Chapter 3
Architectures of Parallel Robotic Machine

3.1 Preamble

One of the objectives of this book is to find the most promising kinematic structures that can be used for machine tool design. Hence, some well-known principles are applied to investigate all the possibilities of structure in detail. A mechanism is defined as a kinematic chain with one of its components (link or joint) connected to the frame. A kinematic chain consists of a set of links, coupled by joints (cylindrical, planar, screw, prismatic, revolute, spherical, and Hooke) between adjacent links. In this chapter, a topological study of different combinations of kinematic chain structures are performed using a graph representation approach. The number of links and joints for the desired system and their interconnections, neglecting geometric details (link length and link shape), are described. The possible architectures that provide 5 degrees of freedom between the tool and the workpiece are generated. In Sect. 3.2, basic kinematic elements of mechanisms are introduced, and the classification of mechanisms is given based on the motion relation. In Sect. 3.3, the basic concept of the graph representation of a kinematic structure is addressed. Then, the Chebychev–Grübler–Kutzbach criterion is introduced in Sect. 3.4. A topological study of the kinematic structures is described in Sect. 3.5. Requirements for possible kinematic structures are set up. Furthermore, the structural representation of kinematic chains and architectures with consideration of parallel and hybrid cases is illustrated. In Sect. 3.6, a remark on the role of redundancy is given. A summary with discussion of related work is presented in Sect. 3.7.

3.2 Fundamentals of Mechanisms

3.2.1 Basic Kinematic Elements of Mechanisms

A mechanism is defined as a kinematic chain with one of its components (link or joint) connected to the frame. A kinematic chain consists of a set of links, coupled by joints between adjacent links.

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3.2.1.1 Prismatic Joint (P, also called sliders)

A prismatic joint allows two components to produce relative displacement along the common axis. The included angle between the two components is a constant value, called deflection angle. The displacement and deflection angle describe the spatial relative relationship of the two components, which forms a prismatic joint. A prismatic joint is a one degree-of-freedom kinematic pair, which provides single-axis sliding function, and it can be used in places such as hydraulic and pneumatic cylinders. The CAD model of a prismatic joint is shown in Fig. 3.1.

3.2.1.2 Revolute Joint (R, also called pin joint or hinge joint)

A revolute joint allows two components produce relative rotation along the joint axis. The vertical dimension between the two components, is a constant value called offset distance. The vertical dimension and offset distance describe the spatial relative relationship of the two components which forms a revolute joint. A revolute joint, as a one degree-of-freedom kinematic pair, provides single-axis rotation function. Revolute joints is the most commonly found joint in industrial and research robots, and it can be found in many classic applications, such as door hinges, folding mechanisms, and other uniaxial rotation devices. The CAD model of a revolute joint is shown in Fig. 3.2.

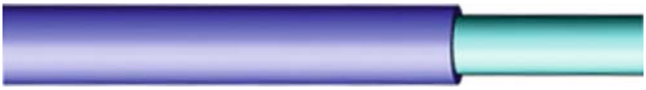


Fig. 3.1 The CAD model of prismatic joint

Fig. 3.2 The CAD model of revolute joint

3.2.1.3 Hooke Joint (H, also called universal joint, Cardan joint or Hardy-Spicer joint)

Hooke joint allows two components to produce two degree-of-freedom relative independent rotation along two perpendicular axes. Generally, a Hooke joint is equivalent to two revolute joints whose axes must be completely perpendicular, namely H = RR. The CAD model of a Hooke joint is shown in Fig. 3.3.

3.2.1.4 Spherical Joint (S, also called ball-in-socket joint)

A spherical joint allows one element to rotate freely in three dimensions with respect to the other about the center of a sphere. The sense of each rotational degree-of-freedom is defined by the right-hand rule, and the three rotations together form a right-hand system. The relative pose of two components can be confirmed by three Euler angles, ϕ (rotate along the original z-axis), θ (rotate along the new x-axis), and φ (rotate along the new z-axis). A spherical joint is kinematically equivalent to three intersecting revolute joints. The CAD model of a Hooke joint is shown in Fig. 3.4.

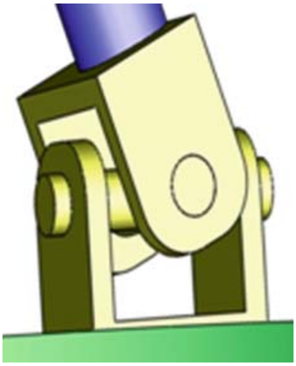


Fig. 3.3 The CAD model of Hooke joint

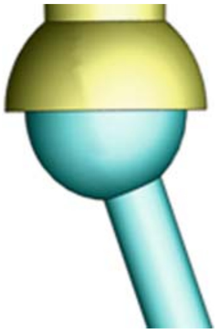


Fig. 3.4 The CAD model of Spherical joint

3.2.2 Classification of Mechanisms

Mechanisms can be divided into planar mechanisms and spatial mechanisms, according to the relative motion of the rigid bodies.

Serial mechanisms have been extensively studied in terms of their design, kinematic and dynamic modeling, and control by many researchers. When properly designed, the serial structure has the benefit of possessing a large workspace volume in comparison to the physical size of the mechanism. Since serial mechanisms only have one open kinematic chain, this means that the serial mechanisms only have one possibility in architecture.

Parallel mechanism is closed-loop mechanism in which the end-effector is connected to the base by at least two independent kinematic chains [106]. This can be further divided into fully-parallel and hybrid mechanism. Fully-parallel mechanism is the one with an n-DOF end-effector connected to the base by n independent kinematic chains, each having a single actuated joint. The hybrid one has the combination of serial and parallel mechanisms.

Because their errors are averaged instead of added cumulatively, parallel robots are more accurate than serial robots. First, since the moving platform of parallel mechanism is supported by several kinematic chains, the system stiffness of the end-effector is largely improved. Furthermore, this also strengthens the structural stability. Contrarily, serial mechanism usually is a single-arm structure. To some extent, a large number of motors increase the burden of the end-effector and affect the structural stability of serial mechanism. Second, the specific configuration of parallel mechanism makes it have obvious advantages in the abilities of reconfiguration, restoration, and payload. Third, the error of the end-effector of serial mechanism will be accumulated and amplified based on each joint error; contrarily, the error of parallel mechanism is smaller and its accuracy is higher. Fourth, the actuators of serial robot usually are located on the end of each rod end. It will increase the inertia and exacerbate the transfer ability of system. For parallel mechanisms, the actuators can be located on the base to decrease the motion load.

With the development of the theory of advanced spatial mechanism and the technology of robotics, parallel robotic machines have been an important branch of robotic technology. Furthermore, the research activities of the theories and applications of parallel robots are becoming increasing. Nevertheless, many scholars have done intensive investigations on the dimension synthesis, kinematics, dynamics, workspace, and singularity of parallel mechanisms, most of the existing work regarding parallel mechanisms was built upon the concept of traditional Gough-Stewart mechanism type. Because of the opposition and unitarian of serial mechanisms and parallel mechanisms in philosophy, the hybrid mechanisms can be built through the combination of parallel and serial mechanisms and play an important role in some specific application background.

The number of independent coordinates to completely determine the location of an object in space can be called the degree-of-freedom of the object. In the Cartesian coordinate system, three independent coordinates *(xyz)* must be used to confirm the position of a particle with random motion. Thus, a free particle has three translational degree-of-freedom. Likelihood, the free motion of a rigid body in three dimensional spaces can be decomposed into the translational motion of its barycenter and the rotational motion with respect to the axis of barycenter. Therefore, a rigid body with random motion totally has six degree-of-freedom: three translations *(xyz)* to measure its position and three rotations (ϕ, θ, φ) to measure its pose. The definition of degree-of-freedom for parallel mechanism, similar to the motion of a rigid body in space, is the sum of independent translational degree-of-freedom and independent rotational degree-of-freedom of end-effector (attached to the moving platform) with respect to a fixed coordinate system. Generally, the fixed coordinate system is attached to the fixed base. Sometimes, the end-effector can only produce motion in a plane. Since a rigid body has three degree-of-freedom in a plane: two translations in *(xy)* and one autogiration, the planar parallel mechanism has at most three degrees of freedom.

A brief introduction of parallel mechanism based on the classification of space dimension and degree-of-freedom is given as follows.

1. Planar two degrees of freedom parallel mechanism
   Cervantes [30] proposed a simplified approach which allowed the generation of the workspace of a complete class of 2-dof manipulators with the type of RPRPR.

   Tensegrity structure is combined by a group of continuous/discontinuous draw bar to form a self-stress, self-supporting reticulated linkage structure. Arsenault [13] designed a planar two degree-of-freedom modular parallel mechanism based on the principle of tensegrity.
2. Planar three degrees of freedom parallel mechanism
   Zhang [179] proposed a planar three degree-of-freedom parallel mechanism with redundant actuation. Since specific driven redundancy method is adopted, the closed-form solution for the forward kinematics was derived.
3. Spatial three degrees of freedom parallel mechanism
   Clavel's delta parallel robot [128] is the classic case of spatial three degree-of-freedom mechanism. The parallelograms are adopted in three symmetrical legs to improve the dynamics performance. Delta robot has 50 gravitational accelerations in the environment of laboratory. Even in the industrial field, it still has 12 gravitational accelerations. In the process of three degree-of-freedom linear motion, the leg in the parallelogram must always keep parallel to its opposite side.
4. Spatial four degrees of freedom parallel mechanism
   Alvarado [50] proposed a four degree-of-freedom CPS+PS+HPS parallel mechanism with three legs. The numerical analysis results showed the efficiency of screw theory when dealing with the issues of kinematics and singularity of simple parallel mechanism.

   Lu [100] analyzed the kinematics and active/passive force of a four degree-of-freedom 3SPU+UPR mechanism with three rotations and one translation.

   Inspired by Clavel's Delta robot, Olivier et al. [39] developed the prototype of H4 robot using parallelogram mechanism.

Kong et al. [85] proposed a general approach for the type synthesis of a class of parallel mechanisms based on screw theory. The common ground of these mechanisms is that they have completely same branch chain, 3T1R.

5. Spatial five degrees of freedom parallel mechanism
Alizade [8] discussed a kind of five degree-of-freedom 4UPS+UPU asymmetry parallel mechanism with three rotations and two one translations.
6. Spatial six degrees of freedom parallel mechanism
Gough-Stewart platform is the original of spatial six degree-of-freedom parallel mechanism.

3.3 Graph Representation of Kinematic Structures

A kinematic chain can be described as a set of rigid bodies attached to each other by kinematic pairs, resulting in a mechanical network containing joints and links [56]. A kinematic structure represents the kinematic chain without considering the detailed geometric, kinematic, and functional properties. The range of kinematic structures given particular constraints on the number and type of joints and links can be examined exhaustively. This range represents a set of logical possibilities for design of a particular type of mechanism. This set is a framework in which designs are to be realized.

A systematic method of enumerating all the possible kinematic chains – kinematic architectures – is needed to meet the required degrees of freedom, i.e. 3-dof, 4-dof, and 5-dof. There were several methods reported in the literature: Hunt [76] used the theory of screw systems to enumerate parallel mechanisms exhaustively; Earl et al. [44] proposed a network approach, which enables consideration of two or more structures into another one. A graph representation will be introduced in this chapter.

Graph theory is a field of applied mathematics [67], which provides a useful abstraction for the analysis and classification of the topology of kinematic chains, and it offers a systematic way of representing the topology of complex kinematic chains. The graph of a kinematic chain consists of a diagram where each link is represented by a point and each joint by a line. Thus, the graph representation of a kinematic chain will take the form of a collection of points connected by lines. The graph representation of kinematic chains has been used by many researchers [16, 56, 146, 180, 181].

3.4 Design Criteria

The degree of freedom (or mobility) of a kinematic chain [76] can be defined as the minimum number of independent variables necessary to specify the location of all links in the chain relative to a reference link. The choice of the reference link does not affect the resulting mobility. A preliminary evaluation of the mobility of a kinematic chain can be found from the Chebychev-Grübler-Kutzbach formula.

$$l = d(n - g - 1) + \sum_{i=1}^{g} f_i, \tag{3.1}$$

where l is the degree of freedom of the kinematic chain, d is the degree of freedom of each unconstrained individual body (6 for the spatial case, 3 for the planar case) [77]; n is the number of rigid bodies or links in the chain; g, the number of joints; and f_i, the number of degrees of freedom allowed by the ith joint.

For example, to design a 5-DOF parallel robotic machine, the possibility of parallel mechanisms can be investigated for the combinations of dofs in (5,0), (4,1), and (3,2). The workpiece can be fixed (0-dof), or move along one axis (1-dof) or move along the X and Y axes (2-dof) or rotate about one or two axes. Hence, one will consider the possibilities of parallel mechanisms with 5-dof, 4-dof, 3-dof, and 2-dof; besides, the case with 6-dof is taken as an option with redundancy. The detail is shown as follows.

1. DOF distributions for each leg
   For a given parallel platform, we can always make the following assumptions:

   number of known bodies = 2 (platform and base),
   number of parallel legs = L, and
   degree of freedom of the ith leg = f_{l_i},

   then one can rewrite (10.1) as

$$\begin{aligned} l &= 6\left[2 + \sum_{i=1}^{L}(f_{l_i} - 1) - \sum_{i=1}^{L} f_{l_i} - 1\right] + \sum_{i=1}^{g} f_i \\ &= 6 - 6L + \sum_{i=1}^{g} f_i. \end{aligned} \tag{3.2}$$

   From this equation, it is apparent that there exist thousands of possibilities for 5-dof or less than 5-dof cases. Hence, some constraints introduced and are specified as follows:

   - From the viewpoint of fully-parallel mechanism, the maximum number of parallel legs are kept equal to the degree of freedom of the mechanism, thus to guarantee the possibility of installing one actuator in each leg, one has

$$L \leq l. \tag{3.3}$$

   - Although two-leg spatial parallel mechanisms are of little direct use independently, they are useful to constructing "Hybrid" mechanisms, the minimum number of the leg is given by

$$L \geq 2. \tag{3.4}$$

Table 3.1 The possible degree-of-freedom distribution for each leg

Degree of freedom	Number of legs	f_{l_1}	f_{l_2}	f_{l_3}	f_{l_4}	f_{l_5}	f_{l_6}
l = 2	L = 2	2	6				
		3	5				
		4	4				
l = 3	L = 2	3	6				
		4	5				
	L = 3	3	6	6			
		4	5	6			
		5	5	5			
l = 4	L = 2	4	6				
		5	5				
	L = 3	4	6	6			
		5	5	6			
	L = 4	4	6	6	6		
		5	5	6	6		
l = 5	L = 2	5	6				
	L = 3	5	6	6			
	L = 4	5	6	6	6		
	L = 5	5	6	6	6	6	
l = 6	L = 2	6	6				
	L = 3	6	6	6			
	L = 4	6	6	6	6		
	L = 5	6	6	6	6	6	
	L = 6	6	6	6	6	6	6

On the basis of the constraints represented by (3.3) and (3.4), and one can enumerate the possible dofs distributions as in Table 3.1. It is noted that these are the basic combinations for different architectures, and one can remove or add legs which have 6-dof for symmetric purpose in any of the basic structures at ease.

3.5 Case Study: Five Degrees of Freedom Parallel Robotic Machine

Since both the tool and the workpiece can be actuated independently and that 5-DOF are required for manufacturing tasks, the possible combinations of 5 dofs are: (5,0), (4,1), and (3,2) as indicated in previous section. For each of these combinations, the kinematic chains involved may lead to several possibilities (serial, parallel, or hybrid). The followings are the details for this enumeration process.

3.5.1 Serial Mechanisms

The serial mechanisms have many drawbacks. Because of the serial nature of actuation and transmission, related masses must be mounted distal to the base of the mechanism leading to a small ratio of payload over machine mass, poor dynamic performance in terms of acceleration capability, and poor system stiffness presented at the end-effector. Since a lower axis has to carry both the loads (in all directions) and the weights of all its upper axes, dynamic behaviors of the lower axes will be poor, especially to machine tools which carry high loads. In addition, the serial structure leads to joint errors being additive, and combined with the inherent low system stiffness, this leads to poor accuracy at the end-effector. Thus, the drawbacks in their structures limit the performance.

3.5.2 Parallel Mechanisms

Among the three possibilities (serial, parallel, and hybrid), the parallel mechanisms are the basic and the most important ones in building all the possible architectures, because of the disadvantages of the serial mechanisms. The hybrid mechanisms will be built through the combination of parallel mechanisms.

1. Possible Structures
   The variables for combining different kinds of architectures are mainly decided by (1) leg length; (2) position of the base points; or (3) both the leg length and position of the base points.

   (a) Possible Legs
   On the basis of the required DOF distributions for each leg, one can find different kinds of legs to meet the requirement through the combination of different joints such as spherical joint (with 3-dof), Hooke joint (with 2-dof), revolute joint (with 1-dof) and prismatic joint (with 1-dof). One can combine them to meet the dof requirements for each leg shown in Table 3.2, where

   S: spherical joint
   R: revolute joint
   H: Hooke joint
   P: prismatic joint

   Table 3.3 shows all the possible legs with a different degree-of-freedom.

   (b) Vertex structures
   From the literature related to the Stewart platform, various architectures have been developed or proposed for the platform mechanisms, such as 3-6, 4-4, 4-5, and 4-6 (the numbers of vertices in the mobile and base plates) platforms [33,48,63,99,169]. Since two spherical joints can be combined to one concentric spherical joint, one can obtain two types of vertices for parallel mechanisms as shown in Fig. 3.5.

Table 3.2 Possible joint combinations for different degrees of freedom

Number of possibilities	DOFs = 2	DOFs = 3	DOFs = 4	DOFs = 5	DOFs = 6
1	2R	1R2P	1S1P	1S2R	2S
2	2P	2R1P	1S1R	1S2P	1S1H1P
3	1R1P	3R	1R3P	1S1R1P	1S1H1R
4	1H	3P	2R2P	1S1H	1S3R
5		1H1R	3R1P	1H3R	1S3P
6		1H1P	4R	1H2R1P	1S2R1P
7		1S	4P	1H1R2P	1S1R2P
8			1H2R	1H3P	1H4R
9			1H2P	5R	1H3R1P
10			1H1R1P	4R1P	1H2R2P
11				3R2P	1H1R3P
12				2R3P	1H4P
13				1R4P	6R
14				5P	5R1P
15					4R2P
16					3R3P
17					2R4P
18					1R5P
19					6P

Table 3.3 Possible leg types with different degrees of freedom

Possible numbers	DOFs = 2	DOFs = 3	DOFs = 4	DOFs = 5	DOFs = 6
1	2R	1S	2R1H	1H2R1P	1S2R1P
2	1R1P	2R1P	1H1R1P	2H1R	1S1H1P
3	1H	1R2P	1S1P	2H1P	1S1H1R
4		3R	1S1R	1S1R1P[1]	1S1R1P
5		3P	2R2P	1S2R[1]	1S1H1P
6		1H1R	1R3P	1S2P	2S
7		1H1P	3R1P	1H3P	1S3P
8			4R	1H1R2P	1S3R
9			4P	1H3R	1H2R2P
10			1H2P	4R1P	1H3R1P
11				5R	1H4R
12				5P	6R
13				1R4P	6P
14				3R2P	1H4P
15				2R3P	3R3P
16					1H1R3P
17					5R1P
18					1R5P
19					2R4P
20					4R2P
Total possibilities	3	7	10	15	20

[1]They are only suitable for those with identical legs, e.g., 3-DOF mechanism.

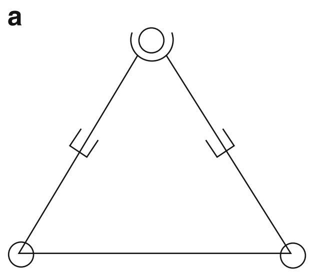
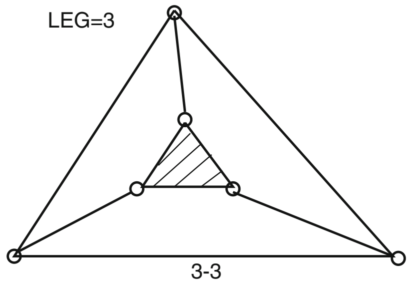


Fig. 3.5 Two types of vertex structures

Fig. 3.6 Possible architectures with 3 legs



On the basis of these two vertex structures, various types of parallel mechanism structures can be obtained through different arrangements of the joints on the base and mobile platforms.

(c) Platform structures
Once the type of vertex structure is decided, one can obtain the platform structure according to the number of vertices.

2. Possible architectures for parallel mechanisms
On the basis of the above analysis, one can assemble all the possible architectures as shown in Figs. 3.6 – 3.9.
3. The most promising architectures
As listed in Tables 3.1 and 3.2, although we have already given constraints to DOF distributions for each leg, there are still lots of possible combinations for parallel mechanisms which meet the machine tool's DOF requirement, *e.g.*, for DOFs = 3, from Table 3.1, there are 3 possible combinations of legs with degree-of-freedom of 3, 4, 5, and 6. Meanwhile, from Table 3.2, there are 7, 10, 14, and 19 possible combinations for legs with dofs of 3, 4, 5, and 6, thus we still have many architectures through the permutation and combination. To find the most promising architectures, the criteria for selection of joints and legs are given as follows

(a) Proper number and type of DOFs
In order to ensure the required motions (i.e., 5-dof between the tool and the workpiece) in Table 3.4, the DOFs distribution numbers and the type of

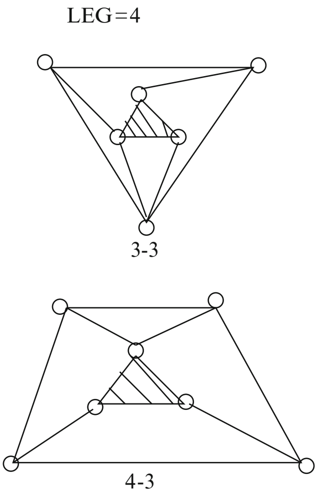


Fig. 3.7 Possible architectures with 4 legs

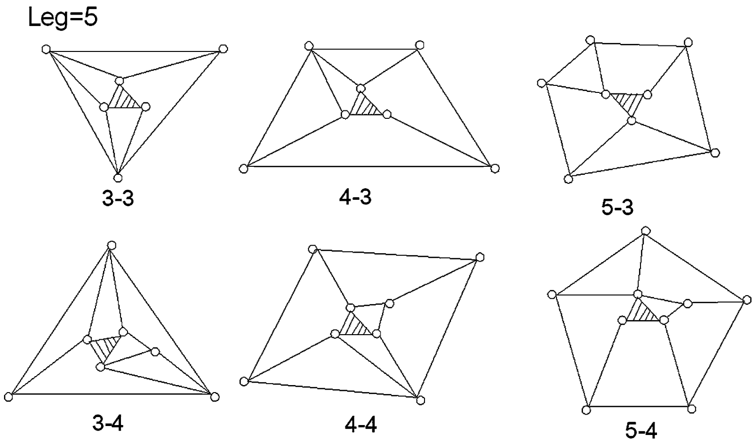


Fig. 3.8 Possible architectures with 5 legs

motions for each leg should be properly arranged. Each leg can be facilitated with spherical, prismatic, Hooke and revolute joints.

(b) Simplicity and practicability
The legs used in machine tools must be simple and practical. For the sake of the simplicity and dexterity of mechanism, we prefer to use "spherical"pairs as the joints between link and platform for those legs with more than 3 dofs. Since the serially connected revolute joints easily lead to "Singularity" and the "manufacturability" is difficult, so we abandon to use of more than 2 revolute joints connected in series.

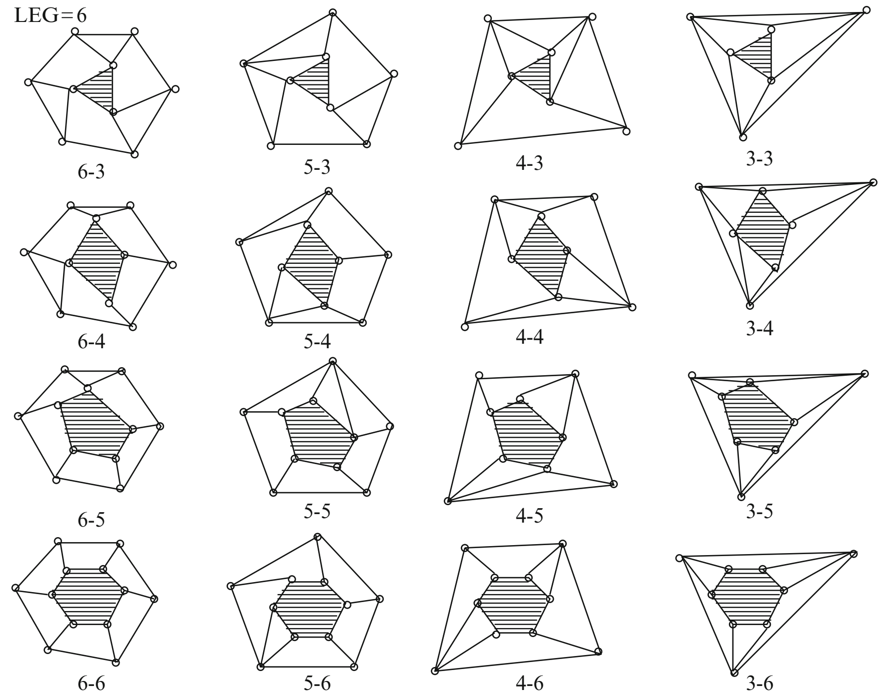


Fig. 3.9 Possible architectures with 6 legs

Table 3.4 The possible motion distributions for required 5-dof between the tool and the workpiece

DOFs (machine tools)	Motion of workpiece	Motion of machine tool
l = 3	X, Y: translation	Z: translation, X, Y: rotation
	X, Y: rotation	X, Y, Z: translation
	combination of R & T	X, Y, Z: combination of R & T
l = 4	X (or Y) translation	X, (or Y), Z: translation; X, Y: rotation
	X, (or Y): rotation	X, Y, Z: translation; X, (or Y): rotation
	combination of R & T	X, Y, Z: combination of R & T
l = 5	fixed	X, Y, Z: translation; X, Y: rotation

(c) Elimination of passive prismatic joints
Because it is difficult to control passive prismatic joints, in order to avoid the existence of passive prismatic joints, we specify

$$\text{Number of actuators} \geq \text{Number of prismatic joints} \tag{3.5}$$

Meanwhile, as we desire to put the actuators at the base of each link, therefore at most one prismatic joint can be used for each leg.

Table 3.5 Possible leg types with different degrees of freedom

Possible numbers	DOFs = 2	DOFs = 3	DOFs = 4	DOFs = 5	DOFs = 6
1	2R	1S	2R1H	1H2R1P	1S2R1P
2	1R1P	2R1P	1H1R1P	2H1R	1S1H1P
3	1H			2H1P	1S1H1R
4				1S1R1P[1]	
5				1S2R[1]	
6				1S1H[1]	
Total possibilities	3	2	2	6	3

[1]They are only suitable for those with identical legs, e.g., 3-DOF mechanism.

Table 3.6 The possible architectures

Degree of freedom	Number of legs	f_{l_1}	f_{l_2}	f_{l_3}	f_{l_4}	f_{l_5}	f_{l_6}	Possible architectures	Possible architectures with identical dof structure	$L = l$
$l = 2$	L = 2	2	6					9	9	9
		3	5					12	12	12
		4	4					3	2	2
$l = 3$	L = 2	3	6					6	6	
		4	5					12	12	
	L = 3	3	6	6				12	6	6
		4	5	6				36	36	36
		5	5	5				56	6	6
$l = 4$	L = 2	4	6					6	6	
		5	5					6	3	
	L = 3	4	6	6				12	6	
		5	5	6				18	9	
	L = 4	4	6	6	6			20	6	6
		5	5	6	6			36	9	9
$l = 5$	L = 2	5	6					9	9	
	L = 3	5	6	6				18	9	
	L = 4	5	6	6	6			30	9	
	L = 5	5	6	6	6	6		45	9	9
$l = 6$	L = 2	6	6					9	3	
	L = 3	6	6	6				10	3	
	L = 4	6	6	6	6			15	3	
	L = 5	6	6	6	6	6		21	3	
	L = 6	6	6	6	6	6	6	28	3	3
Total								429	179	98

(d) Elimination of the rotation around the Z axis
Since the rotation around the Z axis is not needed, we can introduce a n-dof passive leg into the mechanism to reach the desired motion. "Spherical joint" on the movable platform will be replaced by "Hooke joint" + "Prismatic

joint" or "Hooke joint" + "Revolute joint" so as to constrain the rotation around the Z axis. The passive constraining leg will be put in the center of the platform to minimize the torque and force. Since the external loads on the platform will induce a bending and/or torsion in the passive leg, its mechanical design is a very important issue, which can be addressed using the kinetostatic model later. In this case, the actuators are put in each of the identical legs and leave the special one (different DOFs) as the passive link since its structure in design size is larger than the other legs to sustain the large wrench.

(e) Structure of the mechanisms
The study is based on fully-parallel mechanisms, but one can add legs (with 6-dof) to keep the structure symmetric. For the shape of the platforms, one should avoid the use of regular polygon, since it may lead to geometry singularity.

Based on the discussion above, we eliminate some of the impractical joint combinations and obtain the prospective ones as shown in Table 3.5.

Through the combinations of the possibilities, we obtain the number of the most promising possible architectures shown in Table 3.6. When $L = l$, we obtain a fully-parallel mechanism.

3.5.3 Hybrid Mechanisms

A hybrid (serial-parallel) mechanism is a combination of serial and parallel mechanisms. It comprises two parallel actuated mechanisms connected in series, one of them is the upper stage, the other is the lower stage, and the moving platform of the lower stage is the base platform of the upper stage. This special structure results in a mechanism with the attributes of both. It provides a balance between exclusively serial and parallel mechanisms and better dexterity. It can even improve the ratio of workspace to architecture size and the accuracy.

To meet the required 5-dof motion, 2-dof and 3-dof parallel mechanisms are chosen to construct the "Hybrid" mechanisms. Since the upper stage is connected with the end-effector, and it requires high stiffness, so a 3-dof parallel mechanism is considered as the upper stage while a 2-dof parallel stage is taken as the lower stage.

For a 2-dof parallel mechanism – the lower stage of hybrid mechanism – both planar and spatial parallel mechanisms can be considered. Referring to (3.2), for planar mechanisms ($d = 3$), then one has

$$\begin{aligned} l &= 3\left[2 + \sum_{i=1}^{L}(f_{l_i} - 1) - \sum_{i=1}^{L} f_{l_i} - 1\right] + \sum_{i=1}^{g} f_i \\ &= 3 - 3L + \sum_{i=1}^{g} f_i \end{aligned} \tag{3.6}$$

Table 3.7 The possible degree-of-freedom distribution for planar mechanisms

Degree of freedom	Number of legs	f_{l_1}	f_{l_2}	f_{l_3}
l = 2	L = 2	2	3	
l = 3	L = 3	3	3	3
		2	3	4

Therefore, the possible DOFs distribution for planar mechanisms can be found in Table 3.7.

The hybrid motions (5-dof) can be arranged as follows:

- Upper stage: *X*, *Y* axes rotation, *Z* axis translation; lower stage: X, Y axes translation
  One can realize this motion through either the combination of 3SPR as upper stage and "Linear motion components" (LM) as the lower stage (special case) or the combination of 3SPR as upper stage and 3RRR planar parallel mechanism as the lower stage.
- Upper stage: *X*, *Y* axes translation, *Z* axis translation; lower stage: *X*, *Y* axes rotation
  One can realize this motion through the combination of 3SRR as upper stage and 2-dof spherical parallel mechanism as the lower stage. Because of the complexity in manufacturing spherical parallel mechanisms, low stiffness, low precision, and small workspace, we discard spherical parallel mechanisms in our research.

The "Hybrid" mechanisms can also be implemented in an alternative way, i.e., using positioning head (wrist) for machine tools design, this will be described in the next section.

3.6 Redundancy

The main purpose of adopting redundancy is to improve reliability and dexterity. To make the parallel kinematic machines capable of arbitrarily positioning and orienting the end-effector in a three-dimensional workspace, redundancy factor may be considered. In this book, only 3-dof, 4-dof, 5-dof, and 6-dof spatial parallel mechanisms are discussed. Generally, all these types of mechanisms are used for base platform, one can select a positioning head (wrist) with 1-dof, 2-dof, or 3-dof in conjunction with the base platform. This constructs a hybrid mechanism and it will lead to some redundant cases.

3.7 Conclusions

The kinematic structures used for 5-dof or less than 5-dof machine tools design with their underlying design principles have been made more explicit through the discussion and enumeration in this chapter. From the results obtained, it can be seen that both the tool and the workpiece can be actuated independently and that 5-dof is required for manufacturing tasks, the possible combinations of degree-of-freedom are: (5,0), (4,1), and (3,2). Moreover, for each of these combinations, the kinematic chains involved lead to several possibilities (serial, parallel, or hybrid) and additionally, redundancy is taken as an option. Finally, a detailed list of possible topologies has been obtained and the most promising architectures are pointed out under the design criteria.