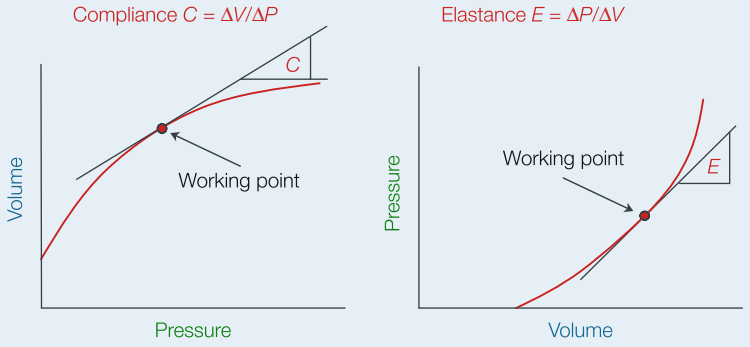
Chapter 11
Compliance



Compliance quantifies the pressure-volume relation. When the transmural or distending pressure is increased, the volume increases (*left figure,* customary in arterial research). When volume is increased pressure increases (*right figure,* customary in cardiac research). The pressure-volume relation is therefore displayed in two ways, each with a different choice of axes. For biological organs the relation is, in general, not straight but convex to the volume axis, implying that both *C* and *E* depend on pressure or volume (see Chapter on Elasticity). The slope of the pressure-volume relation in the chosen "working point" then is compliance, $C = \Delta V/\Delta P$, and elastance, $E = \Delta P/\Delta V$, respectively. Elastance and compliance are the inverse of each other: $E = 1/C$. When organs of different size are to be compared we can normalize both E and C with respect to volume. These normalized descriptions are distensibility, $C/V = (\Delta V/\Delta P)/V$, and bulk modulus or volume elasticity, $EV = V\Delta P/\Delta V$, respectively. The equivalent of the bulk modulus is the "pressure-strain" or "Peterson's modulus", E_p. When cross-sectional area is measured, as is often done in blood vessels, we derive "area compliance" and "area distensibility", where luminal area *A* replaces volume *V*. When diameters or radii are measured the relations are called diameter and radius compliance, respectively. The rules for addition of compliances and elastances are discussed. Compliance and elastance are not material parameters, but structural parameters, they can be derived from the material properties, i.e., elastic modulus, and the geometry, e.g., diameter, wall thickness, etc.

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Description

The advantage of the pressure-volume (or -diameter and -radius) relation (Figures in the box) is that it can be measured *in vivo*. It is important to note that pressure-volume relations do not characterize the material but includes the structure of the organ as a whole.

If the pressure-volume relations were straight and going through the origin, the slope, compliance or elastance would give the full characterization of the organ by a single quantity. However, the pressure-volume relations are never straight. For a small change around a chosen working point, the curve is approximately straight, and the tangent of the pressure-volume curve is used. We can determine compliance in the 'working point' as $C=\Delta V/\Delta P$. The elastance, the inverse of compliance is $E=\Delta P/\Delta V$. Of course, these local slopes depend on the (ventricular) pressure or volume chosen. Thus, when comparing compliance or elastance data one should report the chosen working point, i.e., the pressure at which compliance or elastance was determined. For instance, when the elastance of a heart in diastole is studied and appears increased, the increase can result from either a higher filling pressure but otherwise normal heart, or a normal filling pressure but a hypertrophied heart with thicker wall (Fig. 11.1).

The curvature of the pressure-volume relation is mainly the result of the fact that the Young modulus increases with stretch, therefore C decreases and E increases, with volume.

Measurement of Elastance and Compliance

Elastance is customarily used for the heart while compliance is mostly used to describe blood vessels.

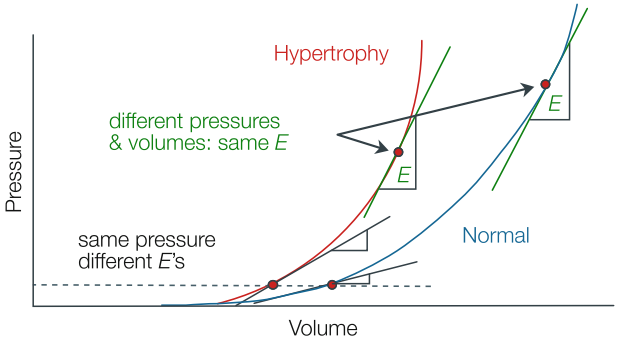

Fig. 11.1 Diastolic pressure-volume relations of a normal and hypertrophied heart are shown. If only the similar elastance values are reported without further information on pressure or volume, it cannot be decided if the heart is normal and overfilled, or hypertrophied, since both have the same E. At similar pressure the hypertrophied heart is stiffer (larger E). The full graph is required to give the complete information

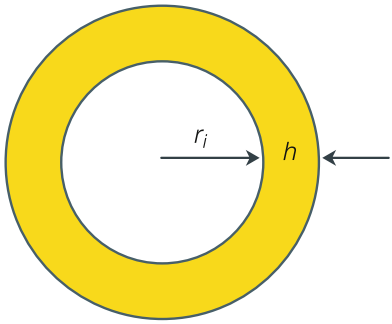

Fig. 11.2 For geometrically simple shapes, like blood vessels, measurement of changes in internal radius or diameter is sufficient to obtain compliance. In complex geometries as that of the heart this cannot be done

Ventricular elastance is best determined by the measurement of pressure and volume. A number of noninvasive techniques are now available to determine volumes, such as Computed Tomography, Magnetic Resonance Imaging, Ultrasound Echo, while the Pressure-Volume catheter allows for determination of elastance in experimental situations. Cardiac elastance determination requires volume and pressure measurements in systole and diastole, because of the varying properties of the cardiac muscle. Diameter as estimate of volume is inaccurate.

Arterial (*volume*) compliance, $C = \Delta V/\Delta P$, is usually determined from pressure and diameter measurements (Fig. 11.2). Diameter changes can be measured noninvasively by wall-tracking and for large vessels like the aorta by MRI. From the local diameter the cross-sectional area is calculated assuming a circular cross-section. When area and pressure are related the term *area* compliance, $C_A = \Delta A/\Delta P$, is used to distinguish it from (volume) compliance. Volume compliance is then lC_A, with l vessel length. For instance, the systolic-diastolic differences in area, ΔA, and pressure, ΔP, i.e., Pulse Pressure, when measured *in vivo*, can be used to obtain the area compliance. When the diameter change is related to the pressure change one obtains *diameter* compliance, $\Delta D/\Delta P$. The area compliance, C_A, and diameter compliance, C_D, are related by: $C_A = \pi \cdot D \cdot C_D/2$.

Distensibility and Bulk Modulus

Compliance depends on the size of the organ under study. To compare properties of blood vessels, or hearts from different animal species we can normalize compliance and elastance with respect to the volume of the organ. We use $C/V = (\Delta V/V)/\Delta P$, called distensibility, and the inverse, $E \cdot V = \Delta P/(\Delta V/V)$, called bulk modulus or volume elasticity. Area and diameter distensibilities are also used, area distensibility is $(\Delta A/A)/\Delta P$ and diameter distensibility is $2 \cdot (\Delta D/D)/\Delta P$.

The Pressure-Strain Elastic Modulus

Peterson et al. [1] introduced the pressure-strain elastic modulus in blood vessel research. This measure of blood vessel elasticity requires the measurement of diameter and pressure only, and can be used to compare vessels of different size. The pressure-strain elastic modulus, or Peterson modulus [1], is defined as $E_p = \Delta P / (\Delta r_o / r_o)$, where usually external radius, r_o, instead of the internal radius r_i is used. The E_p compares to the bulk modulus.

Summary of structural[a] parameters of elasticity for blood vessels

	Volume	Area[b]	Diameter	Radius
Compliance, C	$\Delta V/\Delta P$	$\Delta A/\Delta P$	$\frac{1}{2}\cdot(\Delta D/\Delta P)\pi D$	$2\cdot(\Delta r/\Delta P)\pi r$
Elastance, E	$\Delta P/\Delta V$	$\Delta P/\Delta A$	$2\cdot(\Delta P/\Delta D)/\pi D$	$(\Delta P/\Delta r)/2\pi r$
Distensibility, K	$1/V\cdot(\Delta V/\Delta P)$	$(1/A)\cdot\Delta A/\Delta P$	$2\cdot(\Delta D/\Delta P)/D$	$2\cdot(\Delta r/\Delta P)/r$
Bulk Modulus, BM[c]	$V\cdot\Delta P/\Delta V$	$A\cdot\Delta P/\Delta A$	$\frac{1}{2}\cdot(\Delta P/\Delta D)D$	$\frac{1}{2}\cdot(\Delta P/\Delta r)$[d]

P, V, A, D, r are pressure, volume, area, diameter and radius, respectively.
[a]Structural: parameters depend on organ geometry and material properties.
[b]It is generally assumed that vessel length does not change with pressure.
[c]Bulk modulus or Volume elasticity.
[d]The Pressure-Strain modulus or Peterson's modulus, $E_p = r_0 \cdot \Delta P/\Delta r_0 \approx 2/K \approx 2\cdot BM$; where outer radius is used. In all other relations internal diameter or radius is used.

The Stiffness Index

Compliance, distensibility and Peterson's modulus depend strongly on pressure, because diameter-pressure relationships are nonlinear. To overcome this pressure-dependence, Hayashi et al. [2] introduced a logarithmic stiffness index (or parameter) β, defined by the following relation (β-stiffness)

$$\ln\left(P / P_s\right) = \beta \cdot \left(D_o / D_s\right)$$

where P_s is a reference pressure (working point), typically mean pressure or 100 mmHg and D_s is the outer diameter at pressure P_s. Basic research and also several clinical studies have shown that the stiffness parameter does not depend on pressure within the physiological pressure range. However, outside the physiological pressure range and for pressures far from the reference pressure P_s the β-stiffness indexes are not constant anymore.

Describing the Pressure-Area or Pressure-Diameter Relation of Blood Vessels

The pressure-area and pressure-diameter relations of blood vessels have been described in a number of ways. At a working pressure the slope of the relation

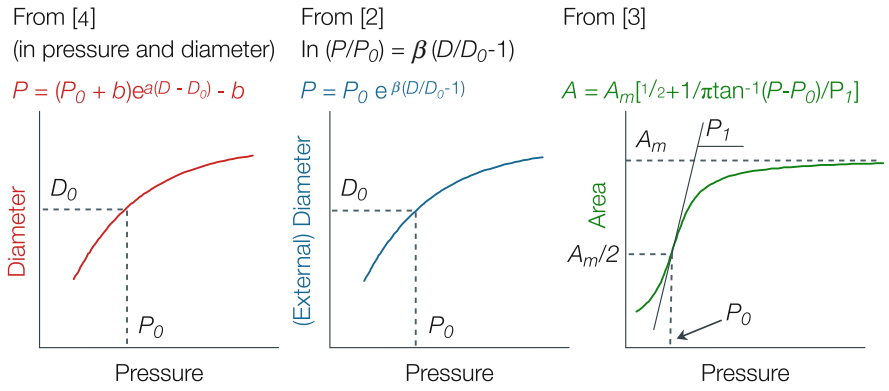

Fig. 11.3 Mathematical models of pressure-diameter and pressure-area relations

gives compliance. However, description of the relation over a range of pressures and volumes gives more insight. Although these descriptions are phenomenological, we mention them here because of their general utility in arterial mechanics (Fig. 11.3).

The relation proposed by Langewouters et al. [3], describes the pressure-area relation over the widest range of pressures, i.e., from 0 to 200 mmHg. The relations of Fung [4] and Hayashi et al. [2] can be applied over the physiological range of pressures. The D_0 and P_0 are reference values for the relations of Fung [4] and Hayashi et al. [2]. In the relation by Langewouters $A_m/2$ and P_0 designate the inflection point and P_1 relates to the slope at the inflection point; A_m is the maximal, asymptotic, vessel area. The relations can also be presented in terms of volumes.

Addition of Compliances and Elastances

Let us consider the compliance of the entire aorta (Fig. 11.4), the individual compliances of three sections of the aorta are shown in this figure. In all sections the pressure is virtually equal (see also Chap. 6). This implies that

$$\begin{aligned} C_1 + C_2 + C_3 &= \Delta V_1 / \Delta P + \Delta V_2 / \Delta P + \Delta V_3 / \Delta P \\ &= (\Delta V_1 + \Delta V_2 + \Delta V_3) / \Delta P = \Delta V_{total} / \Delta P = C_{total} \end{aligned}$$

Thus, simple addition of compliances is allowed and the total compliance is their sum and is therefore larger than the individual compliances.

If an organ with compliance C_1 is enveloped with an organ with compliance C_2, the total volume change equals the individual volume changes (Fig. 11.5). In that case the pressures need to be added. The distending pressure of the inner organ is the luminal pressure minus the pressure in between the organs. The distending pressure of the outer organ is the pressure between the organs minus the pressure of the

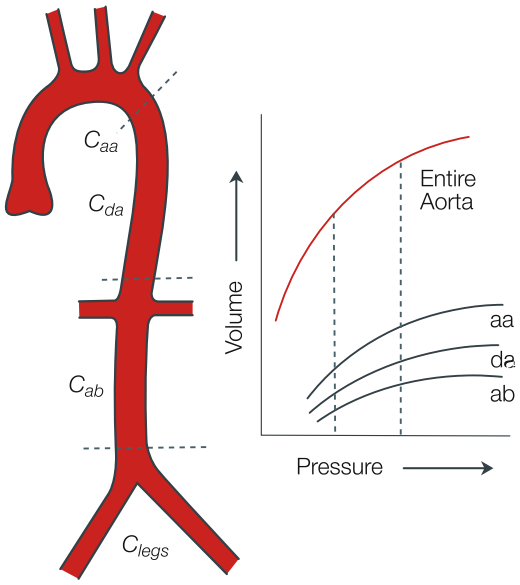

Fig. 11.4 Addition of compliances. Note the choice of axes. The compliances of different sections can be added to obtain total aortic compliance. The whole graphs may also be summed, by adding volumes at similar pressures (*dashed lines*). The aa, da, and ab are ascending, descending, and abdominal aorta

environment. Thus, the distending pressure acting on the two organs combined, i.e., the luminal pressure minus the external pressure is the sum of the distending pressures of each organ. In this situation addition of the elastances is easier. As an example we use the heart with pericardium. When the transmural pressure over the ventricular wall is ΔP_v and over the pericardium is ΔP_{pe}, for the heart inside the pericardium the transmural pressure equals $\Delta P_{total} = \Delta P_v + \Delta P_{pe}$. Therefore

$$E_{total} = \Delta P_{total} / \Delta V = (\Delta P_v + \Delta P_{pe}) / \Delta V = \Delta P_v / \Delta V + \Delta P_{pe} / \Delta V = E_v + E_{pe}$$

with E_v, and E_{pe} the ventricular elastance and pericardial elastance, respectively. When using compliances this would give: $1/C_{total} = 1/C_v + 1/C_{pe}$. The implicit assumption is that the intrapericardial pressure, ΔP_{pe}, is the same at all locations. There are indications that the situation is more complex.

Relating Compliance to Young Modulus

The measurement of pressure-volume or pressure-radius relationships in arteries allows for derivation of compliance but not of the Young modulus or incremental elastic modulus. As discussed in the chapter on Laplace's law, estimation of Young modulus or incremental elastic modulus requires, in addition to radius and pressure,

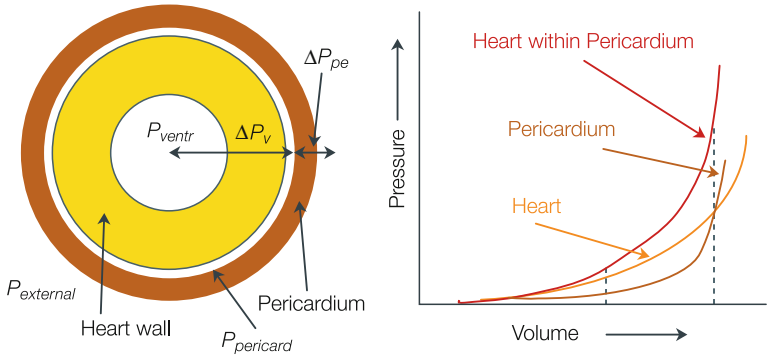

Fig. 11.5 Addition of elastances. Note the choice of axes. Heart and pericardium have their individual elastances and the total elastance of the heart in the pericardial sac can be obtained directly from addition of their individual elastances. Addition of the whole graphs is also allowed, at similar volumes (*dashed lines*). Thus, for structures within each other the elastances can be added directly to obtain overall elastance. Transmural pressure over the ventricular wall is $\Delta P_v = P_{ventr} - P_{pericard}$ and transmural pressure of the pericardium is $\Delta P_{pe} = P_{pericard} - P_{external}$

the measurement of wall thickness. An accurate relation between the Young modulus and compliance is given by Love [5], and used to model transverse impedance of an arterial segment (Appendix 3):

$$C_A = 3\pi \cdot r_i^2 \left(r_i + h\right)^2 / \mathrm{E} \cdot \left(2r_i + h\right)h = 3\pi \cdot r_i^2 \left(a+1\right)^2 / \mathrm{E} \cdot \left(2a+1\right), \quad \text{with } a = r_i / h$$

A simpler formula relating the area compliance with the incremental elastic modulus is:

$$C_A \approx (\mathrm{k}\pi \cdot r_i^3) / (E_{inc} \cdot h)\text{, both k} = 1.5 \text{ and } 2 \text{ have been reported}$$

(Area) compliance, being a structural property, should be plotted against distending pressure. The incremental elastic modulus, being a material property, should be plotted against stress or strain. Plotting E_{inc} against pressure, as is often done, leads to misinterpretation of vessel properties.

An example where the structural aspect of compliance can be seen is the comparison of the elastic properties of veins and arteries. The main reason why pressure-volume relations of veins differ from those of arteries is not the difference in wall material but their difference in wall thickness. More accurately stated, the ratio of wall thickness to radius is much smaller in veins than in arteries.

Physiological and Clinical Relevance

Compliance or elastance gives a quantitative measure of the mechanical and structural properties of an organ. Changes with disease and aging can be quantitatively investigated.

In general arterial compliance decreases with age and this is the main reason why arterial Pulse Pressure, systolic minus diastolic pressure, increases with age. The concomitant increase in systolic pressure is an extra load on the heart possibly leading to (concentric) hypertrophy. Concentric hypertrophy increases the elastance of the left ventricle in both diastole and systole. The increase in diastolic elastance (Fig. 11.6) results in decreased filling for the same filling pressure and filling can only return to near normal values with an increase in diastolic filling pressure, which in turn may lead to pulmonary edema (see Chap. 13).

With the now available wall-track technique arterial diameters can be measured noninvasively in superficial arteries and if pressure is simultaneously determined as well (see Chap. 26), diameter compliance can be derived in large groups of patients. However, we should realize that this is the local area compliance of a single, often peripheral, artery, such as the carotid or radial artery, and may not be a good measure of aortic compliance or total arterial compliance (see below and Chap. 24).

Compliance and elastance depend on volume and pressure. Comparison should thus be carried out at similar pressure. However, compliance and elastance, in contrast to the Young modulus, also depend on the size of the organ. Distensibility and volume elasticity account for vessel size and are often used for comparisons of groups (Fig. 11.6).

Buffering Function of Compliance

Arterial compliance is the buffering element for pressure so that the oscillations in pressure during the cardiac cycle are limited. The Pulse Pressure in the aorta, the

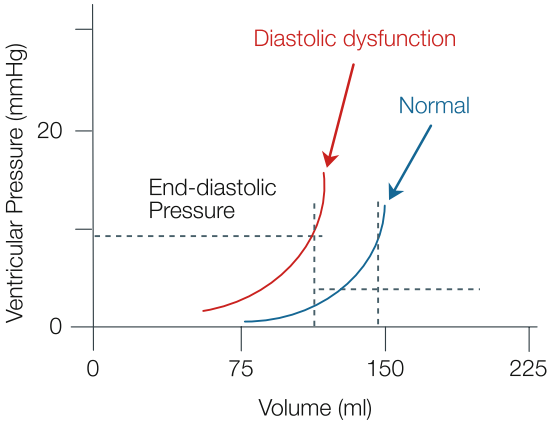

Fig. 11.6 Loss of diastolic ventricular compliance means that distension of the left ventricle in diastole becomes more difficult. Even a higher filling pressure is not capable to reach sufficient end-diastolic volume. An increase in diastolic filling pressure implies an increase in the pulmonary venous pressure, leading to pulmonary edema

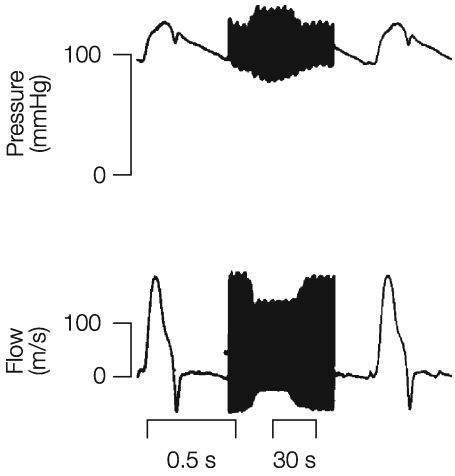

Fig. 11.7 Aortic pressure and flow in the intact dog during an acute decrease in aortic compliance, but constant peripheral resistance. Cardiac filling increases along with the decrease in compliance. The compliance decrease causes an increase in pulse pressure. Adapted from ref. [6], used by permission

difference between systolic and diastolic aortic pressure, is about 40 mmHg in the young healthy adult. It was shown by Randall et al. [6] that, *in vivo*, an acute reduction of total arterial compliance results in a considerable increase in Pulse Pressure (Fig. 11.7). The effect of long term, i.e., ~60 days reduction of aortic compliance to 60%, increased systolic pressure by 31 mmHg and diastolic pressure by 10 mmHg without affecting Cardiac Output and peripheral resistance [7].

It now accepted knowledge that increased Pulse Pressure is the strongest pressure-based indicator of cardiac mortality and morbidity [8, 9]. It has also been reported that diastolic cardiac function is affected when arterial compliance is decreased [10]. The scientific community is becoming more and more convinced that decreased compliance plays a major role in hypertension. In Chap. 29 it is shown that the change in compliance, with age, is considerable and contributes importantly to Pulse Pressure.

References

1. Peterson LH, Jensen RE, Parnell J. Mechanical properties of arteries in vivo. *Circ Res* 1960;8:622–639.
2. Hayashi, K, Handa H, Nagasawa S, Okumura A, Moritake K. Stiffness and elastic behavior of human intracranial and extracranial arteries. *J Biomech* 1980;13:175–184.
3. Langewouters GJ, Wesseling KH, Goedhard WJ. The static elastic properties of 45 human thoracic and 20 abdominal aortas in vitro and the parameters of a new model. *J Biomech* 1984;17:425–435.
4. Fung, Y. C. *Biomechanics. Mechanical Properties of Living Tissues.* 1981, New York & Heidelberg, Springer.

5. Love AEH. *A Treatise on the Mathematical Theory of Elasticity*. 1952, Cambridge, Cambridge University Press, 4th edn.
6. Randall OS, van den Bos GC, Westerhof N. Systemic compliance: does it play a role in the genesis of essential hypertension? *Cardiovasc Res* 1984;18:455–462.
7. Ioannou CV, Morel DR, Katsamouris AN, Katranitsa S, Startchik I, Kalangos A, Westerhof N, Stergiopulos N. Left ventricular hypertrophy induced by reduced aortic compliance. *J Vasc Res* 2009;46:417–425.
8. Benetos A, Safar M, Rudnichi A, Smulyan H, Richard JL, Ducimetieere P, Guize L. Pulse pressure: a predictor of long-term cardiovascular mortality in a French male population. *Hypertension* 1997;30:1410–1415.
9. Mitchell GF, Moye LA, Braunwald E, Rouleau JL, Bernstein V, Geltman EM, Flaker GC, Pfeffer MA. Sphygmomanometrically determined pulse pressure is a powerful independent predictor of recurrent events after myocardial infarction in patients with impaired left ventricular function. *Circulation* 1997;96:4254–4260.
10. Kawaguchi M, Hay I, Fetics B, Kass DA. Combined ventricular systolic and arterial stiffening in patients with heart failure and preserved ejection fraction: implications for systolic and diastolic reserve limitations. *Circulation* 2003;107:714–720.