Appendix D
Bilinear Interpolation

Abstract This appendix describes a mathematical approximation that can be used to interpolate in two dimensions such as in an image. This is a useful operation to compare images of different pixels sizes or spacings.

It is frequently necessary to combine or compare two different digital images with different sampling sizes (Δx or Δy). Interpolation in two dimensions is one method of reconciling the difference in sampling sizes. For example, most simulated electron microscope images will have a rectangular pixel with unequal spacings in x and y (to match the underlying periodicity of the specimen) but many image display devices (computer screens) will have square pixels with equal spacings in x and y. To properly display an image with rectangular pixels on a device with square pixels will require resampling the image.

The basic problem can be stated as: Given a set of image intensities sampled on a two-dimensional grid with spacing Δx_a and Δy_a generate another set of image intensities on a grid with a different spacing Δx_b and Δy_b. Interpolating the initial grid in two dimensions generates a function of two independent variables $f(x,y)$ that is continuous but may not have continuous derivatives. Calculating this function at each point in the new grid effectively samples the first image onto the second grid. This procedure works best if the first and second grids are nearly the sample spacing. If the spacing are dramatically different (i.e., more than about a factor of two different) then various artifacts can be produced.

The basic geometry of interpolation on a grid is shown in Fig. D.1. The initial image is only given at discrete points in (x,y). To find an interpolated value at an arbitrary point requires first locating the four grid points surrounding the point (x,y). In Fig. D.1 the new point is located between x_1 and x_2 in x and between y_1 and y_2 in y. The values of the initial image at the four grid point surrounding (x,y) are f_{11}, f_{12}, f_{22}, and f_{21}.

With four points there are four know conditions and the best interpolation available is a bilinear form:

$$f(x,y) = a + bx + cy + dxy. \tag{D.1}$$

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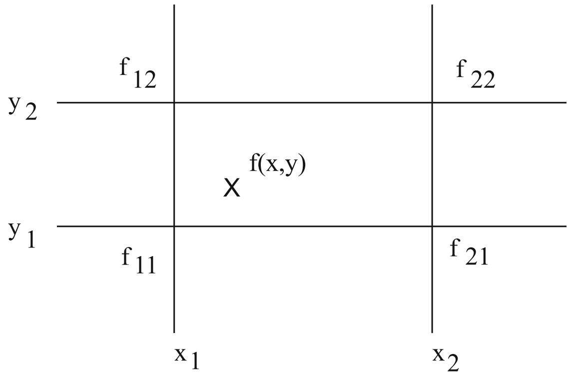

Fig. D.1 Interpolation in a two-dimensional rectangular grid. The function $f(x,y)$ can be found at an arbitrary point (x,y) given only the values sampled at discrete values of $x = x_1, x_2$ and $y = y_1, y_2$ surrounding point (x,y). The spacing in x does not need to be the same as that in y

The a, b, c, and d coefficient are determined by the surrounding image values as:

$$\begin{aligned} f_{11} &= a + bx_1 + cy_1 + dx_1y_1 \\ f_{12} &= a + bx_1 + cy_2 + dx_1y_2 \\ f_{22} &= a + bx_2 + cy_2 + dx_2y_2 \\ f_{21} &= a + bx_2 + cy_1 + dx_2y_1. \end{aligned} \tag{D.2}$$

Combining pairs of equations yields:

$$\begin{aligned} (f_{11} - f_{12}) &= (c + \mathrm{d}x_1)(y_1 - y_2) \\ (f_{21} - f_{22}) &= (c + \mathrm{d}x_2)(y_1 - y_2). \end{aligned} \tag{D.3}$$

The coefficients may be found one at a time. Subtracting (D.3) yields a value for d:

$$d = \frac{f_{11} - f_{12} - f_{21} + f_{22}}{(x_1 - x_2)(y_1 - y_2)}. \tag{D.4}$$

Using this value for d and one of (D.3) yields a value for c:

$$c = \left(\frac{f_{11} - f_{12}}{y_1 - y_2} \right) - dx_1. \tag{D.5}$$

Next combing (D.2) in a slightly different order gives:

$$f_{11} - f_{21} = b(x_1 - x_2) + dy_1(x_1 - x_2) \tag{D.6}$$

from which the a and b coefficients can be found.

$$b = \left(\frac{f_{11} - f_{21}}{x_1 - x_2}\right) - \mathrm{d}y_1 \tag{D.7}$$

$$a = f_{11} - bx_1 - cy_1 - \mathrm{d}x_1y_1. \tag{D.8}$$

Now given a value for all four coefficients (a, b, c, and d) a value for the function $f(x,y)$ at any point inside the four grid points may be obtained from (D.1). Repeating this process at each point of the second grid yields an interpolated value of the first image at each grid point of the second image.