Chapter 3
A Situative Perspective on Cognition and Learning in Interaction

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My goal in this paper, and generally in my research, is to contribute to the development of a theory of cognition and learning in interaction. The ideal result of this would be a theory that explains dynamic aspects of interpersonal interaction with the same degree of rigor and specificity that are achieved by sociolinguistic accounts, and explains the informational contents of interactions with the same degree of rigor and specificity of information-processing accounts.

[1] An effort to develop integration between these two lines of research has been under way for about a decade. The general strategy is to observe and analyze activity involving understanding, reasoning, and learning by groups of people. Different investigators are approaching this in different ways. Some are taking cognitive theory as the basis and extending the analyses of cognitive processes by including interactions between individuals. These include studies by investigators such as Dunbar (1995), Okada and Simon (1997), and Schwartz (1995). Other investigators are taking interactional theories of activity as the basis and incorporating analyses of information structures in analyses of interaction. I have been working with the second of these approaches,[2] as have many others, such as Goodwin (1995), Hutchins (1995), and Ochs, Jacoby, and Gonzalez (1996).[3]

Analyses of cognition in activity differ in their levels of aggregation in two ways. One difference is in whether the analysis focuses on individual information processing, treating interaction with other people and resources in the environment as a context, or focuses on processes of an activity system composed of the several individuals present (if there are more than one) along with their material and informational resources. The other difference in aggregation involves the complexity and time scale of the activity that is analyzed.

Table 3.1 presents a sketch of some research topics in a matrix of these two ways of varying levels of aggregation. The left column mentions kinds of cognitive phenomena that are studied in research on cognition, learning, and cognitive development. These vary in their levels of aggregation involving complexity and

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Table 3.1 Four levels of analysis of cognition in activity

Achievements to be explained	Analysis from cognitive science and psychology	Situative analysis
(4) Conceptual growth, commitment to learning goals, sustained, persistent participation in learnin practices	• Cognitive development, conceptual change • Academic self-esteem, general motivational traits, motivation in subject-matter domains • Ways of knowing	• Changes in discourse practice; legitimate peripheral participation • Intellective identities regarding learning, academic learning, and learning in specific school subjects; positional identities in school and classrooms with mutual engagement and productive agency in relation to a community's joint enterprise of learning
(3) Adopting tasks, expending effort toward accomplishing goals	• Understanding task instructions • Task-level motivation	• Practices that encourage problematizing and resolving and that position students in disciplinary discourse with competence, authority, and accountability in participation structures
(2) Emergent understanding	• Generative (e.g., analogical) reasoning, heterogeneous representations • Flexibility in thinking	• Negotiating different interpretations for mutual understanding • Problematizing, resolving, and positioning in interaction • Explaining
(1) Routine comprehension, conceptual understanding, problem solving, including performing procedures, search in problem spaces, reasoning, planning, skill acquisition	• Information-processing operations • Search heuristics, schemata, strategies • Acquiring components of procedures	• Conversational contributions, mutual attention, understanding propositions and reference • Conceptual common ground, patterns of reasoning in practice • Shared repertoire of schemata and procedures

time scales. The second column mentions theoretical concepts in cognitive science and psychology that provide interpretations and explanations of findings at the various levels. The third column mentions explanatory and interpretive concepts in the situative perspective at the corresponding levels of complexity and time scale. The cognitive-scientific and psychological concepts refer to processes that are usually hypothesized to occur at the level of individual information processing, thinking, and learning. The situative concepts refer to processes that are hypothesized to occur at the level of activity systems and joint participation in communities of practice.

The strategy for integrating these two lines of research that I find promising involves considering information processing as an aspect of interaction in activity systems. Standard cognitive-science analyses hypothesize that structures of information are constructed by individuals as they understand and reason about situations they are in and solve problems. The processes that generate these structures use other, general structures that are hypothesized to be retrieved from memory. Motivations to engage in tasks and to participate in practices are hypothesized as differential traits of individuals. In the theoretical accounts that I call situative, information is assumed to be constructed in the interactive processes of activity systems. More general structures of information, including practices of discourse and problem-solving strategy, are hypothesized to be in the common ground of participants in activity. And differences between individuals in their engagement in tasks and commitments to practices are considered as different ways in which individuals are positioned in their participation.

In an early draft of his commentary, Bredo (Chapter 6) suggested that my analysis focuses primarily on "an integration of 'task' and 'social' aspects of interaction" (cf., pp. 116–117). I believe that he is basically correct in this.[4] I hope that the analyses I review and present here are contributing to a more integrated theory that includes accounts of information structures that are the contents of interaction and the interactional processes in which those structures are generated. This is different from a theory that integrates concepts that refer to processing of information by individuals and concepts that refer to processing of information in activity systems. I believe that a program with that latter goal is feasible. It would involve maintaining the interpretations of concepts in the second column of Table 3.1 as properties of individual cognition, and coordinating those concepts with the kinds of concepts that are in the third column of Table 3.1. That is not the theoretical program I am pursuing. Instead, I am appropriating concepts and representations that have been developed in individual cognitive science and psychology and reinterpreting them as aspects of interaction in activity systems and social practice. This is aimed toward a theory that integrates the task and social aspects of interaction – or the semantic and systemic aspects of interaction – in the sense that it aspires to a theory that is primarily about interaction in activity systems, and includes analyses of structures of information that the participants have in common ground and generate in their activities of accomplishing tasks.

Level 1 Among the contributions of cognitive science, two of the most fundamental are models of text comprehension (e.g., Kintsch, 1998) and models of problem solving (e.g., Brown & Burton, 1980; Greeno, 1978; Newell & Simon, 1972). The information-processing theory explains routine understanding and reasoning as processes of constructing coherent representations that connect various pieces of information according to stored schemata and fill in pieces of information that are missing using stored inference rules The content of understanding depends on schemata that are retrieved from memory. Most understanding that occurs, in this theory, involves assimilation of information to known schemata, guided by comprehension strategies. The information-processing theory of problem solving similarly relies on hypotheses about representations of problem situations in the form of problem spaces that include the problem solver's knowledge of operations, procedures, schemata, and strategies. Problem solving consists of representing goals and subgoals and selecting operations and procedures whose effects will provide progress toward the main problem goal. Learning involves acquisition of new information structures, corresponding to actions that occur during problem solving, in a form that supports performance of the actions appropriately as components of procedures (e.g., Anderson, 1983; Newell, 1990).

The propositions and diagrams that represent information structures in the information-processing theory are usually interpreted as individuals' mental representations. This is not the only interpretation that they may be given. In the situative perspective, information is assumed to be constructed in processes of interaction in activity systems, and the kinds of representations of information that have been developed in cognitive science can be interpreted as referring to structures of information that are used and constructed in the interaction of people and other informational resources in activity systems.

In work that Randi Engle and I began in 1991, our strategy was to take the information-processing analysis of reasoning and understanding, developed in cognitive science, and embed it in a theory of conversational interaction, which we adapted from Herbert Clark's (1996) theory of conversational contributions. Our theoretical move was to hypothesize that constructive processes of understanding and reasoning occur in conversation through the joint actions of the participants. The information structures that are built are in the common ground that the participants construct in their interaction. In this view, the information-processing operations of cognitive theory are assumed to occur as joint actions in conversation, in which the units are what Clark and Schaefer (1989) called *contributions to discourse*. Each contribution includes, minimally, a presentation and an acceptance that signals mutual understanding sufficient for the participants' present purposes. A contribution may also include an action that signals uncertainty or confusion, or presents a question or a challenge to the initiating presentation in the form of an alternative idea or proposal. In that case, there has to be some negotiation to reach mutual understanding and completion of the contribution, to place new information in the common ground. Much understanding depends on the participants sharing of a vast amount of common ground (e.g., Hanks, 1996), a more inclusive version of the cognitive idea of schemata. Using these ideas, Engle and I developed analyses of some examples of conceptual understanding and reasoning in conversation that pairs of students had as they worked on a task of constructing a mathematical representation of a physical system (Greeno & Engle, 1995). The activity included reading instructions for a task in a workbook, setting up a physical apparatus that was designed to operate approximately according to linear functions, and constructing a table that represented the behavior of the apparatus with the parameter values specified in the instructions. Our analysis attempted to identify properties of information structures that students generated, corresponding to their understanding of the task, goals and subgoals that they adopted, and meanings of the symbols and other signifiers that they used in the representations they constructed. To explain these constructions of meaning, we hypothesized several kinds of schemata in the students' prior common ground, including general schemata involving participation in conversational interaction, more specific schemata involving accomplishment of school-like tasks, schemata about the operation of physical systems, and schemata about numbers and arithmetic operations.

Level 2 Level 2 involves understanding and reasoning that produces non-routine insights, which are novel for the participants. Accounting for novel insights has traditionally been challenging. In psychology, non-routine insights require flexible thinking, associated with gestalt analyses such as Duncker's (1935/1945). In more recent work in cognitive science, generative analogical reasoning has been studied and analyzed in detail (e.g., Gentner, Holyoak & Kokinov, 2001). In situations involving more than one person reasoning collaboratively, novel insights (for the participants) can be produced based on negotiation that occurs when they express differing understandings (Engle & Greeno, 1994). The value of diverse opinions in collaborative reasoning and understanding has been studied by Okada and Simon (1997), Schwartz (1995), Tudge and Rogoff (1989), and Rosebery, Warren and Conant (1992).

In detailed analyses of some episodes involving generative reasoning, we have focussed on a kind of interaction that we are calling *problematizing*. Engle and Conant (2002) identified this as an important issue, and Melissa Sommerfeld Gresalfi, Muffie Wiebe Waterman, and I examined some cases in which a group did or did not problematize an issue that they might have, which we have discussed in terms of semantic trajectories in conversation (Greeno, Sommerfeld, & Wiebe, 2000; Stenning, Greeno, Hall, Sommerfeld, & Wiebe, 2002). For an issue to be problematized, we hypothesize, alternative trajectories need to be considered. This can occur if the participants recognize an alternative and create a choice point or if one of the participants questions or challenges the group's current trajectory and succeeds in having the group consider whether a different trajectory might be preferable. This raises another issue in our analyses, the *positioning* of individuals in the participation structure of their classroom activity. In any episode of interaction, different individuals participate in different ways. Quite often, someone is initiating segments of activity, functioning as the director. Others are at least monitoring the actions of the director, providing approval or raising questions. Sometimes agency is distributed across two or more of the participants, with both or all actively contributing to the direction of discourse. This distribution of agency is important in the opportunities to problematize issues. If one of the participants is positioned so as to present virtually all of the information and ideas, with others positioned mainly as bystanders, it is difficult for anyone except the leading participant to introduce a question or challenge that is taken up by the group. However, if one participant initiates a contribution and another participant is positioned with significant agency for questioning or challenging that participant's presentation, rather than being positioned as a bystander, it is much easier for her or him to get the floor and introduce a question or alternative idea and have that taken up by the group.

Level 3 Issues at the next level involve students' engagement with learning tasks. Cognitive theories account for some aspects of these issues with models of students' understanding of problems and setting goals to solve them (e.g., Hayes & Simon, 1974). More generally, psychological theories of motivation include hypotheses about students' motivation to expend effort in tasks of specific subject-matter domains and tasks that have moderate perceived difficulty (e.g., Stipek, 1998).

In situative studies, students' engagement is considered as an aspect of their participation in classroom practices. Recent research on classroom practices has focused on structures of participation in which students are entitled, expected, and obligated to propose conjectures, raise questions and problems, and formulate explanations and arguments, rather than only being entitled to answer questions and solve problems given by the teacher (Ball, 1993; Ball & Bass, 2000; Brown & Campione, 1994; Hatano & Inagaki, 1991; Lampert, 1990). In an analysis based on our study of classrooms in the Fostering Communities of Learners project, Engle and Conant (2002) proposed that authority, accountability, problematizing, and access to resources are critical factors in supporting productive engagement by students in activities of disciplinary learning. Gresalfi, Martin, Hand, and Greeno (2009) studied ways in which students' competence is co-constructed by students and teachers in interaction.

Level 4 Issues at this level involve long-term conceptual growth and orientations toward learning practices. Psychological accounts of these factors include the large literature in cognitive development and general motivational orientations, such as achievement values (e.g., Graham & Taylor, 2002), orientation toward learning goals vs. performance goals (Dweck & Legett, 1988), and individuals' beliefs about themselves as learners and knowers (Belenky, Clinchy, Goldberger, & Tarule, 1986).

In a situative perspective, learning by an individual can be considered as change in her or his ways of participating in the practices of a community (Lave & Wenger, 1991). More specifically, conceptual growth in a domain can be considered as change in the discourse practices of a community, or in the ways an individual participates in discourse, that involve understanding of that conceptual domain. Situative studies have provided analyses of changes in discourse practices, in which participants' conceptual discourse can become more elaborated and integrated, can include representational practices used in conceptual reasoning and understanding, and can include more advanced forms of explanation and argumentation (Bowers, Cobb, & McClain, 1999; Greeno, Benke, Engle, Lachapelle, & Wiebe, 1998; Hall & Rubin, 1998; Lampert, 2001; Rosebery, Warren & Conant, 1992; Strom, Kemeny, Lehrer, & Forman, 2001).

A situative perspective can consider the ways that individuals characteristically participate in learning practices as aspects of their identities, as discussed by Gordon

(2000) and Packer and Goicoechea (2000). This perspective emphasizes "achievements [that] are less focused on what we want learners to know and know how to do, and more sharply focused on what it is that we want learners to become and be, i.e., compassionate and thinking interdependent members of humane human communities" (Gordon, 2000, p. 1), and focuses on the ways in which schools and considers "school as a site for the production of persons" (Packer & Goicoechea, 2000, p. 235).

In currently ongoing research, Melissa Gresalfi and I are beginning to develop ways of characterizing students' identities in their participation in mathematics classrooms. Following Holland, Lachicotte, Skinner, and Cain (1998) and Wenger (1998), we focus on ways in which individual students are positioned in interaction. In our analyses, we distinguish two general aspects of students' positioning. One, their systemic positioning, is in relation to other students and the teacher in the class. The other, their semantic positioning, is in relation the concepts and methods of mathematics. Systemic positioning – in relation to other people – involves the degree to which a student is entitled and expected to initiate contributions, to question or challenge proposals that are made by others, and to be given satisfactory explanations of meanings and methods involved in instructional tasks. Semantic positioning – in relation to the concepts and methods of mathematics – can involve what Pickering (1995) called conceptual agency, in which the individual makes choices and judgments involving meanings and appropriateness of methods and interpretations, or can be limited to disciplinary agency, in which the individual is only involved in performance of procedures that are established in the practices of the domain.[5]

Based on video records and interviews with students in two eighth-grade algebra classrooms, obtained over the period of a school year, Gresalfi (2004) has developed case studies of eight students, characterizing their mathematical identities in terms of persistent patterns of participation in classroom learning activities. She identified ways in which individuals differed regarding their tendencies to work independently or collaboratively, and in their efforts toward individual or mutual understanding of mathematical concepts and principles. The participation structures of the two classrooms differed significantly, with more emphasis and direction for students' working collaboratively toward mutual understanding by one of the teachers. Identities of students were influenced by this difference in ways that are consistent with the idea that identities are constructed in interaction, shaped by both individual students' proclivities and the affordances of the socially organized activity system in which they participate.

Two Examples from Our Previous Research

The holy grail for this quest takes the form of analyses of interaction that require systemic principles of participation in activity systems and semantic principles of meaning and information processing in combination to explain significant aspects of activity. This approach differs from some others in which it is assumed that social interaction provides a context for information processing, or that informational content provides a context for interpersonal interaction. Instead, I expect we will find it most productive to consider activity to be jointly systemic and semantic "all the way down," so that whatever the size of an event we choose to analyze, the appropriate analysis will include principles of both informational and interpersonal interaction that function at that grain size in order to explain the event.[6]

In this section, I present brief reviews of two analyses that we have conducted, in recent years, of episodes of interaction of different sizes. One of the examples involves activity in which students were intensely engaged over several weeks. The account that we have given of this involves principles that are mainly at level (3) in Table 3.1, involving ways that students were positioned in the learning activity. There are significant implications of our analysis for principles at level (4), involving students' positional identities regarding commitments and accountability to each other and to the conceptual domain of their activity. The other example is mainly focused at level (2) in Table 3.1, involving two episodes of classroom work with durations of a few minutes. The episodes contrast in a way that we attribute to the occurrence or nonoccurrence of a discourse feature of problematizing a substantive issue. There are significant implications of this analysis for principles at level (3) involving ways that students are positioned regarding each other and the conceptual domain of the activity.

I hope to accomplish two goals by reviewing these two examples. One goal is to provide cases in which the joint use of semantic and systemic explanatory principles is needed to account for significant aspects of the students' activity and to show that the operation of these principles at different levels of analysis are interrelated. The other goal is to set a stage for the analysis I then present of material from the Wisconsin Fast Plants® tapes. In that analysis, I conclude that we need to distinguish between two aspects of conceptual agency in classroom discourse: for generating variation (problematizing) and for contributing to selecting which alternatives become part of the common ground (resolving).

The explanatory principles that we have offered to account for significant aspects of the two cases in our previous research involve similar systemic and semantic aspects of interaction, functioning at different levels. The systemic principles involve ways in which students are positioned in interaction, that is, how they are expected by others and themselves to participate in relation to the other participants. The semantic principles involve ways of achieving coherence of information, including alignment of the situation with the goal of a task. Systemically, we have hypothesized that positioning students with authority, accountability, and commitment contributes to their productive engagement in the disciplinary activity of the class. Semantically, we have adopted the assumption that successful reasoning corresponds to achieving a coherent network of information, which includes alignment of meanings and propositions that refer to states of affairs in the situation and to concepts and principles in conceptual domains that the participants have access to. We hypothesize that processes that contribute to successful reasoning include detecting inconsistencies in the current information structure, often involving assertion of propositions that are inconsistent with principles of a relevant conceptual domain. Reasoning processes need to include problematizing inconsistencies by taking them up as discourse topics and resolving the alternative interpretations or opinions that are at issue.

These two kinds of principles are inherently interactive. Positioning with authority and accountability for inquiry supports participants in presenting alternative opinions and interpretations and dealing with them seriously, striving to resolve the alternatives in ways that are consistent with the facts at hand and with accepted conceptual principles. And students who are entitled and expected to generate and consider alternative interpretations and proposals for goals and actions in working on tasks need to draw on information, concepts, principles, and practices of explanation and argumentation to formulate and support opinions, which they can express if they are positioned with authority, accountability, and commitment in relation to other people. Being positioned to have opinions and to explain and defend them requires resources of information and practice of explanation and argumentation in the conceptual domain. The actions of forming opinions, explanations, and arguments in a domain require positioning with conceptual agency (in Pickering's (1995) sense), involving active choices, judgments, and evaluations that are not determined by standard practices. And the discursive activities of problematizing issues and resolving different opinions require systemic positioning in interpersonal interaction in which individuals are entitled, expected, and committed to presenting opinions and arguments that differ from those of others, to considering alternative positions and arguments, and being open to the possibility of changing their positions based on evidence and argument in the discussion.

An Extended Controversy

The analysis involving the large episode that I discuss here involves activity that occurred over several weeks in two fifth-grade classroom in Brown and Campione's (1994) Fostering Communities of Learners program. My discussion here draws on the analysis given by Engle and Conant (2002)[7]. The topic was endangered species, and each of the groups studied and wrote a report about a species. There were two classrooms working in parallel on the endangered species unit, and each of them had a group studying whales. A controversy developed about whether killer whales, or orcas, were properly classified as whales or as dolphins. The issue was significant because if orcas are not whales, then information about them would not properly belong in the group's study. The question arose because on a field trip to Marine World, a staff member said that "killer whales aren't whales, they're dolphins," At the time, the group's research included a significant effort of study about orcas, and removing the information about orcas would mean that that work would not contribute to the group's product. But the controversy engaged the students with more intensity than we would expect if it only involved doing some additional work. The two sides had leading advocates, evidence was presented and called into question from both sides, and the shift in opinion from a majority believing that orcas are not whales to almost all the group members believing that they are was an important feature of the group's history of research.

Engle and Conant's (2002) analysis identified features of the classroom practice that they hypothesized were significant in supporting the initiation and maintenance of the extended, intense controversy. Their analysis showed ways in which the teacher and students constructed positioning of the students with authority to form and question opinions on the issue and to construct and evaluate arguments on each side. They also were accountable for presenting their opinions and arguments to each other, for considering the opinions and arguments of other students, and for arriving at a conclusion that all members of the group accepted. The classroom practice generally encouraged problematizing issues in the subject matter, and the students had authority and were accountable for resolving the issues that arose, including the classification of orcas, using evidence and sound argumentation. Their account makes the orca controversy a prime example of an extended learning event whose explanation includes systemic and semantic principles of interaction, functioning integrally. Students were positioned generally to use information from texts and other sources to form opinions about the species they were studying, and when the disagreement about classifying orcas erupted, one of the teachers organized the students studying whales into a discussion, saying that both sides had "good points." The teacher explicitly directed the group to resolve the issue, affirming that they had authority to arrive at a conclusion and were accountable for doing that. They were enjoined to attend to each others' "good points" and to support their positions with evidence, which required them to process information from texts and formulate coherent explanations and arguments (This discussion lasted 27 min and was what the students called their "big ol' argument."). In the students' final report, they wrote that they had disagreed about the proper classification of orcas, and that the issue also had not been resolved by scientists.[8]

Correcting a Course, or Not

My second example involves two brief episodes in a middle school mathematics classroom using a unit of the Middle-school Mathematics through Applications Program curriculum. The unit, called Guppies, uses a software program called Habitech, which supports construction of models of population growth and decline. Students choose parameters for functions that change population size on an annual or other temporal basis, including birth rates and death rates, and the program runs simulations based on the functions that have been defined. Video material obtained by Rogers Hall and his associates were analyzed in a group that included Hall, Keith Stenning, Melissa Sommerfeld Gresalfi, Muffie Wiebe Waterman, and me. This analysis, along with others, is reported in Stenning et al. (2002).

In this example, we used concepts of positioning and information processing in explaining significant aspects of the activity we observed, as we do for the orca controversy. However, our analysis and explanation were at a more detailed level in this example. Here we hypothesized ways in which aspects of moment-to-moment interaction can be explained in terms of students' positioning and their processing of information.

In both of the episodes of this analysis, the group member who was most directive proposed and was beginning to carry out a procedure for solving the problem at hand, and the procedure was incorrect. The episodes provide an interesting contrast because in one of them the group discovered the error in the procedure and adopted one that was more valid, and in the other episode they did not, in spite of the procedure's being questioned by one of the students. We explain the difference in terms of a threshold for problematizing an issue, which is partially determined by the positioning of students in their participation structure. We hypothesize that the threshold was exceeded in one case and not the other because of a difference in the strength of information that was presented by the student who questioned the procedure that was being applied.

The first episode occurred during a pre-unit assessment, when the students worked on a problem involving a population before they were introduced to the computer-modeling environment. A situation was described involving a grain elevator in which 20 mice had been discovered. The students were to make reasonable assumptions about frequency of mouse reproduction and size of litters and predict how many mice there would be after 2 years. They assumed that mice reproduce in couples, and they discussed likely frequencies and sizes of litters, settling on reproduction once every 3-month season (hence, eight cycles in 2 years), and an average of four pups per litter. They inferred that from the initial 20 mice, there would be 40 pups added to the population. In the following excerpt, M and K worked out the number of mice after 2 years based on a linear process, that is, 40 pups each season times 8 seasons, added to the initial 20 mice. L, however, noticed that this calculation did not take into account that the population of mice increased with each cycle, and that the number of pups should take the new members of the population into account. L challenged the procedure by stating an alternative assumption that each cycle would produce four pups per couple of that cycle's current population. (Her explanation included an effective gesture, expressing the pairing of mice that had been born previously and their production of pups.) M and K accepted this alternative, and they proceeded to calculate the number of mice based on the assumption that the mouse population increased exponentially.

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 60: M. so there's ... equals 40 babies each season
 65: M. it's three hundred and twenty
 66: K. (inaudible) is that including adults?
 67: M. no, three hundred and twenty plus twenty
 69: M. by the end of the winter
 70: M. three hundred and forty mouse ...mice ... mices. OK.
 73:    Now we need to make a graph of it ...
182: M. so let's see ... the first season is over here (making
        a mark on the graph)
183: L. xxxxxx wait a minute
186: M. and then sixty plus is going to be a hundred
189: L. wait a minute its forty (gestures a triangular shape)
        OK it's forty right?
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190: L. and then you have to pair those up (brings hands
        together) and then they have kids (spread hands apart,
        while K and M looked at her confused)
192: M. oh yeaaah (embarrassed, laughing at himself) we were
        doing it ...
194: L. That's a lot of mice
195: K. gosh that's a lot of nasty mice
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The second episode in this example involved the same group of students, working later in the unit. They had a worksheet called "Building the birth rate," which directed them to calculate a birth rate that they would then enter in a function on the computer interface. The worksheet asked them to make an assumption about the age and sex distribution of an initial population of ten guppies, then based on data from a reference source to calculate the number of fry that would be born in a reproduction cycle in that population. That number was to be reduced by 95%, the proportion that are eaten by the mother guppies immediately after they are born. This would leave a number of guppy fry that survive. This number should be divided by the initial number of guppies (in this case, 10) to provide a birth rate that the students were directed to enter into the computer model.

The students had arrived at a number of fry produced by their assumed distribution of the initial ten guppies, when M proposed that they could use a shortcut. Rather than dividing that number by the initial population size (10) to arrive at the percentage birth rate, M proposed that the percentage given as the survival rate (5% of those born, which they misremembered as 4%) would be a suitable number to enter into the program. L expressed confusion about this calculation, and M responded in a way that was conversationally appropriate, giving L an explanation of its sufficiency. L did not accept the explanation, but M proceeded to enter the value that he referred to as the survival rate into the program.

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444: M. hey wait wait wait ... no but listen. If 4% of the
        frys survive why don't we just forget about the fry
        survival and just put that amount for the, for how much
        are born
445: L. because the number born are not how much survived
446: M. yes. yes, the ones who survive are the ones we count,
        not the ones who are dead because we don't make room for
        the ones that are dead
453: M. OK you know how 4% the whoooole fry who were born
        survive so why don't we just put 4% on the guppies birth
        because that's how many are going to survive
454: L. I get what you're saying because why put however many
        more guppies in when they're just going to die anyway?
455: K. so why not just put 4% because that's how many are
        surviving/ that's how many we're going to count
497: L. but what's that 4% ?
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498: K. the ones that survive
499: M. The ones that actually survive fryhood
501: L. Yeah, I know, but how many of the guppies are 4% ?
502: M. we don't know, we'll let that mechanical thing work
        and tell us (M moved to the computer to enter the
        parameter in the program)
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To sketch our interpretation of this pair of episodes, we need an explanation of why the group's threshold of problematizing an issue was exceeded in the first episode and not in the second. We believe that both systemic and semantic principles are needed for the explanation. A salient difference between the two episodes is in the significance of the information that L presented. In the first episode, L's challenge included an explicit model of the situation that was more consistent with the students' model of mouse reproduction than the implicit assumption of a linear process. In the second episode, L questioned M's procedure and expressed confusion, but did not offer an account of the semantics of the parameters in the way that she had in the first episode. Providing an explanation makes a stronger case for changing what a group is doing than only expressing uncertainty. We hypothesize, then, that the problematization of Episode 1 and not in Episode 2 can be explained by L's presenting information in Episode 1 with greater force than the information she presented in Episode 2. The interpersonal positioning of the students in both episodes supported M as the main initiator of information and action and supported L and K as having opportunities to question or challenge what M said and did. The evidence they required for grounding contributions was significant, but not as strong as it could be. Weaker evidence of grounding can be used in groups where one of the members simply does the work and others follow without raising questions or disagreements. Stronger evidence for grounding can be used in groups that require strong evidence of mutual understanding, such as restatements, and that take up issues on which someone expresses uncertainty to reach mutual understanding explicitly. If this group's interpersonal positioning had involved significantly weaker evidence for grounding, it might not have taken up the issue of the form of the growth function in Episode 1. If the group had had significantly stronger evidence of grounding, it might have taken up the issue of the base line for calculating birth rates in Episode 2.

Putting this more generally, we hypothesize that the difference between Episode 1 and Episode 2 can be explained by combining principles of systemic and semantic aspects of interaction, working jointly at the same level of analysis.

Planting Abstraction in Representational Practice

In this section I present analyses of two episodes from the tapes that were provided by Schauble and Lehrer for our examination and discussion at the workshop. I hope to show that the analytical scheme that I have presented here provides an appropriate and useful lens for understanding aspects of the interactions in the episodes that I selected. At the same time, accounting for these examples requires a conceptual distinction that was not salient in the examples we have analyzed previously. Therefore, the results of this analysis extend the concepts and empirical materials that I have discussed in this paper.

The conceptual extension distinguishes between two aspects of discourse at level (3) in the scheme shown in Table 3.1. The aspects are problematizing and resolving. Discourse practices include patterns of information that are recognized as problems, and they also include ways of dealing with problems that have been recognized and taken up. We had previously considered practices that encourage problematizing in cases that I have reviewed here. In those examples, issues that were problematized were also resolved productively, with actively engaged participation of students. In the examples I discuss from Schauble and Lehrer's material, there were discrepancies in the ways that students participated in problematiziing and resolving, which requires a more complicated understanding of these concepts.

A concept of resolving is related to the concept of reconciling, which has been developed by Deborah Ball and Hyman Bass (Ball & Bass, 2000) in their analyses of videotapes of Ball's mathematics teaching (Ball, 1999). They pointed out that there are characteristically mathematical ways of proceeding when there are apparent differences between definitions or methods. Mathematical practice requires such differences to be considered as problems, and efforts are made either to show that the apparent difference is not real (the alternatives are equivalent) or to show that there is a difference and to make an appropriate modification in definitions of terms or specifications of procedures. I use the term "resolving" rather than "reconciling" because reconciling by mathematicians depends on technical mathematical analyses that are lacking in the kinds of discourse that we find in ordinary classroom discussion.[9]

More generally, Toulmin (1972) characterized processes of conceptual change as including a process of generating variability and a process of selection, which determines which alternative ideas, methods, findings, and interpretations come to be accepted and established in an inquiring community. He noted that in scientific disciplines and other communities that construct conceptual systems, the selection process involves forums of debate in which members of the community contest candidate concepts, methods, findings, and explanations. In a classroom, practices that generate alternative definitions, solutions, or methods support problematizing, and practices that consider alternatives and select one or some of the alternatives as acceptable or correct are that community's practices of resolving.

The Activity: Designing and Evaluating Representations of Some Data

As I understand the video records that we are studying, the learning activity was designed to advance students' ability and understanding in a representational practice. The instructional strategy was for them to design representations (the teacher called it "inventing displays") for a set of 63 observations that the students had obtained in their project of growing Wisconsin Fast Plants®. The teacher's instruction was to "organize the data" in a way that used a piece of graph paper, with forewarning that the representation would be used to answer two questions: What is the typical Fast Plant height and how spread out are they? The data were presented as an unordered list of numerals. Previous learning apparently included some experience with bar graphs and line graphs, which the students recognized as relevant for their task.

School learning activities generally have an intended conceptual domain located in the curriculum. I would classify the domain of this activity as descriptive statistics. Statistical practice includes representing collections of numerical data and identifying properties of the collections, including central tendency and variation. Statisticians' discussions of these processes do not depend on what the numerical data signify in a domain other than mathematics. References to mathematical entities (numbers, relations between numbers involving ordering and arithmetic and, later, more advanced operations) are sufficient to provide the semantics of statistical concepts. The discussion of graphical representations in the video records that we are studying is consistent with this practice. The central tendency was called the height of a typical plant, but the discussion of the concept was almost entirely about the location of a number in the frequency distribution. The discussion of variation did not refer at all to variation in heights of plants, just the dispersion of numerical data.

Having the assignment be a design activity potentially positioned the students with more conceptual agency than they would have in some other activities that can be used for learning to make graphs. They made choices about the physical arrangement of tokens on the graph and labels of the axes (if they had axes), especially whether the axis was labeled with a numerical scale (rather than just an ordered set of numerals). They were not authors for issues of what the representations would be used for. The questions that would be addressed using the representations were provided as part of the assignment.

Further, although there was not a prescribed form for the representation, it turned out that there was a form that was preferred – a bar graph of the frequency distribution of the 63 data points, with an axis labeled with all the possible heights in the range that had data (i.e., not just the values that had data points), and with data columns corresponding to intervals (called "bins").

Having the assignment be a design activity also ensured that graphs with different features would be constructed, providing opportunities to problematize issues regarding those features. However, in the episodes that I examine here, discussions of alternatives were less productive than in the two positive cases I mentioned earlier. I conjecture that the main difference was in the authority and accountability of students in these episodes, compared to the classrooms that we analyzed previously. In those cases, students were positioned to have to reconcile their differences among themselves. In the episodes I discuss here, an adult (Rich Lehrer in one case, the teacher in the other) provided the authority needed to decide between the alternatives that were presented.

Some Semantic Features of the Graphs

Stenning (2002) has argued that it is useful (actually, essential) to examine logical features of a representational system to understand cognitive issues of its use. By "logical," Stenning explicitly included semantic issues, particularly a distinction between features of the representation that are interpreted directly and others that are interpreted indirectly. For example, if the relevant property of a symbol for some aspect of its interpretation is its presence in the representation or is its location in the representation, then the interpretation is direct. But if a symbol's referent is determined by a convention that only depends on what the symbol is, not on any physical property of the symbol (e.g., the referential meaning of a word), it is indirectly interpreted.

In all of the graphs that students constructed, the heights of individual plants were represented by tokens in the graph. That is, the presence of a token (a numeral or an "x") meant that there was an observation in the data set corresponding to the token. Except for one of the graphs that skipped doubles, there was a one-to-one correspondence between tokens and data points. Although one group began to construct a graph that distinguished the identities of the different plants, in the graphs that the class discussed the tokens represented data points anonymously. That is, the identity of the observation could not be recovered from the token, except in those cases where only one observation had the value represented by the token.

One of the groups constructed an ordered list of the numerical symbols. In this case, the position of a token in the list corresponded to its magnitude relative to the other tokens. For example, the token with an equal number of other tokens to its left and to its right corresponded to the median of the distribution.

In the preferred representation, a set of possible values (limited to the range of observed values) was placed along an axis, either as a string of individual values or as intervals, with positions for all the possible values in the range. This arrangement of numerals was referred to as a scale. Tokens (numeral values or x's) were placed above the axis, forming columns of tokens. Then the horizontal distance of a column from the origin was a feature that could be interpreted directly as the height of plants represented by tokens in the column, and the height of the column was a feature that could be interpreted directly as the frequency of data with that value or set of values. When the symbols along the axis included all of the possible values (in the range of the sample), the horizontal location of any given token was determined by its value. The several tokens in a column collectively represented the set of observations with that value (or set of values), and the specific location of a token vertically in the column did not signify.

A Lesson in Abstraction

An aspect of learning a representational practice is to adopt conventions for what information can be obtained by interpreting a representation and what information cannot be recovered. The design of a representational form and the processes of its construction and interpretation include determining which aspects of the represented events or objects will be included and which will be omitted and, effectively, erased.

In Excerpt 5 in Appendix B, learning an abstraction was a key issue. The episode is an interaction of Lehrer with four students: Anneke, April, Jewel, and Wally. The students' task was to devise a representation that would "organize the data" of plant heights measured on Day 19. The data were presented as an unordered list of numerals (see Excerpt 1). At the beginning of Excerpt 5, RL arrived to see a coordinate graph with intervals of plant heights on one axis (bins of 8) and the other axis set up with values of something from 1 to 63. He got an explanation from April and Anneke, with a follow-up from Wally, that one of the boxes had the "numbers between thirty and thirty-eight." Then he asked for a further explanation of their graph.

RL found out that the labels on the y-axis were intended to refer to numbers that would identify the plants by the vertical placements of their symbols. We don't know how the group had arrived at this. In an earlier excerpt, in which this group worked mainly on designing the spatial array (e.g., there was considerable discussion of which side of the paper should have the values of the plant heights) they had noted that there were 63 data points, and they allowed for 63 positions along one of the dimensions [Excerpt 3, 0:11:34 – 0:11:46].

When Jewel had explained that plant numbers would correspond to positions on the y-axis, Anneke expressed doubt that this information was significant ("but if it doesn't matter"), and after RL confirmed that he understood about the plant numbers, Anneke raised the question explicitly and elaborated, "it doesn't matter what the names of the plants are" [Excerpt 5, 0:40:20], Jewel and April responded in defense of including the plant numbers, and Anneke gave a more elaborate argument for their not being needed [0:40:35 and 0:40:36].[10] In the fragment below, RL forcefully posed a rhetorical question, "How does [the information about plant numbers] help you answer your question?" [0:41:05]. Anneke replied, "It doesn't," RL agreed [0:41:13], marking this as the correct answer.

Excerpt 5 [0:41:05–0:41:13]

```
0:41:05 RL:       How does it help you: answer your question,
0:41:08 Jewel:    Well but ⌈you said-
0:41:09 RL:                ⌊if I call one Plant One and the
                  other Plant Two?
0:41:13 April:    It doesn't.
0:41:13 RL:       It doesn't.
```

April and Jewel held out a bit more, and RL left, cryptically remarking "Well you gotta kinda try to figure out what you're tryin to figure out. Okay, so fix it" [0:41:44]. When RL was gone, April said, "I agree with that, what Anneke's saying," apparently leaving Jewel as the sole supporter of including plant numbers in the representation. Jewel resigned, saying "Who wants to erase this? I don't want to."

[0:41:50]. Anneke and April tried to persuade her that the labels didn't need to be erased, but Jewel was un-reconciled. She and April exchanged "Naah"s, and Jewel erased at least part of the labeled axis [0:42:11].

What are the lessons in this, for the students and for us as theorists about learning in interaction? For the students, I believe there was a significant opportunity to learn about a semantic issue, selectivity that is inherent in representation. Representations have features that can be used to specify some aspects of the objects or events that they refer to, and there are other aspects of referents that cannot be recovered by interpreting features of the representational signs. Choices to include or exclude features for recovering information about referents can be utilitarian (does it help you answer your question?) or they can be built into standard practice of a community (that's the way a line graph really is) or refer to important features of the source of information (you have to organize the data). The students and RL problematized the issue of whether to include a feature that would support identifying individual data points in their representation.

The argument that won in this case was utilitarian (does it matter? does it help you answer your question?) That argument was presented by Anneke and reinforced (or enforced) authoritatively by RL. It is unclear how much of the substantive consideration was mutually understood. The case contrasts with the first episode of the Guppies example, where a student presented an apparently compelling argument for an alternative action, based on a model of the reproduction process that they were representing. In that case, there appeared to be an advance in understanding that led to mutual agreement to assume an exponential process, rather than linear. Here, and in the second episode of the Guppies example, the resolution appears to have been more of a concession and less of resolving alternative opinions. There is more than one way to settle an issue, and the process of reconciling, in which reasons for alternatives are presented and taken seriously, may be an important factor in conceptual learning.[11]

On the systemic side, there were lessons for the students about the permanence of choices made in a collaborative group. Previously, the group had settled on a representation including plant numbers, but at the end of this episode, the representation had changed. There were marks on paper to be erased, but the erasure removed more than the marks. It also erased a property of the representation in which Jewel appeared to be significantly invested.[12] (I imagine that the group's inclusion of plant numbers might have resulted from a discussion in which Anneke expressed some of the doubt that she expressed in the episode we have. In that case, the degree of settlement might have been tenuous, and Anneke may have taken the opportunity of RL's presence to advance her view at the expense of Jewel.)

As for a lesson for us analysts: I believe this provides another example of an event for which is it useful to combine semantic principles of information processing and systemic principles of interpersonal interaction to explain what happened. An account of the group's use of ordered numerical intervals requires hypotheses about their knowledge of the number system, the containment of numbers in intervals, and conventions of graphical representation where locations refer to numerical values. So does an account of their plan to use a second dimension of plant numbers, as well as their eventual decision to omit that variable.

On the other hand, systemic hypotheses about the students' positionings in the participation structure and commitments to positions in the discussion also seem to be required. The shift from including to excluding plant numbers occurred to Jewel's evident displeasure, which makes likely that she was committed significantly to having them. The argument against having them was sound, but it was not presented in much depth, and it is likely that Jewel did not adopt it on its merits. Therefore, the authoritative position that Lehrer held in the participation structure seems needed to account for the group's conclusion to omit the plant numbers from its representation.

Issues of positioning in classroom practice have implications for the opportunities students are afforded for development of learning identities, in the sense that Gresalfi and I are trying to develop that idea (Greeno, 2001; Gresalfi, 2004). One example is the interaction of Anneke, Jewel, and RL around the question of plant numbers. Anneke seems to have been quietly persistent in expressing her view that identifying individual observations "doesn't matter," and then RL joined her and authoritatively settled the issue, with the consequence that Jewel's and, apparently, April's view favoring inclusion of the plant numbers no longer prevailed. It is tempting to speculate that this episode may have exemplified significant aspects of Anneke's and Jewel's positional identities in the class, perhaps involving a pattern in which Anneke's presentations were generally taken up and often prevailed, and Jewel's more often being set aside. But for such a conjecture to be evaluated we would need to examine Anneke's and Jewel's interactions in a collection of episodes, and evaluate whether the suggestive pattern that occurred in this case was generally characteristic of their participation.

Learning to Scale

The other episode I have examined is in Excerpt 9. The class activity was presentation and discussion of graphs that had been constructed by the several working groups of students. The discussion, led by the teacher, considered four graphs in this episode. One was a histogram values in "bins" of ten, Two were ordered lists of numerals corresponding to the data points, and one represented each data point as a vertical line whose length corresponded to the value, with the vertical axis labeled with numerals corresponding to all possible values within the range. In the discussion, the teacher focused on properties of the graphs that "help you see what's typical and how spread out they are," especially the latter. At about the eleventh minute, the teacher introduced a hypothetical data point, asking what would change in the graphs if the point 255 (the largest value in the set) was replaced by 555 [Excerpt 9, 0:10:33]. The graphs that included locations for all the possible values (in the range) would have to be extended, whereas in the graphs that just listed values, the largest numeral would simply be replaced, without having to change its location. The teacher referred to inclusion of all possible values as having a scale, and concluded that a scale is useful "to help see how spread something is" [0:13:51].

Semantic Issues

I consider three semantic issues involving the informational contents of this discussion. One is abstraction, again. The second is the nature of iconic vs. symbolic representation or, as Stenning (2002) put it, whether a representation is directly or indirectly interpreted. The third issue is the introduction of the term "scale," and its meaning in relation to the concept of spread.

Abstraction

The identities of plants and their observers were erased previously, and the semiotics of the graphs was focused on aggregate properties of the collection of observations. In this discussion, the origin of the numbers as properties of a collection of plants was, if not erased, set far in the background. The concept of typicality was, arguably, focused on the typicality of a number or interval of numbers in the collection of numbers that the students were representing. Although it had the label "typical Fast Plant height," or an equivalent, there were no explicit references to the set of plants, in which a height of, e.g., 160 mm, was typical. In the case of spread, there was no reference to the plants (i.e., they did not refer to the "Fast Plant spread" or "spread of the Fast Plant heights"), and the discussion seemed entirely focused on the variability of numbers.

It is remarkable how seamless this abstraction was. The phrase, "How spread out they are," applied to a collection of plants, might be expected to refer to the spatial distances between plants in the ground they grow in. It was obvious to everyone in the classroom that the property of being spread out was not intended as a description of the locations of plants. No one expressed any uncertainty about whether they were talking about the amount of space that the collection of plants occupied.[13]

On the other hand, the intended meaning of the term "spread" apparently was not just how much space the symbols occupied in the graph, either. It turned out that the most approved graphical form had that property, but it was not discussed by the teacher that way. He did not ask, "Which graph has the symbols spread out more (or less)," but instead, "Which graph would be better to help you *see* that spread?" [Excerpt 9, 0:11:26]. If I am right that this did not refer to the collection of plants, what was "that spread" a property of? I believe it was mainly the collection of numbers represented by the numerals that were presented at the beginning of Day 27 (Excerpt 1). This isn't far from a meaning in which "that spread" would refer to a property of the collection of the measured heights of the class's plants. But very little in the discussion (that I found) maintained the connection of the numbers with the observations.[14] There was a set of numbers, and the amount of "spread" referred to differences between the numbers. "Why [that graph would] help you see the spread better" [0:11:26] referred, I believe, to the ease of interpreting a property of the graphical display as a reference to the amount of difference between the numbers that were represented in the graph. The preferred form supported a direct interpretation, with the spatial spread of tokens in the graph interpreted as referring to the spread of numerical values. Of course, this is just the way that statisticians think. But probably not gardeners.

The Concept and Representation of “Spread”

The teacher focused significant attention on the question, “Why [that graph would] help you see the spread better” [Excerpt 9, 0:11:26]. It turned out that he was inviting attention to the extent to which data were distributed across a wide range of values, and how that distribution could be perceived as a feature of the graphical representation. In the standard interpretation of a bar graph, heights of columns refer to frequencies of values of the variable, and distances between columns refer to differences between values, therefore, the spread of a distribution can be perceived directly. Stenning (2002) argued that the availability of direct interpretation is the distinctive feature of diagrammatic representations in contrast to propositional representations, which require indirect interpretations. Diagrammatic representations are constrained in ways that propositional representations are not. For many diagrammatic representations there are spatial constraints, which limit the locations at which symbols or icons can be placed, depending on their referential meanings. The teacher used this feature when he posed the question of whether a representation would be visually changed if 255 was replaced by 555. If the abscissa includes all of the possible values in the range (which the teacher called a scale) the location of a symbol is constrained by its referent in a way that it is not in the more propositional representations (e.g., a list of numerals), or in the representations that do not have the correspondence of distances along the axis with values of the variable. It is interesting that the authors of the list, for whom the typical entry and, especially, the spread, could not be interpreted directly with perceived features, included statements (i.e., propositional representations) to represent their judgments about the features (“We wrote the answer and how we did it and they didn’t.” [0:07:06]).

The favored graphs, which supported direct interpretations to answer the questions the teacher posed, are consistent with pedagogical practices that emphasize modeling, in the sense of constructing representations that allow mental operations that simulate significant conceptual processes. So moving a symbol farther away from the other symbols and gesturing to indicate a region of central tendency or an increase in the spread of a distribution are available in discussions of statistics and are easy to relate to diagrammatic representations, more than with propositional representations or formulas.

The Concept and Term “Scale”

The graph consisting of a list of numerals had just been discussed when the teacher posed the hypothetical situation where 555 was in the set. He indicated that with this value there would be “a much bigger spread.” Then he declared that the change in the number list would involve replacing “2” in “255” with “5,” and asked,

Excerpt 9 [0:11:03–0:11:04]

```
teacher: Would this graph (0.3) help you see that that’s more
         spread out?
```

Two of the graphs had numerals along the x-axis corresponding to values that corresponded to measurements, with 255 at the upper end. The teacher noted that on those graphs, the 255 entry could just be replaced by 555, without changing its location, and asked,

Excerpt 9 [0:11:18–0:11:26]

teacher: Would that >would the graph itself< if you (0.3) could see that or if we did it on this one we had five fifty five here. Is there is there a graph up there that would be better to help you see that spread? than some other ones, and why >would it be< (.) why why would that graph help you see the spread better?

Jewel offered that one of the graphs with only the occupied values along the axis was

Excerpt 9 [0:11:46–0:11:47]

Jewel: ... really good because (0.6) you can like tell if like (1.1) if it goes farther like

There was a bit of defense by the authors of the list, saying that they would have put their numerals in a single line if they had had room. The teacher responded to Jewel's choice, saying

Excerpt 9 [0:12:26–0:12:39]

teacher: Okay so Jewel, you think (.) this graph by looking at it if I wrote the number (.) five hundred fifty-five right here *(pointing to a place on the upper end of the axis))* would be the (.) easiest graph to look at to see that (.) this has a lot of spread. Is that what you think?

Jewel: Um hm, (I) guess.

Without further comment, the teacher called on another student, Kerri, who chose the graph with 10-mm bins and all the possible values in the range of actual values. This was what the teacher apparently had been waiting for. He (implicitly) endorsed Kerri's choice, revoicing her presentation with a significant appropriating move of attributing to her that she was talking about a scale.

Excerpt 9 [0:12:48–0:13:27]

```
Kerri:     ⌈⌈((pointing to Group 3's graph))
Kerri:     ⌊⌊Well I think that probably this graph because (.)
           it lea- they still leave: (0.9) some spaces there,
           (0.8) in case there would be even though there's
           not, so that you can (.) really see how spread out
           it is because it (0.5) goes (0.3) thirties, (0.5)
           up to the most and you can see if when there's like
           (0.7) >how much< ⌈space is there between it
teacher:                    ⌊OH::.
teacher:   >I see what you're saying< you're saying that
           there's some there's a scale down
           ⌈here on the bottom (2.1) an:d if it was
           five-hundred fifty-five they would well two-fifty's
           here so we'd figure it would be five fifty-five
           would be out ⌈here?
teacher:   ⌊((pointing to x-axis of Group 3's graph))
teacher:                ⌊((pointing to a projected point
           beyond the end of Group 3's graph))
Kerri:     °Yeah.°
teacher:   And then ⌈then you would see that number out there,
           and then it the graph itself would actually look
           like spread?
Kerri:              ⌊H(hh)•
```

Actually, Kerri didn't mention a scale or extrapolating the scale beyond the values on the sheet of paper. Even so, the teacher invited Ian to comment on "what helps people see that spread if what, what Kerri is saying is true" [0:13:35].

Ian apparently had listened to Kerri and responded by rephrasing what she had actually said: "Uhm not, not just the numbers that we actually measured that are in between, but all of the numbers that are in between" [0:13:41]. The teacher persisted, and in a turn that had the form of rephrasing Ian's presentation, he added Ian to the group who realized that a scale was important:

Excerpt 9 [0:13:51–0:13:52]

```
teacher: SO:: this (0.2) having a scale down here, which is
         >one two three whatever it is,< (1.4) would help
         you see spread (.) better?
Ian:     Yeah.
```

The teacher elaborated on the point, with a transitional (rhetorical) question, "Does anyone not quite understand what Ian is saying?" [0:14:02]. He finished his elaboration, "so long as you have a scale on the bottom, I think that helps people determine how spread something is" [0:15:15].

The next student, Kristen, selected the graph that represented heights of individual plants as vertical lines with numerals at their tops, with indications of all possible values along the vertical axis.

Excerpt 9 [0:15:22–0:15:37]

```
Kristen: Well I'm not (0.6) sure but (0.5) I'm not I don't
         (0.9) well (0.5) I think (0.7) this: (0.9) graph
         might help you, down there because of the like the
         line (0.2) up here (0.6) might get higher but I'm not
         sure how this graph works really but (      )
```

The teacher moved this graph from its partially obscured position so it was visible and appropriated it for another example of scale.

After establishing that changing the value 255 to 555 would require a point higher up the y axis, the teacher asked questions apparently intended to elicit the concept of scale as the answer. First, he may have started to say, “So they have a scale,” but asked, instead, “What is that, over here, ten twenty thirty . . .” [0:16:06]. Instead of answering, “a scale,” a student replied “The y axes?” [0:16:20]. Trying again, the teacher asked, “and it’s also an, April?” [0:16:14], who answered, “A bar graph” [0:16:19]. He continued with additional scaffolding:

Excerpt 9 [0:16:22–0:16:40]

```
teacher:   And they've done something else to it they just
           didn't write (0.8) >there's< something else that's
           special about it (0.8) that would help (.) that
           you'd also be able to see it
teacher:   Ian?
Ian:       You can tell how high it is
teacher:   Well how can you tell how high it is?
Ian:       (Start)
teacher:   What ⌈did they do to it?
Ian:            ⌊Cuz it's higher on the (.) graph
teacher:   Because they,
Ian:       Put a scale?
```

Finally! The teacher confirmed this contribution. “Yeah, they could put a scale on” [0:16:40]. And after indicating that 555 would be about twice as high as 250, he closed the interchange with, “So are we agreeing that scale is an important thing? To see help see how spread something is?” [0:17:00–0:17:03]. No one objected.

I have focused on three semantic aspects of this episode of classroom discussion; abstraction (it was almost entirely about a distribution of numbers; references to plant heights were far in the background); representation of spread as a directly interpretable property of some graphs more than others; and the feature of having a scale that includes all the possible values, which is a constraint that supports direct interpretation, especially of spread. These are all legitimate aspects of statistical practice, which the students experienced by working with data they had collected.

Systemic Issues

Systemically, students were positioned as designers, interpreters, and evaluators of representations. Collectively, they constructed representations that had significant variety. One form emerged, through the teacher's management of discussion, as being advantageous. Therefore, the activity provided students with an opportunity to learn something about how the form of a bar graph, with values of the variable partitioned in equal intervals and including all the possible values of the variable, provides a representation in which the features of typicality and spread are directly interpretable. The key concepts – typicality, spread, and scale – were in the discussion because the teacher put them there. But he did not introduce these "out of the blue." Instead, he arranged to have student products to ground the discussion of the concepts.

The representational practice that students participated in included significant conceptual agency at the level of designing and explaining graphical representations. This positioned the students differently and, we can hypothesize, resulted in a different relation of agency in the practice, than had the teacher told them how to construct bar graphs of these data and explained the advantageous features didactically.

On the other hand, the opportunity for conceptual agency was also limited. The concepts were illustrated in the discussion, and students participated in the discussion that included the concepts. However, meanings of the concepts seem to have been drawn out from the students, rather than having been initiated by them. Thus, their agency in understanding was primarily animating, rather than authoring (Goffman, 1981), or mastering, rather than appropriating (Herrenkohl & Wertsch, 1999) the discourse patterns (phrases, meanings) that the teacher made available to them.

The issue of alternative formats that defined the issue of scale was problematized in the discussion, but it did not engage students in the ways that we observed in our study of FCL classrooms, at least as far as we can tell from these records. It is unclear whether organizing an extended discussion about the merits of leaving spaces for unobserved values of a variable would have been productive for learning. But resolving the difference by the teacher eliciting a step toward the received view and then presenting it with attributions to the students who took or agreed with the step, created a different positioning toward the concept than could be imagined if the resolution had involved a more symmetrical (and, of course, more time-consuming) discussion.

Conclusions

I have tried to accomplish two things in this paper. First, I have reviewed the current state of research in the perspective that I call situative, emphasizing that this approach treats cognition as an aspect of interaction of individuals with each other and with the material and informational systems in their environments. Analyses of interaction are conducted at multiple levels that differ in the complexity and time scale of the phenomena that are analyzed. I reviewed two cases from our previous research that focused mainly at levels (3) and (2), respectively, of Table 3.1 in this scheme. These analyses emphasized positioning of students with authority, accountability, and competence, and practices that encourage problematizing issues that can lead to conceptual growth when they are taken up.

The new material in this paper came from analyzing two episodes in the Fast Plants videos that Schauble and Lehrer provided. These examples differ from our earlier examples in the way that issues were resolved. In all of the examples I discuss here, the students were positioned with authority and accountability for generating alternatives that were taken up in their group or in class discussion. But in the process of resolution, authority was distributed differently in the Wisconsin examples than it was in our earlier examples from FCL and MMAP. In the orca-controversy and the mouse-reproduction examples, students were positioned with authority and accountability for resolving the issue that was problematized by the alternatives they generated. In the plant-number and graph-scale examples, adults led the students to positions that resolved the issues.

Theoretically, the difference shows a need to differentiate the concepts of authority and accountability in the positioning of students to include what students are authorized and accountable for. In the Wisconsin examples, students had authority and accountability for constructing representations, which were expected to vary and, thus, provide alternatives with differences that could be problematized. But when issues were problematized, the discussions moved toward resolutions with positions that were authorized by adults. As a result, students did not generate arguments based on principles to support the alternatives that they might have had they been positioned with authority and accountability for resolving the issues themselves.

In relation to Toulmin's (1972) characterization of conceptual change, the practices of this classroom positioned students with initiating agency in the process of generating variability. They produced representations that varied significantly. Their role in the process of selection was less generative. The selection of a form, at least in each of these two instances, was guided quite strongly by an adult.

Putting this in another way, the difference also shows a need to differentiate the concept of cognitive demands of instructional tasks, as this has been developed by Stein and her associates (Stein, Grover, & Henningsen, 1996; Stein, Smith, Henningsen, & Silver, 2000). The concept of cognitive demands distinguishes between requirements of reciting from memory, performing a procedure without conceptual connection, performing a procedure with conceptual connection, and "doing mathematics." I would judge that the task of constructing graphical representations involved doing mathematics, but the contrast with our other examples shows that doing mathematics can occur with different participation structures, which may have consequences for what students learn in their activity. The positioning of students in an activity of doing mathematics may include more or less authority in resolving mathematical issues, and this may be significant for their learning outcomes.

This distinction invites counterfactual speculation about the kind of practice that the Fast Plants class might have had in which students would have been positioned to participate with more initiative in the process of resolving differences between alternative representations. It would involve establishing forums of debate about advantages and disadvantages of representational conventions. Like all such issues, this involves dilemmas of how to allocate time to different aspects of classroom activity. It would take significant time to establish a discourse practice of debating properties of different representational forms. To decide to do that would depend on a judgment that the value of students' participation in such debates would be sufficient to offset the loss of participation in other activities that would have to be allocated less time. Perhaps further research that analyzes ways in which such participation contributes to students' understanding and identities as learners would be valuable for us to pursue.

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Notes

1. The following paragraphs are adapted from an earlier presentation of this theoretical approach (Greeno, 2001).
2. I refer to my version of this effort as a "situative" approach. I was introduced to these issues and ideas by Jean Lave, Brigitte Jordan, Lucy Suchman, and others who characterized their perspective as "situated action," "situated cognition," and "situated learning." I make the small syntactic change to "situative perspective" or "situative analysis" to make less likely the unfortunate misconception that only some action, cognition, or learning is situated.
3. I previously (including in a draft of this paper) referred to these two approaches as "working from the inside out" and "working from the outside in." I am grateful to Eric Bredo for pointing out that this is a poor way to characterize the distinction. A very prevalent strategy takes the cognition-to-interaction approach, working from the inside out – it treats individual cognition as the fundamental process and works to account for the influences of other people and systems in the environment as contexts realized as different experimental treatments. The approach taken by Dunbar, Okada and Simon, Schwartz, and others was different; in these studies cognitive processes such as analogical reasoning, generating hypotheses, or representing abstractly were attributed to interacting groups of individuals. Recent interactional analysis of cognition do not "work from the outside in" in the sense of hypothesizing intra-individual processes that are distinguished from processes at the level of activity systems and social practices and that inherit properties of the group processes (although Vygotsky's (1987) writing encourages hypotheses along those lines). Instead, cognitive processes such as perception, remembering, understanding, and reasoning are considered as functions of activity at the level of activity systems and hypotheses to explain cognitive accomplishments are proposed in terms of hypotheses about processes of interaction.
4. I say "basically correct" because calling the aspects "task" and "social" doesn't match my understanding. In my view, tasks are social. What constitutes a task, and which tasks are

important for one to participate in, are aspects of social practice. I prefer a distinction that I have called "informational" vs. "interpersonal," although that is also problematic. Perhaps "semantic" and "systemic" would be useful as terms for this distinction, where semantic aspects involve the referential meanings of concepts and assertions that are made and relied on in activity, and systemic aspects involve the interactive processes of that activity.

5. In disciplinary agency, the result of a process is determined by accepted rules of a practice, if the agent performs the procedure correctly. In conceptual agency, the result of a process depends on choices that the agent makes, including the way a problem is formulated and what procedures to use. Pickering discussed ways in which scientific, mathematical, and engineering practices involve "dances of agency," combining conceptual agency and disciplinary agency, especially in mathematics, or conceptual agency and material agency, where outcomes depend on the way apparatus functions in the world, especially in physics.
6. The assumption that systemic and semantic principles function jointly at all levels is weaker than alternative assumptions that are needed to justify treating either of them as a context for the other. For one of these sets of aspects to function as context for the other, the two sets of aspects have to be factorable in a way that I believe is empirically unwarranted.
7. The initial phase of this analysis was conducted by a group that included Randi Engle, Faith Conant, Muffie Wiebe, Frederick Erickson, and me. Engle and Conant completed the analysis and wrote the report.
8. This isn't quite right, of course. Biologists are clear about how they classify marine mammals. But the students arrived at a reasonable interpretation, given their sources. Whales and dolphins (including orcas) are all members of the order of cetaceans, but are in different families. Thus, at one level, it is appropriate to distinguish orcas from whales. But cetaceans are often referred to collectively as whales; for example, books that are about whales (in this sense) usually include discussions of orcas.
9. I am grateful for a conversation with Hyman Bass in which he corrected my previously inaccurate understanding of the concept of reconciling.
10. It is possible that the group had a previous conversation along these lines. It could be informative to see tape, if there is some, of this group's decision to use plant numbers as the vertical dimension on their graph.
11 It might be useful sometimes, in situations like this, to invite discussion of circumstances that would make it valuable to maintain a representation that lets the identities of individual data points be recovered. For example, if other data were stored with each data point, being able to identify individual data points from the graph of plant heights could be useful for investigating relations between variables. Such a discussion could lead to the same conclusion as this group reached, but it could be based on a judgment that preserved the merits of the rejected alternative, just not for this situation.
12. O'Connor, Godfrey, and Moses (1998) analyzed an extended event in an Algebra Project classroom that focused on a student's "missing data point." An observation by one of the students had been lost, and the class worked on what to do about that over an extended period. O'Connor et al. documented that the class's concern was not limited to having a complete data set for its own sake, but also for not leaving out the student's personal contribution to their joint project.
13. Hall, Stevens, and Torralba (2002) described an instructive (and delightfully interesting) interaction between some biology researchers and a statistician. The biologists had taken samples of termites in a spatial grid, and had data about the chemical composition of wax from the termites collected at each site. They wanted advice about how to evaluate their data to investigate whether there was a previously unidentified species of termite in their woods. The statistician was thinking about distances between populations of data that could be analyzed by some kind of discriminant analysis. The conversation was delightfully confused for some time while the biologists thought he was talking about distances between the locations at which the termites had been captured.
14. When the teacher introduced the hypothetical 555 mm plant, he said, "I'm wondering which graph would show better in (1.3) the spread. So let's let's ignor:e two hundred and fifty five

for a minute and say instead of that plant being >two hundred and fifty five< (0.5) *((writing on board))* it was five hundred fifty five. Oka:y. Does that does that feel like it's quite a bit different(0.4) than two fifty five 'kay?" [Excerpt 9, 0:10:28–0:10:42] This is the only reference to plant heights that I found in the discussion of spread. I found no explicit references to variation in the heights as an aggregate property of the set of plants.

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