Chapter 1
Generalized Continuum Mechanics: What Do We Mean by That?

Gérard A. Maugin

Dedicated to A.C. Eringen

Abstract Discursive historical perspective on the developments and ramifications of generalized continuum mechanics from its inception by the Cosserat brothers (Théorie des corps déformables. Hermann, 1909) with their seminal work of 1909 to the most current developments and applications is presented. The point of view adopted is that generalization occurs through the successive abandonment of the basic working hypotheses of standard continuum mechanics of Cauchy, that is, the introduction of a rigidly rotating microstructure and *couple stresses* (Cosserat continua or *micropolar* bodies, nonsymmetric stresses), the introduction of a truly deformable microstructure (*micromorphic* bodies), "weak" *nonlocalization* with *gradient theories* and the notion of *hyperstresses*, and the introduction of characteristic lengths, "strong nonlocalization" with space functional constitutive equations and the loss of the Cauchy notion of stress, and finally giving up the Euclidean and even Riemannian material background. This evolution is paved by landmark papers and timely scientific gatherings (e.g., Freudenstadt, 1967; Udine, 1970, Warsaw, 1977).

Preliminary note: Over 40 years, the author has benefited from direct studies under, and lectures from, P. Germain, A.C. Eringen, E.S. Suhubi, R.D. Mindlin, W. Nowacki, V. Sokolowski, S. Stojanovic, from contacts with J.L. Ericksen, C.A. Truesdell and D.G.B. Edelen, from friendship with C.B. Kafadar, J.M. Lee, D. Rogula, H.F. Tiersten, J. Jaric, P.M. Naghdi, I.A. Kunin, L.I. Sedov, V.L. Berdichevskii, E. Kröner, and most of the authors in the present volume as co-workers or friends, all active contributors to the present subject matter. He apologizes to all these people who certainly do not receive here the fully deserved recognition for their contribution to the field.

G.A. Maugin (✉)
UMR 7190 CNRS, Institut Jean Le Rond d'Alembert, UPMC Univ Paris 6, Paris, France
e-mail: gerard.maugin@upmc.fr

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1.1 Introduction

The following question is naturally raised with the venue of EUROMECH 510 in Paris in May 2009. What do we understand by generalized continuum mechanics? Note already some ambiguity since the last expression can be alternately phrased as “generalized (continuum mechanics)” or “(generalized continuum) mechanics”. We do not pursue this semantic matter. We simply acknowledge the fact that with the publication of the book of the Cosserat brothers in 1909 a true “generalized continuum mechanics” developed, first slowly and rather episodically and then with a real acceleration. That a new era was borne at the time in the field of continuum mechanics is not obvious if we remember that the Cosserats’ theory was published as a supplement to the French translation (by them for some, I suppose, alimentary purpose) of Chowlson’s Russian Encyclopedia of mathematics (the translation was done from the German edition). Another valid subtitle of the present contribution could be “From the classical to the less classical”. But what is classical? Then the “less classical” or “generalized” will be defined by successively discarding the working hypotheses of the classical case, simple as the latter may be.

1.2 From Cauchy and the Nineteenth Century

Here we consider as a classical standard the basic model considered by engineers in solid mechanics and the theory of structures. This essentially is the theory of continua set forth by A.L. Cauchy in the early nineteenth century for isotropic homogeneous elastic solids in small strains. The theory of continua respecting Cauchy’s axioms and simple working hypotheses is such that the following holds true:

1. *Cauchy’s postulate*. The traction exerted on a facet cut in the solid depends on the geometry of that facet only at the *first order* (the local unit normal); it will be linear in that normal. From this the notion of a *stress tensor* follows, the so-called stress being the only “internal force” in the theory.
2. *It is understood* that both physical space (of Newton) and material manifold (the set of material particles constituting the body) are Euclidean and connected, whence the notion of displacement is well defined.
3. *Working hypothesis (i)*. There are no applied couples in both volume and surface.
4. *Working hypothesis (ii)*. There exists no “microstructure” described by additional internal degrees of freedom.

According to Points 3 and 4, the Cauchy stress tensor is symmetric. This results from the application of the balance of angular momentum. Isotropy, homogeneity, and small strains are further hypotheses but they are not so central to our argument. Then generalizations of various degrees consist in relaxing more or less these different points above, hence the notion of *generalized continuum*. This notion of generalization depends also on the culture and physical insight of the scientists. For instance, the following generalizations are “weak” ones:

- "Generalized" Hooke's law (linear, homogeneous, but *anisotropic* medium);
- Hooke–Duhamel law in thermoelasticity;
- Linear homogeneous piezoelectricity in obviously anisotropic media (no center of symmetry).

These are "weak" generalizations because they do not alter the main mathematical properties of the system. Of course, thermoelasticity and linear piezoelectricity require adding new independent variables (e.g., temperature θ or scalar electric potential φ). In some sense, the problem becomes four-dimensional for the basic field (elastic displacement and temperature in one case, elastic displacement and electric potential in the other). The latter holds in this mere simplicity under the hypothesis of weak electric fields, from which there follows the neglect of the so-called ponderomotive forces and couples, e.g., the couple $\mathbf{P} \times \mathbf{E}$ when electric field and polarization are not necessarily aligned; see Eringen and Maugin [24]. Such theories, just like standard elasticity, do not involve a *length scale*. But classical linear *inhomogeneous* elasticity presents a higher degree of generalization because a characteristic length intervenes necessarily.

From here on, we envisage three true (in our view) generalizations.

1.2.1 The Cauchy Stress Tensor Becomes Nonsymmetric for Various Reasons

This may be due to

(i) The existence of body couples (e.g., in electromagnetism: $\mathbf{P}\times\mathbf{E}$ or/and $\mathbf{M}\times\mathbf{H}$; the case of intense EM fields or linearization about intense bias fields);
(ii) The existence of surface couples (the introduction of "internal forces" of a new type of the so-called *couple stresses*); the medium possesses internal degrees of freedom that modify the balance of angular momentum;
(iii) The existence of internal degrees of freedom of a nonmechanical nature in origin, e.g., polarization inertia in ferroelectrics, intrinsic spin in ferromagnetics (see Maugin's book [57]);
(iv) The existence of internal degrees of freedom of "mechanical" nature.

This is where the Cosserats' model comes into the picture.

The first example in this class pertains to a *rigid microstructure* (three additional degrees of freedom corresponding to an additional rotation at each material point, independently of the vorticity). Examples of media of this type go back to the early search for a continuum having the capability to transmit transverse waves (as compared to acoustics in a pure fluid), i.e., in relation to optics. The works of McCullagh [64] and Lord Kelvin must be singled out (cf. Whittaker [85]). Pierre Duhem [9] proposes to introduce a triad of three rigidly connected directors (unit vectors) to represent this rotation. In modern physics, there are other tools for this, including Euler's angles (not very convenient), quaternions and spinors. It is indeed the Cosserats, among other studies in elasticity, who really introduced internal degrees of freedom of the rotational type (these are *micropolar continua* in the sense of Eringen) and the dual concept of *couple stress*. Hellinger [36], in a brilliant essay, recognized at once the new potentialities offered by this generalization but did not elaborate on these. A modern rebirth of the field had to await works in France by crystallographs (Laval [45–47]; Le Corre [49]), in Russia by Aero and Kuvshinskii [1], and Palmov [71], in Germany by Schaeffer [77], Günther [34], Neuber [67], and in Italy by Grioli [33] and Capriz—see Capriz's book of 1989 [3]. But the best formulations are those obtained by considering a field of orthogonal transformations (rotations) and not the directors themselves, see Eringen [19–21], Kafadar and Eringen [37], Nowacki [70], although we note some obvious success of the "director" representation, e.g., in *liquid crystals* (Ericksen [17]; Leslie [52]) and the kinematics of the deformation of slender bodies (Ericksen, Truesdell, Naghdi)—in this volume see the contribution of Lhuillier. But there was in the mid 1960s a complete revival of continuum mechanics (cf. Truesdell and Noll [82]) which, by paying more attention to the basics, favored the simultaneous formulation of many more or less equivalent theories of generalized continua in the line of thought of the Cosserats (works by Mindlin and Tiersten [66], Mindlin and Eshel [65], Green and Rivlin [32], and Green and Naghdi [31], Toupin [80, 81], Truesdell and Toupin [83], and Eringen and Suhubi [25, 26], etc.).

More precisely, in the case of a *deformable microstructure* at each material point, the vector triad of directors of Duhem–Cosserats becomes deformable and the additional degree of freedom at each point, or micro-deformation, is akin to a general linear transformation (nine degrees of freedom). These are *micromorphic continua* in Eringen's classification. A particular case is that of continua with microstretch. A truly new notion here is that of the existence of a conservation law of *micro-inertia* (Eringen [18], Stokes [79]). In the present volume, this is illustrated by several contributions. A striking example is due to Drouot and Maugin [8] dealing with fluid solutions of macromolecules, while Pouget and Maugin [73] have provided a fine example of truly micromorphic solids with the case of piezoelectric powders treated as continua.

Remark 1.1. Historical moments in the development of this avenue of generalization have been the IUTAM symposium organized by E. Kröner in Freudenstadt in 1967 (see Kröner [41]) and the CISM Udine summer course of 1970 (Mindlin, Eringen, Nowacki, Stojanovic, Sokolowski, Maugin, Jaric, Micunovic, etc. were present).

Remark 1.2. Strong scientific initial motivations for the studies of generalized media at the time (1960–1970s) were (i) the expected elimination of field singularities in many problems with standard continuum mechanics, (ii) the continuum description of *real* existing materials such as granular materials, suspensions, blood flow, etc. But further progress was hindered by a notorious lack of knowledge of new (and too numerous) material coefficients despite trials of estimating such coefficients, e.g., by Gauthier and Jashman [28] at the Colorado School of Mines by building artificially microstructured solids.

Remark 1.3. Very few French works were concluded in the 1960–1970s if we note the exceptional work of Duvaut [10, 11] on finite strains after a short stay in the

USA, the variational principle for micromorphic bodies by Maugin [53] from the USA, and those on micropolar fluids by C. Hartmann [35] under the influence of R. Berker (who had been the teacher of Eringen in Istanbul).

Remark 1.4. The intervening of a rotating microstructure allows for the introduction of wave modes of rotation of the "optical" type with an obvious application to many solid crystals (e.g., crystals equipped with a polar group such as $NaNO_2$, cf. Pouget and Maugin [74]).

1.2.2 The Loss of Validity of the Cauchy Postulate

Then the geometry of a cut intervenes at a higher order than one (variation of the unit normal, role of the curvature, edges, apices and thus capillarity effects). We may consider two different cases referred to as the *weakly nonlocal theory* and the *strongly nonlocal theory* (distinction introduced by the author at the Warsaw meeting of 1977, cf. Maugin [55]). Only the first type does correspond to the exact definition concerning a cut and the geometry of the cut surface. This is better referred to as *gradient theories of the nth order*; it is understood that the standard Cauchy theory is, in fact, a *theory of the first gradient* (by this we mean the first gradient of the displacement or the theory involving just the strain and no gradient of it in the constitutive equations).

1.2.2.1 Gradient Theories

Now, to tell the truth, gradient theories abound in physics, starting practically with all continuum theories in the nineteenth century. Thus, Maxwell's electromagnetism is a first-gradient theory (of the electromagnetic potentials); the Korteweg [39] theory of fluids is a theory of the first gradient of density (equivalent to a second-gradient theory of displacement in elasticity); Einstein's [12] (also [13]) theory of gravitation (general relativity, 1916) is a second-gradient theory of the metric of curved space–time, and Le Roux [50] (also [51]) seems to be the first public exhibition of a second-gradient theory of (displacement) elasticity in small strains (using a variational formulation). There was a renewal of such theories in the 1960s with the works of Casal [4] on capillarity, and of Toupin [80], Mindlin and Tiersten [66], Mindlin and Eshel [65], and Grioli [33] in elasticity.

However, it is with a neat formulation basing on the *principle of virtual power* that some order was imposed in these formulations with an unambiguous deduction of the (sometimes tedious) boundary conditions and a clear introduction of the notion of *internal forces* of higher order, i.e., *hyperstresses* of various orders (see, Germain [29, 30], Maugin [56]). Phenomenological theories involving gradients of other physical fields than displacement or density, coupled to deformation, were envisaged consistently by the author in his Princeton PhD thesis (Maugin [54]) dealing with typical ferroïc electromagnetic materials. This is justified by a microscopic approach, i.e., the continuum approximation of a crystal lattice with medium-range interactions; with distributed magnetic spins or permanent electric dipoles. This also applies to the pure mechanical case (see, for instance, the Boussinesq paradigm in Christov et al. [5]).

Very interesting features of these models are:

F1. Inevitable introduction of characteristic lengths;
F2. Appearance of the so-called capillarity effects (surface tension) due to the explicit intervening of curvature of surfaces;
F3. Correlative boundary layers effects;
F4. Dispersion of waves with a possible competition and balance between nonlinearity and dispersion, and the existence of solitonic structures (see Maugin [60], Maugin and Christov [63]);
F5. Intimate relationship with the Ginzburg–Landau theory of phase transitions and, for fluids, van der Waals' theory.

Truly sophisticated examples of the application of these theories are found in

(i) The coupling of a gradient theory (of the carrier fluid) and consideration of a microstructure in the study of the inhomogeneous diffusion of microstructures in polymeric solutions (Drouot and Maugin [8]);
(ii) The elimination of singularities in the study of structural defects (dislocations, disclinations) in elasticity combining higher-order gradients and polar microstructure (cf. Lazar and Maugin [48]).

Most recent works consider the application of the notion of gradient theory in elastoplasticity for nonuniform plastic strain fields (works by Aifantis, Fleck, Hutchinson, and many others)—but see the thermodynamical formulation in Maugin [58]. In the present volume, this trend is exemplified by the first-hand synthesis contribution of E.C. Aifantis.

Insofar as general mathematical principles at the basis of the notion of gradient theory are concerned, we note the fundamental works of Noll and Virga [69] and Dell'Isola and Seppecher [7], the latter with a remarkable economy of thought.

1.2.2.2 Strongly Nonlocal Theory (Spatial Functionals)

Initial concepts in this framework were established by Kröner and Datta [42], Kunin [43, 44], Rogula [76], Eringen and Edelen [23]. As a matter of fact, the Cauchy construct does *not* apply anymore. In principle, only the case of *infinite bodies* should be considered as any cut would destroy the prevailing long-range ordering. Constitutive equations become integral expressions over space, perhaps with a more or less rapid attenuation with distance of the spatial kernel. This, of course, inherits from the action-at-a-distance dear to the Newtonians, while adapting the disguise of a continuous framework. This view is justified by the approximation of an infinite crystal lattice; the relevant kernels can be justified through this discrete approach. Of course, this raises the matter of solving integro-differential equations instead of

PDEs. What about boundary conditions that are in essence foreign to this representation of matter-matter interaction? There remains a possibility of the existence of a “weak-nonlocal” limit by the approximation by gradient models.

The historical moment in the recognition of the usefulness of strongly nonlocal theories was the EUROMECH colloquium on nonlocality organized by D. Rogula in Warsaw (cf. Maugin [55]). A now standard reference is Eringen’s book [22], also Kunin [44]. A recent much publicized application of the concept of nonlocality is that to *damage* by Pijaudier–Cabot and Bazant [72].

Note in conclusion to this point that any field theory can be generalized to a nonlocal one while saving the notions of linearity and anisotropy; but loosing the usual notion of flux. Also, it is of interest to pay attention to the works of Lazar and Maugin [48] for a comparison of field singularities in the neighborhood of structural defects in different “generalized” theories of elasticity (micropolar, gradient-like, strongly nonlocal or combining these). In this respect, see Lazar’s contribution in this volume.

1.2.3 Loss of the Euclidean Nature of the Material Manifold

Indeed, the basic relevant problem emerges as follows. How can we represent *geometrically* the fields of structural defects (such as *dislocations* associated with a loss of continuity of the elastic displacement, or *disclinations* associated with such a loss for rotations). A similar question is raised for *vacancies and point* defects. One possible answer stems from the consideration of a *non-Euclidean material manifold*, e.g., a manifold without curvature but with an affine connection, or an Einstein–Cartan space with *both* torsion and curvature, etc. With this, one enters a true “geometrization” of continuum mechanics of which conceptual difficulties compare favorably with those met in modern theories of gravitation. Pioneers in the field in the years 1950–1970 were K. Kondo [38] in Japan, E. Kröner [40] in Germany, Bilby in the UK, Stojanovic [78] in what was then Yugoslavia, W. Noll [68] and C.C. Wang [84] in the USA. Modern developments are due to, among others, M. Epstein and the author [14, 15], M. Elzanowski and S. Preston (see the theory of material inhomogeneities by Maugin [59]). Main properties of this type of approach are (i) the relationship to the multiple decomposition of finite strains (Bilby, Kroener, Lee) and (ii) the generalization of theories such as the theory of volumetric growth (Epstein and Maugin [16]) or the theory of phase transitions within the general *theory of local structural rearrangements* (local evolution of reference; see Maugin [62], examining Kröner’s inheritance and also the fact that true *material inhomogeneities* (dependence of material properties on the material point) are then seen as *pseudo-plastic effects* [61]. All local structural rearrangements and other physical effects (e.g., related to the diffusion of a dissipative process) are reciprocally seen as pseudo material inhomogeneities (Maugin [62]). An original geometric solution is presented in the book of Rakotomanana [75] which offers a representation of a material manifold that is everywhere dislocated. Introduction of the notion of fractal sets opens new horizons (cf. Ostoja-Starzewski's contribution in this volume). An antiquated forerunner work of all this may be guessed in Burton [2], but only with obvious good will by a perspicacious reader.

1.3 Conclusion

Since the seminal work of the Cosserats, three more or mess successful paths haven been taken towards the generalization of continuum mechanics. These were recalled above. They are also fully illustrated in the various contributions that follow. An essential difference between the bygone times of the pioneers and now is that artificial materials can be man-made that are indeed generalized continua. In addition, mathematical methods have been developed (homogenization techniques) that allow one to show that generalized continua are deduced as macroscopic continuum limits of some structured materials. This is illustrated by the book of Forest [27].

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