2 Differentiation: Geometric Diversity and Design Classification

2.1 Need for Classification

The wide-ranging diversity of Islamic geometric patterns is a testimony to the degree of understanding that early Muslim pattern artists had of geometry and symmetry. Their inspired use of geometry led to the development of multiple varieties of pattern, symmetrical stratagems, and generative methodologies; the likes of which no other ancient culture came close to equaling in ingenuity and beauty. The diversity and complexity of this design tradition make it difficult to categorize, and indeed, no systematized method of comprehensive classification has been established. At best, writers and scholars addressing this subject employ descriptive analysis; for example, "The design . . . is a fully developed star pattern based upon a triangular grid. Its primary unit is a six-pointed star inscribed within a hexagon, which is surrounded by six five-pointed stars whose external sides form a larger hexagon."[1] However, detailed descriptions rarely elucidate beyond the visually obvious star types and square or triangular repeat units. Other fundamental features are frequently unaddressed when examining a given geometric pattern, including the symmetrical schema for more complex designs, the crystallographic plane symmetry group, the generative methodology, the incorporation of culturally associated additive features and treatments, and identification of the specific pattern family. The absence of appreciation for these less obvious, but nonetheless significant design features obscures the extraordinary scope of this design tradition, and it is only through a more nuanced and differentiated approach to this study, with its myriad cultural and geometric attributes, that a thorough understanding and appreciation of Islamic geometric patterns can be achieved.

The benefits of a more comprehensive approach to the classification of Islamic geometric patterns are wide ranging, and highly relevant to historians of Islamic art and architecture, as well as to contemporary artists, designers, and architects who use such designs in their work. In addition to the more general enhanced appreciation of the width and breadth of this ornamental tradition, the highly detailed classification of geometric patterns according to their overall symmetry, repetitive schema, numeric qualities, generative methodology, family type, and additive pattern variations and treatments has very specific relevance to each Muslim culture and dynasty. From the perspective of art and architectural history, the ascription of these differentiated qualities to the ornamental use of geometric designs allows for a far greater understanding of the artistic practices of a given Muslim culture, as well as an enhanced comparative appreciation for the subtle differences between the design conventions of neighboring and succeeding cultures. What is more, the categorization of the diverse geometric characteristics that comprise this design tradition provides the necessary methodological knowledge for those who wish to more fully explore the range of possibilities and unlimited potential for creating fresh original geometric designs that these historical methodologies still offer. It is only through such knowledge that this once great ornamental tradition can be rekindled into a contemporary artistic movement endowed with creative vitality.

Despite the expressed rationale for a more detailed categorization of Islamic geometric patterns, there is no evidence to suggest that Muslim designers of the past were particularly concerned with a need to systematically organize their geometric patterns into differentiated categories. The design scrolls that have survived to the present day are a random collection of diverse ornamental motifs that include *Kufi* calligraphy, *muqarnas*, star net vaulting, domical gore segments with geometric designs, and a wide variety of two-dimensional geometric patterns. The fact that these pattern scrolls have no logical sequence in the placement of their many individual designs obviously does not imply that Muslim designers had no appreciation for geometric

[1] This quotation references a geometric pattern used on a door at the Bimaristan al-Nuri in Damascus (1154). Tabbaa (2001), 88.

J. Bonner, *Islamic Geometric Patterns*, DOI 10.1007/978-1-4419-0217-7_2

differentiation within this ornamental tradition. On the contrary, the full range of sophistication in this Islamic design tradition, in and of itself, provides clear evidence that Muslim artists had a highly sophisticated knowledge of geometric diversity, but did not require this knowledge to be outwardly systematized. The history of collecting and classifying Islamic geometric patterns is closely associated with nineteenth-century orientalism: frequently with the objective of making illustrated representations of specific patterns available to Western artists working with this novel aesthetic.[2] The publication of *The Grammar of Ornament* by Owen Jones in 1856 included numerous geometric designs from Muslim sources.[3] The organizing principle behind this work was loosely ethnographical rather than geometric; with chapters dedicated to *Arabian*, *Turkish*, *Moresque*, *Persian*, and *Indian* ornament, and the examples of geometric design within these sections are arbitrarily sequenced alongside their floral and calligraphic neighbors. The earliest work to organize geometric patterns into geometric categories was published in 1879 by the ornamental theoretician and architect Jules Bourgoin.[4] The 190 geometric designs that comprise this collection are divided into eight numeric and geometric categories: hexagonal patterns; octagonal designs; dodecagonal designs; patterns with two different star forms; designs with squares and octagons; patterns with three and four different star forms; sevenfold patterns; and fivefold patterns with ten-pointed stars. While these categories may seem somewhat limited today, at the time this collection was a significant contribution to the spread of interest in this subject throughout Europe, and continues to be a standard reference book for Islamic geometric pattern to this day.[5] The history of the classification of Islamic geometric design is an interesting study in itself, and has mostly built upon the overtly obvious categories identified by Bourgoin. This organizational refinement began during the beginning of the last quarter of the twentieth century with the publication of several books on the subject of Islamic geometric patterns.[6] For the most part, these more recent studies have included the ordering of patterns that repeat upon the isometric and orthogonal grids by complexity, as well as patterns that have fivefold symmetry. In the isometric examples the least complex designs are comprised of triangles, hexagons, and six-pointed stars. These are followed by patterns that place increasingly large star forms upon the vertices of the repetitive grid (and/or its hexagonal dual) whose local symmetry is always a multiple of 3: e.g., 9-, 12-, and 15-pointed stars. In some studies, recognition is also given to patterns with greater complexity that exhibit more than a single region of local symmetry, for example, the well-known designs with 9- and 12-pointed stars. Orthogonal patterns are similarly organized by repeat unit and increasing complexity: the least complex being comprised of squares, octagons, and eight-pointed stars, followed by more complex designs with star forms that are multiples of 4. The more thorough studies include designs with more than one region of local symmetry, such as 8- and 12-, 8- and 16-, as well as 8- and 24-pointed stars.[7] The most comprehensive twentieth-century catalogue of Islamic geometric design was published by Gerd Schneider in 1980.[8] This study focuses exclusively on the geometric ornament of the Seljuk Sultanate of Rum, under whose patronage this ornamental tradition produced many of the most sophisticated and complex geometric designs. Schneider illustrates 440 patterns that are not differentiated according to their repetitive structure, but placed within a broad set of visually explicit categories that include square *Kufi* calligraphy; orthogonal brick designs; domical brick designs; three-, four-, five-, and six-fold swastika designs; border designs; additive designs; superimposed polygonal designs; star patterns with extended points; patterns made up of a single repetitive device; patterns with 6-pointed stars; patterns with 6- and 12-pointed stars; patterns with hexagonal centers; fourfold patterns with square centers; 8-pointed star patterns with octagons; complicated 8-pointed star patterns; 9-pointed star patterns; pentagonal designs with 5- and 10-pointed stars; 10-pointed star patterns; 12-pointed star patterns; patterns with 8- and 12-pointed stars; patterns with 9- and 12-pointed stars; 12-pointed star patterns with additional star forms; 12- and 14-pointed star patterns; 16-pointed star patterns with additional star forms; 15- and 18-pointed star patterns with additional star forms; 24-pointed star patterns; and geometric patterns on domes and hemispheres. Many of

[2] Necipoğlu (1995), Chapter 4. *Ornamentalism and Orientalism: the Nineteenth and Early Twentieth Century European Literature*, 61–87.

[3] Jones (1856).

[4] Bourgoin (1879).

[5] The ongoing availability of Bourgoin's work is due to its being kept in print as part of the Dover Pictorial Archive Series (printed without original text). In creating the illustrations for his book, Bourgoin does not appear to have used a traditional methodology for recreating the patterns in his collection. As a consequence, the proportions within many of his illustrations—especially those with greater complexity—are inaccurately represented, and have clearly discernable distortion. Being that this has been an artist's reference for over 150 years, the direct copying of such problematic designs has occasionally promulgated these errors by their application within the applied and architectural arts.

[6] –Critchlow (1976).
–El-Said and Parman (1976).
–Wade (1976).

[7] Wade (1976), 63–79.

[8] Schneider (1980).

the designs illustrated in this valuable study are not represented elsewhere. Equally impressive is the work of Jean-Marc Castéra, dating from 1999, which focuses upon the geometric design diversity found in the Moroccan ornamental tradition.[9] This work includes orthogonal designs with increasingly large numbers of primary star forms (up to 96-pointed stars), the hexagonal family, the pentagonal family, and patterns with two varieties of star. This is also one of the earliest publications to categorize a subset of dual-level geometric designs, herein referred to as Type D, as having self-similar properties: a type of design created from what Castéra calls the "Alhambra method."[10] The differentiation between systematic and nonsystematic generative methodologies was first introduced by the author in 2003,[11] with the identification for the first time of four historical systems used for creating geometric patterns: one that produces patterns that can be either threefold or fourfold; two that produce patterns that are fourfold; and one that produces patterns that are fivefold. Further, this work also identified three geometrically and aesthetically distinct varieties of dual-level design with self-similar characteristics that were reliant upon these systems for their creation. This was expanded upon in 2012 to include the historical use of a system that generates sevenfold geometric patterns.[12] The application of Islamic geometric patterns to the parameters of the 17 plane symmetry groups is a particularly interesting development in the efforts toward methodical categorization. Beginning in 1944 a number of mathematicians and crystallographers have published works devoted to this topic.[13] Of especial note is the work of Syed Jan Abas and Amer Shaker Salman, dating from 1995, that identifies some 248 Islamic geometric patterns with their respective crystallographic plane symmetry group.[14]

The abounding diversity of this design tradition necessitates categorization according to several criteria. The standard classification of Islamic geometric patterns has provided a useful means for descriptive dialogue, and is certainly relevant to art historians and contemporary artists alike. However, this does not provide any insight into the methods used in the creation of these designs. The categorization according to methodology and pattern family that concludes this chapter is a subject that has been largely overlooked by previous studies, but is fundamental to the thorough understanding of this ornamental tradition.

2.2 Classification by Symmetry and Repetitive Stratagems

In examining this tradition, the most fundamental category to which all patterns must ascribe is *geometric symmetry*. Most Islamic geometric patterns exhibit threefold, fourfold, or fivefold symmetry, although other more obscure symmetrical systems were also developed within this tradition. Directly related to a pattern's symmetry is its *repeat unit*. Islamic geometric patterns are able to continuously fill the plane through the repetitive use of a single element. These repeat units will always contain the minimum portion of a pattern that is able to seamlessly fill the plane through repetitive edge-to-edge translation symmetry. In this way, the repeat unit is essential to a pattern's ability to successfully fill two-dimensional space. The laws that govern the science of repetitive two-dimensional space filling, or tiling, are universal, and apply no less to Islamic geometric design than to any other pattern-orientated ornamental tradition. All periodic covering of the two-dimensional plane must conform to the symmetrical determinants of one or another of the 17 plane symmetry groups. Yet these limits offer tremendous scope for symmetrical and aesthetic diversity. And no artistic tradition explored symmetrical potential with the degree of passion and ingenuity as those of successive Muslim cultures.

A regular polygon is defined as having equal included angles and common edge lengths. As illustrated in Fig. 1, only three of the regular polygons are able to infinitely cover a plane on their own: the triangle, square, and hexagon. Figure 1a illustrates the isometric grid made from equilateral triangles, along with its hexagonal dual (green); Fig. 1b shows the orthogonal grid made from squares, with the dual grid (green) being the same orthogonal grid; and Fig. 1c shows the hexagonal grid made from regular hexagons, along with the isometric dual grid (green). The majority of Islamic geometric patterns repeat upon either the isometric or the orthogonal grids. Each vertex of the isometric grid has sixfold symmetry comprised of six acute angles of 60° where the six edge-to-edge equilateral triangles meet. The dual of the isometric grid is the grid of regular hexagons. This grid has threefold symmetry that results from the three coincident hexagons with 120° included angles that meet at each vertex. Islamic geometric patterns that repeat upon the isometric grid will invariably exhibit sixfold symmetry at the vertices of this grid, and threefold symmetry at the centers of each triangular repeat unit—which is to say the vertices of the dual-hexagonal grid. As such, these patterns have regions of both sixfold and

[9] Castéra (1996).

[10] Castéra (1996), 276–277.

[11] Bonner (2003).

[12] –Bonner and Pelletier (2012).
–Pelletier and Bonner (2012).

[13] –Müller (1944).
–Weyl (1952).
–Lalvani (1982).
–Lalvani (1989).
–Lovric (2003), 423–431.

[14] Abas and Salman (1995).

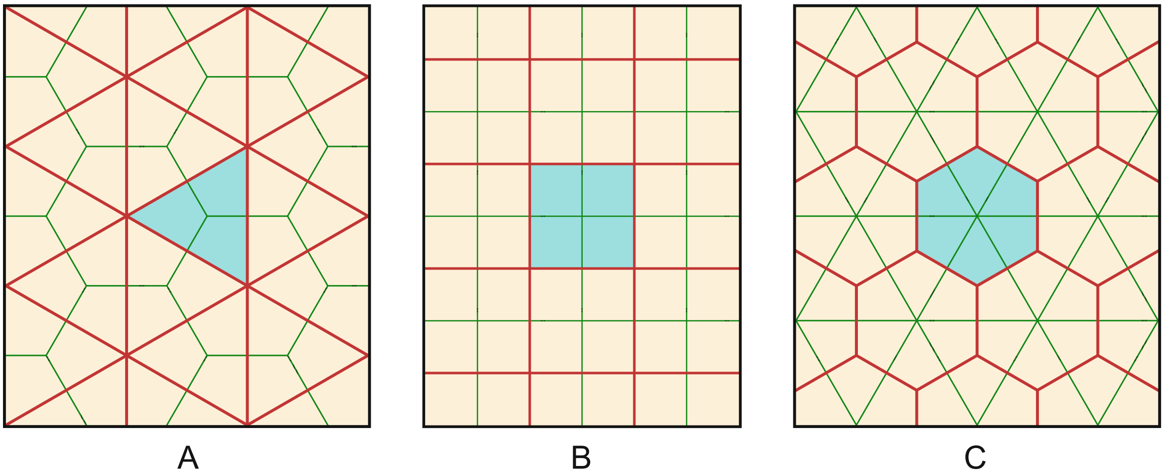


Fig. 1

threefold symmetry. However, for purposes of convenience, this category of geometric design is referred to as simply threefold. This is acceptable due to the fact that, three being a devisor of six, the sixfold vertices also have threefold symmetry. More complex threefold patterns will place higher ordered star forms at the vertices of either or both these grids, and the number of points of these stars will always be a multiple of the threefold or sixfold symmetry of the vertex. The vertices of the orthogonal grid have fourfold symmetry resulting from the four coincident squares, with 90° included angles, that meet at each vertex; and the dual of the orthogonal grid is an identical orthogonal grid whose vertices are located at the center of each square repeat unit of the original grid. Patterns that employ square repeat units are therefore referred to as fourfold. Similarly with threefold patterns, fourfold designs will often place star forms at the vertices of both the orthogonal grid and its dual that are multiples of 4, thus creating regions of higher order local symmetry.

It is important to differentiate between the repeat unit of a given pattern and its *fundamental domain*. The fundamental domain is the minimal essential repetitive component of a design. By the singular or combined functions of rotation, reflection, and glide reflection, the fundamental domain will populate the repeat unit. It is remarkable how little visual information is contained within the fundamental domains of many highly successful, albeit less complex geometric designs. Figure 2 illustrates the classic threefold *median* pattern comprised of 6-pointed stars located upon the vertices of the isometric grid. The relationship between the design and both the isometric grid (green) and its hexagonal dual (red) is clearly evident; and indeed, the triangles or hexagons are equally capable of being used as the underlying generative polygons responsible for this design. The fundamental domain for this pattern is a right scalene triangle (blue) with a single applied pattern line. This is reflected and then rotated to complete the repeat unit: ×3 for the triangle, and ×6 for the hexagon. Figure 3 illustrates the classic fourfold star-and-cross *median* pattern that places eight-pointed stars at the vertices of the orthogonal grid (green) and fourfold crosses at the vertices of the dual of this grid (red). The fundamental domain is a right isosceles triangle (blue) with just two applied pattern lines. By reflecting the fundamental domain upon its hypotenuse, and rotating this four times at the vertex of the dual grid, the square repeat unit will be completed. Alternatively, rotating four times at the vertex of the repeat unit will fill a unit cell of the dual grid.

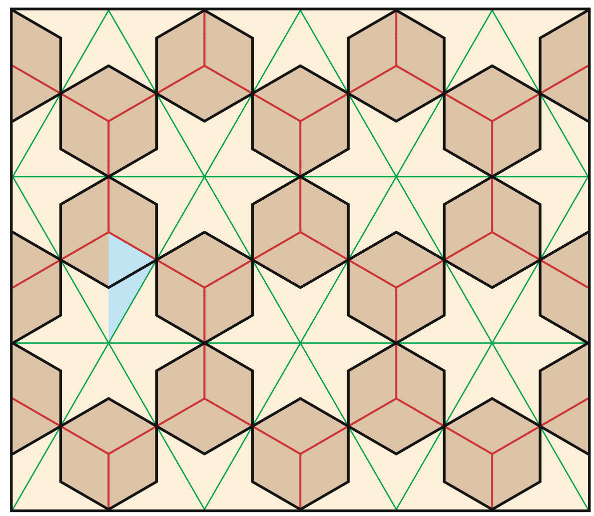


Fig. 2

As said, the historical record is rife with Islamic geometric patterns based upon threefold and fourfold symmetry that respectively utilize the triangle, hexagon, and square as repeat units. Yet as early as the eleventh century Muslim artists began working with distinctive patterns characterized by fivefold and even sevenfold symmetry. This was made possible through the use of rhombic, rectangular, and elongated hexagonal repeat units and resulting repetitive grids with proportions that directly relate to fivefold and sevenfold symmetry. What is more, the proportions of these types of alternative repeat units could also conform to symmetries more commonly associated with the regular polygons. In this way, it was possible to create patterns with higher order star forms with points that are multiples of 3 and 4 (for example: 8, 12, 15, 16, 18, and 24) that were not confined to the isometric and orthogonal grids, yet often shared visual characteristics with their more conventional counterparts. These three less common repetitive stratagems are elongated corollaries of the three grids produced from the regular polygons: the isometric grid sharing properties with rhombic grids; the orthogonal grid with rectangular grids; and the regular hexagonal grid with elongated hexagonal grids. Changing the edge lengths and/or included angles of the polygonal components of these three regular grids such that the new angles and edge lengths correspond with the inherent proportions of specified polygons opened this tradition to the creation of designs with all manner of symmetries, including fourfold patterns with 8-pointed stars set upon a rhombic grid; fivefold patterns with 10-pointed stars set upon both rhombic and rectangular grids; sevenfold patterns with 14-pointed stars set upon both rhombic and rectangular grids; and patterns with 12-pointed stars set upon a rectangular grid. This more flexible approach to repeat units with specific inherent proportional properties also allowed for the creation of more complex designs with multiple centers of local symmetry that would ordinarily be incompatible. Such designs are invariably nonsystematic and include a pattern with 7- and 9-pointed stars set upon an elongated hexagonal grid; a pattern with 9- and 11-pointed stars set upon an elongated hexagonal grid; and a pattern with 11- and 13-pointed stars that is also set upon an elongated hexagonal grid.

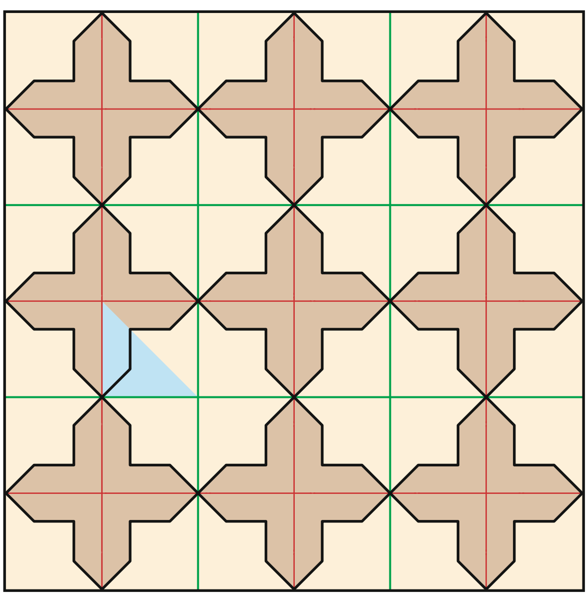


Fig. 3

The isometric grid is made up of three sets of parallel lines. By removing one of these sets a rhombic grid is produced. Each rhombus becomes a repeat unit with the proportion of two edge-to-edge equilateral triangles. The location and number of vertices remain identical to the original isometric grid. Figure 4 illustrates a very successful example of a class of pattern that uses this rhombic repetitive schema by placing nine-pointed stars at each isometric vertex. Whereas nines will work nicely at the vertices of the regular hexagonal grid, they do not conform to the vertex constraints of the regular triangular grid (because 9 is not evenly divisible by 6). The placement of nines upon the vertices of the rhombic grid elegantly overcomes this limitation. The fundamental domain for this design is an equilateral triangle (blue) that is reflected to create the rhombic repeat unit with translation symmetry. Figure 5 illustrates the proportional determinants for the two rhombi with fivefold symmetry that were used historically for patterns with 5- and 10-pointed stars. The opposing included angles of both these rhombi are multiples of 36°: a 1/10 division of the circle. Figure 5a illustrates the wide rhombus with two opposing acute angles with 2/10 included angles, and two opposing obtuse angles with 3/10 included angles. The acute included angles of the thin rhombus in Fig. 5b are a 1/10 segment, and the obtuse angles are 4/10 segments. Figure 6 illustrates the obtuse and acute fivefold grids that these two rhombi produce; and Fig. 7a shows how the wide rhombi relates to the pentagon, and Fig. 7b demonstrates how the thin rhombi relates to the decagon. Figure 8 illustrates two geometric patterns that repeat with these two rhombi.

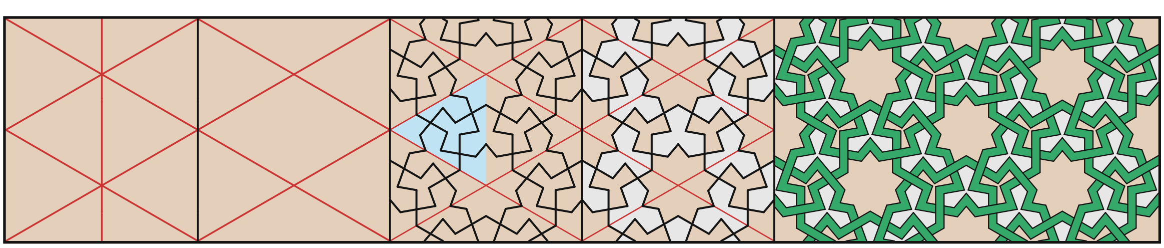


Fig. 4

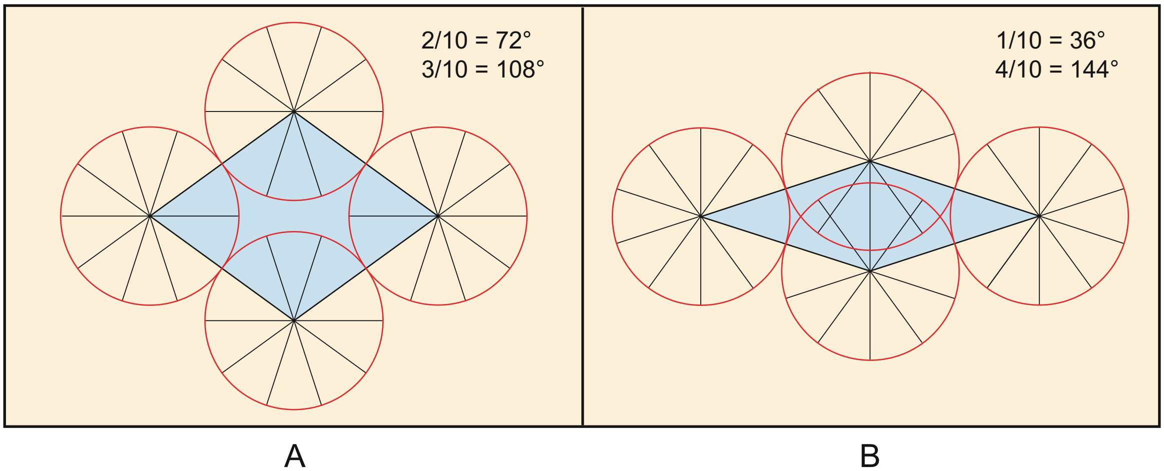


Fig. 5

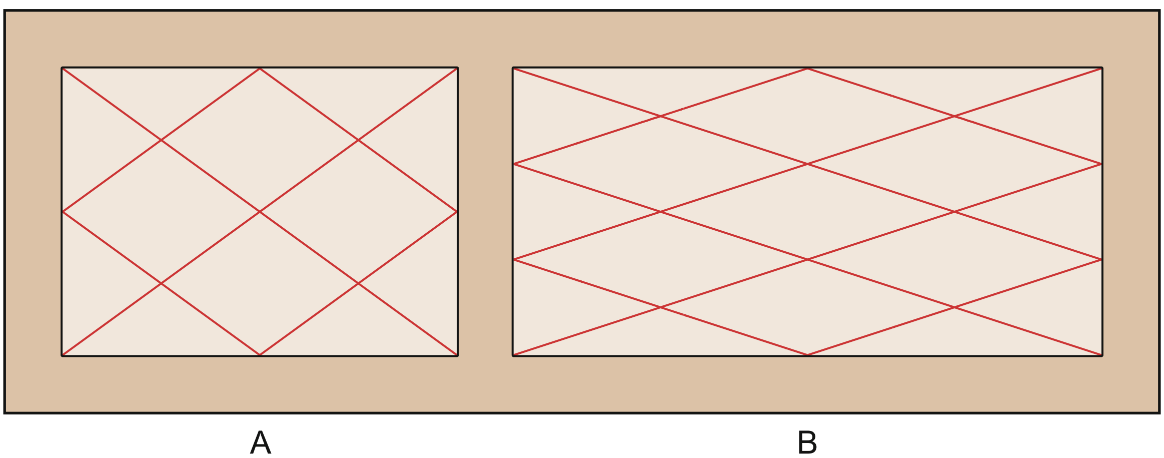


Fig. 6

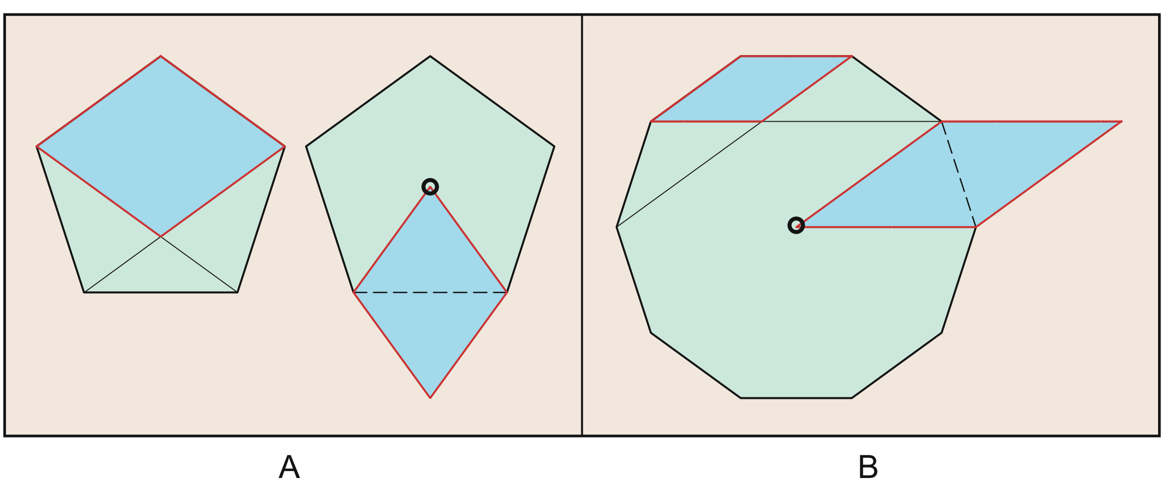


Fig. 7

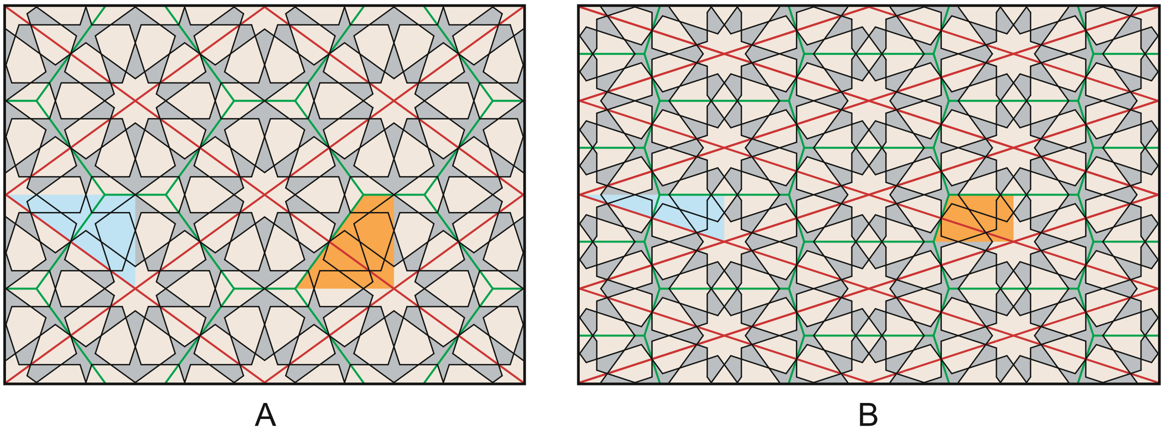


Fig. 8

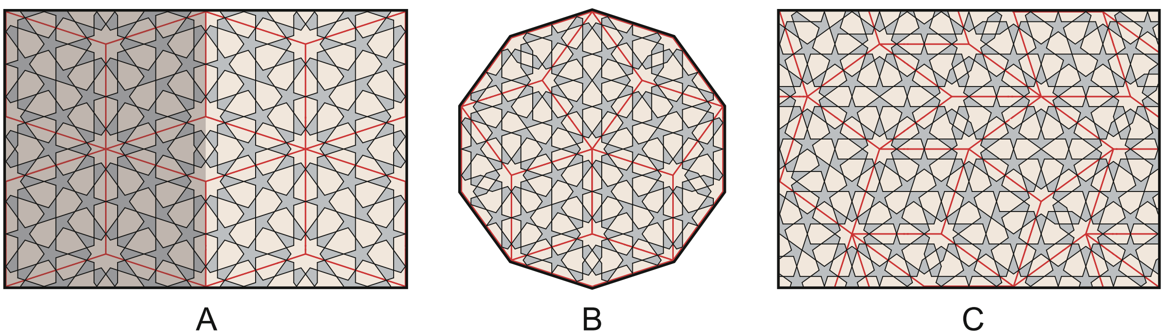


Fig. 9

Figure 8a is the classic fivefold *acute* pattern that repeats on the obtuse rhombic grid (red) and has a dual-hexagonal repetitive grid (green). The pattern in Fig. 8b (by author) repeats upon the acute rhombic grid (red) and also has a dual-hexagonal repetitive grid (green). The fundamental domain of both varieties of rhombic repeat unit is a 1/4 triangular segment (blue) that requires reflection 4×, while the fundamental domain of both types of the dual-hexagonal repeats is a 1/4 quadrilateral (orange) that also requires reflection 4×. Figure 9 demonstrates how these 2 fivefold rhombi can be used together to tessellate the plane in several fashions. The combined use of more than a single repetitive element qualifies each of the three examples in this figure as a hybrid design. Figure 9a is an example of a periodic tessellation with translation symmetry that is provided by a rectangular repeat unit (shaded) made up of four obtuse rhombi and two acute rhombi; Fig. 9b is an example of a radial tessellation; and Fig. 9c is an example of a non-periodic tessellation devoid of translation symmetry.[15] The success of such hybrid designs is conditioned upon the applied pattern lines along the edge of each rhombus being congruent. Although no historical examples of fivefold hybrid designs that use just these two rhombi are known, several periodic fivefold hybrid designs were produced that employ multiple repetitive elements, including rhombi, pentagons, triangles, and non-regular hexagons, always with the requisite matching edge configurations within the applied pattern lines [Figs. 261–268].

The same rhombic repetitive logic applies to the generation of sevenfold geometric patterns. Figure 10 illustrates the proportional determinants for the three rhombi with sevenfold symmetry, with the obtuse rhombus in Fig. 10a being

[15] These 2 fivefold rhombi are the same as those identified by Sir Roger Penrose in his groundbreaking research into aperiodic tilings. However, the application of the geometric patterns to the two rhombi in Fig. 9 does not include Penrose's matching rules for forced aperiodicity and the design in Fig. 9c is therefore referred to herein as non-periodic rather than aperiodic. While never occurring within the historical record, it is certainly possible to populate these 2 fivefold rhombi with patterns that conform to the Penrose matching rules, thereby forcing the geometric design to be aperiodic [Figs. 480 and 482].

–Penrose (1974), 266–271.

–Gardener, Martin (January 1977), "Extraordinary nonperiodic tiling that enriches the theory of tiling," *Scientific America*, pp. 110–121.

–Penrose (1978), 16–22.

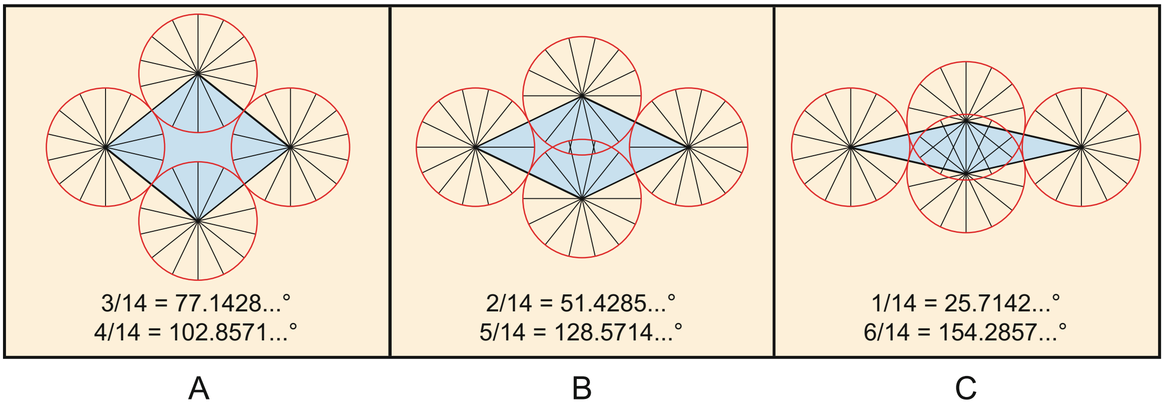


Fig. 10

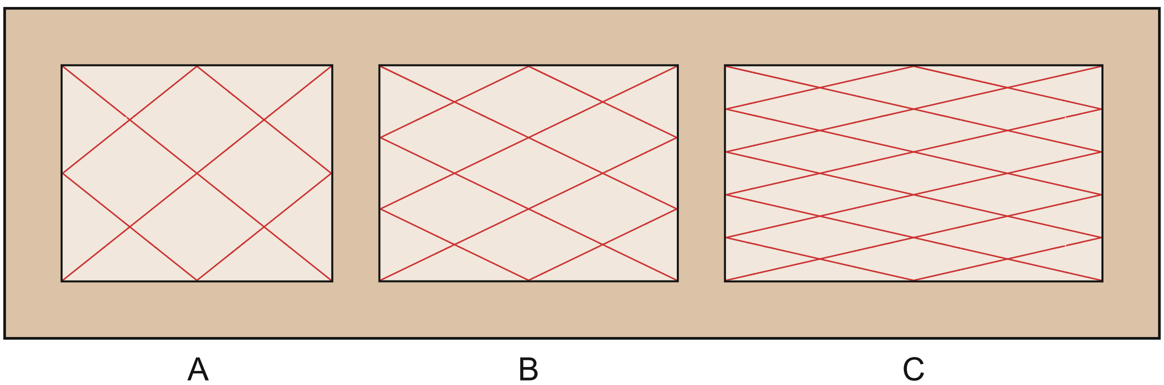


Fig. 11

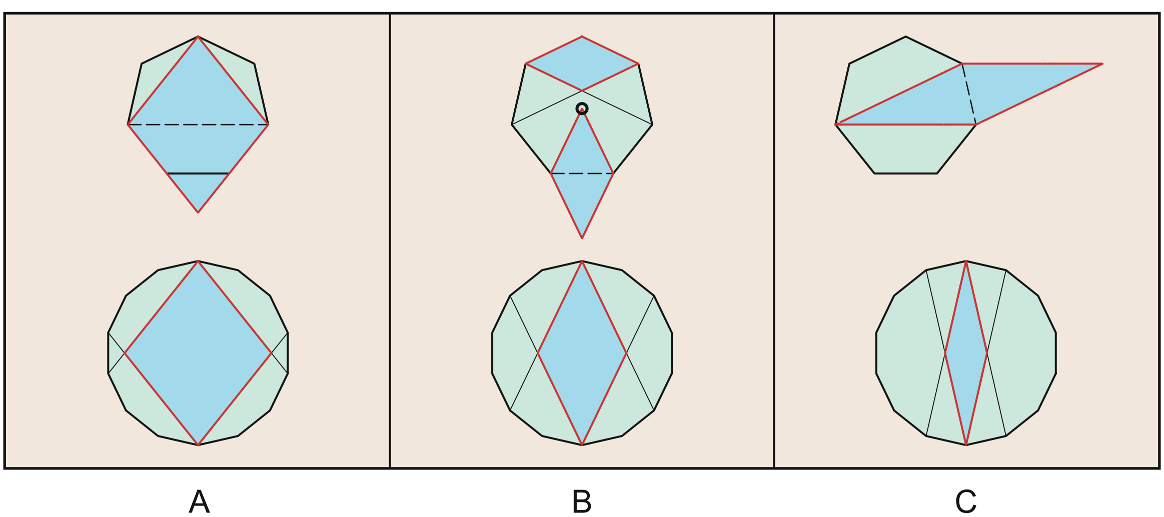


Fig. 12

comprised of 3/14 and 4/14 included angles; the median rhombus in Fig. 10b having 2/14 and 5/14 included angles; and the acute rhombus in Fig. 10c having 1/14 and 6/14 included angles. Figure 11 shows the three rhombic grids that these three rhombi produce. Figure 12 demonstrates several simple methods for creating the 3 sevenfold rhombi from the heptagon and tetradecagon. Figure 12a shows two ways of creating the obtuse rhombus; Fig. 12b shows two

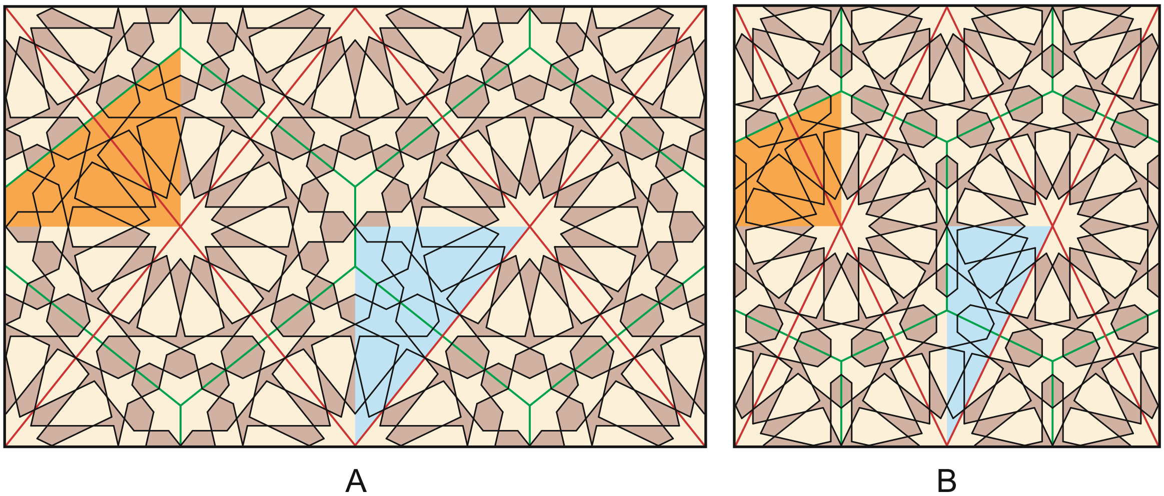


Fig. 13

ways of producing the median rhombus; and Fig. 12c shows two ways of producing the acute rhombus. Figure 13 illustrates two historical examples of patterns that employ the obtuse and median sevenfold rhombic repeat units. The design in Fig. 13a is from the ’Abd al-Ghani al-Fakhri mosque in Cairo (1418). This repeats with either the obtuse rhombic grid (red) or the dual-hexagonal grid (green). The fundamental domain of the rhombic repeat unit is a 1/4 segment right triangle (blue) that requires reflection 4× to fill the repeat. The fundamental domain of the dual-hexagonal repeat unit is a 1/4 segment quadrilateral (orange) that also requires reflection 4×. The design in Fig. 13b is from the Bayezid Pasha mosque in Amasya, Turkey (1414-19). This utilizes the median sevenfold rhombic grid (red), and also has a hexagonal dual grid (green). Like the previous example, the fundamental domain of the rhombic repeat is a 1/4 segment right triangle (blue) that requires reflection 4× to fill the repeat, and the fundamental domain of the dual-hexagonal repeat (orange) is a 1/4 segment quadrilateral that also requires reflection 4×. The acute sevenfold rhombus does not appear to have been used historically. As with the fivefold rhombi, the 3 sevenfold rhombi can be used with one another to create more complex periodic [Fig. 284], radial [Fig. 285], and non-periodic hybrid designs,[16] and indeed, these two historical examples have the requisite identical edge configuration to produce hybrid variations [Fig. 26d]. However, no examples of sevenfold hybrid designs are known from the historical record.

The use of rectangular repeat units for geometric designs with symmetries that do not readily conform with either the isometric or the orthogonal grids began in Khurasan during the late twelfth century. Figure 14 illustrates such an

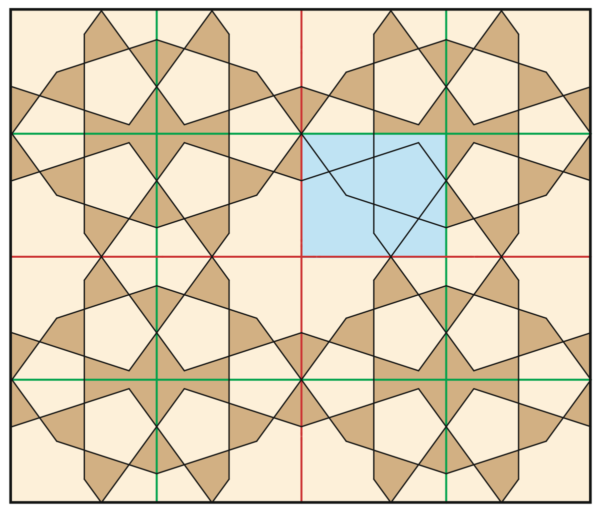


Fig. 14

example from the Maghak-i Attari mosque in Bukhara, Uzbekistan (1178-79). This is one of the earliest examples of a pattern that repeats upon a rectangular grid, and is also one of the least complex rectangular fivefold patterns. This design places ten-pointed stars at the vertices of the rectangular grid (red), and the specific proportions of the rectangular repeat unit are determined by the arrangement of the underlying generative polygonal modules from the *fivefold system* that are responsible for this pattern [Figs. 203 and 245a]. As with the orthogonal grid, the dual of a rectangular grid is the same rectangular grid (green). Fundamental domains for designs that utilize rectangular repeat units are almost always a 1/4 rectangular segment (blue) that fills the repeat unit through reflection 4×.

Figure 15 illustrates a design from the Sultan al-Mu’ayyad Shaikh complex in Cairo (1412-22) that is created from the *sevenfold system* and repeats upon both a

[16] Pelletier and Bonner (2012), 141–148.

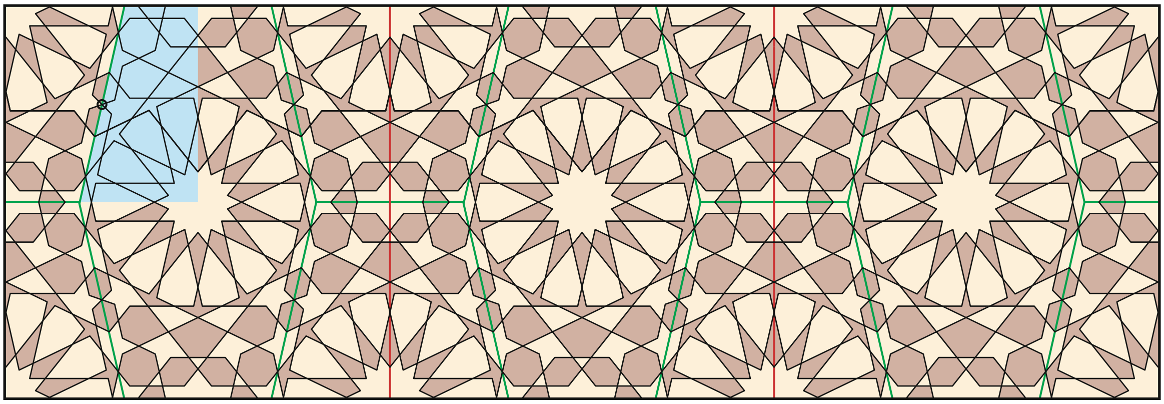


Fig. 15

rectangular grid (red) and a hexagonal grid (green) [Fig. 294] [Photograph 50]. The hexagonal repeat unit is half the area of the rectangle, and is the true minimal repetitive cell. However, the rectangular repeat units place the 14-pointed stars at their vertices, and this is frequently more convenient for practical application. When considered from the perspective of the rectangular grid, this design has a second unusual characteristic: applied pattern lines on the rectangular repeats that are precisely the same as on its dual grid. (Note: were it not for the skewed orientation between the 14-pointed stars at the vertices and center of each rectangular repeat unit, this pattern would repeat upon a rhombic grid.) The fundamental domain of the rectangular repeat is a quadrilateral (blue) that must rotate 180° upon the center point of the long edge before filling the remaining repeat unit through reflection 4× to fill the repeat. As said, the hexagonal grid (green) is the true minimal repeat unit with translation symmetry. This shares the same fundamental domain, but only requires reflection 4× to fill the repeat.

Rectangular repeat units were also used with geometric designs that have more than a single region of local symmetry. Such patterns will typically place one variety of star at the vertices of the rectangular grid, and another star form at the center of each repeat unit; which is to say, upon the vertices of the dual grid. Figure 16 illustrates a particularly successful example of this type of compound pattern from the *minbar* of the Great Mosque of Aksaray in Turkey[17] (1150-53). This places 12-pointed stars upon the vertices of the primary grid (red), and 10-pointed stars on the vertices of the identically proportioned dual grid (green). The proportions of this rectangular repeat unit are the direct product of the correlation between the 12- and 10-fold local symmetries as they relate to the underlying polygonal tessellation that generates this design [Fig. 414]. The fundamental domain (blue) of this pattern is a 1/4 segment of the repeat unit that fills the unit by reflection 4×.

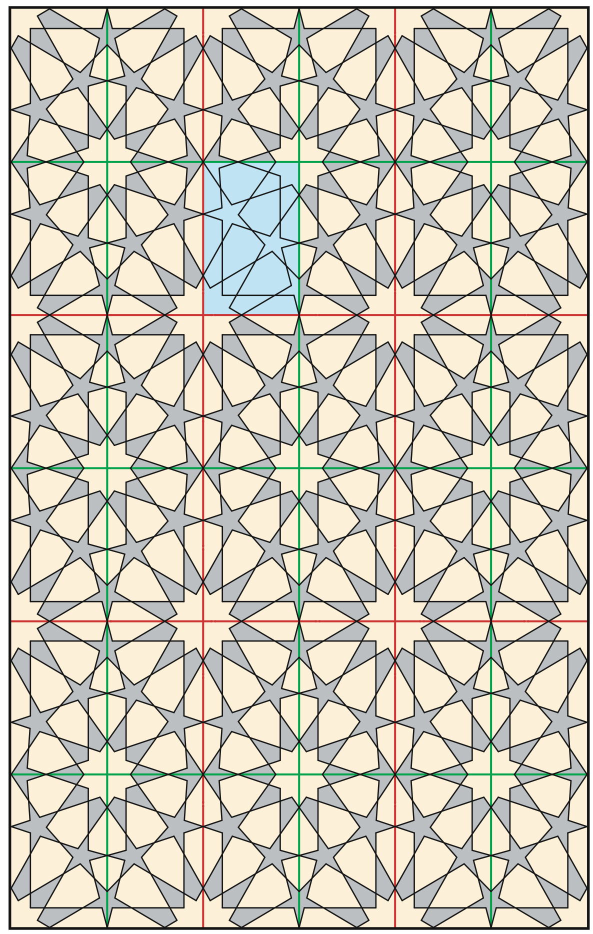


Fig. 16

The use of non-regular hexagonal repeat units encompasses a wide variety of design types, including systematic and nonsystematic patterns, more simplistic field patterns, and very complex patterns with compound local symmetries and multiple star forms. The discovery that this repetitive

[17] Schneider (1980), pattern no. 416.

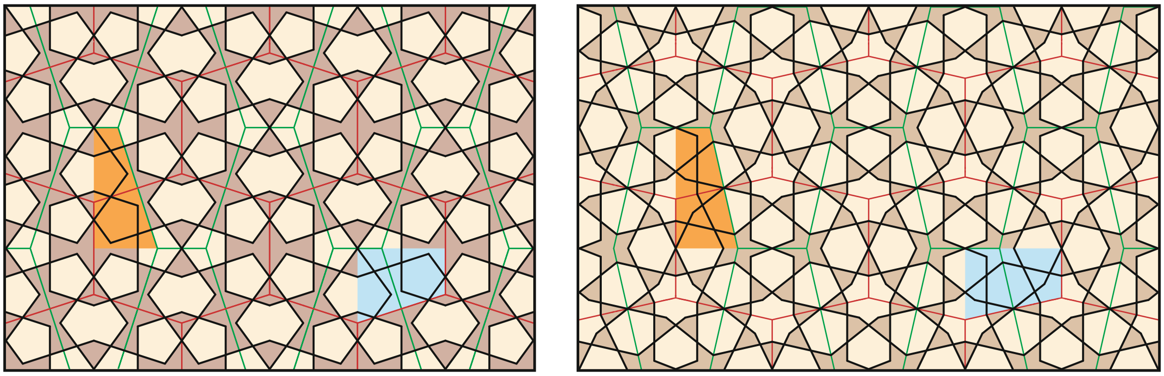


Fig. 17

stratagem was applicable to symmetries that do not conform to the convenient tessellating properties of the regular triangle, square, and regular hexagon can be traced back to the sevenfold design used by Seljuk artists in the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Fig. 279] [Photograph 26], and the sevenfold designs created by Ghaznavid artists on the tower of Mas'ud III (1099-1115) [Figs. 280 and 281]. Figure 17 illustrates two rather simple, but nonetheless elegant, field designs that repeat upon non-regular hexagonal grids. Figure 17a is created from the *fivefold system*, and Fig. 17b from the *sevenfold system*. Similar to rhombic repeat units, each of the included angles of the hexagonal repeat units for both of these patterns (red) are multiples of a 10- and 14-fold division of a circle, respectively. The perpendicularly orientated dual of each of these hexagonal grids is also a hexagonal grid (green), and their included angles are likewise multiples of 10- and 14-fold divisions of a circle. Both the repetitive grid and its dual for each of these patterns have their own quadrilateral fundamental domain, and each fills the repeat unit through reflection 4×. Both of these patterns are from the Seljuk Sultanate of Rum: the fivefold pattern from the Sitte Melik tomb in Divrigi (1196) [Fig. 213], and the sevenfold design from the Great Mosque of Dunaysir in Kiziltepe (1204) [Fig. 282a]. Many of the more complex patterns that utilize a non-regular hexagonal repeat unit will have a combination of differing star forms that are seemingly irreconcilable in their geometric symmetry. Figure 18 is a remarkable design from the Mu'mine Khatun in Nakhichevan, Azerbaijan (1186). This design has two regions of local symmetry: 13-fold placed upon the vertices of the hexagonal primary grid (red), and 11-fold located at the vertices of the perpendicularly orientated hexagonal dual grid (green). The combination of equal numbers of 13- and 11-pointed stars requires a geometric dexterity that pushes the limits of two-dimensional space filling. The fundamental domain for each type of hexagonal repeat is a right-angled quadrilateral that is a 1/4 segment of their respective repeat unit requiring reflection 4× to fill their respective repeat unit. Figure 19 illustrates the origin of the included angles of the 13-fold and 11-fold hexagonal repeat units from this design. Figure 19a illustrates a 1/13 division of a 13-fold tridecagon. Four of the included angles of the 13-fold hexagonal repeat unit are made up of three-and-a-half 1/13 segments, and two are made up of six 1/13 segments. Figure 19b illustrates a 1/11 division of a 11-fold hendecagon. Four of the included angles of the 11-fold hexagonal repeat unit are comprised of three 1/11 segments, while the remaining two included angles have five 1/13 segments.

The use of parallelograms as a repetitive device in the Islamic geometric tradition is extremely rare. One such example is from the Khwaja Atabek mausoleum in Kerman (1100-1150) [Fig. 211]. Figure 20 illustrates several repetitive features of this design. Figures 20a and b illustrate two distinct chevron repeat units, each comprised of mirrored parallelograms. The fundamental domains for these repeat units are rotated 180° and then reflected to fill the chevron repeat units. Figure 20c shows a 1/5 decagonal kite repetitive element that must be rotated 180° for edge-to-edge translation symmetry. The fundamental domain for this kite is a 1/10 triangular segment of a decagon that is reflected to fill the repetitive element.

Most motifs with a radial symmetry in Islamic ornament tend to be floral. However, Muslim designers also created many geometric radial patterns; mostly created from one or another of the polygonal systems (although the *sevenfold system* does not appear to have been used for such designs). Radial geometric designs that are applied to the gore segments of domes are a special category within this tradition. Whether on a two-dimensional plane or on the curved surface of a dome, a pattern with radial symmetry is significantly different from patterns that employ translation symmetry to cover the plane. Radial patterns work by dividing a circle into n number of equal divisions, and treating each

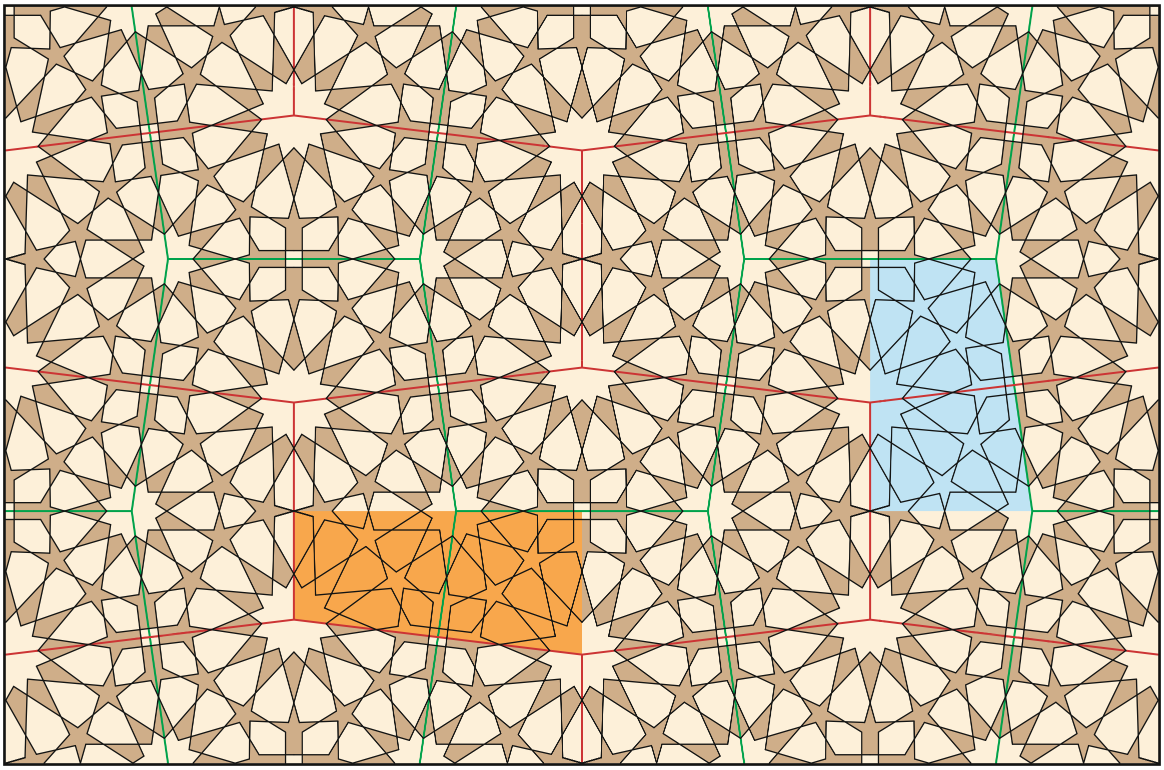


Fig. 18

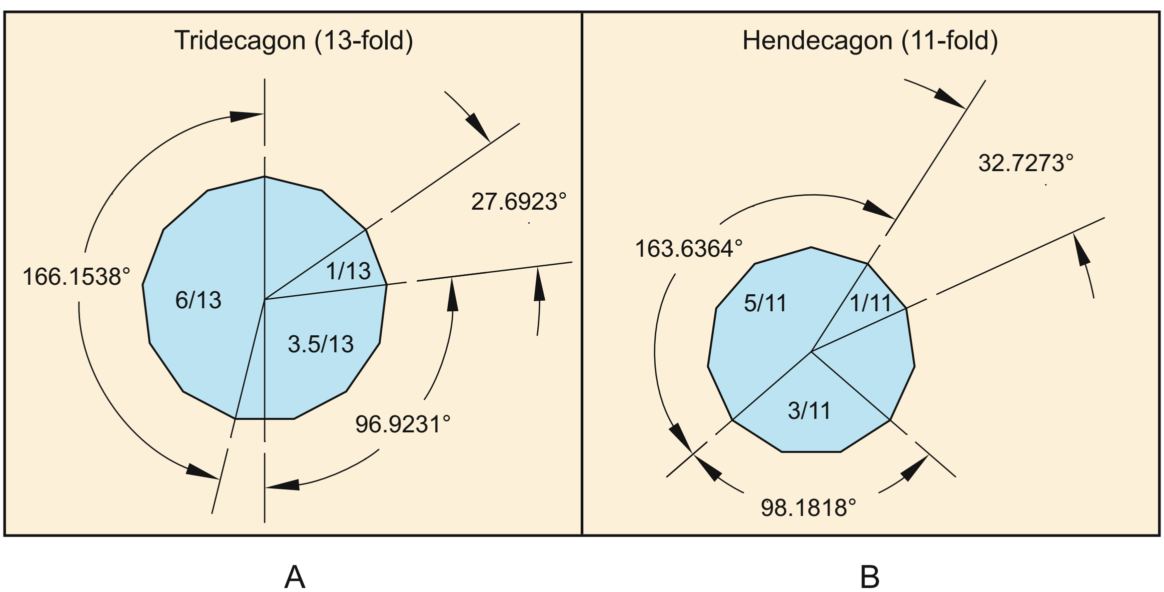


Fig. 19

segment as a distinct repetitive element that is copied and rotated *n* times around the center of the circle. In this way, the pattern is made to repeat through rotation along the radius of the circle. The Islamic conventions for applying radial geometric patterns onto domes most commonly employ 8-, 12-, 16- or 24-fold gore segments. Each of these relates comfortably to the square or octagonal base upon which domes most commonly rested. In his 1925 publication *The Drawing of Geometric Patterns in Saracenic Art*, E. H. Hankin demonstrates a Mughal

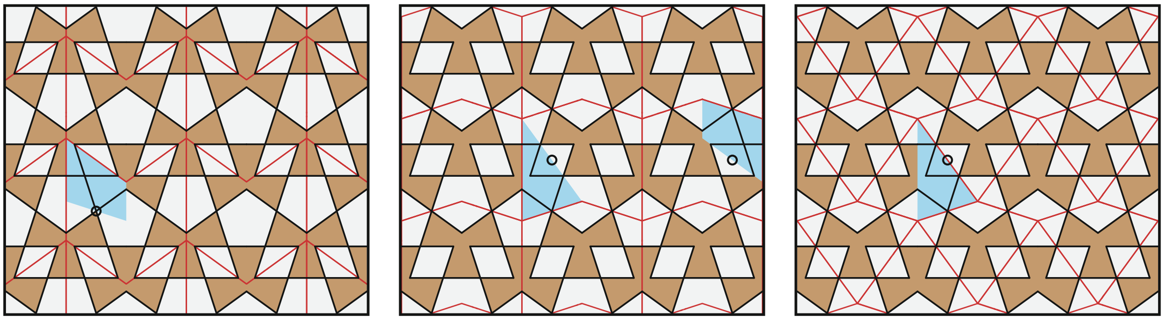


Fig. 20

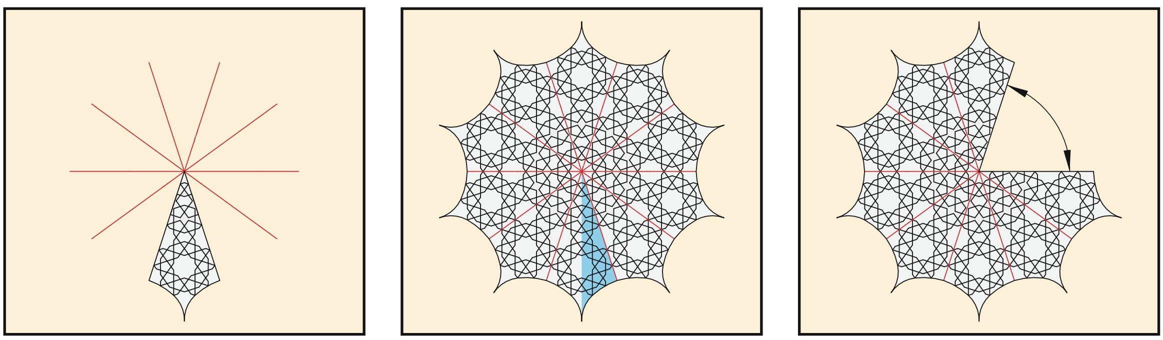


Fig. 21

technique for designing the gore segments of four domes in Fatehpur Sikri, India.[18] Each example that Hankin cites is created from the *fivefold system*, and characterized by the judicious use of ten-pointed stars. Figure 21 illustrates his analysis of the dome in the Samosa Mahal at Fatehpur Sikri, India (sixteenth century). This illustration shows how the geometric pattern is designed to fit the 1/10 division of a circle, and demonstrates how this segment can be arrayed around the central point ten times to create a very satisfactory radial pattern. Figure 21b shows how the fundamental domain of this radial design (blue) is half of the 1/10 segment divided through its central axis and reflected to fill the segment. As reported by Hankin, the Mughal technique for applying such patterns onto the three-dimensional interior surface of a dome called for the removal of two of the ten segments, and adjoining the remaining eight segments into a conical form that could then be applied to the curved surface of the dome with minimal distortion. This technique has the benefit of maintaining the integrity of this type of fivefold pattern even while being applied to a three-dimensional surface. Other than the minimal distortion, the only real change to the pattern is that the central star will have eight points rather than the original ten. A feature of this methodology is the fact that the curvature of the dome is a direct result of the chosen geometric design. This is distinct from the more common approach to designing domical geometric patterns wherein the design is applied to a predetermined gore segment. Each segment of the design from the Samosa Mahal has acute projections at the periphery that, when applied to the dome, extend downward into eight arched pendentives. Historical examples of geometric designs with radial symmetry are far less common than two-dimensional patterns that repeat with translation symmetry. Some of the more interesting examples of two-dimensional radial design are from the flat stellate soffits that were incorporated into Persian *muqarnas* vaults during the Safavid period [Figs. 440 and 441], and from the secondary infill of dual-level designs [Fig. 447]. Figure 22 illustrates two radial designs with tenfold rotation symmetry from the Topkapi Scroll. Figure 22a is an *obtuse* pattern from the ten-pointed star component of a dual-level design from this collection of designs,[19] and Fig. 22b is the full dodecagonal infill of the

[18] Hankin (1925a), Figs. 45–50.

[19] Necipoğlu (1995), diagram no. 29.

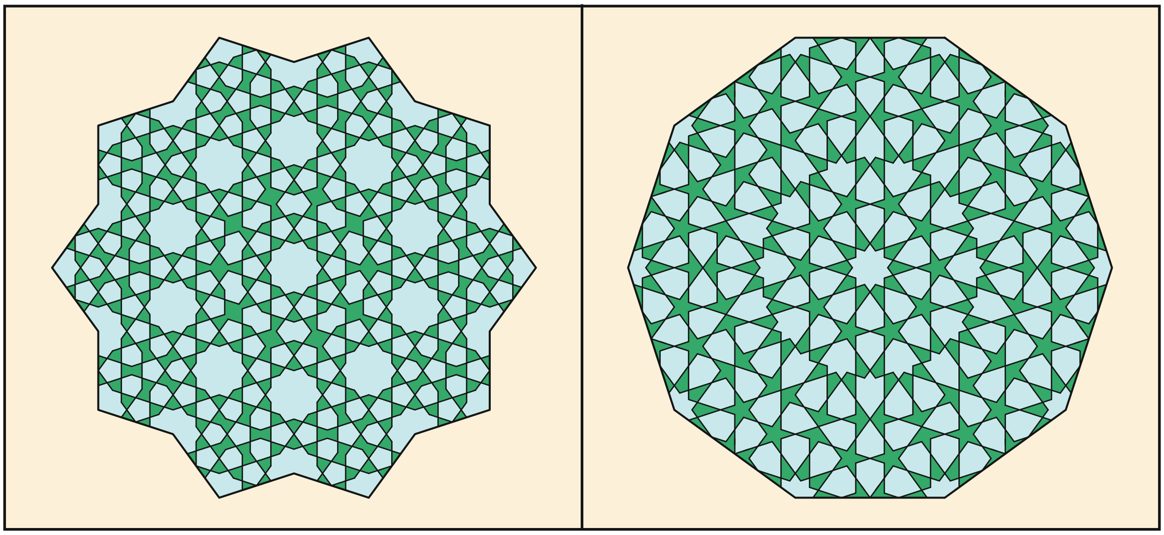


Fig. 22

1/10 segment of an *acute* pattern[20] that was possibly intended for use on a dome—as per the Mughal technique illustrated in Fig. 21. Both designs were produced from the *fivefold system* of pattern generation.

Another means of incorporating unusual symmetrical relationships in Islamic geometric patterns utilizes a particular tessellation of squares and rhombic repetitive units. This variety of tessellation is characterized by the square elements oscillating in orientation, and the rhombi being placed in an alternating perpendicular layout. Designs based upon this configuration of squares and rhombi are orthogonal, but eccentric; and are herein referred to as *oscillating square* patterns. The geometric structure of this variety of Islamic geometric pattern invariably adheres to the *p4g* plane symmetry group. This rather simple geometric repetitive device was occasionally used as ornament in and of itself, and an early example is found in the carved stucco panels of the Khirbat al-Mafjar outside Jericho (eighth century). However oscillating square tessellations can also provide the repetitive structure for more complex Islamic geometric patterns. In this class of design, the angular proportions of the rhombic elements will always inform the geometric characteristics of the completed pattern. Figure 23 illustrates the geometric principle behind two historical oscillating square patterns. Figure 23a illustrates the square-within-a-square motif. In this particular oscillating square tessellation, the rhombi are made up of two contiguous equilateral triangles, and the distribution of squares and rhombi is effectively the 3^2.4.3.4 semi-regular tessellation [Fig. 89]. Oscillating square patterns are characterized by multiple lines of symmetry, leading to a surprising number of equally valid repeat units with translation symmetry for a single design. Figure 23b places diagonal lines (green) within the square elements of this tessellation. This produces a grid comprised of concave octagonal shield shapes (orange) that tessellate through 90° rotation. The fundamental domain (blue) is rotated 4× to populate the minimal square region, and this is reflected 4× to complete a square repeat with translation symmetry. Each of these square repeat units has 16 fundamental domains. However, this is not the minimal repeat unit with translation symmetry. This grid also produces two types of hexagonal repeat unit (brown) with translation symmetry, each comprised of just eight fundamental domains. These are identical except for their respective 90° orientations, and the proportions of each are based upon 105° and 150° included angles. Figure 23c places an alternative set of lines (green) within the oscillating square elements that bisect the midpoints of each set of parallel edges. This creates the dual of the original square and rhombic grid (green) in Fig. 23a, and is similarly comprised of alternating concave octagonal shield repetitive units (orange), although with very different proportions as those from Fig. 23b. The fundamental domain (blue) for this dual grid is likewise a 1/4 segment of the minimal square region that is rotated 4× and reflected 4× to complete the square repeat unit with 16 fundamental domains. This dual grid also produces two types of hexagonal repeat units (brown) that are perpendicularly orientated and comprised of just eight fundamental domains. Like the regular hexagon, these non-regular hexagonal repeat units have 120° at each included angle: the difference being the two edge lengths rather than uniform

[20] Necipoğlu (1995), diagram no. 90a.

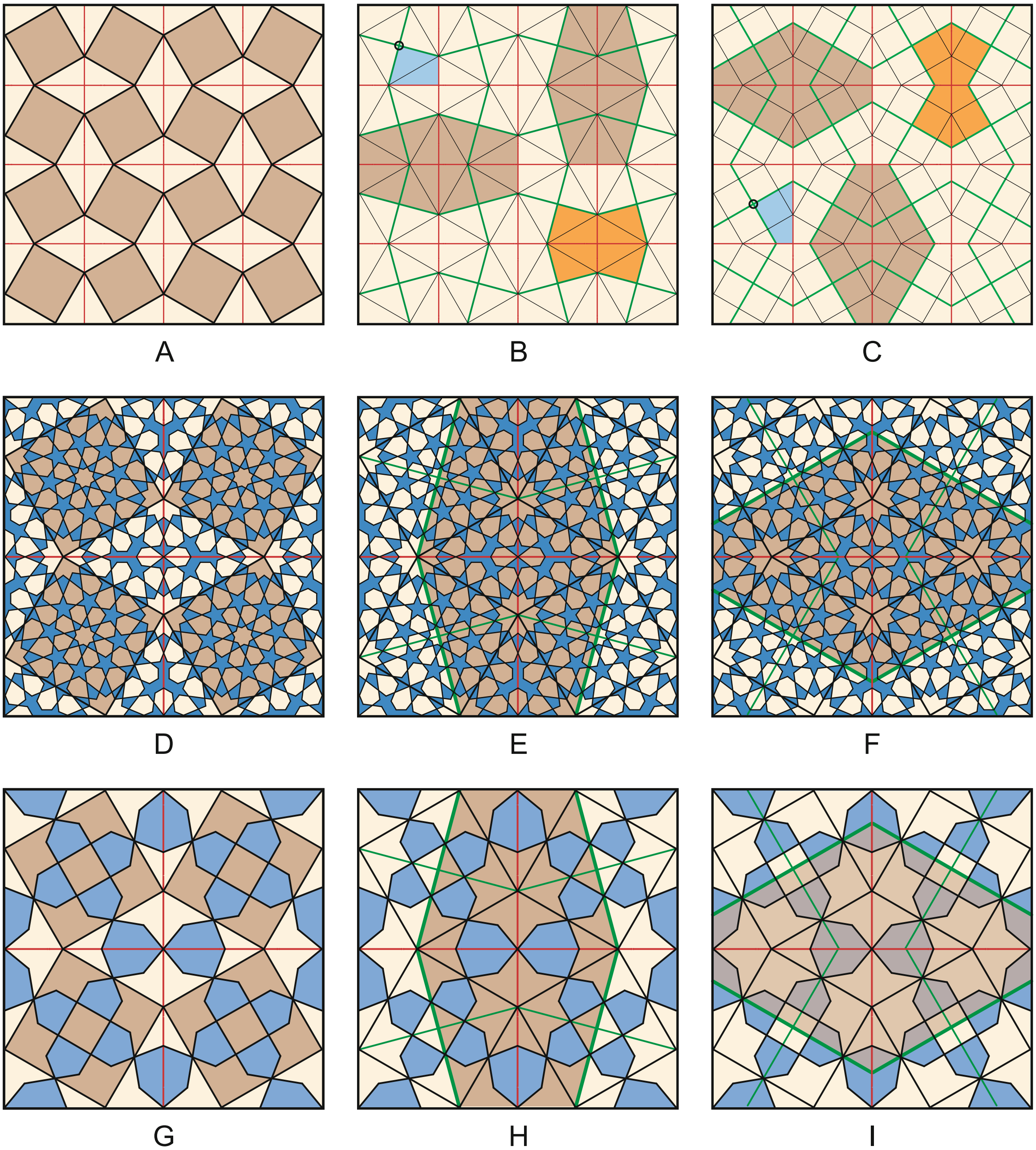


Fig. 23

edges. The two historical designs in this figure are imbued with each of these repetitive characteristics. The pattern in Fig. 23d through **f** is from the exterior stucco ornament of the Mustansiriyah *madrasa* in Baghdad (1227-34), as well as the Topkapi Scroll.[21] Figure 23d emphasizes the oscillating square and rhombic repetitive cells that govern the geometry of this pattern. The applied pattern lines within both the oscillating squares and rhombi are able to fill the plane independently with very acceptable designs, and their combined use in these examples qualifies this example as a hybrid design. The pattern within just the rhombus is a variant of an isometric nonsystematic design with 12-pointed stars at the vertices of the isometric grid [Fig. 321b], while the design within each square cell is a very well-known nonsystematic design that places 12-pointed stars at the vertices of the orthogonal grid and 8-pointed stars at the center of each repeat unit [Fig. 379b]. Figure 23e demonstrates the placement of this pattern within the hexagonal repeat unit with 105° and 150° included angles, and Fig. 23f shows how the pattern fits

[21] Necipoğlu (1995), diagram no. 35.

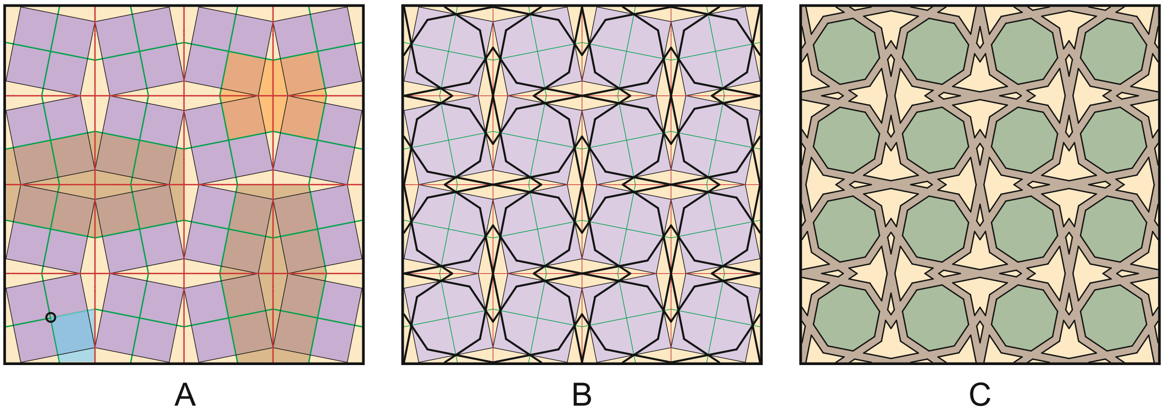


Fig. 24

within the repetitive hexagon with exclusively 120° included angles. The design with non-regular seven-pointed stars in Fig. 23g through **i** is found in several locations, including the Malik mosque in Kerman, Iran (eleventh century), as well as the Topkapi Scroll.[22] Figure 23g places this pattern into the oscillating squares and rhombi. The pattern is composed of 90° angular openings placed at the midpoints of the $3^2.4.3.4$ grid. This produces the non-regular seven-pointed stars that are a primary feature of this design. As with the previous pattern from the Mustansiriyah *madrasa*, the pattern lines in both the square and rhombic elements produce very-well-known designs on their own: the squares making the classic star-and-cross design [Fig. 124b], and the rhombi making a pattern with point-to-point six-pointed stars [Fig. 95c]. Figure 23h shows this pattern placed within the hexagonal repeat unit with 105° and 150° included angles, and Fig. 23i demonstrates the placement of this pattern into the repetitive hexagon comprised of just 120° included angles.

As mentioned, the proportions of the rhombic cells in oscillating square patterns are not restricted to 60° and 120° included angles. Muslim geometric artist discovered that the angles of the rhombic elements within oscillating square tessellations can be adjusted to conform to other polygonal symmetries, thereby introducing the visual characteristics inherent to these forms. Figure 24 demonstrates an oscillating square pattern from the Sultan Han in Aksaray, Turkey[23] (1229). The rhombic repetitive cells in Fig. 24a have 22.5° and 157.5° included angles. The fundamental domain (blue) is rotated 4× to fill the square cell that is reflected 4× to produce a square repeat unit with translation symmetry that is made up of 16 fundamental domains. As with the previous designs, the dual of this tessellation (green) provides for the two perpendicular elongated hexagonal repeat units (brown) comprised of eight fundamental domains. This figure also illustrates the concave octagonal shield element (orange) that requires alternating 90° rotations to cover the plane. And as with the examples in Fig. 23, this oscillating square grid will also repeat with perpendicular hexagonal grids created from diagonal lines placed within each oscillating square (not shown). Figure 24b illustrates how the pattern can be derived from simply placing octagons within each of the oscillating squares and extending the pattern lines until they meet with other extended pattern lines. The specific proportions of the rhombi provide for the pattern lines to extend uninterrupted from octagon to octagon through the center point of each rhombus. Figure 24c is a representation of the historical design with widened pattern lines. Figure 25 illustrates two additional historical examples of oscillating square tessellations that use rhombi that are proportioned differently from that of the double-equilateral triangle. The proportions of the rhombic elements used in Fig. 25a are associated with sevenfold symmetry and can be derived from either the heptagon or the tetradecagon [Fig. 12c], with the acute angles being 1/14 divisions of a circle, and the obtuse angles being 6/14 [Fig. 10c]. The use of this rhombus elegantly provides for the incorporation of regular seven-pointed stars within a pattern matrix that is otherwise orthogonal in structure. This design was used in numerous locations, including the Mirjaniyya *madrasa* in Baghdad (1357), and the Amir Qijmas al-Ishaqi mosque in Cairo[24] (1479-81). Figure 25a illustrates the fundamental domain (blue) that is rotated 4× to populate the square that is then reflected 4× to produce a square repeat unit with translation symmetry. This repeat has 16 fundamental domains and, as with the previous examples, is not the

[22] Necipoğlu (1995), diagram no. 81a.

[23] Schnieder (1980), pattern no. 297.

[24] Bourgoin (1879), pl. 170.

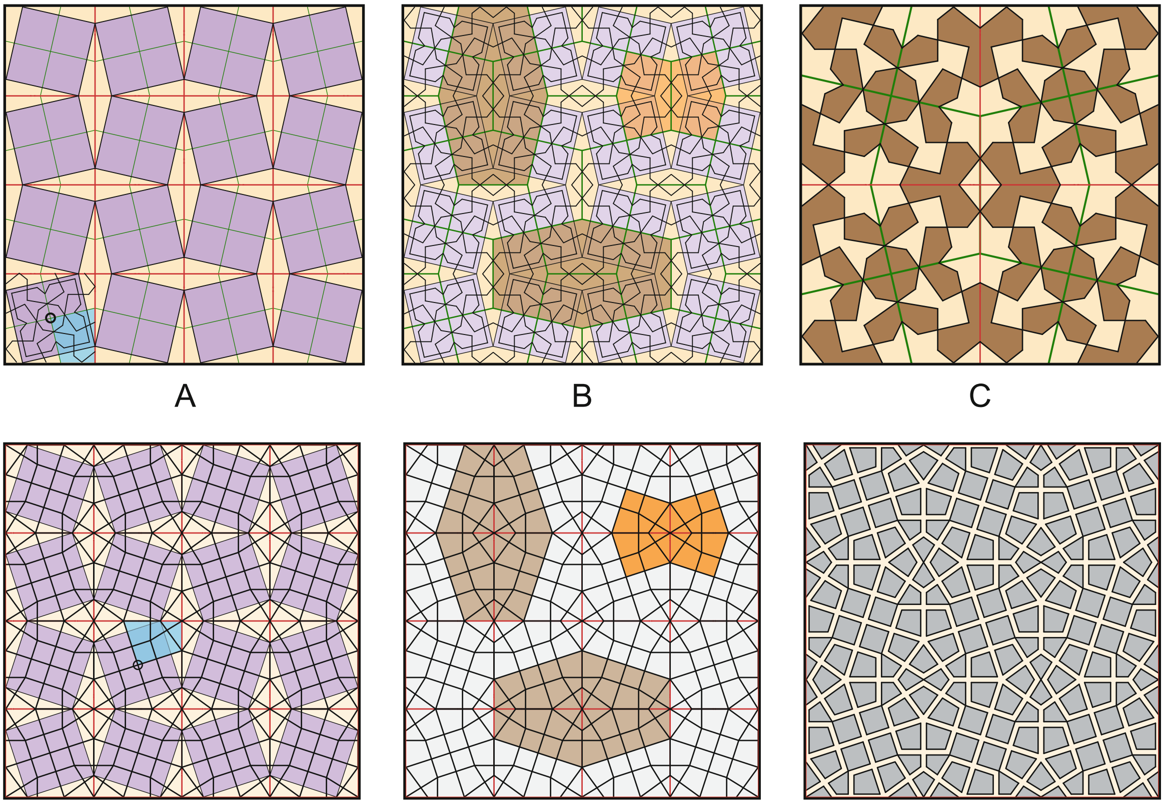


Fig. 25

minimal repeat unit. Figure 25b illustrates the two perpendicular hexagonal grids (green) that, together, are the dual of the square and rhombus tessellation. The two elongated perpendicular hexagons (brown) are the minimal repeat units with translation symmetry, comprised of eight fundamental domains. This figure also illustrates the repetitive shield element (orange) that requires alternating 90° rotations to cover the plane. This pattern will also repeat with the two perpendicularly orientated hexagonal grids created from the diagonal lines applied to each oscillating square (not shown). Figure 25c is a representation of the very successful historical design based upon this repetitive geometric schema. Figures 25d through f illustrate an interesting oscillating square raised brick design from the western tomb tower at Kharraqan in northwestern Iran (1093). This incorporates squares, near-regular pentagons, and near-regular triangles into the pattern matrix. Figure 25d illustrates the oscillating squares located within the orthogonal repetitive element. The fundamental domain (blue) requires rotation 4× to populate each square cell, and this, in turn, is reflected 4× to produce a square repeat unit with translation symmetry. The included angles of the rhombi are 36° and 144°: equaling 1/10 and 4/10 segments of the decagon. The applied pattern lines include lines from the dual grid that bisect the midpoints of the oscillating squares, as well as an arbitrary network of pentagons, squares, and rhombi that complete the design. While visually becoming, the aesthetics of this design are atypical to this tradition. Figure 25e illustrates both orientations of the identical elongated hexagonal repeat unit. These are minimal repeat units with translation symmetry, and comprised of eight fundamental domains. This figure also illustrates the repetitive shield element (orange) that requires alternating 90° rotation to fill the plane. This is comprised of just four fundamental domains. The alternative hexagonal repeat unit produced from diagonal lines within each oscillating square also works as a repeat unit with translation symmetry (not shown). Figure 25f represents the widened line expression of this design as per the historical example.

Although not known to the historical record, a variation of oscillating square designs will make interesting patterns with unique repetitive structures. Figure 26a demonstrates the ability of two varieties of rhombus to tessellate together in a similar manner as the squares and rhombi of oscillating square configurations. In this variation, obtuse rhombi replace the square modules, and acute rhombi are placed on each of the edges of the obtuse rhombi such that they are in a rotational pinwheel arrangement. This can be thought of as a

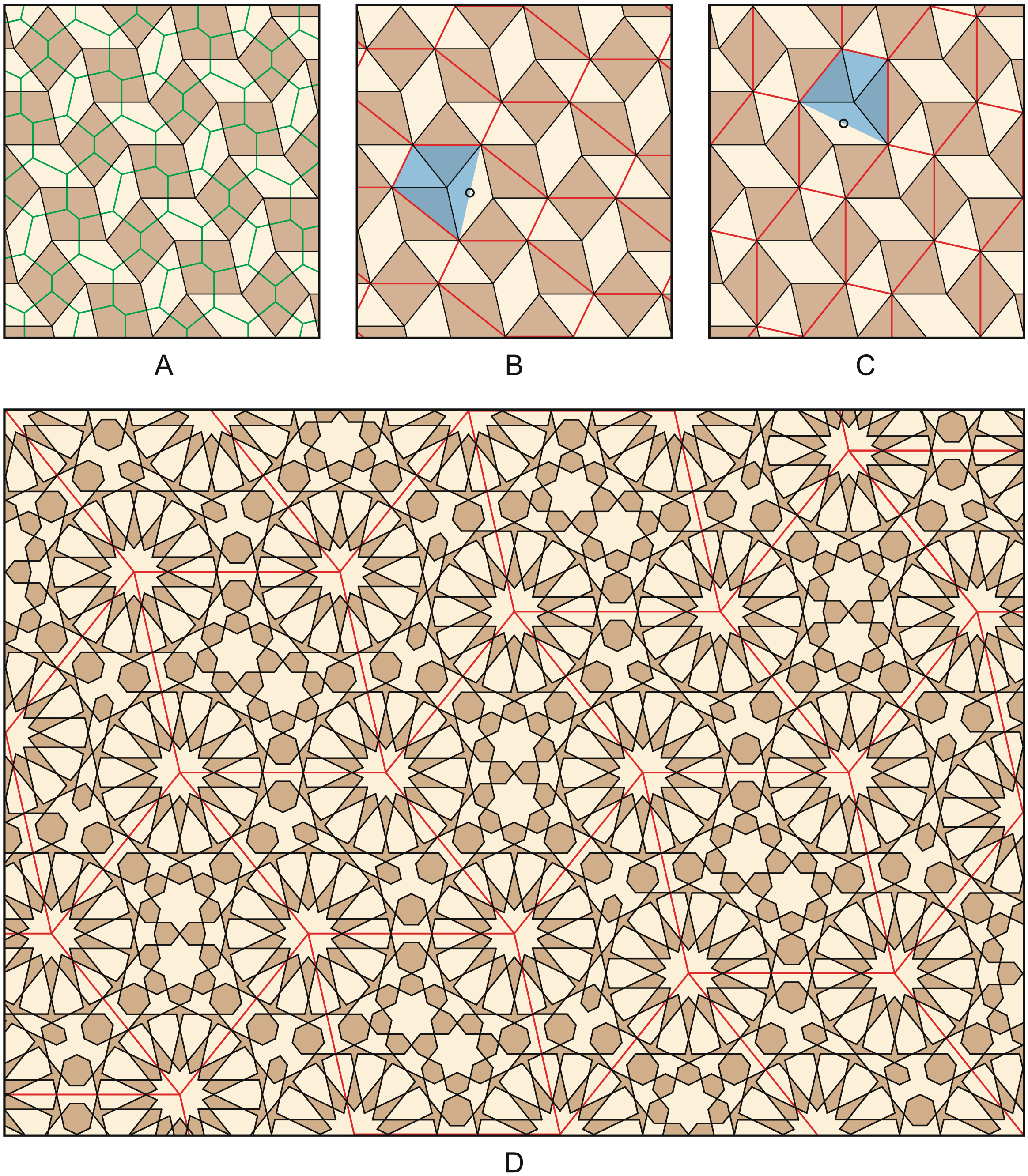


Fig. 26

skewed oscillating square tessellation, and the angular proportions of both the rhombi in this particular example are derived from sevenfold symmetry [Figs. 10a and b]. The dual of this grid (green) creates an interesting tessellation of irregular hexagons. Figure 26b illustrates how each hexagonal repeat unit (red) contains the area of two obtuse rhombi and two acute rhombi. Because of the skewed nature of the rhombic tessellation, the hexagonal repeat does not have reflection symmetry, and the fundamental domain (blue) requires rotation ×2 (point symmetry) to fill the repeat. This changes the *p4g* plane symmetry group typical of oscillating square patterns to *p2*: one of the least common plane symmetry groups among Islamic geometric patterns. Figure 26c shows how the repetitive grid for this rhombic tessellation can also be orientated in a roughly perpendicular direction. This is analogous to the perpendicular hexagonal repeats of standard oscillating square designs. Figure 26d applies a sevenfold geometric pattern to each of the two varieties of rhombus in this unusual tessellation. The combined use of two types of rhombus qualifies this as a hybrid design (by author). Although examples of this variety of hybrid design are unknown within the historical record, this is a successful variation of traditional design methodology. The pattern placed within the obtuse rhombi is from the 'Abd al-Ghani al-Fakhri mosque in Cairo (1418) [Fig. 13a], and the pattern within the acute rhombi is from the Bayezid Pasa mosque in Amasya, Turkey (1414-19) [Fig. 13b].

Other historical examples of particularly fine oscillating square designs include a Khwarizmshahid example from the Zuzan *madrasa* in northeastern Iran (1219) [Fig. 103] [Photograph 39], and a remarkably complex Anatolian Seljuk example from the Huang Hatun complex in Kayseri (1237) created from the *fourfold system A*[25] [Fig. 156]. As with the above-mentioned oscillating square patterns from the Mustansiriyah *madrasa* in Baghdad and the Malik mosque in Kerman, the design from the Huang Hatun complex in Kayseri is also represented in the Topkapi Scroll.[26] Other oscillating square patterns in the Topkapi Scroll include a simple but affective design that incorporates floating squares and rhombi in a matrix of four-pointed stars with swastika centers[27], and a design that is suitable for polychrome *ablaq* inlaid stone that places swastikas inside the square elements and utilizes rhombi with 45° and 135° included angles.[28] While oscillating square patterns are essentially fourfold in that they repeat upon a square grid, their distinctive arrangement of local symmetries creates the topsy-turvy quality that is a hallmark of this unusual category of Islamic geometric design. It is worth noting that this same repetitive schema can be used to create substantially more complex designs than those found within the historical record. This category of design requires the star or regular polygon at the center of the square cell to have fourfold symmetry, and for a set of four additional centers of local symmetry to be located upon the edges of the square cell. These additional centers of local symmetry are required to have bilateral symmetry so that they mirror upon the square edges. This type of geometric construction is relatively unexplored, and lends itself to contemporary pattern making [Figs. 406–411].

Another historical method for introducing seemingly incompatible symmetries into an orthogonal repetitive structure employs the placement of four quadrilateral kites rotated around a central square. As with oscillating square designs, the *rotating kite* motif is mirrored into adjacent square cells, creating an overall reciprocating structure, and like oscillating square designs, this closely related variety of Islamic geometric pattern is invariably of the *p4g* plane symmetry group. This is a well-known ornamental motif in its own right, but was occasionally used as a repetitive stratagem for more complex designs. Figure 27a demonstrates two simple methods of constructing a common form of the rotating kite motif: one from a single square, and the other from a 3 × 3 grid of nine squares. In addition to the two mirrored 90° included angles, these kites have acute angles of 53.1301…° and obtuse angles of 126.8698…°. Figure 27b shows the structural composition of the standard rotating kite design, and examples with this proportion and widened line thickness abound, including a Mughal high-relief red sandstone panel at the Agra Fort (1550). Among the earliest examples are several Ghurid raised brick panels from the exterior of both the western (1167) and eastern (late twelfth century) mausolea at Chisht in Afghanistan. These have wider pattern lines, but are otherwise identical. The fundamental domain (blue) is rotated 4× to create a square that is then reflected 4× to create a square repeat unit with translation symmetry. Figure 27c represents a Seljuk variation from the brickwork façade of the western tomb tower at Kharraqan, Iran (1093). This utilizes the same fundamental domain (not shown). As with oscillating square designs, the included angles of the kites can be adjusted for specific proportions and symmetries. The proportions of these two examples are the most commonly found within the historical record, and are characterized by the length of the square elements being equal to the short edges of each kite. Figure 28 illustrates two rotating kite patterns that utilize a kite and square tessellation with 60° and 120° included angles accompanying the two obligatory 90° angles. Rather than being used as the design itself, these two examples make use of this repetitive schema to construct a far more elaborate design. Just as the included angles of the rhombic elements in oscillating square patterns can be adjusted to conform to *n*-fold symmetry, the acute and obtuse angles of the kites in rotating kite designs can also be adjusted to accommodate local symmetries that are ordinarily incompatible with orthogonal patterns. While the 90° included angles of the kite are invariable, the *n*-fold symmetry of the kite's acute and obtuse included angles is required to be even numbered, thus imposing fourfold reflective symmetry on the pattern elements centered upon the vertices of each kite's acute and obtuse included angles. This includes *n*-fold symmetries that are not divisible by 4, such as 6 and 10. Figure 28a is from a stone *jali* screen at the Taj Mahal in India (1632-48). The 60° and 120° included angles of each kite provide the sixfold local symmetry at each vertex of the orthogonal grid. This allows for six-pointed stars to be located at each vertex of the kite's acute and obtuse angles, and the orientation of these stars is rotated by 90° from alternating vertices. The design in Fig. 28b is from the Topkapi Scroll.[29] This employs the

[25] Schneider (1980), pattern no. 330.

[26] Necipoğlu (1995), diagram no. 61.

[27] Necipoğlu (1995), diagram no. 41.

[28] Necipoğlu (1995), diagram no. 69b.

[29] Necipoğlu (1995), diagram no. 59.

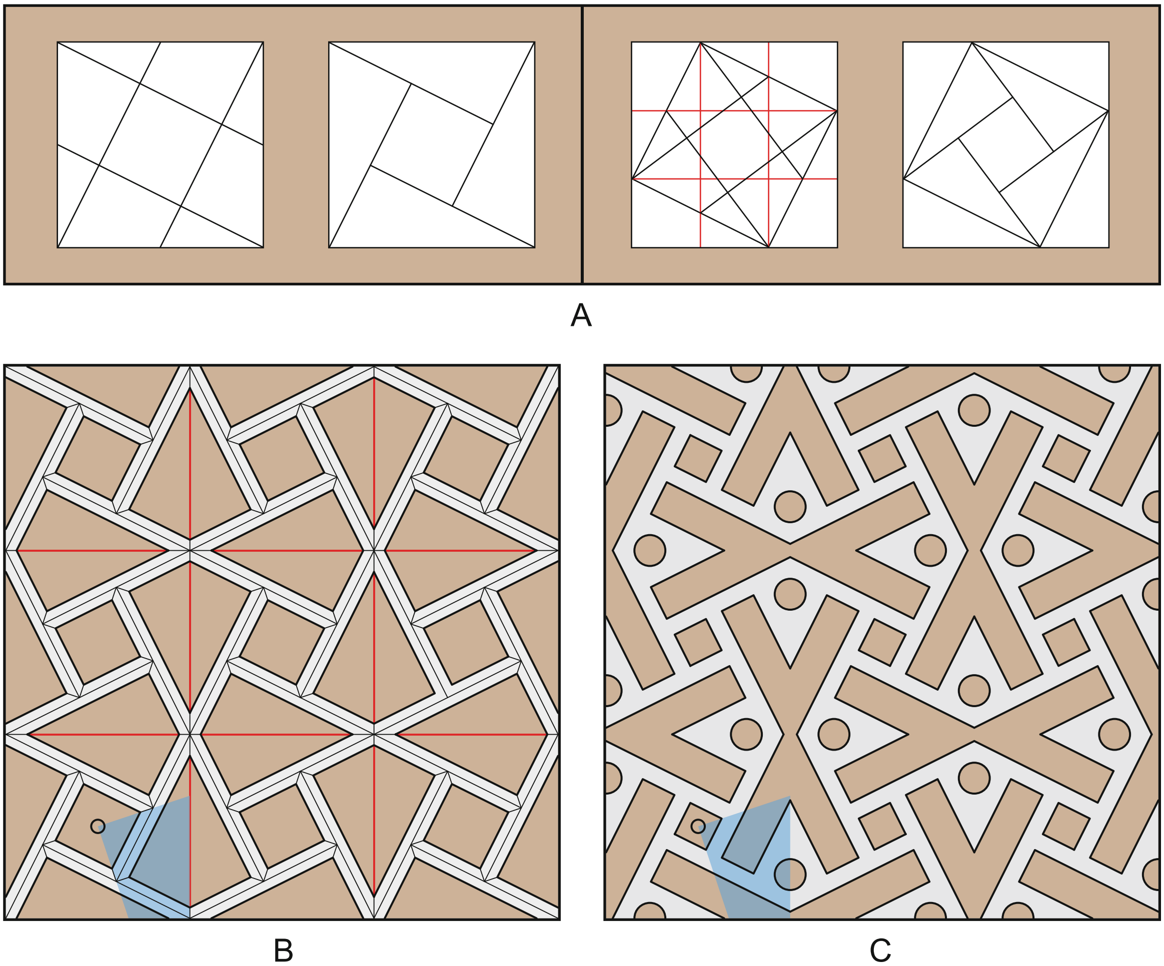


Fig. 27

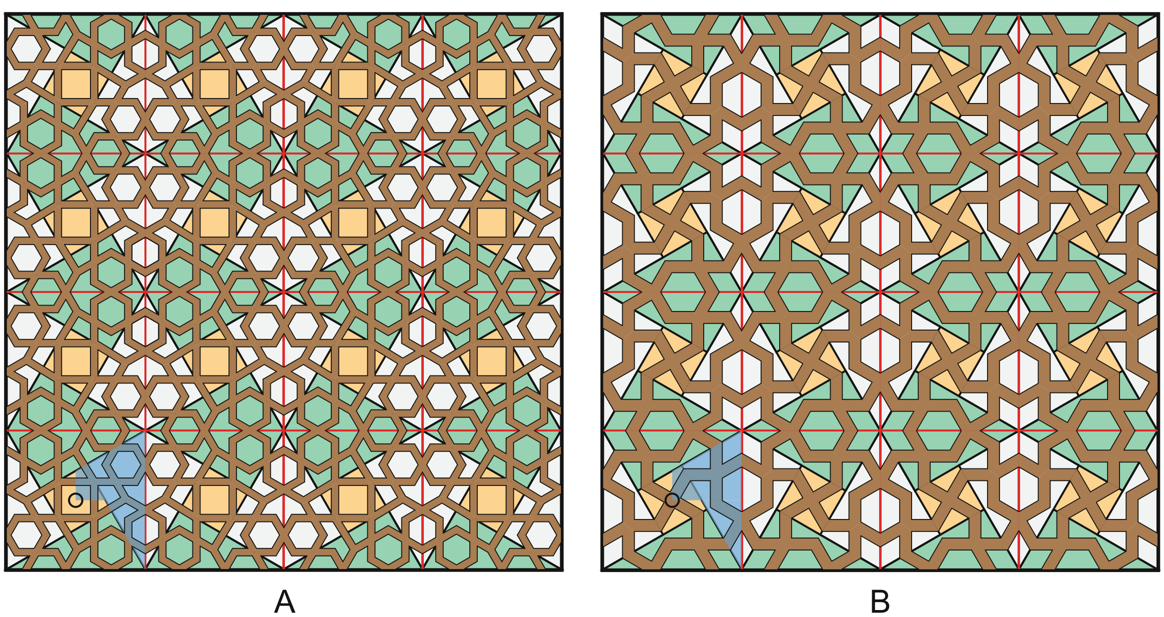


Fig. 28

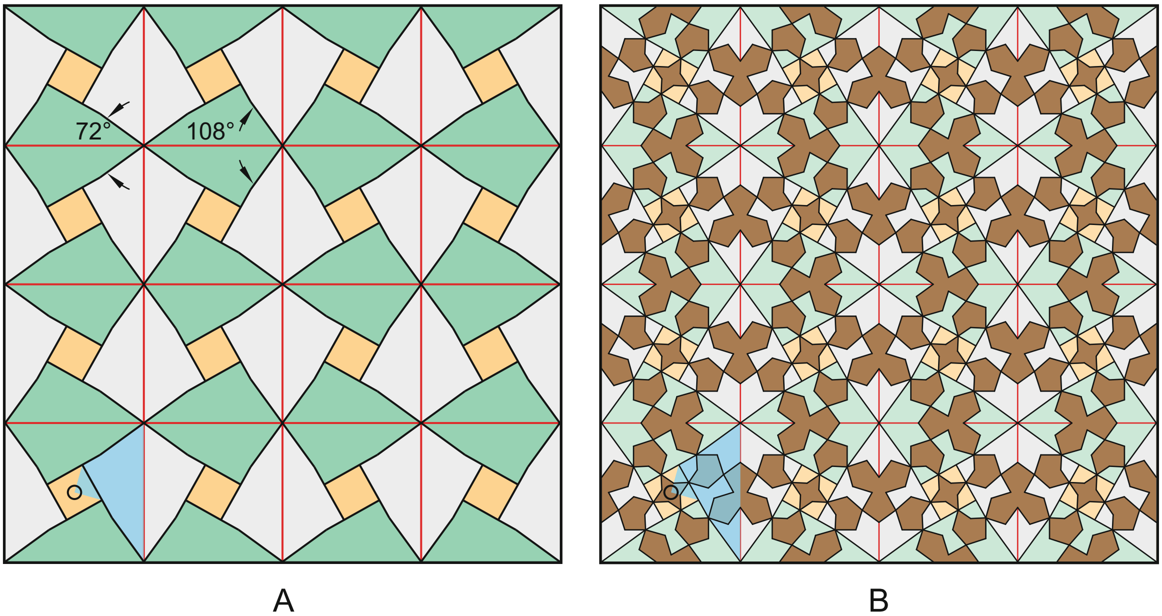


Fig. 29

same repetitive schema, and is also characterized by an alternating distribution of six-pointed stars. The fundamental domain for each of these designs (blue) is rotated 4× followed by reflection 4× to fill the square repeat with translation symmetry. Figure 29 illustrates a design based upon the repetitive structure of rotating kites that has ten-pointed stars placed at the vertices of the orthogonal grid, and four-pointed stars within the square elements of the tessellation. This unusual pattern is also from the Topkapi Scroll.[30] Figure 29a indicates the 72° and 108° included angles of the kites that correspond to tenfold symmetry. Each of the lines emanating from the square elements are slightly kinked so that they are not collinear, with 6.7783...° off 180°, providing each kite with six sides rather than four. While somewhat forced, this allows for the tenfold symmetry of the acute and obtuse angles to combine with a large fourfold center within each square cell, which in turn produces a four-pointed star that is balanced with the other elements within the pattern matrix. This example is testament to the flexible methodological practices employed by artists engaged in this tradition. Figure 29b shows the design from the Topkapi Scroll along with its repetitive schema. As with the six-pointed stars from the previous example, the ten-pointed stars are placed in 90° alternating orientation at the vertices of the orthogonal grid. As with other examples of this variety of pattern, the fundamental domain for this design (blue) rotates 4× followed by reflection 4× to fill the square repeat with translation symmetry.

2.3 Classification by Numeric Quality

Another means of classifying Islamic geometric patterns takes into account their prevalent numeric qualities. Because of the variables within this design tradition, this type of classification requires descriptive text rather than a singular nomenclature. When categorizing geometric patterns from this perspective, the numbers of points found in the characteristic star forms with *n*-fold rotation symmetry are particularly significant. The least complex and easiest to describe are those patterns with only a single variety of primary star; for example, the classic star-and-cross pattern that can be described as a *fourfold pattern, with point-to-point eight-pointed stars that repeat upon an orthogonal grid*: or more concisely, 8s on squares [Fig. 3]. As patterns become more complex, the identification of their numeric qualities becomes a useful tool for differentiating the particular attributes of a given design, as well as qualifying the scope and potential within this tradition overall. In addition to star forms, many patterns will incorporate regular polygons as key elements of the design that are located at the vertices of the primary grid or its dual. In abbreviating the numeric description of a given design that includes such polygons it is useful to follow the nomenclature of Gerd Schneider[31] by using Roman numerals for distinguishing these primary regular polygons. This is especially helpful in differentiating between regular polygonal features and stars with *n*-fold

[30] Necipoğlu (1995), diagram no. 72c.

[31] Schneider (1980).

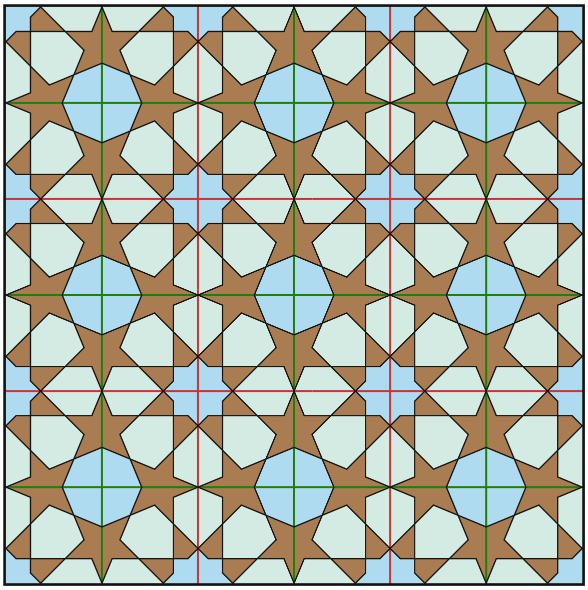


Fig. 30

symmetry. Figure 30 illustrates a widely utilized fourfold design comprised of eight-pointed stars and regular octagons. Describing this pattern as having *fourfold symmetry, with eight-pointed stars placed at the vertices of the orthogonal grid and octagons upon the vertices of the dual grid*, or simply 8s on squares/VIIIs at center, does not uniquely apply to this pattern alone, but identifies it within a category into which only a select number of other patterns fall [Figs. 173a, 175a, 176b, 177a, 177c, etc.]. Figure 31 illustrates a fourfold design that repeats on a rhombic grid (red) with 45° and 135° angles. This was used in several locations historically, including: the Lower Maqam Ibrahim in the citadel of Aleppo, Syria; and the Izzeddin Kaykavus hospital and mausoleum in Sivas, Turkey (1217). This design places eight-pointed stars on the vertices of a rhombic grid, and octagons upon the vertices of the hexagonal dual grid (green): or 8s on rhombic vertices/VIIIs on hexagonal dual vertices [Fig. 181]. This example also illustrates how the duals of rhombic grids are always hexagonal grids.

Not all Islamic geometric patterns employ star motifs. Some are composed of a repetitive field of polygonal forms. Such *field patterns* are most commonly made up of either threefold or fourfold symmetry, although fivefold field patterns are also well known, and especially appealing. Figure 32 shows a well-known threefold pattern comprised of two sizes of hexagons, the larger placed at the vertices of the isometric grid and the smaller upon the vertices of the hexagonal dual grid: or VIs on triangle/smaller VIs at center [Fig. 96d]. Figure 33 illustrates a fourfold field pattern comprised of two sets of differently sized octagons, one set placed upon the vertices of the orthogonal grid, and the other on the vertices of the orthogonal dual grid: or simply, VIIIs

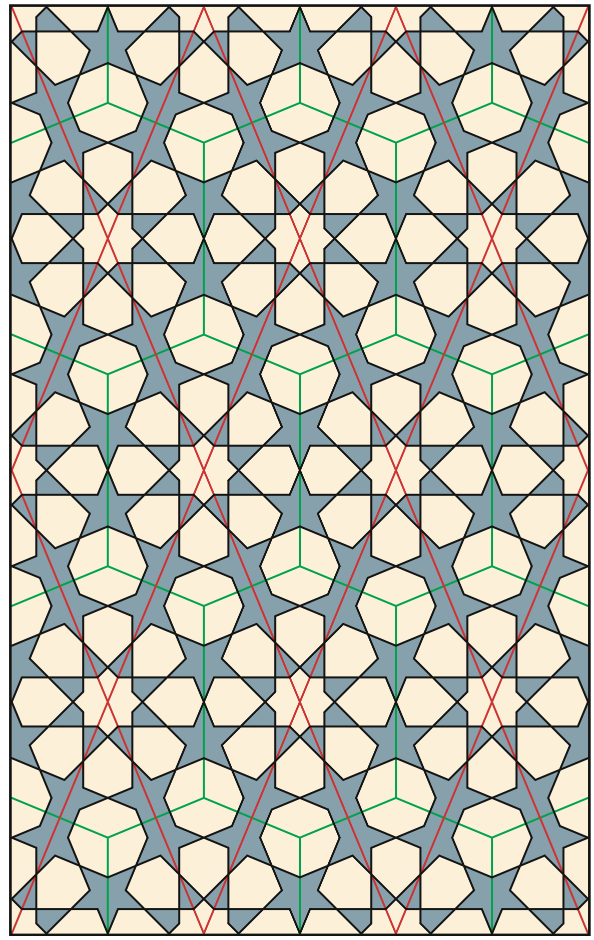


Fig. 31

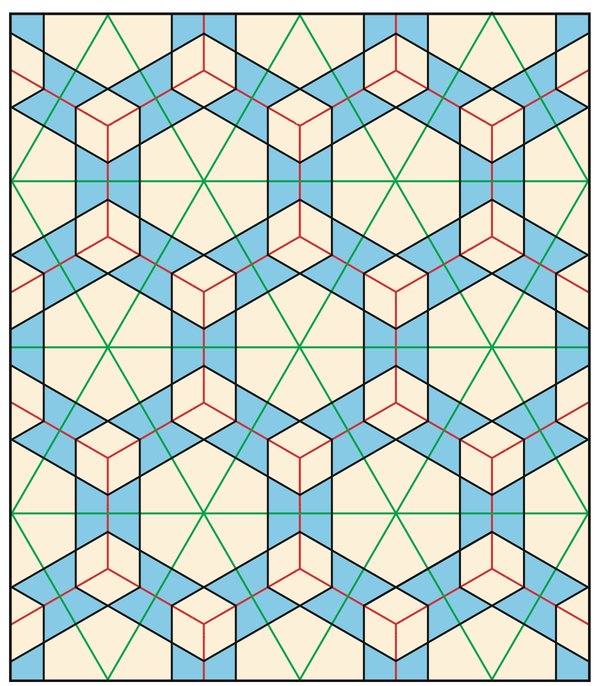


Fig. 32

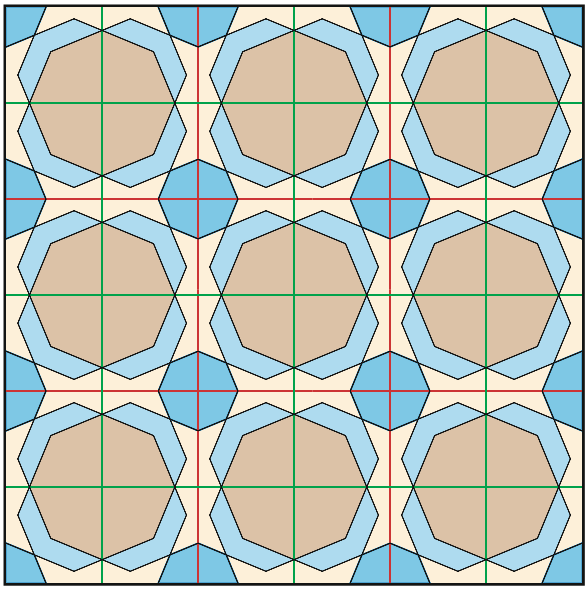


Fig. 33

on square/smaller VIIIs at center. This was used in the celebrated Baghdad Quran (1001) produced by Ibn al-Bawwab [Photograph 6] [Figs. 127c and 128d]. Figure 34 illustrates a fivefold field pattern from the Great Mosque at Malatya (1237-38). This design can be described as a matrix of regular pentagons, concave octagonal shields, kites, and decagonal hourglass figures that will repeat upon several alternative grids with translation symmetry. These include a rectangular grid (red); the rectangular dual of this grid (green), both with eight fundamental domains; and a grid of hexagonal repeat units (brown) made up of just four fundamental domains (blue). The lack of higher order polygons or stars at the vertices of the different repetitive cells makes it more difficult to ascribe an abbreviated description with the tools discussed thus far. To populate the rectangular repeat units, the fundamental domains are rotated 2× and then reflected 4×, and to fill the hexagonal repeat unit the fundamental domain is simply reflected 4×. The plane symmetry group is *cmm*. The lack of stellar centers and the similarity in size of the polygonal elements give this example a pleasing homogeneous aesthetic that is a common quality of fivefold field patterns [Fig. 220].

As discussed previously, there is a direct corollary between the *n*-fold rotational symmetry at the vertices of a repetitive grid and the numeric quality of geometric star patterns. With threefold patterns, the vertices of the isometric grid support the application of stars with *n*-fold rotational symmetry that are multiples of 6. The vertices of the hexagonal dual grid similarly support stars whose points are multiples of 3. The simplest threefold star patterns employ six-pointed stars; and more complex designs will have higher numbered stars, such as 9, 12, and 15. The most

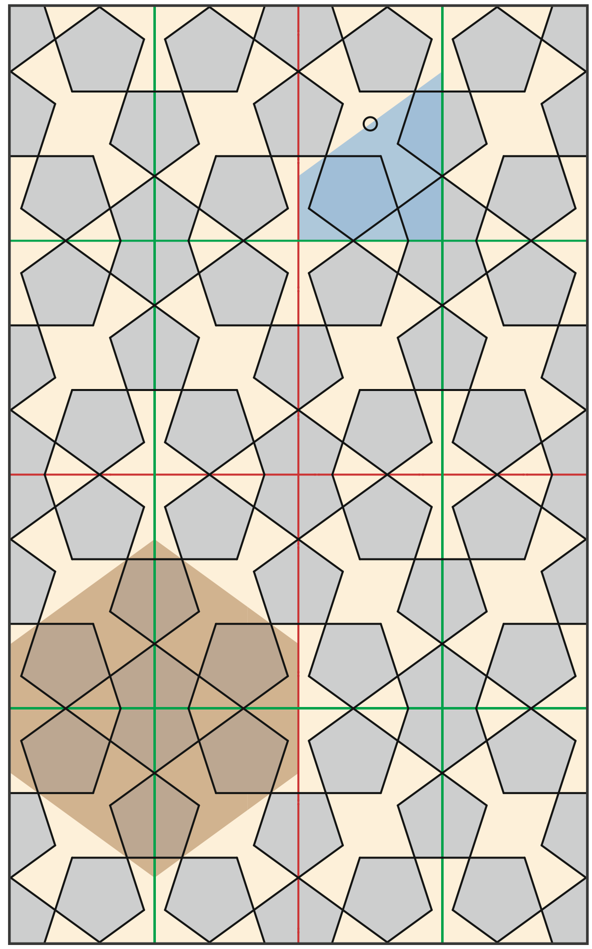


Fig. 34

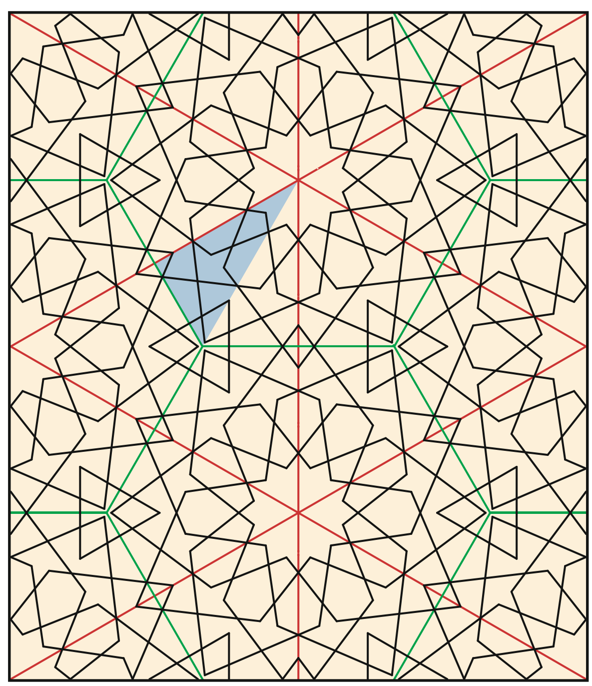


Fig. 35

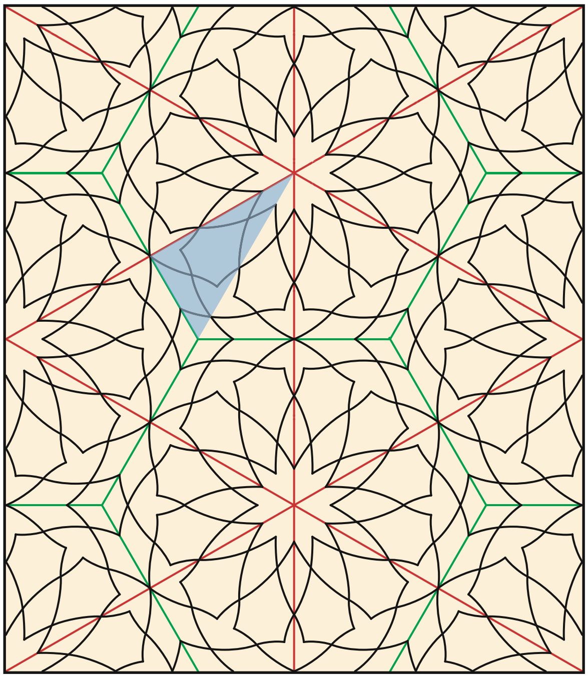


Fig. 36

common of the more complex threefold designs employ 12-pointed stars placed at the vertices of the isometric grid. Figure 35 is just such a design, and can be concisely described simply as 12s on triangle, for which it is one among many designs that fit this simple description [Figs. 300, 320, 321, etc.]. This example was used above the entrance to the tomb of Umar al-Suhrawardi in Baghdad (1234) [Fig. 300a, *two-point*]. Figure 36 shows a similar design [Fig. 321j], but with a curvilinear treatment. This is from a Turkish miniature (1558) painted during the reign of Süleyman the Magnificent.[32] This is a threefold curvilinear pattern, with 12-pointed stars at the vertices of the isometric grid: or simply curvilinear 12s on triangle. While less frequently used, patterns with nine-pointed stars at their repetitive vertices are particularly interesting. Figure 4 is an example of this type of pattern from the Great Mosque at Malatya (1237-38) comprised of threefold symmetry, with nine-pointed stars at the vertices of a rhombic grid: or simply 9s on rhombus [Fig. 311]. As mentioned, threefold patterns with *n*-pointed stars that are higher multiples of 6 and 3 were also widely used. Figure 37 shows an exquisite design with 24-pointed stars in the vertices of the isometric grid and 7-pointed stars within the field: or 24s on triangle/7s in field. This pattern was executed in the carved stone relief

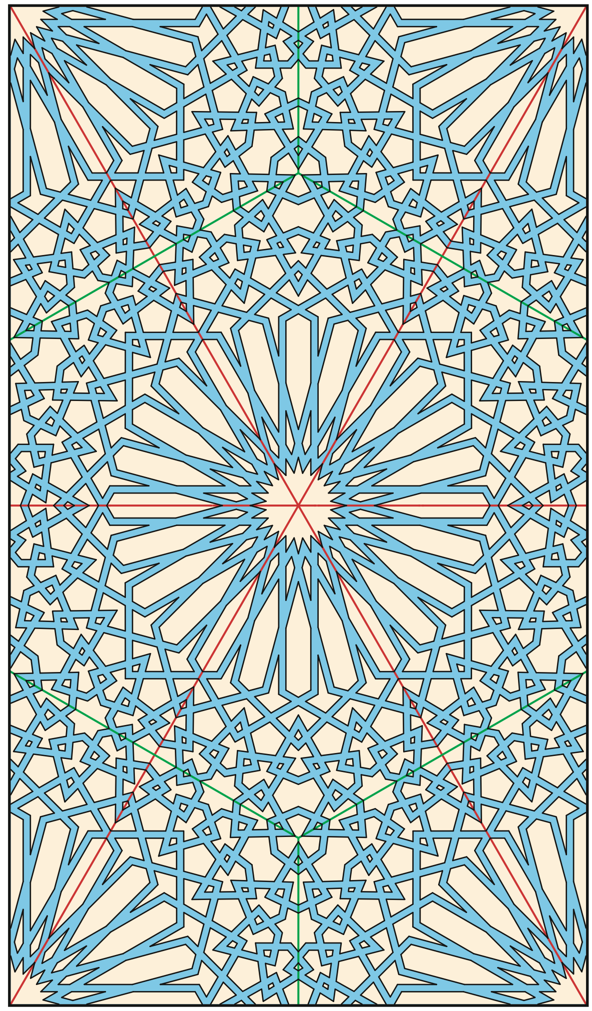


Fig. 37

of the portal at the Nalinci Baba tomb and *madrasa* in Konya, Turkey (1255-65), and in the cut-tile mosaic *mihrab* niche at the Esrefoglu Süleyman Bey mosque in Beysehir, Turkey (1296-97) [Fig. 327] [Photograph 44].

Numeric description becomes especially relevant when differentiating patterns with more than a single region of local symmetry. Figure 38 is a classic threefold compound pattern used throughout the Islamic world that uses 12-pointed stars set upon the vertices of the isometric grid, and 9-pointed stars at the vertices of the hexagonal dual grid: or 12s on triangle/9s at center [Fig. 346a]. Figure 39 illustrates a more complex threefold compound pattern from a carved stone lintel at the Qartawiyya *madrasa* in Tripoli, Lebanon (1316-26). This pattern has 12-pointed stars at the vertices of the isometric grid and 15-pointed stars at the vertices of the hexagonal dual grid: or simply, 12s on triangle/15s at center [Fig. 355d].

Similar to threefold designs, the application of stars to the vertices of the orthogonal grid, as well as to the center of each square repeat unit, will invariably exhibit *n*-fold

[32] Süleymanname*: Presentations of gifts to Süleyman the Magnificent on the occasion of the circumcision of his sons Bayezid and Cihangir in 1530* by Ali b. Amir beg Sirvani. Topkapi Museum, Istanbul TKS H. 1517. See: Rogers and Ward (1988), 45c (f. 360a).

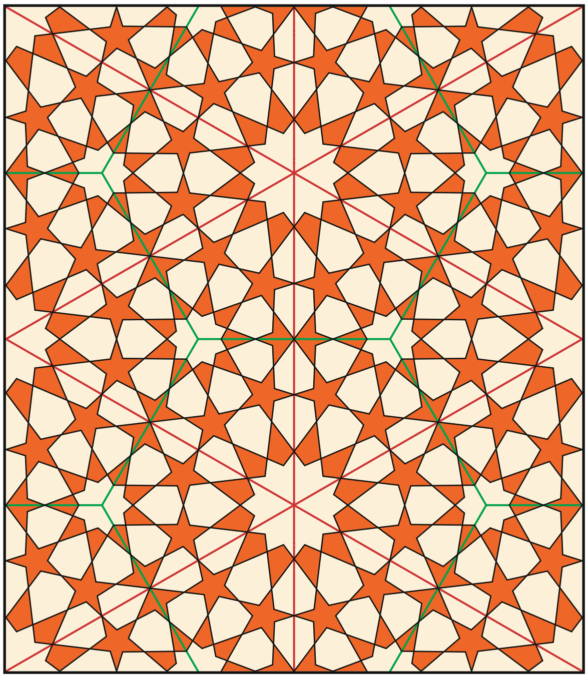


Fig. 38

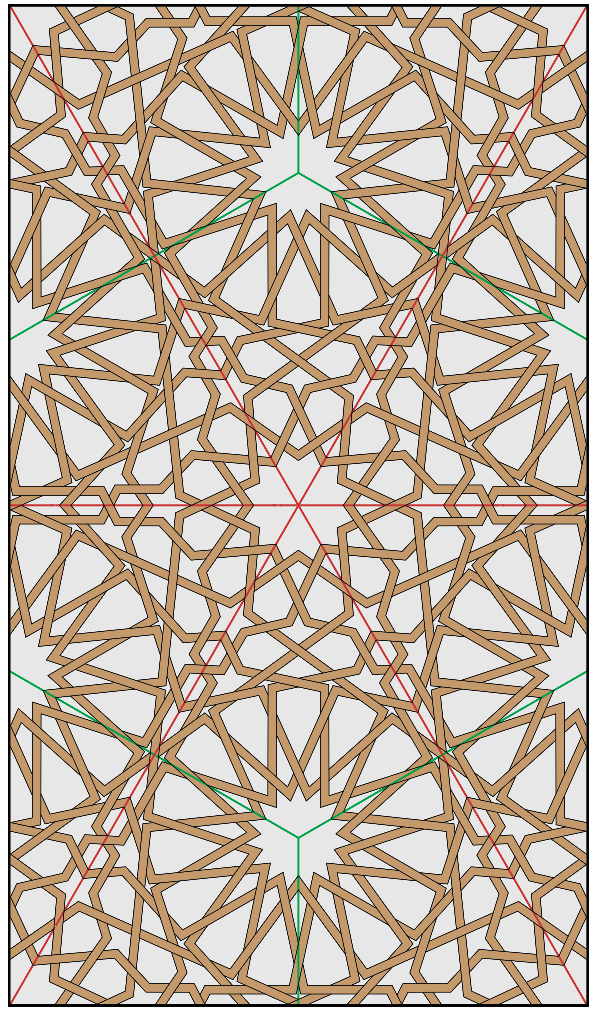


Fig. 39

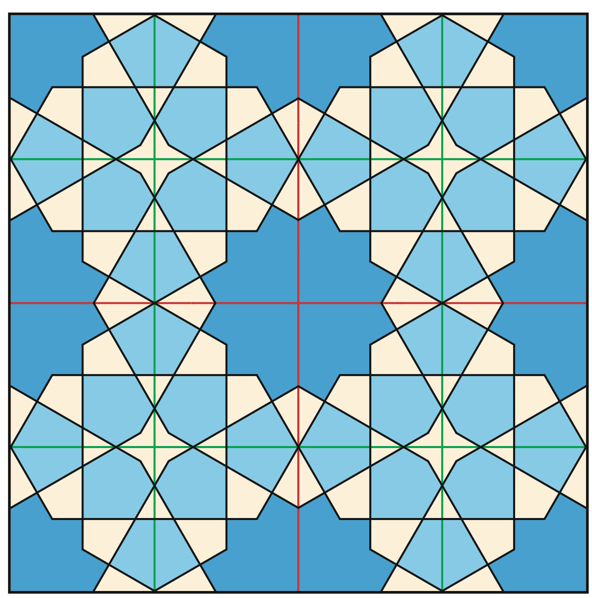


Fig. 40

rotational symmetry that is a multiple of 4. In this way, patterns with 8-, 12-, and 16-pointed stars are very common, and higher order stars, such as those with 24 points, are not unusual. Figure 40 is an illustration of a fourfold pattern that was used throughout the Islamic world and is made up of 12-pointed stars placed at the corners of a square repeat unit with a 4-pointed star placed at the center of the repeat: or simply, 12s on square/4s at center [Fig. 113a]. Figure 41 illustrates an orthogonal design with 16-pointed stars at the vertices of the orthogonal grid with octagons at the vertices of the dual grid: or 16s on square/VIIIs at center. This fine design was used to illuminate a Mamluk Quran commissioned by Sultan Sha'ban in Cairo[33] (1369) [Fig. 344d].

Many of the more complex fourfold geometric patterns will incorporate higher order star forms at the vertices of both the orthogonal grid and its orthogonal dual: each constrained by the same multiple-of-four numeric mandate. Figure 42 shows a variant of a compound pattern with 12-pointed stars at the vertices, and 8-pointed stars at the center points: or just 12s on square/8s at center. This particular version of this well-known design is located at the Kale mosque in Divrigi (1180-81) [Fig. 379b]. Figure 43 is a fourfold compound pattern with 16-pointed stars at the vertices and 8-pointed stars in the centers: or 16s on square/8s at center. This example was used in the Quran of Uljaytu[34]

[33] Cairo, National Library, 7, ff. IV-2r.

[34] This Ilkhanid Quran is in the National Library in Cairo: 72, pt.19.

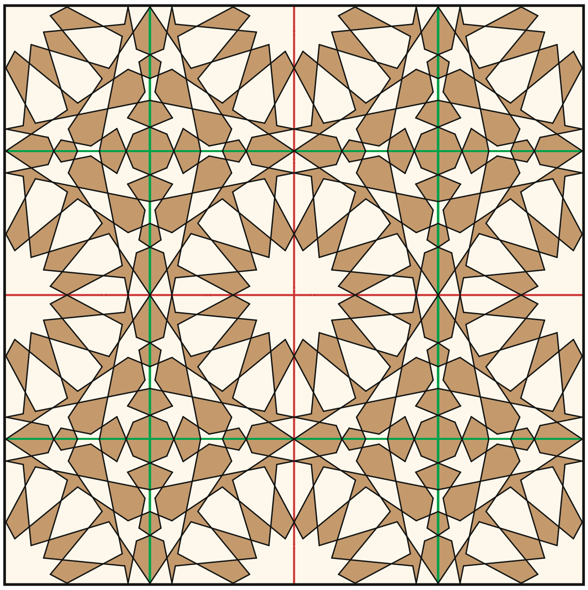


Fig. 41

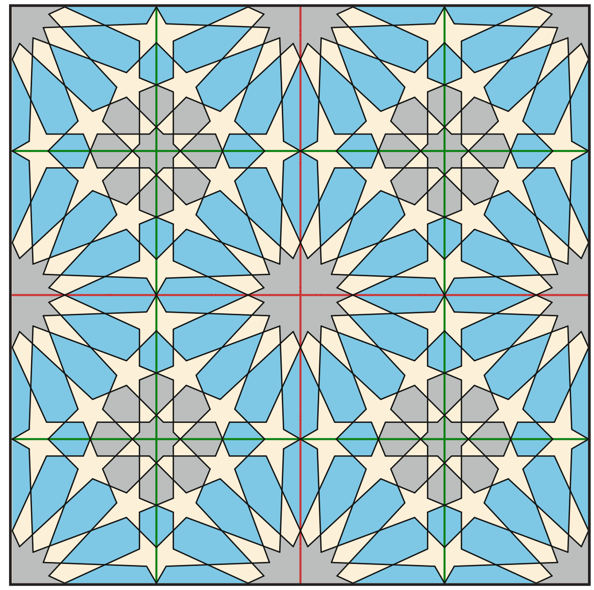


Fig. 43

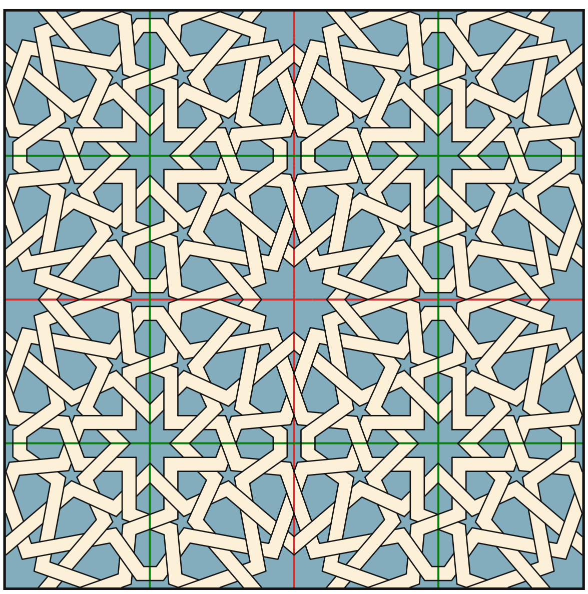


Fig. 42

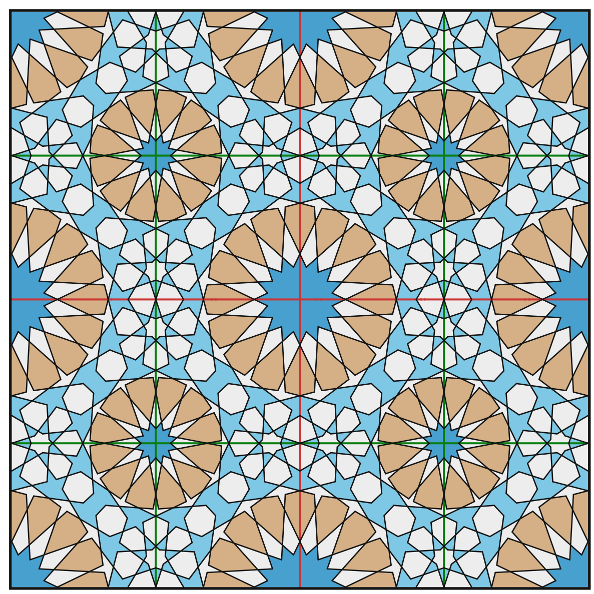


Fig. 44

(1313), written and illuminated by ’Abd Allah ibn Muhammad al-Hamadani [Fig. 389a]. Figure 44 shows a considerably more complex fourfold compound pattern with 16-pointed stars at the vertices of the orthogonal grid and 12-pointed stars at the vertices of the dual grid: or 16s on square/12s at center. This beautiful pattern was used on the minaret of the Mughulbay Taz mosque in Cairo (1466) [Fig. 396b]. Figure 45 shows a fourfold design with 20-pointed stars at the vertices of the orthogonal grid, and 8-pointed stars in the center of each square repeat unit: or 20s on square/8s at center. However, it is relevant to further note that the eight-pointed star at the center of the repeat unit is an arbitrary feature. This exceptional Sa’did pattern is from the Badi’ Palace in Marrakesh, Morocco (1578-1594). These examples are but a few of the vast number of fourfold designs with two primary regions of local symmetry employed frequently within the tradition of Islamic geometric patterns. It is worth noting that both of the previous examples employ seven-pointed stars within their pattern matrix, but these have not been included in categorizing according to their primary stars. This is due to the fact that in both cases the seven-pointed stars are not regular, are not placed upon nodal centers, and are, therefore, not primary star forms.

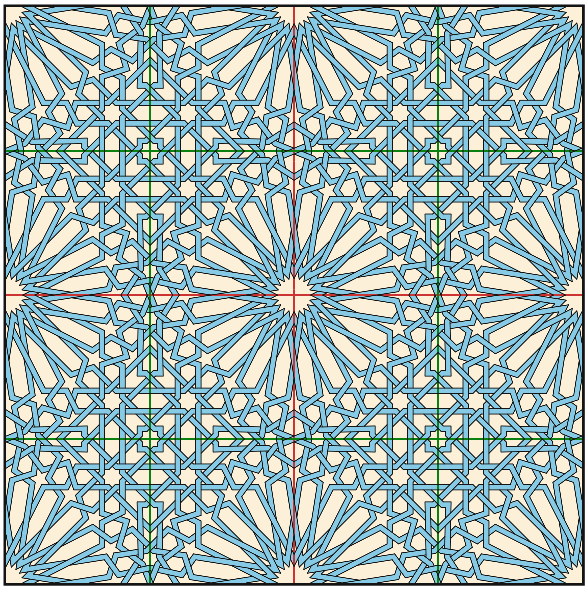


Fig. 45

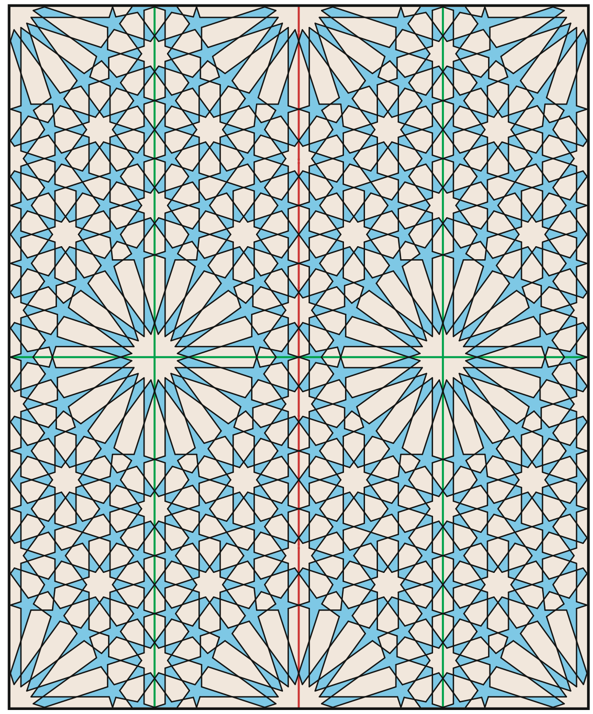


Fig. 46

Historically, star patterns with fivefold symmetry are typically limited to a single variety of primary star: ten-pointed. Because these patterns are derived from the *fivefold system* they are limited by the polygonal modules that comprise this system, including the decagon as the generative module for the ten-pointed stars. Occasionally, 20-pointed stars were incorporated into patterns created from this system: creating fivefold designs with two varieties of primary star. Figure 46 is a very successful design that places 20-pointed stars at the vertices of a rectangular grid, 20-pointed stars at the vertices of the rectangular dual grid, and a network of 10-pointed stars upon the repetitive edges and within the field of the design: or more concisely, 20s on rectangle/20 at center/10s on edges/10s in field [Fig. 268]. As with many, but not all, patterns that repeat with a rectangular grid and have the same star at the center of the repeat as at the vertices, the pattern within the repeat is exactly the same as that of the dual. This feature can be described as *self-dualing*. This outstanding fivefold pattern was employed in the ornament of the Bu 'Inaniyya *madrasa* in Fez (1350-55). Figure 13a illustrates a Mamluk pattern from the 'Abd al-Ghani al-Fakhri in Cairo (1418) that is created from the *sevenfold system*, and characterized by two varieties of primary star: the 14-pointed stars located on the vertices of the rhombic grid, and the 7-pointed stars placed upon the vertices of the hexagonal dual grid. This can be abbreviated as 14s on rhombus/7s on hexagonal dual [Fig. 292a].

Among nonsystematic designs with two varieties of primary star are those that are neither threefold nor fourfold, and utilize other repetitive structures such as rectangular grids and irregular hexagonal grids. Figure 16 shows a design from the Great Mosque of Aksaray in Turkey (1150-53) that has 12-pointed stars at the vertices of its rectangular grid and 10-pointed stars at the vertices of the rectangular dual grid: or simply, 12s on rectangle/10s at center [Fig. 414]. Figure 47 shows one of the more geometrically complex nonsystematic designs from the Topkapi Scroll.[35] This repeats with equal efficiency upon either the irregular hexagonal grid with 11-fold proportional angles at the vertices (red), or the perpendicular irregular hexagonal dual grid with 9-fold proportional angles at the vertices (green). This allows for the placement of 11-pointed stars at the vertices of the former irregular hexagonal grid, and 9-pointed stars at the vertices of the latter irregular hexagonal dual grid: or 11s on hexagons/9s on dual hexagons [Fig. 431]. Figure 18 is a conceptually similar design from the Mu'mine Khatun in Nakhichevan, Azerbaijan (1186), with 13- and 11-pointed stars at the vertices of the dualing hexagonal grids: or 13s on hexagons/11s on dual hexagons [Fig. 434].

Triangles and squares as repeat units also support considerably more complex compound patterns with three or more regions of local symmetry. These will often have unusual, and seemingly irreconcilable, combinations of star forms. As with less complex compound patterns, these will place appropriately numbered *n*-pointed stars at the vertices of

[35] Necipoğlu (1995), diagram no. 42.

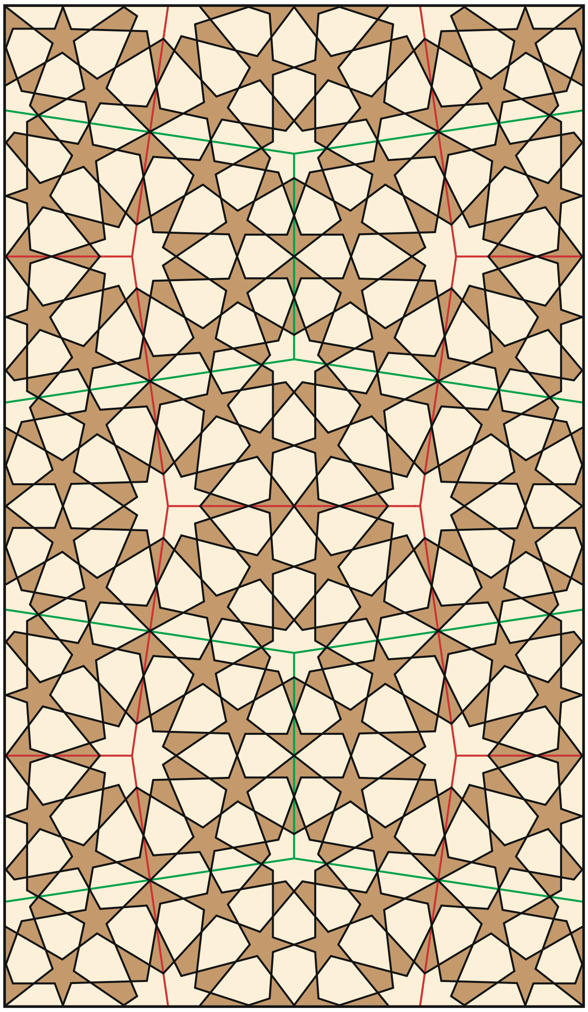


Fig. 47

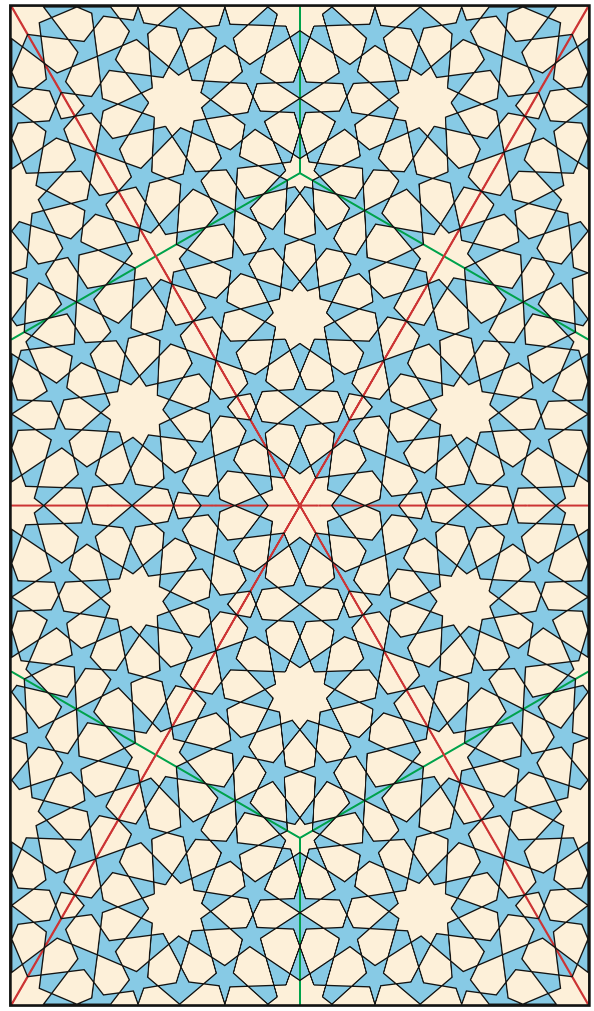


Fig. 48

both the repetitive grid and its dual. However, these more complex compound patterns are fixed upon these locations by placing added primary stars with regular *n*-fold rotation symmetry upon the edges of the repeat unit and/or along the bisecting radii of the repeat unit. The number of points for the stars at these secondary locations is less constrained by predetermined local symmetries, often resulting in star forms with unexpected numeric qualities. Figure 48 shows a threefold design from the Karatay Han near Kayseri, Turkey (1235-41) that has 12-pointed stars at the vertices of the triangular repeat unit, 9-pointed stars at the center of the repeat, 10-pointed stars at the midpoint of each edge of the repeat, and 11-pointed stars upon the bisecting radii of each corner of the repeat unit. This can be described more concisely as 12s on triangle/9s at center/10s on edge/11s on bisecting radii (not shown) [Fig. 367]. The stars that are located at the midpoints of the repetitive edges of such patterns are required to be even numbered, such as the ten-pointed stars in this example, while those located upon the bisecting radii can be either even or odd numbered. Figure 49 shows an orthogonal design from the Agzikarahan near Aksaray, Turkey (1231), that places 12-pointed stars at the vertices of the square repeat unit, an octagon at the center of the repeat, 10-pointed stars at the midpoints of the edges of the repeat, and 9-pointed stars along the bisecting diagonals of the repeat unit. This can be described more briefly as 12s on square/VIII at center/10s on edges/9s on diagonals [Fig. 400]. Figure 50 illustrates a design with three varieties of higher order star that repeats upon a rectangular grid of unusually long proportions. This was reportedly used at the Lower Maqam Ibrahim in the citadel of Aleppo[36] (1168). This highly complex nonsystematic pattern places 12-pointed stars at the vertices of the rectangular repeat unit

[36] The wooden panel described and drawn by Herzfeld is no longer present at the Lower Maqam Ibrahim, and its current location is unknown. Herzfeld (1954-56), Fig. 56.

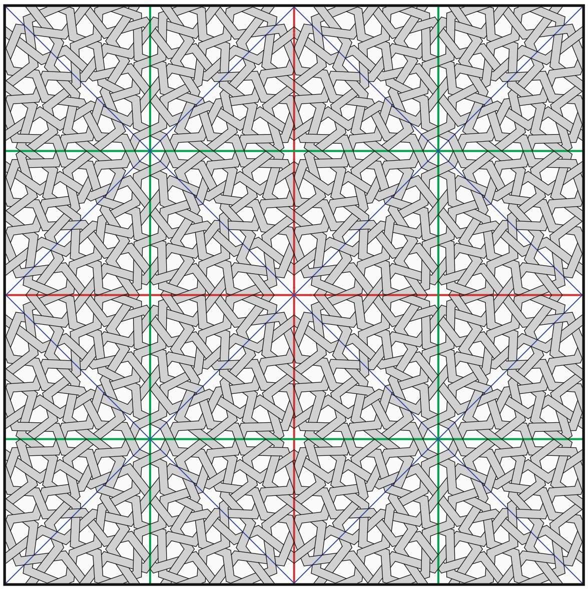


Fig. 49

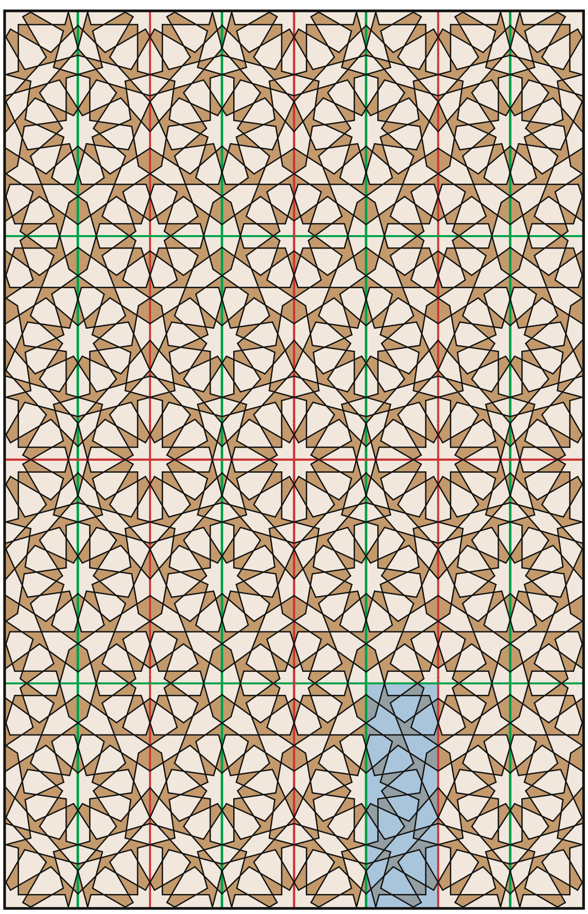


Fig. 50

(red), 10-pointed stars at the midpoints of each long edge of the repeat, and two 11-pointed stars within the field of the pattern matrix, or 12s on rectangle/10s on long edges/11s in field [Fig. 427]. The plane symmetry group of this design is *pmm*, and the fundamental domain (blue) is a rectangle that is reflected 4× to fill the repeat unit.

Designs that use repetitive stratagems that allow *n*-pointed stars, with an otherwise incompatible number of points, to be placed at the vertices of the orthogonal grid can also be categorized according to their numeric qualities. As discussed earlier, oscillating square patterns and rotating kite designs will occasionally have local symmetries such as 6-, 7-, 8-, 10-, and 12-fold. The design in Fig. 23d through **f** is an oscillating square pattern with 12-pointed stars at vertices of the square and rhombus tessellation, and 8-pointed stars at the center of each square element. However, from the perspective of the overall orthogonal repeat, this design places 12-pointed stars upon each edge of the square repeat and 8-pointed stars at the centers: or 12s on square edges/8s at centers. Other historical oscillating square and rotating kite designs can be described in a similar fashion: the design in Fig. 23g through **i** can be described as irregular 7s on square edges/IVs on centers; Fig. 24 as VIIIs at centers; Fig. 25c as 7s on square edges/VIIIs at centers; Fig. 28a as alternating 6s on squares/IVs at center; Fig. 28b as alternating 6s on squares; and Fig. 29 as alternating 10s on square/4s at center.

Another category of design that elegantly utilizes local symmetries that are ordinarily incompatible with the repetitive structure is *imposed symmetry* designs. These do not have oscillating characteristics, but achieve their inclusion of otherwise atypical regular polygons or stars by (1) only using forms that have two perpendicular lines of reflected symmetry, and (2) placing the imposed stars or polygons upon the edges of the repeat unit rather than the vertices. Figure 51 illustrates three related imposed symmetry designs that introduce octagons into an isometric structure: each octagon being placed at the midpoint of each repetitive triangular edge. Figure 51a shows a design from the Çifte Minare *madrasa* in Erzurum, Turkey (late thirteenth century), and is comprised exclusively of superimposed octagons. The size and distribution of the octagons are determined by the constraints of the underlying 3.4.6.4 generative grid [Fig. 107a]. The included angles of the octagons produce the ditrigonal hexagons at the centers of each triangular repeat. Figure 51b shows a design from the Cincikh mosque in Aksaray, Turkey (1220-30). This maintains the identical octagonal structure as in Fig. 51a, but with the addition of hexagons into the superimposed polygonal design matrix [Fig. 107b]. Figure 51c shows a design from

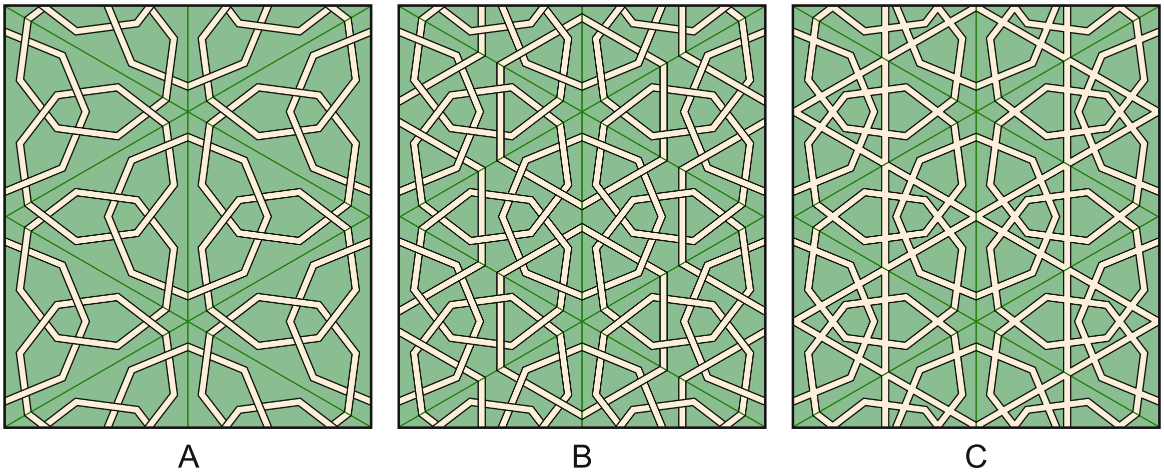


Fig. 51

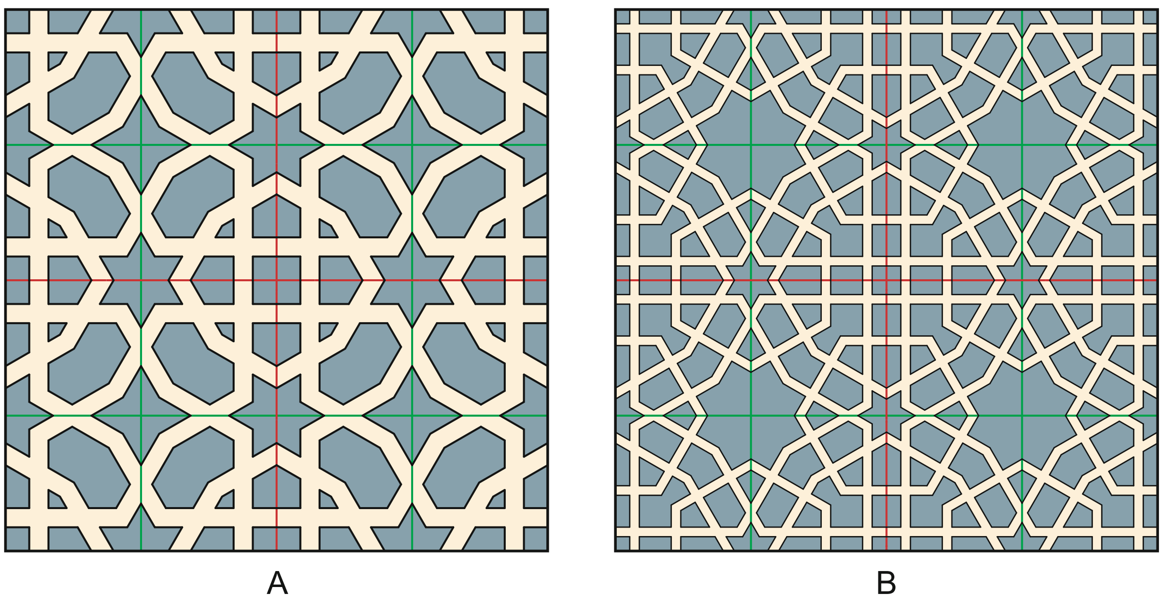


Fig. 52

the mausoleum of Yusuf ibn Kathir in Nakhichevan, Azerbaijan (1161-62). This also maintains the same octagonal structure, but includes the 3.6.3.6 tessellation of triangles and hexagons into the design matrix. In this case, the octagons are located at the vertices of the 3.6.3.6 tessellation. As such, this tessellation can be regarded as equally generative of the overall design as the 3.4.6.4 tessellation. Figure 52 represents two orthogonal imposed symmetry designs that are generated from the deployment of six-pointed stars upon the midpoints of each square repeat unit. Figure 52a shows a design from the original portal of the Palace of Malik al-Zahir at the citadel of Aleppo (before 1193). The parallel lines of the six-pointed stars extend outward to create a four-pointed star at the center of the square repeat, an irregular octagon centered upon the corners of the repeat, and the small square at the corners of the repeat unit: IVs on square/6s on edge/4s at center. Figure 52b shows a design from the mausoleum of Yusuf ibn Kathir in Nakhichevan (1161-62) that is similarly produced from the extension of the parallel lines of the identically placed six-pointed stars. However, the smaller size of these stars provides for the inclusion of a hexagon that bounds each six-pointed star. The corners of this hexagon, together with the extended lines of the six-pointed stars, create an irregular eight-pointed star at the center of each repeat unit, or IVs on square/6s on edge/irregular 8s at center.

2.4 Classification by Plane Symmetry Group

In the late nineteenth century, scientists working in the field of crystallography determined that there are just 17 symmetrical conditions by which the plane can be periodically tiled. The two-dimensional periodic space filling characteristics of Islamic geometric patterns are, ipso facto, governed by the constraints of these 17 plane symmetry groups. As such, the inherent symmetry of all two-dimensional Islamic geometric patterns conforms to an imposition of a fundamental domain to the singular or combined isometric forces of translation, rotation, reflection, and glide reflection. This is not to suggest that artists knowingly applied these four isometric functions to pre-identified fundamental domains as part of their generative methodology. Rather, these are inherent geometric features that govern all periodic two-dimensional space filling and are, therefore, more relevant to the geometric analysis of these patterns than to questions of design methodology and historicity.

The crystallographic discoveries advanced by pioneering scientists including Yevgraf Fyodorov, Arthur Schönflies, William Barlow, and later George Pólya[37] were soon to find artistic expression. George Pólya is particularly relevant for his pronounced influence on Maurits Cornelis Escher.[38] Escher traveled twice to the Alhambra in Spain and was heavily influenced by the geometric designs there, recording many patterns in his workbooks. The same year of his first visit to the Alhambra (1924) he was sent a copy of Pólya's publication that included line drawings of repetitive patterns in each of the 17 plane symmetry groups, some of which were derived from Muslim architectural sources. Pólya and especially Escher appear to be the first individuals to examine Islamic geometric designs from the perspective of their crystallographic group. Later ethnomathematical studies of Islamic geometric design focused more specifically upon their crystallographic characteristics,[39] and historical examples of all 17 plane symmetry groups have been identified. The works of Syed Jan Abas and Amer Shaker Salman,[40] as well as Emil Makovicky,[41] are particularly significant to this study.

Figure 53 shows a flowchart that identifies the four isometric conditions of rotation, reflection, glide reflection, and translation for each of the 17 plane symmetry groups,[42] and from which existing designs can be analyzed to readily identify their specific symmetry group. Figure 54 represents a geometric design from each of the plane symmetry groups with 120° rotational centers and/or 60° rotational centers. The *p3* symmetry group has three types of 120° rotation center (threefold), and is without reflections or glide reflections. Islamic geometric patterns that conform to this group are uncommon (the example shown is the author's creation). The *p31m* symmetry group has three types of 120° rotation center (threefold), and three directions of reflection. The lines of reflection form the isometric grid (red) and two of the points of rotation are located at the center of each triangular cell, while the third is located at each vertex of this grid. This structure also has three directions of glide reflection with lines that are parallel to and located in the middle of adjacent parallel lines of reflection. The design representing this symmetry group is a design that is easily created from the 6^3 underlying tessellation. The *p3m1* symmetry group has three types of 120° rotation center (threefold) and three directions of reflection that comprise the isometric grid. Each point of rotation is located at a vertex of this grid. This structure has three directions of glide reflection that are identical to the previous group. Mughal artists frequently used the design representing this symmetry group in the production of *jali* screens. The additive threefold lines at the center of each six-pointed star provide the stone screen with greater uniformity in the size of the openings, as well as greater structural integrity, an important consideration in this pierced stone medium. This additive device also changes the plane symmetry group of the well-known pattern of superimposed dodecagons from *p6m* to *p3m1*. The *p6* symmetry group has one variety of 60° rotation center (sixfold), and two types of 120° rotation center (threefold), and a single 180° rotation center. There are no reflections or glide reflections. The design that represents

[37] –Fedorov (1891), 345–291.
–Schönflies (1891).
–Barlow (1894), 1–63.
–Pólya (1924), 278–298.

[38] Schattschneider (1990).

[39] –Müller (1944).
–Bixler (1980).
–Lalvani (1982).
–Grünbaum, Grünbaum, and Sheppard (1986), 641–653.
–Mamedov (1986), 511–529.
–Pérez-Gómez (1987), 133–137.
–Lalvani (1989).
–Chorbachi (1989), 751–789.
–Abas and Salman (1995).
–Lovric (2003), 423–432.

[40] Abas and Salman (1995).

[41] –Makovicky and Makovicky (1977), 58–68.
–Makovicky (1989), 955–999.
–Makovicky (1994), 1–16.
–Makovicky (1995), 1–6.
–Makovicky (1997), 1–40.
–Makovicky (1998), 107–127.
–Makovicky (1999), 143–183.

[42] This flowchart replicates that of Donald Crowe, Department of Mathematics, University of Wisconsin-Madison, and is included in his book on symmetry in cultural artifacts: See Washburn and Crowe (1988).

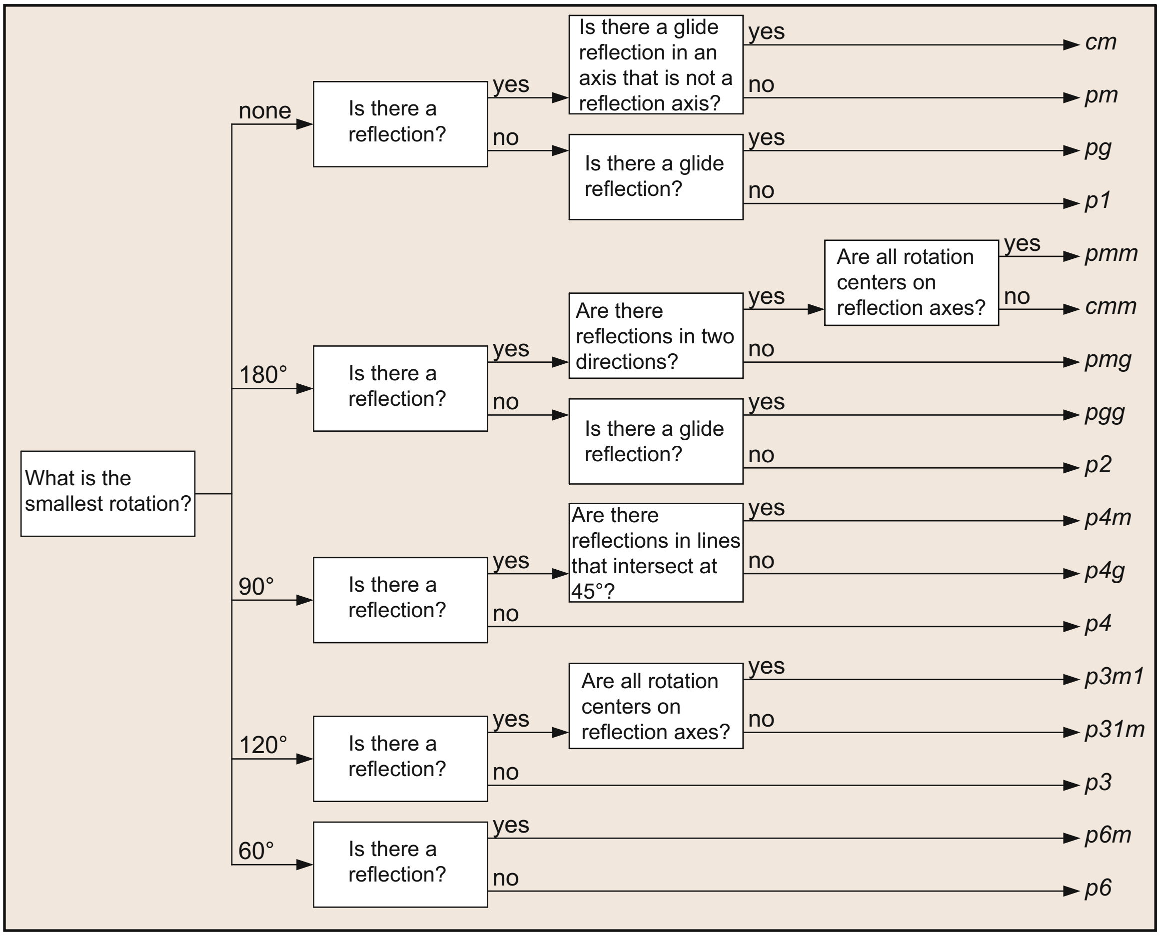

What is the smallest rotation?
none
180°
90°
120°
60°
Is there a reflection?
yes
no
Is there a glide reflection in an axis that is not a reflection axis?
Is there a glide reflection?
Are there reflections in two directions?
Are all rotation centers on reflection axes?
Are there reflections in lines that intersect at 45°?
cm
pm
pg
p1
pmm
cmm
pmg
pgg
p2
p4m
p4g
p4
p3m1
p31m
p3
p6m
p6

Fig. 53

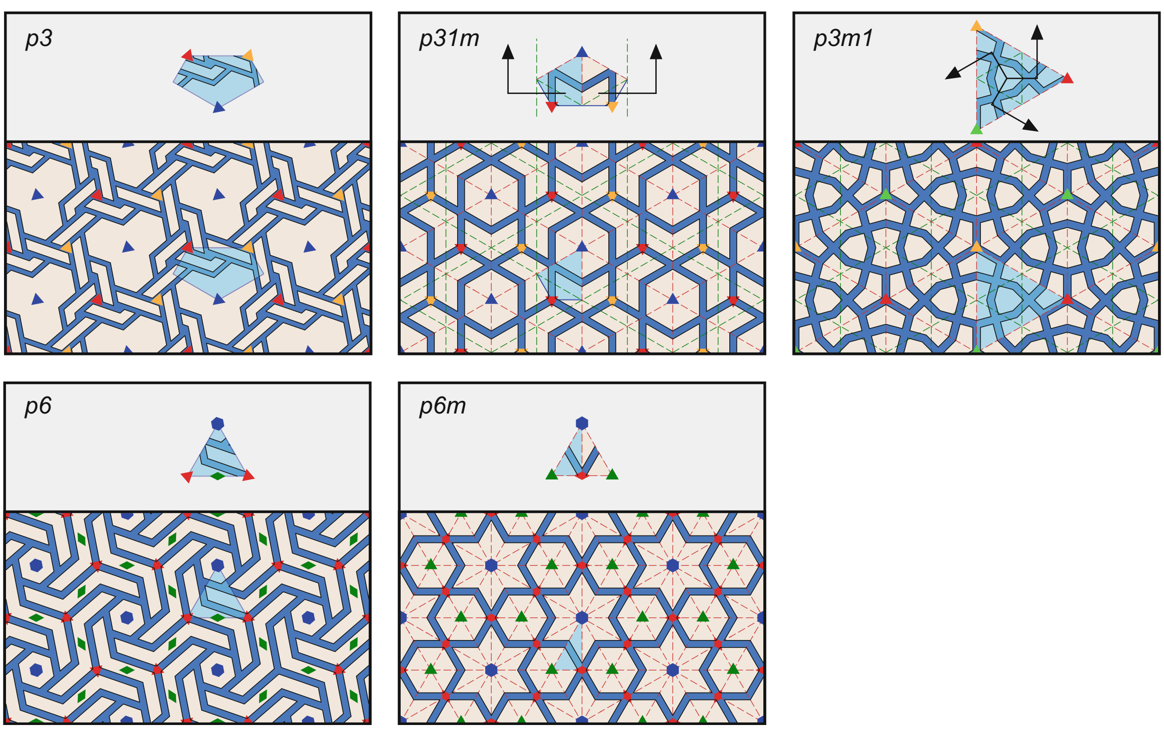

p3
p31m
p3m1
p6
p6m

Fig. 54

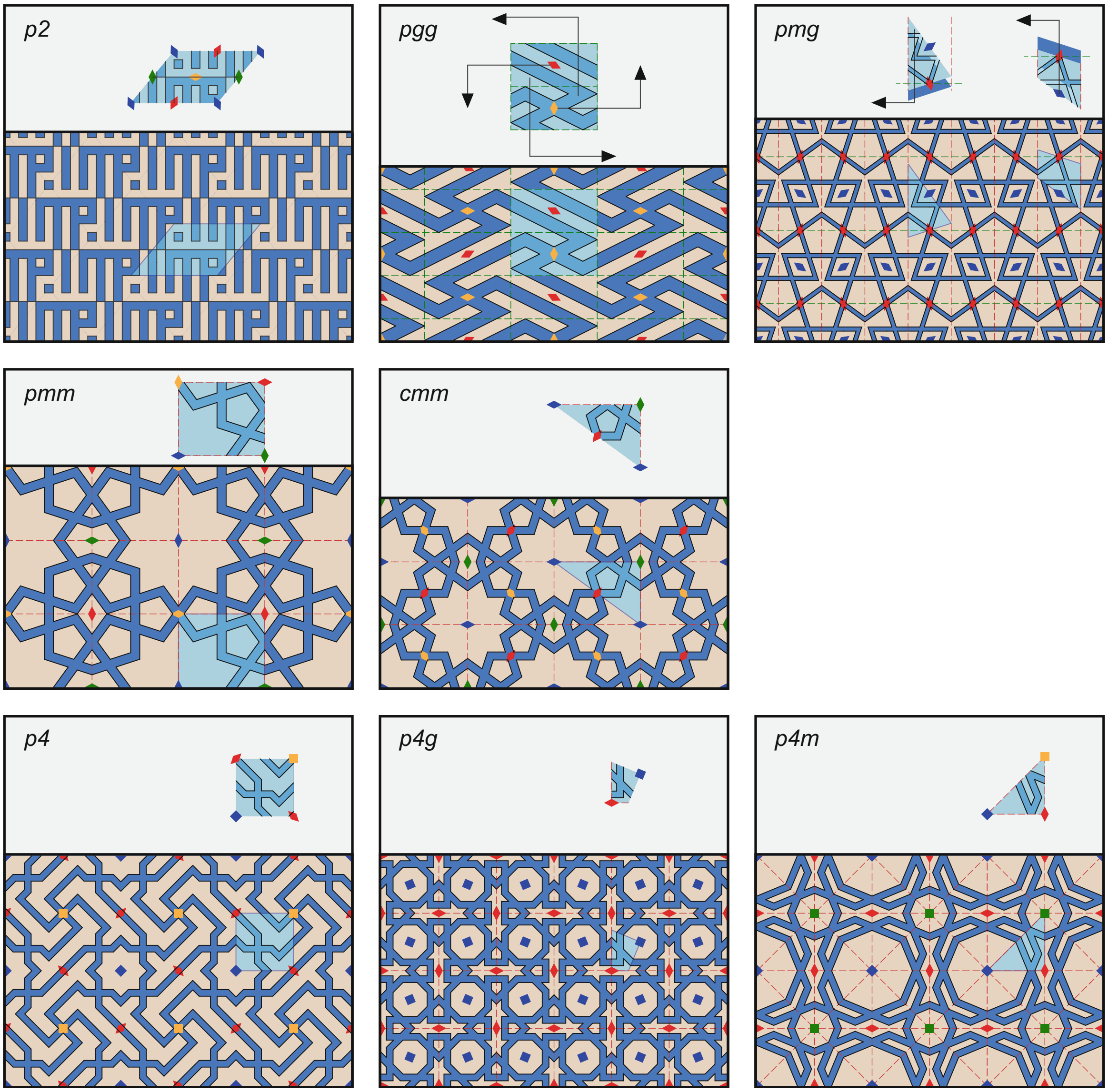


Fig. 55

this symmetry was used widely throughout Muslim cultures, and one of the earliest examples is from the brickwork ornament of the western tomb tower at Kharraqan, Iran (1093). The *p6m* symmetry group has one variety of 60° rotation center (sixfold), two types of 120° rotation center (threefold), which are perpendicularly orientated; and three types of 180° rotation center. This group has six directions of reflection, and six directions of glide reflection (not shown) that are parallel with, and centered between, the lines of reflection. The example shown is one of the most common threefold geometric patterns. Figure 55 represents a geometric design from each of the plane symmetry groups with 180° rotational centers and/or 90° rotational centers. The *p2* symmetry group has four types of 180° rotation center, with no reflections or glide reflections. Islamic geometric patterns structured on this symmetry group are unusual. Among the more interesting examples are a variety of square *Kufi* calligraphic designs, in this case with a simple *Allah* motif (the example shown is the author's creation). The *pgg* symmetry group has two types of 180° rotation center, with two glide reflections in perpendicular directions. There is no reflection symmetry. The example shown is a well-known key pattern with swastikas in glide reflection. The *pmg* symmetry group has two types of 180° rotation center, with parallel lines of reflection in just one direction. It also has glide reflections that are perpendicular to the lines of reflection, and the rotation centers are located on the lines of glide reflection. Islamic geometric designs with this symmetry group are ordinarily very simple. The example shown is from the Khwaja Atabek mausoleum in Kerman (1100-1150) and is one of the more complex historical designs with this crystallographic structure. The *pmm* symmetry group has four types of 180° rotation center, each located at a vertex of the perpendicular lines of reflection. There are no glide reflections. The example shown is ubiquitous throughout the Islamic world. The *cmm* symmetry group has four types of 180° rotation center: two located on the vertices of the perpendicular lines of reflection, and two that are not located on lines of reflection. The example shown is a very common fivefold *obtuse* pattern that repeats upon a rhombic grid. The *p4* symmetry group has two types of 90° rotation center, and two types of 180° rotation center (twofold). There are no lines of reflection or glide reflection. The example shown is well known from the historical record. The *p4g* symmetry group has two types of 90° rotation center (fourfold), and two types of 180° rotation center (twofold). The 90° rotation centers are not located on lines of reflection, while the 180° rotation centers are located on the vertices of the orthogonal lines of reflection. There are diagonally oriented glide reflections that run halfway between the vertices of the lines of reflection (not shown). This design was used in numerous locations historically. The *p4m* symmetry group has two types of 90° rotation center (fourfold), two types of 180° rotation center (twofold), and four directions of reflection. All rotation centers are located at the vertices of the lines of reflection. This design is from a frontispiece from a Quran produced in 1001 by ibn al-Bawwab (d.1022). Figure 56 represents a geometric design from each of the plane symmetry groups with no rotation symmetry. The *p1* symmetry group relies solely upon translation symmetry, with no rotations, reflections, or glide reflections. Islamic geometric patterns based upon this group are very rare, and the example shown (by author) avoids reflection symmetry by the introduction of chirality with the interweaving lines. The *pg* symmetry group is defined by glide reflection only, with no rotation or reflection. This variety of pattern is also very rare in Islamic geometric design, and the illustrated example (by author) is a rather complex design that is otherwise indicative of the early brickwork ornament of Khurasan. The *pm* symmetry group only has parallel lines of reflection, with no rotation or glide reflection. This design places an additive pentagonal device within the otherwise central ten-pointed stars. Without this added fivefold device there would be the additional

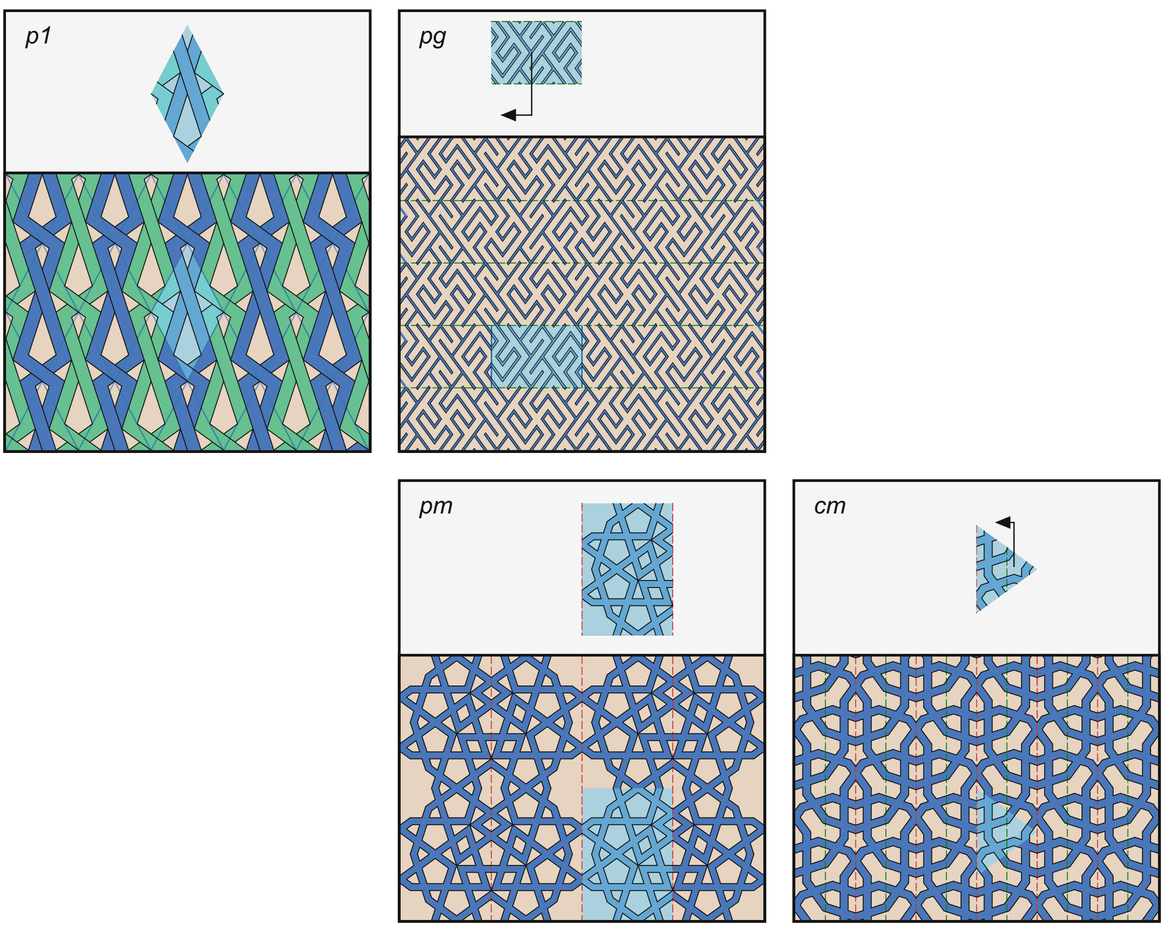


Fig. 56

lines of reflection and points of rotation of the *pmm* symmetry group. The *cm* symmetry group has parallel lines of reflection and parallel lines of glide reflection located halfway between the reflections. There are no points of rotation. Islamic ornament in this group is predominantly floral, such as certain ogee designs. The classic geometric patterns of Muslim cultures do not ordinarily conform to this symmetry group, although decent designs are possible (example shown by author).

It is beyond the scope of this work to quantify the distribution of historical geometric designs within a given Muslim culture, let alone the totality of Islamic art, according to their plane symmetry group. However, without question, certain isometric transformations occurred with greater frequency within this tradition, while others are less common or very rare. According to Abas and Salman, the *p6m* and *p4m* symmetries are the most widely distributed; the *cmm*, *pmm*, and *p6* are also significantly represented; the *p4*, *p31m*, *pm*, and *p3m1* are significantly fewer; and the *p4g*, *p3*, *cm*, *p2*, *pgg*, *pmg*, *p1*, and *pg* are very rare.[43] The question of why certain symmetry groups were favored over others appears to have more to do with methodological practices than aesthetic predilections. The vast majority of Islamic geometric patterns are readily created from the polygonal technique wherein a tessellation of diverse edge-to-edge polygons is used to extract the design. The symmetry group of an underlying generative tessellation directly determines the symmetry group of the extracted pattern. This is not to say that the two are always identical, especially when additive design features alter or cancel the rotation and reflection, or lines of reflection are annulled through the introduction of chirality with interweaving lines. Creating successful polygonal tessellations that are well suited to extracting patterns that conform to the aesthetic standards of this tradition typically involves the placement of higher order primary polygons at strategic locations of the repetitive grid. These invariably have *n*-fold rotation symmetry and their placement at the vertices, centers, and edges of the repeat unit insures compliance with those symmetry groups that are similarly structured, and is generally less suited to symmetry groups without rotation or reflection. Field patterns created from the polygonal technique eschew regions of local symmetry created from higher order polygons affiliated with strategic locations within the repeat. This lack of affiliation occasionally allows field patterns to be structured upon symmetry groups that are less common to this tradition. Islamic geometric designs that are not created from the polygonal technique will also occasionally employ these less commonly used symmetry groups. These can include key patterns, designs with swastika motifs, and square *Kufi* brickwork designs.

[43] The methodology behind the gathering of the data points for this statistical analysis of the distribution of the 17 symmetry groups within the tradition of Islamic geometric patterns is not provided in this study. See Abas and Salman (1995), 138.

2.5 Classification by Design Methodology

2.5.1 The Polygonal Technique

The aesthetic character of a given geometric design is greatly determined by the method used in its creation. Generative methodology is therefore an important criterion for better understanding of Islamic geometric patterns. Some of the less complex geometric patterns are able to be produced from more than a single generative methodology, and it is not always possible to ascertain with certainty which was used in the creation of a particular historical example. As stated previously, surviving evidence indicates that the most widely used and, therefore, historically relevant design methodology was the *polygonal technique*, wherein strategic points of a polygonal tessellation, such as the midpoints of each polygonal edge, are used to locate pattern lines, after which the tessellation is discarded, leaving behind the completed design. Depending on the angles of the applied pattern lines, multiple designs can be created from a single underlying tessellation. Insofar as Islamic geometric patterns are concerned, no other design methodology provides the level of flexibility and consequent design diversity, and other approaches used over the centuries are of significantly less importance to this overall tradition.

It appears that the artists responsible for the development and furtherance of Islamic geometric patterns were discriminating in their need to balance generational transferral with protection of the highly specialized design practices required of this art form. There are no known historical sources that speak to the methodological secrecy employed by individuals, ateliers, and artists' guilds employed in the geometric arts. One must assume that the ongoing development of geometric design flourished under the same sort of protectionist control as other arts reliant upon patronage for their survival. This might explain the paucity of geometric artists' reference scrolls (*tumar*) and design manuals currently known to art historians. Of the few such documents, one is particularly significant in that it is very likely the earliest depiction of a geometric pattern accompanied by its underlying generative polygonal tessellation. Figure 57 illustrates a design created from one of the many figures contained in the anonymous Persian language treatise titled *On Similar and Complementary Interlocking Figures* in the

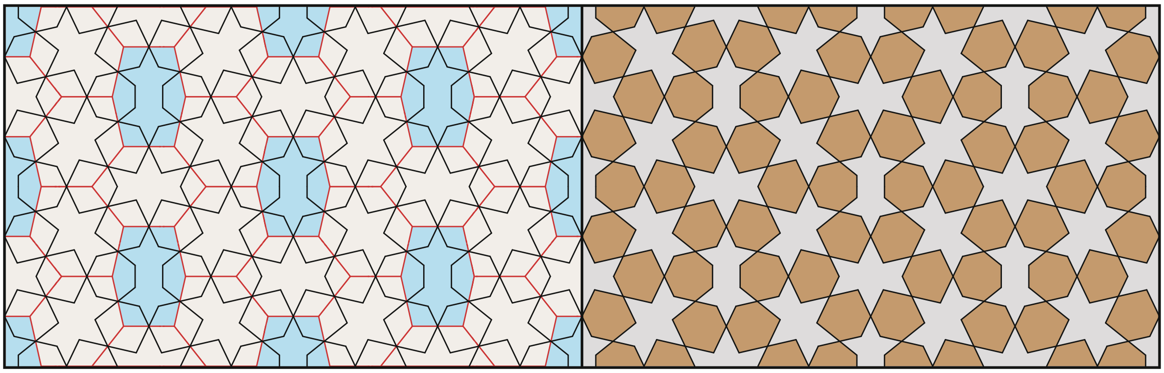


Fig. 57

Bibliothèque Nationale de France in Paris.[44] This pattern is all the more remarkable in that its only known architectural use is from one of the blind arches in the Seljuk northeast dome chamber of the Friday Mosque in Isfahan[45] (1088-89) [Photograph 26]. Figure 57a represents the manuscript's depiction of the polygonal tessellation comprised of two types of irregular hexagon, as well as the generated *acute* pattern whose intersecting lines rest upon the midpoints of each polygonal edge: the classic formulation of the polygonal technique. Figure 57b illustrates the pattern on its own. This heptagonal design is all the more interesting in that it is the earliest known example of a design created from the *sevenfold system* of pattern generation, and its use in Isfahan precedes later extant examples created from this system by 100 years. One of the remarkable features of *On Similar and Complementary Interlocking Figures* are the written instructions that accompany most of the illustrations, and the step-by-step instructions that accompany this figure are revealing in that they provide instructions for the creation of the polygonal tessellation, but not the pattern that this tessellation creates. The absence of secondary instructions for the application of the pattern lines onto the tessellation may indicate that this process was a given: sufficiently understood so as not to warrant further instruction. The many illustrations and instructions for geometric patterns in this anonymous manuscript are better known as one of the very few historical sources of evidence for what is herein referred to as the point-joining methodology. The historical relevance of this aspect of the manuscript is examined below; but the inclusion of this one representation of the polygonal technique is significant for four reasons: (1) it is one of the earliest known examples of a pattern accompanied by its underlying generative tessellation; (2) it includes written instructions for creating the generative tessellation; (3) it is one of the earliest examples of a pattern created from the *sevenfold system* of pattern generation; and (4) it is one of the very few historical documents that overtly demonstrates the polygonal technique.

On its own, the example of the polygonal technique from the anonymous manuscript might be regarded as merely an interesting anomaly. However, in association with the many additional examples from diverse media, wide-ranging regions, and over prolonged periods of time, the validity of the polygonal technique as the preeminent historical methodology used in creating Islamic geometric patterns becomes unassailable. The earliest architectural examples include numerous patterns that maintain the generative tessellation as part of the completed design. Even during the eleventh and twelfth centuries when this ornamental tradition was in the process of rapid development, it was far more common for the generative tessellations to be discarded after completion of the design process. However, some early examples of patterns created from the *system of regular polygons* include the generative tessellation within the completed design. The least complex of these are based upon the 6^3 tessellation of regular hexagons, and include a Qarakhanid *two-point* pattern from the southern portal of the Maghak-i Attari mosque in Bukhara, Uzbekistan (1179-79) [Fig. 96f], and a Sultanate of Rum *two-point* pattern from the Great Mosque of Bayburt in northeastern Turkey (1220-35) [Fig. 97b]. A Mamluk *two-point* design from the *mihrab* of the Aqbughawiyya *madrasa* (1340) at the al-Azhar mosque in Cairo similarly expresses the 3.6.3.6 generative tessellation as part of the completed design [Fig. 100d]. The design of a brickwork panel in the portal of the anonymous southern tomb in the complex of three adjoining Qarakhanid mausolea in Uzgen (1186) includes the depiction of its 3.4.6.4 generative tessellation [Fig. 104d]. Several patterns that overtly express their 4.8^2 generative tessellation of squares and octagons are known to the historical record,

[44] MS Persan 169, fol. 192a.

[45] The author is indebted to Professor Jan Hogendijk at the University of Utrecht for pointing out the connection between the panel with sevenfold symmetry at the Friday Mosque at Isfahan and the design from folio 192r in the anonymous manuscript at the Bibliothèque Nationale de France in Paris.

including a Timurid variation of the classic star-and-cross pattern from the Ghiyathiyya *madrasa* in Khargird, Iran (1438-40) [Fig. 126d].

One of the most compelling examples of architectural evidence for the polygonal technique is from the main entry portal of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63). In one of the sidewalls of this Mamluk *iwan* is an arched niche with a *muqarnas* hood. This niche is decorated with an interesting nonsystematic pattern with six- and eight-pointed stars that repeats on a rectangular grid [Photograph 58]. The pattern in this niche is produced in white marble inlaid into a beige limestone background. The artist also inlaid a black stone representation of the generative tessellation of octagons, distorted hexagons, and rhombi [Fig. 413]. This is significantly different from the previously cited examples in that the geometric pattern is distinctly independent of the generative tessellation rather than being incorporated into the finished design. The presence of the tessellation is highly unusual in that it reveals the methodological key to this design specifically, and to almost all Islamic geometric patterns generally.

In addition to the panel from the Sultan al-Nasir Hasan funerary complex in Cairo, the most overt architectural examples of geometric designs accompanied by their generative tessellations come from several *jali* screens from Mughal India. A marble *jali* in the tomb of I'timad al-Daula in Agra (1622-28) expresses the generative tessellation in high relief as the primary visual motif, and the resulting geometric design as secondary elements. This example is the classic fivefold *acute* pattern created from the *fivefold system* [Fig. 226c]. Additional Mughal examples are located in the *jali* screens of the tomb of Salim Chishti at Fatehpur Sikri (1605-07), including a very-well-known *acute* pattern created from the *fourfold system B* [Fig. 173a] [Photograph 77]; a widely used nonsystematic *acute* design with 12-pointed stars on vertices of the isometric grid [Fig. 300a *acute*]; and an unusual example wherein the fivefold pattern generator is, itself, a field pattern created from the *fivefold system*. This field pattern is made up of just two design elements, pentagons and hourglass decagons. The simplicity of this design allows for it to be used as a generative tessellation for the secondary pattern.

Evidence of the polygonal technique is occasionally found in objects that employ comparatively complex polygonal tessellations without the presence of one of the geometric designs that can be generated from the tessellation. A particularly early example of such an item is a Persian fritware tile (c. 1250-1300) in the collection of the Los Angeles County Museum of Art[46] [Photograph 104]. The date of origin suggests that this is either late Khwarizmshahid or early Ilkhanid. The molded relief decoration boldly depicts a polygonal tessellation comprised of dodecagons, decagons, and nonagons, with concave hexagonal secondary interstitial polygons that function within the tessellation much like the concave hexagons within the *fivefold system* [Fig. 232]. There are no known historical designs created from this nonsystematic polygonal tessellation. Rather than the midpoints of the polygonal edges being used to locate geometric pattern lines, these points determine the construction of a floral design. This is the only known example of an Islamic floral design being extracted from a complex polygonal substructure, and the fact that the polygonal midpoints are similarly used as location points is significant. Another significant example of a complex nonsystematic polygonal tessellation being used as ornament without the depiction of one of the geometric designs that the tessellation can create is from a Karamanid walnut door from the Imaret mosque in Karaman, Turkey[47] (1433) [Photograph 105]. The repetitive structure is orthogonal, and the local regions of symmetry are 8- and 12-fold, separated by irregular pentagons and barrel hexagons. This particular nonsystematic tessellation is one of the most commonly employed historically, and was used to produce innumerable geometric designs in all four of the principal pattern families throughout the Islamic world [Figs. 379–382].

By far the most convincing evidence of the polygonal technique as the primary historical method used by artists for creating complex Islamic geometric patterns is the Topkapi Scroll.[48] According to Gülru Necipoğlu, the prominent authority on the historical significance of this scroll:

> The Topkapi Scroll was probably compiled in the late fifteenth or sixteenth century somewhere in western or central Iran, possibly in Tabriz, which served as a major cultural capital under the Ilkhanids, the Qaraqoyunlu, and the Aqqoyunlu Turkman dynasties, as well as the early Safavids. Its geometric designs in all likelihood were produced under Turkman patronage, but an early Safavid date is also a possibility as the international Timurid heritage would still have been very much alive.[49]

The Topkapi Scroll contains 157 different designs that represent the full range of geometric ornament in the regions directly influenced by Timurid aesthetics. These include *muqarnas* vaulting, star-net (*rasmi*) vaulting, geometric ornament for domes, *Kufi* script, square or chessboard (*shatranji*) *Kufi* motifs, and numerous examples of geometric patterns. Among the many geometric patterns is a wide range of diverse types, including three designs produced

[46] Los Angeles County Museum of Art, the Madina Collection of Islamic Art, gift of Camilla Chandler Frost (M.2002.1.285).

[47] In the collection of the Museum of Turkish and Islamic Arts, Istanbul, Turkey, accession no. 244.

[48] Topkapi Palace Museum Library MS H. 1956.

[49] Necipoğlu (1995), 37–38.

Photograph 104 Persian fritware relief tile with a polygonal tessellation comprised of quarter dodecagons and a half decagon as the primary ornament, and a floral motif with symmetry that is governed by the polygonal structure (The Los Angeles County Museum of Art: the Madina Collection of Islamic Art, gift of Camilla Chandler Frost (M.2002.1.285): www.lacma.org)

from the *fourfold system A* (nos. 1, 67, 61)[50]: one from the *fourfold system B* (no. 57); eight designs with rhombic repeat units made from the *fivefold system* (nos. 8, 52, 53, 54, 55, 62, 64, 73); six with rectangular repeats from the *fivefold system* (nos. 33, 48, 50, 56, 58, 60); five *Type A* dual-level designs created from the *fivefold system* (nos. 28, 29, 31, 32, 34); one *Type B* dual-level design produced from the *fivefold system* (no. 49); one Type B dual-level design that uses hybrid square and triangle repetitive elements with 8- and 12-pointed stars (no. 38); an additional hybrid design with square and triangle repeats with 8- and 12-pointed stars (no. 35); two nonsystematic designs with 12-pointed stars located at the vertices of the isometric grid, one with rotating square swastikas (nos. 63 and 70); two nonsystematic orthogonal compound patterns, one with 8- and 12-pointed stars (no. 72d), and the other with 13- and 16-pointed stars (no. 30); three nonsystematic designs that do not use either

[50] The indicated numbers in this paragraph follow the numbering protocol in the Topkapi Scroll—Geometry and Ornament in Islamic Architecture. See Necipoğlu (1995).

Photograph 105 A Karaminid walnut door from the Imaret mosque in Karaman, Turkey, that depicts a nonsystematic tessellation associated with the polygonal technique that includes partial octagons and dodecagons surrounded by pentagons and barrel hexagons (© Dick Osseman)

the isometric or the orthogonal grids, including one with 8-, 10-, and 12-pointed stars (no. 39), one with 10- and 12-pointed stars (no. 44), and one with 9- and 11-pointed stars (no. 42); one rotating kite design with 6-pointed stars (no. 59); two oscillating square patterns with swastikas (nos. 41 and 69b); three designs with forced 10-pointed stars in a square repeat unit (nos. 66, 68, 72c); and three designs for application onto domical surfaces, including two created from the *fivefold system* (nos. 4, 90a), and one compound design with 8- and 10-pointed stars (no. 10b). The Topkapi Scroll is drawn primarily in black and red ink. These two colors are used to differentiate the features of a given illustration. This frequently involves the contrast between the pattern and its generative tessellation. Further differentiation is occasionally achieved through using dotted lines. What is more, many of the geometric patterns that do not overtly show the generative tessellation in ink reveal this important methodological feature in finely scribed layout lines made with a steel stylus and referred to as *dead drawing*. With the exception of the oscillating square, rotating kite, and forced patterns, virtually all of the geometric designs in the Topkapi Scroll visually represent their generative tessellation in either ink or inscribed lines. This is by far the largest known single repository of geometric designs represented in association with their underlying generative tessellations. The fact that this scroll was an artist's reference intended for practical application is an incontrovertible evidence for the polygonal technique being the preeminent methodology employed in the creation of Islamic geometric patterns during the time and place of the scroll's production, and by extrapolation, to this tradition more generally. The designs from the Topkapi Scroll range in complexity between the more basic systematic patterns and those that are highly complex with more than one region of local symmetry, as well as dual-level designs with self-similar characteristics. As demonstrated so aptly in this scroll, the polygonal technique is uniquely capable of creating these exceptionally complex designs.

Further scroll evidence for the historical use of the polygonal technique is found among the scroll fragments at the Institute of Oriental Studies in Tashkent. These range in date between the fifteenth to seventeenth centuries. Like the Topkapi scroll, these depict a combination of vaulting systems for three-dimensional application, and two-dimensional geometric patterns, and also include the use of colored inks and scribed lines. Of particular interest to the question of design methodology are a series of geometric patterns that include the underlying generative tessellation. These examples are estimated to date from the sixteenth or possibly seventeenth century, and employ only black ink with the underlying tessellation represented in the un-inked incised lines produced with a steel stylus.[51] The geometric patterns include *median* and *two-point* designs created from the *fivefold system*, as well as a nonsystematic *median* pattern with 9- and 12-pointed stars [Fig. 346b].

Much like the incised lines from the Topkapi and Tashkent scrolls, Quranic illuminators also used a steel stylus to lay out their designs prior to painting the final illumination. The relevance of Quranic illumination to the understanding of traditional design methodology has not received the research it deserves. The likely significance of this artistic discipline in providing further corroboration of the prevalent use of the polygonal technique is found in an outstanding Mamluk illuminated frontispiece (c. 1399-1411) at the British Library [Photograph 48].[52] This is decorated with an *obtuse* pattern created from the *fivefold system* [Fig. 233b] that was used in many locations over the years, including the Izzeddin Kaykavus hospital and mausoleum in Sivas, Turkey (1217-18); the Agzikarahan in Turkey (1242-43); and the Sultan Qala'un funerary complex in Cairo (1284-85). Upon close inspection with oblique lighting, the fine incised lines beneath the paint that were used for laying out this illumination are faintly detectable with the naked eye. In this example, these painted-over incised lines reveal the underlying generative tessellation that was used to produce the pattern [Fig. 233c]. Unless this example is an anomaly, considering that this is just one of a very large number of illuminated pages with geometric ornament, it is entirely possible that a study of Quranic examples will reveal further evidence that illuminators used this design methodology when laying out their compositions.

The last piece of evidence for the historicity of the polygonal technique comes from the published observations of Ernest Hanbury Hankin, a bacteriologist working in India in the latter part of the nineteenth century. His observations of a deteriorating stucco ceiling in a bathhouse (*hammam*) at Fatehpur Sikri led to his discovery that Islamic geometric designs were constructed from underlying polygonal tessellations:

> During visits to Fathpur-Sikri many years ago, I spent much time in measuring the angles and making tracings of these designs but always failed to find any rational scheme by which they could be constructed. At last, by good fortune, I happened to enter a small Turkish bath attached to Jodh Bai's Palace. It had previously been inhabited by Indians, who had only just been evicted, and I was probably the first European to visit the place. In one of the rooms of the bath was a half dome decorated by a straight-line pattern. In addition to the pattern, some faint scratches were discovered on the plaster. Obtaining a table and chair and a piece of tracing paper I succeeded in making a copy. On closer examination these scratches were found to be parts of polygons, which, when completed, surrounded the star-shaped spaces of which the pattern was composed, and it turned out that these polygons were the actual construction lines on which the pattern was formed.[53]

Hankin first published his finding in a 1905 article in the Journal of the Society of Arts[54] entitled *On some Discoveries of the Methods of Design employed in Mahomedan Art*, and in greater detail in his 1925 article *The Drawing of Geometric Patterns in Saracenic Art* for the Memoirs of the Archeological Survey of India. In these

[51] There are relatively few sources of photographs of the geometric patterns from the Tashkent scrolls. See
–Rempel' (1961).
–Necipoğlu (1995), 12–13.

[52] British Library, London, BL Or. MS 848, ff. 1v-2.

[53] Hankin (1925a), 3–4, no. 15.

[54] Hankin (1905), 461.

works and others,[55] Hankin describes in considerable detail the design process for the polygonal technique, and analyzes a number of historical designs. He is the first European to have discovered this design methodology, yet the significance of his discoveries has had far less impact than deserved upon the prevailing views regarding traditional design methodologies that came after him. Since the turn of the millennium, recognition of the polygonal technique has gradually gathered momentum for its historicity and methodological flexibility.[56]

Regarded as a group, the above-cited methodological examples provide compelling evidence for the efficacy and historicity of the polygonal technique as a primary tool for generating the diverse range of designs that characterize this geometric art form. While the more basic designs can often be produced with alternative methods, the polygonal technique is the only methodology that will produce the more complex patterns within this tradition. This preponderance of evidence provides the certainty that the polygonal technique was the preeminent design methodology used historically. Without this evidence, the relevance of this method of creating geometric designs would be based solely upon common sense and experience.[57]

The earliest Islamic geometric patterns are easily created from regular, semi-regular, and occasionally *two-uniform* tessellations comprised of regular polygons. These polygons include the triangle, square, hexagon, octagon, and dodecagon. Being that the octagon is limited to only one semi-regular tessellation, and that other than the square, it will not tessellate with any of the other regular polygons, the 4.8^2 semi-regular tessellation is, for practical purposes, regarded within this study as its own group with its own distinctive aesthetic merits. This is why, for the purposes of this discussion, the octagon is not included within the modules of the *system of regular polygons* [Fig. 92]. The 4.8^2 tessellation is one of the most widely used, versatile, and prolific underlying generative substrates used within this ornamental tradition [Figs. 124–129]. Figure 58 demonstrates just two of the many designs that can be produced from the orthogonal 4.8^2 semi-regular tessellation of octagons and squares. Figure 58a shows the classic *median* star-and-cross pattern used frequently throughout the Islamic world. Employing the polygonal technique to produce this pattern involves drawing lines through the edge at every second midpoint within the octagons, creating two superimposed squares with 90° crossing pattern lines at each midpoint. These lines are trimmed so that the interior octagons are converted to the characteristic eight-pointed stars. The design in Fig. 58b is also very well known within this tradition, and is produced in a similar fashion, except that the applied pattern lines transect the midpoints of every third octagonal edge: creating 45° crossing pattern lines at these midpoints, and four-pointed stars within each underlying square. The interior octagonal region is similarly trimmed to create the eight-pointed stars. The 45° crossing pattern lines in Fig. 58b qualify this as an *acute* pattern.

As mentioned, there are four standard techniques for extracting geometric designs from underlying polygonal tessellations. In addition to the *acute* and *median* families illustrated in Fig. 58, this tradition also includes *obtuse* and *two-point* patterns. Each of these four families has an identifiable visual quality that is independent of its symmetrical characteristics or repeat unit. These appear with such regularity, and are sufficiently distinct from one another that it is appropriate for each to be included as a distinct category of pattern created from the polygonal technique. Figure 59 highlights the distinctive features of each of these four pattern families. For the purposes of demonstration, the four examples shown are created from the *fivefold system*, but the four pattern families are equally relevant to all systematic and nonsystematic geometric designs created from the polygonal technique. What is more, the visual characteristics of the pattern elements created from the *fivefold system* have direct analogs to those created from other types of geometric design, and the examples provided in this illustration are therefore representative of this tradition generally. Figure 59a demonstrates the characteristics of the *acute* family, with stars, darts, kites, and bilateral shield-shaped hexagons. Quick visual references for identifying *acute* designs are the acute angles of the points that surround the primary star forms, as well as the acute angles of the five-pointed stars. Figure 59b demonstrates the more open character of the pattern elements of the *median* family, especially as pertains to the points of the primary stars and five-pointed stars. The angles of the stars, darts, overlapping darts, kites, and shields are recognizably less acute, and fall between the angles of the *acute* family and the *obtuse* family: hence the name *median*. Figure 59c demonstrates the characteristics of the *obtuse* family. The pattern elements in this type of

[55] In addition to the two articles mentioned above, E. Hanbury Hankin, M.A., Sc.D., also published occasional articles concerning Islamic geometric pattern derivation in the Mathematical Gazette. I am indebted to Dr. Carl Ernst for first bringing the work of Hankin to my attention in 1980. See
–Hankin (1925b), 371–373.
–Hankin (1934), 165–168.
–Hankin (1936), 318–319.

[56] –Bonner (2003).
–Kaplan (2005).
–Lu and Steinhardt (2007a).

[57] In 1987 I had the good fortune to see and photograph the Topkapi Scroll while it was on temporary display at the Topkapi Museum in Istanbul. Other than the publications of Ernest Hanbury Hankin, this was my first corroboration that the polygonal methodology I had developed independently, and had been employing as an artist for many years, was, in fact, historical.

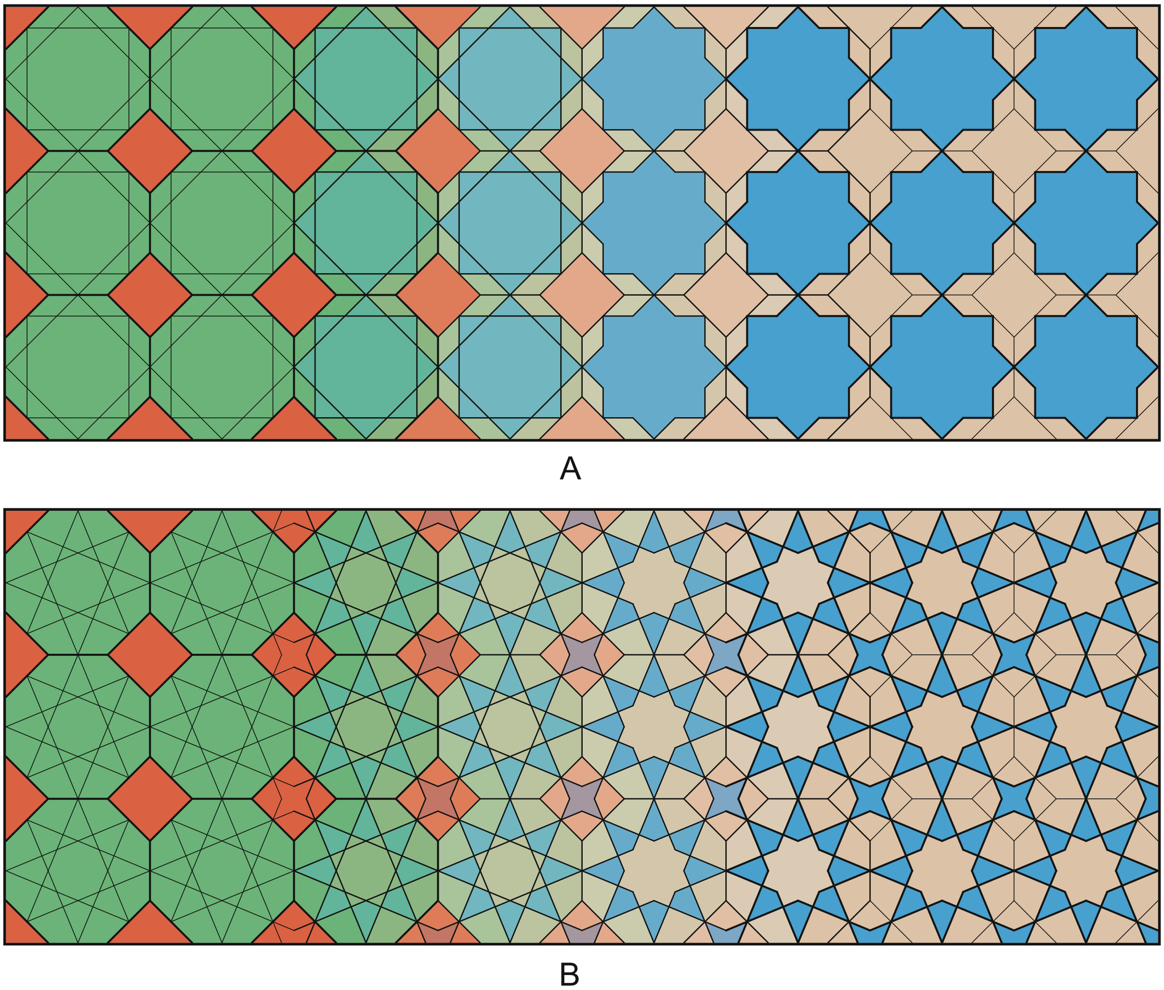


Fig. 58

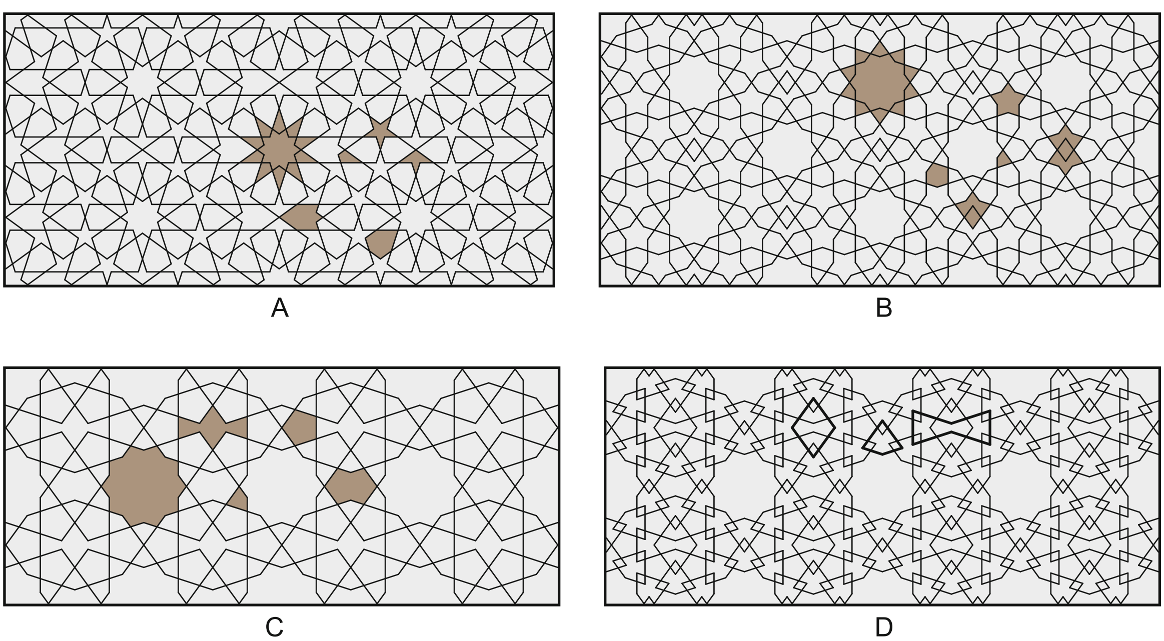


Fig. 59

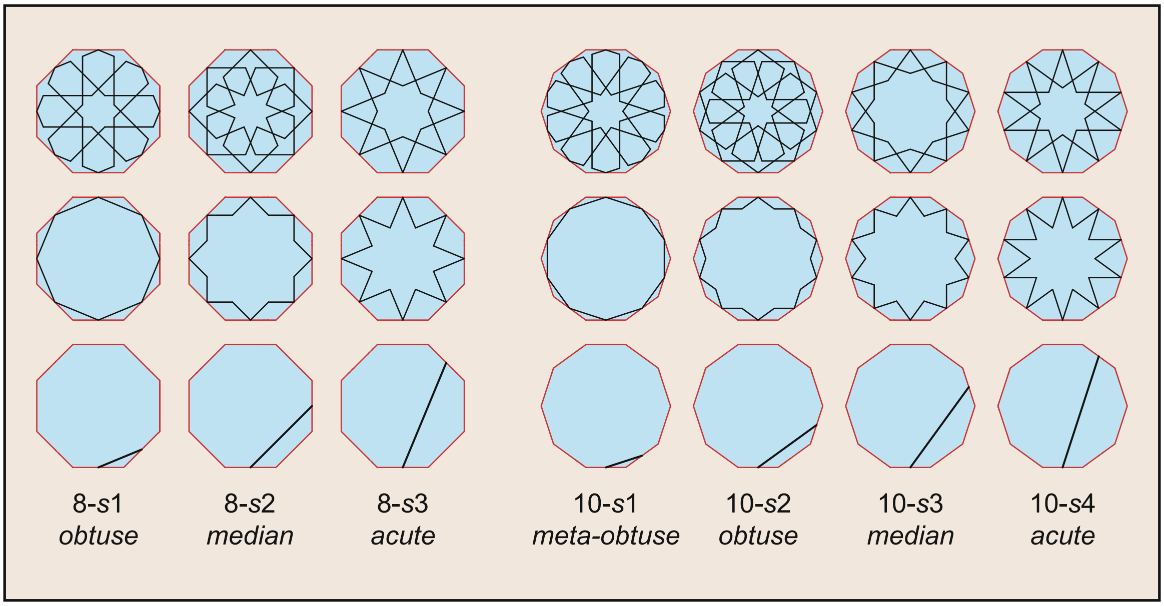


Fig. 60

pattern will typically have primary stars with points that are appreciably more obtuse, pentagons rather than five-pointed stars, kites, concave octagons, and distinctive hourglass polygons with ten sides. Figure 59d demonstrates the characteristics of the *two-point* family. This type of pattern is recognizable for its matrix of overlapping closed polygons, including kites, rhombi, and concave hexagons. Each of these families is subject to considerable stylistic variation and additive treatment, especially to the primary star forms.

In creating the primary stars, the application of pattern lines is most frequently determined by drawing lines that connect the midpoints of the primary underlying polygons. Figure 60 illustrates this process as applied to octagons and decagons as representative examples. The four pattern families are only descriptive of an aesthetic quality. For a more precise design classification it is often helpful to define the specific variety of star contained within the primary underlying polygons. The method illustrated roughly follows the nomenclature of Anthony J. Lee by identifying the number of sides of the primary polygon in relation to the number of sequential sides in midpoint-to-midpoint line application for creating a given star.[58] This way of identifying primary star forms is especially relevant to the regularity of systematic patterns. However, it is important to note that the application of pattern lines to primary underlying polygons does not always follow the convenient midpoint-to-midpoint method in this illustration. In some cases, especially with nonsystematic designs, the supplemental angles of the pattern lines that are placed at the midpoints of each edge of the primary underlying polygons are not determined by a straight line that connects to another midpoint. Rather, the precise angle of the pattern lines that are applied to these points, that ultimately determines the visual character of the primary star, is arrived at through aesthetic evaluation on the part of the artist. This decision is greatly influenced by how the extended lines behave within the adjacent secondary underlying polygonal cells. When this aesthetic approach is used, the identifying nomenclature of Fig. 60 is not applicable.

Every underlying generative tessellation is capable of producing a pattern from each of the four families. However, this is a nuanced discipline and not all of the patterns so generated will be acceptable to the aesthetic standards of this tradition. Prior to the maturity of Islamic geometric patterns, the approach to applying pattern lines onto underlying tessellations was less codified and more experimental. During the twelfth and thirteenth centuries, as part of the overall maturing of this artistic tradition, these four pattern families were established as distinct methodological aspects of the polygonal technique, each producing designs that were recognizably distinct from one another, and each with its own aesthetic appeal. What is more, the aesthetic predilections of different Muslim cultures favored, to a lesser or greater extent, specific pattern families over others, as well as certain additive variations that were frequently applied to these designs. The *acute*, *median*, and *obtuse* families differ according to the angle of the crossing pattern lines that are located at, or near, the midpoints of each underlying polygonal edge, while the *two-point* family has applied pattern lines placed on two points of each edge. Figure 61 illustrates

[58] Lee (1995), 182–197.

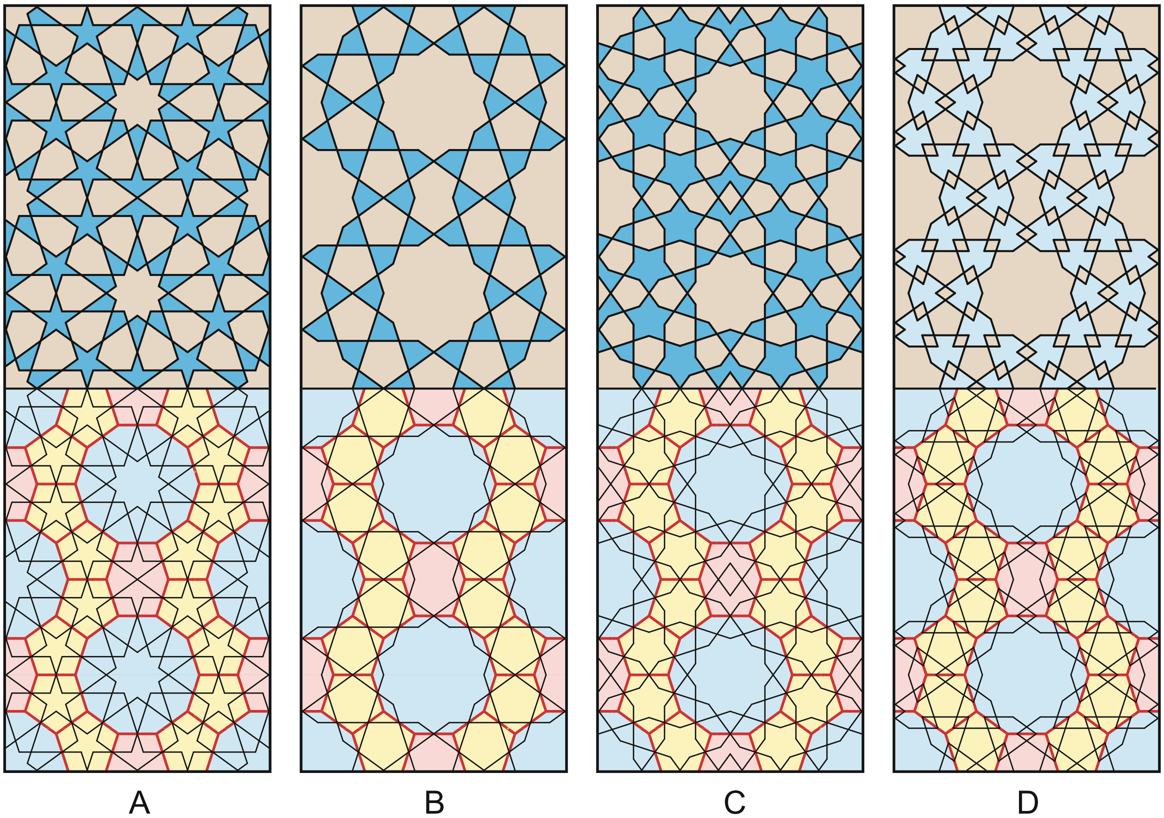


Fig. 61

these four pattern families as associated with an underlying tessellation of decagons, pentagons, and hexagons that repeat upon a rhombic grid. This is one of the most basic rhombic repeats produced from the *fivefold system*, and each of the four patterns created from this tessellation was used widely. Figure 61a shows the *acute* pattern created from this tessellation. The crossing pattern lines of *acute* patterns created from the *fivefold system* have a 36° angular opening at each midpoint of the polygonal edge. The bisector of the angular opening is perpendicular to the polygonal edge. These crossing pattern lines are easily determined by their transecting every second midpoint of the pentagons (5-*s*2), and every fourth midpoint of the decagons (10-*s*4). Figure 61b illustrates the application of the pattern lines of the *obtuse* family wherein the pattern lines transect the underlying pentagonal edges at adjacent midpoints (5-*s*1), and the decagons at every second midpoint (10-*s*2), creating crossing pattern lines at these midpoints with 108° angular openings. As stated, one of the visual characteristics of *obtuse* patterns is the occurrence of pentagons nested within the pentagons of the underlying tessellation—creating a more open aesthetic. Figure 61c shows the *median* pattern created from this tessellation. As the name implies, the angle of the crossing pattern lines is between the *acute* and *obtuse* angles. Within the *fivefold system* this is 72°. These lines are conveniently determined by transecting every third midpoint of the decagon (10-*s*3). Figure 61d illustrates the *two-point* pattern created from this tessellation. This variety of design employs two points on each underlying polygonal edge rather than just one, and the resulting designs are almost always given a widened line or interweaving line treatment (rather than the colored tiling treatment in this illustration). The above-mentioned angular openings in the four pattern families mentioned above are standard to the *fivefold system*. In other systems, and indeed in nonsystematic designs, the angles of the crossing pattern lines employed in each pattern family will vary according to the inherent geometry of the system. For example: in the *system of regular polygons* the *acute*, *median*, and *obtuse* angular openings are typically 60°, 90°, 120° respectively, whereas those of both fourfold systems will have 45°, 90°, and 135°, respectively. In each case, the aesthetic character of each pattern family is essentially the same.

The extraordinary design diversity provided by the polygonal technique necessitates further subcategorization beyond the four standard pattern families. As mentioned previously, Islamic geometric patterns created from the polygonal technique fall into two distinct categories: systematic and nonsystematic. Differentiation between these two types of design is a primary form of classification and is fundamental to a thorough understanding of this tradition. Systematic designs employ a limited set of polygonal modules, with associated applied pattern lines, that are assembled into different tessellations to create myriad designs. There are five polygonal design systems that were used historically: I have named these the *system of regular polygons*, the *fourfold system A*, the *fourfold system B*, the *fivefold system*, and the *sevenfold system*. The numeric values in the names of the four-, five-, and sevenfold systems reference the smallest value of rotational symmetry within the primary star forms (other than 2). The patterns that Hankin analyzed in his publications include both systematic and nonsystematic examples. Hankin's groundbreaking work on the polygonal technique did not identify systematic characteristics or differentiate between these two categories of design. As such, he does not appear to have recognized the systematic nature of the underlying polygonal modules in many of his reconstructions. The use of methodological systems provided geometric artists with a fast and accurate means of producing new and original geometric designs with great ease. With few exceptions, each of the five pattern families has a specific set of pattern lines associated with each polygonal module. The simplicity of creating systematic geometric patterns explains the vast number of examples from all but the *sevenfold system*. The patterns produced from this latter system are very beautiful, and the paucity of examples found within the historical record is more likely due to a limited number of artists trained in this system rather than any aesthetic distaste for this variety of design. With the exception of the *sevenfold system*, these design systems were first discovered by the author as systems *per se* in the late 1970s and early 1980s while working on polygonal design methodologies. These findings were first recorded in an unpublished manuscript in 2000,[59] and later published in 2003 in the paper *Three Traditions of Self-Similarity in Fourteenth and Fifteenth Century Islamic Geometric Ornament*.[60] In addition to Hankin, several authors had previously identified some of the underlying polygonal modules that make up the *fivefold system*, but only in relation to individual tessellations rather than as components of a flexible modular system with associated pattern lines.[61] More recently, in 2007 a limited subset of the *fivefold system* received significant public acclaim as the methodological basis employed in the production of an allegedly quasicrystalline design at the Imamzada Darb-i Imam in Isfahan some 500 years before the discovery of fivefold aperiodic tilings by Sir Roger Penrose.[62] The first published account of the *sevenfold system* as a historical design methodology was in 2012.[63]

Figure 62 illustrates a design, along with its generative tessellation, from each of these five polygonal systems. Figure 62a shows a design created from the *system of regular polygons* that is located at the Gök *madrasa* and mosque in Amasya, Turkey (1266-67); Figure 62b shows a design created by the *fourfold system A* that was used widely, with a particularly early example at the eastern tomb tower at Kharraqan, Iran (1067-68); Fig. 62c shows a design produced by the *fourfold system B* that was used ubiquitously by Muslim cultures; Fig. 62d shows a design created by the *fivefold system* from the Patio de las Doncellas at the Alcazar in Seville (1364); and Fig. 62e shows a design created by the *sevenfold system* that comes from the Sultan al-Mu'ayyad Shaykh complex in Cairo (1412-22). Except for the *fourfold system B*, which is more restricted due to the smaller number of modules, the polygonal modules in each of these systems can be assembled in an infinite number of tessellations, providing for an unlimited number of possible geometric patterns. And, as demonstrated, depending upon the angular opening of the crossing pattern lines located on the edges of the underlying polygons, no less than four distinct designs can be produced from any single tessellation, thus augmenting the already significant design potential within each of these systems.

Another variety of design classification is the differentiation within dual-level designs. These are associated most directly with the use of one or another of the design systems. As discussed in the previous chapter, these place scaled-down secondary modules from a given system into the pattern matrix of a design that was created from the same set of non-scaled-down modules. This variety of design is especially beautiful, and is the last of the great innovations associated with Islamic geometric patterns. Furthermore, many of these designs have geometric self-similarity whereby the qualities of the primary pattern are replicated within the scaled-down secondary pattern, and this recursive diminution can, in theory, be applied *ad infinitum*.

[59] Bonner (2000).

[60] Bonner (2003), 1–12.

[61] –Wade (1976) (after Hankin).
–Pander (1982).
–Makovicky (1992), 67–86.

[62] In 2007 Paul Steinhardt and Peter Lu published a paper citing their discovery of a set of "girih tiles" that share the inflation symmetry characteristics of the set of two prototiles with matching rules discovered by Sir Roger Penrose in the 1970s. The five "girih tiles" presented by the authors are, in fact, a subset of the ten polygonal modules with associated pattern lines detailed in my 2003 paper. Lu and Steinhardt's pattern lines for each "girih tile" are identical to the pattern lines of the *median* pattern family (with characteristic 72° crossing pattern lines located at the midpoints of each polygonal edge) that is described in detail in my 2003 paper. See
–Bonner (2003).
–Lu and Steinhardt (2007a).

[63] –Bonner and Pelletier (2012), 141–148.
–Pelletier and Bonner (2012), 149–156.

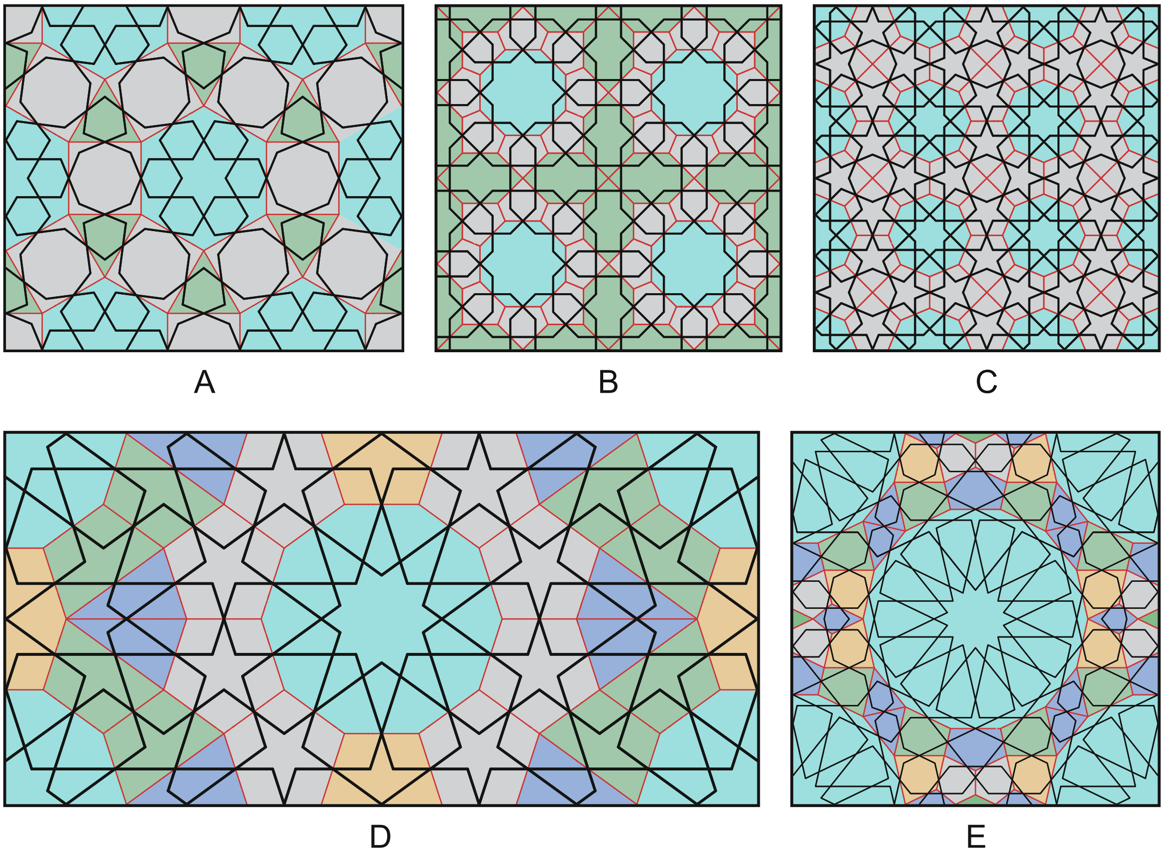


Fig. 62

This variety of Islamic geometric design has four distinct classifications: *type A, type B, type C, and type D* [Fig. 442].[64]

Nonsystematic designs created from the polygonal technique utilize underlying tessellations that include irregular polygons with proportions that are specific to the circumstances of the tessellation, and will not reassemble into other tessellations. This variety of geometric design is characterized by greater geometric complexity, often combining multiple centers of higher order local symmetry. As mentioned previously, the characteristic *n*-pointed stars are typically placed at the vertices of the repeat unit, and greater complexity is frequently achieved through placing further higher order star forms at the center of the repeat, at the midpoints of the edges of the repeat, and/or within the field of the repeat. These higher order stars are the product of their associated *n*-sided primary polygons within the underlying generative tessellation. A polygonal matrix comprised of smaller polygons, such as irregular pentagons and hexagons, separates the primary polygons from one another. Figure 63 illustrates three nonsystematic orthogonal tessellations in sequential levels of complexity. Figure 63a shows dodecagons at the vertices of the square repeat unit with a connecting matrix of pentagons. Four of the pentagons (yellow) are clustered at the center of the repeat and have a different proportion than the two separating the dodecagons (dark blue). A feature of this underlying tessellation is the *ring of pentagons* that surrounds each dodecagon. This is a common motif in both systematic and nonsystematic tessellations, and reliably provides distinctive and desirable visual characteristics to the completed designs in each of the four pattern families. This tessellation was used to produce a number of very fine geometric designs [Figs. 335 and 336], including an *acute* pattern from the Great Mosque of Siirt in Turkey (1129); a *median* pattern from the Great Mosque of Silvan in Turkey (1152-57); and an *obtuse* pattern that was used frequently throughout the Islamic world. Figure 63b also employs dodecagons at the vertices of the square repeat, with added octagons at the centers of each repeat. This tessellation has a ring of pentagons around each octagon, as well as a ring of pentagons and barrel hexagons around each dodecagon. This ring of pentagons with included hexagons is also commonly encountered in both systematic and nonsystematic underlying tessellations. This tessellation was used to create very successful designs in all four pattern

[64] See footnote 241 from Chap. 1.

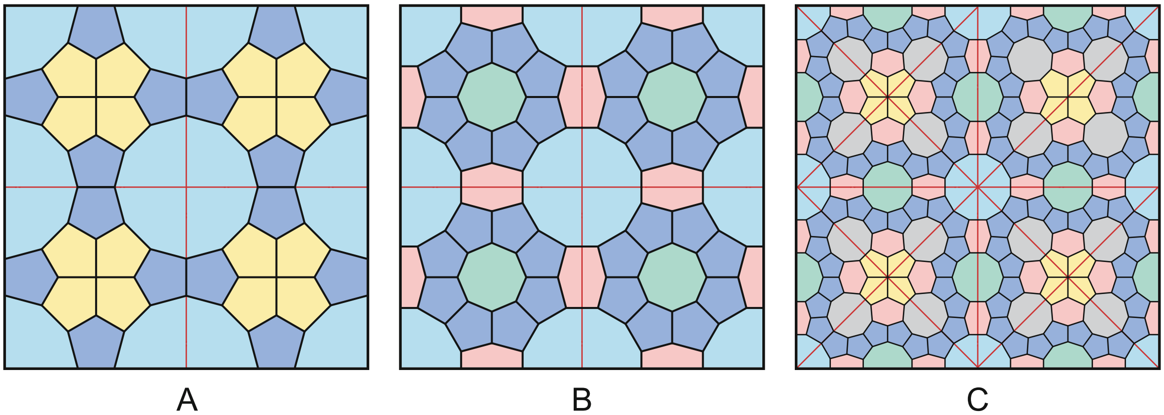


Fig. 63

families [Figs. 379–382]. The two regions of local symmetry allow for the 8- and 12-pointed stars that characterize these patterns. In addition to the dodecagons placed on the corners of the square repeat, Fig. 63c includes decagons placed at the midpoints of each repeat and enneagons placed upon the diagonal of the repeat unit. This is one of the most complex orthogonal generative tessellations employed within this overall tradition, and the *acute* pattern that this tessellation produces was used in several locations by Seljuk artists working during the Sultanate of Rum [Fig. 400]. The earliest example is from the Kayseri hospital (1205-06). This underlying tessellation also creates very attractive designs from the other three pattern families, although no historical examples are known. Patterns produced from this tessellation combine 9-, 10-, and 12-pointed stars.

A further subcategory of design created by the polygonal technique achieves greater complexity through added secondary pattern elements to an already existing design. Almost all such *additive patterns* are produced from one or another of the design systems: most frequently the *system of regular polygons* or the *fourfold system A*, although the *fivefold system* was occasionally used for additive modification. As with so many geometric design innovations, *additive patterns* were initially developed under the auspices of Seljuk influence, and early examples are found at the Gunbad-i Surkh in Maragha, Iran (1147-48), and the Gunbad-i Qabud in Maragha, Iran (1196-97). This additive practice was especially popular among artist working under the Ilkhanids in Persia. Figure 64 illustrates an Ilkhanid additive pattern from the portal of the Khanqah-i Shaykh ’Abd al-Samad in Natanz, Iran (1304-25), that is created from a very simple design from the *system of regular polygons*. The primary *median* pattern (blue) is generated from the underlying 6^3 tessellation of regular hexagons [Fig. 95c], and the additive component places octagons at each vertex of the primary design. The incorporation of octagons into a design with sixfold symmetry works by virtue of the 180° rotational symmetry at the vertices of the

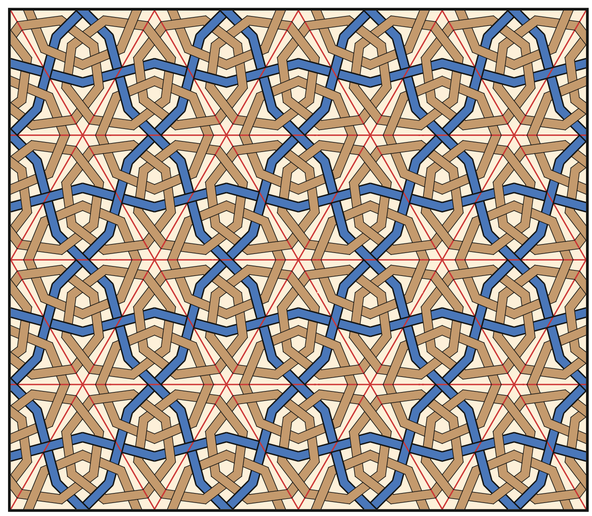


Fig. 64

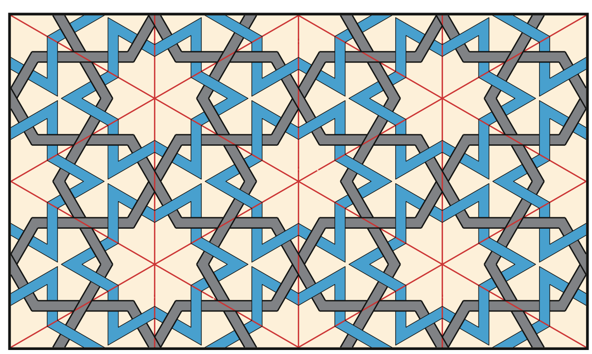


Fig. 65

primary design. Without the primary design, this arrangement of octagons placed upon the isometric grid is identical to the imposed symmetry design in Fig. 51a. The design in Fig. 65 is an Ilkhanid additive pattern from the interior of the Friday Mosque at Varamin, Iran (1322). The initial *acute*

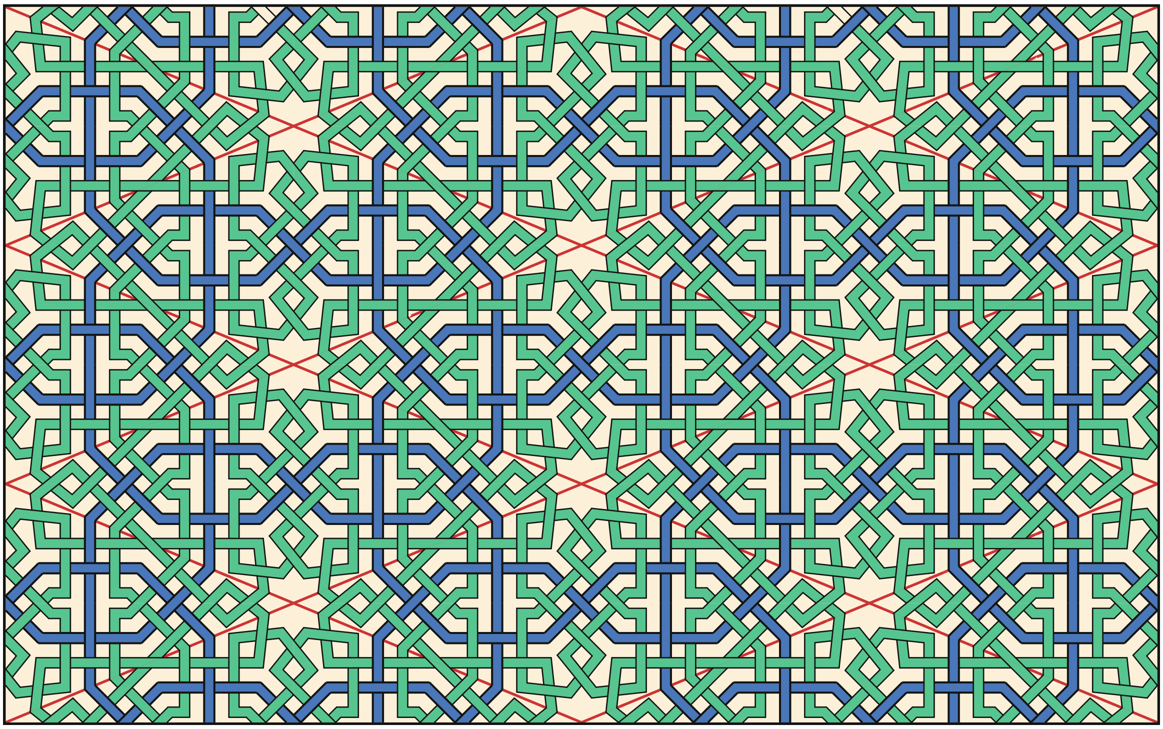


Fig. 66

pattern is created from the same simple hexagonal grid as the previous example, the difference being in the 60° angular openings of the crossing pattern lines at the midpoints of each underlying hexagonal edge [Fig. 95b]. The addition of a second 6-pointed star in 30° rotation placed on the same center point as in the original design creates a more complex pattern of 12-pointed stars. Figure 66 shows an outstanding Ilkhanid additive pattern from the mausoleum of Uljaytu in Sultaniya, Iran (1305-1313). The original design (blue) is created from the *fourfold system A* [Fig. 157b], and the secondary additive elements provide this otherwise rather simple design with a feeling of far greater complexity. Figure 67 is a detail of an *obtuse* additive pattern created from the *fivefold system* that was used at the Gunbad-i Qabud in Maragha, Iran (1196-97) [Photograph 24]. This is one of the most ambitious examples of additive pattern making, and is characterized by its unusually large repeat unit [Figs. 239 and 240]. This design anticipates the dual-level aesthetic that developed in the same approximate region some 250 years later.

Additive patterns are similar to a less common class of patterns that are comprised of two superimposed designs that are otherwise distinct from one another. And like additive patterns, most of these are derived from the *system of regular polygons*. These *superimposed patterns* are most common to Anatolia during the Sultanate of Rum, and Gerd Schneider provides multiple examples in his book devoted to the geometric ornament of this region.[65] Figure 68 illustrates a threefold superimposed pattern with one component being the classic threefold *acute* pattern with six-pointed stars [Fig. 95b], but with additive six-pointed star rosettes, created from the 6^3 hexagonal grid, and the second a well-known design of superimposed dodecagons created from the 3.6.3.6 underlying tessellation of triangles and hexagons [Fig. 99b]. This very fine superimposed pattern was used during the Sultanate of Rum at the Karatay Han (1235-41), 50 km east of Kayseri, Turkey,[66] as well as by the Timurids on a door from the mausoleum of Sayf al-Din Bakharzi in Bukhara[67] (fourteenth century).

To summarize, only the polygonal technique has been demonstrated to provide for so many fundamental features of this ornamental tradition, including a method for producing nonsystematic geometric patterns with greater complexity characterized by multiple centers of local symmetry; the occurrence of a multitude of designs with identical visual characteristics that result from the use of systematic polygonal methodologies; the extraordinary range of symmetrical and repetitive diversity that results from manipulations of this

[65] Schneider (1980), pl. 22–23.

[66] Schneider (1980), pattern no. 253.

[67] In the collection of the Victoria and Albert Museum, London, acc. No. 437–1902.

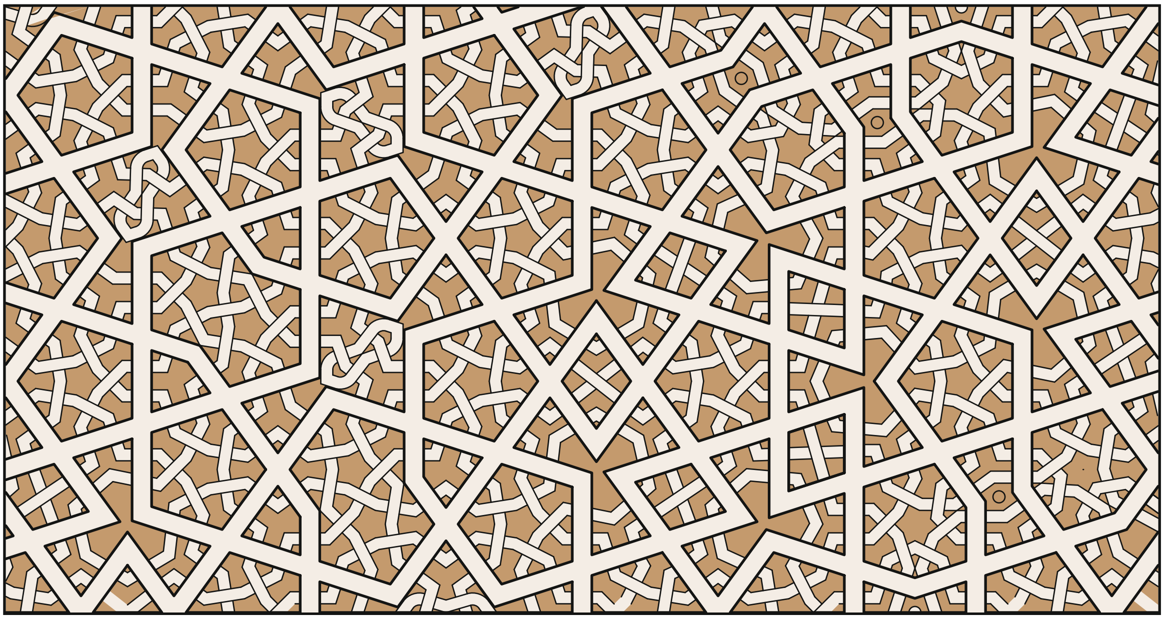


Fig. 67

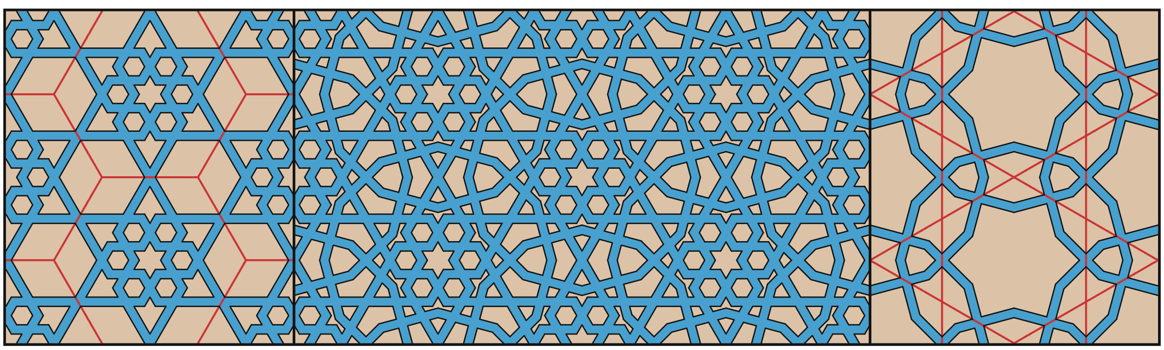


Fig. 68

design methodology; the four pattern families that are ubiquitous to this tradition; and the means for producing highly complex dual-level designs with recursive characteristics.

2.5.2 The Point-Joining Technique

In addition to the polygonal technique, many of the less complex Islamic geometric patterns can also be produced from alternative design methodologies. This is especially true with the more basic patterns created from the *system of regular polygons* and *fourfold system A*. However, there is sparse historical evidence for the use of these other methodologies. By contrast, and as demonstrated, the historical significance of the polygonal technique is supported by a preponderance of evidence. Nonetheless, among some of the less complex historical designs, it is not possible to know categorically whether they were produced from the polygonal technique or an alternative methodology. There are two alternative design methodologies that have been proposed as traditional and each has merit as a vehicle for constructing geometric patterns. For the purpose of descriptive clarity, these are referred to herein as the *point-joining technique*, and the *graph paper technique*.

Since the 1970s, the point-joining technique has been advanced by a number of proponents,[68] causing it to gain support as the dominant historical design methodology

[68] –Maheronnaqsh (1976).
–El-Said and Parman (1976).
–Critchlow (1976).
–Wade (1976).
–Bakirer (1981).
–El-Said (1993).
–Marchant (2008), 106–123.
–Broug (2008, 2013).

among the interested public. However, the multiple publications that advance the point-joining technique do not provide evidence for the historical use, let alone primacy, of this methodology. Much of the enthusiasm during the 1970s for the point-joining technique stemmed from its conflation with esotericism[69] wherein "the harmonious division of a circle is no more than a symbolic way of expressing *tawhid*, which is the metaphysical doctrine of Divine Unity as the source and culmination of all diversity."[70] The problem with ascribing metaphysical symbolisms to this design methodology, and indeed to the tradition of Islamic geometric design generally, is similar to that of point-joining itself: the authors provide no evidence for its historicity.[71] The general method behind point-joining constructions involves the use of a compass and straight edge: typically starting with a circle that is divided and subdivided to produce a square or regular hexagonal repeat unit, from which further divisions lead to the construction of a matrix of geometric coordinates. Lines that connect selected intersection points within this matrix will produce the completed design within its repeat unit. A fundamental feature of this technique is that each individual pattern has its own unique step-by-step construction. This is a formal process that lacks flexibility, and while it is well suited to reproducing existing designs with low to moderate complexity, it is not an especially convenient method for creating original designs. This limitation is exponentially true for creating designs with greater complexity, such as those with multiple centers of local symmetry. Even the reconstruction of preexisting patterns with particularly complex compound local symmetries via step-by-step point-joining constructions is extraordinarily cumbersome at best, and for all intents and purposes impracticable. What is more, to use this methodology to *originate* such designs begs credulity. The required independent point-joining construction for each individual pattern is in marked distinction to the inherent flexibility of the polygonal technique. With point-joining, an artist is limited by the number of patterns that have been put to memory, or that have been recorded with instructions on paper. By contrast, an artist with knowledge of the polygonal technique is able to create an unlimited number of original designs, or easily recreate existing designs as needs be. What is more, the polygonal technique is ideally suited to creating exceptionally complex patterns with multiple centers of local symmetry.

[69] –El-Said and Parman (1976).
–Critchlow (1976).
–Burckhardt (1976).

[70] From the forward by Titus Burckhardt: El-Said and Parman (1976).

[71] The popularized claims, advanced during the 1970s, that Islamic geometric patterns are inherently associated with perennial symbolisms have been convincingly refuted as ahistorical by several scholars of note: See
–Chorbachi (1989), 751–789.
–Necipoğlu (1995), 73–83.

Yet despite its limitations the point-joining technique appears to have played an important role in the history of this artistic tradition. The polygonal technique requires a high level of commitment to master, and clearly not all artists working in diverse media, and at varying degrees of geometric skill, would have received training in this design methodology. What is more, it is reasonable to assume that the transmission of the polygonal technique was formal and controlled, thereby protecting the patronal support and financial interests of the practitioners. As such, the primary role of the point-joining technique may have been as a means of providing specific instruction for individual designs to artists and craftsmen who needed access to geometric patterns, but were not privy to the methodological practices of the polygonal technique. In this way, a wide variety of geometric designs could have been introduced into the canon of general artists and craftspeople, thereby disseminating these designs into the wider cultural milieu while simultaneously protecting the interests of the specialized artists responsible for their original creation.

It is also likely that the point-joining technique occasionally provided a convenient means of scaling up patterns for their transferral to architectural surfaces. Due to the complexity limits of point-joining, this would only have been suitable with patterns of low or intermediate complexity, and artists working with more complex designs would have required alternative methods for accurately transferring scaled-up patterns for architectural locations—as per the above-referenced evidence of the polygonal technique revealed in the ceiling at Fatehpur Sikri.

Historical evidence for the point-joining technique is sparse. One rather amusing early twentieth-century anecdotal example comes from Archibald Christie who wrote:

> Oriental workers carry intricate patterns in their heads and reproduce them easily without notes or guides. There is a story that tells of an English observer, seeing a most elaborate design painted directly on a ceiling by a young craftsman, (the Englishman) sought the artist's father to congratulate him on his son's ability, but the father replied that he regarded the boy as a dolt for he knew only one pattern, but his brother was a genius—he knew three![72]

All humor aside, this story is revealing in that it relays the mnemonic practices of artists working with geometric patterns: albeit very late in the history of this tradition. While this anecdote tells us that at least some artists were reliant upon memory to recreate patterns within their limited repertoire, it also implies that such artists lacked the necessary skills that would allow them to create original designs.

[72] Christie (1910).

However, considering the vast number of patterns from the historical record, it is unlikely that these specific point-joining constructions were held within memory alone, and it must be assumed that design scrolls and manuals were employed to a greater or lesser extent in propagating the recreation of existing designs. Regrettably few artists' scrolls (*tumar*) or bound manuscripts are known to have survived to the present, and one hopes that more will turn up with time.[73] Two are of particular importance to the question of traditional geometric design methodology: the aforementioned Topkapi Scroll and the anonymous Persian language treatise *On Similar and Complementary Interlocking Figures* in the Bibliothèque Nationale de France in Paris,[74] henceforth referred to as *Interlocking Figures*. The exceptional significance of this treatise is that the illustrations are accompanied with written step-by-step instructions for constructing the diverse range of geometric figures, including multiple geometric patterns. Except for those more complex examples that involve either conic sections or verging procedures, some of these instructions are very similar in concept to the point-joining methodology advocated since the 1970s. This is currently the only known ancient treatise that provides written instructions for constructing geometric patterns, some of which are found within the historical record. *Interlocking Figures* illustrates over 60 geometric constructions, most of which are accompanied with written instruction. Like the Topkapi Scroll, the illustrations are inked in black and red, with occasional dotted lines that provide further differentiation. The provenance of *Interlocking Figures* is uncertain and speculations for its date of origin have been based upon both linguistic analysis and comparisons with identical or near-identical geometric patterns within the architectural record.[75] Estimates for its date range between the eleventh and thirteenth centuries during either the Great Seljuk or Khwarizmshahid periods, with some portions added as late as the Timurid period when the Paris manuscript was copied. More recent research estimates its origin to circa 1300, the later end of this spectrum.[76] The problem with comparing specific patterns from *Interlocking Figures* to architectural examples from the historical record as a means of estimating the approximate date of its original compilation is that it is impossible to know whether (1) the manuscript may have preceded and possibly influenced an architecture example, and, if so, by how long; (2) the manuscript and architectural examples were produced concurrently, possibly by the same individuals; or (3) the production of a given architectural example may have preceded and possibly influenced the manuscript, and, if so, by how long. Adding to this uncertainty is the fact that it is not known how many times the original manuscript may have been copied, and to what extent the copyists may have included examples of patterns from later dates. Nonetheless, at the very least, comparisons to the architectural record are a valuable means of contextualizing the geometric patterns in *Interlocking Figures*.

The illustrations in *Interlocking Figures* fall into several categories, including mathematical dissections of polygonal figures that can be reassembled into other figures, and can be regarded as sophisticated geometric puzzles; instructions pertaining to the construction of geometric figures such as triangles, pentagons, heptagons, and nonagons; three figures without explanatory text that appear to be *muqarnas* plan projections; and multiple examples of geometric designs ranging from the simple to moderately complex. The question naturally arises: Who created *Interlocking Figures*, and for what purpose? Alpay Özdural makes a compelling case for this treatise having possibly been compiled by a scribe as a record of meetings, or *conversazioni*, between artists and mathematicians over an unspecified period of time.[77] Gülru Necipoğlu suggests that the "anonymous author . . . seems to have been a *muhandis* with practical rather than theoretical training in geometry," and that some of the more complex mathematically precise constructions requiring an angle-bracket and conic sections were followed by instructions for simplified constructions that rely on approximations.[78] Both of these scholars place *Interlocking Figures* into context with other more widely known collaborations between medieval Muslim artists and mathematicians whereby the edification of the geometric arts was facilitated in part through the direct influence of mathematicians. Of particular note is the celebrated treatise by Abu al-Wafa al-Buzjani (940-998): *About that which the artisan needs to know about geometric constructions*. In fact, along with other works on geometry, *Interlocking Figures* is appended to a copy of this work by al-Buzjani. The general consensus among art

[73] The most comprehensive study of known pattern manuals and scrolls is that of Gülru Necipoğlu. See Necipoğlu (1995).

[74] MS Persan 169, fol. 180a–199a. For a thorough account of the significance of this manuscript as one of the very few historical Muslim sources of geometric analysis and instruction for Islamic geometric designs, and for its place among other historical documents concerned with the practical application of mathematics, see
–Chorbachi (1989), 751–789.
–Chorbachi (1992), 283–305.
–Necipoğlu (1995), 131–175.
–Özdural (1996), 191–211.
–Necipoğlu [ed.] (forthcoming).

[75] –Necipoğlu (1995), 168–169.
–Özdural (1996), 191–211.

[76] Necipoğlu [ed.] (forthcoming).

[77] Özdural (1996), 192.

[78] Necipoğlu (1995), 169.

historians is that *Interlocking Figures* was intended, at least in part, to assist artists to better understand more advanced geometric principles, and where necessary to familiarize them with approximate constructions of figures that otherwise require more complex procedures. Through this lens, the multiple geometric designs included in *Interlocking Figures* are similarly seen as instructions intended for artistic application. However, there is another interpretation of this important historical treatise. It is also possible that the focus upon geometric patterns within *Interlocking Figures* was the result of a fascination among some mathematicians to better understand the underlying geometry of an art form that was pervasive throughout their culture. Seen from this perspective, the step-by-step constructions in *Interlocking Figures* are not so much instructions for artists as exercises for students of geometry. Were this the case, these medieval constructions would be analogous to contemporary point-joining constructions promoted by multiple Western authors since the 1970s whereby people with an interest and facility with geometry and an appreciation for Islamic geometric patterns analyzed specific designs to better understand their geometric nature by creating step-by-step construction sequences.

The fact that *Interlocking Figures* is the only known historical treatise that accompanies the illustrations of geometric patterns with instructional text, coupled with the simplified instructions for creating approximate constructions that would otherwise require far greater mathematical sophistication, would appear to be a persuasive argument for the step-by-step instructions being representative of the primary methodology responsible for this geometric art form.[79] Even prior to *Interlocking Figures* becoming known to the public through the work of Wasma'a Chorbachi,[80] as mentioned, the conviction that point-joining was the preeminent design methodology employed by Muslim geometric artists had been promoted by several authors since the 1970s. More recently, the polygonal technique has become increasingly accepted as especially relevant to the development of Islamic geometric patterns—especially considering the multiple examples of historical evidence for this methodology. Despite the growth in acceptance of the polygonal technique, the historical significance of the point-jointing methodology is central to any serious study of Islamic geometric design, and all the more so in light of the constructions contained within *Interlocking Figures*.

The argument for the more exulted significance of this treatise, whereby artists were provided with necessary approximate solutions to geometric figures through collaboration with mathematicians, runs as follows: (1) there was a desire to create designs with geometric figures, such as heptagons and nonagons, that required advanced mathematical skill, such as intersecting conic sections, to produce mathematically correct constructions; (2) these mathematically correct constructions were beyond the intellectual or practical abilities of artists working in the geometric idiom; (3) and therefore mathematicians working with artists produced simplified step-by-step instructions for various geometric figures and individual patterns that would approximate true mathematical accuracy through what is described herein as point-joining constructions. The first two parts of this proposition assume that artists, in their wish to produce more complex designs employing less straightforward *n*-fold rotational symmetries, would not have conceived the very simple method of dividing the circumference of a circle or arc into a desired number of equal segments, or modular units, using a pair of dividers or compass [Fig. 295]. While this does not provide true mathematical precision, it is very fast, and no less accurate from a practical standpoint. Whether working with intersecting conic sections or the simple division of a circle's circumference into equal units, the drawing of any figure, let us say a nonagon, requires the use of tools such as dividers, straightedge, and set squares. The use of these tools can never be mathematically precise: the point of the divider will never fall at the theoretically correct intersection; the opening of the divider will never precisely conform with the precise mathematical distance; and a line between two points will never connect with absolute mathematical precision. The more steps in a handmade geometric construction, the greater the compounding error. Maintaining our example, the simple division of a circle's circumference into nine segments requires fewer steps than creating a nonagon through intersecting conic sections, and the end result is no less accurate from a practical standpoint. In addition to more complex formulae, *Interlocking Figures* indeed makes reference to this type of mathematical approximation.[81] But to assume that the presence of this divisional methodology in some of the provided instructions is an indication that artists needed to be taught this very simple procedure is disingenuous to the intelligence and innovative spirit of artists, who were, let us not forget, already well advanced in producing highly complex patterns by the time of this treatise's likely creation. This calls into question the third part of the above

[79] –Chorbachi (1989), 776.
–Bulatov (1988), 52.

[80] Chorbachi (1989), 751–798.

[81] For example: "But we have found a technique of approximation (*taqrīb*) that, whenever we divide a right angle into nine equal parts, four parts of that angle are آ ح ب and five parts are ب و د. And this is the limit of approximation." MS Persan 169, fol. 190a (upper right corner, diagonal text of four lines). Translation by Carl W. Ernst, Kenan Distinguished Professor of Religious Studies, The University of North Carolina at Chapel Hill.

proposition: that artists required mathematicians to produce simplified instructions for the construction of individual patterns. If artists' innate practical skills meant that they were not reliant upon mathematicians to create such geometric masterpieces as the sevenfold designs on the façade of the minaret of Mas'ud III in Ghazna, Afghanistan (1099-1115) [Figs. 280 and 281], or the design with seven- and nine-pointed stars that surrounds the *mihrab* at the Friday Mosque at Barsian, Iran (1105) [Fig. 429], then the direct contribution of mathematicians toward the growth of sophistication and maturity in this ornamental tradition becomes less significant. And if other design methodologies, such as the polygonal technique, are demonstrably superior in their ability to generate new and original designs, and if this is supported by the preponderance of historical evidence, then elevating the methodological significance of *Interlocking Figures* would appear open to question.

Many features of the anonymous *Interlocking Figures* do not support the premise that this was a manual prepared for use by artists to better equip them in their use of these construction sequences, herein referred to as point-joining, as a primary design methodology for creating new patterns. Nowhere within the text does it state that the work is intended for artists. In fact, the only references to artists within this document pertain to specific constructions used by some artists to construct rather simple designs.[82] In short, the author appears to be more influenced by artists than influence upon them. And while certainly intriguing, the large portion of this treatise dedicated to geometric dissections does not appear to be of any practical use to artists working with geometric design. Similarly, many of the instructions are of questionable relevance to artists. For example, the multiple permutations on the construction of the pentagon would have no practical value to geometric artists who it can be presumed would be very familiar with the construction of this simple figure. The inclusion of these instructions appears to corroborate a fascination with diverse geometric solutions as intellectual exercises. Significant attention is also given to the construction of the heptagon and nonagon; but as mentioned, segmenting the circumference of a circle with a pair of dividers was a more practical way of accurately producing these polygons.

One possible reason for the preponderance of point-joining instructions in *Interlocking Figures* could have to do with the very different functions that these two design methodologies appear to have within this ornamental tradition. The polygonal technique, in both its systematic and nonsystematic variants, is predisposed to the creation of new designs. By contrast, point-joining does not conveniently lend itself to designing original patterns, but is an effective method for recreating existing designs. As proposed above, if indeed the point-joining technique was used principally for reproducing existing patterns by artists and craftspeople not otherwise trained in the very specific methodology of the polygonal technique, then it would appear reasonable to consider the possibility that the intention behind the constructions for specific geometric patterns in *Interlocking Figures* may have been to develop step-by-step instructions for such non-specialized artists and craftspeople. If this was indeed the case, *Interlocking Figures* provides important evidence of how specific geometric patterns were introduced and disseminated to artists and craftspeople throughout Muslim cultures without jeopardizing the exclusivity of methodological knowledge among the actual originators of such patterns.

Several of the geometric patterns included in *Interlocking Figures* are also found within the architectural record. Of particular interest is the presence of two notable examples from this treatise that are also found within the northeast dome chamber of the Friday Mosque at Isfahan (1088-89). Indeed, there appears to be more than a coincidental relationship between *Interlocking Figures* and this remarkable architectural monument. If the 1300 date attributed to *Interlocking Figures* is correct, the examples within the northeast dome chamber precede this treatise by approximately 200 years.[83] Figure 57 illustrates one of the most remarkable patterns from *Interlocking Figures*: the aforementioned design with sevenfold symmetry that is the only example from this treatise that includes an underlying generative tessellation typical to the polygonal technique. Considering the possibility of an earlier date of origin, Jan Hogendijk has suggested that the occurrence of this heptagonal pattern in both the anonymous treatise[84] and the northeast dome chamber of the Friday Mosque at Isfahan [Photograph 26] may indicate that the same individuals produced both during the same period, and that the presence of Omar Khayyam (1038-1141), the great Persian mathematician and poet, in Isfahan during the construction of the

[82] –"Some craftsmen (*ṣunnā*') draw this problem in such a way that they take its height as seven portions and its width as six portions. The magnitude ('*uẓm*) is close." MS Persan 169, fol. 187b (four lines of upside down text at the corner of the large rectangle). Translation by Carl W. Ernst, Kenan Distinguished Professor of Religious Studies, The University of North Carolina at Chapel Hill.

–"Masters perform a test of the proportion of this problem, and Abu Bakr al-Khalil has performed the test by several methods (*wajh*, lit."face") and has achieved it. One of those [methods] is the following, which has been commented upon." MS Persan 169, fol. 189a. (bottom three lines of main text). Translation by Carl W. Ernst, Kenan Distinguished Professor of Religious Studies, The University of North Carolina at Chapel Hill.

[83] This 200-year discrepancy diminishes arguments for the importance of this treatise to the development of this geometric idiom.

[84] MS Persan 169, fol. 192a.

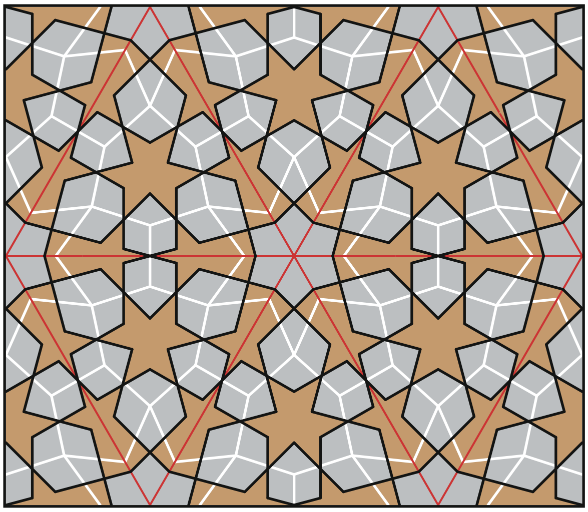


Fig. 69

northeast dome suggests the possibility that he may have been associated with this process.[85] This heptagonal pattern was used in the tympanum of one of the eight recessed arches beneath the cupola. Whether or not *Interlocking Figures* dates to this earlier period, or involved Omar Khayyam in its preparation, it would appear significant that another one of these eight arches from the northeast dome chamber employs a pattern that is almost identical to a design represented within *Interlocking Figures*.[86] This nonsystematic design is represented in Fig. 69, and is characterized by six-pointed stars placed at the vertices of the isometric grid. The only differences between the design from *Interlocking Figures* [Fig. 309b] and that from the recessed arch in Isfahan [Fig. 309a] [Photograph 27] are slightly different angles in the layout of the pattern lines, as well as the absence of regular hexagons centered at each vertex of the isometric grid. Although only small changes, the slightly adjusted pattern angles and the inclusion of the hexagons in the architectural example from Isfahan result in a significant improvement to what is already a successful design. In particular, these changes produce regular heptagons and attractive five-pointed stars within the pattern matrix. The inclusion of the regular heptagons would appear to be a willful corollary with the above-referenced heptagonal pattern in one of the neighboring recessed arches in this domed chamber. A nearly identical example of this nonsystematic design in Isfahan is also found in the Zangid doors in the portal of the Nur al-Din Bimaristan in Damascus (1154) [Fig. 309c]. The pattern in these doors has slightly more acute angles, as well as added geometric rosettes in place of the six-pointed stars. All three of these examples are easily created from the same underlying generative tessellation. However, the illustration and written instructions in *Interlocking Figures* do not include the underlying generative tessellation. The point-joining construction sequence provided in the text of this manuscript is insufficient to complete the design, although a person familiar with this design tradition could reasonably extrapolate the complete design from the instructions provided. However, this extrapolation requires advanced knowledge of the desired end result, making the instructions unsuitable for teaching this design to anyone not already familiar with it. Be that as it may, the fact that both the heptagonal design in Fig. 57 and the nonsystematic isometric pattern in Fig. 69 were used in the Seljuk ornament of the northeast dome chamber of the Friday Mosque at Isfahan suggests the possibility that the compilers of *Interlocking Figures* were very likely familiar with this building.

There are two patterns in *Interlocking Figures* that are characterized by 10- and 12-pointed stars. With the exception of a very unsuccessful pattern with six-, seven-, and eight-pointed stars (see below), these are the only patterns represented with compound local symmetry. The first example with 10- and 12-pointed stars[87] is identical to a design from the Great Mosque of Aksaray in Turkey (1150-53) [Fig. 414]. Nonsystematic patterns that employ two seemingly incompatible regions of local symmetry within the pattern matrix were an early twelfth-century innovation; and notable twelfth-century examples include a very successful design with 7- and 9-pointed stars at the Friday Mosque at Barsian, Iran (1105) [Fig. 429], and an outstanding design with 11- and 13-pointed stars at the mausoleum of Mu'mine Khatun in Nakhichevan, Azerbaijan (1186) [Fig. 434] [Photograph 35]. As with the design from both *Interlocking Figures* and the Great Mosque of Aksaray, compound patterns with disparate local symmetries frequently rely upon more complex repetitive stratagems that transcend the more prosaic orthogonal and isometric grids. As demonstrated previously, this variety of more complex geometric design will frequently utilize either rectangular or non-regular hexagonal repeat units. Compound patterns are not a feature of Islamic geometric ornament prior to the early twelfth century. Both of these patterns with 10- and 12-pointed stars from *Interlocking Figures* repeat upon a rectangular grid, and it is worth noting that the first of the two is also represented in the Topkapi Scroll[88] wherein it is illustrated along with its underlying generative tessellation. The second design from *Interlocking Figures* with 10- and

[85] Hogendijk (2012), 37–43.

[86] MS Persan 169, fol. 193a.

[87] MS Persan 169, fol. 195b.

[88] Necipoğlu (1995), diagram no. 44.

12-pointed stars[89] is not presently known within the architectural record, although it is a beautiful design, and fully in keeping with the mature style of compound patterns. This also repeats upon a rectangular grid, but its underlying generative tessellation is completely different. While the stylistic character of this design is fully in keeping with similar patterns with compound local symmetry that were created during the period of heightened maturation, the fact that this illustration from *Interlocking Figures* is not accompanied by any additional construction lines or instructional text, and that it is near the end of the manuscript, suggests that this may have been added when the manuscript was copied at a later date. Similarly, this is likely true of the very last design in the manuscript, whose swastika aesthetic suggests a later Timurid origin.[90]

Some of the examples in *Interlocking Figures* that have been identified as geometric patterns intended for ornamental use appear to actually be mathematical exercises without ornamental utility. In particular are two varieties of motif based upon the rotational application of quadrilateral kite shapes. Great emphasis was given within *Interlocking Figures* to these two types of motif, with multiple constructions in each case. In their ideal form, both require conic sections to accurately construct the triangles that comprise these motifs, and in each case, the multiple construction sequences provide approximate solutions for their production without conic sections. Ten of the figures in *Interlocking Figures* are step-by-step point-joining instructions for constructions comprised of quadrilateral kites in fourfold rotation within a square. The kites are subdivided into secondary quadrilaterals and triangles.[91] Without their subdivision, most of these figures are similar to the rotating kite constructions that were frequently used in Islamic architectural ornament, with early examples found in the brickwork façade of the western tomb tower at Kharraqan (1093) [Fig. 27c], and on a wooden door at the Imam Ibrahim mosque in Mosul[92] (1104). The compilers of *Interlocking Figures* paid considerable attention to approximate constructions of a rotating kite design with the specific geometric proportion wherein the altitude of the right triangle plus the shortest edge is equal to the hypotenuse. In the text associated with one such construction, differentiation is made between Ibn e-Heitham's method of constructing this triangle with conic sections, hyperbola and parabola, and the provided construction using a T-square.[93] In his paper that references *Interlocking Figures* Jan Hogendijk has pointed out that Ibn e-Heitham is the Persian form of Ibn al-Haytham (965-1041), an important Arab mathematician and astronomer (Alhazen) who was interested in conic sections, but whose work on this triangle is missing. Jan Hogendijk also points to the fact that Omar Khayyam (1038-1141) was also concerned with this triangle, describing it in his *treatise on the division of the quadrant*.[94] It is important to note that the fundamental constituent of a rotating kite design is a right triangle that is mirrored on its hypotenuse to create the kite motif, and that a rotating kite pattern can be made from any right triangle.[95] There are, therefore, a theoretically infinite number of rotating kite patterns—each with common 90° angles and differing pairs of acute angles.[96] It is worth noting that the proportions of the rotating kite patterns used at both Kharraqan and Mosul, and in fact almost all examples from the ornamental record, have a very simple construction that produces a specific proportion wherein the length of the edge of the central square is equal to the shortest edge of the surrounding kites. This proportion is very pleasing to the eye, but is not present in the multiple examples in *Interlocking Figures*. There are a number of ways that this visually more pleasing rotating kite motif can be easily constructed, including from a simple square or a 3 × 3 grid of nine squares [Fig. 27]. Similarly, Abu al-Wafa al-Buzjani (940-998) provided an elegant and equally simple square-based method for drawing the identical fourfold rotating motif in his *About that which the artisan needs to know about geometric constructions*.[97] *Interlocking Figures* includes five approximate constructions for the rotating kite motif created from the triangle described by Omar Khayyam. The compiler's reference to Ibn al-Haytham is a clear indication that they knew that this triangle required conic sections for a precise mathematical construction, and their reason for including the multiple approximate constructions has been proposed as a simplified approach

[89] MS Persan 169, fol. 196a.

[90] Several scholars have suggested the possibility that some of the illustrations in *Interlocking Figures* may date to the Timurid period. Gülru Necipoğlu has specifically referenced the final illustration, with its distinctive swastika aesthetic, as likely of Timurid origin, but goes on to mention the possibility of an earlier origin: "even this last pattern is not inconsistent with an earlier medieval repertory," Necipoğlu (1995), 180 [Part 4, note 113].
–MS Persan 169, fol. 199a.
–Bulatov (1988).
–Golombek and Wilber (1988).

[91] –MS Persan 169, fols. 188a, 189b, and 19a.
–Jan Hogendijk refers to this motif as the 12 kite pattern. See: Hogendijk (2012), 37–43.

[92] Wasma'a Khalid Chorbachi compares the rotating kite designs from the anonymous manuscript to multiple historical examples, including the door from Mosul. See Chorbachi (1989), 751–789.

[93] MS Persan 169, fol. 191a.

[94] Hogendijk (2012), 37–43.

[95] The one exception to the rule that all right triangles will produce rotating kite designs is in the case of the isosceles triangle with equal 45° acute angles. When this is mirrored along its long side it produces a square rather than a kite.

[96] Cromwell and Beltrami (2011), 84–93.

[97] Chorbachi (1989), 769.

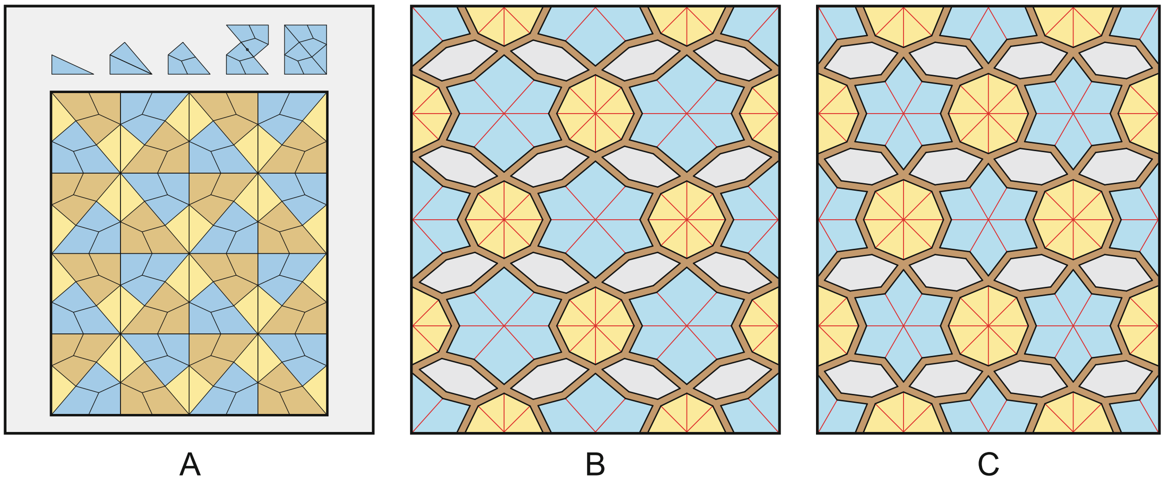


Fig. 70

for artists who would not have known the methodology of conic sections.[98] However, there are two problems with this assertion. Firstly, the visual character of the subdivided rotating kite designs in *Interlocking Figures* does not conform to the many examples of fourfold rotating kite designs from the architectural record. These are invariably non-subdivided. Secondly, despite assertions to the contrary,[99] the very specific geometric proportions of the rotating kite motif that results from using Omar Khayyam's elusive triangle do not appear to have been used within the architectural record. Furthermore, the supposed need to educate artists in the difficulties of creating the rotating kite motif is not supported by the fact that the examples from the architectural record are very easy to produce. It would therefore appear that the focus upon approximate constructions for these subdivided rotating kite designs with Khayyam-like proportions was more an intellectual exercise on the part of the mathematicians who compiled this section of the treatise, and less a product for use by artists in their ornamental constructions.

The other rotational kite motif that received almost equal attention in *Interlocking Figures* is comprised of two kite motifs in twofold rotational point symmetry that are placed within a bounding rectangle. This treatise contains seven separate constructions for this motif, and as with several of the fourfold rotational kite examples, these kites are subdivided into three polygons that maintain the bilateral symmetry of the original kite. Figure 70a is constructed from the point-joining instructions given in fol. 185v from *Interlocking Figures*. As with the fourfold rotational kite motif, the basic constructive component is another triangle of specific proportion that requires conic sections to draw with mathematical precision, and like the fourfold examples, the multiple instructions for this rectangular motif make use of approximate constructions that side step the use of conic sections. This is confirmed in the written instructions wherein the author ends by stating that the triangular component is difficult to create, that it is outside Euclid's *Elements*, and otherwise requires the use of conic sections.[100] As demonstrated in the upper portion of Fig. 70a, the completed rectangle is produced from the generating triangle through mirroring the triangle on its hypotenuse to create the distinctive quadrilateral kite. This is subdivided into three secondary quadrilaterals. The subdivided kite is rotated with point symmetry such that the secondary approximate quarter octagons on adjacent sides of the two kites are contiguous. The specific proportions of the original triangle allow for the acute angles of each kite to align with the extended edges of the other, thus creating a bounding rectangle. Alpay Özdural suggests

[98] –Chorbachi (1989), 765.
–Özdural (1996), 191–211.
–Hogendijk (2012), 37–43.

[99] Some scholars who have written on the significance of the fourfold rotating kite designs in *Interlocking figures* have failed to differentiate between the geometric proportions of the examples from this treatise and the proportions of the examples from the architectural record. By conflating all rotating kite designs into a single complex construct requiring conic sections for mathematically accurate construction, the need for mathematicians to assist artists in the construction of simplified approximations is corroborated. See Chorbachi (1989), 751–789.

[100] "Producing a triangle such as this is difficult, and it falls outside of the Elements of Euclid. It belongs to the science of conics (*makhrūṭāt*) and it is produced by the action of moving the ruler (*misṭara*). When the height of the vertical is postulated (*mafrūḍ*) as in this example, postulated as half of segment آ ب, it produces the square آ ح ه و." MS Persan 169, fol. 185b. (Bottom three lines of diagonal text in upper right). Translation by Carl W. Ernst, Kenan Distinguished Professor of Religious Studies, The University of North Carolina at Chapel Hill.

that the multiple constructions for this figure were intended for artists to create a geometric design that mirrors the rectangular motif to cover the plane.[101] Figure 70b shows a widened line version of this pattern, thereby providing a more typical ornamental treatment to the line work in Fig. 70a. The deficiencies as an ornamental design are readily apparent: the six-pointed stars lack sixfold rotational symmetry, and the irregular octagons are not in conformity with the aesthetic standards of this ornamental tradition. Not surprisingly, no examples of this poorly proportioned pattern are known from the historical record. However, this basic conceptual arrangement, but with symmetrically regular six-pointed stars and regular octagons, can result in an acceptable design. Figure 70c is just such an idealized version of this otherwise unsatisfactory design, and the incorporation of regular octagons and six-pointed stars completely solves the visual imbalance of the design created from the multiple constructions in *Interlocking Figures*. This improved version is very easily created by using the exterior angles of a regular octagon to produce the angular proportions of the six-pointed stars. In light of the geometric complexity of designs produced during the period of *Interlocking Figures* estimated origin, this idealized version would have posed no intellectual challenge to a competent geometric artist of the period. While no examples of this idealized version are known in the ornamental arts of the Seljuks or the direct inheritors of their geometric traditions, this particular combination of octagons, six-pointed stars, and irregular hexagonal interstice regions was used as a generative tessellation for a design on the back wall of the previously mentioned niche in the Mamluk entry portal of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63) [Fig. 413] [Photograph 58]. Unlike almost all other geometric patterns within the architectural record, this example employs both the generative tessellation and the design itself, and is an important source of evidence for the historicity of the polygonal technique. While this unusual arrangement of octagons and six-pointed stars is responsible for the very lovely design at the Sultan al-Nasir Hasan funerary complex, it does not appear to have been used otherwise as ornament. The simplicity of construction for the idealized version in Fig. 70c is in marked contrast to the complexity of the seven constructions in *Interlocking Figures*. Had these seven constructions indeed been intended for artistic application, it is reasonable to assume that the originators of these constructions, be they mathematicians working either with or without artists, would have known that the resulting octagons and six-pointed stars would not have had regular eightfold and sixfold symmetry. It can also be assumed that if the objective had been to create an aesthetically acceptable design, a construction sequence that provided for regularity of the octagons and six-pointed stars would have been provided—especially considering the relative simplicity of such a construction. As with the previous examples of fourfold rotational kite motifs from this treatise, it would appear that the interest in this construction was less for the purpose of informing artists of a construction sequence for producing a satisfactory geometric pattern, and more as a series of geometric exercises in their own right.

The greatest reason for calling into question the view that *Interlocking Figures* is an exposition of the primary design methodology employed during this developmental period of the geometric idiom is in the prescribed instructions for each included pattern. Many of the step-by-step constructions are rife with inaccuracies that lead to distortions, and the resulting designs are not indicative of the geometric accuracy within the vast canon of historical geometric ornament. For this reason, several of the patterns from *Interlocking Figures* fall short of their potential for creating designs that are aesthetically acceptable to this ornamental tradition. A case in point is a construction that places seven-pointed stars on the edges and near the vertices of a square repeat unit.[102] As with previous examples, this design utilizes the geometry of a fourfold rotating kite motif to structure the design upon. As clearly observed in the compilers' illustration, rather than their placement on each vertex of the square repeat, the points of each of the 4 seven-pointed stars extend beyond the vertices of the square repeat. Another failing is particularly problematic in that historical examples of geometric patterns based upon a structure of fourfold rotating kites invariably have bilateral symmetry within the kite element [Figs. 28], whereas the dotted lines that make up the kites in this example do not. What is more, some of the pattern lines that extend from the points of the seven-pointed stars are not collinear with the star, and change direction at the point of intersection. These problems are generally not in keeping with the aesthetics of this design tradition, and the inclusion of this construction in the manuscript indicates a certain naïveté on the part of the creator of this example. This design is similar in principle to an orthogonal pattern found at both the Mirjaniyya *madrasa* in Baghdad (1357), and the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) [Fig. 25c]. This also places seven-pointed stars on the edges of a square repeat unit, but has well-balanced proportions throughout. The first of the above-referenced designs with 10- and 12-pointed stars[103] is particularly revealing of the inconsistencies that result from the flawed point-joining

[101] Figure 70a illustrates the repetitive application of this construction, and, except for color, is identical to the prior representation by Alpay Özdural. See Özdural (1996), Fig. 7.

[102] MS Persan 169, fol. 194b.

[103] MS Persan 169, fol. 195b.

construction sequence. This example in *Interlocking Figures* is in marked contrast to the successful use of this design at the Great Mosque at Aksaray, Turkey (1150-53). As per the example of this pattern in the Topkapi Scroll, this is easily and accurately created using the polygonal technique [Fig. 414]. However, a quick study of the anonymous author's point-joining construction reveals the design to be incomplete, with a series of false starts, overdrawing, and poor angles in the unresolved region. In point of fact, the written instructions do not produce a successful design. This is a stark example of the inadequacy of the point-joining technique to accurately provide an easily followed construction for complex patterns with multiple centers of local symmetry: especially when these centers have seemingly disparate rotational symmetries such as the 10-fold and 12-fold regions within this design. The failings of this construction appear to indicate the limited scope of the author's knowledge and technical mastery of the more complex designs that were already a feature of this tradition at the likely time of the manuscript's preparation.

Other than the fivefold swastika design at the end of the treatise that was likely a Timurid addition, the only pattern with fivefold symmetry from *Interlocking Figures* is a *median* field pattern.[104] This is surprising in that fivefold patterns are an immensely important feature of this geometric art form, and were widely employed by its estimated date of origin. As with so many of the designs in this treatise, this fivefold example also fails to achieve the stylistic aesthetic to which it aspires. The greatest visual failing is in the lack of collinearity in the crossing pattern lines. With adjustments, this pattern could be made successful, and its portrayal in *Interlocking Figures* appears to be the product of someone lacking a refined understanding of the aesthetic requisites of this design tradition generally, and of the fivefold design discipline specifically.

An interesting, but ultimately disappointing, geometric pattern from *Interlocking Figures* is made up of six-, seven-, and eight-pointed stars placed into a rectangular repeat unit.[105] A cursory examination of this figure reveals several problems, most notable being the points of the seven-pointed stars not intersecting with the edges of the rectangular repeat. The lack of collinearity in the crossing pattern lines that connect the seven-pointed star with both the six- and eight-pointed stars is also problematic. A number of examples of geometric designs with sequential numbers of star types are known to the historical record: for example, a very-well-conceived pattern with five-, six-, seven-, and eight-pointed stars from the *mihrab* of the Friday Mosque at Barsian (1105) [Fig. 332]. However, the arrangement of the sequential star forms in the example from *Interlocking Figures* appears arbitrary and contrived by comparison, and falls far short of achieving the already well-established aesthetic standards of this ornamental tradition.

Despite the presence of problematic designs in *Interlocking Figures*, there are several that are very successful, with accurate and useful point-joining instructions. Without a doubt, the most successful and remarkable design from this manuscript is the above-mentioned heptagonal design represented in Fig. 57. The design illustrated in Fig. 71a is another successful, if considerably less remarkable, pattern from *Interlocking Figures*.[106] This places six-pointed stars upon the vertices of an orthogonal grid with 90° alternating orientations much in the fashion of the historical designs in Fig. 28. Coinciding with four of the points of these 4 six-pointed stars are four of the points of an eight-pointed star centered within the square repeat unit. This central eight-pointed star is, by force, rotated out of an orthogonal alignment by 11.4254...°. Except for the fact that the 120° exterior angles of the eight-pointed stars that match those of the six-pointed stars must be inferred (no written instructions are provided), the point-joining construction for this design is complete and accurate. The oscillating orientation of the eight-pointed stars, and rotating concave octagonal shield elements of the repetitive structure (red), shares geometric properties with typical oscillating square and rotating kite designs. However, rather than the angle of declination being governed by a single polygonal element, it is the direct product of the six-pointed stars in 90° rotation. This design is a pleasing juxtaposition of six- and eightfold rotational symmetry, and its attractive qualities may well have led to its use historically, although no examples are known. Figure 71b slightly changes the design so that the proportions are determined by regular heptagons.[107] The original design suggests these heptagons within the interstices of the two star forms. By utilizing regular heptagons, the exterior obtuse angles of the six- and eight-pointed stars become the product of this polygon, as do the proportions of the concave hexagonal repetitive shield elements (red). This change is attractive in that the eye readily recognizes and appreciates the regular heptagon; but this is at the loss of the sixfold rotational symmetry of the six-pointed stars.

[104] MS Persan 169, fol. 193b.

[105] MS Persan 169, fol. 190b.

[106] This manuscript includes a second design (fol. 191r) that places six-pointed stars in 90° rotation around the vertices of the square repeat unit. However, the construction of the six-pointed stars is problematic in that it does not provide for the desired sixfold rotational symmetry. MS Persan 169, fol. 194a.

[107] This experimental change to the original design is the work of the author, but was inspired by an observation by Jan Hogendijk.

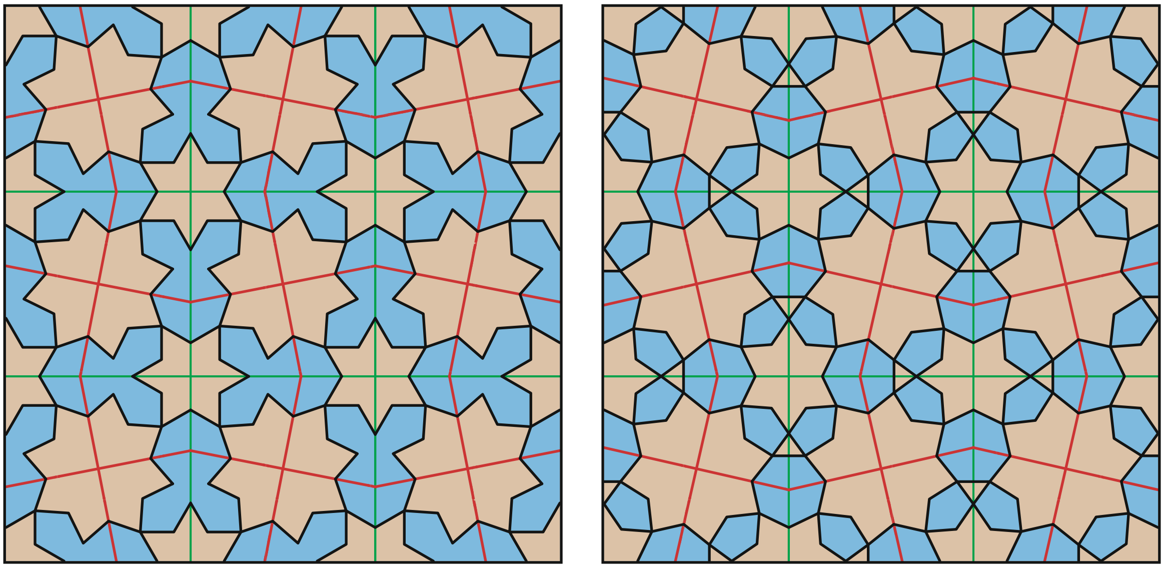


Fig. 71

One of the orthogonal designs from *Interlocking Figures* is particularly successful both aesthetically and in its point-joining construction. This is the one fourfold rotational kite figure from *Interlocking Figures* that is an actual geometric pattern, and although no examples of its use are known within the architectural record, its aesthetic character is fully in keeping with this ornamental tradition. Of particular significance is the central fourfold rotational motif comprised of a square surrounded by four rotating kites separated by four rotating chevrons, all within a bounding square [Fig. 112d]. This isolated motif is found in a number of historical patterns from Persia and Khurasan to which this example from *Interlocking Figures* is closely related. While this design is a point-joining construction, it is more easily produced using the polygonal technique. From this perspective, it is a *two-point* pattern that is created from the *system of regular polygons* through the use of the *two-uniform* $3^3.4^2$ $-3^2.4.3.4$ underlying tessellation of triangles and squares [Fig. 112c]. The point-joining instructional text that accompanies this illustration in *Interlocking Figures* states:

> Masters perform a test of the proportion of this problem, and Abu Bakr al-Khalil has performed the test by several methods (*wajh*, lit."face") and has achieved it. One of those [methods] is the following, which has been commented upon.[108]

[108] MS Persan 169, fol. 189a. (bottom three lines of main text). Translation by Carl W. Ernst, Kenan Distinguished Professor of Religious Studies, The University of North Carolina at Chapel Hill.

Abu Bakr al-Khalil and his associates do not appear to have been knowledgeable of the less complex approach to constructing this pattern using underlying triangles and squares, or if they were, they cared not to reveal it. A comparison between the point-joining instructions in *Interlocking Figures* and the derivation of this design using the polygonal technique provides clear evidence of the superiority of the polygonal technique as a design methodology. This is especially true not just for its inherent simplicity, but in its greater flexibility: the ability of rearranging the polygonal modules of the *system of regular polygons* into other tessellations, thereby producing new patterns, as well as the ability to create additional patterns by applying alternative pattern lines from the other historical pattern families to each new underlying tessellation. When using the polygonal technique to create this design, the characteristic rotational motif is produced from a central square contiguously surrounded by four triangles. The underlying squares within this generative tessellation are provided with two perpendicular sets of parallel pattern lines placed on each edge, thereby identifying this as a variety of *two-point* pattern. These pattern lines extend into the adjacent underlying triangles until they meet with other extended pattern lines. An early example of a design associated with this variety of *two-point* design methodology, with the central fourfold rotational motif, was produced by Khwarizmshahid artists for the Zuzan *madrasa* in northeastern Iran (1219) [Fig. 112b] [Photograph 38]. Being that Abu Bakr al-Khalil has not yet been identified through other sources, the similarity between the example from *Interlocking Figures* and that of the Zuzan

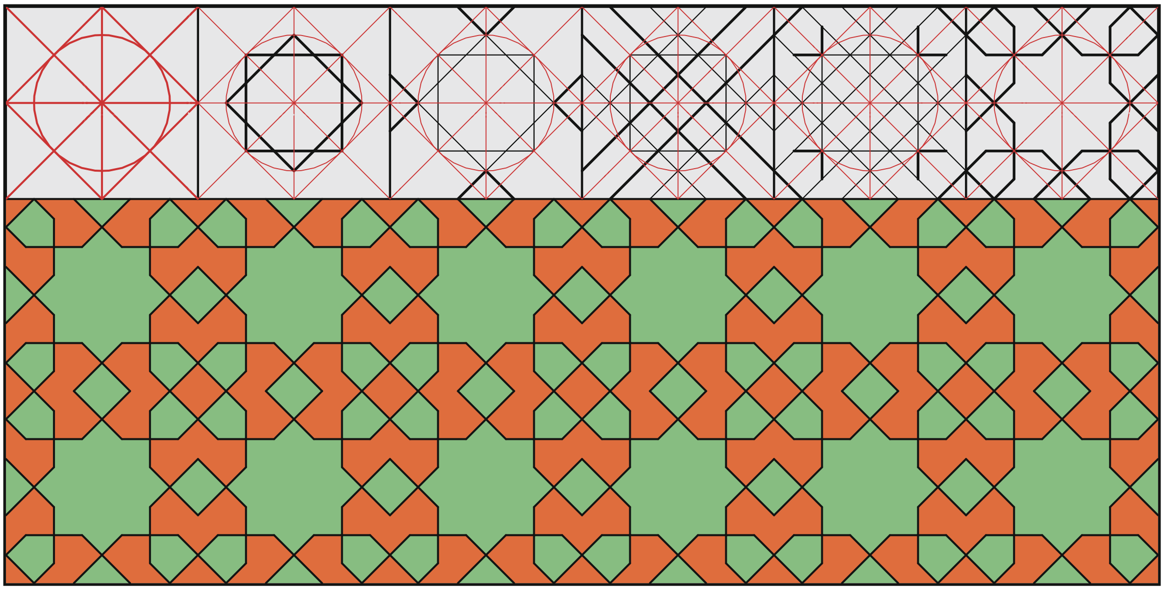


Fig. 72

madrasa raises the possibility first proposed by Alpay Özdural that portions of *Interlocking Figures* may have been produced during the Khwarizmshahid period.[109]

Another example of a successful point-joining construction from *Interlocking Figures* produces a very-well-known orthogonal design that was used widely throughout the Islamic world.[110] This *median* design is easily produced from the *fourfold system A* [Fig. 145], and was particularly popular in Khurasan during the late eleventh and early twelfth centuries. The illustration and instructions for this design in *Interlocking Figures* are an accurate, if slightly incomplete, point-joining method for its reproduction, although it is worth noting that the illustration, as drawn in this treatise, includes point-joining layout lines that obscure the actual pattern to the point of being difficult to initially identify. This presumably explains why this figure from *Interlocking Figures* has not been recognized as this particularly well-known fourfold pattern in previous studies. The inclusion of this design is significant in that it was used so widely throughout Khurasan and eastern Persia preceding the likely date of origin of this treatise.

By the time *Interlocking Figures* was written, the geometric ornamental idiom was fully mature, and the need for artists to have direct mathematical input would have been less of an aesthetic imperative than during the earlier time of Abu al-Wafa al-Buzjani (940-998). As stated, the many problems with the manuscript's constructions lead one to question the assumption of its significance to the methodological development of this tradition. And yet, as demonstrated, this anonymous manuscript also has numerous constructions that accurately produce geometric patterns that are very acceptable, and even outstanding—as per the pattern with heptagons that is also found in the northeast dome chamber in the Friday Mosque at Isfahan.

The question of by and for whom *Interlocking Figures* was produced remains intriguing. Was it written by mathematicians for artists, or perhaps by mathematicians, inspired by and seeking to better understand the geometric work of artists? Or did the point-joining constructions of geometric patterns result from artists privy to the polygonal technique requesting assistance from mathematicians to devise step-by-step instructions that could be provided to artists and craftspeople more widely so that this art form could be adopted more pervasively in a wide range of media? These uncertainties are augmented by the lack of cohesion throughout this treatise, by the inconsistencies of mathematical sophistication, and disparities between naively conceived simplistic geometric patterns on the one hand, and well-realized construction sequences for very acceptable designs on the other. One is led to conclude that this treatise was the work of multiple individuals with variable levels of mathematical and artistic proficiency, and possibly for more than a single intent.

Recent publications that focus upon the point-joining technique include well-conceived construction sequences for numerous orthogonal and isometric designs, as well as some fivefold patterns. A typical example of a sequential point-joining construction (by author) is shown in Fig. 72. This is for a simple design that was used in a brickwork border at the tomb of Nasr ibn Ali (1012-13), the earliest of the three

[109] Özdural (1996).

[110] MS Persan 169, fol. 196a.

adjoining mausolea at Uzgen, Kyrgyzstan [Photograph 15]. This same design can also be created from the polygonal technique, and indeed is one of the earliest examples of a design that can be created from the *fourfold system A*. Another relatively early example of this well-known *median* design is from the mausoleum of Sultan Sanjar in Merv, Turkmenistan (1157) [Fig. 159]. As with other designs with low or moderate complexity that were very likely produced originally with the polygonal technique, this illustration demonstrates how such designs can frequently be recreated easily from point-joining methodology.

To summarize, the point-joining technique may well have been used historically as a means to recreate existing geometric patterns among artists and craftspersons working in the geometric idiom who were not privy to the more specialized and highly versatile design methodologies of the polygonal technique. While the point-joining technique is less convenient than other generative methodologies for designing original patterns of moderate complexity, and especially impractical for recreating patterns with greater complexity, it nonetheless provides a particularly useful method of recreating existing patterns of low and moderate complexity. The dissemination of specific point-joining constructions to artists and craftspeople that were not otherwise privy to some of the more esoteric design methodologies would have allowed for the widespread and repeated use of a wide variety of specific designs. In this way, point-joining constructions were likely an important contributor to the ongoing spread and consolidation of the geometric aesthetic throughout Muslim cultures.

2.5.3 The Grid Method

Some of the less complex threefold patterns can be created directly from the isometric grid. Through simple trial and error, repetitive interlocking and overlapping figures can be found that make very acceptable designs. Most of the patterns created from the isometric grid will have pattern lines that are congruent with the grid itself. More complex isometric grid designs will have pattern lines in six directions: three with the grid, and three perpendicular to the grid. Figure 73 illustrates three isometric grid designs. The first of these, Fig. 73a, is an interlocking design with three- and sixfold centers of rotational symmetry that is typical of the *p6* plane symmetry group. All of the pattern lines in this design are congruent with the grid. An example of this design is found at the Gök *madrasa* in Amasay (1266-67). The parallel pattern lines in Fig. 73b are also congruent with the grid, but interweave with one another to produce a more conventional Islamic geometric aesthetic effect. This design can also be produced very readily using the *system of regular polygons*, and a fine example was used at the Shah-i Mashhad in Gargistan, Afghanistan (1176) [Fig. 104c]. The design in Fig. 73c employs lines that are both congruent with the grid and perpendicular to the grid. This design can also be produced from the *system of regular polygons*, and the imbalance between the large and small background elements can be improved by widening the pattern lines in one direction rather than both directions. This design was used on the façade of the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan (1186) [Fig. 101d].

The orthogonal grid can also be used to create geometric designs. At the most basic level, designs produced on this grid will maintain congruency with the orthogonal grid. Designs of this variety include the many swastika and key patterns, as well as square *Kufi* calligraphic motifs. The orthogonal nature of this variety of design made them especially relevant to the brickwork ornament championed by the Ghaznavids, Qarakhanids, Ghurids, and Seljuks. Later expressions included Timurid polychrome cut-tile mosaic. Diagonal lines were also introduced to patterns created from

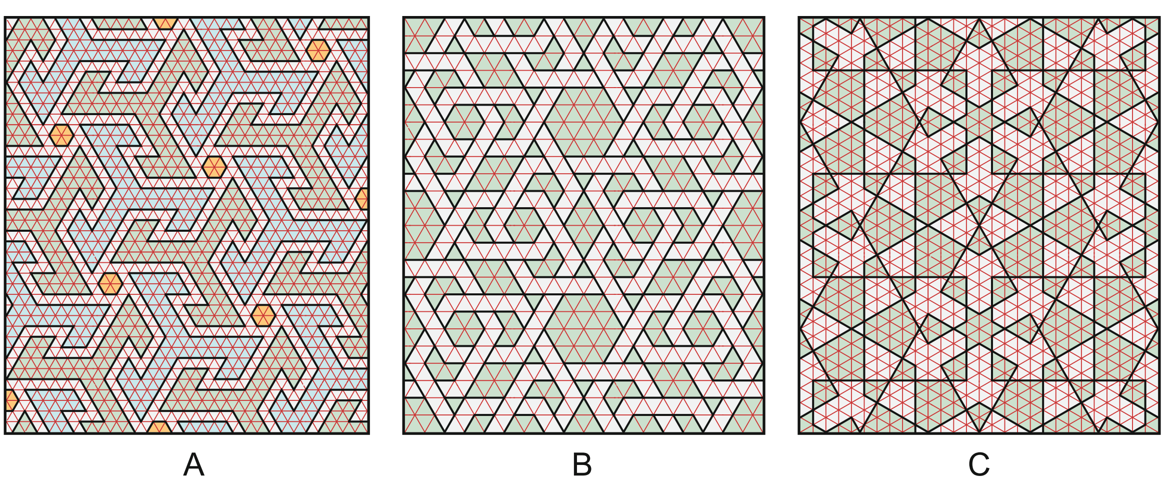


Fig. 73

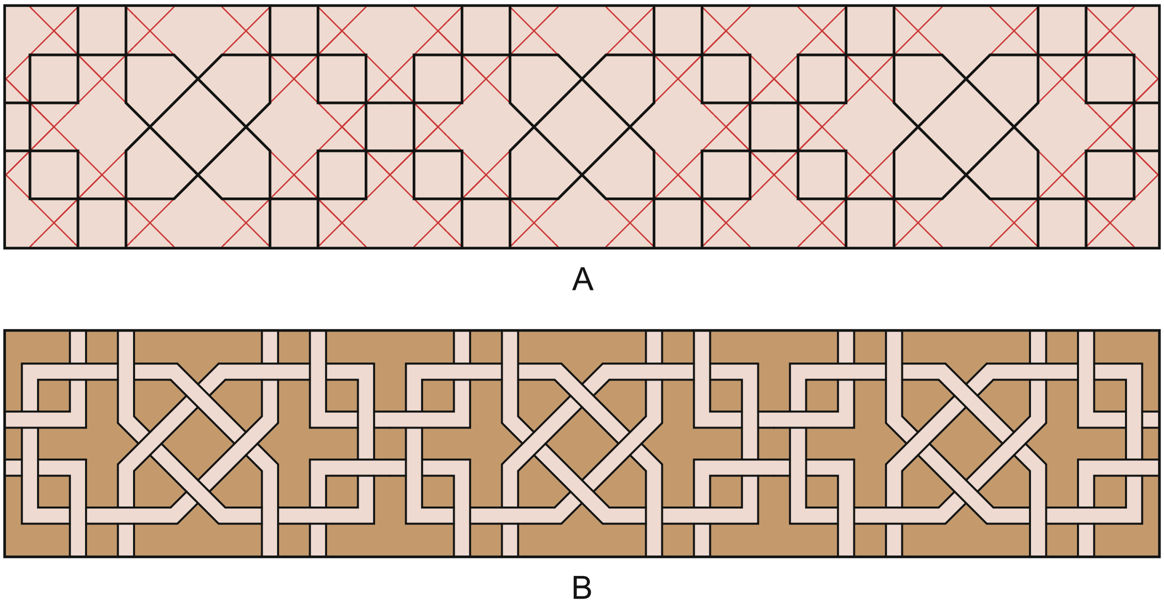


Fig. 74

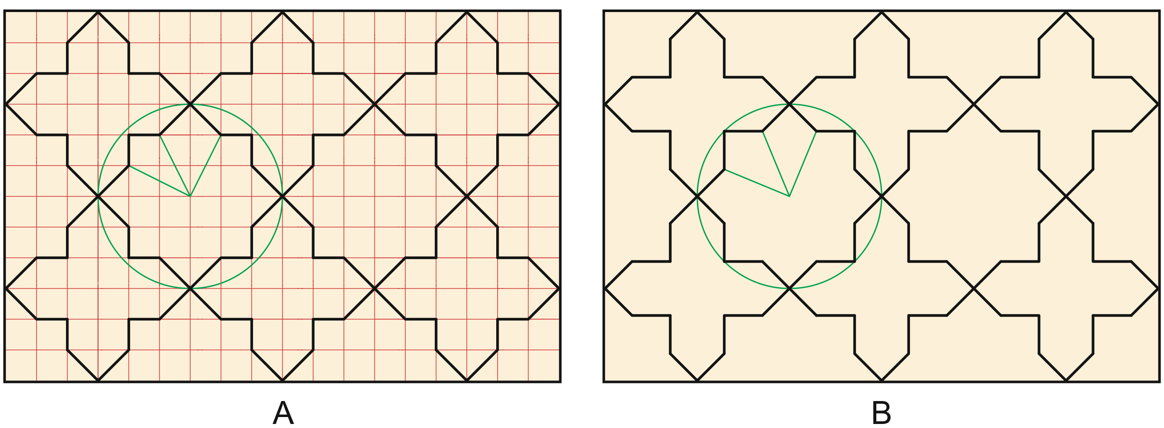


Fig. 75

the orthogonal grid, adding two more directions of pattern line. Figure 74 illustrates a border design from the minaret of Uzgen in Kyrgyzstan (twelfth century) that combines two directions of lines that are congruent with the orthogonal grid, and two directions with 45° diagonal lines. This type of design can be used to create designs with eight-pointed stars, and Fig. 75 illustrates the use of the orthogonal grid to construct the well-known star-and-cross design. Figure 75a demonstrates the problem with constructing designs with eight-pointed stars using the grid method. Because four of the points for each star are congruent with the grid, and the other four are diagonals, there are two sizes of points. The finished star does not have eightfold rotational symmetry. Examples of the star-and-cross pattern with this distortion are occasionally found in the architectural record, but almost always from a much later date after this ornamental tradition had begun to decline. Figure 75b illustrates the correct proportion for the eight-pointed stars. More complex designs produced from the orthogonal grid frequently have the character of the *fourfold system A*. Figure 76 illustrates the orthogonal grid derivation of a widely used design, along with the same design with the correct proportions as generated from the *fourfold system A* [Fig. 145]. As mentioned above, point-joining instructions for this design were included in *Interlocking Figures*, and several examples (with correct proportions) are found in the early brickwork ornament of Khurasan, including the minaret of the Friday Mosque at Damghan, Iran (1080); the *mihrab* of the Friday Mosque at Golpayegan, Iran (1105-18); and the minaret of Daulatabad in Afghanistan (1108-09) [Photograph

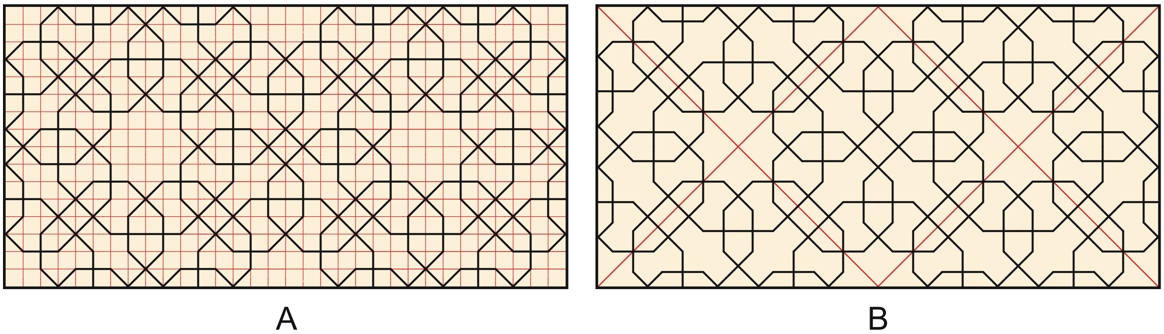


Fig. 76

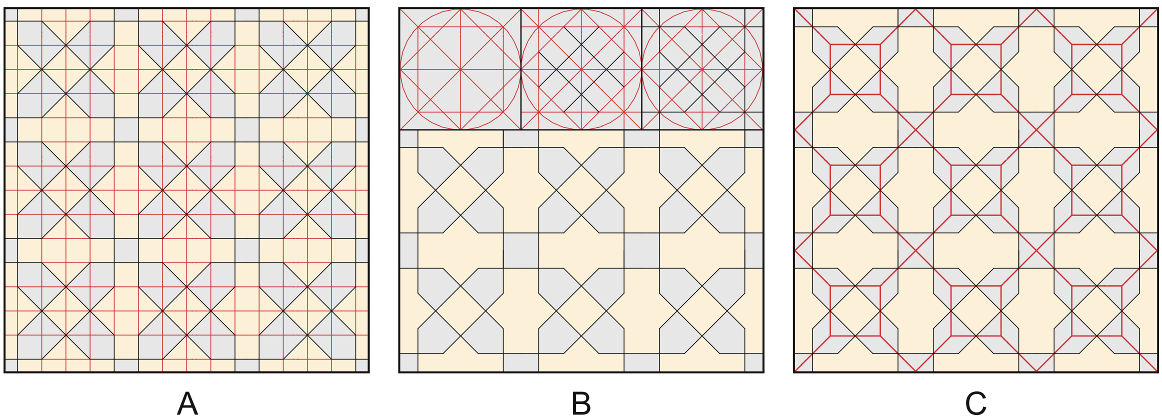


Fig. 77

20]. Working with the orthogonal grid in Fig. 76a can be a fast way of testing ideas and exploring design options. However, these designs will always have distortions that result from the difference between the length and diameter of each square cell of the orthogonal grid. As demonstrated in Fig. 76b, when using the orthogonal grid, once a design has been arrived at, it is necessary to draw it anew so that the distortions are eliminated.

As stated, there are multiple options for designing the less complex patterns in this geometric art form, and with less complex designs it is impossible to say with certainty exactly how a particular example was constructed. The same design may have been constructed one way at a given location, and another way elsewhere. Figure 77 illustrates construction solutions for a very simple fourfold design in all three of the methods discussed herein. Figure 77a demonstrates the orthogonal grid method; Fig. 77b provides a simple construction sequence for the point-joining technique; and Fig. 77c shows the generation of this pattern from an underlying tessellation associated with the polygonal technique. The benefit of the grid method is that it is a fast way to explore design options, but requires correction. The benefit of point-joining is that it is an accurate way of recreating existing designs, but lacks flexibility as a means of creating original designs. By contrast, the polygonal technique is accurate and extremely flexible, and, as pertains to the polygonal systems, very fast. What is more, the polygonal technique also provides for the generation of at least four distinctly different patterns from each underlying tessellation. By way of example, Fig. 78 illustrates designs in each of the four pattern families for the tessellation shown in Fig. 77c.

In the hands of an experienced practitioner, the orthogonal grid can be used to generate increasingly complex geometric patterns with fourfold repetitive symmetry. This especially pertains to the distinctive geometric style of Morocco and al-Andalus, and can be applied to patterns with higher order n-pointed stars that are multiples of 8—even up to and including 64-pointed stars. Jean-Marc Castéra has deftly demonstrated the versatility of this advanced orthogonal grid technique,[111] which he refers to as the "freehand method." His methodology takes into

[111] Castéra (1996).

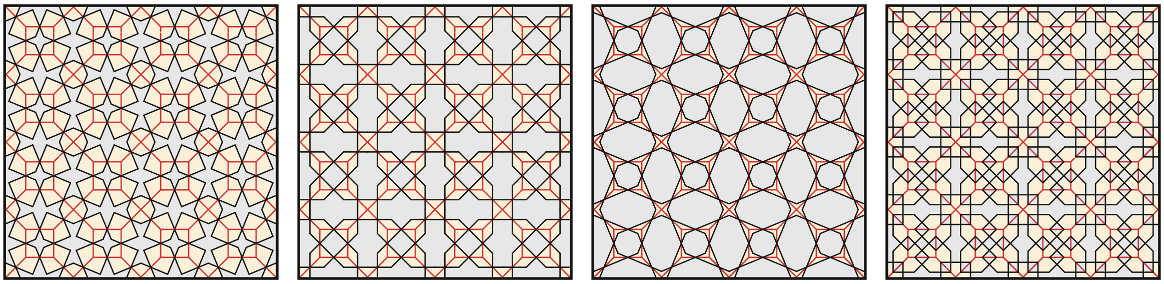
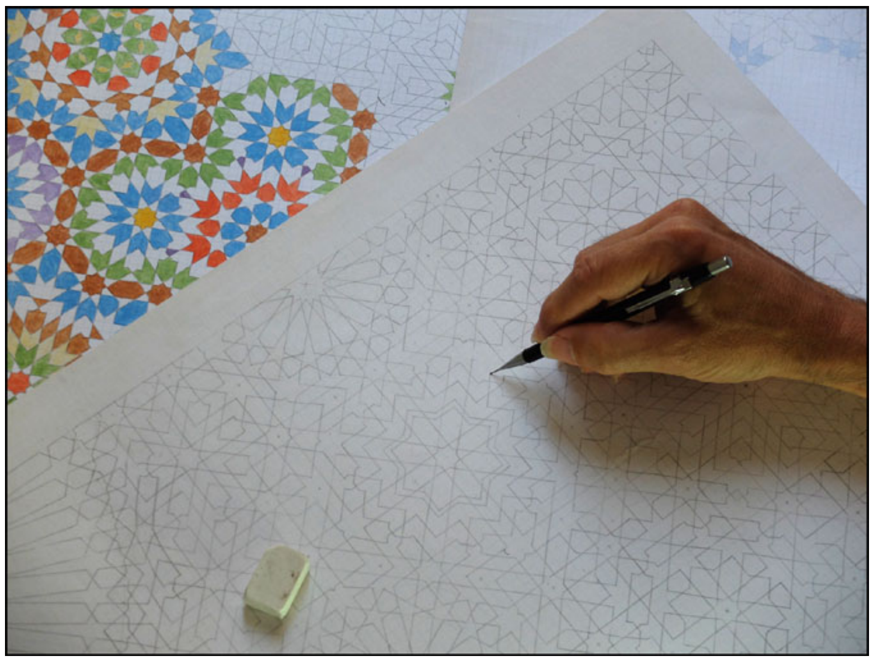


Fig. 78

Fig. 79



account the disparity between the orthogonal and diagonal coordinates, and the need to correct these approximations:

> Since we are aware of the errors brought about by these approximations, it is quite simple for us to correct them when necessary, if, for example, we wish to make an actual mosaic. In this manner, each time we create a new piece, it adopts the correct proportions, which have been geometrically deducted from those of the pieces that have been already made.[112]

Figure 79 is an example of this more complex Maghrebi form of grid method construction requiring approximate coordinates of the orthogonal grid.[113] While this method of constructing patterns is currently used in Morocco, the extent to which Maghrebi artists of the past employed this methodology is unclear. Certainly the preponderance of geometric patterns from Morocco and southern Spain is of a geometric nature that would allow for their creation in this manner. However, no ancient pattern books or scrolls from the Western regions have confirmed the historicity of this methodology, and the patterns created from the more advanced grid method can, almost always, also be created using the polygonal technique with relative ease.

2.5.4 Extended Parallel Radii

There is a category of geometric pattern that is rarely encountered, but sufficiently unusual, and indeed beautiful, as to justify methodological analysis. While no examples of *extended parallel radii* designs appear in historical scrolls or design reference books such as the *Topkapi Scroll* or

[112] Castéra (1996), 99. (note: this quotation is from the English edition of 1999).

[113] This photograph shows the hand of Jean-Marc Castéra using the orthogonal grid to construct the design by drawing freehand.

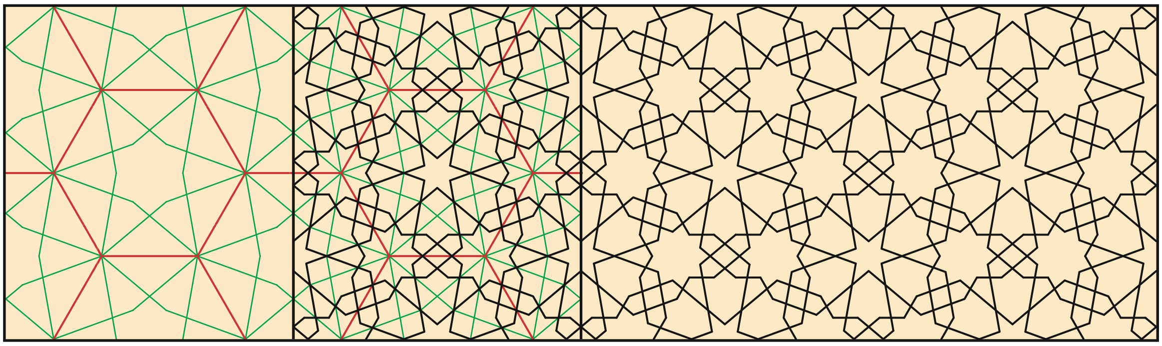


Fig. 80

Interlocking Figures, the method for constructing these patterns can be intuited by their geometric character. The essential feature of this methodology offsets the lines of a radii matrix in both directions, eliminates the original radii, and extends these parallel offsets until they meet with other extended offset lines. In its most simple form, the lines of a common grid are offset on both sides by an amount that will create a visually acceptable pattern. The very simple design in Fig. 32 can be produced in this manner, although this replicates the most basic *two-point* functionality of the polygonal technique [Fig. 96d]. As with this example, a number of early and uncomplicated geometric patterns that are easily created from the *system of regular polygons* have these parallel grid line characteristics, including a Ghurid brickwork panel from the façade of the western mausoleum at Chisht, Afghanistan (1167) [Fig. 105a]. This design is characterized by parallel offsets of the hexagonal grid and its triangular dual. An example from the synagogue in Cordoba, Spain (1316), constructed during the Nasrid period, achieves greater complexity through additional parallel offsets of lines that connect the vertices of the hexagonal and triangular grids [Fig. 105h]. When created from the polygonal technique, the distance between the parallel lines in each of these examples is determined geometrically through the lines being located at determined points within the underlying generative tessellation. By contrast, with the extended parallel radii technique the distance between the parallel lines is generally an arbitrary determination based upon the aesthetic predilections of the artist. Though less formal than other design methodologies, through trial and error, this alternative technique will nonetheless create very beautiful designs.

The more interesting extended parallel radii designs employ more complex radii matrices. As an example, Fig. 80 illustrates a design (by author) that utilizes a radii matrix with ninefold symmetry at each vertex of the regular hexagonal grid. A close inspection of this design reveals each line of the pattern to be an equally distanced parallel offset to the generative radii matrix. This is not a historical design, although it falls within the acceptable aesthetics of this ornamental tradition. Radii matrices are a fundamental methodological component of the polygonal technique. They provide the structure upon which the generative polygonal tessellations are created. This is especially relevant to nonsystematic patterns with compound regions of *n*-fold rotational symmetries. The historical use of radii matrices is confirmed in the *Topkapi Scroll*, where they appear as incised reference lines produced with a steel stylus. Figure 81a shows a radii matrix with local regions of 8-, 10-, 12-, and 16-fold rotational symmetry set within a square repeat unit. Figure 81b illustrates an *acute* pattern produced from an underlying tessellation that can be made from this radii matrix [Figs. 404 and 405]. This *acute* pattern was used in the *iwan* of the Kemaliya *madrasa* in Konya, Turkey (1249). This same radii matrix, with its very particular combination of local symmetries, was used to create two extended parallel radii designs that date to the same approximate time and place during the Seljuk Sultanate of Rum. Figure 81c represents the extended parallel radii design from the Kaykavus hospital in Sivas, Turkey[114] (1217-18), and Fig. 81d illustrates the closely related design from the Sultan Han near Aksaray, Turkey[115] (1229). These three designs with identical symmetrical structure, but distinctly different aesthetics, all come from central Anatolia and were produced within 32 years of one another: conceivably by the same artist or artistic lineage.

2.5.5 Compass Work

The process of laying out a design with a compass or dividers was inherited by Muslim artists from their Christian counterparts who were actively engaged in the Hellenistic aesthetic that survived well into the Late Antique period.

[114] Schneider (1980), pattern no. 426.
[115] Schneider (1980), pattern no. 425.

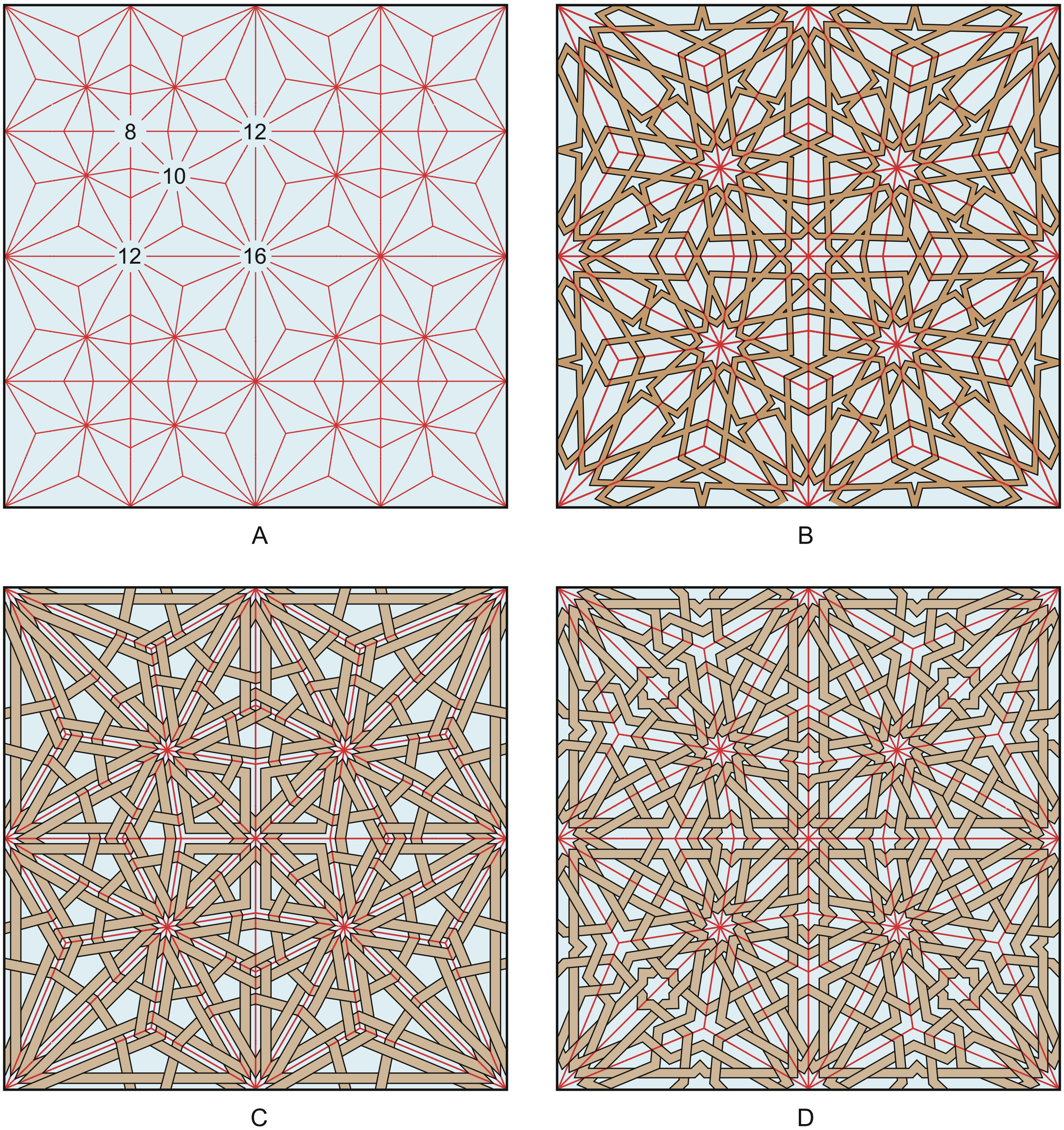


Fig. 81

The earliest examples of Islamic *compass-work* ornament are associated with several surviving Umayyad buildings: the most notable examples being several pierced stone window grilles from the Great Mosque of Damascus (706-15). While geometrically undemanding, these are significant in their application of what had previously been an ornamental device used primarily for mosaic pavements to a new expression in pierced stonework. What is more, the visual quality of these windows helped to establish interweaving geometric designs as a primary feature of Muslim aesthetics.

Compass-work ornament has its own visual character and curvilinear appeal, and it is not surprising that this artistic practice continued among succeeding Muslim cultures, even if reduced to a role of relatively minor significance. In addition to repetitive patterns, these later expressions included nonrepetitive, stand-alone, ornamental panels primarily composed from circles within a rectangular frame. Such compass-work constructions were occasionally used as Quranic illuminations, including in the Quran produced by ibn al-Bawwab in 1001. However, this study is concerned expressly with geometric compass-work creations that have repetitive characteristics. The methodology behind these compass-work patterns is overtly apparent upon examination, and involves the drawing of circles at set points of a

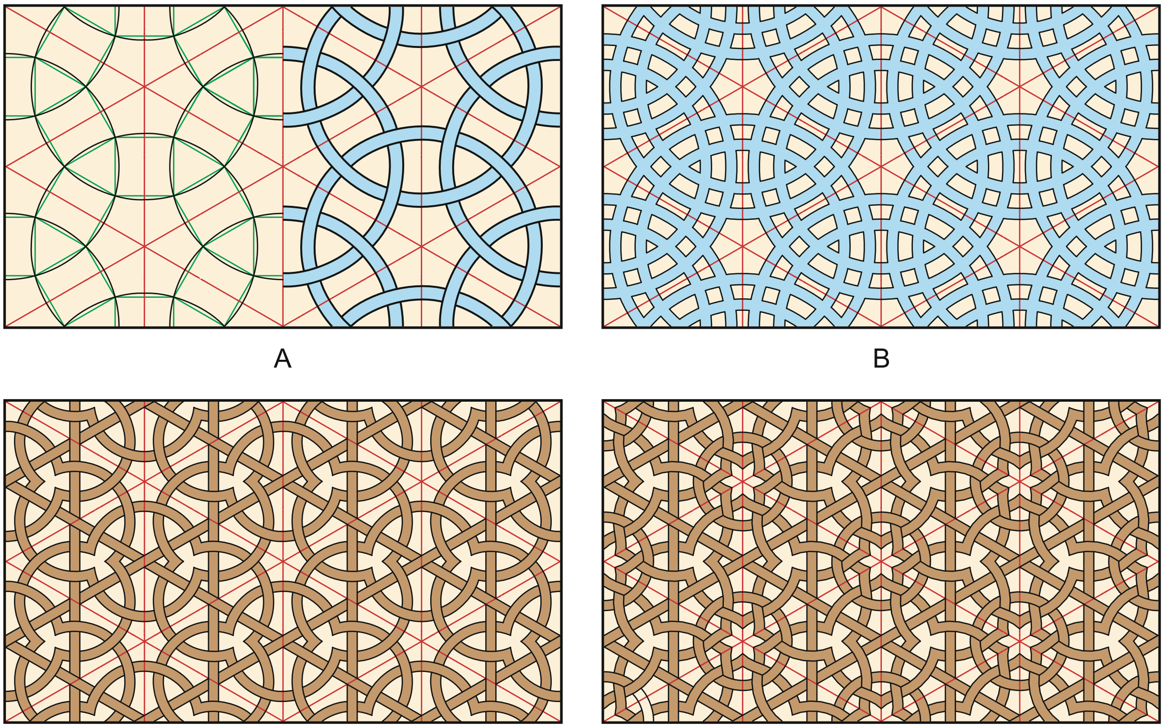


Fig. 82

given geometric grid.[116] These circles can be uninterrupted or trimmed where they intersect with other circles. Compass-work designs will sometimes incorporate s-curves within their overall matrix. Unlike patterns created from the polygonal technique, compass-work patterns will often include the generative grid along with the circular elements, thereby creating designs that include both an angular and a curvilinear quality. Figure 82a illustrates a very basic compass-work pattern made up of interweaving circles set upon the vertices of the isometric grid. The proportions of this design are easily determined by locating the center of each circle upon the vertices of the isometric grid and the radius at a point that is past the midpoint of each edge of the triangular cells that make up the isometric grid. In this illustration, the size of the circles is determined by their circumference being placed upon the vertices of the 3.4.6.4 semi-regular tessellation of triangle, squares, and hexagons. Figure 82b shows essentially the same design, but with a double-line treatment. The radii and width of the parallel circles are carefully contrived to create the distinctive network of similarly sized background elements. This compass-work design was used in the Ottoman inlaid stone ornament of the Sehzade Mehmet complex in Istanbul (1544-48), and is an excellent example of the continued use of compass-work patterns among later Muslim cultures. Figure 82c is a representation of one of the many compass-work patterns used in the stone window grilles found in the Great Mosque of Damascus (715). This early example maintains the 3.6.3.6 semi-regular grid as part of the finished design, and the circles are located at the vertices of this grid. The circles have been trimmed where they intersect with one another, thereby opening up the design in an aesthetically pleasing fashion that also allows for greater light penetration. The trimming of these circles produces the distinctive trilobed motif at the centers of each triangular cell of the 3.6.3.6 grid. Figure 82d illustrates a slightly later Umayyad window grille from palace of Khirbat al-Mafjar in Jordon (c.743). This is identical to the previous example except for two added features: the replacement of the arcs with s-curves that create distinctive six-pointed stars at the vertices of the isometric repeat, and the small interwoven circles that surround these six-pointed stars. Given their geometric similarity, and the fact that they were produced within 30 years of one another, it is very likely that the design of the window grille from Khirbat al-Mafjar was directly influenced by the earlier example at the Great Mosque of Damascus.

Figure 83 shows an example of Tulunid compass-work ornament set on a square grid. This pattern was used on one of the arch soffits of the ibn Tulun mosque in Cairo (876-79). The circles in this example are set upon the vertices of the

[116] Creswell (1969), 75–80, Figs. 12 and 15.

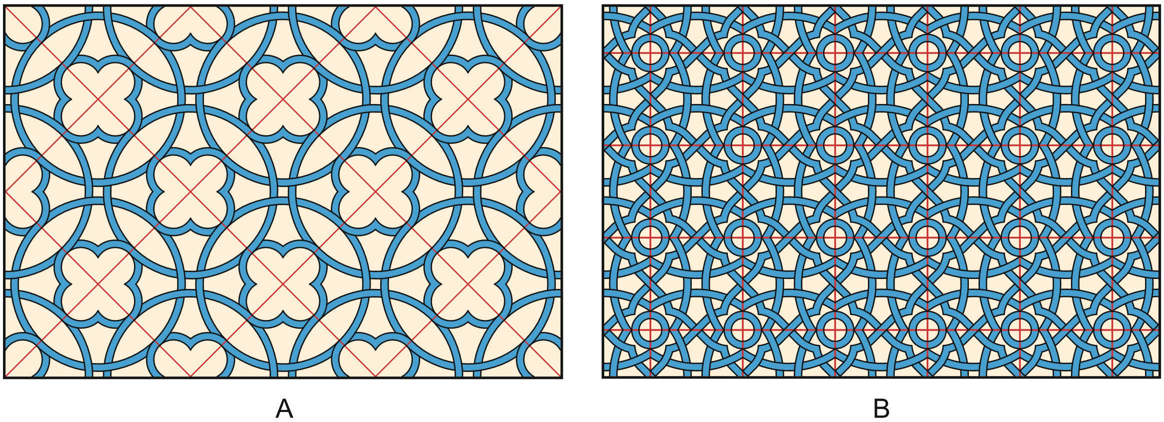


Fig. 83

orthogonal grid, which is not incorporated into the completed design. Each circle connects with a four-lobed motif that is also placed at the vertices of the repetitive grid. Figure 83b is from the Great Mosque of Damascus, and is also comprised of circles set upon the orthogonal grid [Photograph 5]. A close examination of this design reveals an unexpected similarity with the design from the ibn Tulun. Included in the orthogonal design from the Great Mosque of Damascus are circles of the same relative size and location as those from the ibn Tulun, except that the earlier Umayyad example has double the number of circles and incorporates diagonal s-curve elements within its overall structure—thereby creating a design with far greater density. However, unlike the patterns in Figs. 82c and d, given the distance over time and territory, it is unlikely that the similarity in circular layout is the product of any direct causal influence.

2.6 Classification by Line Treatment

This final form of classification is perhaps the most obvious in that each category is readily apparent when first viewing any given design. Categorization by line treatment falls into three basic forms: (1) the basic line without widening that is almost always provided with differentiated background colors for a tiling treatment; (2) widened lines; and (3) interweaving lines. The thickness of the widened and interweaving lines is variable and determined by several criteria, including the constraints of the artistic medium; cultural conventions; geometric concordance; and aesthetic preferences of the artist [Figs. 85–88]. Differences in line treatment can greatly alter the overall appearance of a pattern, sometimes to the point of obscuring similitude between examples of the same design. While line treatment is a very basic and obvious classification, it is nonetheless important as it greatly impacts the aesthetic quality of each historical example, and as such becomes part of the descriptive analysis that accompanies any in-depth examination of this tradition, just as it is a fundamental concern to any artist engaged in working with these geometric patterns.

To conclude this discussion of classifications within Islamic geometric patterns, the wide range of diverse criteria within this tradition requires a high degree of description to fully differentiate a given example, and place it into context with the tradition as a whole. That said, the formal identification of specific and critically important aspects of this tradition allows for greater clarity and understanding of both the bold and subtle features that permeate this remarkable art form. Such formal classifications include repetitive schema; design methodology; specific pattern family; whether a pattern is nonsystematic or systematic, and if systematic, which generative system; and when relevant, the type of dual-level design. More nuanced considerations include stylistic variables such as arbitrary additive or subtractive features.