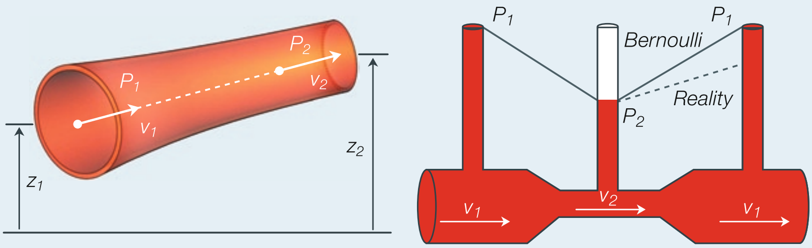
Chapter 3
Bernoulli's Equation



Bernoulli's equation relates blood pressure, *P*, and blood flow velocity, *v*. It expresses the conservation of energy in the flowing blood. If pressure losses due to friction or turbulence are neglected, Bernoulli's equation states that the sum of mechanical, *P*, kinetic, $1/2\rho v^2$, and potential energy, $\rho g z$, stays constant. In any organ filled with blood the sum of pressures or total energy is constant. For a blood vessel in the supine human the term $\rho g z$ is usually neglected. Thus, when velocity is high, $v_2>v_1$, pressure is low (*right hand figure*). In reality pressure distal to the narrowing section does not recover completely as suggested by Bernoulli's equation. The law helps to understand the effect of valvular stenosis and coarctation. The pressure drop over a stenosed valve can be estimated by $\Delta P=4v_s^2$ with ΔP in mmHg and with v_s, the maximal velocity in the stenosis, given in m/s.

Description

The Bernoulli equation can be viewed as an energy law. It relates blood pressure (*P*) to flow velocity (*v*). Bernoulli's law says that if we follow a blood particle along its path (dashed line in left Figure in the box) the following sum remains constant:

$$P + \tfrac{1}{2} \cdot \rho \cdot v^2 + \rho \cdot g \cdot z = \text{constant}$$

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where ρ is blood density, g acceleration of gravity, and z elevation with respect to a horizontal reference surface (i.e., ground level or heart level). The equation of Bernoulli says that as a fluid particle flows, the sum of the hydrostatic pressure, P, potential energy, $\rho \cdot g \cdot z$, and the dynamic pressure or kinetic energy, $½ \cdot \rho \cdot v^2$, remains constant. One can easily derive Bernoulli's equation from Newton's law: Pressure forces + gravitational forces = mass × acceleration.

Strictly speaking, the Bernoulli equation is applicable only if there are no viscous losses and blood flow is steady.

Physiological and Clinical Relevance

Bernoulli's law tells us that when a fluid particle decelerates pressure increases. Conversely, when a fluid particle accelerates, such as when going through a severe stenosis, pressure drops.

Because of the direct relationship between pressure and velocity, the Bernoulli equation has found several interesting clinical applications, such as the Gorlin [1] equation for estimating the severity of an aortic or mitral valve stenosis. Let us consider flow through a stenosed valve, s, as shown in Fig. 3.1.

Applying Bernoulli's Law

$$P_v + ½ \cdot \rho \cdot v_v^2 = P_s + ½ \cdot \rho \cdot v_s^2$$

and

$$P_v - P_s = ½ \cdot \rho \cdot (v_s^2 - v_v^2)$$

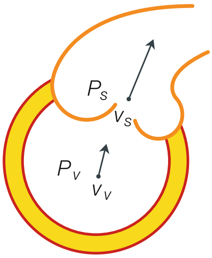

Fig. 3.1 Pressures, P_v and P_s, and velocities, v_v, and v_s, in ventricular lumen and valvular stenosis

The flow Q is the same at both locations, thus $A_v \cdot v_v = A_s \cdot v_s = Q$, where A_v and A_s are the cross-sectional areas of ventricle and valve, respectively. Substituting this into the Bernoulli's equation we obtain:

$$\Delta P = P_v - P_s = ½ \cdot \rho \cdot Q^2 \cdot (1/A_s^2 - 1/A_v^2)$$

Since the cross-sectional area of the stenosed valve A_s is much smaller than the cross-sectional area of the ventricle ($A_s << A_v$), the equation can be simplified to:

$$\Delta P = ½ \cdot \rho \cdot Q^2/A_s^2 = ½ \cdot \rho \cdot v_s^2$$

When velocity in the stenosis, v_s, is expressed in m/s the pressure drop (P, in mmHg) is approximately $4 \cdot v_s^{\,2}$.

Earlier this approach was used to estimate effective area [1], A_s, of the valvular stenosis by measuring flow and pressure gradient (e.g., using a pressure wire).

$$A_s = Q\sqrt{\frac{\rho}{2\Delta P}}$$

When the pressure is in mmHg and flow in ml/s, this gives an effective area: A_s (in cm^2) $= 0.02 \cdot Q/\sqrt{\Delta P}$. If pressure recovery downstream of the vena contracta (see below) is included then: $A_s = 0.0225 \cdot Q/\sqrt{\Delta P} = Q/(44\sqrt{\Delta P})$, [2].

Calculation of Aortic Valvular Area

Doppler velocimetry applied to both the valvular annulus and the aorta allows for the direct calculation of valve area (Fig. 3.2). Since volume flow is the same, the product of velocity and area is also the same at both locations. Thus

$$A_{valve} = A_{aorta} \cdot v_{aorta} / v_{valve}$$

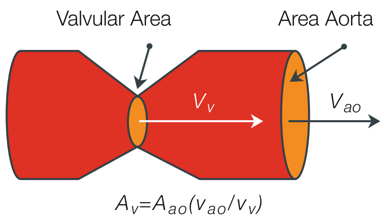

Fig. 3.2 Aortic valve area, A_v, can be derived from Doppler velocity measurements, in aorta and valve, v_{ao} and v_v, and aortic area, A_{ao}

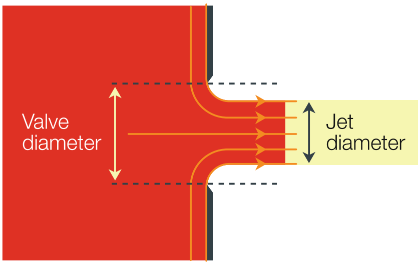

Fig. 3.3 Vena contracta effect is the result of the inability of the fluid to turn a sharp corner. The contraction coefficient A_{jet}/A_{valve} depends on the anatomical shape

Jets and Vena Contracta

Jets and vena contracta (Fig. 3.3) are formed when blood flow emerges from an opening such as a valve, and play a role in valvular stenosis and regurgitation. The contraction coefficient, i.e., the area ratio of the jet (color) and the valve (dashed lines) depends on the shape of the valve. The coanda effect is the phenomenon that a jet along the atrial or ventricular wall appears smaller than a free jet. Estimation of valvular area from the jet area is therefore not straightforward. Computational flow dynamics, i.e., the numerical solution of the Navier-Stokes equations (Appendix 5), allows the calculation flow velocity in complex geometries and makes it possible to learn more about jets.

Kinetic Energy

Bernoulli's equation pertains to conservation of energy. The term $½ \cdot \rho \cdot v^2$ is the kinetic energy. At peak systole ($P=130$ mmHg), the blood flowing in the lower abdominal aorta with a velocity $v=1$ m/s hits the wall of the apex of the iliac bifurcation. When it would come to a rest there, velocity is negligible ($v=0$). On the basis of the Bernoulli equation this implies a pressure rise of $½ \cdot \rho \cdot v^2 = 1/2 \cdot 1{,}060 \cdot 1^2 = 530\ N/m^2 \approx 0.5$ kPa. With 1 kPa = 7.5 mmHg, this pressure due to flow deceleration is thus about 3.5 mmHg.

The Hydrostatic Pressure

Most measurements are performed in the supine position. However, most activity takes place in the standing position. Figure 3.4 shows the pressures in the arterial and venous systems when a person is in the supine and the (motionless) standing position. It may be seen that the arterio-venous pressure gradients are not much affected.

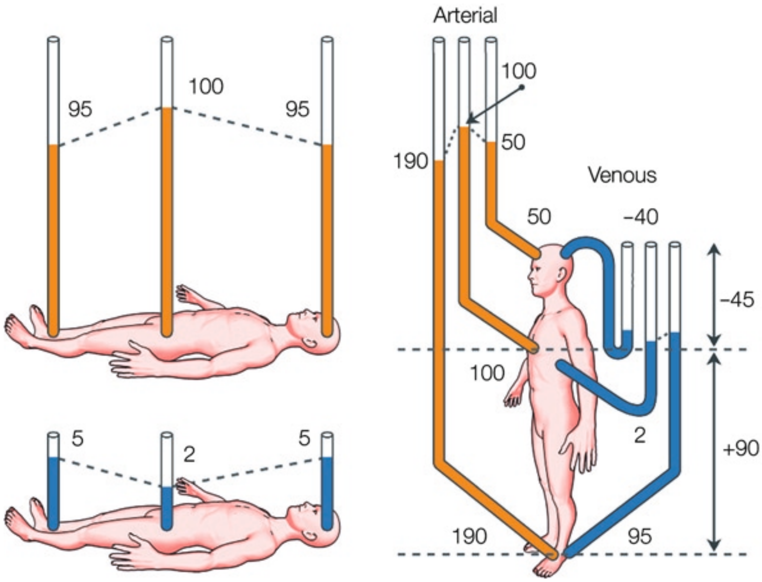

Fig. 3.4 Effect of posture on arterial and venous pressures (estimates, in mmHg). Effect of level is given by the hydrostatic pressure ρ g h, with ρ blood density, g, acceleration of gravity, and h height difference $z_1 - z_2$. *Dashed line* indicates the heart level. Adapted from ref. [3], used by permission

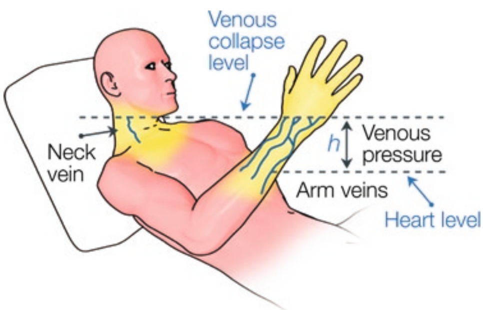

Fig. 3.5 Estimation of venous pressure by collapse. The level above the heart where collapse takes place, h, is measured in cm. The central venous pressure is then h/1.33 mmHg. Adapted from ref. [3], used by permission

Thus the driving forces for the flow are not much different in the two positions. The transmural pressures are strongly different and this mainly has an effect on the venous and capillary systems since the arterial system is rather stiff. The venous pooling of blood reduces cardiac filling and therefore has a, temporary, effect on the pump function of the heart. The capillary transmural pressure increase gives rise to edema formation.

When a person is lying in a reclined position the venous pressure can be estimated in the veins of the neck and hand (Fig. 3.5). The height difference between the point of collapse of superficial veins and the heart is the venous pressure. If the height difference is z in cm, the venous pressure can be calculated as $\rho \cdot g \cdot z = 1.05 \cdot 980 \cdot z$ dynes/cm^2 or $1.05 \cdot 980 \cdot z/1{,}360 = z/1.33$ mmHg, and thus for $z = 10$ cm the pressure equals ~7 mmHg.

References

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