred into or out of the system:*
>
> $$\Delta E = E_{e,net} \tag{2.16}$$

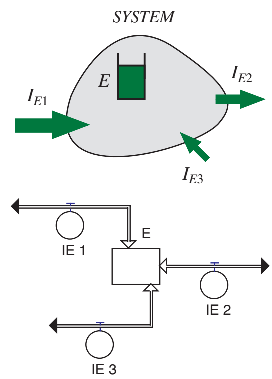

Figure 2.19: The law of balance of energy resembles the law of balance of amounts of water. The energy content of a system can only be changed as the result of flows of energy into and out of the system. Bottom: Graphical representation of the law of balance in a system dynamics diagram.

E_e is called an amount of energy exchanged as the result of a process. Note that one of the properties of energy—namely that it can be released and bound—does not appear in a law of balance. Releasing and binding take place inside the system being considered whereas a law of balance only speaks of the relation between amounts stored and amounts flowing into and out of the system.

2.4.2 Storing Energy with the Help of Gravity

We know how to calculate energy transfers. If we add to this the knowledge contained in the law of balance, we can determine changes of quantities of energy contained in particular systems. A particularly useful and graphically intuitive example is the storage of liquids in containers in the gravitational field. If we fill a tank with water, we add energy to the system along with the fluid, and this energy can usually be regained if the water is let flow out.

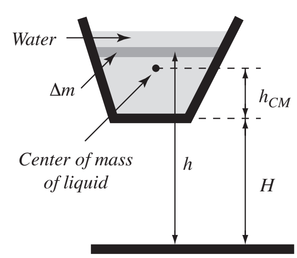

Figure 2.20: Water stored in an artificial lake contains a certain amount of energy relative to an arbitrary zero level.

Imagine a storage device such as an artificial lake having a certain shape. Water contained in it can flow down to a power station which is located at a certain level H below the bottom of the lake (Fig. 2.20). If we imagine a small amount of water having mass Δm at level h lowered to $h = 0$, the quantity of energy flowing out of the system is equal to $E_e = gh\Delta m$. This quantity is different for different layers of water in the lake. It is quite intuitive that, on average, all the water is lowered from the level of the center of mass of the liquid to the bottom. In other words, the water comes from an average level $H + h_{CM}$. The total energy that flows out, and therefore the change of the energy of the storage system, equals

$$\Delta E = g\left(H + h_{CM}\right)m \tag{2.17}$$

There is a special form of this for a straight walled tank sitting at level $h = 0$ and being filled to level h_0. With $H = 0$, $h_{CM} = h_0/2$, and $m = \rho A h_0$, Equ.(2.17) becomes

$$\Delta E = g\left(0 + \frac{1}{2}h_0\right)\rho A h_0 = \frac{1}{2}\rho g A h_0^2$$

Here, A is the cross sectional area of the tank. If we introduce a gravitational capacitance of the storage device:

$$C_G = \frac{\Delta m}{\Delta\left(gh\right)} = \frac{\Delta\left(\rho A h\right)}{\Delta\left(gh\right)} = \frac{\rho A}{g}$$

the former expression can be converted to

$$\Delta E = \frac{1}{2}C_G\left(gh_0\right)^2 = \frac{1}{2}C_G \Delta\varphi_G{}^2 \tag{2.18}$$

2.4.3 Storing Energy in Pressure Vessels

The derivation for the change of energy of a pressure vessel resulting from the change of volume of liquid stored in it, proceeds along similar lines to what we just did. Let me do it here in the general form. A pressure vessel is described by its elastance or its (hydraulic) capacitance $C_V(P)$ which, in general, is a function of pressure (see Section 1.4.2). If we add fluid to the vessel at pressure P, there is an energy current equal to PI_V accompanying the current of liquid. The integral of this energy flux over time equals the energy communicated to the tank which is equal to the change of energy stored:

$$\Delta E = \int_{t_0}^{t_f} I_E\, dt = \int_{t_0}^{t_f} P\dot{V}\, dt = \int_{t_0}^{t_f} PC_V\dot{P}\, dt$$

or, after a transformation of the integral,

$$\Delta E = \int_{P_0}^{P_f} C_V P\, dP \tag{2.19}$$

If we consider the case of constant capacitance, this results in

$$\Delta E = \frac{1}{2}C_V\left(P_f^2 - P_0^2\right) \tag{2.20}$$

Compare this to Equ.(2.18). We see that it is equivalent to what we obtained for a straight walled open tank in the gravitational field which corresponds to C_G = const.

2.4.4 The Energy Content of Capacitors

The derivation of the energy content (or the change of energy) of electrical capacitors does not add anything new. The result for capacitors having constant capacitance C is:

$$\Delta E = \frac{1}{2} C\left(U_f^2 - U_0^2\right) \tag{2.21}$$

We can now summarize the results for storage of energy in simple gravitational, hydraulic, or electric systems having constant capacitances (see Table 2.4). Note that the results are given in terms of energy changes. Commonly, absolute energy contents are not known and are not needed, but we can always speak of an *energy content* relative to an arbitrarily chosen zero level.

Table 2.4: Capacitors with constant capacitance

	Gravitation	Hydraulics	Electricity
Capacitance	C_G	C_V	C
Potential difference	$\Delta(gh)$	ΔP	U
Stored quantity	$\Delta m = C_G \Delta(gh)$	$\Delta V = C_V \Delta P$	$\Delta Q = CU$
Stored energy	$\Delta E = 1/2 C_G \Delta(gh)^2$	$\Delta E = 1/2 C_V \Delta P^2$	$\Delta E = 1/2 CU^2$

2.4.5 Storing Energy in Inductors

Energy can also be stored in inductive electric and hydraulic elements. We can use Fig. 2.12a to demonstrate how to calculate the energy content of inductors. The derivation goes along the line of what we have seen in Section 2.4.3. The result for inductors having constant inductance is:

$$\Delta E = \frac{1}{2} L\left(I_{X2}^2 - I_{X1}^2\right) \tag{2.22}$$

Here, I_X represents either the electric current I_Q or the current of volume I_V. In the electromagnetic case, the energy is stored in the magnetic field. In a fluid system, the energy is the energy of motion of the fluid.

2.5 Analogy Once More: Simple Rotational Motion

To demonstrate the power of analogical reasoning in physics once more, let us take a brief look at some simple phenomena from rotation. Now we can include the energy concept as well. If we restrict our discussion to the motion of bodies around a fixed axis, models turn out to be particularly simple, having a structure very similar to those we constructed for fluid and electric systems in Chapter 1.

Rather than developing a formal description of concepts of rotation, I shall limit myself to the construction of a few dynamical models. Ideas will be discussed and listed as we go along.

2.5.1 Rotational Collision of Two Flywheels

A simple phenomenon demonstrates the nature of rotation or, put differently, the source of a successful conceptualization of such phenomena. In an experiment, two plexiglass disks are mounted on the same vertical axis (Fig. 2.21). They are attached to the axis with ball bearings which allows them to spin more or less freely. The upper wheel can be lifted slightly from the lower one. If it is made to rotate, the lower one stays at rest. When the upper flywheel is let fall onto the second one, it interacts with it in a way that the former slows down as the latter spins up (Fig. 2.21, center).

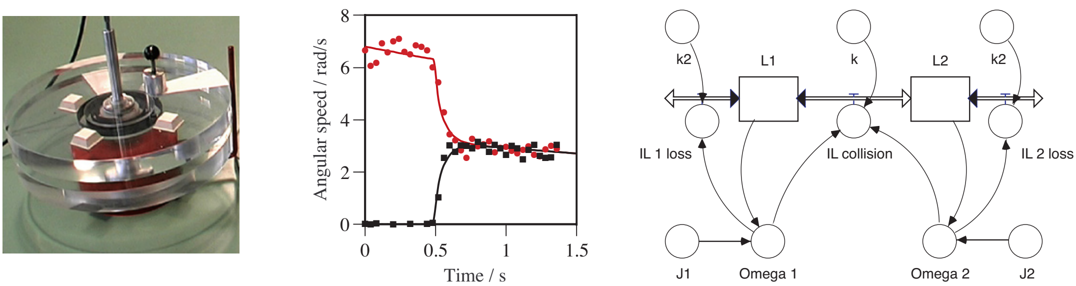

Figure 2.21: Two identical plexiglass flywheels rotate about the same vertical axis and interact (photograph on left). In an experiment, one wheel makes a second one spin up as it is slowing down (see the graph at the center which shows the angular speed of the wheels; dots: experimental data). Right: System dynamics diagram of a model of this system. Simulation of model: Solid lines in the graph (center).

Take a closer look at the data of the experiment in Fig. 2.21. The gross features are these. The upper wheel spins at constant rate—its angular speed is (almost) constant. When it touches the lower wheel, its angular speed goes down while the angular speed of the lower flywheel goes up. Within a very short period of time the angular speeds of the two wheels become the same, roughly half of the original speed. The wheels continue to spin at constant angular speed.

A second look confirms the first with the exception of the fact that angular speeds are not constant during the phases when the first wheel spins freely and when they spin together. During these periods, the angular speeds decrease.

This looks suspiciously like phenomena we have seen before in fluid or electric systems. Think about it—which simple fluid system would yield data similar to the one we have here? A little consideration reveals that the levels of a liquid in two communicating tanks each having an additional outflow will behave quite similarly.

All of this suggests that we can conceptualize rotational processes as follows. A spinning body possesses a "quantity of rotational momentum," the more it has the faster it moves. This "quantity of rotational momentum" which is officially called spin or angular momentum can be communicated to other bodies through rotational interaction. The flow of angular momentum measures the strength of the interaction. So we expect a law of *balance of angular momentum* for a body:

$$\frac{dL}{dt} = I_{L,net} \tag{2.23}$$

L is the symbol for angular momentum (spin) and I_L denotes fluxes of this quantity. In the case of our experiment this means we should represent two storage elements for spin with their associated flows (Fig. 2.21, right). There is the flow of angular momentum from wheel to wheel and one flow from each wheel to the environment representing the effect of friction (we know that friction makes a wheel slow down):

$$\frac{dL_1}{dt} = -I_{L,collision} - I_{L1,loss}$$
$$\frac{dL_2}{dt} = I_{L,collision} - I_{L2,loss} \qquad \textbf{(2.24)}$$

We expect the fluxes of spin to somehow depend upon the speed of rotation. The speed at which a wheel spins is called angular speed. The simplest idea for a relation between spin and angular speed is

$$L = J\omega \qquad \textbf{(2.25)}$$

ω is the symbol for angular speed, and J (the moment of inertia) is the measure of how much angular momentum a wheel needs to rotate at a given speed. The latter quantity is clearly analogous to a *capacitance* (Section 1.4.2). The intuitive meaning of angular speed is a *level*: levels adjust in communicating reservoirs. When Equ.(2.25) is applied to both wheels, the angular speeds can be calculated from the angular momenta.

This allows us to formulate ideas for the fluxes of angular momentum. If we simply apply the ideas from fluids or electricity, we might start with linear relations between flows and speed differences:

$$I_{L,collision} = -k(\omega_2 - \omega_1)$$
$$I_{L1,loss} = k_2\omega_1 \qquad \textbf{(2.26)}$$
$$I_{L2,loss} = k_2\omega_2$$

All we still need are proper initial values for the angular momenta of the two wheels. These are chosen according to the observed initial angular speeds. Choosing values for J and adjusting the flow constants k in Equ.(2.26), we can try to fit simulation results to data as in the graph of Fig. 2.21 (center). Clearly, the agreement between model and experiment is not bad at all. This does not mean, however, that we should already be satisfied with details of the model such as the forms for the flows in Equ.(2.26). Observations are not detailed enough to make a final judgement, but we can be sure that the structure of the model of rotational motion leads in the right direction.

We need values for the moments of inertia of the wheels to actually make the calculations, but we know that we can choose them arbitrarily—a change of J translates into a change of the flow constants k by the same factor. This means that, on the basis of considerations from rotational mechanics alone, we could arbitrarily define units for the moment of inertia or, equivalently, for angular momentum. However, rotation can couple to other phenomena and if these have been specified already, unit values must agree to make this coupling unique.

We have seen in this chapter that energy provides for a means of quantifying this coupling, so here is an example of the utility of the energy principle. Consider how we might apply this principle. Simply on the basis of analogy, we can formulate the expression for the energy stored in a spinning wheel; it should take the form found in

Table 2.4. Now we can write an expression for the balance of energy of the wheels before and after the interaction:

$$\Delta E_{i \to f} = \frac{1}{2}\left(J_1 + J_2\right)\omega_f^2 - \left(\frac{1}{2}J_1\omega_{1i}^2 + \frac{1}{2}J_1\omega_{2i}^2\right) < 0$$

The energy of the spinning wheels is smaller after the collision: energy has been released and used to produce heat. If we could measure the energy released, say by measuring how fast a body of water is getting warmer and comparing the result to what we know from how electricity warms the water (Section 2.2.1), we have additional information which lets us quantify the moment of inertia of the wheels.

2.5.2 Electric Breaking of a Flywheel

Here is a practical example that demonstrates the use of the energy principle in a dynamical model. A flywheel is attached to an electric generator (Fig. 2.22, left). It spins and drives the generator. If we hook up a resistor to the generator, the wheel will spin down as shown in Fig. 2.22 (right). As expected, the resistor will get warm. The angular velocity of the flywheel is close to an exponentially decaying function. (The angular speed is measured with the help of a second generator whose voltage is an indication of how fast the wheel spins.) Let us build a model for this experiment and experience how energy considerations become an integral part of the solution of the problem.

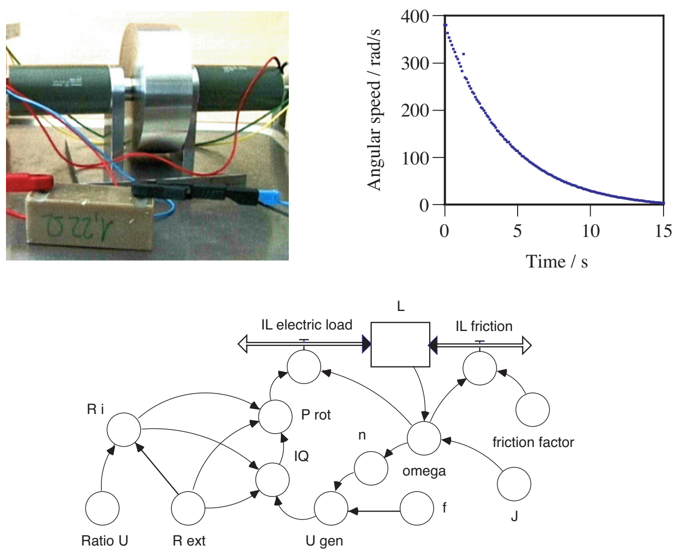

Figure 2.22: A flywheel drives a generator which is hooked up to a resistive load (top left). The angular speed decreases (top right). SD model diagram (bottom). n: number of revolutions per minute; f: conversion factor for rpm to open circuit voltage of the generator (U_gen); Ratio_U: ratio of open circuit voltage to voltage with load; R_ext: resistance of external load; R_i: resistance of generator; P_rot: rotational power of the flow of angular momentum.

The phenomenon reminds us of a container or an electric capacitor discharging. From the previous model in Section 2.5.1 we can be assured that the idea of discharging can be transferred to a spinning wheel as well. There are two phenomena affecting the behavior of the flywheel. First, the body would slow down even if we did not have a load connected to the generator. There is friction which, by the way, could or even should be quantified in an independent experiment. Then there is the effect of the electric circuit upon the rest of the system which can be best understood if we draw a process diagram of the devices making up the system (Fig. 2.23). The wheel emits angular momentum through the generator—this is why it decelerates. In the generator, the angular momentum flows from the high level, i.e., the angular speed of the spinning body, to the ground. If analogy can be used as a guide, energy must be released in this process which then drives the electric process of the flow of charge through the resistors in the circuit (there is the external load, but just as importantly, the wires making up part of the generator have and electric resistance as well). Energy is released in the electric process which is used to produce heat.

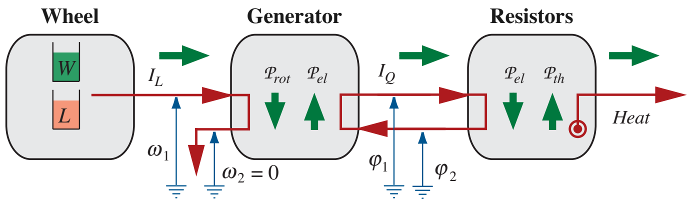

Figure 2.23: Process diagram of wheel, generator, and resistive load(s).

By applying the argument in reverse we can calculate the angular momentum flux from the wheel through the generator. The angular momentum of the flywheel lets us find the angular speed which converts to the open circuit voltage of the generator. With values for the resistances in the electric circuit we calculate the electric current. Finally, electric current and open circuit voltage of the generator yield the electric power which is equal to the rotational power. The important new idea that completes the model concerns the expression for rotational power:

$$\mathcal{P}_{rot} = -\Delta\omega \, I_L \quad \textbf{(2.27)}$$

By now we have become accustomed to formulating expression for the power of a process (Section 2.2). Rotation is not any different. If angular momentum flows through a difference of angular speeds, energy is released at a rate given by Equ.(2.27). The idea is visualized by the standard waterfall diagram in Fig. 2.24.

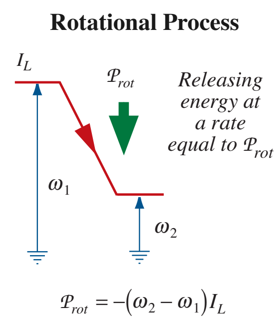

Figure 2.24: Energy released in a rotational process. The rate at which energy is released (the power of the rotational process) depends upon the flow of angular momentum and the angular speed difference.

2.6 THE EXCHANGE OF ENERGY IN MAGNETIC SYSTEMS

Consider the concrete example of a paramagnetic substance filling the interior of a long straight coil. If we turn on an electric current through the coil, a magnetic field will be set up which leads to the magnetization of the body inside. Naturally, this process involves the transfer of energy to the magnetized body.

It is known from electromagnetic theory that the rate of transfer of energy may be expressed in terms of the product of the *magnetic tension* U_{mag} and the Hertz *magnetic current* I_{mag}:[5]

5. Herrmann and Schmid (1986).

$$I_{E,mag} = U_{mag} I_{mag} \tag{2.28}$$

The magnetic tension and the magnetic current are defined as follows:

$$U_{mag} = \int_C \mathbf{H} \cdot \mathbf{s} dr \tag{2.29}$$

$$I_{mag} = \int_{\mathcal{A}} \dot{\mathbf{B}} \cdot \mathbf{n} dA \tag{2.30}$$

These definitions are similar to the quantities known from electricity. $\mathcal{A}$ and C stand for *surface area* and *curve*, respectively. The former is the path integral of the magnetic field **H**, while the latter is the rate of change of the magnetic flux. Obviously, the magnetic flux plays the role of the extensive magnetic quantity, and its rate of change replaces the rate of flow of electric charge in this analogy.

Let us now derive these quantities for the special example mentioned above. The magnetic tension in the uniform field of the coil is equal to

$$U_{mag} = LH \tag{2.31}$$

where L is the length of the coil. Since the magnetic flux density **B** is taken to be uniform over the cross section of the coil, the magnetic current turns out to be

$$I_{mag} = A\dot{B} \tag{2.32}$$

so that the magnetic energy current is equal to

$$I_{E,mag} = VH\dot{B} \tag{2.33}$$

With a paramagnetic substance in the field, the magnetic flux density may be expressed as follows:

$$B = \mu_o \left(H + \frac{M}{V} \right) \tag{2.34}$$

M is the total magnetization of the body. If we consider only the body as the physical system and neglect the field in empty space, the magnetic energy current associated with the magnetization of the paramagnetic substance is

$$I_{E,mag} = \mu_o H \dot{M} \tag{2.35}$$

There is an interesting point to be made about the example just treated and the compression of a gas (Section 2.3.2). The power involved in the compression of a simple fluid and in the magnetization of a body involves the production rate or the rate of change of an extensive quantity rather than the transfer of a quantity such as charge or mass. Obviously, there are physical processes in which quantities are not transported. Rather, they change their values directly at the locations where they are to be found. Such processes may be interpreted in terms of the creation or the destruction of the quantity involved. Production and destruction join transport processes in our description of nature.

EXERCISES AND PROBLEMS

1. Viscous oil is to be pumped from a shallow container into one lying 10 m higher up. The pipe has a diameter of 5.0 cm and a length of 20 m. If the mass flux is required to be 10 kg/s, how large should the power of the pump be? Draw the process diagram of the system and the processes. Neglect the acceleration of the fluid. Take values of 800 kg/m^3 and 0.20 Pa · s for the density and the viscosity of the fluid, respectively.

2. A large oil tank is filled through a pipe at its bottom (see Fig. P.2). The flow of oil is assumed to be laminar. (a) Derive the instantaneous power of the ideal pump in terms of the length and the radius of the pipe, the viscosity and density of the oil, and the height of the oil in the tank. (b) Express the energy needed to fill the tank up to a certain height in terms of the hydraulic capacitance. (c) Where has the energy that was supplied gone to?

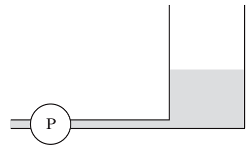

Figure P.2

3. If you fill the tank of Problem 2 through a pipe which leads to the top of the tank (Fig. P.3), how much energy is required? How does this compare to the results of those problems? Has energy been lost?

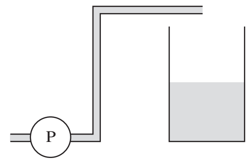

Figure P.3

4. Derive the expression for the energy stored in a charged capacitor by considering the process of charging. Compare the result to the analogous hydraulic expression.

5. Consider two capacitors, one of them charged, connected in a circuit. (a) Calculate the final charges and voltages of the capacitors in terms of the initial charge and the capacitances. (b) Is the energy of the capacitors conserved? (c) Translate the problem into an equivalent hydraulic one.

6. A capacitor (capacitance 150 μF) and a resistor (resistance 1500 Ω) are connected in series to a battery (voltage 50 V) at time $t = 0$ s. The initial charge of the capacitor is equal to zero. (a) Derive the equation of balance of the charge of the capacitor. Derive the formula for the electric current as a function of time from its solution. (b) Draw the process diagrams for the battery, the resistor, and the capacitor. (c) What are the values of the electrical power of the three elements at 0.15 s? (d) What are the values of the corresponding electrical energy currents at that point in time? (e) Calculate the rate of change of the energy of the capacitor. (f) How large is the rate of change of the energy of the resistor?

7. A small photovoltaic panel consisting of 21 cells arranged in series is exposed to sunlight. (The surface area of a single cell is about 15 cm^2.) It is connected to a load resistor with variable resistance. Voltage and electric current for the load resistor have been measured for different values of the resistance (see Fig. P.7). Irradiation was about 60 W/m^2 for the first, 200 W/m^2 for the second, and nearly 400 W/m^2 for the third (the highest) curve. (a) Calculate the electric power of the panel for a voltage of 4.0 V for the three characteristic curves. (b) Determine the maximum values of the electric power for the three cases. What are the values of the load resistance for the maximum power point for the three curves? (c) Determine the efficiency of the panel for maximum power point conditions for the three cases.

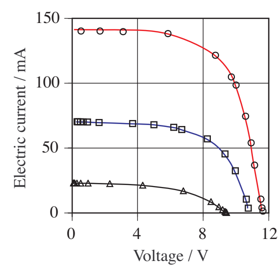

Figure P.7

8. Imagine an artificial mountain lake in the shape of a cuboid of 10.0 km^2 surface area, and 50 m depth. The turbine station of a power plant is located 150 m below the bottom of the lake. Assume that the lake can be filled and drained once a year. (a) How large is the energy stored with the water if we take the bottom of the lake as our reference level? (Assume the lake to be full.) (b) How large is the energy stored with the water if we take the turbine station as our reference level? (c) How large is the power of the water flowing out of the lake to the power plant if the lake is full? If it is almost empty? Take a flow of 20.0 m^3/s. (d) How much energy is released by the water flowing out of the lake and down to the power plant if the lake is drained completely once a year? (e) Now we cover the lake with photovoltaic cells. How much energy can we gain from them in one year if we assume the cells to have an efficiency of 10%.

9. Assume that 4 capacitors of 1.0 F capacitance each are connected in parallel. We want to charge them with the help of the photovoltaic panel of Problem 7 (Fig. P.9.1). The sun shines at 400 W/m^2 which yields the characteristic curve shown in the diagram (FIg. P.9.2).

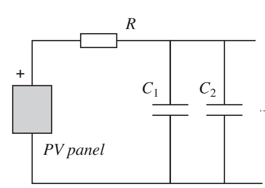

Figure P.9.1

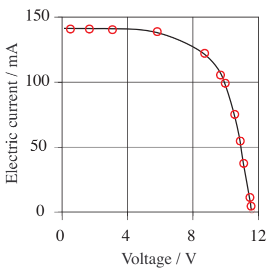

Figure P.9.2

A resistor is between the panel and the capacitors. (a) Choose the resistor so that if it were the only element in the circuit, we would have maximum power conditions. (b) What will the electric current be right at the beginning (when the capacitors are still uncharged)? What is the energy current flowing into the capacitors at that moment? (c) At a certain moment, the current through the circuit is 80 mA. What is the voltage across the capacitors at that time? What is the energy current flowing into the capacitor at that time? What are the electric power of the cells and the power of the resistor? (d) At a certain moment, the voltage across the capacitors is 5.0 V. What is the current through the circuit at that moment? (You will have to solve a set of nonlinear equations.) What will the energy current flowing into the capacitor be?

10. A large and shallow lake is going to be filled through a horizontal pipe with a length of 10 km. Initially, the lake is empty; in the end it is supposed to contain 10^5 m^3 of water. Assume the hydraulic resistance to be modeled by the law of Hagen and Poiseuille; i.e., take the volume flux to be proportional to the pressure difference across the pipe. The pressure drops by 10^2 Pa per meter of length at a volume flux of 1.0 m^3/s. While the lake is being filled, water evaporates from its surface at a rate of 0.10 m^3/s. (a) If the volume flux is constant and equal to 0.50 m^3/s how much energy is required for pumping while filling the lake? (b) How large should the (constant) volume flux be for the energy required to fill the lake to be minimal?

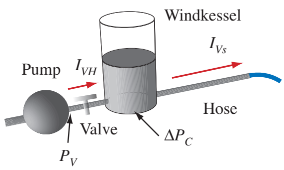

Figure P.11.1

11. Fig. P.11.1 shows a windkessel model of the systemic blood flow circuit. Resistances and capacitance are assumed to be constant. The capacitance of the aorta is $C = 2.0 \cdot 10^{-9}$ m^3/Pa. The resistance between pump (heart) and container (aorta) is $4.0 \cdot 10^7$ Pa·s/m^3, the one for the systemic vessels is $4.0 \cdot 10^8$ Pa·s/m^3. The diagrams (Fig. P.11.2) give data of a simulation of the model for one cardiac cycle of 0.60 s. In Fig. 11.2 (left), we see the pressure at the exit of the pump (P_V) and the capacitive pressure difference for the blood in the aorta (ΔP_C). The volume currents out of the heart (I_{VH}) and out of the aorta (I_{Vs}) are shown in Fig. P.11.2 (right). Data apply to the case of a sheep.

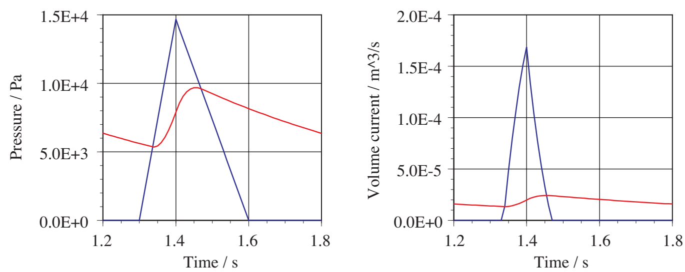

Figure P.11.2

(a) Identify the functions in the diagrams. (b) Determine the energy current associated with the blood flow from the heart and sketch the result as a function of time. Use this to determine the amount of energy flowing from the heart in one cycle. (c) How can the result from (b) be used to estimate the energy released in one cycle by the heart? What is a realistic value for the energy use of the heart of a sheep? (d) Determine the (lost) power for the flow from the heart to the aorta and use this to calculate the energy lost due to friction. (e) From when until when does the energy of the blood in the aorta increase? Determine the maximum change of energy of the blood in the aorta. (f) Formulate the law of balance of energy of the blood in the aorta in instantaneous form and add constitutive expressions to the equation.

12. Derive the expression for the energy contained in an inductive element (consider the process of starting a current flowing through a circuit containing a battery, an inductor, and a resistor). Translate the result for hydraulics. Show that you can obtain the formula for the inductance of a pipe with fluid by comparing the energy of the inductive element with the kinetic energy of the fluid in the pipe.