Chapter 8
Sequential Location Models

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8.1 Introduction

Competitive location models have been discussed in the location literature since Hotelling's (1929) seminal paper. As other location contributions, his model includes customers, who are located in some metric space and who have a demand for some good. This demand may be satisfied by firms that offer the product, given some pricing policy. The difference between standard location problems and competitive location models is that in the competitive case, there are at least two competing firms, who offer the same product. Depending on the complexity of the model under consideration, the differences between the firms may include their different locations, prices, pricing policies, or the attractiveness of their respective facilities.

In their simplest form, competitive location models are based on the assumption that customers will patronize the firm that offers them the best value, in terms of price, transportation costs, and general attractiveness. Given some objective function, each firm will then attempt to determine the optimal value of the variables that are under their respective jurisdictions, such as its location, price, and possibly other features. Models including some of these features can be found in literature, see, e.g., Eiselt and Laporte (1996) and Plastria (2001).

There are two main types of analyses that have been performed on competitive location models. The first asks whether or not there exists a stable situation for the model, i.e., an equilibrium. Depending on the tools available to the decision makers of the firms, we may have location equilibria, price equilibria, etc. In the context of location, the equilibrium question was first addressed by Hotelling (1929) and a summary of his contribution can be found in Chap. 7 in this volume. His analysis assumes that the competitors play a simultaneous game, in the sense that they

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choose their strategies at the same time. Another type of analysis involves sequential moves, i.e., the competitors make their choices one after the other in a prescribed sequence. For simplicity, this chapter will concentrate on location choices, assuming that the firms' prices are set at a fixed and equal level.

One type of analysis starts with an arbitrary locational arrangement of the firms on the market and applies short-term optimization by allowing them to relocate one by one so as to maximize their profit. The main question is what location pattern will result from such a process, and whether or not it is stable. The first to investigate such process appears to be Teitz (1968), which is the first paper reviewed in this chapter.

Another type of analysis that employs a sequential location process was first proposed by the economist Freiherr von Stackelberg (1943). His analysis assumed that in any industry, there are firms that lead (in innovation, product development, or in any other way), while there are others that follow. This concept was later extensively used in marketing, where leaders and followers were referred to as "first movers" and "second movers." In our analysis, we will consider a firm the leader, if it acts (most prominently: locates) first, while a follower is a firm that acts after the leader has chosen his strategy.

Note the asymmetry in the decision making processes of leaders and followers: The follower faces a situation in which the values of his opponent's decision variables are known to him, so that he faces possibly a number of restrictions, but deals with certainty, at least in regard to his competitor's key decisions. On the other hand, taking into account his opponent's decision, the leader faces uncertainty, as he usually does not know what his competitor's objectives are. Even if he did, he first has to establish what is known as a *reaction function*, i.e., for each of his own potential decisions, the leader must establish the reaction of his competitor and determine the outcome based on this pair of decisions. Given the complete reaction function, i.e., having established his competitor's reactions to each of his own possible actions, the leader can then choose the course of action that benefits him most. The setting here is a straightforward application of bilevel programming problems, (see, e.g., Dempe 2002), in which the follower's solution becomes the input in the leader's problem. If the model setting is simple, there may be a (closed-form) description of the reaction function. However, in most practical cases, the reaction function consists of solutions that are much more complex, e.g., solutions of integer programming problems, making the leader's problem very difficult, to say the last.

Another aspect of von Stackelberg solutions is that firms are not necessarily designed to be leaders or followers. As a matter of fact, this choice may be up to the firm as part of the decision-making process. In order to be a leader, there are essentially two requirements: First, a firm must have the *capability* to be a leader, and secondly, it must have an *incentive* to become a leader. For instance, the capability could require a firm to have a large research and development lab, to have a foothold in a country they want to compete in, or similar advantages in the industry. Typically, only firms that have significant capital can possibly be leaders. The second requirement has nothing to do with the firms themselves, but with the way the process is structured. For instance, if the system does not protect the leader, it may not be beneficial to become a leader. As an example, take the pharmaceutical industry. A leader would be a firm that develops new drugs for certain illnesses. The requirement of capability is clear. Consider now the need for appropriate protection. In this example, protection is provided in the form of patenting laws. In case a patent lasts for a very long time, then there is a strong incentive for a capable firm to develop all sorts of new medicines, as they will be able to reap the benefits for a very long time. On the other hand, if the time of a patent elapses after only a few years, the firm will have little time to introduce the drug, publicize it, and recover some of its costs before the patent runs out, allowing other firms to produce generic versions of the drug, thus dramatically cutting down the leader's market share. Knowing this in advance, the leader may not consider the time sufficient to recover costs and make a profit, so that he may not conduct the research and consequently will not introduce the product. In other words, the leader-follower game will not be played. Another aspect concerns the existence of more than two firms in the market. It has been suggested that in such a case, there will be a waiting line of firms, the first being the leader, the second will follow thereafter, and so forth. However, the question is why any firm would accept to take an specific place in line rather than choose what is most beneficial for his firm (other than may be first in line, which requires special capabilities). It is much rather likely that the firm will group into leaders and followers, depending on their abilities and the system's incentives.

The major assumptions of the sequential location model are that

1. Location decisions are costly and are made once and for all. Relocating is considered prohibitively costly and is not permitted.
2. Firms enter in sequence, one after another.
3. The leader and the follower have full and complete knowledge about the system, and the follower will have complete knowledge about the leader's decisions, once they have been made.

Among the first to propose the sequential entry of firms to the market are Teitz (1968), Rothschild (1976) and Prescott and Visscher (1977). The paper by Prescott and Visscher (1977) introduced the sequential entry of firms in a competitive location model from the perspective of operations research. Their ideas are illustrated through a set of examples, which are covered in this chapter.

Prior to von Stackelberg's work, other theories regarding market competition were known, mainly the one by Cournot. In his analysis two firms A and B are competing to supply the market with a homogeneous product at the same price. The two firms compete in the amounts of the product that they will put on the market. Each firm's objective is to determine the amount of the product it will make and sell in order to maximize its profit. In order to do so, each firm will determine its own supply reaction to the other firm's supply. Cournot stability assumes that each firm will move along its reaction curve. Cournot asserts that if each supplier takes the amount offered by his rival as a parameter of action, then the two firms can reach a point of equilibrium as the point of intersection of the firm's reaction curve to its competitor's supply. While these contributions provide the basic ideas for sequential location problems, their main emphasis is in economics, which is why we have chosen not to review them in detail.

The remainder of this contribution is organized as follows. Section 8.2 will review two classic contributions that use sequential location processes: one by Teitz (1968), and the other by Prescott and Visscher (1977). Section 8.3 will then assess the major impact of these contribution and outlines some directions of future research.

8.2 Classic Contributions

In this paper, we have chosen to survey two of the major contributions to the field. The first is a paper by Teitz (1968), in which he discusses a sequential relocation problem for two firms, one of which locates a single facility, while the other locates multiple facilities. The stability of the locational arrangement is investigated. The second paper is by Prescott and Visscher (1977). Its main contribution is the description of the sequential location of three facilities with foresight. This paper was the first to use von Stackelberg solutions in the context of competitive location models. Many contributions have used the basic ideas put forward in this work and extended them.

8.2.1 Teitz (1968): Competition of Two Firms on a Linear Market

While Prescott and Visscher (1977) are usually credited as the pioneers of sequential location, the contribution by Teitz (1968) predated their work by more than a decade. Teitz's paper considers the usual competitive system on a linear market, but with fixed and equal prices. In contrast to other contributors, the author does not consider simple competition between firms that locate one branch or facility each, but competition, in which each firm locates a given number of branches. The main thrust of the paper deals with repeated short-term optimization of the facilities' locations. For simplicity, the space customers and firms are to be located in a "linear market" of length 1, i.e. a line segment, on which customers are uniformly distributed.

The paper starts with the simple case of each firm locating a single branch each and, starting with initial random locations, use "short-term optimization" to relocate. This is done in order to maximize the firm's maximal profit, which, given fixed prices and fixed demand, reduces to the maximization of market share. The author uses sequential and repeated optimization by the two firms. In each step, the relocating firm takes the location of its competitor as fixed and optimizes. Given short-term maximization of market share, the relocation rule is to locate next to the competitor on the "longer" side of the market. Once this is accomplished, the other firm relocates in the same fashion. In this way, the two firms will cluster in each step and slowly move towards the center of the market, where neither of them has any more incentive to relocate further. This central agglomeration solution recreates Hotelling's "central agglomeration."

The paper then investigates the case of firm A locating two facilities A^1 and A^2 and firm B locating a single facility by the same name. Either firm has two choices: either locate directly next to one of the other two branches on the outside and thus capture the hinterland of that branch, or locate between the two branches and capture half of what is called the "competitive region." Clearly, if a branch were to relocate to the outside, it would chose the branch with the larger hinterland and locate right next to it.

In our example shown in Fig. 8.1, the branches relocate in the sequence B, A^1, A^2, B, and so forth. At first, the two branches of firm A are located arbitrarily on the market. Then firm B locates its firm. It does so directly to the left of A^2, as its hinterland is larger than that of A^1 or half the competitive region. In the next step,

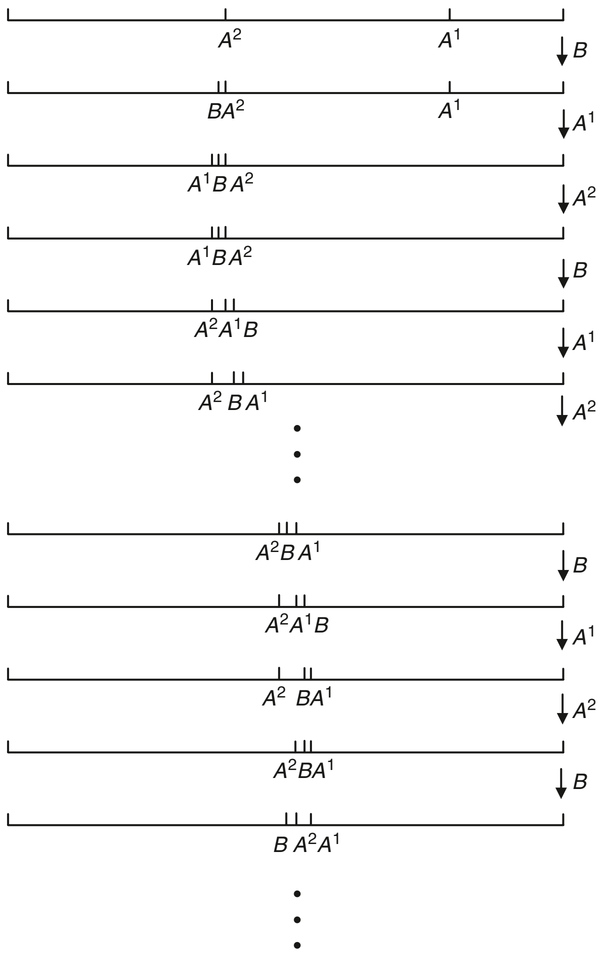

Fig. 8.1 Repeated relocation of two firms

branch A^1 relocates by moving directly to the left of B, even though A^1 has a larger hinterland. The reason is that both branches belong to the same firm, and by locating next to B on its outside, firm A will capture almost the entire market with both of its branches. Thus, when it is branch A^2's turn, it will not move as it is already located in its optimal place.

The next round of relocations starts again with B. It will move to the right of A^1, as this is the longer side of the two outside branches. Branch A^1 will then counter by moving just outside of B, reducing its market share again to a very small segment of the market. Branch A^2 will then move just a bit to the right towards B, so as to almost reduce its market share to zero again. The relocation process will continue in this fashion until all three branches are clustered at or near the center of the market.

At the center, the branch at the center will move to the outside; if it is branch B, it moves to the longer of the two hinterlands, if it is one of firm A's branches, it will move next to branch B and locate on its outside. Teitz referred to this relocation process as a "dancing equilibrium." The market shares of the two firms can be determined as follows. Firm A captures the entire market after one of its branches relocated, while it gets half the market after firm B relocates. Assuming equal relocation speed, firm A captures an average of ¾ of the market, while B obtains an average of ¼ of the market.

The instability of the solution leads to the author's conclusion that short-term optimization may not be the best solution. Instead, he suggests "long-term or conservative maximization." This can be explained as follows. Suppose that firm A locates both of its facilities at the first and third quartiles. Firm B can then either locate adjacent to either of A's branches on the outside (thus capturing the entire hinterland), or anywhere in between A's branches and capture half of the central region. Either way, firm B will capture ¼ of the market. Once this has happened, the author suggests that A does not relocate his branches (although such a relocation would benefit firm A in the short run), but stay put, thus ending the location process. That way, firm A will capture ¾ of the market, while firm B obtains the remaining ¼. This, incidentally, is the same market share that the two firms had obtained if they engaged in short-term optimization, giving the two firms an incentive to behave in this manner (especially when relocation costs are introduced, which are ignored in this discussion). While the author mentions that firm A uses a minimax objective, there is no mention of von Stackelberg and his leader-follower model. There is also no mention of what would happen if firm B were to locate first (which will always result in firm A capturing the entire market, as the two branches of A would "sandwich" firm B regardless of its location).

The analysis is then extended to include one facility of firm B, but 3 branches of firm A. As long as firm A knows that its competitor will locate only a single branch, it is aware that the branches will either follow the pattern *BAAA* or *ABAA*, all other location patterns reduce to these two based on symmetry. If firm A locates its branches at the first, third, and fifth sextiles, firm B will again either locate adjacent to either of the two outside facilities and capture the hinterland of length ⅙, or anywhere between any of firm A's branches and capture also ⅙.

This result can be further generalized to the case in which firm B locates a single facility, while firm A locates n_A branches. Firm A will then subdivide the market, so that the two hinterlands are half as long each as the region between any two of its branches, so that it will locate at $1/2n_A$, $3/2n_A$, $5/2n_A$, …, $(2n_A-1)/2n_A$, while firm B will locate its single facility again either adjacent to A's outside facilities in one of the two hinterlands, or anywhere between A's facilities. With that locational arrangement, firm B will capture $1/2n_A$, while firm A will capture the remaining $1-1/2n_A$ of the market.

The next step in the analysis is to allow firm B to locate more than one facility. In general, we now allow firm B to locate n_B branches, so that $n_B<n_A$. Following a reasoning similar to that above, we find that firm A locates again at the odd $2n_A$-tile points, while firm B locates its branches in the same manner prescribed above. The results are market shares of $M(B)=1/2n_A$ for firm B and $M(A)=1-n_B/2n_A$, for firm A. Figure 8.2 plots Firm A's market share against the number of facilities it locates. Even though n_A must obviously be an integer, we plot for all values for improved visibility. The solid, broken, dashed-and-dotted, dashed-and-double-dotted, and dotted lines show firm A's capture for $n_B=1$, 2, 3, 4, and 10.

A bit of elementary algebra leads to another interesting result. We can rewrite firm A's capture function $M(A)=1-n_B/2n_A$ as $n_A = \frac{n_B}{2[1-M(A)]}$ and then determine the number of branches firm A must locate in order to obtain the desired market share. For instance, for $M(A)=0.5$, we obtain $n_A=n_B$ (an obvious result), for $M(A)=0.75$, we obtain $n_A=2n_B$, for $M(A)=0.99$, $n_A=50n_B$, and so forth. One of the author's conclusions of this process is that the results do not exhibit agglomeration, but are quite similar to the social optima that minimize overall transportation costs. As Teitz put it, "Even a small gadfly can keep the big operator 'honest'." The remainder of the contribution deals with an investigation into equilibria for models for firms with fixed locations and variables prices, which is not of interest in this context.

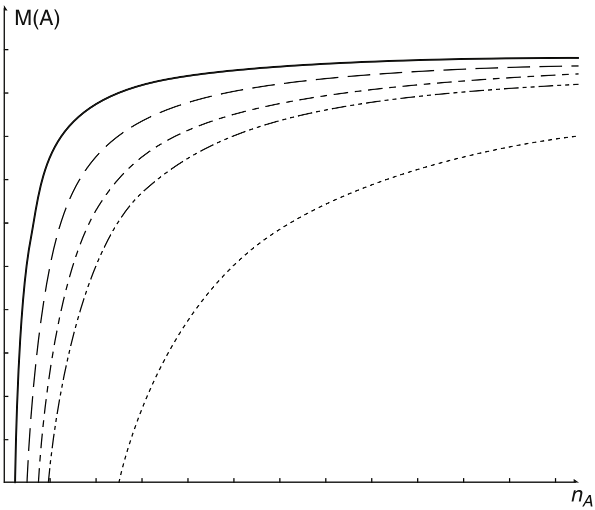

Fig. 8.2 Firm A's market share against the number of facilities

8.2.2 Prescott and Visscher (1977): Extensions of the Model on a Linear Market

The paper by Prescott and Visscher examines a number of different scenarios by way of five examples. Each such “example” relates to a competitive location model. Of specific interest in the context of sequential location models is Example 1, which demonstrates the complexities of the process for the case of three facilities that enter the market sequentially.

The novelty of Prescott and Visscher’s approach is the use of foresight. In other words, the leader of the location game locates first, knowing that the follower will locate, so that its profit will be optimized. Such sequential location problems are typically solved in recursive fashion. If, for instance, n facilities are to be located by n independent decision makers, we first assume that $n-1$ facilities already are located and we are to locate the last facility. This will result in some general location rules. These rules, commonly called “reaction function,” will then be used as input by the $(n-2)$nd facility. In particular, the decision maker at that facility will consider all possible location configurations of the first $(n-3)$ facilities and plan his location, taking into account the reaction function of the n-th facility. It is apparent that this process will get exceedingly tedious once the number of facilities increases.

For now, suppose there are two firms located somewhere on the market. Without loss of generality, assume that firm A is located to the left of firm B. As in our discussion of Teitz’s paper, the area to the left of A is called A’s hinterland, the area to the right of B is referred to B’s hinterland, and the region between A and B is called the competitive region. Finally, that part of the market that is closer to a facility is said to be *captured* by that facility. (Note that this phrase was coined later by ReVelle (1986).

Consider first the simple case of two firms that locate a single branch each. Suppose that firm A is the leader who locates at some point x_A, while firm B is the follower who locates at x_B. The recursive argument assumes for the time being that x_A is fixed and that firm B’s task is to optimally locate its facility. Then there are two cases: either firm B (the follower) will now locate to the left of A (i.e., $x_B<x_A$), or it will locate to the right of A (i.e., $x_B>x_A$). In the former case, firm B will capture the hinterland on its left in its entirety and half of the competitive region between itself and its competitor, i.e., $x_B+½(x_A-x_B)=½(x_A+x_B)$. Since its capture depends positively on x_B, firm B will choose the largest possible value of x_B. Since its location is only limited by x_A, it will choose $x_B=x_A-\varepsilon$ for some arbitrarily small $\varepsilon>0$. In other words, firm B will locate directly to the left of the leader A. In doing so, firm B will capture x_A, while firm A will capture the remaining $1-x_A$ of the market. A similar argument applies to the case, in which firm B locates directly to the right of its competitor A. In this case, firm B locates at $x_B=x_A+\varepsilon$ and captures $1-x_A$, while firm A captures the remaining x_A.

In summary, firm B will now locate directly to the left of A, if $x_A>1-x_A$ or, equivalently, $x_A>½$, while B will locate directly to the right of A, if $x_A<1-x_A$ or

$x_A < ½$. Or, even shorter, if $x_A < ½$, then B locates at $x_A + \varepsilon$ and A will receive x_A, while if $x_A > ½$, then B will locate at $x_A - \varepsilon$ and A will receive $1 - x_A$. Knowing this to be firm B's reaction function, firm A will then decide as follows. In the former case, firm A's capture depends positively on its location, so that it will locate at $x_A = ½ - \varepsilon$, while its capture in the latter case depends negatively on its location, so that it will locate at $x_A = ½ + \varepsilon$. this means that the leader's location is best chosen at the center of the market, and the follower will then locate to either side, so that both capture about half of the market each.

This is the type of argument employed by Prescott and Visscher in their contribution. Below, we discuss two examples that constitute the major contribution of their paper to sequential location theory.

Example 1: Sequential location of three firms

This is example is a straightforward (albeit tedious) extension of the argument put forward above for two firms. Here, three facilities locate in sequential fashion. The facilities are A, B, and C, and their respective locations are x_A, x_B, and x_C. The facilities are going to locate in the order A, B, and C. Again, the length of the market has been generalized to 1. All facilities charge fixed and equal prices, so that we deal with pure location competition. Without loss of generality, we assume that $x_A < ½$. Starting with firm C, we note the C will either locate in one of the two hinterlands or in the competitive region, created by the already existing locations of firms A and B.

We now consider the following four cases.

Case 1: Facility B is located in the left half of the market, i.e., $x_B < ½$. This case allows two Subcases 1a and 1b: either B is located to the left of A, or B is located to the right of A.

Subcase 1a: If B is located at some point to the left of A (i.e., $x_B < x_A$), then C can either locate directly to the left of B (and capture x_B, which, by assumption, satisfies $x_B < x_A \leq ½$), or C can locate between A and B (thus capturing $½(x_B - x_A)$, which, by virtue of the assumptions concerning x_A and x_B, is less than ¼), or locate immediately to the right of A, which results in C capturing $1 - x_A > ½$. Clearly, this option dominates, so that C will locate immediately to the right of A at $x_A + \varepsilon$ for some arbitrarily small $\varepsilon > 0$.

Subcase 1b: Suppose now that B is located to the right of A at some point $x_B > x_A$, while still maintaining that $x_B \leq ½$. Again, facility C's best option is to locate directly to the right of firm B at $x_B + \varepsilon$, thus capturing $1 - x_B \geq ½$.

Summarizing Case 1, we find that Firm C will always locate at $x_C = \max\{x_A, x_B\} + \varepsilon$ and capture about $1 - \max\{x_A, x_B\}$ of the market. Incidentally, given firm C's behavior, firms A and B capture $½(x_A - x_B)$ and $½(x_A + x_B)$ in Subcase 1a, and $½(x_A + x_B)$ and $½(x_B - x_A)$ in Subcase 1b, respectively.

In the remaining three cases, we assume that firm B has located to the right of the mid-market point at $x_B > ½$. The cases differ in that in Case 2, firm C best locates in A's hinterland just to the left of firm A; in Case 3, Firm C best locates in B's hinterland just to the right of firm B; and in Case 4, Firm C's best option is to locate between the two firms A and B. The three cases establish the conditions that the chosen solution provides a larger capture to Firm C than the other two options.

Case 2: Firm B locates at $x_B > ½$, and $x_A > \max\{1 - x_B, ½(x_B - x_A\}$. In this case, Firm C will locate at $x_A - \varepsilon$, capturing somewhat less than x_A. The three firms then capture:

Firm A captures $½(x_B - x_A)$,
firm B captures $1 - ½(x_A + x_B)$, and
firm C captures x_A.

Case 3: Firm B locates at $x_B > ½$, and $1 - x_B > \max\{x_A, ½(x_B - x_A)\}$. Here, Firm C will locate at $x_B + \varepsilon$ and capture somewhat less than $1 - x_B$. The captures of the firms in this case are:

Firm A captures $½(x_A + x_B)$,
firm B captures $½(x_B - x_A)$, and
firm C captures $1 - x_B$.

Case 4: Firm B locates at $x_B > ½$, and $½(x_B - x_A) > \max\{x_A, 1 - x_B\}$. In this case, Firm C can locate anywhere between its competitors A and B and capture about half of the competitive region. Prescott and Visscher assume that Firm C will locate in the middle of the competitive region at $x_C = x_A + ½(x_B - x_A) = ½(x_A + x_B)$ and capture $½(x_B - x_A)$. The captures of the three firms are then:

Firm A captures $¾\, x_A + ¼x_B$,
firm B captures $1 - ¼x_A - ¾x_B$, and
firm C captures $½(x_B - x_A)$.

This completes the reaction of firm C. Consider now the reaction of firm B, which will depend on what firm A has done (something that firm B can observe) and the anticipated reaction of firm C derived above. Note for firm B in Case 1, Subcase 1a dominates Subcase 1b. Table 8.1 shows firm B's options, where *LB* and *UB* denote the bounds derived from the conditions imposed in the four cases.

Table 8.1 Summary of cases in example 1

Case #	Conditions (in addition to $x_A < ½$)	Firm B's capture	Strongest condition for x_B
1	$x_B < x_A < ½$	$½(x_A + x_B)$	$x_B < x_A$
2	$x_A > 1 - x_B$ or $x_B > 1 - x_A$ $x_A > ½(x_B - x_A)$ or $x_B < 3x_A$	$1 - ½(x_A + x_B)$	$x_B \in [1 - x_A, 3x_A]$ ($x_A \geq ¼$, as $3x_A \geq 1 - x_A$)
3	$1 - x_B > x_A$ or $x_B < 1 - x_A$ $1 - x_B > ½(x_B - x_A)$ or $x_B < ⅔ + ⅓\, x_A$	$½(x_B - x_A)$	$x_B < ⅔ + ⅓\, x_A$, if $x_A \leq ¼$ $x_B < 1 - x_A$, if $x_A \geq ¼$ $x_B > ½$ in both cases
4	$½(x_B - x_A) > x_A$ or $x_B > 3x_A$ $½(x_B - x_A) > 1 - x_B$ or $x_B > ⅔ + ⅓\, x_A$	$1 - ¼x_A - ¾x_B$	$x_B > ⅔ + ⅓\, x_A$, if $x_A \leq ¼$ $x_B > 3x_A$, if $x_A \geq ¼$

In Case 1, firm B's capture is positively correlated with its location, so that B will choose as large a value of x_B, which is achieved at $x_B = x_A - \varepsilon$. Firm B's gain is then about x_A, while firm A is wedged in between B and C and will get nothing.

Note that in Case 2, firm B's capture is negatively correlated with its location, so that B will attempt to decrease x_B as much as possible. The same argument applies in Case 4, while firm B will increase x_B as much as possible in Case 3.

Assume now that $x_A < ¼$. Then firm B's choice is to either locate at $x_B = x_A - \varepsilon$ and capture $½(x_A + x_B) \approx x_A$ (Case 1), at $x_B = ⅔ + ⅓\, x_A$ and capture $⅓(1 - x_A)$ (Case 3), or at $x_B = x_B = ⅔ + ⅓ x_A$ and capture $½(1 - x_A)$ (Case 4). Note that (Case 2) does not apply. Clearly, Case 4 dominates, so that

- if $x_A < ¼$, firm B will locate at $x_B = ⅓(2 + x_A)$ and capture $½(1 - x_A)$.

In case $x_A > ¼$, firm B can either locate at $x_B = x_A - \varepsilon$ and capture x_A (Case 1), locate at $x_B = 1 - x_A$ and capture ½ (Case 2), locate at $1 - x_A$ and capture $½ - x_A$ (Case 3), or locate at $3x_A$ and capture $1 - 2½x_A$ (Case 4). As $x_A \geq ¼$ in case 4, firm B's capture in that case cannot exceed ⅜, so that Case 2 dominates. This results in the location rule for firm B:

- If $x_A > ¼$, firm B will locate at $x_B = 1 - x_A$ and capture ½.

This now completely describes the reaction function of firm B.

On the last, and highest, level, consider now firm A's planning. Note that firm A knows exactly how its two competitors will react to any of its own actions. In particular, our above discussion reveals that if firm A locates somewhere at $x_A < ¼$, then firm B will locate at $x_B = ⅓(2 + x_A)$ and firm C will locate at $x_B + \varepsilon$ or at $⅓(1 + 2x_A)$. As for firm C, Cases 3 and 4 dominate. In both cases, firm A will maximize its own capture by choosing as large a value of x_A as possible, so that $x_A = ¼$ (resulting in $x_B = ¾$ and $x_C = ¾ + \varepsilon$ or $x_C = ½$).

As the former case requires some distance between firms B and C, firm C's capture will be somewhat less than in the latter case, so that we assume that firm C locates at the center of the market. Given locations at ¼, ¾, and ½ for the firms A, B, and C, their captures are ⅜, ⅜, and ¼, respectively.

Suppose now that firm A locates at some point $x_A > ¼$. As derived above, firm B will then locate at $x_B = 1 - x_A$, while firm C will either apply Case 2 and locate at $x_A - \varepsilon$, or apply Case 3 and locate at $1 - x_B + \varepsilon$. Each of these two cases result in firm C capturing x_A. Given that, firm A will capture $½(x_B - x_A) = ½(1 - 2x_A)$ or $½(x_A + x_B)$. In the former case, firm A's best option is to choose x_A as small as possible, so that $x_A = ¼$ (resulting in $x_B = ¾$ and $x_C = ½$), given the same argument used above), while in the latter case, firms A and B will cluster about the center with firm C locating next to either one of them, cutting out one of the firms. This outcome is unlikely, leaving again the symmetric locations of the firms at the first and third quartile and the center respectively. As demonstrated above, the captures of the three firms are then ⅜, ⅜, and ¼ for the firms A, B, and C. One question not addressed by Prescott and Visscher is why firm C would agree to be the third firm to locate, given that its capture is one third less than that of the second firm.

Example 2: Sequential location of infinite number of firms

Prescott and Visscher's second example considers the case in which an infinite number of firms, with a fixed cost of locating, can be potential entrants. Given that α is the market share needed to cover the fixed costs, then the largest number of firms that can enter the market with positive profit is $1/\alpha$, assuming again a market of length one. The authors describe three basic rules for the location. The first rule considers the case, in which two facilities are located at x_A and x_B, respectively, where, without loss of generality, x_A is located to the left of x_B. It is assumed that the two facilities are direct neighbors, i.e., there exists no facility between them.

Then, if the two facilities are no more than 2α apart, no new facility will ever locate between them, as the space is not sufficient to make a positive profit. If the space is more than 2α but no more than 4α, then a facility may profitably locate between x_A and x_B and, as in their previous example, the authors claim that the new facility would locate halfway between the two existing facilities. Finally, if there is more than 4α between the two existing facilities, the authors assert that a new facility would locate at a distance of 2α to the right of A or to the left of B with equal probability.

The second rule (and, by virtue of symmetry, the third rule) considers the situation that a facility exists at some point x and no other facility is located to its left. Clearly, if the space to the left of x is less than α, no facility will be able to profitably locate to the left of x. On the other hand, if the space left of x is larger than α, a new facility can locate there, which, as the authors assert, will happen at point α. This point, of course, guarantees that there will be no additional facility locating at any point in time to the left of the newly entering facility.

After some algebra, the authors determine that the model will locate facilities, so that the outside firms are α distance units from the two ends of the market, and each subsequent firm is located at a distance of 2α from its neighbor. The only disruption of the uniformity is the last interval that is either too short for an additional facility to locate in, or in which a facility will locate in the center.

Example 3: Competitive location model with product quality as "location"

This example introduces a revised Hotelling problem, in which firms choose a level of product quality in addition to the price. The product quality characteristic in this example is waiting time. The introduction of such a product characteristic enables the authors to formulate the classical Hotelling problem without discontinuities in the reaction function, thus avoiding the disequilibrium problem that is inherent to Hotelling's classical model. The solution of the duopoly model is made by numerical models, in which the equilibrium is unique and, as opposed to Hotelling's assertion of "minimal differentiation," the locations are widely dispersed. Equilibria with more than two competitors cannot be guaranteed.

Example 4: An extension of a competitive location model with product quality as "location"

This example expands the model introduced in Example 3. In particular, it is assumed that once the facilities have chosen their location, they can no longer be moved. First, the authors observe that if the duopolists were to choose location and price simultaneously and irreversibly, then the follower firm always has an advantage, as it can locate at the same site as the leader, but charge a slightly lower price, thus being guaranteed higher profits than the leader. This raises the questions if any location will actually take place at all, as both firms may wait for the other to lead, so they can have the advantage to follow.

However, prices are not very likely to be as inflexible as locations, so that while locations (waiting times) are chosen once and for all, prices can be determined subsequently, so that they constitute a Nash equilibrium. The authors describe a recursive procedure that includes the possibility that the leader firm decides not to enter the market. Given some specific parameters, the authors then compute equilibrium solutions. The authors obtain some interesting results.

- Fixed costs are a barrier to the entry of additional facilities.
- Increasing fixed costs allows duopoly firms to locate farther apart, thus realizing local monopolies, so that the firms' profits actually increase.
- Earlier entrants hive higher profits.
- If the first firm to enter is allowed and has the resources to locate multiple branches, it will locate branches at all profitable locations, resulting in a monopoly.

Example 5: A competitive location model with plant capacity as "location"

This final example assumes that the price of a good in an industry is determined by the total capacity of all firms in the industry. If the number of firms that enter the market is not set in advance, the first entrant will build just as much capacity so as to ensure that no subsequent firms enter the market, thus resulting in a monopoly. This case is reminiscent of the last observation in the previous example. The results are very different, if a predetermined number of firms will enter the market. In such a case, early entrants will chose smaller capacities, so that subsequent entrants increase the total capacity of the industry to a level that is beneficial to the early entrants.

8.3 Impact of the Classic Contributions and Future Research

Competitive location models can be and have been applied to a variety of problems in marketing, political science, product positioning, and others. Sequential location procedures are appealing for many of these applications, so that it is not surprising that many researchers have discussed different aspects of sequential location models. In what he called the von Stackelberg equilibrium problem, Drezner (1982) introduced the planar sequential location problem and offered a polynomial time algorithm for this problem. Macias and Perez (1995) used rectilinear distance for planar competitive problem with an $O(n^5)$ algorithm. The case of asymmetric distance was first studied by Nilssen (1997) and more recently by Lai (2001). Lai's results show that equilibrium results cannot be attained in continuous location. Teraoka et al. (2003) considered the case of the two-firm planar von Stackelberg problem with customers distributed continuously according to a random distribution with a probability that a customer would patronize a certain facility. Bhadury et al. (2003) describe heuristic solution methods for centroid and medianoid problems in the plane. Eiselt and Laporte (1996) studied the case where the facilities have different levels of attractiveness based on certain characteristics. Plastria (1997) introduced a competitive model based on location and attractiveness level.

Neven (1987) and Anderson (1987) investigate sequential location models from an economist's perspective. Both authors determine—as Prescott and Visscher did before them—that locations are much more difficult to change than prices and are therefore much more likely to be permanent. Prices, on the other hand, can easily be changed without cost. This lead them to a "first location, then price" game. The contribution by Ghosh and Buchanan (1988) allows duopoly firms to locate multiple facilities each. The authors also introduce the marketing concept of "first mover advantage" into the discussion.

Eiselt and Laporte's review (1996) of the sequential location problem listed the major contributions that employ a linear market or two-dimensional real space. The authors identify three main research issues: different objectives for firms; endogenizing the leader/follower choice; and the position of firms in a queue for entrance to the market.

One of the major contributions that uses the concept of sequential location choice is by Hakimi (1983). In his paper, the author defines centroid problems and medianoid problems, the former pertaining to the leader in the location game, while the latter is the decision problem by the follower. While his paper deals with the location of facilities on a network, the concepts easily translate to other spaces. In Hakimi (1990), the author further develops specific results given different customer choice rules. For further details on Hakimi's results and an in-depth discussion thereof, readers are referred to Chap. 9 of this volume.

One major assumption of the sequential location model is that the firms in the model enter the market at different points of time, an issue closely related to that of a firm's position in the entry queue. The time between entries enables leader firms to gain more profit and market share for a certain period, while the followers decide on the timing of their entry. Important issues for future studies include the effects of changes of the cost of entry over time due to different factors such as fixed cost change, inflation or a reduction in the cost of technology.

Other factors to be included could be market penetration costs for followers and customer retention costs for leader firms. Models that take such factors into account should allow for uncertainty of these factors. A stochastic approach to the competitive environment was introduced by Choi et al. (1990), who produced a model with one leader and multiple followers and customers with a stochastic utility function.

Another open area of research is the incorporation of the concepts of competitive location in the context of supply chain models. A model based on competition for customers can be considered as a model which is looking forward in the supply chain, while a model looking backward would consider competing for suppliers, e.g., manufacturers for retailers or natural resources such as oil. Sequential location problems will include location decisions with respect to both suppliers and customers.

An interesting aspect of competitive location models concerns cases, in which customers cannot arbitrarily switch between competitors without incurring an early termination fee. As an example, this situation applies to the cell phone industry. Other examples involve suppliers of mineral water or heating gas, where customers are bound by annual contracts with a supplier. Adding switching costs as well as binding contracts time to the models would create more realistic models for certain industries.

A sign of globalization is the tendency of competing firms to form bigger companies through consolidations, acquisitions and mergers. Questions in this context include: what location factors would lead a firm to decide to consolidate with firm *A* and not firm *B*? What are the impacts of such mergers and acquisitions on the market and on present and future competitors?

Finally, an issue that could be included in competitive location models includes the privatization of services such as of waste management and disposal. While the location of undesirable facilities (for details, see Chap. 10 of this volume) is a well-studied area of location theory, it has not been investigated in the context of competitive location.

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