Chapter 1
Introduction

Recent years have witnessed an enormous growth of interest in dynamic systems that are characterized by a mixture of both continuous and discrete dynamics. Such systems are commonly found in engineering practice and are referred to as hybrid or switching systems. The widespread application of such systems is motivated by ever-increasing performance requirements, and by the fact that high-performance control systems can be realized by switching between relatively simple LTI systems. However, the potential gain of switched systems is offset by the fact that the switching action introduces behavior in the overall system that is not present in any of the composite subsystems. For example, it can be easily shown that switching between stable subsystems may lead to instability or chaotic behavior of the overall system, or that switching between unstable subsystems may result in a stable overall system. In this book, we closely examine two classes of systems: switched systems (SS) and time-delay systems (TDS), which will eventually pave the way toward studying a new class of systems, switched time-delay systems (STDS).

1.1 Introduction

Motivated by the desire for a high degree of automation and excellent performance capabilities, control system design has been the focal point of extensive research work during the past several decades. Increasingly sophisticated tools from modern control theories have been developed for improved and better tracking performance. Concurrent advances in microprocessor technology have made the implementation of complex nonlinear control algorithms practically feasible. To meet the explosive social demands, contemporary engineering applications and real-life systems are becoming more complex, interconnected, and spatially distributed. By careful consideration of such systems and phenomena, it turns out that they have a distinct property that the future evolution of the systems states is affected by their previous values, this is frequently called the time-delay effect or simply time delay. This effect can be produced from different sources and in some cases it may affect the system behavior and performance and complicate the system analysis. By and large, the delays are perhaps the main causes of instability and poor performance in

M.S. Mahmoud, *Switched Time-Delay Systems*,
DOI 10.1007/978-1-4419-6394-9_1, © Springer Science+Business Media, LLC 2010

dynamical systems and frequently encountered in various engineering and physical systems [24, 108, 216]. Formally, a system with time delay can be defined as the system in which the future states depend not only on the present but also on the past history of the system [304] and there are many names used in literature for these phenomena, such as system with aftereffect, system with time lag, and hereditary system. In general, such systems are often described by functional differential equation; *a functional equation* is an equation involving an unknown function for different argument values [304]. When this is a differential equation we have a functional differential equation (FDE) or delay differential equation (DDE), where the rate of change of the state in a system model is determined not only by the present state but also by past values. The wide appearance of DDE as a model for several physical and man-made systems is especially important for control systems where actuators, sensors, and transmission lines introduce time delays.

On the contrary, a switched system is a wide class of dynamical systems that are comprised of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switching between the subsystems. This class of systems has an inherent multi-modal behavior in the sense that several dynamical subsystems are required to describe their behavior that might depend on various environmental factors. Switched systems provide an integral framework to deal with complex system behaviors such as chaos and multiple limit cycles and gain more insights into powerful tools such as intelligent control, adaptive control, and robust control. Switched systems have been investigated for a long time in the control and systems literature and have increasingly attracted more attention for the past three decades.

In the remainder of this chapter, we will review some basic notions of dynamical system representation before providing an organization chart of the book.

1.2 Functional Differential Equations

Let $\mathbf{C}_{n,\tau} = \mathbf{C}([-\tau, 0], \Re^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\Re^n$ with the topology of uniform convergence and designate the norm of an element $\phi \in \mathbf{C}_{n,\tau}$ by

$$||\phi||_* \stackrel{\Delta}{=} \sup_{\theta \in [-\tau, 0]} ||\phi(\theta)||_2 \tag{1.1}$$

If $\alpha \in \Re$, $d \geq 0$ and $x \in \mathbf{C}([\alpha - \tau, \alpha + d], \Re^n)$ then for any $t \in [\alpha, \alpha + d]$, we let $x_t \in \mathbf{C}$ be defined by $x_t(\theta) := x(t + \theta), \ -\tau \leq \theta \leq 0$. If $\mathcal{D} \subset \Re \times \mathbf{C}$, $f : \mathbf{D} \to \Re^n$ is a given function, the relation $\dot{x}(t) = f(t, x_t)$ is a retarded functional differential equation (RFDE) [109] on $\mathbf{D}$ where $x_t(t), t \geq t_o$ denotes the restriction of $x(\cdot)$ to the interval $[t - \tau, t]$ translated to $[-\tau, 0]$. Here, $\tau \geq 0$ is termed the *delay factor*. In the sequel, if $\alpha \in \Re$, $d \geq 0$ and $x \in \mathbf{C}([\alpha - \tau, \alpha + d], \Re^n)$ then for any $t \in [\alpha, \alpha + d]$, we let $x_t \in \mathbf{C}$ be defined by $x_t(\theta) \stackrel{\Delta}{=} x(t + \theta), \ -\tau \leq \theta \leq 0$. In addition, if $\mathcal{D} \subset \Re \times \mathbf{C}$, $f : \mathbf{D} \to \Re^n$ is given function, then the relation

$$\dot{x}(t) = f(t, x_t) \tag{1.2}$$

is a retarded functional differential equation (RFDE) on $\mathbf{D}$ where $x_t, t \geq t_0$ denotes the restriction of $x(.)$ on the interval $[t-\tau, t]$ translated to $[-\tau, 0]$. A function x is said to be a *solution* of (1.2) on $[\alpha - \tau, \alpha + d]$ if there $\alpha \in \Re$ and $d > 0$ such that

$$x \in \mathbf{C}([\alpha - \tau, \alpha + d], \Re^n), \quad (t, x_t) \in \mathbf{D}, \quad t \in [\alpha, \alpha + d] \tag{1.3}$$

and $x(t)$ satisfies (1.2) for $t \in [\alpha, \alpha + d]$. For a given $\alpha \in \Re,\ \phi \in \mathbf{C},\ x(\alpha, \phi, f)$ is said to be a solution of (1.2) with *initial value* ϕ at α.

In the linear case, the RFDE (1.2) assume the form

$$\dot{x}(t) = A_o x(t) + A_d x(t - \tau), \quad x(\theta) = \phi(\theta),\ -\tau \leq \theta \leq 0 \tag{1.4}$$

We note from [108] that when $\phi(.)$ is continuous then there exists a unique solution $x(\phi)$ defined on $[-\tau, \infty)$ that coincides with ϕ on $[-\tau, 0]$ and satisfies (1.4) for all $t \geq 0$. By the Lagrange's formula, this solution is given by

$$\begin{aligned} x(t) &= \exp^{A_o t}\ x(0) + \int_0^t \exp^{A_o(t-\theta)}\ A_d x(t-\theta)\ \mathrm{d}\theta \\ &= \exp^{A_o t}\ x(0) + \int_{-\tau}^{t-\tau} \exp^{A_o(t-\theta-\tau)}\ A_d x(\theta)\ \mathrm{d}\theta \end{aligned} \tag{1.5}$$

In the case where $\tau \equiv 0$, system (1.4) reduces to

$$\dot{x}(t) = (A_o + A_d) x(t) \tag{1.6}$$

which is asymptotically stable when all the eigenvalues of $(A_o + A_d)$ have negative real parts.

1.3 Piecewise Linear Dynamical Systems

Piecewise linear (PL) systems are naturally due to the presence of a range of system nonlinearities, such as dead zones, saturation, relays, and hysteresis. Indeed, stability properties of system components, especially actuators which are piecewise linear, have been studied for decades. However, in recent times engineers have started to employ control laws that are piecewise linear in nature. Important examples are rule-based control, gain scheduling, and programmable logic control [334]. There has also been a recent interest in what has been termed hybrid systems [99]. Indeed, this term has been used for a wide range of systems, from timed finite state automation to complete integrated factory control and scheduling problems to the extent that some definitions used would encompass the piecewise linear systems. In [334], a computational tool for the analysis of PL dynamical systems was developed. An example of such a system is the ABS (anti-skid braking) system in a car, where the controller is rule-based and designed using the engineer's knowledge of the system. The only current viable approach to testing such a system is by using extensive simulation and prototype testing, which must be repeated for each of the different car models on which it is installed. The work reported in [334] provides useful insight into the logic and dynamic interaction of such a system. Similarly, systems with programmable logic controllers and gain schedulers also fall into the class of piecewise linear systems.

By taking ideas and known results from linear systems, convex set theory, and computational geometry, the work of [334] aims to synthesize an analysis tool for studying a class of systems that mix logic and dynamics.

The attractions of piecewise linear (PL) systems in control can be recognized by representing a PL system as a set of convex polytopes $\Pi_j \; \Re^n$, each containing some linear system of the form

$$\dot{x} = A_m x + b_m, \quad x \in \Pi_m \tag{1.7}$$

where the Π_j form a partition of $\Re^n$ such that

$$\cup \; \Pi_j = \Re^n, \quad \Pi_j \cap \Pi_k = \emptyset, \quad j \neq k \tag{1.8}$$

In a geometric setting, the problem has a complex picture of *boxes* stacked together in state space with each box containing a different linear dynamic system. Any global analysis must somehow identify the behaviors in each box and then link them together to form a global picture of the dynamics. Loosely speaking, the associated state space will comprise of $n \times p$ linear regions, where n and p represent the number of states and number of PL functions, respectively. Note that the PL functions of the system would eventually result in switching surfaces in the state space. These surfaces act as the boundaries of the convex polytopes that contain each linear dynamic region. The difficulties presented in analyzing this setup are bound up in the need to manipulate high-dimensional convex polytopes and the dynamic systems within them. One analysis technique, using the phase portrait, fulfills many of the analysis aims. In the phase portrait, PL functions can be represented as lines in the plane and trajectories or isoclines plotted to represent the dynamics. The result is a graphical plot of the system dynamics that gives global stability information and shows how the dynamic patterns change due to the switching lines and hence the PL functions. The major drawback is the limitation of the phase portrait to two states.

In [334], the idea of mapping a piecewise linear system into a connected graph was developed, the idea being based on the phase portrait. Each convex polytope or region in the state space will have dynamics entering and exiting that region. If the boundaries of every region were partitioned into sections containing only dynamics entering a region (termed an *Nface*) and only dynamics exiting a region (termed an *Xface*) then the boundaries can be characterized into sections of homogeneous dynamic behavior. Each section thus identified is then represented as a node of a graph. The connections between nodes are then characterized by tracking the set of trajectories (or trajectory bundle) entering via some *Nface* and identifying which (if any) *Xface* the trajectory bundle leaves that region.

Piecing together the nodes and connections for each region results in a directed graph that captures the global dynamic patterns of the system. The nodes of the graph represent the PL functions and the directed connections represent the interaction of the PL functions with the system's dynamics. As will be explained in the subsequent sections, the realization of this apparently simple idea is not easy.

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More about piecewise linear (PL) systems with time delays will be provided later in the book.

1.4 Fundamental Stability Theorems

In this section, we present the fundamental stability theorems that can be used in studying the stability behavior of switched systems and time-delay systems. Further details of these theorems can be found in the classical books [96, 108, 109, 171].

1.4.1 Lyapunov–Razumikhin Theorem

Here the idea is based on the following argument: because the future states of the system depend on the current and past states' values, the Lyapunov function should become functional – more details in Lyapunov–Krasovskii method – which may complicate the condition formulation and the analysis. To avoid using functional, Razumakhin made his theorem, which is based on formulating Lyapunov functions, not functionals. First, one should build a Lyapunov function $V(x(t))$, which is zero when $x(t) = 0$ and positive otherwise, then the theorem does not require $\dot{V} < 0$ always but only when the $V(x(t))$ for the current state becomes equal to $\bar{V}$, which is given by

$$\bar{V} = \max_{\theta \in [-\tau, 0]} V(x(t+\theta)) \tag{1.9}$$

The theorem statement is given by [105]:

suppose f is a functional that takes time t and initial values x_t and gives a vector of n states $\dot{x}$ and u, v, w are class $\mathcal{K}$ functions, $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0$, v is strictly increasing. If there exists a continuously differentiable function $V : \Re \times \Re^n \rightarrow R$ such that

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|) \tag{1.10}$$

and the time derivative of V along the solution $x(t)$ satisfies $\dot{V}(t,x) \le -w(\|x\|)$ whenever $V(t+\theta, x(t+\theta)) \le V(t,x(t))\theta \in [-\tau, 0]$, then the system is uniformly stable.

If, in addition, $w(s) > 0$ for $s > 0$ and there exists a continuous nondecreasing function $p(s) > s$ for $s > 0$ such that

$$\dot{V}(t,x) \le -w(\|x\|) \text{ whenever } V(t+\theta, x(t+\theta)) \le p(V(t, x(t)))$$

for $\theta \in [-\tau, 0]$, then the system is uniformly asymptotically stable. If in addition $\lim\ u(s)_{s\to\infty} = \infty$ then the system is globally asymptotically stable.

The argument behind the theorem is like this: $\bar{V}$ is serving as a measure for the V in the interval $t-\tau$ to t then if $V(x(t))$ is less than $\bar{V}$ then it is not necessary that $\dot{V} < 0$, but if $V(x(t))$ becomes equal to $\bar{V}$ then $\dot{V}$ should be < 0 such that V will not grow.

The procedure can be explained by the following discussion: consider a system and a selected Lyapunov function $V(x)$, which is positive semidefinite. By taking the time derivative of this Lyapunov function we get $\dot{V}$. According to the Razumikhin theorem this term does not always need to be negative, but if we add the following term $a(V(x) - V(x_t))\ \ a > 0$ to $\dot{V}$, then the term

$$\dot{V} + a(V(x) - V(x_t)) \tag{1.11}$$

should always be negative. Then by looking at this term we find that this condition is satisfied if $\dot{V} < 0$ and $V(x) \le V(x_t)$, meaning that the system states are not growing in magnitude and it is approaching the origin (stable system) or $a(V(x) < V(x_t))$ and $\dot{V} > 0$ but $\dot{V} < |a(V(x) - V(x_t))|$ then although $\dot{V}$ is positive and the states increasing, the Lyapunov function is limited by an upper bound and it will not grow without limit. The third case is that both of them are negative and it is clear that it is stable. This condition insures uniform stability, meaning that the states may not reach the origin but it is contained in a domain, say ε which obeys the primary definition of stability. To extend this theorem for asymptotic stability, we can consider adding the term $p(V(x(t))) - V(x_t)$, where $p(.)$ is a function that has the following characteristics:

$$p(s) > s$$

and then the condition becomes

$$\dot{V} + a(p(V(x(t))) - V(x_t)) < 0,\ a > 0 \tag{1.12}$$

By this, when the system reaches some value, which makes $p(V(x(t))) = V(x_t)$, requires $\dot{V}$ to be negative but at this instant $V(x(t) < V(x_t)$ then in the coming τ interval the $V(x)$ will never reach $V(x_t)$ and the maximum value in this interval is the new $V(x_t)$, which is less than the previous value, and with time the function keeps decreasing until the states reach the origin.

1.4.2 Lyapunov–Krasovskii Theorem

The Razumikhin theorem attempts to construct the Lyapunov function while the Lyapunov–Krasovskii theorem uses functionals because V, which can be considered as an indicator for the internal power in the system, is function of x_t, then it is logical to consider V, which is a function of function and hence a functional. The terms of $V(x_t)$ should contain terms for the x in the interval $(t - \tau)$ to t and $\dot{V}$ should be < 0 to ensure asymptotic stability. This method will be covered in more detail in the next section.

In many cases, the Lyapunov–Razumikhin theorem can be found as a special case of Lyapunov–Krasovskii, theorem which makes the former more conservative. The Lyapunov–Krasovskii method tries to build a Lyapunov functional, which is function in x_t, and the time derivative of this Lyapunov function should be negative for the system to be stable. Previously there were criticism on the Lyapunov–Krasovskii method that it can be used for systems with the third category of delay mentioned in Section 2.2.2 only when $\dot{\tau} \le \mu \le 1$ [338], but the recent results resolve this problem as we see in the next chapter. Another criticism is that the Krasovskii methods cannot deal with delay in the second category and also the recent results in this method succeed to include this case [153, 155, 168–170]. The remaining advantage of the Razumikhin method is its simplicity, but the Krasovskii method proved to give less conservative results, the object of interest of most of the researchers in the recent years. Before going to the theorem we have to define the following notations

$$\begin{aligned} \phi &= x_t \\ \|\phi\|_c &= \max_{\theta \in [-\tau, 0]} x(t+\theta) \end{aligned} \tag{1.13}$$

Lyapunov–Krasovskii theorem statement [105]:

Suppose f is a functional that takes time t and initial values x_t and gives a vector of n states $\dot{x}$ and u, v, w are class $\mathcal{K}$ functions $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0$, v is strictly increasing. If there exists a continuously differentiable function $V : R \times R_n \rightarrow R$ such that

$$u(\|\phi\|) \le V(t, x) \le v(\|\phi\|_c) \tag{1.14}$$

and the time derivative of V along the solution $x(t)$ satisfies

$$\dot{V}(t, x) \le -w(\|\phi\|) \text{ for } \theta \in [-\tau, 0]$$

then the system is uniformly stable. If in addition $w(s) > 0$ for $s > 0$ then the system is uniformly asymptotically stable. If in addition $\lim\ u(s)_{s \rightarrow \infty} = \infty$ then the system is globally asymptotically stable.

It is clear that V is a functional and $\dot{V}$ should always be negative.

When considering a special class of systems that considers the case of linear time invariant system with multiple discrete time delay, which is given by [167]

$$\dot{x}(t) = A_o x(t) + \sum_{j=1}^{m} A_j x(t - h_j) \tag{1.15}$$

where h_j $j = 1, 2, ..., m$ are constants then this case is a simplified case, and in spite of that the Lyapunov–Krasovskii functional that gives a necessary and sufficient condition for the system stability is given by

$$\begin{aligned} V(x_t) = {} & x'(t)U(0)x(t) \\ & + \sum_{k=1}^{m} \sum_{k=1}^{m} x'(t + \theta_2) A'_k \times \int_0^{-h_k} U(\theta_1 + \theta_2 + h_k - h_j) \\ & \times A_j x(t + \theta_1) d\theta_1 d\theta_2 \\ & + \sum_{m}^{k=1} \int_0^{-h_k} x'(t + \theta)[(h_k + \theta)R_k + W_k]x(t + \theta)d\theta \end{aligned} \tag{1.16}$$

where $W_0; W_1; ...; W_m;\ R_1, R_2; ...; R_m$ are positive definite matrices and U is given by

$$\frac{\mathrm{d}}{\mathrm{d}\tau} U(\tau) = U(\tau)A_0 + \sum_{k=1}^{m} U(\tau - h_k)A_k \qquad \tau \in [0, \max_k(h_k)] \tag{1.17}$$

This theorem were found by trying to imitate the situation of delay-free systems by finding the state transition matrix and then using it to find P that makes

$$x'(t)(PA + A'P)x(t) = -Q, \quad Q > 0, \quad P > 0$$

This Lyapunov functional gives a necessary and sufficient condition for the system stability, but finding the U for this equation is very difficult "and involves solving algebraic ordinary and partial differential equations with appropriate boundary conditions which is obviously unpromising" [105]. Even if we can find this U, the resulting functional leads to a complicated system of partial differential equations yielding infinite dimension LMI. Thus, many authors considered special forms of it and thus derived simpler but more conservative, sufficient conditions, which can be represented by an appropriate set of LMIs.

This is the case for LTI system with a fixed time delay, then considering time varying delay or a generally nonlinear system makes it more difficult. But looking at these terms one can have an idea about the possible terms that can be used in the simplified functional.

1.4.3 Halanay Theorem

The following fundamental result plays an important role in the stability analysis of time-delay systems. Suppose the constant scalars k_1 and k_2 satisfy $k_1 > k_2 > 0$ and $y(t)$ is a nonnegative continuous function on $[t_o - \tau, t_o]$ satisfying

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \leq -k_1\, y(t) + k_2\, \bar{y}(t) \tag{1.18}$$

for $t \geq t_o$, where $\tau \geq 0$ and

$$\bar{y}(t) = \sup_{t-\tau \leq s \leq t} \{y(s)\}$$

Then, for $t \geq t_o$, we have

$$y(t) \leq \bar{y}(t_o)\, \exp(-\sigma(t - t_o))$$

where $\sigma > 0$ is the unique solution of the following equation

$$\sigma = k_1 - k_2 \exp(\sigma\tau)$$

It must be emphasized that the Lyapunov–Krasovskii theorem, Lyapunov–Razumikhin theorem, and Halanay theorem can be effectively used to derive stability conditions when the time delay is time varying and continuous, but not necessarily differentiable. Experience and the available literature show that the Lyapunov–Krasovskii theorem is more usable particularly for obtaining delay-dependent stability and stabilization conditions.

In this book we are going to adopt the use of a simplified sufficient condition Lyapunov–Krasovskii method for continuous-time as well as discrete-time nominally linear system, with single time-varying delay. Of course, the general case is to consider

- nonlinear system
- distributed delay.

When one looks at the real application, it is found that dealing with a nonlinear system cannot give a general result because every family of nonlinear systems has its own characteristics, so trying to build a method of a nonlinear system is not useful, in addition to the difficulties of dealing with a nonlinear system even in delay-free systems. The general practice is to linearize around some operating point and to use the linearized model and treat the nonlinearities as perturbations. In spite of this, the proposed method in Chapter 5 can be used for some families of nonlinear system, which are and not necessarily coming from a linearized mode. Regarding the distributed delay, again the difficulties in obtaining a good result in this field prevent one from selecting this direction in addition to the fact that many systems not only have discrete delay but also there are techniques to approximate [338] or even transform [140] the distributed delay into discrete delays, and the problem becomes of multiple discrete time-delay types, but as we will see in Chapter 3 if the Lyapunov functional is selected properly then a theorem made for single delay can be easily extended to multiple delays. The reason behind selecting time-varying delay is that it can cover a large class of systems and it can also be modified to cover fixed delay.

1.5 Outline of the Book

Toward our goal, this book has been carefully tailored to

(i) give a comprehensive study of STD modeling and dynamics,
(ii) present theoretical explorations on several fundamental problems for switched time-delay systems, and
(iii) provide systematic approaches for switching design and feedback control by integrating fresh concepts and the state-of-the-art results to the distinct theories on switched systems and time-delay systems.

Essentially, a basic theoretical framework is formed toward a switched time-delay theory, which not only extends the theory of time-delay systems, but also applies to more realistic problems.

In dealing with STDS, we follow a systematic modeling approach in that a convenient representation of the system state would be by observing a finite-dimensional vector at a particular instant of time and then examining the subsequent behavior to arrive at the dynamical relations. Looked at in this light, the primary objective of this book is to present an introductory, yet comprehensive, treatment of STDS by jointly combining the two fundamental attributes: the system dynamics possesses an inherent time delay and the system behavior is managed by a switching signal. Although each attribute has been examined individually in several texts, the integration of both attributes is quite unique and deserves special consideration. Additionally, STDS are nowadays receiving increasing attention by numerous investigators as evidenced by the number of articles appearing in journals and conference proceedings.

1.5.1 Methodology

Throughout the monograph, our methodology in each Chapter/section is composed of five steps:

- *Mathematical Modeling*
  in which we discuss the main ingredients of the state-space model under consideration.

- *Definitions and/or Assumptions*
  here we state the definitions and/or constraints on the model variables to pave the way for subsequent analysis.
- *Analysis and Examples*
  this signifies the core of the respective sections and subsections, which contains some solved examples for illustration.
- *Results*
  which are provided most of the time in the form of theorems, lemmas, and corollaries.
- *Remarks*
  which are given to shed some light of the relevance of the developed results vis-a-vis published work.

In the sequel, theorems (lemmas, corollaries) are keyed to chapters and stated in *italic* font with **bold titles**, for example, **Theorem 3.4** means Theorem 4 in Chapter 3 and so on. For convenience, we have grouped the reference in one major bibliography cited toward the end of the book. Relevant notes and research issues are offered at the end of each chapter for the purpose of stimulating the reader.

We hope that this way of articulating the information will attract the attention of a wide spectrum of readership.

1.5.2 Chapter Organization

Switched linear systems have been investigated for a long time in the control literature and have attracted increasingly more attention for more than two decades. The literature grew progressively and quite a number of fundamental concepts and powerful tools have been developed from various disciplines. Despite the rapid progress made so far, many fundamental problems are still either unexplored or less well understood. In particular, there still lacks a unified framework that can cope with the core issues in a systematic way. This motivated us to write the current monograph. The book presents theoretical explorations on several fundamental problems for switched linear systems. By integrating fresh concepts and the state-of the-art results to form a systematic approach for the switching design and feedback control, a basic theoretical framework is formed toward a switched system theory, which not only extends the theory of linear systems, but also applies to more realistic problems.

The book is primarily intended for researchers and engineers in the system and control community. It can also serve as complementary reading for linear/nonlinear system theory at the postgraduate level.

The book is divided into six parts:

Part I covers the mathematical ingredients needed for switching systems and time-delay systems and comprised of two chapters: Chapter 1 introduces the system description and motivation of the study and presents several analytical tools and stability theories that serve as the main vehicle throughout the book. Chapter 2 reviews some basic elements of mathematical analysis, calculus and algebra of matrices to build up the foundations for the remaining topics of stability, stabilization, control, and filtering of switched time-delay systems.

Part II treats switched stability and consists of three chapters: Chapter 3 establishes an overview of the recent progress of time-delay systems and presents a comprehensive picture about the contemporary results and methods. Chapter 4 gives a general framework of switched systems and addresses the main concepts and ideas. Chapter 5 draws the picture of switched time-delay systems with emphasis on the major properties.

Part III deals with switching stabilization and feedback control and contains two chapters: Chapter 6 includes delay-dependent switched stabilization techniques using different switching strategies and Chapter 7 gives different delay-dependent switched feedback techniques and compares among their merits, features, and computational requirements.

Part IV focuses on switched filtering and summarizes the results in two chapters: Chapter 8 is devoted to switched systems and the corresponding methods for switched time-delay systems are presented in Chapter 9. In both chapters, the design of Kalman, $\mathcal{H}_\infty$, and $\mathcal{H}_2$ filters are presented.

Part V treats switched interconnected systems by concentrating on switching decentralized control in Chapter 10. In this chapter, pertinent materials are selected and presented in a unified way.

Part VI provides applications of switched time-delay systems in terms of water-quality studies and control policies in streams as the subject of Chapter 11. Multirate control is presented in Chapter 13.

An appendix containing some relevant mathematical lemmas and basic algebraic inequalities is provided at the end of the book.

We selected the arrangement of references to be in alphabetical order for the purpose of convenience and easy tracking.

Throughout the book and seeking computational convenience, all the developed results are cast in the format of a family of LMIs. In writing up the different topics, emphasis is primarily placed on the major developments attained thus far and then reference is made to other related work.

In summary, this book covers the analysis and design for switched time-delay systems supplemented with rigorous proofs of closed-loop stability properties and simulation studies. The material contained in this book is not only organized to focus on the new developments in the analysis and control methodologies for such STD systems, but it also integrates the impact of the delay factor on important issues such as delay-dependent stability and control design. After an introductory chapter, it is intended to split the book into self-contained chapters with each chapter being equipped with illustrative examples, problems, and questions. The book will be supplemented by an extended bibliography, appropriate appendices, and indexes. It is planned while organizing the material that this book would be appropriate for use either as a graduate-level textbook in applied mathematics as well as different engineering disciplines (electrical, mechanical, civil, chemical, systems), a good volume for independent study, or a suitable reference for graduate students, practicing engineers, interested readers, and researchers from a wide spectrum of engineering disciplines, science, and mathematics.