Chapter 15
Wave Propagation in Damaged Materials Using a New Generalized Continuum Model

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Abstract An approach is proposed that allows formulating self-consistent problem that includes equations of the material's dynamics and conditions of its damage. It is shown that damage in the material introduces a frequency-dependent damping and dispersion of the phase velocity of ultrasonic acoustic waves that allows estimating damage using the acoustic method. Applied field of deformations leads to the accumulation of damage. A kinetic equation is obtained, whose analysis shows that damage grows exponentially. The parameters of the system for which accumulation of damage can be considered as linear are estimated.

15.1 Introduction

Nowadays, ensuring the safety of mechanical engineering structures is performed with the help of non-destructive control methods. The acoustic method is currently considered as the most promising one. The precision of parameter measurement and further interpretation of the condition of structural material depend on the numerous factors, including the exploitation conditions. The durability and longevity of a structure depend on strength characteristics of the material in local zones that experience highest loading. During the exploitation, the material experiences structural changes, and the degree of degradation depends on the exploitation conditions of loading. Clearly, during the diagnostics of structures exploited for a long time period, such structural changes lead to a significant change of data compared to the original calibration data.

The goal of this work is to develop methods of deciphering the data of acoustic measurement tools to account for the damage of structural material. An approach

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is proposed that allows formulating a self-consistent problem that includes equations of the material's dynamics and conditions of its damage. It is shown that damage in the material introduces a frequency-dependent damping and dispersion of the phase velocity of ultrasonic acoustic wave that allows estimating damage using the acoustic method. Applied field of deformations leads to the accumulation of damage. A kinetic equation is obtained, whose analysis shows that damage grows exponentially. Parameters of the system for which accumulation of damage can be considered as linear are estimated.

15.2 Mathematical Model

The mechanics of damaged continuum is being actively developed, starting with the fundamental works of L.M. Kachanov [1] and Y.N. Rabotnov [2].

What is usually meant by damage is the shortening of elastic response of the body due to a reduction of effective area that transmits internal stress from one part of the body to the other. This reduction, in turn, is caused by appearance and development of a disperse field of micro-defects (micro-cracks in elasticity, dislocations in plasticity, micro-pores for creepage and surface micro-cracks for fatigue).

In traditional approaches to damageability computations, a measure of damageability during deformation development is represented by a scalar parameter $\psi(x,t)$ which characterizes the relative density of equally-spread micro-defects in the unit volume. This parameter is equal to zero when there is no damage, and is close to 1 at the moment of destruction.

We study a sample, shaped as a rod, through which a longitudinal wave can propagate. We denote middle-line particle's dislocation by $u(x,t)$. Let us consider that the rod undergoes static and cyclic testing and its material can accumulate damage. To measure damage, we introduce a function $\psi(x,t)$.

As a rule, in mechanics of deformable solid body, dynamic problems are studied separately from damage accumulation problems. During these method developments, it is common to postulate that velocity of an elastic wave is a function of damageability and then determine the proportional coefficients experimentally.

Phase velocity (υ_{ph}) of the wave and its dissipation (frequency-dependent damping) are considered to be power functions of frequency (ω) and linear functions of damageability (ψ):

$$\upsilon_{\mathrm{ph}}(\omega) = c_0\big(1 - h_1\psi - h_2\psi\omega^2\big), \tag{15.1}$$

$$\alpha(\omega) = (h_3 + h_4\psi)\omega^4, \tag{15.2}$$

where $C_0 = \sqrt{\mathrm{E}/\rho}$ is the velocity of a longitudinal wave in the main medium if there were no damage; E Young modulus; ρ material density, h_i, $i = 1, \dots, 4$, coefficients that need experimental determination.

Damageability evolution can be described with the following kinetic equation:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = f(\sigma, \psi), \tag{15.3}$$

where σ is the external stress.

The function $f(\sigma, \psi)$ is most often approximated as a linear function. Sometimes as a polynomial.

While this method, based on (15.1) and (15.2), has an important advantage, namely, it is rather simple, it also has drawbacks like all methods that are not based on mathematical models of processes and systems.

Let us consider that the studied problem is self-consistent and includes, besides an equation for damageability development which we rewrite as

$$\frac{\partial\psi}{\partial t} + \alpha\psi = \beta_2 E \frac{\partial u}{\partial x}, \tag{15.4}$$

an equation for the rod dynamics

$$\frac{\partial^2 u}{\partial t^2} - C_0^2 \frac{\partial^2 u}{\partial x^2} = \beta_1 \frac{\partial \psi}{\partial x}. \tag{15.5}$$

Here α, β_1, β_2 are constants that characterize material damageability and a connection between cyclic testing and damage accumulation.

15.3 Damping and Dispersion of Elastic Waves

We look for a solution of the system (15.4) and (15.5) in the form of harmonic waves $u, \psi \approx \mathrm{e}^{i(\omega t - kx)}$, where ω is the circular frequency, $K = 2\pi/\lambda$ the wave number (λ the wave length), and arrive at the dispersion equation

$$\omega^2 - \left(C_0^2 + \frac{E\beta_1\beta_2}{\alpha}\right)K^2 + \frac{i}{\alpha}\omega^3 - \frac{iC_0^2}{\alpha}\omega K^2 = 0. \tag{15.6}$$

Please note that (15.6), which connects special and temporal scales of the longitudinal wave, also contains complex coefficient, which means that the wave will not only propagate through medium, it will also dissipate.

Let us consider the wave number to be complex $K = K^1 + iK^{11}$, where K^1 characterizes the propagation constant ($\upsilon_{\mathrm{ph}} = \omega/K^1$ is the phase velocity of the wave), and $K^{11} = \alpha(\omega)$ characterizes wave dissipation.

Solving algebraic equation (15.6) allows the determination of both components of the complex wave number

$$K^1 = \pm\sqrt{\frac{a\omega^2 + C_0^2\omega^4/\alpha^2 \pm \sqrt{a^2\omega^4 + C_0^4\omega^8/\alpha^4 + (a^2 + C_0^4)\omega^6/\alpha^2}}{2[a^2 + C_0^4\omega^2/\alpha^2]}}, \tag{15.7}$$

$$K^{11} = \pm\sqrt{\frac{(a - C_0^2)\omega^3/\alpha}{a\omega^2 + C_0^2\omega^4/\alpha^2 \pm \sqrt{a^2\omega^4 + C_0^4\omega^8/\alpha^4 + (a^2 + C_0^4)\omega^6/\alpha^2}}}. \tag{15.8}$$

Here a denotes $a = C_0^2 + E\beta_1\beta_2/\alpha$.

From (15.7) and (15.8), it follows that the existence of damage causes dispersion, i.e., the phase velocity of the wave depends on the wave frequency $\upsilon_{\rm ph} = \upsilon_{\rm ph}(\omega)$ (Fig. 15.1) and frequency-dependent dissipation $K^{11} = K^{11}(\omega)$ (Fig. 15.2).

In the lower frequency range ($\omega \to 0$), the wave velocity works for

$$\upsilon_{\rm ph}(0) \approx \sqrt{C_0^2 + E\beta_1\beta_2/\alpha}.$$

The wave dissipation is proportional to the square of wave frequency:

$$K^{11}(0) \approx \frac{E\beta_1\beta_2\omega^2}{\alpha^2\sqrt{C_0^2 + E\beta_1\beta_2/\alpha}}.$$

In the higher-frequency range ($\omega \to \infty$), the phase velocity works for C_0: $\upsilon_{\rm ph}(\infty) \approx C_0$, and dissipation is proportional to the frequency:

$$K^{11}(\infty) \approx \frac{E\beta_1\beta_2\omega}{\alpha C_0}.$$

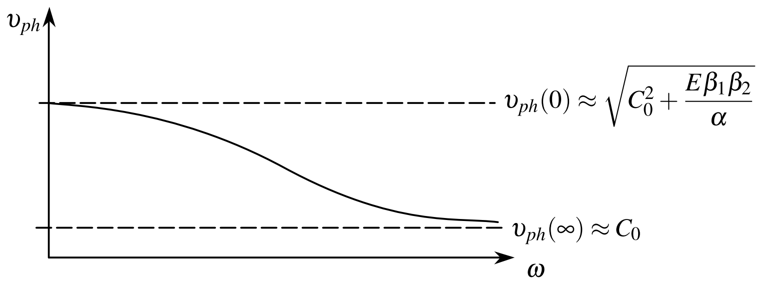

Fig. 15.1 Dispersion of the phase velocity

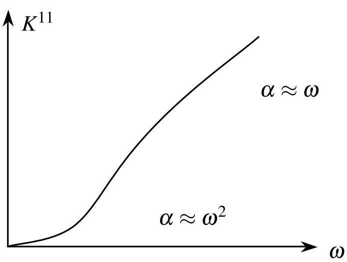

Fig. 15.2 Frequency-dependent damping

In the lower frequency range,

$$\frac{K^1(0)}{K^{11}(0)} = \left(1 + \frac{C_0^2\alpha}{\beta_1\beta_2 E}\right)\frac{\alpha}{\omega^2} \to \infty,$$

which means that the wave propagates almost without dissipation.

In the higher frequency range,

$$\frac{K^1(\infty)}{K^{11}(\infty)} = \frac{C_0^2\alpha}{\beta_1\beta_2 E\omega} \to 0,$$

which means that in this case dissipation is a dominant factor.

Note that the imaginary part of the wave number K^{11} can be measured in both the low-frequency and higher-frequency ranges. Therefore, the constants introduced in (15.4) and (15.5), namely α, β_1, β_2, can be computed from measured parameters:

$$\alpha = \frac{K^{11}(\infty)\omega}{K^{11}(0)\sqrt{1 + K^{11}(\infty)/(C_0\omega)}}, \tag{15.9}$$

$$\beta_1\beta_2 = \frac{C_0(K^{11}(\infty))^2}{EK^{11}(0)\sqrt{1 + K^{11}(\infty)/(C_0\omega)}}. \tag{15.10}$$

The system (15.4)–(15.5) can be rewritten as one equation in terms of ψ, characterizing damageability:

$$\frac{\partial^2\psi}{\partial t^2} - \left(C_0^2 + \frac{E\beta_1\beta_2}{\alpha}\right)\frac{\partial^2\psi}{\partial x^2} + \frac{1}{\alpha}\frac{\partial^3\psi}{\partial t^3} - \frac{C_0^2}{\alpha}\frac{\partial^3\psi}{\partial x^2\partial t} = 0. \tag{15.11}$$

Equation (15.11) represents a kinetic equation of damage accumulation. From its analysis it follows that damage grows exponentially. The exponent factor is determined from expression (15.8), and only with a few values of these parameters the process can be approximated as a linear function.

15.4 Conclusions

The suggested approach has allowed obtaining the new dependencies relating the dynamical equations of a material and the kinetics of its damage. This fact enables one to consider a problem about a deformation of a structural material and its damage as a unified self-consistent process.

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References

1. Kachanov, L.M.: Introduction to Damage Mechanics. Martinus Nijhoff, Dordrecht (1986)
2. Rabotnov, Y.N.: Elements of Hereditary Solid Mechanics. Mir, Moscow (1980)