# 1 The Historical Antecedents, Initial Development, Maturity, and Dissemination of Islamic Geometric Patterns

## 1.1 Geometry in Islamic Art

Since the earliest period of Islamic history the ornamental traditions of Muslim cultures have found expression in a highly diverse range of styles and media. Throughout this broad sweep of ornamental diversity and historical longevity there remained an essential Islamic quality that differentiates this tradition from all others. One of the primary characteristics responsible for such cohesion is the pervasive triadic nature of Islamic ornament. From its onset, this ornamental tradition employed three principal design idioms: calligraphy, geometry, and stylized floral.[1] It can be argued that figurative art depicting both human and animal forms is also characteristic of Islamic art. This additional feature of Islamic art requires brief mention, if only to legitimately dismiss it for the purposes of this discussion. During the Umayyad period figurative motifs were widely used in both architecture and the applied arts, and virtually all subsequent Muslim cultures used figurative depictions to a greater or lesser extent. Such work has always been anathema to Islamic religious sentiments and frequently to Muslim cultural sensibilities.[2] Even among the Umayyads, who inherited the figural traditions of the late antique period, the use of figurative depictions was invariably secular and often associated with courtly life. The eighth-century Umayyad palaces of Qusayr ʻAmra and Khirbat al-Mafjar are replete with figurative decoration, the former carried out in fresco and the latter in mosaic and carved stucco. Such notable examples notwithstanding, the surviving religious architecture of the Umayyads is evidence of the interdiction in the use of human and animal depiction within mosques. It is significant that the Umayyad architectural motifs in the mosaics of the Great Mosque of Damascus and the Dome of the Rock were entirely devoid of human and animal figures; and the one area of carved stone ornament that is entirely without animal representation at the eighth century Umayyad palace of Qasr al-Mshatta is a wall directly adjacent to the mosque. The figurative restraint in the ornament of Mosques was adhered to strictly throughout succeeding Muslim cultures, and the continued use of human and animal figures was generally limited to the decoration of utility objects such as ceramic pottery, textiles, metal vessels, furniture, wood and ivory boxes, and the occasional architectural expression in murals and carved relief in such secular locations as palaces, private homes, and bath houses. However, with the exception of the miniature traditions,[3] this form of decoration was certainly never a primary feature in Islamic art or architectural ornament. For all of their beauty and refinement, the figurative aesthetics of the Persian, Mughal, and Turkish miniature traditions were for the most part insular, and did not significantly overlap with other artistic traditions within these Muslim cultures. Notable exceptions include the "miniature" style of the enameled *minai'i* ware of late twelfth- and thirteenth-century Kashan[4]; the so-called Kubachi painted ceramic vessels created in northeastern Persia during the Safavid period[5]; and the many Persian painted tile panels produced during the Qarjar period. Perhaps the greatest indication of the lesser role that figurative imagery played throughout the history of Islamic art and architecture is the fact that the non-miniature figurative art of Muslim cultures was not subject to the concerted effort

[1] Hillenbrand (1994a), 8.

[2] Allen (1988), 17–37.

[3] The depiction of the human figure in the fine art traditions of Persian, Turkish, and Mughal miniature painting is an exception to the conventions of human representation occasionally found in the ornamental and applied arts of various Islamic cultures. The reconciliation of these miniature traditions with the Islamic religion and Islamic cultures is a fascinating study, but outside the scope of this work.

[4] –Watson (1973–75), 1–19.
–Watson (1985).
–Hillenbrand (1994b), 134–41.

[5] Golombek et al. Chap. 4: "The Kubachi Problem and the Isfahan Workshop."


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toward continued refinement and stylistic development that is a hallmark of the calligraphic, geometric, and floral traditions. As such, with the exception of the miniature traditions, figurative art can be regarded as tangential rather than integral to Islamic art, and to have been occasionally employed rather than part of an ongoing developmental evolution.

It may seem remarkable that such an apparently limited palette of calligraphy, geometric patterns, and floral design should have provided the basis for such a rich and varied artistic tradition. Yet each of these separate disciplines benefits from unlimited developmental opportunities, and when used together provide an inexhaustible supply of aesthetic variation. The continued adherence to the triadic quality of Islamic ornament provided a governing mechanism whereby the aesthetic expressions of multiple Muslim cultures, spanning great divisions of distance and time, were able to be both culturally distinct yet identifiably Islamic. Similarly, this also served as a form of regulator, or cohesive principle, through which Muslim artists could appropriately assimilate specific ornamental conventions from non-Muslim cultures. This assimilative process contributed greatly to the tremendous stylistic diversity found in Islamic art and architectural ornament.

The historical development of all three of the primary ornamental idioms is characterized by an evolving refinement and increased complexity. This process was aided by any number of influences, not the least of which include contacts with other mature artistic traditions; concomitant improvements in fabricating technologies (e.g., a brocade loom allows patterns to be woven that would otherwise not be possible); vainglorious patronal expectations that commissioned works should exceed that of their predecessors or neighbors; and the natural tendency for an artist to strive for creative excellence by challenging personal limitations and pushing the boundaries of an artistic tradition. Such criteria are common to all cultures, but the refinement and growth in complexity within the ornamental traditions of Muslim cultures were also greatly aided by the ongoing fascination with and influence of geometry.

That geometry should be at the root of the geometric idiom goes without saying. Yet the role of geometry in the aesthetic development of both Islamic calligraphy and floral design was also of paramount importance. The tradition of Islamic calligraphy is, first and foremost, a book art. Within Muslim cultures, calligraphy is regarded as the highest art form, and the copying of the Quran is as much a spiritual discipline as it is an artistic activity. The creative heights to which Muslim calligraphers refined this tradition were directly driven by their need to adequately express their deep reverence for the Quran.[6] In copying the Quran, calligraphers were motivated by the need for legibility and beauty. This twofold concern led to the continual refinement and eventual preference of the more easily read cursive scripts over the older and overtly angular *Kufi* scripts. The ability for geometry to positively influence the legibility and beauty of an artistic tradition characterized by non-repetitive rhythmic undulation might appear unlikely. However, the use of geometry as an underlying governing principle for the more legible cursive scripts was not just appropriate, but crucial to the beauty and longevity of this tradition. In the first half of the tenth century the renowned calligrapher Abu ‘Ali Muhammad ibn Muqlah (d. 940) developed a system of calligraphic proportion that was applied to the development of the six principal cursive scripts.[7] His system was complex and highly rational, and so successful in creating balanced writing that the rules he established have been universally employed through successive centuries by Muslim calligraphers. The rules he established for cursive calligraphy relied upon the application of carefully contrived mathematical proportions. His standard unit was the rhombic dot, produced by moving the pen nib diagonally the same distance as the nib is wide. The rhombic dots were, in turn, used to determine the height of the *alif*, and the height of the *alif* was then used to determine the diameter of a circle, all of which were used to determine the precise proportions of each letter.[8] Ibn Muqla was *vizier* to three successive Abbasid Caliphs during a time when interest in mathematics and geometry was acute. The training he received in the geometric sciences is evident in the refinement he brought to bear upon the tradition of calligraphy, and is fully consistent with the intense interest in geometry during this period.[9]

The geometric angularity of *Kufi* is in marked contrast with the flowing movement of the cursive scripts, and together these stylistic trends create a dynamic complementarity that was used to great aesthetic effect. This is especially evident within the realm of Islamic architectural ornament. In contrast to the writing of the Quran, conventions for the use of text on buildings were less rigid, and the traditions of architectural calligraphy allowed greater latitude for ornamental stylization. Being less bound by governing rules, *Kufi* scripts were particularly suited to ornamental elaboration. From as early as the eighth century, the letters of the *Kufi* script were embellished with floral extensions that encroach upon the background space

[6] Schimmel (1990), 77–114.

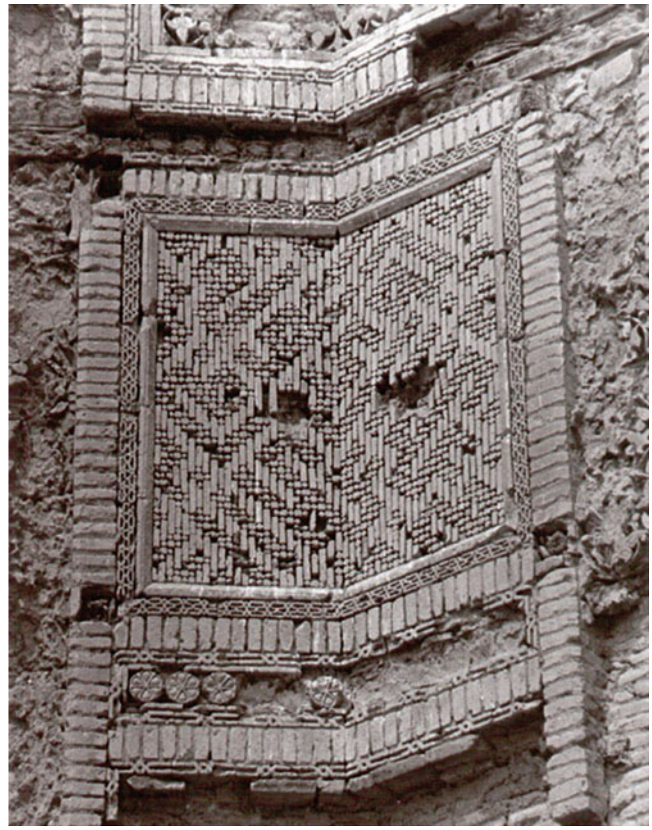
[7] Each of the six principal cursive scripts (*al-Aqlam as-Sitta*) is associated with a student of Yaqut al-Musta’simi. These are *Naskh* (‘Abdallah as-Sayrafi), *Muhaqqaq* (‘Abdullah Arghun), *Thuluth* (Ahmad Tayyib Shah), *Tauqi* (Mubarakshah Qutb), *Rihani* (Mubarakshah Suyufi), and *Riqa* (Ahmad as-Suhrawardi). See Schimmel (1990), 22.

[8] Tabbaa (2001), 34–35.

[9] Tabbaa (2001), 34–44.

between the letters. Similarly, plaited *Kufi* intertwines the vertical letters into elaborate knots. In time, the traditions of foliated and plaited *Kufi* became highly elaborate, and a prominent feature of architectural ornament and decorative arts throughout the Islamic world.

The geometric quality of *Kufi* received its most extreme expression in the development of the principally epigraphic style of *Shatranji Kufi* (chessboard *Kufi*). This calligraphic style forces each letter of the alphabet to conform to the orthogonal grid; and the resulting geometric nature of this style endows it with a quality which appears, at first glance, more akin to geometric key patterns than written words. As an ornamental device, this expressly geometric calligraphic style is highly effective and can be found in buildings from al-Andalus to India. The orthogonal nature of *Shatranji Kufi* was ideally suited to the technical constraints that governed early Islamic brick ornament.[10] Among the earliest examples of this calligraphic style are the raised-brick ornament of the Ghaznavids and Seljuks in the regions of Khurasan and Persia. Two fine examples of this art are included in the ornament of the minaret of Mas'ud III in Ghazni, Afghanistan (1099-1115) [Photograph 1], and the interior façade of the Friday Mosque at Golpayegan, Iran (1105-18). Each of these examples place the bricks that define the calligraphy at 45° from the direction of the orthogonal script, and the direction of the background bricks perpendicular to those of the calligraphy. This creates the herringbone brick aesthetic that remained popular for many centuries, and received particular attention during the Timurid period, as well as subsequent Qajar and Uzbek periods. *Shatranji Kufi* is notoriously difficult to read, and its deliberate obscuration requires the viewer to stop and consider the text in an attempt to unveil its meaning. It is an interesting fact that the development of the virtually illegible *Shatranji Kufi* took place during the same period as the refinements to the legibility of the cursive scripts. Just as religious sentiments were an impetus for calligraphers to better reflect the gravity of the Quran by refining their art to be ever more beautiful and legible, it is possible that the developers of the willfully illegible *Shatranji Kufi* script may also have been motivated by religious convictions in creating an epigraphic corollary of the *Hadith* (saying of the Prophet) wherein Allah replies to the prophet David "I was a hidden treasure, and I longed to be known."[11]



**Photograph 1** *Shatranji Kufi* at the Minaret of Mas'ud III in Ghazni, Afghanistan (© Bernard O'Kane)

The role of geometry within the traditions of Islamic floral ornament is primarily structural: providing symmetrical order upon which the stylized tendrils, flowers, and foliation rest. Most obvious are the innumerable examples of floral design with reflective symmetry. Floral designs with bilateral symmetry are commonly used as infill motifs within the individual cells of a geometric pattern. The use of floral patterns as fillers in an otherwise geometric pattern was certainly part of the pre-Islamic, Late Antique ornamental vocabulary that assisted in the formation of Islamic art as a distinct tradition. However, with the Muslim development of increasingly sophisticated geometric patterns comprised of far more complex and diverse polygonal elements and multiple regions of diverse local symmetry, over time, the floral fillers followed this growth in complexity by becoming considerably more symmetrically varied than their antecedents. Both as polygonal fillers and as stand-alone motifs, floral designs with multiple lines of reflected symmetry were widely employed within Islamic architecture, manuscript illumination, and applied arts. Within architecture, floral designs with reflected symmetry were frequently used for dome ornamentation. In such examples the number of radial lines of symmetry will invariably be divisible by the number of side walls of the chamber that the dome is covering: e.g., if the plan of the chamber is a square, the reflected symmetry will be a multiple of 4.

The use of rotational symmetry was also common; and floral designs with twofold, fourfold, fivefold, sixfold, and eightfold rotational symmetry frequent this tradition. Such

[10] Schimmel (1990), 11.

[11] –Furuzanfar, Badi' al-Zaman (1956), no. 70.
–Ernst (1997), 52.

designs were typically used as roundel motifs, on tiles, or as fillers within the background elements of geometric patterns. As with higher order reflective symmetries, rotational floral designs were also used for dome ornamentation—typically with 8-, 12-, or 16-fold symmetry, albeit less frequently.

Geometric patterns were occasionally provided with an additive floral device that meanders throughout the geometric design rather than being contained as fillers within the individual polygonal background elements. The movement and symmetrical order of this variety of floral design are always in strict conformity to the symmetry of the governing geometric pattern. This hierarchic relationship is visually emphasized by the fact that the floral element invariably flows beneath the geometric pattern. Several notable examples of this type of ornamental device include the wooden *mihrab* from the mausoleum of Sayyidah Nafisah, Cairo (1138-46); the central arch of the stucco *mihrab* at the Reza'iyeh mosque in Orumiyeh, Iran (1277); the carved stucco ornament above a niche in the Pir-i Baqran mausoleum in Linjan, Iran (1299-1311); the carved marble entry facade of the Hatuniye *madrasa* in Karaman, Turkey (1382); and the dome of the mausoleum of Sultan Qaytbay, Cairo (1472-74) [Photograph 2].

Since its onset, Islamic architectural ornament frequently made use of floral scrollwork border designs. This form of floral design was widely used in all media throughout the long history of this ornamental tradition, receiving distinctive interpretations throughout the breadth of Muslim cultures. Such designs employ a single repetitive unit to populate a linear spatial expansion, and without exception adhere to the symmetrical constraints of the *seven frieze groups* that are comprised of different combinations of translation symmetry, reflection symmetry, rotation symmetry, and glide reflection.[12] All linear repeat patterns, be it floral, geometric, figurative, etc., must conform to one or another of these seven frieze groups. There is no indication that Muslim artists, or indeed Muslim mathematicians, had specific knowledge of the seven frieze groups, but the inherent genius for empirical geometric exposition nonetheless led Muslim artists to create border designs that repeat according to the symmetry of each of the seven frieze groups.

Within the science of two-dimensional tiling, just as translation symmetry, reflection symmetry, rotation symmetry, and glide reflection provide the constraints for the seven frieze groups, so do they also provide for the symmetrical conditions for the *17 plane symmetry groups* (also referred to as the *wallpaper groups* or *plane crystallographic groups*).

**Photograph 2** A dome with a geometric and floral design at the mausoleum of Sultan Qaytbay, Cairo (Sébah and Joaillier photograph, courtesy of Special Collections, Fine Arts Library, Harvard University)

All two-dimensional repetitive space filling follows the symmetrical order of one or another of the 17 plane symmetry groups. These were first enumerated by crystallographers and mathematicians in the late nineteenth and early twentieth centuries.[13] There is no evidence that Muslim artists or mathematicians were knowledgeable of this branch of crystallography. However, the art history of all pattern-making cultures is evidence of the fact that an artist does not have to understand the science of two-dimensional space filling in order to make efficacious use of its principles. Several studies have sought to identify examples of all 17 plane symmetry groups within the Islamic ornamental tradition,[14] or even within the single architectural complex of the Alhambra in Spain.[15] Within the Islamic floral idiom, foliage net designs

[12] –Weyl (1952).
–Hargittai (1986).
–Washburn and Crowe (1988).
–Farmer (1996).

[13] –Fedorov (1891), 345–291.
–Pólya (1924), 278–282.

[14] –Lalvani (1982).
–Lalvani (1989).
–Abas and Salman (1995).

[15] –Müller (1944).
–Grünbaum, Grünbaum and Shepherd (1986).
–Pérez-Gómez (1987), 133–137.

are always predicated upon one or another of these 17 plane symmetry groups, as are most of the many repetitive floral scrollwork designs.

The Muslim use of overtly geometric ornament dates back to the earliest period of military expansion. The rapid acquisition of territories previously governed by the Byzantines, Copts, and Sassanians availed the Muslim conquerors to a wide range of artistic and ornamental influences. These included several mature geometric design conventions that were readily appropriated into the ambitious architectural projects of the Muslim conquerors, and that were to prove highly influential to subsequent Muslim dynasties. In this way, the ornamental art of early Islamic cultures can be considered as inheritors of the geometric traditions of their conquered subjects, as well as progenitors of the extraordinary advances in the geometric arts that followed. Among the geometric design conventions that the Muslim conquerors inherited were stellar mosaics, compass-work compositions, braided borders, key patterns, and polygonal tessellations. Each of these continued to be used to a greater or lesser extent throughout the history of this ornamental tradition, and part of the genius of Muslim artistry was the ability to assimilate foreign artistic conventions by reworking them to fit within its own distinctive aesthetic. Under the patronage of the Umayyads, the inherited design conventions employed in the creation of stellar mosaics, compass-work mosaics, and polygonal tessellations were particularly relevant to the development of the preeminent form of overtly geometric Islamic ornament: the star patterns that characterize this ornamental tradition.

## 1.2 The Rise to Maturity

The history of Islamic geometric design can be regarded as a sequential evolution from simplicity to complexity. From its onset in the ninth and tenth centuries, this new form of ornament was characterized by an overall geometric matrix with primary stars or regular polygons located upon the vertices of a repetitive grid. The geometric designs from this early period have either threefold or fourfold symmetry: the former characterized by hexagons or six-pointed stars located on the vertices of either a triangular or a hexagonal repeat unit, and the latter generally characterized by eight-pointed stars, octagons, or squares placed on the vertices of a square repeat unit. Geometrically simple patterns of these varieties are found in several of the early monuments in the central and western regions of Abbasid influence, including the Great Mosque of Shibam Aqyan near Kawkaban in Yemen (pre-872); the mosque of ibn Tulun (876-79) in Fustat, Egypt (now part of greater Cairo); and the Baghdadi *minbar* (c. 856) at the Great Mosque of Kairouan in Tunisia.

Several techniques for creating geometric patterns appear to have been used historically, and many of the less complex geometric designs can be created from more than a single methodological approach.[16] It is therefore not always possible to know for certain which generative technique was used for a given historical example. Of particular interest, and the primary focus of this study, is the *polygonal technique*. Almost all of the early geometric patterns can be easily produced with this design methodology. However, placing stars or polygons in simple point-to-point configurations will also create many of the earliest patterns known to this tradition. Additionally, simple tracings upon the isometric grid will easily create many of the early threefold designs, as is similarly possible with the orthogonal grid for some of the least complex fourfold designs. The fact that the polygonal technique is a more demanding design methodology requiring two distinct steps would appear to argue for the greater relevance of less complicated and more immediate methodologies. However, the strength of the polygonal technique is in its inherent flexibility, providing the high level of design diversity and range of complexity that characterize this tradition. By the close of the tenth century geometric patterns were being created that were significantly more difficult to produce using other methodologies. With the growth in complexity, the polygonal technique became an increasingly important force behind the evolving sophistication of Islamic geometric star patterns that took place between the onset of this tradition during the ninth and tenth centuries and its full maturity during the thirteenth century.

The distinctive feature of this methodology is the employment of a polygonal tessellation that acts as a substructure from which the geometric pattern is derived. This process involves the placement of the pattern lines upon specified points along the edges of each polygon within an underlying generative tessellation. By the twelfth century, four distinct families of geometric pattern had evolved. Three of these are determined by placing crossing pattern lines that intersect on, or occasionally near, the midpoints of the underlying polygonal edges. The specific angle of these crossing pattern lines, referred to herein as the *angular opening*, determines the overall character of the design. For purposes of descriptive clarity these three families are referred to as *acute*, *median*, and *obtuse*. The fourth historically common pattern family places the pattern lines upon two points of each underlying polygonal edge, and is hence referred to as the *two-point* family. These two contact points are frequently determined by dividing the polygonal edge into either thirds or quarters. With rare exception, the

[16] A comparison and descriptive analysis of differing generative methodologies is covered in Chap. 2.

underlying polygonal tessellation is dispensed with after the pattern is created, leaving behind only the derived pattern with no easily discernable indication for how the pattern was constructed. Any one of the four pattern families can be used when extracting a geometric pattern from an underlying polygonal tessellation. The fact that each underlying formative tessellation can generate patterns from each family significantly augments the generative design potential of this methodology.

During the period of rising maturity, Muslim artists discovered several polygonal *systems* for creating geometric patterns. It is impossible to know for certain exactly when and where these systems were discovered, and without definitive historical evidence, it could be argued that use of these systems by Muslim artists remains conjecture. However, the systematic mode of the polygonal technique is the only practical explanation for the fact that such large numbers of patterns were created that strictly adhere to a common set of visual features that are associated with specific pattern families within one or another of these design systems. For example, the large number of fivefold *acute* patterns with ten-pointed stars that are ubiquitous to this ornamental tradition share very specific design characteristics within their pattern matrix, and these similarities are difficult, if not impossible, to explain with anything other than the existence of the *fivefold system* of pattern generation. Each of the historical design systems relies upon a limited set of polygonal modules that combine together in an edge-to-edge configuration to make the underlying generative tessellation. As described above, pattern lines in either the *acute*, *median*, *obtuse*, or *two-point* families are applied to the edges of these polygonal modules. The strength of designing patterns with polygonal systems is the ease of exploring new assemblages and resulting patterns. If one considers that the modules that make up each system can be combined in an infinite number of ways, and that each of the four pattern families can be applied to each tessellation, there are an unlimited number of geometric patterns available to each system.

The surviving architectural record indicates that the earliest methodological system to have been developed relies upon regular polygons to create the underlying generative tessellations. This is referred to herein as the *system of regular polygons*. The construction of geometric patterns from underlying tessellations made up of regular polygons appears to have begun in the ninth century and continued throughout the length and breadth of this ornamental tradition. From as early as the eighth century, Muslim artists employed tessellations made from the regular polygons as ornamental motifs in their own right. Noteworthy among the early examples of this form of geometric ornament are the Yu'firid ceiling panels of the Great Mosque of Shibam Aqyan near Kawkaban in Yemen (pre-871-72). Considering the interest in polygonal ornament generally, it is entirely reasonable to allow for the inventive leap from using such tessellations as ornamental motifs to employing them as a substratum from which pattern lines can be extracted. As said, the precise date for the methodological discovery of using underlying tessellations to create geometric patterns is uncertain. This is due to the aforementioned fact that the simplicity of ninth- and tenth-century examples allow for their creation with either the polygonal technique, the iterative placements of simple star forms, or simply the tracing of lines from the isometric grid. What is certain is that almost all of the ninth- and tenth-century prototypical geometric patterns *can* be easily created using the polygonal technique. It is significant that when considered from the perspective of the polygonal technique, the underlying generative tessellations for virtually all of these early examples are comprised of regular polygons. As the use of this regular polygonal methodology became more sophisticated, the resulting geometric patterns became more diverse and more complex; and the prevalence of such patterns became sufficiently common to warrant their own descriptive classification: the *system of regular polygons*. The growth in complexity of geometric patterns made from the *system of regular polygons* is directly associated with the expansion of knowledge of the tessellating potential of the regular polygons.

Only five of the regular polygons will combine to uniformly fill the two-dimensional plane in an edge-to-edge configuration: the triangle, square, hexagon, octagon, and dodecagon [Figs. 89–91]. *Regular*, *semi-regular, two-uniform*, and *three-uniform* tessellations were all used historically to generate geometric patterns [Figs. 95–115]. Depending on the arrangement of the polygonal modules, patterns with either threefold or fourfold symmetry were constructed. Similarly, the variety of repeat units found within this system includes the equilateral triangle, square, and regular hexagon, as well as rectangles, rhombi, and non-regular hexagons. As this ornamental tradition matured the *system of regular polygons* occasionally included additional non-regular polygons. These non-regular modules are created as interstitial spaces in a tessellation of otherwise regular polygons. These interstice modules create distinctive pattern characteristics that augment the beauty of patterns made from this system [Figs. 116–122]. The historical record demonstrates a high level of symmetrical and repetitive variety within this design system, resulting in the surprising degree of design diversity that is emblematic of this systematic methodology.

The regular polygons that tessellate together include the octagon. However, unlike the other regular polygons from this system, the octagon only tessellates in one combination: the semi-regular $4.8^2$ tessellation of squares and octagons [Fig. 89]. The octagon and square are also components of the *fourfold system A* [Fig. 130]; and patterns made from this semi-regular tessellation of octagons and squares are justifiably associated with either of these two systems. However, patterns such as the ubiquitous classic star-and-cross design [Fig. 124b] that are easily created from the $4.8^2$ tessellation precede the earliest known patterns associated with the *fourfold system A* by some 200 years. Furthermore, the early examples of patterns created from the octagon and square share the same approximate time and place with other designs that are created from the *system of regular polygons*. Patterns created from the underlying $4.8^2$ tessellation are therefore more appropriately considered as part of the group of designs that originate from the *system of regular polygons*. However, due to that fact that the octagon has only a single tessellation within the theoretically infinite number of possible combinations of the other regular polygons within this system, for the purposes of this discussion, patterns derived from this tessellation of octagons and squares are regarded as a separate generative category. It is worth noting that the design diversity produced from this one tessellation is truly remarkable, and its historical use very likely exceeds that of any other single underlying tessellation from this design tradition [Figs. 123–129].

Two of the polygonal systems used regularly throughout Muslim cultures have fourfold symmetry and employ the octagon as the primary polygonal module. These are referred to herein as the *fourfold system A* and the *fourfold system B*. Most of the patterns that these two systems create repeat upon the orthogonal grid, although patterns with 45° and 135° angled rhombic repeat units were occasionally employed, as were patterns with rectangular repeat units. Patterns with radial symmetry are also possible with these systems, although infrequently used. The *fourfold system A* has three modules that are regular polygons: a large octagon, a small octagon, and a square. Other than these, all of the polygons within this system are irregular [Fig. 130]. The geometric construction for each shape within this system is easily derived from the large octagon [Fig. 131]. The *fourfold system A* is comprised of a relatively large number of polygonal modules, resulting in a particularly high level of diversity in the potential underlying generative tessellations [Figs. 136–168]. The *fourfold system B* has fewer polygonal modules: allowing for less tessellating variation than that of the *fourfold system A*. This system is nevertheless responsible for a wide variety of distinctive and beautiful designs from the historical record [Figs. 173–186]. The octagon is the only regular polygon within the set of generative modules of the *fourfold system B* [Fig. 169]. The polygonal modules of this system are easily constructed from this primary polygon, or through identifying interstice regions through tessellating with the octagon and irregular pentagon [Fig. 170]. The large number of historical patterns that are associated with both these fourfold systems has by no means exhausted the generative potential for making new and original designs.

Ghaznavid and Qarakhanid artists produced the earliest patterns associated with the *fourfold system A* during the first quarter of the eleventh century. Seljuk and Ghurid artists adopted this methodological system within half a century, and the diversity of patterns created by these eastern cultures is remarkable. The rapid westward spread of Seljuk influence introduced this system to the Artuqids, Zangids, and the Seljuk Sultanate of Rum, by which time it had become part of the standard geometric design repertoire of these regions. This system was not adopted by the Fatimids of Egypt, and even their Ayyubid and Mamluk successors made significantly less use of this variety of pattern than their contemporaries to the north and east. By the fourteenth century, the *fourfold system A* was also an integral feature of the western Islamic ornamental tradition, and the number of examples found in Nasrid, Marinid, and *Mudéjar* monuments is remarkable. Artists working in the Maghreb developed this system to further levels of refinement and complexity through the incorporation of 16-pointed stars. Even more remarkable was the innovative use of this system to create the astonishing dual-level designs that are the earliest expressions of complex self-similar art ever produced.[17]

The architectural record indicates that development of the *fourfold system B* took place during the first half of the twelfth century. These earliest examples are Qarakhanid, Seljuk, and Ghurid, but there were far fewer patterns produced from this system during this early period in the eastern regions than those of the *fourfold system A*. The predominance of early patterns created from the *fourfold system B* is found in the western regions of Seljuk influence, and these were produced under the patronage of the Ildegizids, Zangids, Ayyubids, and the Seljuk Sultanate of Rum. The Mamluks were far more disposed to this system than to the *fourfold system A*. Following the innovations in the western regions under Seljuk influence, the *fourfold system B* was readily incorporated into the ornament of the eastern regions following the Mongol destruction, and fine examples were produced under Ilkhanid, Kartid, Muzaffarid, Timurid, and Mughal patronage. And in the western regions of the Maghreb, as with the *fourfold system A*, the Nasrids and Marinids also used this system widely in their architectural ornament. The rapid adoption of the *fourfold system B* into the body of geometric expression among diverse Muslim cultures suggests a transcultural mechanism wherein artistic innovations were willfully shared between artists under the patronage of both friendly and rival dynasties. At the very least, it must be concluded that the currency of artistic knowledge was highly valued and facilitated the movement of specialists from region to region.

The differences in appearance between the *fourfold system A* and the *fourfold system B* are readily apparent to a trained eye. Both incorporate eight-pointed stars and

[17] Bonner (2003).

octagons as standard features, and the vast majority of patterns from both systems repeat upon the orthogonal grid. However, the very different characteristics of the respective underlying polygonal modules from each system result in geometric designs with concomitant differentiation. In particular, the pentagonal and hexagonal modules from the *fourfold system B* create distinctive pattern qualities that are entirely dissimilar to the geometric characteristics associated with the *fourfold system A*. By the twelfth century, artists working with the *fourfold system B* discovered that the application of *acute* pattern lines to the elongated hexagonal modules could be varied to allow for the creation of octagons within the pattern matrix [Fig. 172].

Almost all of the innumerable patterns with fivefold symmetry and ten-pointed stars that are found throughout the Islamic world have their origin in the *fivefold system*. The repeat units of patterns generated from this system are predominantly either rhombic or rectangular. There are two rhombi associated with fivefold symmetry that function as repeat units for patterns made from this system [Fig. 5]: the wide rhombus with 72° and 108° included angles, and the thin rhombus with 36° and 144° included angles. The wide rhombus was used more extensively as a repeat [Figs. 232–240], but many patterns were also created that repeat with the thin rhombus [Figs. 241–244]. The proportions of the rectangular repeat units used with this system varied considerably [Figs. 245–256]. Less common are patterns with irregular hexagonal repeat units [Figs. 257–259], and those with radial symmetry [Fig. 260]. Occasionally, greater complexity was achieved through using several repetitive components within a single design, any one of which is able to create patterns on its own [Figs. 261–266]. In this study, these are referred to as *hybrid* patterns, and the earliest known example was produced by Seljuk artists for one of the recessed arches in the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Fig. 261] [Photograph 25]. Most of the subsequent fivefold hybrid examples were produced under the patronage of the Seljuk Sultanate of Rum.

The *fivefold system* has a greater number of components than either of the two fourfold systems [Figs. 187–188]. The limited set of polygonal modules that comprise the *fivefold system* includes two that are regular: the decagon and pentagon. The polygonal modules of this system can be easily produced from the decagon [Fig. 189], or through interstice regions when tessellating with other modules [Figs. 190–191a]. Some modules can also be created from overlapping the pentagon or decagon, and a further set of components is created from the union of two conjoined decagons [Fig. 191b]. There are two edge lengths among the polygonal modules within this system: the shorter being the length of the edges of the regular decagon and pentagon, and the longer being equal to the distance from the center of the decagon to one of its vertices. The ratio of these two edge lengths is the golden section (1:1.618033987. . .); and indeed, the proportional relationships inherent within fivefold geometric patterns are imbued with this geometric ratio [Fig. 195].

A subcategory of fivefold patterns forgoes the decagon within the underlying tessellation, thereby eliminating the characteristic ten-pointed stars from the overall pattern matrix. Such designs are referred to as *field patterns*, as the absence of the ten-pointed stars produces more uniform density within the pattern matrix [Figs. 207–220]. This variety of design was especially popular in the architecture of the Seljuk Sultanate of Rum. Such field patterns are both aesthetically distinct from standard fivefold patterns, and pleasing to the eye. The repeat units of fivefold field patterns are predominantly either rectangles or irregular hexagons.

The versatility and visual appeal of patterns made from the *fivefold system* led to its rapid spread throughout Muslim cultures; and outstanding examples are to be found in diverse ornamental media throughout the length and breadth of this ornamental tradition. The earliest extant fivefold geometric designs were produced by Seljuk artists during the close of the eleventh century. Within a decade, the architectural ornament of the Ghaznavids also incorporated patterns with fivefold symmetry. By the middle of the twelfth century Ghurid artists also made use of patterns created from the *fivefold system*, followed by the Qarakhanids some 30 years later. And as with the *fourfold systems A* and *B*, the *fivefold system* spread westward from Khurasan and Persia into regions under Seljuk influence, subsequently becoming an ubiquitous feature of the ornamental arts of Muslim cultures generally.

Among the most fascinating systematic geometric patterns to have been created by Muslim artists are a relatively small number of designs with sevenfold symmetry [Figs. 279–282 and 286–294]. However, the small number of surviving historical examples of such patterns begs the question as to the extent to which geometric artists were aware of the systematic repetitive potential of the underlying polygonal components that made up the generative tessellations. This variety of patterns is very beautiful, and had the systematic potential for these components been known by those artists working with geometric patterns generally; one would assume that, as with fivefold patterns, there would be far more examples found throughout the Islamic world. This paucity of examples appears to indicate the rarity of knowledge of this system among geometric artists. However tenuous our understanding of past sevenfold methodological knowledge is, it is nonetheless a fact that the relatively few sevenfold patterns in the historical record would have been relatively easy to create from a limited set of repetitive polygonal modules that include associated pattern lines in each of the four standard pattern families. The earliest known sevenfold geometric patterns include a Seljuk field pattern from the northeastern domed chamber (1088-89) of the Friday Mosque at Isfahan [Fig. 279], and two Ghaznavid examples from the minaret of Mas'ud III in Ghazna, Afghanistan (1099-1115) [Figs. 280 and 281]. Interestingly, these same monuments also include the earliest known examples of two-dimensional fivefold patterns. Each of these early Seljuk and Ghaznavid sevenfold patterns repeat upon irregular hexagonal grids. The underlying generative tessellation of the earlier Seljuk example employs two varieties of hexagon to create the sevenfold field pattern, and the hexagonal repeat unit is a product of the specific arrangement of underlying hexagons. The hexagonal repeat units of the two Ghaznavid examples have touching edge-to-edge heptagons placed at each vertex of the repeat unit. The interstice of these six edge-to-edge regular heptagons is comprised of two irregular pentagons that likewise touch edge to edge. The first of these Ghaznavid sevenfold geometric designs incorporates a set of primary pattern lines placed upon the vertices of the underlying heptagons, thereby producing a set of seven-pointed stars whose points touch those of adjacent seven-pointed stars [Fig. 280b]. The second set of pattern lines are placed upon two points of each heptagonal edge [Fig. 280c]. This is a remarkably complex design, especially considering its very early date. The second sevenfold design from Ghazna is no less impressive. The primary pattern lines of this design are located upon the midpoints of the heptagons in the same underlying tessellation [Fig. 281b], while the secondary pattern lines are an arbitrary addition that makes this design considerably more complex [Fig. 281c]. Approximately a hundred years later, artists in Anatolia created several sevenfold geometric patterns using the same underlying generative tessellation of heptagons and irregular pentagons. These three Anatolian examples differ in that they are less complex, and fully systematic—in that all of the pattern lines are the direct product of the underlying polygonal tessellation [Fig. 282]. In time, this *sevenfold system* developed in its increased use of a larger number of polygonal components with a resulting increase in complexity. A noteworthy feature that distinguishes these later examples from the earlier designs is the use of underlying tetradecagons (14-sided regular polygons) that produce 14-pointed stars. There is a marked increase in the number of polygonal modules associated with the *sevenfold system* over the other historical systems [Fig. 271], and as a general rule, the greater the number of sides to the primary polygon, the greater the number of modules within a given system. An added feature of the growth in complexity of this system was the use of additional repeat units beyond the initial elongated hexagon described above. These included patterns with rectangular repeats, and patterns based upon one or another of the three rhombi associated with 14-fold symmetry [Fig. 10]. As with the other systems, the primary star forms (in this case 14-pointed stars) were typically placed upon the vertices of each repeat unit. These more complex sevenfold geometric patterns originated among the Mamluks in Egypt and the Levant during the fourteenth and fifteenth centuries, and to a lesser extent were also employed by a select number of artists working for the Ottomans and Timurids.

Perhaps the most remarkable historical use of generative polygonal design systems was in their application to multiple level designs. During the fourteenth and fifteenth centuries, the innovative dual-level use of the *system of regular polygons*, both fourfold systems and the *fivefold system*, brought about the last great creative leap in the historical development of Islamic geometric star patterns [Figs. 442–477]. Through careful manipulation of these polygonal systems, Muslim artists produced several varieties of geometric design that are consistent with the modern geometric criteria for self-similarity whereby an entity or a structure is recursively present within an analogous scaled-down substructure that, in turn, provides for the possibility of infinite further recursive iterations. While this recursive process is mathematically infinite, be it cosmological, geographical, biological, or anthropogenic, the manifestation of self-similar recursion is constrained by the medium in which it occurs. The historical examples of self-similar star patterns never exceed a single recursion, and are characterized by two levels of design: the visual character and methodological origins of each being either identical or very similar to the other. Can an object be self-similar if it has only a single recursion? The answer is yes, provided the relationship between both levels satisfies the criteria for self-similarity, and the recursion has the theoretical capacity for infinite reiteration. The recursive scaling ratio is always a product of the geometric schema, and the placement of the secondary pattern is determined through the application of scaled-down underlying polygonal modules from the same system that were used to create the primary design. These scaled-down elements typically place the primary polygonal modules, such as an octagon or a decagon, upon the crossing lines of the primary pattern. Most of the examples of Islamic self-similar ornament were fabricated in cut-tile mosaic, and a few examples were produced in wood. The fact that the Muslim artists responsible for these masterpieces of geometric art limited themselves to just two levels of self-similarity is more to do with the material constraints of their chosen medium than with any lack of geometric ingenuity.

Islamic self-similar design developed along two distinct historical paths. The earliest occurrence of such patterns was during the fourteenth century in the western regions of Morocco and al-Andalus under the patronage of the Marinid and Nasrid dynasties. A century later, highly refined self-similar patterns were introduced to the architectural ornament of Transoxiana, Khurasan, and Persia under rival

Timurid, Qara Qoyunlu, and Aq Qoyunlu patronage. It is unknown whether these two design traditions developed in isolation, or the preceding design methodologies from the Maghreb directly influenced the development of this design convention in the eastern regions. While the methodology in the creation of self-similar designs from both regions is essentially the same, their respective stylistic character is very different. As mentioned, this methodology is reliant upon the recursive tessellating properties inherent to these design systems. When considered from the perspective of Islamic art history, the self-similar designs created in these western and eastern regions represent the pinnacle of systematic geometric design, and, as said, the last great innovation in the illustrious tradition of Islamic geometric star patterns. As pertains to the history of mathematics, these fourteenth- and fifteenth-century designs are no less significant in that they appear to be the earliest anthropogenic examples of sophisticated self-similar geometry.

In addition to geometric star patterns being produced via a systematic design methodology, Muslim artists expanded the polygonal technique to include *nonsystematic* designs. These are generated from underlying tessellations that include polygons that are irregular and specific to the tessellation [Figs. 309–441]. In contrast to the various generative systems, many of the polygonal components of such tessellations will not reassemble into other tessellations, and as such, patterns made from this variety of underlying tessellation are therefore nonsystematic. One of the virtues of a systematic design methodology is the ease of creating new patterns through new assemblages of the polygonal modules. One has only to produce a new tessellation from a predetermined set of compatible decorated polygonal modules. The creation of nonsystematic geometric patterns is entirely different. Muslim artists developed a precise design methodology that produced a wide range of underlying tessellations with polygonal components that are specific to the construction. As with the systematic approach, each nonsystematic tessellation will produce geometric designs in each of the four pattern families. Although only conjecture, similarities between nonsystematic designs and those created from the *fivefold system* suggest the possibility that the mature expression of nonsystematic patterns was directly influenced by the aesthetics and working practices found within the *fivefold system*. Fundamental to the creation of nonsystematic underlying tessellations is the use of radii matrices as an initial foundation for the construction sequence. Evidence that radii matrices were an integral feature of the nonsystematic use of the polygonal technique is found in many of the geometric star patterns illustrated in the Topkapi Scroll. This is a unique and immensely important document in many respects, including the insight it provides into the methodology employed for constructing complex geometric star patterns. The maker of the Topkapi Scroll used a steel graver to scribe non-inked "dead drawing" reference lines into the paper, and included among these barely visible scribed lines are radii matrices.[18] These articulate the regions of primary and secondary local symmetry, and relate directly to the construction of the underlying polygonal tessellations, most frequently illustrated in finely dotted lines of red ink, upon which the typically black pattern lines are positioned.

The tradition of nonsystematic geometric star patterns is immensely diverse and covers a wide range of symmetries and variety of repeat units. Most commonly, nonsystematic geometric patterns will repeat on either the isometric or the orthogonal grids. The least complex examples of this type of geometric design employ a single variety of star that is located upon each vertex of the repetitive grid [Figs. 309–345]. The number of points for these stars is governed by the number of angles at each vertex as a multiplier, with *n*-points being the product. In this way, patterns that repeat upon the isometric grid will typically have 6, 12, 18, 24 (etc.) pointed stars at each vertex [for example: Fig. 320], while the vertices of patterns that repeat on the orthogonal grid will typically have 8, 16, 24 (etc.) pointed stars [for example: Fig. 337]. The regular hexagonal grid was also employed, and such patterns will commonly have 6, 9, 12, 15, 18 (etc.) pointed stars at the vertices of this repetitive grid [for example: Fig. 313]. The isometric and orthogonal grids also provide for patterns with greater complexity that have additional varieties of local symmetry beyond those located at the vertices of the repeat unit [Figs. 346–411]. These are generally referred to as *compound patterns*, and the least complex will place additional stars at the vertices of the dual of the isometric or orthogonal grid—which is to say at the centers of each repeat unit. The dual of the isometric grid is the regular hexagonal grid, and examples of compound local symmetry for such patterns can include star combinations of 6 and 9 points, 12 and 9 points, 12 and 15 points, etc. [for example: Fig. 346]. The dual of the orthogonal grid is of course another orthogonal grid, and compound patterns of this variety will typically include star combinations of 8 and 12, 8 and 16, 12 and 16, etc. [for example: Fig. 379]. Still further complexity was achieved through additional centers of local symmetry being incorporated into the isometric or orthogonal repeat units. The locations for these additional regions of local symmetry are typically at the center points of each edge of the repeat unit [for example: Fig. 402], or within the field of the repeat unit [for example: Fig. 400]. It is worth mentioning that the center point of the repeat unit's edge is also the intersection of the grid and its dual [Fig. 1]. These additional locations provide the designer with greater latitude in determining the variety of local

[18] Necipoğlu (1995), 239–283.

symmetry and resulting star forms. When these additional star forms are located at the midpoint of the edge of the repeat unit, they tend to have an even number of points, while the use of additional local symmetries within the field of the repeat unit is less rigid.

Some of the most remarkable nonsystematic geometric patterns are characterized by their incorporation of two seemingly incompatible varieties of primary local symmetry. As mentioned above, compound patterns that repeat upon either the isometric or the orthogonal grids will most commonly place regions of local symmetry at the vertices of both the repetitive grid and its dual. The relationship between the grid and its dual provides for star forms at these locations that are compatible and predictable. By contrast, this more elusive variety of compound pattern brings together two varieties of *n*-fold local symmetry that would not ordinarily work with one another to fill the two-dimensional plane: for example 9- and 11-pointed stars [Fig. 431]. This variety of compound pattern typically employs either a rectangular grid [Figs. 412–428] or an elongated hexagonal repetitive grid [Figs. 429–439]. As with the more complex compound patterns that adhere to the isometric, regular hexagonal, and orthogonal grids, patterns that repeat with rectangular and elongated hexagons will occasionally incorporate additional centers of local symmetry upon the edges or within the field of the repeat unit [for example: Fig. 427].

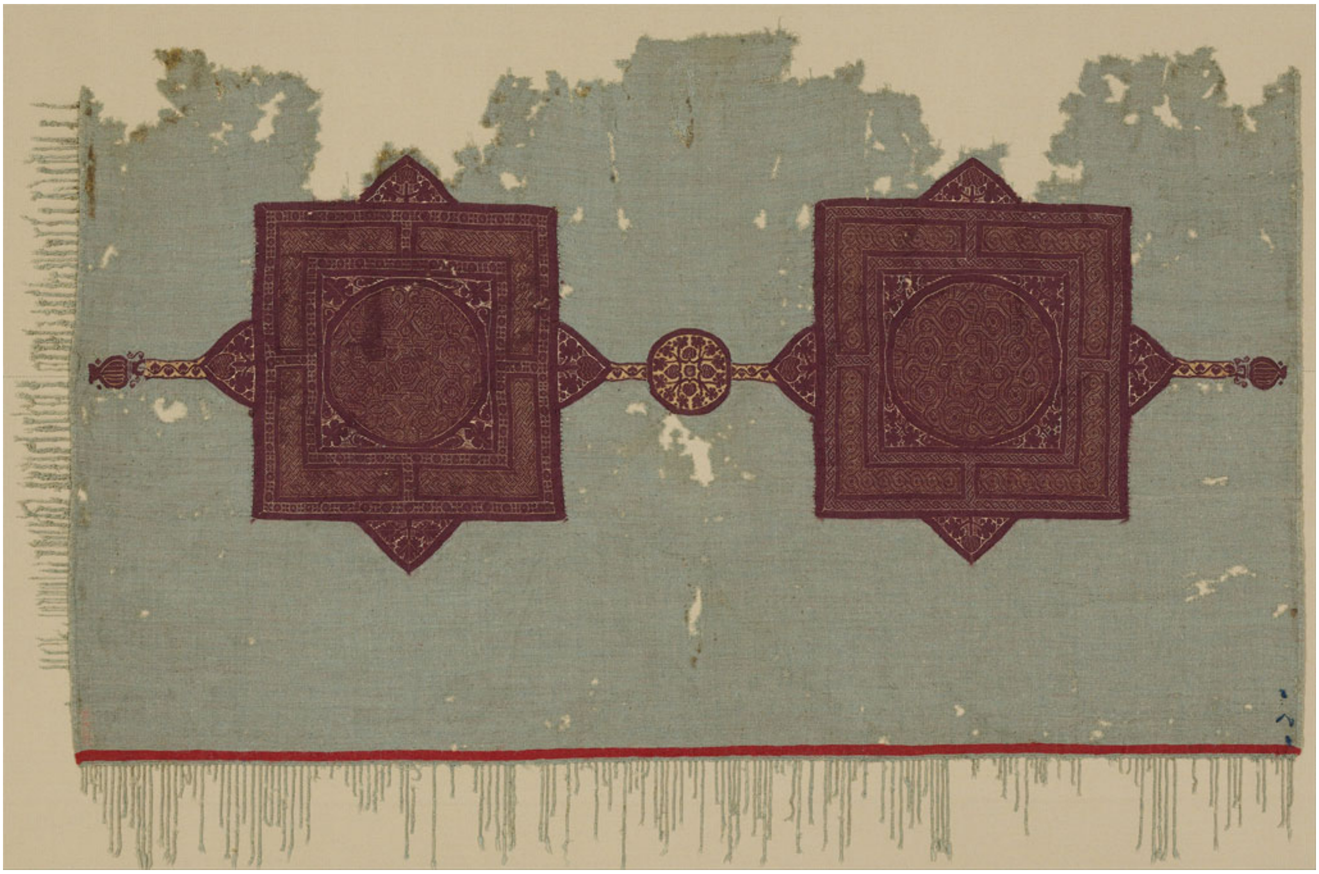
The beauty of nonsystematic compound star patterns is, in large part, the direct consequence of their geometric sophistication. Indeed, this highly refined utilization of the polygonal technique is responsible for the creation of the most geometrically complex Islamic star patterns throughout the length and breadth of the Islamic world. Many patterns created from one or another of the historical design systems can also be produced via alternative methodologies, for example, point joining or through the use of grid-based constructions. However, these alternative methodologies become less and less relevant as complexity increases, and the only practicable method for constructing the considerably more complex nonsystematic designs with multiple regions of local symmetry is via the polygonal technique. Other historically demonstrable design methodologies do not have the flexibility to work seamlessly with the diverse complexities associated with multiple regions of local symmetry.

The continued development of the polygonal technique allowed Muslim artists to raise the geometric arts to an unsurpassed level. The versatility of this methodology facilitated the remarkable diversity that characterizes this tradition, including the discovery of new and ingenious repetitive formulae for covering the two-dimensional plane; the establishment of the four principal pattern families; the discovery of several tessellating *systems* that employ a limited set of decorated polygons that iteratively combine in an infinite number of ways; the development of nonsystematic compound patterns wherein centers of differing local symmetry allow for the placement of different star types within a single pattern; and the discovery of the recursive application of the polygonal systems to create two-level geometric patterns that conform to the modern criteria of self-similarity.[19] Each of these is a separate and significant aspect of this overall tradition, and each is unlikely to have developed and flourished without the inherent flexibility of the polygonal technique.

## 1.3 Umayyads (642-750)

In 635, just 3 years after the death of the Prophet Muhammad, Muslim forces of the Rashidun Caliphate conquered the Byzantine vassal state of Syria. Within two years the Sassanian Empire of Persia fell, followed by Byzantine-held Egypt in 642. The succeeding Umayyad Caliphate continued this rapid expansion: taking control of a contiguous region from Spain and North Africa to the Indus River. The vast territorial expanse of this empire created the need for a more central administrative capital. This brought about the move of their capital from Medina to Damascus. The conquering of Byzantine Syria, Persia, and Egypt brought the Muslim conquerors into contact with several cultures with highly developed architectural and ornamental traditions. By contrast, the artistic heritage of the conquering Arabs was far less sophisticated. The Umayyad rise to power and wealth facilitated an ambitious emphasis upon the construction of large monumental buildings. They were prolific builders, and were quick to employ the superior skills of their non-Muslim subjects. When the Great Mosque of Kufa was rebuilt in 670, a Persian architect was employed who had worked for the Sassanid kings; and Persian masons were used in rebuilding the Kaaba in 684. Builders and craftsmen from Egypt, Greece, and Syria were employed in rebuilding the Masjid al-Nabawi (Prophet's Mosque) in Medina during the period of 707-709; and Coptic Christians from Egypt were likewise used in building both the al-Aqsa mosque in Jerusalem, and the Dar al-Imara palace from 709 to 715. Very little remains of the original al-Aqsa mosque, and among the most important existing examples of early Umayyad ornament, are the Qubbat al-Sakhra (Dome of the Rock) in Jerusalem (685-92), the Great Mosque of Damascus (706-15), the excavated palace of Khirbat al-Mafjar near Jericho in the Jordan Valley (739-44), and the archeological site of Qasr al-Mshatta in Jordan (744-50).

[19] Bonner (2003).



**Photograph 3** A fifth-century Coptic textile with eight-pointed stars (The Metropolitan Museum of Art: Gift of George F. Baker, 1890: www.metmuseum.org)

The precise origins of Islamic geometric star patterns are impossible to establish categorically. There are too many ornamental influences, and too few remaining buildings or objects of art from the early formative period to know definitively when or precisely how this intrinsically Islamic ornamental convention began. The use of stars as a decorative motif was practiced by the pre-Islamic cultures of Byzantium, Coptic Egypt, and Sassanid Persia, and included their use as either singular motifs within a decorative schema or constellations wherein multiple stars provide the primary character of the design. Within the pre-Islamic Coptic textile tradition the eight-pointed star was frequently used as an independent element, often filled with an elaborate profusion of embroidered interweaving knot-work [Photograph 3]. A pre-fifth-century Hellenic mosaic pavement from the Sardinian town of Nora may be relevant to the later development of Islamic geometric star patterns. This design incorporates multiple eight-pointed stars composed of two interweaving squares that are placed upon a square grid in such a manner that two adjacent points from each star touch the equivalent points from each orthogonally placed neighboring star. The interstices of this stellar formation are regular octagons and rhombi. A conceptually similar design from a Roman settlement in El Djem, Tunisia (third century), uses 12-pointed stars in a similar arrangement [Photograph 4].[20] These also repeat upon the square grid and are orientated so that two adjacent points touch the equivalent two points of their orthogonal neighbor. This arrangement of 12-pointed stars results in the background shapes being rhombi and a cross-like element that is further filled with regular hexagons and central 4-pointed star. Prior to the advent of Islam, Byzantine artists continued working with the long-established conventions of Hellenic mosaics, including the geometric idiom that forms part of this overall tradition.[21] Exposure to the architectural ornament and mosaic pavements from the historic centers of Byzantine culture in the Middle East, such as Jerusalem, Antioch, Madaba, Tel Mar Elias, al-Maghtas, and Tell Hesban, would have familiarized the early Arab conquerors with the Hellenic practice of creating designs from an assemblage of stars. Moreover, the Umayyads had ready access to the aesthetic

[20] From the collection of the El Djem Archaeological Museum, El Djem, Tunisia.

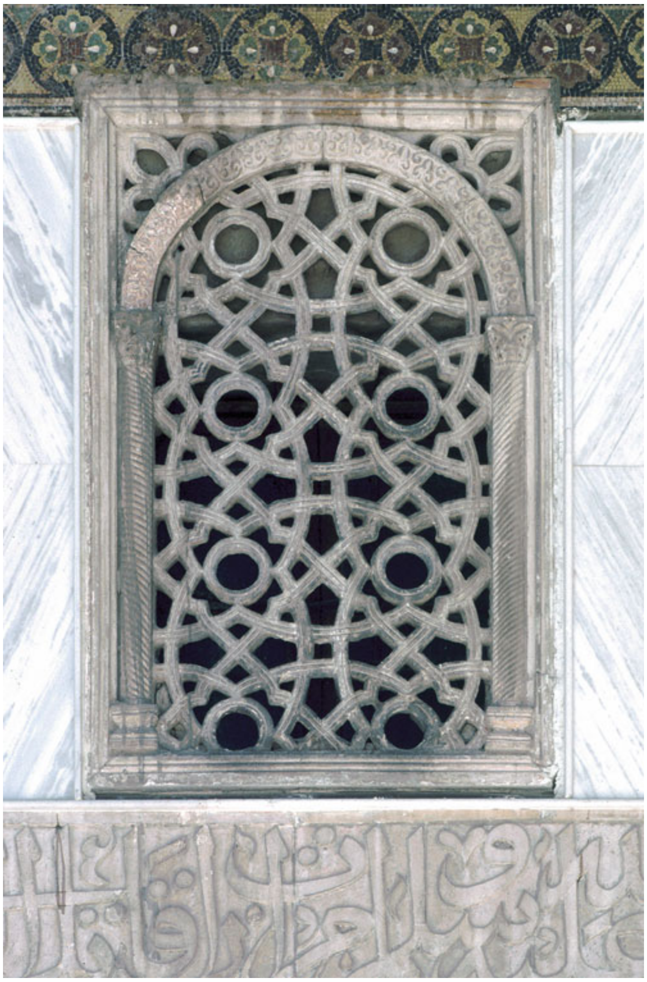
[21] Kitzinger, Ernst. (1965), 341–352.

**Photograph 4** A third-century Roman mosaic with 12-pointed stars from El Djem, Tunisia (© Damian Entwistle)

conventions of Byzantine artisans living in the newly conquered territories who were now their subjects. From a design standpoint, the primary difference between the Hellenic examples of patterns with multiple stars and those subsequently developed by Muslim artists is in the cohesiveness of the overall design. In the earlier Hellenic work, the stars are independent elements scattered across the plane in a repetitive staccato fashion, relating to one another through geometric proximity and similitude. By contrast, within the Islamic star pattern aesthetic, the lines of each star proceed outward to join with the similarly extended lines from adjacent stars to produce an interconnected network wherein each star is an integral part of a unified whole. Secondary to the geometric pattern itself is the treatment of the lines. More often than not, the geometric matrix was given an interweaving treatment wherein the pattern lines are widened to a desired thickness, often informed by material constraints, and made to flow over and under one another. Interweaving lines were a common feature of the pre-Islamic decoration of the Byzantines, Copts, and Sassanids, and were similarly employed in the braided borders, compass-work motifs, and key patterns of the Umayyads. Over the centuries, the geometric star pattern aesthetic was broadened by the introduction of further forms of line treatment [Figs. 85–88], but the primacy of interweaving widened lines continued throughout the long history of this tradition and helped to provide aesthetic continuity within the ornamental arts of Muslim cultures for centuries to come.

The Umayyad innovation of applying Byzantine compass-work mosaic conventions to their pierced window grilles was another progenitor of the tradition of Islamic star patterns. The methodology used for constructing these Umayyad window grilles was described and aptly named by K. A. C. Creswell as *compass work*.[22] The Hellenic art of the Byzantines included a distinctive geometric device constructed from overlapping circles. This form of ornament was employed widely in the embellished mosaic pavements throughout the Hellenic world. The diverse range of ornamental motifs in the fourth-century mosaic paving at Mount

[22] Creswell (1969), 75–79.



**Photograph 5** A fourfold compass-work design with eight-pointed stars from a pierced stone window at the Great Mosque of Damascus (© David Wade)

Nebo, a Christian site in the mountains of Jordan, includes this variety of geometric motif. The Umayyads were quick to employ compass-work designs in their tesserae mosaic architectural decoration. Of particular note is the mosaic pavement in the Umayyad palace of Khirbat al-Mufjar near Jericho (724-43). This is of outstanding quality in both design and execution, and is the largest mosaic floor to have survived from antiquity. Among the multiple ornamental panels that make up this pavement are several that employ interweaving circular elements. The earliest extant Umayyad window screens with this type of geometric design were executed in stucco and several are found in the Great Mosque of Damascus (c. 715) [Photograph 5], and at Khirbat al-Mafjar.[23] While the general geometric schema of these window grilles is fundamentally the same as seen in Hellenistic mosaics, the aesthetic effect is distinct and original. The interweaving line work of the mosaic pavements is heavily elaborated with secondary elements such as interior braided bands. By contrast, the stucco window grilles rely on a more austere geometric exposition that is highly effective and beautiful. It is possible that the inspiration for applying the compass-work design methodology of mosaic paving to Umayyad window grilles derived in part from Sassanid sources. Excavations at the Sassanid fortified township of Qasr-i Abu Nasr near Shiraz revealed a stucco window grille dated from the sixth or seventh century that, while very simple in its pierced honeycomb design, is identical in architectonic concept to later Umayyad window grilles. It is also significant that Sassanid artists were masters of carved stucco ornament. The mixed cultural milieu of the Umayyads wherein artists from Byzantium were working alongside those from Persia may have led to the amalgamation of these two separate ornamental traditions. Whoever the originators were, this innovation was undoubtedly driven as much by technical and functional constraints as by aesthetic consideration. Being a pierced window grille, the ratio of foreground to background had to be carefully determined so that adequate light would filter into the building, yet with interweaving line work thick enough, and so designed, as to provide adequate structural integrity. Adding to the more austere aesthetic is the fact that the resulting line work is too thin for much in the way of secondary elaboration: the added surface decoration frequently limited to a simple carved groove that creates the over/under interweave, and narrower incised grooves that run parallel to the central line work. The austere geometric aesthetic born from such constraints provided a very successful and, one can argue, much-needed counterpoint to the highly ornate Umayyad floral conventions. Furthermore, despite methodological differences, the overtly geometric aesthetic of these window grilles undoubtedly influenced the cultural predilections that eventually led to the development of Islamic geometric star patterns. However, the rudimentary geometry and simple techniques of construction for these early compass-work geometric patterns are in marked contrast to the geometric complexity of the tradition that was to follow.

The Umayyad compass-work window grille aesthetic was also appreciated in their western territories. Begun in the eighth century, the Great Mosque of Córdoba is one of the masterpieces of Umayyad architecture. The many pierced marble window screens that adorn this mosque include a compass-work example just north of the Puerta de San

[23] Several stucco window grilles were found in the ruins of the Umayyad palace of Qasr al-Hayr al-Gharbi (724–727): now in the National Museum of Damascus. The design of each of these is comprised of a central palm motif flanked by floral scrollwork rather than the overlapping circles under discussion.

Esteban[24] (855-6) that is very similar in conceptual design to the earlier Umayyad compass-work window grilles from their Syrian homelands.

The abundant application of a diverse range of geometric motifs in Umayyad ornament provides clear evidence of the Islamic fascination for applied geometry dating back to the earliest period of Islamic architectural accomplishment. Despite their appreciation for geometric ornament, none of the Umayyad geometric designs exhibited the distinctive qualities found in the mature tradition of Islamic geometric star patterns. Without a working knowledge of the precise methodology that allows for the creation of Islamic geometric star patterns this stylistic disconnect was inescapable. However, the Umayyad geometric convention of employing polygonal tessellations as ornament appears to have been critical to the later development of Islamic geometric star patterns. Their familiarity with polygonal tessellations is significant in that such knowledge was essential to the eventual development of the *polygonal technique* of geometric pattern generation wherein a polygonal tessellation is used as scaffolding upon which pattern lines are located, and, like scaffolding, discarded once the pattern is completed.

This form of geometric ornament utilizes an edge-to-edge configuration of one or more regular or irregular polygons to create a tessellating field pattern, typically with secondary floral designs contained within each polygonal cell. An early Umayyad example of this type of inherited ornament is found in the portal of the palace of Qasr al-Hayr al-Gharbi near Palmyra, Syria[25] (724-27). The carved stucco ornament in this highly elaborate entry portal includes two panels that employ regular hexagons and rhombi with 60° and 120° included angles that are the equivalent of the $3^2 6^2$-3.6.3.6 *two-uniform* tessellation of triangles and hexagons [Fig. 90]. As with other polygonal designs composed of regular hexagons, triangles, and double-triangle rhombi, this design can be easily constructed using the isometric grid. The fact of this form of isometric design being an established motif among the pre-Islamic peoples of the eastern Mediterranean is confirmed in the surviving ornamental ceilings from the second-century Roman ruins of Baalbek in Lebanon. The Umayyad mosaic pavement of the Khirbat al-Mafjar includes multiple panels with polygonal tessellations, including designs made up of elongated hexagons and squares, as well as panels with regular octagons and squares. Each of these tessellations received continued use by subsequent Muslim cultures. The Umayyads also combined simple polygonal tessellations with compass-work patterns. While both of these ornamental themes were derived from pre-Islamic sources, their combined use was an original development. Umayyad examples of this form of geometric ornament are found in two of the pierced stucco window grilles at the Great Mosque of Damascus [Fig. 82c] and one of the window grilles from Khirbat al-Mafjar [Fig. 82d]. These two examples employ the 3.6.3.6 tessellation of triangles and hexagons as the polygonal component of the composition. The use of polygonal tessellations as an ornamental device continued among artists in later Muslim cultures, but the great innovation in the ornamental use of polygonal tessellations was the discovery of the *polygonal technique* wherein these tessellations could be used as generative structures from which geometric patterns were extracted.

## 1.4 Abbasids (750-1258)

The forces of Abu'l-Abbas as-Saffah (721-754) defeated the Umayyads in 750. This marked the beginning of the Abbasid dynasty: one of the longest lasting and most influential dynasties in Islamic history. The Abbasids were descended from the Prophet Muhammad through the Prophet's uncle, Abbas ibn Abd al-Muttalib (566-653); and this kinship to the Prophet allowed them to assert greater religious authority and right to the caliphate over that of their Umayyad predecessors. Over the centuries, the respect bestowed upon the Abbasid Caliphate was to have a profound and continuing influence on Islamic politics and culture, including the arts. With control over Egypt and North Africa to the west, and Persia, Khurasan, and much of Transoxiana to the east, the Abbasids chose to move their capital eastward to a place more central to their empire. In 762 Al-Mansur founded his capital of Baghdad. He brought hundreds of builders, engineers, and craftsmen to Baghdad from areas throughout his empire. It can be assumed that this influx of artists and architectural specialists into a single location would have contributed greatly in creating the atmosphere of ornamental innovation that took place at this time. With Baghdad as the center of the Abbasid Empire, Persian influence became a major aspect of Abbasid culture. Persian customs were adopted as part of royal protocol; Persians were placed in important positions of power and influence within the government and military; and Persian artistic and architectural traditions were enthusiastically embraced by the otherwise Arab culture of the Abbasids.

The earliest Islamic geometric star patterns date to the ninth century at a time when Baghdad was the preeminent center of Arab culture. The rise of the Abbasid Caliphate heralded a period of great sophistication and refinement, creating a legacy for which subsequent Islamic cultures, and indeed the entire world, must be forever indebted. Baghdad became the foremost center for Islamic religious

[24] Originally known as Bab al-Wuzara.

[25] The portal of the palace of Qasr al-Hayr al-Gharbi is now in the National Museum of Dasmascus.

studies and scholarly learning, attracting the most learned scholars and theologians from far and wide. It was during this early Abbasid period that the four primary orthodox Sunni religious doctrines were developed: *Hanafi*, *Maliki*, *Shafi'i*, and *Hanbali*. Great emphasis was given to the translation of earlier Greek texts; and these works laid the groundwork for the following 800 years of Muslim achievements in the sciences. Great advances in the fields of philosophy, chemistry, medicine, zoology, botany, mathematics, geometry, astronomy, geography, linguistics, and history augmented the course of human knowledge. Many of these scientific works were introduced to Europe in the fourteenth and fifteenth centuries, and provided a significant influence upon the Italian Renaissance. Indeed, it was largely through Arabic translations that Europeans regained their knowledge of Greek science and philosophy. The Abbasid cultural milieu provided the background for such important philosophical thinkers as al-Kindi (d. c. 874), al-Farabi (d. 951), al-Haytham (d. 1021), and ibn Sina (d. 1037). Similarly, the cultural richness of this period engendered the blossoming of Islamic mysticism with such luminaries as Rabia of Basra (d. 801), Bayazid Bastami (d. 874), al-Junayd Baghdadi (d. 910), and al-Hallaj (d. 922), to name but a few. This was also an environment in which poetry thrived. In fact, the lines of demarcation separating poetry, mysticism, philosophy, and science were not so clearly delineated as experienced in the present era.

The Abbasids were equally committed to the further development of the arts and architecture: calligraphy and Quranic illumination were developed into a discipline of great beauty and originality; new architectural forms were assimilated from a variety of pre-Islamic sources,[26] bringing ever-greater diversity to the Islamic architectural tradition; and aesthetic innovation within the ornamental arts benefited greatly from patronal attention. Architectural ornament was a primary beneficiary of this commitment to innovation: both in terms of an increased availability to wider range of materials and fabricating technologies, and in the ever-expanding diversity of decorative motifs and themes. This included the development of the incipient tradition of Islamic geometric star patterns during the ninth century. Over the course of some 300 years, this design tradition developed to its full maturity, characterized by exceptional versatility, great beauty, unparalleled geometric ingenuity, and pan-Islamic appreciation.

It is generally agreed that the sophisticated culture of Baghdad was central to the initial development of Islamic geometric star patterns. Even with the early rise in prominence of other early centers of Muslim culture such as Córdoba, Cairo, Shiraz, Nishapur, Bukhara, and Merv the preeminence of Baghdad as the seat of Abbasid religious authority and cultural influence remained undisputed. While the surviving brickwork, woodwork, and stucco ornament from such widespread locations as Kairouan, Cairo, Balkh, Na'in, Tim, Qala-i-Bust, Uzgen, Damghan, and Kharraqan provide some of the best evidence of the early development of Islamic geometric star patterns, the broad distribution of so many stylistically similar examples during the same approximate period argues for the centrality of Baghdad as the principle place of origin and dissemination for this discipline.[27] Furthermore, knowledge of the importance of Baghdad in the historical development of other allied and highly influential artistic traditions is well known. Notable examples include the calligraphic innovations of Abu 'Ali Muhammad ibn Muqlah (886-940), the inventor of the geometric system of calligraphic proportion that was critical to lifting this tradition to the level of fine art[28], the development of the highly distinctive beveled style of floral ornament (Samarra style C) that appears to have originated in nearby Samarra and was used widely throughout the vast regions of Abbasid influence[29], and the technically sophisticated lusterware ceramics that also developed in and around Samarra.[30] The case can similarly be made for Baghdad as an important center in the ongoing development of ornamental brickwork. Relatively little architecture survived the succession of Mongol invasions during the thirteenth and fourteenth centuries and Baghdad did not escape this destruction. While most extant pre-Mongol ornamental brickwork architecture is found in the regions of Persia, Khurasan, and Transoxiana, the fact that older, albeit less complex, examples of ornamental brickwork façades are found in locations near Baghdad supports the theory that this tradition grew out of the cultural vitality of Baghdad, and was disseminated from there to regions under Abbasid influence.

Another case for Baghdad as the principal place of origin in the development of Islamic geometric star patterns is the central importance of Baghdad in the study of mathematics and geometry during this period. These disciplines were provided a practical emphasis in such areas as geography, land surveying, navigation, taxation, commerce, and the arts. Abu al-Wafa al-Buzjani (940-998) was a leading mathematician of his time. As a young man he moved from Khurasan to Baghdad where he lived the remainder of his life. He is best known for his work with plane and spherical trigonometry. More prosaically, al-Buzjani was also concerned with

[26] For a detailed analysis of pre-Islamic influences upon the development of early Islamic architecture, see Hillenbrand (1994a).

[27] Necipoğlu (1995), 99–100.

[28] Schimmel (1990), 18–19.

[29] Creswell (1969), 75–79.

[30] Caiger-Smith (1985), 21–31.

the practical application of mathematics and geometry,[31] and is associated with the work *About that which the Artist needs to Know of Geometric Constructions*.[32] This work details practical solutions to geometric problems posed by members of the professional classes, including people working in the arts. Perhaps most significantly, the Abbasid caliph al-Mu'tadid (r. 892-902) founded a royal atelier within his palace dedicated to the furtherance of theoretical and practical sciences and their application to diverse artistic disciplines.[33] It was during this approximate period that geometric star patterns began to emerge as a distinct ornamental aesthetic. It is reasonable to speculate that this interaction between mathematicians and artists may have played an influential role in the development of the methodologies required in the construction of complex geometric star patterns. Certainly the place and time are significant.

The fact that most extant examples of early Islamic geometric star patterns are architectural should not lead one to conclude that this ornamental discipline developed solely as part of the architectural traditions of Islam. The book arts, and specifically the concerted attention paid to Quranic illumination, appear to have also played a significant role in the progressive development of Islamic geometric patterns. It is regrettable that so few Qurans have survived from the early formative period of this ornamental tradition, and knowledge of the degree of interplay between geometric artists working on Qurans and those working on buildings is limited to conjecture. However, the cultural centrality and royal patronage of this tradition, coupled with the few evidentiary examples that have survived, do indeed indicate the likelihood that these artists were involved in the development of geometric patterns. During the ninth and tenth centuries, the art of Quranic ornamentation evolved from simple border devices, emphasizing *surah* headings and *ayah* markers, to fully illuminated pages. The work of the great calligrapher ibn Muqla (d. 940) is an example of the successful application of geometric principles to the arts of the Baghdadi cultural milieu. He is known to have studied geometry, and his prescribed use of geometric proportion to perfect the cursive scripts profoundly influenced the trajectory of Islamic calligraphy. Preeminent among artists, calligraphers would have participated in the exploratory exchanges between scientists and other artists, and it is certainly possible that calligraphers were involved in discussions that may have assisted in the development of geometric star patterns. Significantly, the first such pattern

**Photograph 6** A frontispiece comprised of two varieties of octagon from a Quran produced in Baghdad by Ibn al Bawwab (© The Trustees of the Chester Beatty Library, Dublin: CBL Is 1431, ff. 7b-8a)

known to have been created by a specific individual is from a matching set of illuminated frontispieces from the celebrated Baghdad Quran produced by 'Ali ibn Hilal, better known as ibn al-Bawwab (d. 1022). Like ibn Muqla, he is regarded as one of the great masters of Arabic calligraphy. His Baghdad Quran was produced in 1001 and includes several illuminated pages that are believed to be his own creations. Most of these illuminations are compass-work creations, but the matching frontispieces employ a beautifully executed geometric pattern comprised of a network of interweaving lines that create a series of large and small octagons[34] [Fig. 127c] [Photograph 6]. The pattern that ibn al-Bawwab incorporated into his surviving Quran is an alternative treatment of the well-known, and less complex, design comprised of octagons touching corner to corner. Creswell has written of the use of this less complex octagonal pattern in pre-Islamic architecture,[35] and cites the example of a ceiling coffer from the Great Temple of Palmyra

[31] Necipoğlu (1995), 123.

[32] *Kitab fima yahtaju ilayhi al-sani min a'mal al-handasa*, MS Persan 169, sec. 23, folios. 141b–179b, Bibliotheque Nationale, Paris.

[33] –Özdural (1995), 54–71.
–Necipoğlu (1995), 123.

[34] Chester Beatty Library Ms. 1431, fol. 7b–8a.

[35] Creswell (1969), 77.

(c. 36). This simple octagonal design from Palmyra is easily constructed by iteratively applying octagons in a corner-to-corner orientation upon an orthogonal grid. This same basic octagonal pattern was widely used by many generations of Muslim artists, and in keeping with the rise to dominance of the polygonal technique within this pattern tradition, it can be conveniently produced through the application of pattern lines onto an underlying $4.8^2$ tessellation of squares and octagons [Fig. 124c]. However, prior to the earliest known use of this very simple octagonal design by Muslim artists, ibn al-Bawwab had incorporated his more complex version into his celebrated Quran. By contrast, the added complexity of the pattern produced by ibn al-Bawwab is not so easily produced via simple iteration, and is more readily created from the underlying $4.8^2$ tessellation. This more complex pattern can be extracted from the tessellation in either of the two pattern line arrangements [Figs. 127c and 128d]. His use of this design strongly suggests that ibn al-Bawwab was knowledgeable of the advances in geometric design methodology generally, and quite possibly the polygonal technique specifically, that were taking place during this period.

As with calligraphy and illumination, bookbinding also received decorative emphasis during the Abbasid period. Paper technology was introduced from China, allowing for books to be lighter weight than either parchment of papyrus. This new material was less susceptible to the adverse effects of humidity, providing greater technical efficiency and allowing for lighter bindings. Abbasid artists developed bindings that were made from leather-covered pasteboard. From as early as the ninth century, the leather coverings were decorated with blind tooling: a process whereby the leather binding was dampened and stamped with metal tools and dies, and burnished to completion. A surviving Aghlabid example of an early interweaving geometric pattern being used to decorate such a binding is from a ninth-century Quran in the library of the Great Mosque of Kairouan. The design is interesting in that it is an embossed leather representation of an ancient cane weave that is still used to this day in the furniture industry, wherein it is known as the standard cane weave. The geometric structure of this design is produced from a four-directional weave made up of parallel interweaving double lines in the vertical and horizontal directions, and over-under single lines in the diagonal directions. The interweaving lines create regular octagons that are located upon the vertices of the repetitive orthogonal grid. An unusual feature of this design is the nonuniform structure of the interweave, wherein the individual lines will skip over-over-under-under, rather than over-under-over-under: the standard of this tradition. In this respect, the bookbinding design is faithful to the cane weave. The interweaving aesthetic of this design is nevertheless similar to that of later Islamic geometric star patterns, and indeed, the geometric structure of this cane weave design relates directly to the classic star-and-cross design [Fig. 124b]: the difference being that the horizontal, vertical, and diagonal lines from the Aghlabid book binding are continuous, and are widened to their maximum extent. This is similar to a design from a wooden ceiling at the Alhambra, but without the small arbitrarily included eight-pointed stars [Fig. 126b].

The aesthetic similarity between the pattern on the Aghlabid bookbinding and later Islamic geometric ornament is indicative of an emerging aesthetic orientation that took form under the auspices of Abbasid patronage during the ninth and tenth centuries. Among the earliest extant examples of Islamic geometric star patterns are the multiple pierced wood panels from the *minbar* (c. 856) at the Great Mosque of Kairouan. This *minbar* was manufactured in Baghdad and exported to North Africa. The sides of this *minbar* are a veritable cornucopia of early geometric design, and provide the best surviving evidence for the emerging geometric aesthetic of early Abbasid Baghdad. Among the diverse multitude of designs are key patterns, compass-work patterns, polygonal tessellations, and several panels with very basic prototypical star patterns. Each of these is very simple compared with the characteristic complexity that eventually became a hallmark of this tradition. The star patterns from the Kairouan *minbar* all have eight-pointed stars as their central feature, and repeat upon the square grid. One of these is a particularly early occurrence of the classic star-and-cross design that went on to become the most ubiquitous geometric star pattern throughout the Islamic world [Fig. 124b]. As discussed, using the polygonal technique, this classic orthogonal design can be easily created from the $4.8^2$ tessellation of squares and octagons.

Being that the region of greater Baghdad during the ninth and tenth centuries was central to the development of several significant artistic traditions, that the arts were informed by mathematics and geometry under royal patronage, and that significant artistic trends and objects were exported from this region to diverse regions of Abbasid influence, it appears likely that the cultural milieu of Baghdad provided the impetus for the development of Islamic geometric star patterns, and that knowledge of this incipient tradition was dispersed widely from this region throughout the Islamic world. However, the few remaining Abbasid buildings from the region of Baghdad that date from the early formative period are devoid of geometric star patterns, and the Kairouan *minbar* notwithstanding, it is impossible to know categorically the extent of methodological knowledge of the polygonal technique enjoyed by the artists working in Baghdad during this period.

## 1.5 Tulunids (868-905)

In 868 Ahmad ibn Tulun, originally from Bukhara, was sent with an army from Iraq to Egypt to be deputy to the viceroy of Egypt. Within a short time he became the Abbasid governor of Egypt and Syria, founding the Tulunid dynasty. While only ruling until 905, the Tulunids had a significant influence on subsequent Egyptian ornament. The mosque of ibn Tulun (876-79) is located in Fustat, Egypt (now part of greater Cairo). The geometric patterns at the ibn Tulun mosque are collectively the most significant ninth-century examples from the western regions of Abbasid influence. Of particular note are the soffits of the multiple arches that surround the large courtyard. These are decorated with a wide variety of patterns, including compass-work, polygonal tessellations, and early examples of geometric star patterns. Like the Great Mosque of Kairouan, the ornament in the ibn Tulun mosque was transitional: incorporating elements from the earlier Umayyad period with more contemporary ornamental devices such as the Samarra floral styles and simple geometric star patterns. Several of these are noteworthy in that they are among the earliest extant Islamic geometric patterns to have threefold symmetry, although this cannot be regarded as innovative in that such patterns are known from the pre-Islamic architecture of the Byzantines and Sassanians. One of these soffit designs is comprised of interweaving rhombi and hexagons and can be regarded as pure polygonal ornament [Photograph 7]. One of the distinctive characteristics of this pattern is the six-pointed star motif that is placed upon the vertices of the isometric grid. This pattern of hexagons and six-pointed stars (without the rhombic emphasis) was used with great frequency throughout the Islamic world in succeeding centuries; so much so, in fact, that it can be regarded as the classic threefold pattern. This classic design is easily constructed using the polygonal technique from the hexagonal grid as the underlying generating tessellation [Fig. 95b]. As said, many of the simple threefold geometric patterns that are characterized by 60° and 120° angles can be constructed from either the *system of regular polygons*, by the simple assembly of design elements, or by simply tracing over the isometric grid. However, as this tradition developed, the increase in complexity required a constructive methodology that surpassed the limitations of simple grid tracing or assembly, but were amply met with the polygonal technique. One of the more intriguing soffit designs at the ibn Tulun mosque uses a 3.6.3.6 tessellation of triangles and hexagons as polygonal ornament, with interweaving circles of greater line thickness located at the centers of each hexagon [Photograph 8]. As each circle approaches the center of a hexagon, its curvature is tweaked toward this center, creating a distinctive flower with six petals. This beautiful design has an innovative playfulness that qualifies it as one of the outstanding ninth-century examples from this burgeoning tradition. The conceptual similarity of this design to two of the window grilles from the Great Mosque of Damascus and the one from Khirbat al-Mufjar is striking: all include the 3.6.3.6 polygonal tessellation with added circular elements

**Photograph 7** A Tulunid threefold pattern with six-pointed stars from a carved stucco arch soffit at the Ibn Tulun Mosque, Cairo (© David Wade)

positioned at key points within the geometry of the tessellation. The straight-line component of this composition is equally attributable to either of the two categories: the *system of regular polygons* [Fig. 95d], or polygons as pattern. Several additional curvilinear patterns were used on the soffits of the ibn Tulun mosque, including one with threefold symmetry that was also constructed from the 3.6.3.6 tessellation: although this example does not include the tessellation as part of the finished design. The swing points for the compass were conveniently left as a subtle design feature, providing evidence of the 3.6.3.6 tessellation having been used to create the design. The center point of the semicircular pattern line is located upon the center point of each polygonal edge of the underlying 3.6.3.6 tessellation, and the radius of each curved pattern line is equal to half the polygonal edge length.

Several different *fourfold* designs that are easily constructed with the $4.8^2$ underlying tessellation of squares and octagons are found at the ibn Tulun mosque, including

**Photograph 8** A Tulunid threefold pattern with six-pointed stars from a carved stucco arch soffit at the Ibn Tulun Mosque, Cairo (© David Wade)

**Photograph 9** A Tulunid fourfold pattern with eight-pointed stars that can easily be created from the $4.8^2$ tessellation of squares and octagons from a carved stucco arch soffit at the Ibn Tulun Mosque, Cairo (© David Wade)

the classic star-and-cross pattern from one of the carved stucco arch soffits [Fig. 124b]. This pattern was contemporaneously used by Yu'firid artists in the ceiling ornament of the Great Mosque of Shibam Aqyan near Kawkaban in Yemen (pre-871-72). Another of the arch soffits at the ibn Tulun mosque is decorated with a distinctive interweaving design of eight-pointed stars and two sizes of square in the background [Fig. 129a] [Photograph 9]. This design is easily created by drawing the eight-pointed stars from vertex to vertex within the octagons of the underlying tessellation, resulting in a pattern composed of two sizes if interweaving squares that surround the eight-pointed stars. As noted, the methodology of the polygonal technique typically places the pattern lines upon the midpoints, or upon two points, of each edge of the generative polygons, and the use of polygonal vertices is relatively rare, and when found is almost always associated with the early formative period. The third design from the ibn Tulun arch soffits that is easily created from the underlying $4.8^2$ tessellation of squares and octagons is a compass-work design comprised of interweaving curvilinear pattern lines. This design replaces the 90° angles in the eight-pointed stars of the standard star-and-cross pattern with a network of *s*-curves that weave together to create eight-lobed rosettes within each underlying octagon, and full circles within each underlying square and at the center of each eight-lobbed rosette.

## 1.6 Umayyads of al-Andalus (756-929)

Muslim conquerors first landed in Spain in 711, and by 714 had wrested control of the greater portion of the Iberian Peninsula from the Christian Visigoths. Until the arrival of Abd er-Rahman I in 755, Islamic Spain was ruled by an assortment of governors under the authority of the Umayyad Caliphate in Damascus. Following the Abbasid overthrow of the Umayyads in Syria, the heirs to the Umayyad Caliphate were rounded up and executed. The only survivor was Abd er-Rahman I, the grandson of the last Umayyad Caliph. Along with many of his Syrian supporters, he successfully escaped to the Iberian Peninsula where he was accepted as the Amir in 756. This continuation of the Umayyad dynasty in al-Andalus was the beginning of one of the great epochs of Islamic civilization. Centered in Córdoba, this dynasty was to rule over most of the Spanish peninsula for over 250 years. In 929 the eighth Umayyad Amir, Abd er-Raham III, declared himself Caliph, directly challenging the authority of the Sunni Abbasids in Baghdad, and the Shi'a Fatimids in North Africa and Egypt.

The Great Mosque of Córdoba was founded by Abd er-Rahman I between 784 and 786, and expanded by the Umayyad Caliph al-Hisham II and his minister al-Mansur between 987 and 990.[36] This expansion included the introduction of a number of marble window grilles. In addition to the purely functional benefits of these grilles, their incorporation into this mosque may have served as a homage to the cultural greatness of their Umayyad ancestors from Syria, and more specifically to the window grilles of the Great Mosque of Damascus. These Iberian window grilles are designed in a variety of geometric styles, and included patterns easily created with the polygonal technique. One of these is a threefold pattern that can be produced from either the $6^3$ hexagonal grid [Fig. 96e] or the 3.6.3.6 underlying tessellation [Fig. 99e], although the precise proportions of this example relate to the former generative schema. The earliest known example of this pattern is from one of the original window grilles at the al-Azhar mosque in Cairo (970-76). Later historical examples for the use of this pattern include one of the raised brick panels on the exterior of the Seljuk eastern tomb tower of Kharraqan (1067) [Photograph 17]; a pierced wood screen on the Seljuk *minbar* at the Friday Mosque of Abyaneh (1073); and a Fatimid pierced marble grille from the al-Aqmar mosque in Cairo (1125) that is stylistically identical to the earlier example from Córdoba. Two other surviving window grilles from Córdoba are in the collection of the Museo Arqueológico de Córdoba. These are thought to be from the same period as the examples from the Great Mosque of Córdoba, and possibly from the same workshop.[37] Like the above-cited window grille from the Great Mosque of Córdoba, one of these has the distinctive quality of being made up of interweaving superimposed hexagons that are placed upon the isometric grid, yet their design characteristics are noticeably distinct from one another. And like the example in the Great Mosque of Córdoba, the isometric window grille from the Museo Arqueológico is easily created from the $6^3$ grid of regular hexagons [Fig. 96d], and is conceptually the same, but with different proportions, to a pattern created from the 3.6.3.6 arrangement of triangles and hexagons [Fig. 99c]. The simple and easily discerned arrangement of interweaving hexagons is responsible for the compelling beauty of this pierced marble window grille, and it is not surprising that it was also used frequently throughout the history of Islamic ornament, including the Ghurid portal at the Friday Mosque at Herat (1200). The second marble window grille from this workshop in Córdoba places eight-pointed stars upon the orthogonal grid, and, like the previously referenced example from the ibn Tulun mosque, locates the pattern line upon the vertices of the underlying octagons within the $4.8^2$ tessellation of squares and octagons. This design is methodologically identical to a later Ghurid raised brick panel on the exterior of the western mausoleum at Chisht, Afghanistan (1167) [Fig. 129c]. However, the design from Córdoba is differentiated by the additive inclusion of two varieties of semicircular scallops incorporated into the otherwise uninterrupted straight lines of the pattern. These scallops serve several functions: by touching their neighbors, they provide added structural integrity to the pierced marble; they open up an otherwise dense area of the design where three interweaving lines would otherwise touch at a single point (as per the example in Chisht); and their curvilinear quality adds visual dynamism to the overall design. Additional examples of this design, sans scallops, are found on the Ghurid minaret of Jam in central Afghanistan (1174-75 or 1194-95), and the *minbar* of the al-Aqsa mosque in Jerusalem (1187). Another example of a design from the Great Mosque of Córdoba that can be created from the underlying $4.8^2$ tessellation is from the celebrated tessera mosaic *mihrab* (971) that was ordered by al-Hashim II [Fig. 126c]. This is a variation of the star-and cross design with curvilinear four-pointed stars within the underlying squares, and a second eight-pointed star within the primary eight-pointed star.

## 1.7 Abbasids in the Eastern Provinces

During the period when the tradition of geometric star patterns was advancing to full maturity, Persia, Khurasan, Sindh, and Transoxiana were beset with political turmoil. Out of this turmoil rose and fell a series of great empires. The Buyids wrested control over Persia and Iraq from the Abbasids, placing them in direct confrontation with the

[36] King Ferdinand III of Castile converted the Great Mosque of Córdoba into a cathedral in 1236.

[37] It is speculated that these two window screens may have been made for a country residence outside Córdoba: possibly that of al-Mansur. See Dodds [ed.] (1992), 252.

Samanids in Transoxiana and Khurasan. The Samanids suffered defeats at the hands of the Qarakhanids and Ghaznavids, the latter of whom were eventually defeated in turn by the Ghurids and Seljuks. The Seljuk overthrow of the Buyid dynasty, and the liberation of the caliphate in Baghdad, brought on the Sunni Revival. Following the defeat of the Buyids, the Seljuk sphere of influence spread across Persia into the Caucasus and Anatolia, al-Jazirah, Mesopotamia, much of Syria, and the Levant. The Qara Khitai defeated the Seljuks in the northern regions of Transoxiana, only to be overrun by the Khwarizmshahs who went on to defeat the last of the Great Seljuks and Ghurids, consolidating control over an empire that stretched across Persia and Khurasan, and northward across the vast regions of Transoxiana. Soon after their multiple victories, the Khwarizmshahs fell to the Mongol onslaught in the thirteenth century. Yet throughout this history of military conquest and political upheaval great cities thrived, intercontinental trade continued, great wealth was amassed, and the arts and sciences flourished. It was in this tumultuous yet culturally refined environment that the development of Islamic geometric star patterns matured beyond the simplistic modalities that were characteristic of the patterns used in the *minbar* of the Great Mosque of Kairouan and the arch soffits at the ibn Tulun mosque. Over time, the advances made in the eastern regions were disseminated throughout the Islamic world, where they were, in turn, readily incorporated into the palette of ornamental themes and applied to a broadening range of materials and techniques.

Following the defeat of the Umayyads, the Abbasids soon came under increased pressure to more effectively govern the vast regions of their empire by founding the more centrally located capital of Baghdad. As in North Africa and Egypt, in Persia, Khurasan, and Transoxiana, governorships were granted, leading to several powerful semiautonomous vassal states. During the ninth century, Abbasid suzerainty over its eastern provinces began to break apart. This challenge to the Abbasid authority in Baghdad did not stem solely from the desire for independence, but, in many cases, was driven by fundamental religious differences. The Mu'tazilite reform doctrine of the created Quran caused deep schisms within Abbasid Sunni orthodoxy; and further pressure resulted in the growing Shia movement that regarded the descendants of the Prophet Muhammad through the line of 'Ali ibn Abi Talib, the Prophet's son-in-law, as the only legitimate heirs to the caliphate. The erosion of the Abbasid dynasty in the ninth and tenth centuries led to the rise in importance of various regional centers; and this was to have a profound effect on the history and development of Islamic architecture and ornament.

The uncertainty as to exactly when, where, and under what circumstances the polygonal technique for creating geometric patterns originally developed is compounded by the fact that the simplicity of the earliest geometric star patterns allows for their creation by other generative techniques beyond just the polygonal technique. The degree of overlap between competing methodologies, and the point at which the polygonal technique assumed its role as the preeminent design methodology, is, therefore, impossible to definitively determine. Regardless of whether this seminal design methodology first originated and possibly matured in and around Baghdad,[38] the architectural record clearly indicates that the earliest extant mature expression of this ornamental tradition is found in the eastern regions of the Islamic world. If the maturity of Islamic geometric patterns corresponds with the surviving architectural record, and indeed occurred in the eastern provinces, this shift in creative vitality would have paralleled the waning influence of the Baghdadi caliphate in the face of the de facto independence of those outlying regions that had previously come under direct Abbasid control.

Like the ibn Tulun mosque, the surviving architecture in the eastern regions of Abbasid suzerainty provides some of the best indications for the early use of the polygonal technique as a generative methodology during the initial developmental period of geometric star patterns. The ruins of the No Gumbad mosque in Balkh, Afghanistan (800-50), are extensively ornamented with carved stucco geometric and floral designs. Among the many ornamental motifs is an example of the classic star-and-cross design with eight-pointed stars at each vertex of the orthogonal grid [Fig. 124b] [Photograph 10]. It is significant that the use of this design at the No Gumbad mosque is contemporaneous with its use on the wooden *minbar* at the Great Mosque of Kairouan. Clearly, ninth-century Abbasid ornamental conventions disseminated quickly throughout their vast territories; helping to create an ornamental style that, while engendering distinct regional variations, nonetheless exhibited remarkable aesthetic cohesion. The No Gumbad mosque was built during the same approximate period as the floral examples found in excavations of the Bab al-'Amma (836-7) and the Bulawara Palace (849-59) in Samarra, Iraq. Both the floral infill of the geometric designs at the mosque of ibn Tulun in Egypt and the carved stucco floral infill designs in the No Gumbad mosque have much in common with the Samarra style B floral designs; providing added evidence of the rapid dissemination of newly developed ornamental innovations throughout Abbasid territories. The ninth-century incorporation of the Samarra floral conventions in regions as far flung as Cairo and Balkh, as

[38] For a detailed exposition on the importance of Baghdad in the development of Islamic science and mathematics, and the influence of these developments upon the origins of the geometric design idiom, see Necipoğlu (1995), 131–166.

**Photograph 10** An early Abbasid example of the classic star-and-cross pattern at the No Gunbad in Balkh, Afghanistan (Horst P. Schastok photograph, courtesy of Fine Arts Library, Harvard University)

well as the utilization of the star-and-cross pattern during the same period in both the east and west, strongly supports the argument for the centrality of Baghdad in this process of dissemination. However, the surviving architectural record strongly indicates that the early developmental innovations and maturation of Islamic geometric star patterns took place primarily in the eastern regions of Khurasan and eastern Persia and spread westward during the period of Seljuk expansion.

Advances in the tradition of Islamic geometric patterns must be regarded in the context of the brickwork ornament of the eastern dynasties. The Persian term for this brickwork ornament is *banna'i*, or *work of brick builders*.[39] The Persian term *hazarbaf* for woven rush matting is occasionally applied to brickwork when the design resembles this variety of interweaving woven structure. The earliest examples of Islamic ornamental brickwork are found in present-day Iraq, and make use of very simple geometric motifs that rely upon the rectilinearity of the brick module. Among the earliest surviving examples are the Baghdad Gate in Raqqa (772) and the Court of Honor at the desert palace of Ukhaidir (c. 764-778), some 120 miles south of Baghdad. Both these examples employ simple brickwork designs such as chevrons and swastikas inside a series of horizontally aligned blind arches. Of particular interest is the minaret of Mujda (mid-eighth century), situated between the two Abbasid palaces of Ukhaidir and Atshan. While little of this minaret still stands, and while the geometric brickwork is very basic, it is remarkable for its conceptual similarity to the beautiful ornamental brick minarets produced by the Ghaznavids and Seljuks some 300 years later.[40] In the eastern regions, the rise in sophistication of ornamental brickwork began with the Samanids and Buyids, and can be seen in such buildings as Samanid mausoleum in Bukhara (c. 914-43) and the Jurjir mosque in Isfahan (976-85). The rival Qarakhanids and Ghaznavids built upon the brick-building heritage of their predecessors; and the Ghaznavids in particularly were especially innovative in their use of this medium. Artists working for both of these dynasties pioneered the application of geometric star patterns to brickwork ornament. The three adjoining mausolea in Uzgen, constructed between 1012 and 1186, exhibit some of the finest Qarakhanid geometric ornament, while the brick minaret Mas'ud III in Ghazna, Afghanistan (1099-1115), has some of the most sophisticated geometric patterns of its period. Their Seljuk and Ghurid successors further expanded upon this decorative device, creating works of exceptional beauty and originality in such buildings as the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) and the minaret of Jam in central Afghanistan (1174-75 or 1194-95). As this tradition matured the variety of patterns employed became increasingly diverse and complex. What began as simple key patterns and interlocking devices that firmly adhered to the 90° orthogonal angularity of the brick module transformed into an ornamental medium with tremendous design flexibility. The repertoire of the brick artist was expanded to include cast ceramic inserts, often with either a glazed or an unglazed decorative relief, as well as specially cut or specially molded bricks that allowed them to break free of the orthogonal rigidity that otherwise constrained this medium. In this way, the rise in technical

[39] The Persian terms *hazarbaf* and *parceh* are also used for brickwork ornament. It is interesting that these terms are also associated with the woven rush matting and textile industries.
–Wolff (1966), 118.
–Creswell (1969), 186.

[40] Hillenbrand (1994a), 144.

mastery of ornamental brickwork in Khurasan and Persia during the tenth, eleventh, and twelfth centuries provided an ideal vehicle for the growth in complexity of geometric patterns with angles other than 90°. This was equally the case for the brickwork application of cursive calligraphic scripts, increasingly elaborate forms of knotted and floriated *Kufi*, and the floral idiom. Added to this integral evolution of ornamental motifs and materials was the revival of glazed ceramic faience, and the continuation of carved stucco, carved stone, carved wood, and, less commonly, fresco painting. All of these architectural media were exceptionally well suited to the burgeoning tradition of geometric star patterns.

## 1.8 Samanids (819-999)

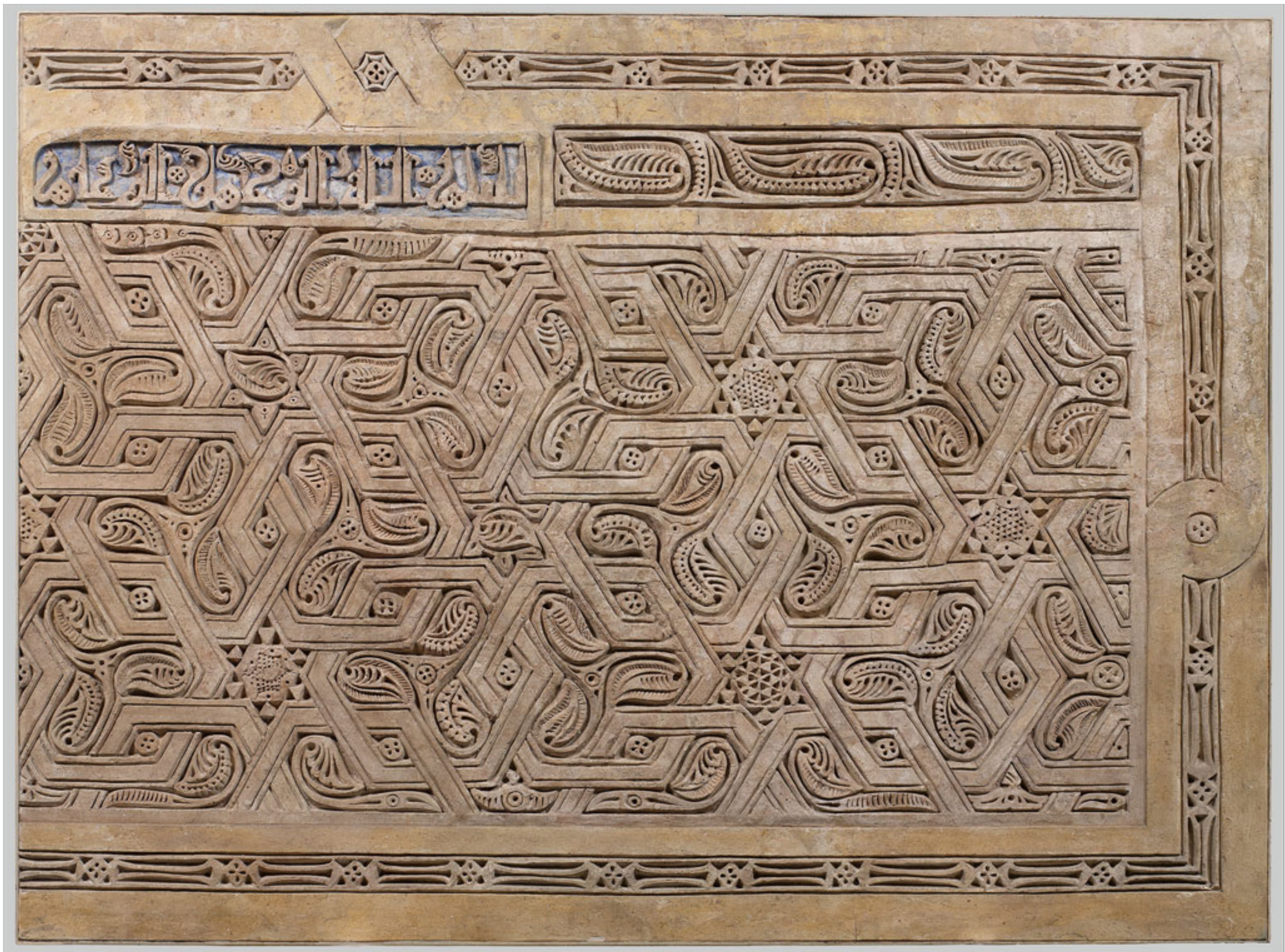
The Samanid Empire was founded with the appointment of four brothers to rule over the regions of Samarkand, Ferghana, Herat, and Shash (Tashkent) by the Abbasid Governor of Khurasan in 819. These brothers were granted their positions of leadership as an award for their military support in putting down a revolt against the caliph al-Ma'mun. The Samanid Empire reached its political and cultural height during the reign of Isma'il Samani (892-907). During this period the Samanids controlled a vast region that included most of modern-day Afghanistan, the eastern half of Iran, parts of Pakistan, and much of Tajikistan, Kyrgyzstan, Uzbekistan, Kazakhstan, and Turkmenistan. The Samanids originated from the region of Balkh, and were strict adherents of Sunni Islam. At the height of power they ceased paying tribute to the Abbasids, but continued to recognize the religious authority of the caliph in Baghdad. Like the Saffarids whom they vanquished, the Samanids revived Persian language and culture. Their first capital was Bukhara, and their principal cities were Samarkand, Herat, and Nishapur. Bukhara in particular became a great cultural center of learning and the arts, rivaling Baghdad, Cairo, and Córdoba. Such luminaries as 'Ali Sina Balkhi (Avicenna), Muhammad al-Bukhari, Rudaki, and Ferdowsi received patronage from the Samanid court. Clearly this was a highly sophisticated culture where religion, sciences, and arts flourished. Indeed, it was during the Samanid period that Nishapur became one of the great Islamic centers of ceramic art. The few Samanid buildings that have survived to this day give clear indication that this was also an architecturally innovative period, and written accounts from this period make repeated reference to the architectural wonders and great ornamental beauty of these early Islamic buildings.

Excavations of a private residence at Sabz Pushan outside Nishapur revealed a number of finely carved stucco panels dating from 960 to 985 during the period of Samanid rule over Nishapur.[41] These include a threefold geometric panel that is easily constructed from the $6^3$ grid of underlying regular hexagons[42] [Fig. 96c] [Photograph 11]. This same pattern was used a century later by Seljuk artists at the eastern tomb tower at Kharraqan (1067-68). It is interesting to note that, like the two isometric window grille designs from Córdoba, this contemporaneous design is also made up of superimposed hexagons. The floral infill designs from Sabz Pushan are derivative of the Samarra style C—the beveled style. The carved stucco geometric ornament from Sabz Pushan also includes an interweaving classic star-and-cross linear border design [Fig. 124b]. The *pishtaq* of the mausoleum of Arab Ata (977-78) at Tim, Uzbekistan, 85 km southwest of Samarkand, employs several geometric designs in incised stucco, including a typical key pattern, an example of polygons as pattern, and two threefold geometric patterns easily constructed from the *system of regular polygons*. This Samanid building is also noteworthy for having the earliest extant example of a trilobed squinch. This particularly attractive solution to the structural challenge of placing a circular dome upon a square chamber became a regular feature of Seljuk brick architecture, achieving its apogee in the northeast and southwest domes of the Great Mosque of Isfahan (1072-92). This architectonic device is thought to be an important influence upon the development of *muqarnas* vaulting. The ornament of the mausoleum of Arab Ata is considerably more sophisticated than that of its better known predecessor, the Samanid mausoleum in Bukhara[43] (c. 914-43). This earlier example of Samanid funerary architecture is remarkable for its simple beauty wherein the entire surface of both the interior and exterior walls is replete with decorative brickwork. While the individual designs are, in and of themselves, very simple, the overall effect is of a wholly ornamented building. Such abundant use of ornamental texture was soon to become a predominant characteristic of Islamic architecture in the eastern regions. The mausoleum of Arab Ata, by contrast, limits its ornament to the front façade of the *pishtaq*, and employs geometric designs that are noticeably more complex. Of especial interest is the geometric pattern created from the *system of regular polygons* set within the arched tympanum above the entry *pishtaq* [Photograph 12]. This very successful design is conveniently created from the basic $6^3$ underlying generative tessellation [Fig. 96g]. This same design was used by the Seljuks on both the eastern tomb

[41] Blair (1991), 55.

[42] This panel from Sabz Pushan, Nishapur, is now in the permanent collection of the Metropolitan Museum of Art, New York: Accession Number 40.170.442.

[43] The Samanid Mausoleum is also known as the Tomb of Ismail the Samanid.



**Photograph 11** A Samanid period carved stucco panel with six-pointed stars that is easily created from the *system of regular polygons* that was found at the Sabz Pushan excavation near Nishapur, Iran (The Metropolitan Museum of Art: Rogers Fund, 1940: www.metmuseum.org)

tower at Kharraqan, Iran (1067-68), and the minaret in Daulatabad outside Balkh, Afghanistan (1108-09), and by the Seljuk Sultanate of Rum at the Izzeddin Kaykavus hospital and mausoleum in Sivas (1217). The other isometric design used on the front façade of the mausoleum of Arab Ata can likewise be created from the $6^3$ underlying tessellation [Fig. 95d], but is, in and of itself, a widened line version of simple 3.6.3.6 polygonal tessellation [Fig. 89]. This is one of the most widely used threefold patterns, with one of the earliest examples found at the ibn Tulun mosque in Cairo (876-79).

**Photograph 12** A Samanid threefold pattern with six-pointed stars easily created from the *system of regular polygons* in the tympanum over the *pishtaq* of the mausoleum of Arab Ata in Tim, Uzbekistan (© Bernard O'Kane)

## 1.9 Buyids (945-1055)

While the Saminids were flourishing in the regions of Khurasan and Transoxiana, the rise of the Shia Buyids had tremendous impact upon the political and military authority of the Abbasid Caliphate. The Buyids were a Persian tribe originally from the mountainous region of Daylam, south of the Caspian Sea. Around the year 932 they set out to conquer large areas of central Persia, and by 945 had conquered Baghdad, most of Iraq, Oman, and parts of Syria. Under the Buyids, the temporal power of the Abbasid Caliphate was reduced to a position of political subjugation.

Like the Saminids to the northeast, the Buyids were greatly influenced by earlier Persian culture. Under Buyid patronage, their capital cities of Isfahan and Shiraz became important centers of Islamic culture, with a great emphasis on the arts and architecture. Adud ad-Dawla, who reigned between 936 and 983, was a great patron of the arts and learning, and was reputed to have been an avid calligrapher. He was a prolific builder, and is reported to have ordered the construction of 3000 mosques in his lifetime, although this must certainly be an exaggeration.[44] Among the many architectural achievements of Adud ad-Dawla was his palace in Shiraz. This no longer exists, but was said to have 360 rooms, one for each day of the year, and each decorated in a differing style. For all the monumental architecture built by the Buyids, regrettably little has survived to the present.

The Friday Mosque at Na'in, Iran (960), was built by the Buyid Dynasty some 25 years after their seizing control of Baghdad. This mosque includes two carved stucco geometric star patterns that can be produced from the $4.8^2$ tessellation of squares and octagons. One is a bold linear band treatment of the classic star-and-cross design [Fig. 124b] with floral background infill that is similar in concept to the Samarra style A. This linear border runs vertically and horizontally around the *mihrab* as a framing device. The second wraps around one of the circular supporting piers in the prayer hall and is interesting in that the interweaving pattern lines are curvilinear [Fig. 127b]. The fact that the two examples from the Friday Mosque in Na'in share the same generative polygonal tessellation would not appear to be coincidental, and are certainly not the only architectural examples where two or more patterns are placed in close proximity that share the same underlying generative tessellation. The curvilinear treatment of the design on the pier has a softening effect on the rigid angularity that is otherwise a standard feature of this tradition. This is an early curvilinear design easily produced from the polygonal technique, and while known in the work of succeeding Muslim cultures, the allure of such designs is augmented by virtue of their rarity.

The degree of Buyid involvement in the development of Islamic geometric patterns is difficult to establish due to the paucity of ornamental examples that have survived from this dynasty. The most significant example of Buyid involvement in the maturation of this geometric tradition is the above-cited illuminated frontispiece from the celebrated Quran created by ibn al-Bawwab [Figs. 127c and 128d] [Photograph 6]. This design of interweaving octagons set upon the orthogonal grid is significant in several respects: as an indication of the relevance of the book arts in the early development of Islamic geometric patterns; as an indication of the ongoing importance of Baghdad to the development of this tradition; and as an early example of a geometric pattern that likely employed the polygonal technique in its creation. While the Quran of ibn al-Bawwab was certainly a product of the Abbasid cultural continuity associated with Baghdad, the Buyid patronage of ibn al-Bawwab is nonetheless significant.

## 1.10 Ghaznavids (963-1187)

The Ghaznavid dynasty was founded by Turkic military commanders of the Saminids who, in 977, took control of the Samanid territories in Afghanistan, setting up their capital in Ghazna. While politically autonomous, as staunch Sunnis, they were closely allied with the Abbasids. Under the command of Mahmud of Ghazna, who ruled between 988 and 1030, the Ghaznavids were victorious over both the Buyids in central Persia, and the Saminids in Khurasan. At the height of their power, they governed over an immense empire encompassing much of Azerbaijan, Persia, Transoxiana, and Khurasan, as well as large portions of the Indus Valley and northern India. In 1040, just 10 years after the death of Mahmud of Ghazna, the Ghaznavids were to loose much of their western territories to the Seljuks. In 1161 they lost Ghazna to the Ghurids, a rival central Afghan dynasty. Following this defeat, the Ghaznavids moved their capital to Lahore, and held control of their Indian provinces until their final overthrow in 1182, again at the hands of the Ghurids.

The Ghaznavids were ambitious patrons of science and the arts. Abū Rayḥān al-Bīrūnī, one of the great Muslim polymath scientists, rose to prominence within the Ghaznavid cultural milieu, and it was the commission by Mahmud of Ghazni that prompted Firdausi to write his epic poem *Šāh-nāma*. The Ghaznavid Empire was immensely wealthy, both by virtue of their precious metal resources and the plunder they amassed in the conquering of northern India. This wealth was poured into architecture and the arts. Ghaznavid metalwork included highly refined work in silver and gold, as well as utilitarian objects in bronze. While little has survived the passage of time, their metalwork was likely to have also included architectural components such as lamps, locks, hinges, door-pulls, and knockers. The acclaimed Persian poet and scholar Nasir-i Khusraw wrote that large silver door-pulls produced in the workshops of Ghazna were sent to Mecca for the door to the

[44] Hillenbrand (1994a), 373.

**Photograph 13** A Ghaznavid threefold pattern with six-pointed stars created from the *system of regular polygons* originally located at the South Palace at Lashgari Bazar near Bust, Afghanistan (© Thalia Kennedy)

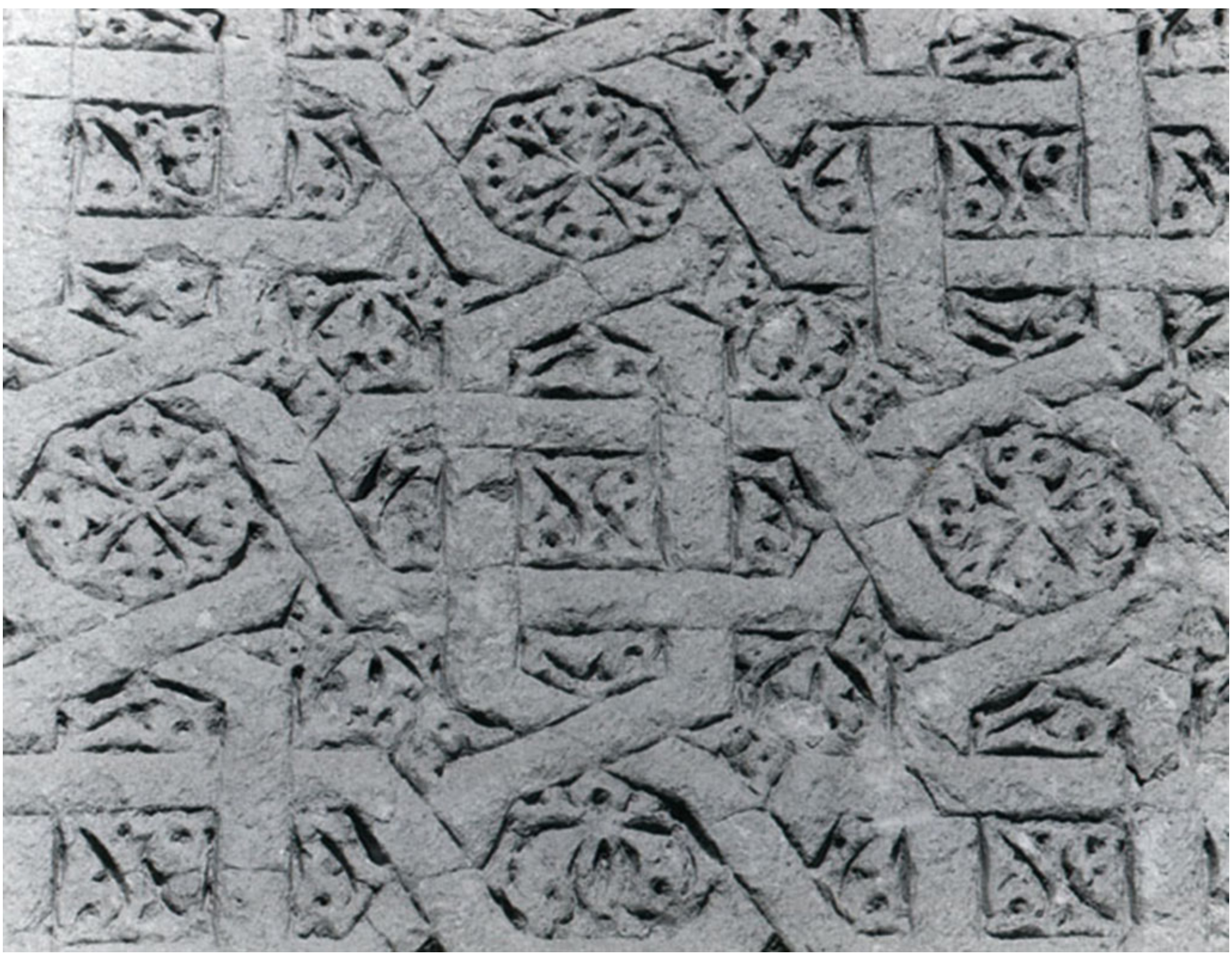
Kaaba.[45] Nishapur continued to thrive as a center for ceramic production under Ghaznavid rule. Ceramic tiles with molded relief decoration and vivid turquoise glaze have been found in excavations at Ghaznavid sites. However, the relatively few surviving examples of Ghaznavid architecture are devoid of ceramic faience decoration. Nonetheless, the remaining examples of Ghaznavid architecture show a remarkable degree of ornamental sophistication, and with their Samanid antecedents it is not surprising that the Ghaznavid architectural aesthetic would be largely characterized by ornamental brickwork. The quality of Ghaznavid design and fabrication surpassed that of their Samanid predecessors and set the standard for the outstanding Ghurid and Seljuk architectural brickwork that followed. Ghaznavid artists also employed carved stucco and marble to great effect, as well as painted fresco—although very little has survived. Each of these media was used in giving expression to the remarkable innovations in the art of geometric star patterns that transpired during this period. The remaining examples of Ghaznavid architecture include the Lashkar-i Bazar near Bust, Afghanistan (early eleventh century); the minaret and palace of Mas'ud III in Ghazna, Afghanistan (1099-1115); the ruins of the Ribat-i Mahi Caravanserai near Mashhad, Iran (1019-20); and the Arslan Jadhib tomb and minaret in Sangbast (997-1028).

Ghaznavid artists in Khurasan played a significant role in the development of geometric star patterns. It was under the auspices of this empire that the polygonal technique was expanded to include a greater range of geometric design, opening the door to the maturity and diversification of this tradition. These experimental innovations led to the creation of geometric patterns that expanded the stylistic boundaries and geometric underpinnings of this burgeoning tradition. An excellent Ghaznavid example of a geometric pattern derived from the *system of regular polygons* is a carved stone panel from the audience hall in the South Palace at Lashgari Bazar (completed in 1036), near Bust [Photograph 13]. This has a number of interesting and unusual

[45] Ward (1993), 57.

characteristics [Fig. 102a1]. The underlying polygonal tessellation is comprised of triangles and hexagons in a 3.6.3.6 configuration. What sets this pattern apart is the unusual manner in which the pattern lines relate to the underlying tessellation. The application of crossing pattern lines to the edges of each underlying polygon within a given tessellation typically employs the same angular treatment throughout. In this way, the pattern lines will relate equally to each underlying polygon in an identical manner. In the design from Lashgari Bazar, two different pattern line arrangements have been applied to the midpoints of the edges of alternating underlying hexagons: one set that cross with 60° angular openings, and another with 90° angular openings. This alternating innovation adds a further level of design diversity to what is already a highly versatile methodology. This alternating methodology never became widely practiced, and those patterns that employ this design technique are almost exclusively created from the *system of regular polygons*. This variant practice was mostly employed during the formative years in the eastern regions, and by the time this design tradition reached its full maturity in the thirteenth and fourteenth centuries, such patterns were seldom used. The carved stone panel that employs this design dates from before the Ghurid destruction of Lashkar-i Bazar in 1151. This very distinctive Ghaznavid geometric pattern was also used on the door of the Zangid *minbar* at the al-Aqsa mosque in Jerusalem (1168-74) [Fig. 102a3]. The very unusual derivational methodology of this particular pattern suggests the possibility of the Zangid example being produced by an artist familiar with the panel at Lashkar-i Bazar: perhaps having fled the political turbulence in Khurasan during the period of Ghurid conquest, or conceivably on the pilgrimage route to Mecca via Jerusalem.

A carved stucco panel from the intrados of the arched portal at the Ribat-i Mahi Caravanserai near Mashhad, Iran (1019-20), employs a fourfold *acute* pattern with octagons at the vertices of the square repeat unit [Fig. 138b] [Photograph 14]. This is one of the earliest designs associated with



**Photograph 14** A Ghaznavid carved stucco panel with octagons created from the *fourfold system A* in the arched portal at the Ribat-i Mahi Caravanserai near Mashhad, Iran (© Bernard O'Kane)

the *FourFold System A*, and uses only the large hexagon and square elements from the multiple components of this system. The lack of large octagons within the underlying modules qualifies this as a field pattern. *Acute* patterns within this system are characterized by 45° crossing pattern lines set on the midpoint of each edge of the underlying polygons. This example from the Ribat-i Mahi is unusual in that the underlying hexagonal modules employ two pattern line conditions at their edges. In addition to the 45° crossing pattern lines placed at the midpoints of the edges that are contiguous with other underlying hexagons, the hexagonal edges that are contiguous with the underlying squares have arbitrarily placed pattern lines with 90° angles. These 90° angled lines create a distinctive diagonally orientated square within the underlying square module, while the 45° crossing pattern lines create an octagon at each vertex where four elongated hexagons meet.

The significance of the Ghaznavid minaret of Mas'ud III in Ghazna, Afghanistan (1099-1115), looms large in the history of Islamic geometric star patterns. Like its nearby neighbor, the minaret of Bahram Shah (1117-58), this minaret is not associated with an adjacent mosque, and is possibly a victory tower commemorating successful military campaigns in the Indus Valley and northern India, and possibly inspired by their exposure to Hindu commemorative towers.[46] All that remains of the minaret of Mas'ud III is the magnificent stelliform shaft of the lower half, the upper cylindrical shaft having been destroyed by an earthquake in 1902. The lower shaft is an eight-pointed star in plan, and each pair of vertical flanges is divided into a series of ornamental panels of elaborate raised brick ornament; each divided in half at the 135° included angle of the eight-pointed star. The diverse ornamental treatment of these multiple raised brick panels includes herringbone *shatranji Kufi* calligraphy, knotted *Kufi*, and several linear bands of the classic fourfold star-and-cross design [Fig. 124b]. The horizontal grouping of eight geometric star patterns around the base of the shaft, as well as an array of eight similar patterns that circle the midsection of the shaft are remarkable for their level of complexity at this early date. Each of these 16 patterns has either fivefold or sevenfold symmetry, and includes patterns with 7- and 10-pointed stars [Fig. 206], 10- and 20-pointed stars, and 5- and 7-pointed stars [Figs. 280 and 281]. The fivefold patterns repeat on a rhombic grid, the 72° and 108° angles of which correspond to the symmetry of the decagon [Fig. 5a]. Throughout the subsequent history of this tradition, this was the most frequently used repetitive foundation for fivefold patterns, and these two-dimensional examples from Ghazni were among the earliest occurrences of patterns with fivefold symmetry, the only known earlier examples being from the northeastern domed chamber of the Friday Mosque at Isfahan (1088-89). It is also significant that these Ghaznavid designs are, collectively, of considerably greater complexity than most all other Islamic geometric patterns from this same period, with the only contemporaneous examples of equal geometric complexity being the Seljuk work at the Friday Mosque in Isfahan, and the Friday Mosque at Barsain near Isfahan (1105). It would appear that the artist who designed the raised brick panels of the minaret of Mas'ud III was a pioneer of outstanding ability. The fivefold patterns of this minaret employ the polygonal technique in their construction [Fig. 206]. However, they differ from the contemporaneous Seljuk fivefold patterns in Isfahan, as well as subsequent fivefold geometric patterns generally, in that they do not employ a systematic methodology: relying upon a less rigid approach to the application of the pattern lines to the edges of the generative polygonal tessellation of decagons. These decagons are placed in a vertex-to-vertex arrangement that repeats upon a rhombic grid. This is in marked contrast to edge-to-edge polygons that eventually became standard practice for underlying generative tessellations. What is more, these decagons were kept as part of the completed design, thereby providing telltale evidence for the polygonal schema of this pattern. The seven-pointed stars within the pattern matrix are non-regular, but nonetheless add appreciably to the beauty of the design.

As with fivefold geometric patterns, the earliest extant sevenfold pattern is found within the northeastern domed chamber of the Friday Mosque at Isfahan. This single example is a field design of relative simplicity [Fig. 279]. By contrast, each of the two patterns with sevenfold symmetry from the minaret of Mas'ud III is considerably more complex. Both of these Ghaznavid examples utilize an elongated hexagon as the repeat unit [Figs. 280 and 281]. This has four 2/7 and two 3/7 included angles. In a manner that is similar to their neighboring fivefold designs, the application of the pattern lines to their respective generative polygonal tessellations was nonsystematic, involving a higher level of arbitrarily determined design components than frequently found in this tradition. The first of the sevenfold designs [Fig. 280] places a set of pattern lines that connect every other heptagonal corner, creating a matrix of seven-pointed stars that touch point to point. Into this matrix is added a secondary set of arbitrary pattern lines that complete the design. The first set of pattern lines in the second sevenfold pattern [Fig. 281] are placed upon the midpoints of the heptagonal edges and extend into the interstice region of twin pentagons. On its own, this initial set of pattern lines is a very acceptable *median* pattern that follows the midpoint conventions of this design tradition, and qualifies as being systematic. The artist responsible for this masterpiece of geometric design added a secondary set of pattern lines to

[46] Hoag (1977), 189.

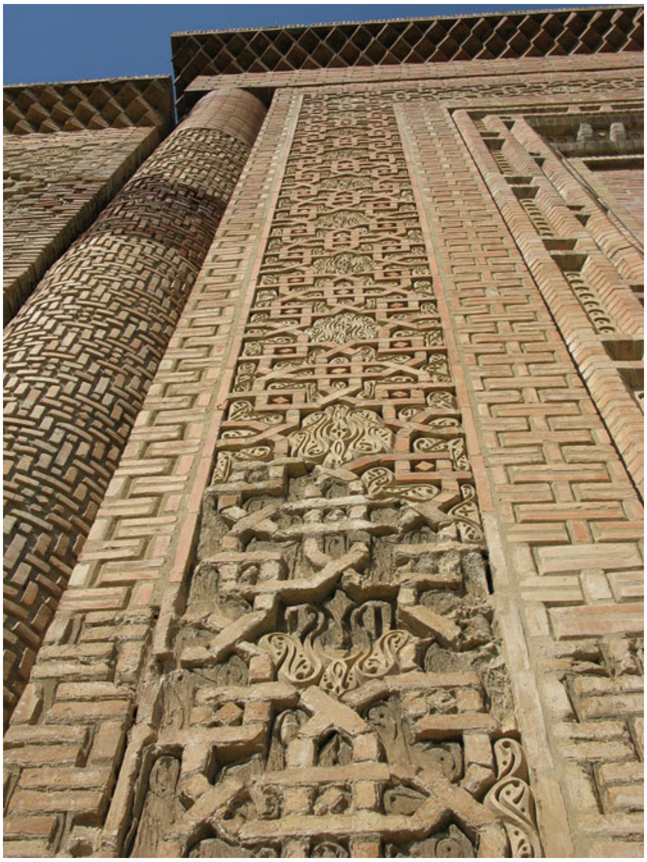
the initial design, thereby making it far more complex, but also creating a design that more affectively balanced with the aesthetics of the neighboring geometric panels. The early occurrence of designs with fivefold and sevenfold symmetries, coupled with the comparatively greater complexity of the patterns themselves to other geometric star patterns of similar date, gives this building great significance to the historical development of Islamic ornamental art. Adding to this importance is the fact that, along with the single example from Isfahan, these sevenfold geometric designs predate the next earliest extant examples by approximately a hundred years.

## 1.11 Qarakhanids (840-1212)

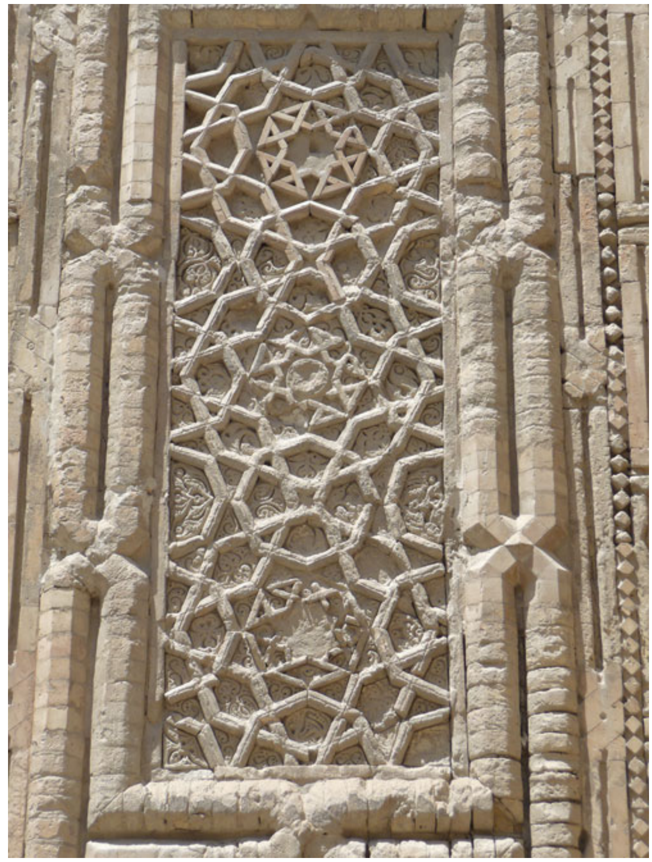
The Qarakhanids began as a confederation of Turkic tribes who rose to power in Central Asia during the ninth century. At the close of the tenth century, Qarakhanid and Ghaznavid forces defeated the Samanids: with the Ghaznavids taking control of the Samanid territories in Khurasan and the Qarakhanids taking control of Transoxiana. The boundary between these two Turkic rivals was the Amu Darya (Oxus River). Their capitals included Kashgar in western China, Balsagun, and Uzgen in Kyrgyzstan. The Qarakhanids concentrated their power in Central Asia, and were rivals with the Seljuks as well as the Ghaznavids. In 1140 they became subjects of the Kara-Khitan Dynasty from northern China, and were finally defeated by the Khwarizmshahid Dynasty in 1212.

Qarakhanid architectural ornament generally followed the monochrome brickwork and stucco practices prevalent in eastern regions of the Islamic world during the eleventh and twelfth centuries. Along with the geometric ornament of the Ghaznavids, the Qarakhanids were among the first Muslim cultures to expand the repertoire of geometric design to include patterns of greater complexity and diversity. Despite the very few remaining Qarakhanid buildings, the range of extant geometric patterns provides strong evidence for the important role they played in the development of the mature style of Islamic geometric patterns. The architectural record indicates that the Qarakhanids were particularly fond of geometric patterns made from both the *system of regular polygons* and the *fourfold system A*. Examples of Qarakhanid patterns made from the *system of regular polygons* include a very simple design constructed from the underlying $6^3$ tessellation of regular hexagons located within the corners of the quarter dome of the southern portal at the Maghak-i Attari mosque in Bukhara, Uzbekistan (1178-79). This is a *two-point* pattern that uses the $6^3$ hexagonal grid both as part of the completed pattern and as the formative schema [Fig. 96f]. The anonymous southern tomb in the complex of three adjoining Qarakhanid mausolea in Uzgen (1186) has two patterns with threefold symmetry that are constructed from the 3.4.6.4 underlying tessellation of triangles, squares, and hexagons [Fig. 89]. The first of these is located on the wide soffit of the entry arch and includes an overt expression of the underlying tessellation within the pattern itself. This pattern employs the square module of the generative tessellation as a primary feature of the completed design, thereby indicating the underlying triangles and hexagons as implied background elements [Fig. 104d]. The second pattern from the southern tomb at Uzgen to use the underlying 3.4.6.4 tessellation is located beneath the arch soffit on the sidewall of the arched portal [Fig. 105b]. As an added design feature, this example arbitrarily places six-pointed stars at the vertices of the isometric grid. This additive variation is similar in concept to a Ghurid example of the same design at the minaret of Jam, dating from just 20 years earlier [Fig. 105c]. The 3.4.6.4 pattern from Jam differs in that it places additive hexagons into these same positions. The northern tomb of the Jalal al-Din Hussein (1152-53) at this complex of three mausolea at Uzgen features a particularly delicate interpretation of the classic fourfold star-and-cross design on the intrados of the main arch of the portal. The eight-pointed stars touch point to point rather than their interweaving with one another. The visual impact of this less typical arrangement is augmented by a secondary interweaving motif of finer line thickness that results in an overall design that is unique, delicate, and extremely effective.

The earliest of the three adjoining mausolea at Uzgen, Kyrgyzstan, is the middle tomb of Nasr ibn Ali (1012-13). The entry portal of this tomb is framed with a *median* pattern in raised brick that was constructed from the *fourfold system A* [Fig. 159] [Photograph 15]. Along with the above-cited Ghaznavid design from the portal of the Ribat-i Mahi, this is one of the earliest examples of an Islamic geometric pattern that can be created from this generative system, and multiple later examples are found within the historical record. It is important to once again emphasize that when regarding more basic designs produced during the early formative period of this design tradition, it is impossible to know for certain which methodological practice was used in a given circumstance. This design from the tomb of Nasr ibn Ali could have been produced just as readily from either the grid method or the point joining technique as from the *fourfold system A* [Fig. 72]. An identical Seljuk use of this Qarakhanid design is found at the Sultan Sanjar mausoleum in Merv, Turkmenistan (1157). The points of the eight-pointed stars in the later example from Merv are irregular (as per Fig. 75a), and indicate that this example of the pattern may have been produced using the orthogonal graph paper technique. At the base of the sidewalls in the entry portal of the anonymous southern tomb at Uzgen is a small square carved stone panel with a pattern constructed from the



**Photograph 15** A Qarakhanid raised brick pattern with eight-pointed stars created from the *fourfold system A* in the entry façade of the tomb of Nasr ibn Ali in Uzgen, Kyrgystan (© Igor Goncharov)



**Photograph 16** A Qarakhanid pattern with eight-pointed stars created from the *fourfold system A* in the entry façade of the entry of the Maghak-i Attari mosque in Bukhara, Uzbekistan (© Thalia Kennedy)

*fourfold system A* [Fig. 160]. The underlying tessellation for this geometric pattern includes modules that are atypical to this system, but add a very acceptable dynamic to the completed design. This design dates from the early period of the *fourfold system A* when experimentation with both polygonal components and application of their associated pattern lines was prevalent. The ornamental banding in the nearby minaret of Uzgen (twelfth century) includes a geometric pattern created simply from the orthogonal grid that has design characteristics similar to the *fourfold system A* [Fig. 74]. The entry *pishtaq* of the Maghak-i Attari mosque in Bukhara has several raised brick geometric panels with patterns constructed from the *fourfold system A*. The two most basic of these are located closest to the ground. The absence of eight-pointed stars in both of these qualify them as field patterns, and the underlying generative tessellation is made up of just large hexagons and squares. One of these is a *median* pattern with 90° crossing pattern lines [Fig. 138c], and the other is a design comprised of superimposed dodecagons, each of which is centered upon the vertex of the four large hexagons [Fig. 138f]. These dodecagons relate to the underlying tessellation through 90° crossing pattern lines located upon the midpoints of each hexagonal edge, and 120° crossing pattern lines are located at the midpoints of the square edges. The highest panel on this façade from Bukhara is a *median* pattern created from the *fourfold system A* with considerably greater complexity than its neighbors [Fig. 155]. The relationship between the pattern lines and the underlying generative tessellation is less consistent than normally found within this tradition: with some midpoints of the underlying polygonal edges having crossing pattern lines; others having lines that meet, but do not cross these midpoints; and still others having no pattern lines at all. The completed geometric design is as much a product of subjective artistic license as systematic methodology. Specifically, the completed design is the result of a subtractive process whereby pattern lines are strategically removed to create a new pattern matrix with background regions that would not have otherwise been there. While orthogonal, the repetitive schema of this design is comprised of the $4.8^2$ semi-regular grid, upon which octagons are located at each vertex. The middle panel on the façade of the Maghak-i Attari mosque also employs a design created from the *fourfold system A* [Fig. 151] [Photograph 16]. This is an elegant *acute* pattern that is created from the square and triangle modules from the *fourfold system A*, and with added large octagons with sides equal to the longer edges of the triangle. The octagon with this edge size is atypical to patterns made from this system. The end result is an exceptional pattern comprised of two sizes of interweaving octagons, and arbitrarily added octagons set within the large octagons (not shown in Fig. 151).

The design used on each of the circular columns that flank the entry portal of the anonymous southern tomb at Uzgen is a *two-point* pattern generated from the *fourfold system B* [Fig. 176c]. The perpendicular parallel pattern lines at the center of the square repeat unit are an additive device that was particularly popular in the eastern regions: for example, the central region of the repeat unit for the Ghurid raised brick design from the Friday Mosque at Herat (1200) [Fig. 174b]. The underlying generative tessellation for this design employs octagons, small pentagons, and small hexagons from this system.

The back wall of the south entry portal at the Maghak-i Attari mosque in Bukhara has two adjacent carved stone relief panels with identical geometric patterns created from the *fivefold system*. This is an *obtuse* design with rectangular repeat units [Fig. 245a]. Along with the more complex contemporaneous Seljuk example from the Seh Gunbad in Orumiyeh, Iran (1180), these are among the earliest extant examples of purely systematic fivefold patterns that repeat upon a rectangular grid. With the adoption of the *fivefold system* by subsequent Muslim cultures, this Qarakhanid design became the most widely used fivefold pattern that repeats upon a rectangular grid. Several especially fine examples include an Ilkhanid arch soffit at the Friday Mosque at Ashtarjan, Iran (1315-16); a Timurid cut-tile panel from an entry portal at the Shah-i Zinda funerary complex in Samarkand, Uzbekistan (1386); and a Timurid running mosaic wainscoting panel at the Abdullah Ansari complex in Gazargah, Afghanistan (1425-27).

## 1.12 Great Seljuks (1038-1194)

The art and architecture of the Ghaznavids profoundly influenced their Seljuk and Ghurid successors. These rival dynasties vied for power within the tight confines of greater Khurasan. Each adhered to Sunni Islam, and each had a strong affinity with Persian customs and culture. The Seljuks rose to power as military commanders of the Qarakhanids who fought against the Ghaznavids. As an independent force, they conquered Merv and Nishapur in 1028-1029, followed by Ghazna in 1037. In 1038, Tughril adopted the title of Sultan of Nishapur: officially founding this immensely influential dynasty. In 1040, they defeated the Ghaznavids at the Battle of Dandanaqan, taking control of the Ghaznavid's western territories. Upon securing the greater portion of Khurasan, Seljuk forces expanded their conquest further westward against the Buyids. Allied with the Abbasid Caliph, Tughril defeated the Buyid forces in Baghdad. Within 20 years of his declaring himself Sultan, Tughril had wrested control over a broad swath of land that extended from the Levant and most of Anatolia in the west, all of Persia, large tracks of Transoxiana in the north, to western Khurasan in the east.

The eleventh-century advances in geometric design methodology made in Khurasan and Transoxiana spread westward during the twelfth century. During the first half of the eleventh century the Ghaznavid Empire gained control over eastern Persia. This was rapidly eclipsed by the military successes of the Seljuks, whose rule and hegemony profoundly influenced the architectural ornament throughout their vast territorial holdings for over a century. It was during this period of Seljuk cultural dominance that complex geometric design became a dominant feature of the architectural ornament in their western territories. Furthermore, the twelfth-century westward spread of evermore complex geometric patterns is evidence that the polygonal technique—the only viable method of creating particularly complex patterns—was wholeheartedly embraced throughout the regions of Seljuk influence, beyond to Egypt, and across North Africa to Morocco and al-Andalus—effectively establishing a pan-Islamic geometric aesthetic. Throughout this westward expansion, the use of the polygonal technique continued to be employed as a primary methodology for creating geometric patterns.

Surviving examples of early Seljuk architectural ornament include numerous fine geometric patterns created easily with the polygonal technique. Of particular note are the two tomb towers of Kharraqan in Qazvin Province, Iran: the eastern tower (1067) and the western tower (1093-94). These two towers are decorated with a variety of geometric patterns executed in raised brick, including key patterns, polygonal tessellations as pattern, and an assortment of geometric star patterns. Several of these geometric patterns were used earlier at the mausoleum of Arab Ata [Fig. 96g] [Photograph 12], the Great Mosque of Córdoba [Fig. 96e], and Sabz Pushan near Nishapur [Fig. 96c] [Photograph 11], while other patterns from Kharraqan appear at their earliest known date. One of the most interesting geometric patterns from Kharraqan appears on the eastern tower, and is very likely the earliest surviving example of an Islamic geometric pattern with 12-pointed stars [Photograph 17]. The stars are located on each vertex of the isometric grid, and the underlying polygonal tessellation that produces this pattern is made up of triangles and dodecagons in a $3.12^2$ configuration [Fig. 108a]. This exact same design was used as part of the interior ornament for the Friday Mosque of Golpayegan, Iran (1105-18); the Sayyid Ruqayya Mashhad in Cairo (1133); the Great Mosque at Kayseri, Turkey (1205); one

**Photograph 17** A Seljuk example of a threefold pattern with 12-pointed stars created from the *system of regular polygons* at the eastern tomb tower at Kharraqan, Iran (© Reza Roudneshin)

of the Mamluk window grilles from the restoration of the Ibn Tulun mosque in Cairo (1296); and the interior of the Mamluk door (1303) of the Vizier al-Salih Tala'i mosque in Cairo. Indeed, over time, this design came to enjoy great popularity throughout the Islamic world. A particularly beautiful curvilinear example was used as an illumination in the celebrated 30-volume Quran of Uljaytu,[47] written and illuminated by ʻAbd Allah ibn Muhammad al-Hamadani in 1313 [Fig. 108c]. Another early Seljuk pattern with 12-pointed stars that is easily created from the $3.12^2$ tessellation is from the southern *iwan* of the Friday Mosque at Forumad in northwestern Iran (twelfth century) [Fig. 108d]. This example was also popularly used in later periods, including a frontispiece from a Baghdadi Quran illuminated by Muhammad ibn Aybak ibn ʻAbdullah (1303-07), and a Mamluk stone mosaic panel from the Amir Aq Sunqar funerary complex in Cairo (1346-47) [Photograph 45]. One of the very successful isometric patterns from the east tower at Kharraqan is easily made from the simple hexagonal grid with an additive six-pointed star motif at the centers of the underlying hexagon [Fig. 96h]. This was a very popular design that was used in many succeeding locations: including the wooden *minbar* of al-Aqsa mosque in Jerusalem (1168); the *mihrab* of the Lower Maqam Ibrahim at the citadel of Aleppo (1168); the entry portal of the Izzeddin Kaykavus in Sivas, Turkey (1217-18); and an archivolt at the Zahiriyya *madrasa* in Aleppo (1217). The wooden *minbar* at the Friday Mosque at Abyaneh, Iran (1073), includes another Seljuk pattern created from the 3.6.3.6 underlying tessellation that places six-pointed stars at the vertices of the isometric grid [Fig. 99a]. As with several other designs made from this system, this example is comprised of superimposed hexagons. The earliest extant example of this design is one of the window grilles at the Great Mosque of Córdoba (987-99), and over time, this came to enjoy wide popularity throughout the Islamic world. A raised brick border that surrounds the *mihrab* of the Friday Mosque of Golpayegan (1105-18) can also be derived from the 3.6.3.6 underlying tessellation of triangles and hexagons [Fig. 99b]. This *median* pattern places 90° crossing pattern lines at the midpoints of each edge of the underlying polygons. The resulting design is characterized by superimposed dodecagons that repeat upon the isometric grid. The earliest known use of this pattern is from a Fatimid window grille at the al-Azhar mosque in Cairo (970-72), and over time, it was widely used by succeeding Muslim cultures. A conceptually similar design with superimposed dodecagons can be created from the simple $6^3$ tessellation of hexagons [Fig. 97c]. While the placement of the dodecagons within the pattern matrix is identical, their size relative to the isometric repeat is slightly larger. This produces differently proportioned concave octagons and ditrigonal shield-shaped background modules. This subtle variation was also widely used by diverse Muslim cultures, and two fine Seljuk examples include the surrounding border of the *pishtaq* of the Seh Gunbad in Orumiyeh, Iran (1180), and a carved stucco panel from the Friday Mosque at Forumad in Iran (twelfth century). Other notable examples of these closely related designs are found at the Sirçali *madrasa* in Konya, Turkey (1242-45); the Shah Rukn-i-ʻAlam tomb in Multan, Pakistan (1320-24);[Photograph 69]; and the fourteenth-century ceramic tile work added to the main *iwan* of the tomb of Abu Sa'id Abul Khayr in Mayhaneh, Turkmenistan.[48] A very beautiful Seljuk example of an additive variation of this pattern was used in the celebrated tympanum over the door in the main portal

[47] This Ilkhanid Quran is in the National Library in Cairo: 72, pt. 19.

[48] The Tomb of Abu Sa'id Abul Khayr in Mayhaneh, Turkmenistan, is known locally as the Tomb of Meana Baba.

**Photograph 18** A Seljuk example of a threefold pattern with six-pointed stars from the northeast dome chamber in the Friday Mosque at Isfahan (© Tom Goris)

of the Gunbad-i Surkh in Maragha, Iran (1147-48). This example is credited with being the earliest extant Islamic ornamental panel to incorporate glazed faience ceramics[49]: a precursor to the tradition of cut-tile mosaics wherein the whole ornamental surface is covered with specially cut ceramic pieces that fit together to make the design. Prior to this example, faience was used as an ornamental accent, frequently to emphasize the calligraphic component of the ornament. There are two separate additive motifs in this panel: the first being a series of superimposed nonagons in turquoise faience, and the second being a series of parallel pattern lines that emphasizes the hexagonal repetitive grid. A very similar additive design was used on the Kaykavus hospital in Sivas (1217-18), as well as the tomb of Sahib Ata in Konya (1283-93). The difference between these two later examples from the Seljuk Sultanate of Rum in Anatolia is that their secondary additive elements are parallel pattern lines that emphasize the isometric repeat rather than the hexagonal dual.[50]

A very simple, but highly effective, Seljuk geometric pattern from one of the multiple blind arches in the upper portion of the northeast dome chamber in the Friday Mosque at Isfahan (1088-89) is created from the $6^3$ hexagonal grid [Photograph 18]. This example places 45° crossing pattern lines at the center points of each underlying hexagonal edge, and is hence categorized as a variation of the standard 30° angular openings of an *acute* design from this system [Fig. 95a]. These crossing pattern lines create six-pointed stars at the centers of each hexagonal repeat unit. It is surprising that this dynamic design was not used nearly as often as those composed of either 60° or 90° crossing pattern lines that were created from this same underlying tessellation. Other Seljuk examples of this pattern are found in one of the small blind arches in the upper *muqarnas* squinches of the Friday Mosque in Barsian, near Isfahan (1105), and in the carved stucco on the intrados of an arch at the Friday Mosque at Sin in Iran (1134). Contemporaneous with the example from Sin is an example from the niche of the Fatimid portable wooden *mihrab* of the Sayyid Ruqayya Mashhad in Cairo[51]

[49] Wilber (1939), 35.

[50] Schneider (1980), pattern no. 225.

[51] Currently in the collection of the Islamic Museum in Cairo.

(1133), and a later example from Cairo is from a Mamluk carved stone relief at the Imam al-Shafi'i mausoleum (1211).

The thin border that surrounds the entry door of the Gunbad-i 'Alaviyan in Hamadan, Iran (late twelfth century), employs a rather clever design with six-pointed stars and nonagons. This is relatively easy to construct from the 3.6.3.6 underlying tessellation of triangles and hexagons [Fig. 100c] [Photograph 22]. Later examples of this design were produced by artists working in Anatolia, and include a pattern in the Great Mosque of Divrigi (1228-29), the Muzaffar Barucirdi *madrasa* in Sivas (1271-72), and the central panels on the interior of the Mamluk door (c. 1303) at the Vizier al-Salih Tala'i mosque.

Like their Ghaznavid predecessors, Seljuk artists occasionally employed the atypical variation to this design methodology whereby two varieties of pattern-line configuration are applied to adjacent underlying polygonal cells of the same type. As mentioned, the earliest known example of such a design is the above-cited Ghaznavid pattern from the Audience Hall in the South Palace Lashkar-i Bazar (before 1036) [Fig. 102a1] [Photograph 13]. This uses the 3.6.3.6 underlying tessellation as its generative schema. A panel above one of the exterior blind arches of the west tower at Kharraqan (1093) appears to be the earliest Seljuk pattern to similarly employ differentiated treatments to adjacent polygonal cells from the underlying generative tessellation: in this case the simple $6^3$ hexagonal grid [Fig. 98c]. The primary underlying hexagons have six-pointed stars with 60° crossing pattern lines located at the center points of the polygonal edges: the standard pattern line application of the *median* family. Each of these primary hexagonal cells is surrounded by six secondary underlying hexagons that place pattern lines that connect each vertex through the center of the underlying hexagon, as well as extend the 60° crossing pattern lines from the primary underlying hexagons. Somewhat surprisingly, this fine pattern is not known elsewhere within the historical record. A Seljuk example of a 3.6.3.6 design with alternating pattern application to the underlying hexagons is immediately adjacent to the door within the portal of the Seh Gunbad tomb tower in Orumiyeh, Iran (1180) [Fig. 101a]. This utilizes 90° crossing pattern lines located at the center points of the active underlying hexagons. The constructive methodology of this design is unusual. The pattern is created by extending the 90° crossing patterns that are placed upon the primary underlying hexagons into the pattern matrix where they are met by the extended 60° crossing pattern lines that originate from designated active underlying triangles. Locations of later examples of this Seljuk design include the Sirçali *madrasa* in Konya, Turkey (1242), and a variation from the Çifte Minare *madrasa* in Sivas, Turkey (1271) [Photograph 41].

Another type of atypical pattern line application simply widens the lines of the underlying tessellation itself, rather than following the standard convention of the pattern lines being located upon the midpoints of the generative polygonal edges. A Seljuk isometric design of this type was used on the façade of the west tower at Kharraqan [Fig. 110]. This design is created from the $3^2.4.3.4\text{-}3.4.6.4$ *two-uniform* tessellation of regular triangles, squares, and hexagons [Fig. 90]. All of the polygonal edges in this tessellation are widened to create the interweaving design except the coincident edges of the twin triangles. In this respect, the twin triangles are treated as a single rhombus. This Seljuk pattern from Kharraqan appears to be the earliest example of the use of a *two-uniform* tessellation in Islamic art. Another design created from a *two-uniform* underlying polygonal tessellation is one of the multiple patterns from the anonymous Persian language treatise *On Similar and Complementary Interlocking Figures* in the Bibliothèque Nationale de France in Paris.[52] It is speculated that this was produced circa 1300 and was influenced by earlier Seljuk and possibly Khwarizmshahid artistic practices and sources.[53] One of the designs included in this treatise is a *two-point* pattern created from the $3^3.4^2\text{-}3^2.4.3.4$ underlying tessellation of triangles and squares [Fig. 112d]. This is a rather remarkable orthogonal design that has oscillating squares and rotating kite motifs within each repetitive square component. While unknown to the architectural record, the aesthetic style of this design is very closely related to examples from the Khwarizmshahid [Fig. 112b] [Photograph 38] and Ilkhanid periods [Fig. 111].

Among the many Seljuk geometric patterns in the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) is the earliest example of a design that employs the distinctive ditrigonal shield module in its underlying generative tessellation [Fig. 118a] [Photograph 19]. This underlying tessellation is, in and off itself, identical to the classic *median* pattern created from the $6^3$ tessellations of hexagons [Fig. 95c]. This pattern is one of several patterns that decorate the series of small blind arches in the upper portion of the square base of the northeast dome chamber. This same underlying generative tessellation was used in several other locations to produce patterns that are very similar to the example from Isfahan. These include two examples from the Seljuk Sultanate of Rum: one from the *mihrab* of the Karatay *madrasa* in Antalya (1250) [Fig. 118d], and the other from the Ahi Serafettin mosque in Ankara (1289-90) [Fig. 118c]. A later Ottoman example of inferior quality was used in the *mihrab* of the Yesil mosque in Bursa, Turkey (1419-21). A Mamluk variation that can be created from this underlying tessellation was used in a window grille at the

[52] MS Persan 169, fol. 188b.

[53] –Özdulral (1996).
–Necipoğlu [ed.] (Forthcoming).

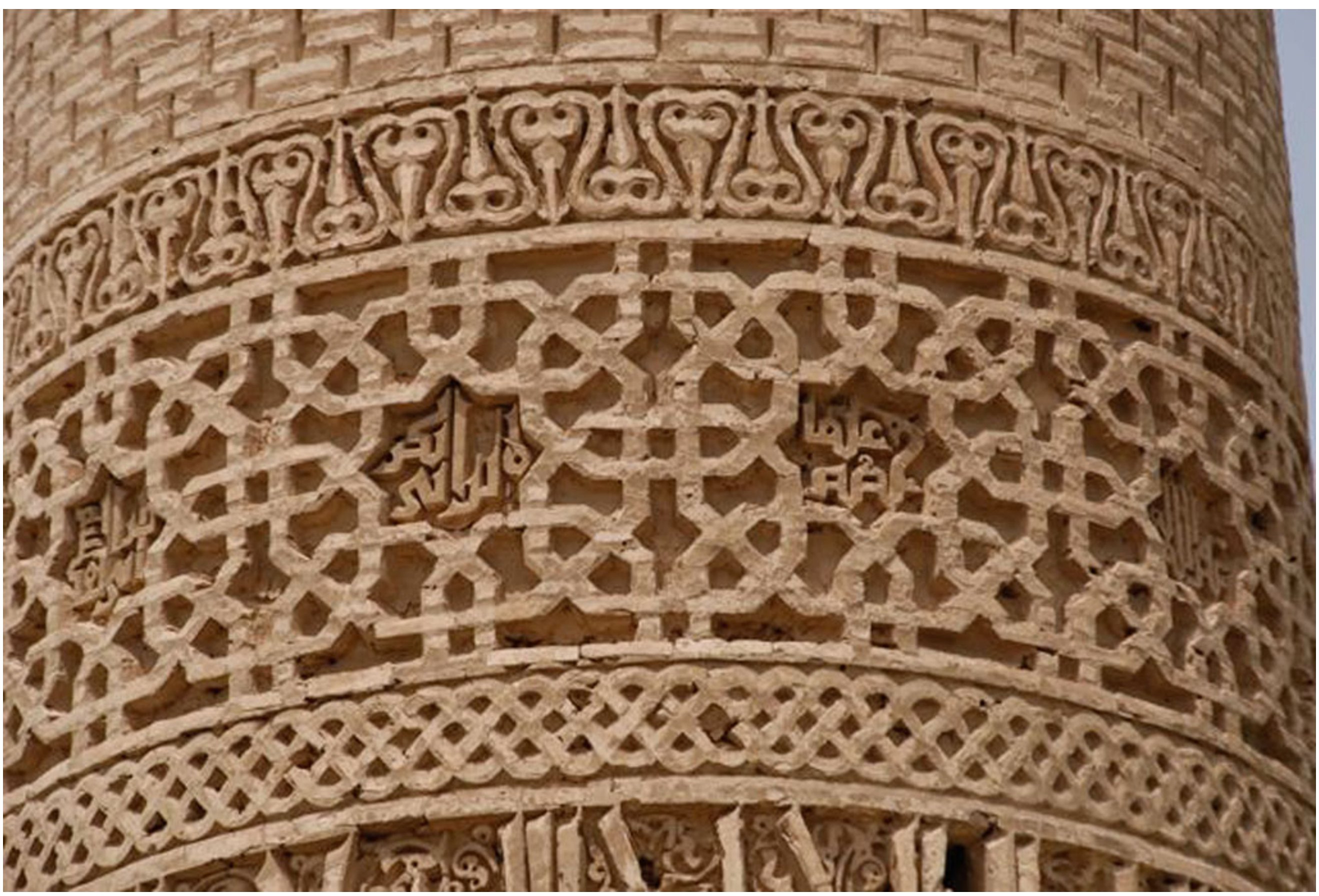
**Photograph 19** A Seljuk example of a threefold pattern with six-pointed stars and octagons from the northeast dome chamber in the Friday Mosque at Isfahan (© Tom Goris)

Tabarsiyya *madrasa* (1309) at the al-Azhar mosque in Cairo [Fig. 118b].

Surviving examples of Seljuk patterns that can be derived from the $4.8^2$ underlying tessellation are relatively uncommon. An example of the classic star-and-cross design was used in the wooden ceiling of the Friday Mosque of Abyaneh (1073) [Fig. 124b]. A raised brick border from the Gunbad-i ʻAlayvian in Hamadan (late twelfth century) employs a pattern that can be constructed in several ways, including from the $4.8^2$ tessellation [Fig. 125c]; and a Seljuk carved stucco panel at the Tehran Museum can also be created from this tessellation [Fig. 127e]. It is important to stress that these latter two examples have alternative methods of construction that may well have been employed at the time of their creation.

Along with their Qarakhanid and Ghaznavid counterparts, Seljuk artists were among the first to explore the design potential of the *fourfold system A*. The diverse range of patterns used in the decoration of the two tomb towers at Kharraqan includes a very simple border design on the earlier eastern tomb tower (1067-68) that is conveniently constructed from this generative system and was used widely by succeeding Muslim cultures [Fig. 138c]. The underlying polygonal tessellation that creates this pattern is comprised of just the elongated hexagonal and square polygonal modules. On its own, the basic tessellation of squares and elongated hexagons (but with different proportions) had been used as early as 300 years previous by Umayyad artists at Khirbat al-Mufjar, as well as by Abbasid artists at Samarra some 200 years previous. A contemporaneous Abbasid example of the ornamental use of this polygonal tessellation, with approximately the same hexagonal proportions as used in the *fourfold system A*, was used at the No Gunbad mosque in Balkh. The example from Kharraqan uses this tessellation to generate a *median* geometric design that places crossing pattern lines with 90° angular openings at the midpoints of each polygonal edge. The absence of large octagons within the underlying generative tessellation means that this pattern from Kharraqan does not have eight-pointed stars, and therefore qualifies as a field pattern. In addition to the polygonal technique, this well-known design can also be produced using either the orthogonal grid method or the point joining method [Fig. 77]. Field patterns associated with the *fourfold system A* were pioneered in the eastern regions during the early developmental period of this artistic tradition. The *acute* pattern created from this same underlying tessellation of large hexagons and squares was used by two western subordinate dynasties that were part of the sphere of Seljuk influence during the twelfth century [Fig. 138a]: the Artuqid *mihrab* of the Maqam Ibrahim at Salihin in Aleppo (1112) and the Tepsi minaret in Erzurum (1224-32) produced by the Saltukids. A *median* field pattern created from a tessellation of just large hexagons was used as a border that surrounds the entry portal of the Khwaja Atabek mausoleum in Kerman (1100-1150) [Fig. 137d].

A more complex Seljuk example of the early use of the *fourfold system A* is a raised brick pattern surrounding the midportion of the shaft of the minaret of the Friday Mosque at Damghan, Iran (1080) [Fig. 145]. It is interesting to note that this *median* design can be created from two separate sets of underlying tessellations from this same system: the first comprised of large octagons, large hexagons, and pentagons; and the second comprised of small octagons, small hexagons, pentagons, squares, and interstice rhombi. The reason for this unusual reciprocal feature is that the underlying polygonal edges can bisect the 90° crossing pattern lines in two perpendicular directions. These two underlying generative tessellations are essentially duals of one another. This same design enjoyed great popularity among Seljuk artists, and the many examples include: the Friday Mosque at Golpayegan, Iran (1105-18), that is particularly interesting for its being interwoven into the ascending letters of a band of *Kufi* script; the minaret of Daulatabad outside Balkh, Afghanistan (1108-09) [Photograph 20]; the minaret of the Friday Mosque at Saveh, Iran (1110); the Friday Mosque at Sangan-e Pa'in (second half of the twelfth century); and the Friday Mosque at Gonabad, Iran (1212). The Ghurids used this design during the same early period in several panels from the minaret at Jam, Afghanistan (1174-75 or 1194-95). The *mihrab* of the Malik mosque in Kerman (eleventh–twelfth century) is decorated with a more complex *median* pattern created from the *fourfold system A* [Fig. 153]. This introduces the triangular module that is 1/8 of an octagon into an underlying tessellation of large octagons and pentagons. A strong visual feature of this design is the set of large orthogonally placed octagons that are orientated vertex to vertex. This orientation relates to the classic *obtuse* pattern of octagons and four-pointed stars that is derived



**Photograph 20** A Seljuk example of a pattern with eight-pointed stars created from the *fourfold system A* on the minaret of Daulatabad outside Balkh, Afghanistan (© Thalia Kennedy)

from the $4.8^2$ underlying tessellation of octagons and squares. A second *fourfold system A* pattern from the Friday Mosque at Gonabad is an *acute* pattern that is unusual in that it uses an eight-pointed star as a primary component of the underlying generative tessellation that is created from the arrangement of square modules [Fig. 147a]. The use of this eight-pointed star creates a design that is atypical to this generative system. It is interesting that essentially this same design can be produced from an altogether separate underlying tessellation of different components from this same system [Fig. 146]. Ordinarily, when a given design can be produced from two different underlying tessellations, they are duals of one another. In this case, the two generative tessellations are not duals. This alternative tessellation employs octagons, pentagons, and small hexagons that combine together to create a large dodecagonal interstice region at the center of each repeat. As with other systematic designs created during this formative period, these two examples from Gonabad exemplify the ongoing experimentation that led toward the full maturity of this design tradition.

Despite the Seljuks being highly influential innovators of the geometric idiom, examples of their extant architectural ornament do not include a representative quantity of patterns created from the *fourfold system B*. This is surprising in that their allied Zangid, Ildegizid, and Sultanate of Rum neighbors to the west made wide use of this variety of geometric design. One notable exception to the rarity of Seljuk designs created from this system is an example of the classic *acute* pattern found within the *mihrab* arch spandrels at the Friday Mosque at Sin, Iran (1134) [Fig. 173a]. This is not only the earliest known use of this highly popular Islamic geometric design, but also the earliest known example of a pattern constructed from the *fourfold system B*. Were it not for the Mongol destruction, it is possible that a far greater number of Seljuk *fourfold system B* designs may have survived to the present, and our knowledge of the origins and dissemination of this important variety of geometric design would be more complete.

The architectural record indicates that the Seljuks were also the first to develop geometric patterns created from the *fivefold system*. This methodological system for creating Islamic geometric patterns is of particular significance to the history of Islamic art and architecture. Over time, this form of design spread throughout the Islamic world, receiving ongoing innovative attention and lasting popularity. The earliest fivefold designs date from the close of the eleventh century, and within a hundred years this variety of systematic design was making full use of rhombic, rectangular, and hexagonal repeat units, as well as fully mature patterns in each of the four pattern families: *acute*, *median*, *obtuse*, and *two-point*. The earliest Islamic geometric patterns created from the *fivefold system* are three examples from the northeast dome chamber of the Friday Mosque at Isfahan (1088-89). One of these three is the classic *obtuse* pattern that repeats upon a rhombic grid with 72° and 108° included angles [Fig. 229a] [Photograph 21]. This early example includes the additive star rosette infill of the ten-pointed stars that, in time, became a common feature of *obtuse* patterns [Fig. 221]. An interweaving version of this same design (without the additive infill) was used very soon after at the Friday Mosque at Golpayegan, Iran (1105-1118) [Fig. 229b], and indeed this design was used with great frequency throughout Muslim cultures. Other early Seljuk patterns created from the *fivefold system* that employ this same rhombic repeat unit include a *two-point* pattern in the magnificent entry tympanum at the Gunbad-i ‘Alaviyan in Hamadan, Iran (late twelfth century) [Fig. 231d] [Photograph 22], and a classic *acute* pattern from the Friday Mosque at Gonabad (1212) [Fig. 226c] [Photograph 23]. Indeed, each of these three early Seljuk examples employs the same underlying generative tessellation. A late Abbasid *median* pattern at the mausoleum of ‘Umar al-Suhrawardi in Baghdad (early thirteenth century) also uses the same underlying generative tessellation, but uses only selected midpoints of the underlying tessellation for locating the pattern lines [Fig. 228d]. This caliphal building dates to when Baghdad was no longer ruled by the Seljuks but was still under the aesthetic influence of Seljuk culture. A particularly complex Seljuk design that employs the rhombic repeat with 72° and 108° included angles wraps nine of the ten sides of the Gunbad-i Qabud in Maragha, Iran (1196-97) [Photograph 24].[54] The rhombic repeat unit of this design holds an unusually large number of polygonal modules that comprise the underlying generative tessellation [Figs. 239 and 240]. The continuous flow of this pattern across the nine sides of this decagonal tomb tower includes coverage of the ten engaged columns at each corner of the tomb tower, and is only discontinued on the side of the tower with the entry portal. This remarkable geometric design has been the subject of considerable interest in recent years, with arguments and counterarguments as to whether it is an example of quasicrystalline geometric design.[55] While this design has clear Penrose tiling characteristics, it nevertheless repeats in nine linear units, each of which is a unit cell, thereby disqualifying it from being an aperiodic quasicrystalline structure. Although the linear repeats of this design appear as rectangular, corresponding to the rectangular façades of the building, when considered more broadly it becomes clear that the actual repeat units are the fivefold

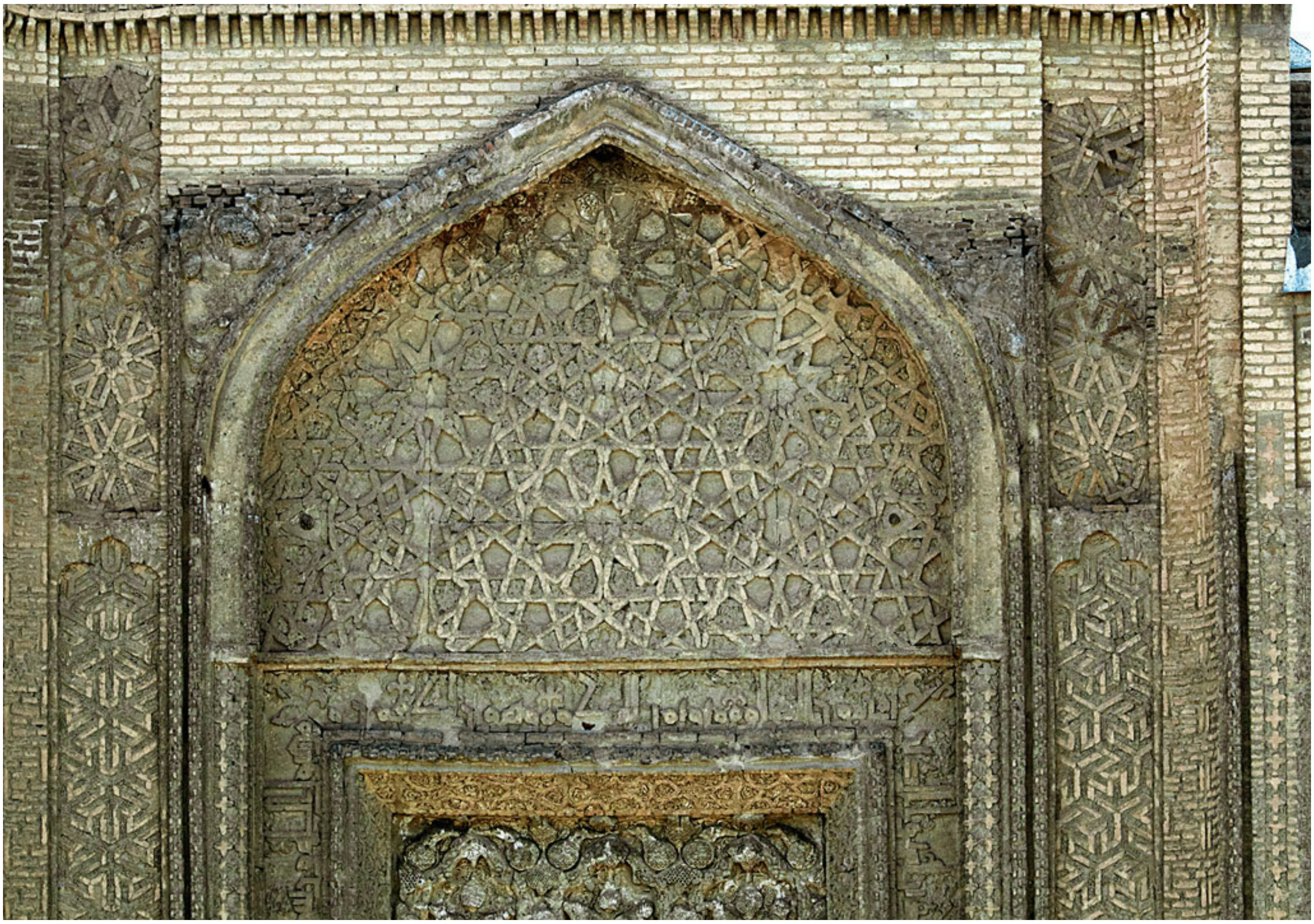
[54] Bier (2012).

[55] –Makovicky (1992), 67–86 and (2007).
–Lu and Steinhardt (2007b), 1106–1110.
–Cromwell (2009), 36–56.
–Cromwell (2015), 1–15.

**Photograph 21** A Seljuk example of the classic *obtuse* pattern with ten-pointed stars created from the *fivefold system* in the northeast dome chamber of the Friday Mosque at Isfahan (© Tom Goris)

rhombus with 72° and 108° included angles. The apparent complexity and randomness of the underlying tessellation bely what is actually a well-ordered geometric schema comprised of rings of ten edge-to-edge decagons placed upon each vertex of the rhombic grid. This arrangement of decagons is then treated to a secondary application of polygonal modules from the *fivefold system*: with most of the decagons being filled, and some remaining unfilled. To further complicate this secondary application, the infill of the secondary polygons only has reflection symmetry along the vertical line of axis within the rhombic repeat units. This is highly unusual, and creates a geometric pattern that is certainly eccentric, but not aperiodic. Still further complexity is achieved by the arbitrary infill of the ten-pointed stars within the remaining unfilled decagons with an additive infill motif that was popularly used among Seljuk artists in Persia and Anatolia [Fig. 224a]. This effectively disguises the ten-pointed stars, and transforms the overall design into a field pattern. This is the earliest extant example of this well-used transformative variation to the ten-pointed star. The pattern from the Gunbad-i Qabud incorporates yet a further degree of complexity through the introduction of a secondary design element that is arbitrarily added into the pattern matrix [Fig. 67]. This is the most elaborate example of a Seljuk additive pattern, and the dual-level quality of this design can be regarded as an aesthetic precursor of the recursive geometric patterns that were developed in the same region some 250 years later.

Among the earliest examples of patterns created from the *fivefold system* that repeat upon the more acute rhombic grid of 36° and 144° angles [Fig. 5b] is a remarkable *two-point* design from the late Abbasid main entry portal of the Mustansiriyah *madrasa* in Baghdad (1227-34) [Fig. 243b]. This was built after the collapse of the Seljuk Empire, but during the period when the Seljuk artistic heritage was still influential on the ornamental arts of Baghdad. As with other late Abbasid geometric designs that survived the Mongol destruction of Baghdad in 1258, this *two-point* pattern is highly innovative. This is one of the earliest designs to employ truncated decagons within its underlying generative tessellation. What is more, the angles of the applied pattern lines to each of the two points of the underlying polygonal edges have 54° angles of declination rather than the 72° or 36° that are standard among *two-point* patterns created from the *fivefold system*.



**Photograph 22** A Seljuk example of the classic *two-point* pattern with ten-pointed stars created from the *fivefold system* in the arched tympanum of the Gunbad-i ʻAlaviyan in Hamadan, Iran (© Daniel C. Waugh)

Designs created from the *fivefold system* that employ rectangular repeat units also appear to have been a Seljuk innovation. The *median* field pattern from the Khwaja Atabek mausoleum in Kerman (1100-1150) [Fig. 211] is interesting not just for its early date, but also for its unusual geometry. The inventive arrangement of the polygons that comprise the underlying tessellation employs just one module from the *fivefold system*: the 1/10 decagonal triangle. By placing two of these triangles edge to edge along their long edges, and applying the 72° crossing pattern lines to the short edges (as per convention), the pattern lines allow for the creation of a distinctive trefoil device within the two adjacent triangles [Fig. 188]. Another Seljuk example with rectangular repeat units is an *obtuse* border design that frames the *pishtaq* of the Seh Gunbad in Orumiyeh, Iran (1180). This places ten-pointed stars at the vertices of each rectangular repeat unit. This design is similar to a later Seljuk Sultanate of Rum example from the Sirçali *madrasa* in Konya (1242-45) [Fig. 247].

In addition to the above-cited *obtuse* pattern from one of the small blind arches in the northeast dome chamber of the Friday Mosque at Isfahan, there is also a very interesting *acute* pattern created from the *fivefold system* in another of the set of arches that surround the dome [Fig. 261b] [Photograph 25]. This design is remarkable in that it is the earliest example of a hybrid design known to this tradition. Of the three varieties of repetitive cell that comprise this design, the most visually apparent is the large central pentagon, the base of which rests upon the horizontal spring line of the arch. Attached to the four exposed edges of this pentagon are rhombi with 72° and 108° included angles. It is noteworthy that the pattern contained within these rhombic regions is the classic *acute* design, and the occurrence of this rhombic motif is the earliest known representation of this classic *acute* pattern, albeit not as a continuous surface coverage in its own right. The pattern within the large central pentagon is noteworthy on two counts: it is the earliest example of a fivefold design with rotation symmetry, and it is the earliest fivefold pattern to employ the motif of a central pentagon surrounded by 5 nine-sided flattened five-pointed star motifs that are derived from the five underlying irregular pentagons [Fig. 261a]. The use of two or more otherwise

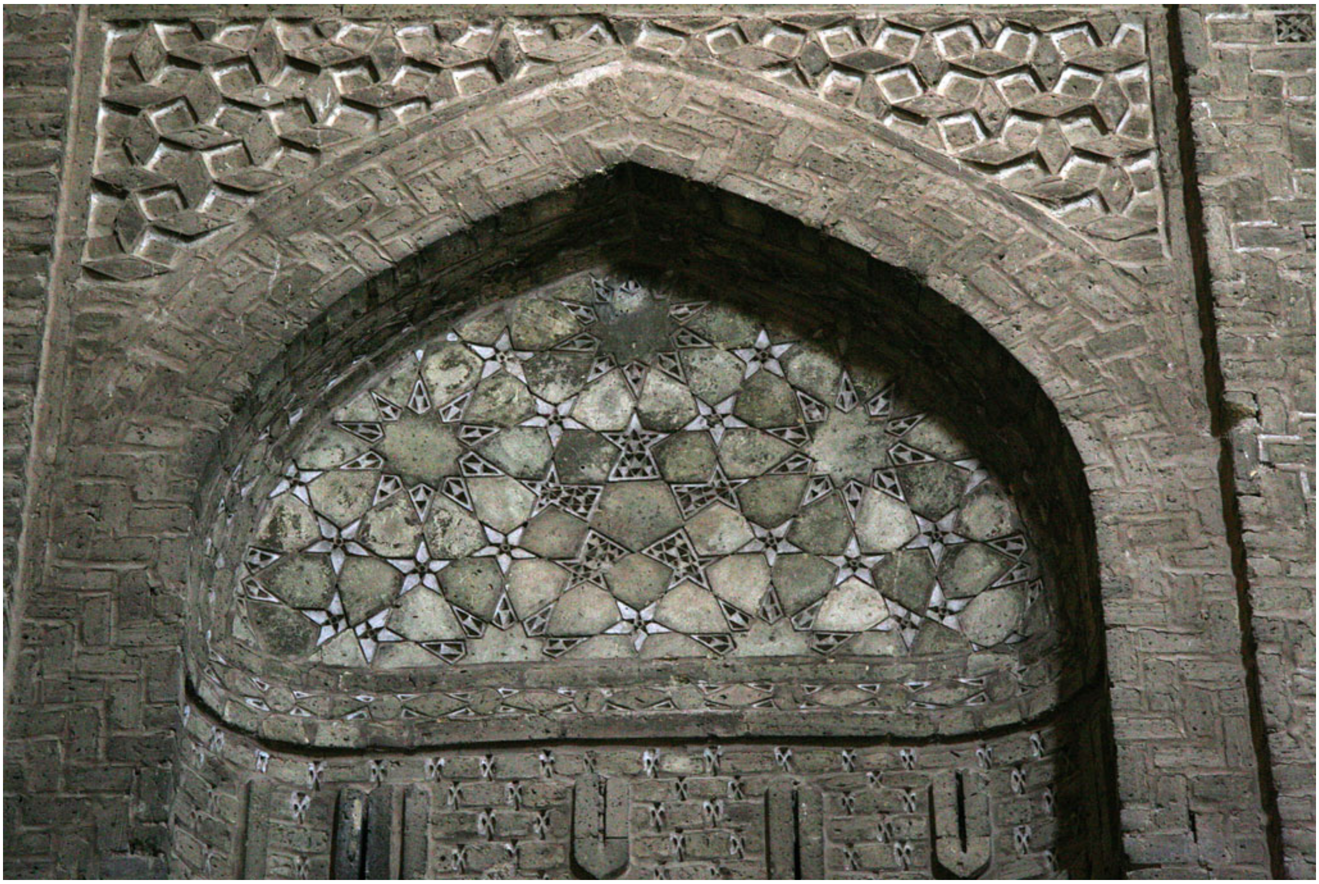
**Photograph 23** Seljuk unglazed brick and terra-cotta geometric ornament from the exterior façade of the Friday Mosque at Gonabad, Iran (photograph by Farshid Emami; courtesy of the Aga Khan Documentation Center at MIT)

independent repetitive cells to create a hybrid geometric design with greater complexity was used in several Anatolian locations during the Seljuk Sultanate of Rum [Figs. 262–265]. Marinid artists in Morocco employed this practice at a later date, and a small number of examples were also produced by both Mamluk artists in Egypt and Mughal artists in India. However, this design from the northeast dome chamber of the Friday Mosque in Isfahan appears to be the first historical hybrid design that employs more than a single repetitive cell within a single construction. The sophistication of this hybrid design presupposes an earlier origin of the *fivefold system* than the 1088-89 date of the northeast dome chamber in Isfahan; and one can assume that prior to the inventive discovery that otherwise distinct repeat units were able to work together to produce a more complex geometric design, artists would have already been familiar with the independent application of these individual repeat units for standard surface coverage. It is important to note that in analyzing this hybrid design a certain amount of conjecture has been used to fill the two-dimensional plane beyond the obvious central pentagon and adjacent rhombic cells. The artist who created this remarkable design may well have used a different combination of repetitive cells in the peripheral regions that extend beyond the central pentagon and contiguous rhombi. Indeed, the artist did not have to work with a continuous two-dimensional coverage at all, and may have just worked with the three rhombic cells on each side of the central pentagonal cell. This would have been enough to complete the design. It is interesting to consider that one way or another, an artist clever enough to have developed the use of the pentagon and rhombi with 72° and 108° included angles used in this design would have likely also discovered the need of the further rhombus with 36° and 144° included angles for full two-dimensional coverage. These more acute rhombi are implicit within this construction (for example, the upper point of the ten-pointed star at the apex of the arch), and were the artist who devised this design aware of the more acute rhombus, this individual may have been the first to discover the contiguous tiling potential of these two "Penrose rhombi." Although these rhombi have the ability for non-periodic application, or even aperiodic tiling with Penrose's matching rules [Fig. 480], the historical examples of Islamic geometric designs are invariably periodic with translation symmetry,

**Photograph 24** A Seljuk dual-level design from the façade of the Gunbad-i Qabud in Maragha, Iran, wherein the primary pattern is created from the *fivefold system* (© Richard Mortel)

and there is no evidence that Muslim artists were aware of the non-periodic potential of the design methodology they employed. The application of repetitive pentagonal and rhombic cells in the design from the northeast dome chamber has bilateral symmetry that reflects upon the vertical line that bisects the arch. The applied repetitive cells do not fall into a recognizable periodic structure. This is due to the fact that there is too little information within the arch to determine whether there is a larger meta repeat that is unseen. As said, the artist may have just added rhombic cells to the central pentagonal cell until the arched region was covered. If the latter, which would seem likely, the three varieties of repetitive cell within this example could be extended outward from the line of symmetry to produce either a periodic repeat with translation symmetry or a non-periodic structure without translation symmetry. However, considering the very limited cellular exposition in this design, the question of whether this design is one or the other is essentially moot.

Still greater evidence for the significance of the northeast dome in the Friday Mosque at Isfahan to the history of Islamic geometric art is once again found in the multiple blind arches that surround the cupola. Like the fivefold examples cited above, one of these arches contains the earliest example of a sevenfold pattern known to this ornamental tradition [Fig. 279] [Photograph 26]. Unlike the nonsystematic Ghaznavid sevenfold designs from the minaret of Mas'ud III in Ghazni (1099-1115), this Seljuk pattern is systematic, which is to say that the underlying generative polygonal modules that make up the particular tessellation are part of a limited set of sevenfold elements that tessellate in innumerable ways [Fig. 271]. This underlying generative tessellation is made up of just two of these polygonal modules, both being irregular hexagons of differing proportions. At this early stage of development, it is impossible to know to what extent the artist was aware of these two generative hexagons as systematic modules with greater

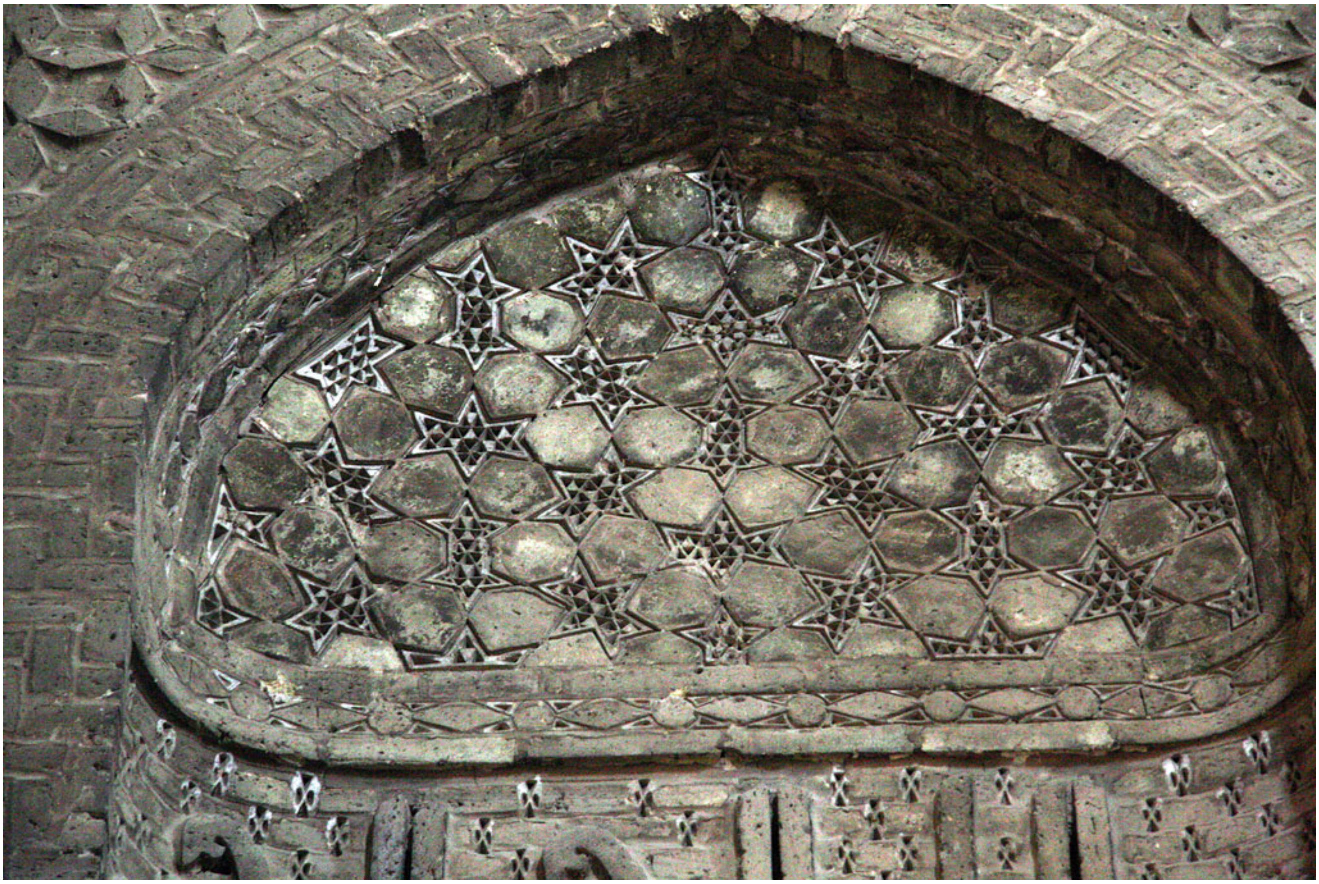


**Photograph 25** A Seljuk hybrid *acute* design with ten-pointed stars created from the *fivefold system* in the northeast dome chamber of the Friday Mosque at Isfahan (© Tom Goris)

tessellating potential. However, in light of the fact that the two fivefold examples from this same chamber are most distinctly systematic, it can be assumed that the same artist would have also known the systematic potential of the sevenfold polygonal modules. The example from the northeast dome chamber is an *acute* pattern with crossing pattern lines of 51.42857...°. This angular opening is determined from the inherent geometry of the heptagon. The absence of regular polygons, such as heptagons or tetradecagons (14 sides), within the underlying tessellation, and the concomitant absence of star forms with matching radial symmetry within the generated pattern, places this example into the *field pattern* category. The only other historical example of this sevenfold pattern is, significantly, an illustration from the anonymous Persian treatise *On Similar and Complementary Interlocking Figures* at the Bibliothèque Nationale de France in Paris.[56] This illustrated example, and its accompanying step-by-step instructions, is all the more interesting in that the underlying generative polygonal tessellation is visually represented and textually described. Such depiction of the generative schema is extremely unusual, and this illustration is hence one of the rare examples of a primary source for the historicity of the polygonal technique.

In addition to creating designs from underlying polygonal tessellations that were systematic, Seljuk artists also derived patterns from tessellations that were nonsystematic. Among the earliest Seljuk examples of nonsystematic pattern making is also from one of the blind arches that surround the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Photograph 27]. The underlying generative tessellation for this design places edge-to-edge regular pentagons upon each triangular edge of the isometric grid [Fig. 309a]. This creates two interstice elements that are specific to this pentagonal arrangement: a six-pointed star at the vertices of the isometric grid, and an irregular ditrigon at the centers of each triangular repeat unit. The *acute* design that was extracted from this underlying tessellation is very successful, with incorporated regular heptagons within the

[56] MS Persan 169, fol. 192a.



**Photograph 26** A Seljuk *acute* pattern created with underlying modules included in the *sevenfold system* from the northeast dome chamber of the Friday Mosque at Isfahan (© Tom Goris)

pattern matrix. Of particular interest is the pattern treatment within the central ditrigonal element. This motif—with slightly different proportions—became a relatively common feature within the *system of regular polygons* [Figs. 117–120]. Zangid artists used a variant of this nonsystematic design at the Nur al-Din Bimaristan in Damascus (1154) [Fig. 309c], but what is especially interesting is that the anonymous Persian treatise *On Similar and Complementary Interlocking Figures* also contains an illustration of a close variation of the nonsystematic design from the northeast dome chamber[57] [Fig. 309b]. The occurrence of these two examples in both locations provides further evidence of a likely direct association between this manuscript and the architectural ornament of this portion of the Friday Mosque in Isfahan.

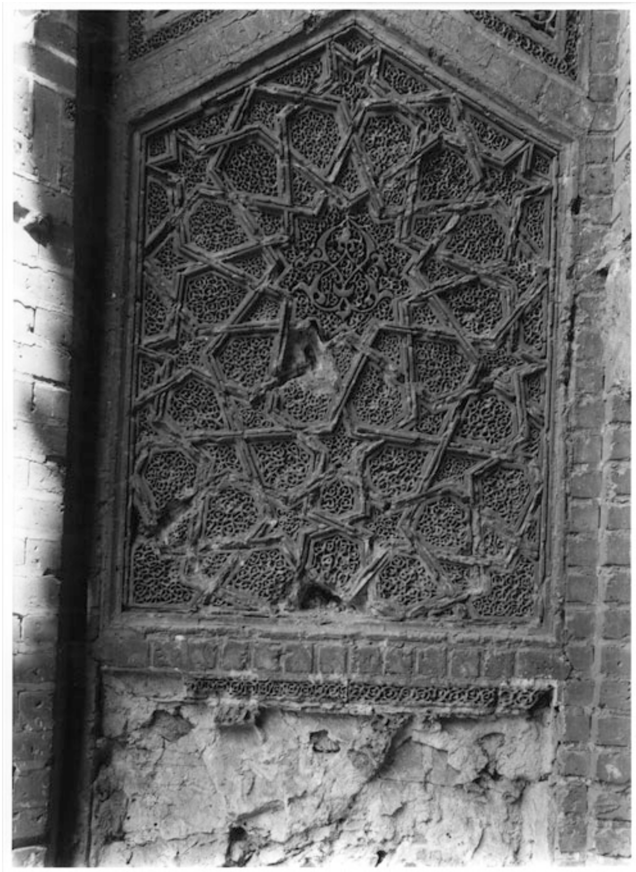
Among the most common nonsystematic patterns are those with just 12-pointed stars as the higher order star form. As with designs with 12-pointed stars created from the *system of regular polygons*, this variety of nonsystematic pattern will invariably place the 12-pointed stars upon the vertices of the repetitive grid. Being that 12 is divisible by 6, 4, and 3, nonsystematic designs with 12-pointed stars can repeat on the regular hexagonal grid, the orthogonal grid, and the isometric grid. Seljuk locations of fourfold nonsystematic patterns with 12-pointed stars include the arched tympanum over an entry gate near the northeastern *iwan* at the Friday Mosque at Isfahan[58] (after 1121-22) [Fig. 335a]; the minaret of the Great Mosque of Siirt, Turkey (1129) [Fig. 335b]; and a pair of bronze doors from the Seljuk *atabeg* of Cizre, Turkey (thirteenth century) [Fig. 337]. An Artuqid example, with strong Seljuk influences, is found in the carved stucco back wall of the *mihrab* niche of the Great Mosque of Silvan, Turkey (1152-57) [Fig. 336a]. Two Seljuk-influenced examples of nonsystematic threefold patterns with just 12-pointed stars located at each vertex of the isometric grid are found on the Mengujekid *minbar* at the Great Mosque of Divrigi in Turkey (1228-29). The *acute* design within the triangular side panel employs an

[57] MS Persan 169, fol. 193a.

[58] Ettinghausen, Grabar and Jenkins-Madina (2001), 141, pl. 215.

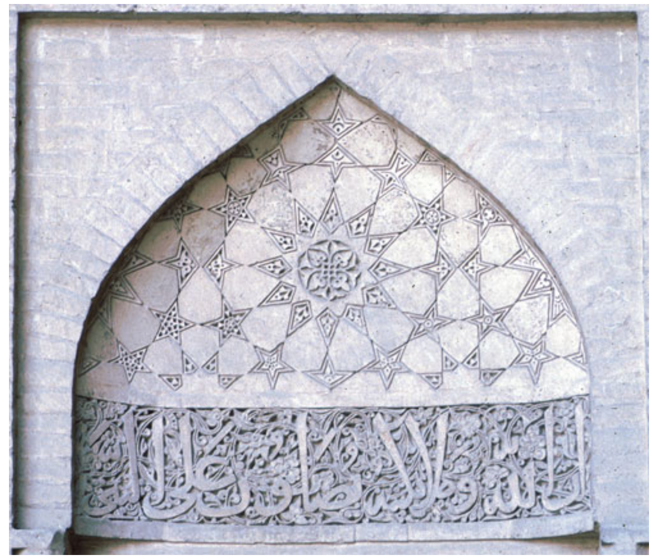
**Photograph 27** A Seljuk nonsystematic threefold *acute* pattern with six-pointed stars and heptagons in the northeast dome chamber of the Friday Mosque at Isfahan (© Tom Goris)

underlying tessellation that separates the underlying dodecagons with edge-to-edge pentagons [Fig. 300a *acute*]. Immediately adjacent to this is a vertical panel with an *acute* design created from a modified version of this same underlying tessellation wherein the pentagons are truncated into trapezoids [Fig. 320]. The use of two designs with such closely related methodological origin would appear to have been a willful decision on the part of the artist. Other examples of Seljuk-influenced threefold patterns that place 12-pointed stars at the vertices of the isometric grid include the carved stucco ornament of the Abbasid Palace of the Qal'a in Baghdad (c. 1220) [Fig. 300b *acute*] [Photograph 28], and a carved stucco wall panel at the Mustansiriyah in Baghdad (1227-34) [Fig. 300a *acute*]. These two late Abbasid buildings were constructed only decades after the overthrow of Seljuk dominion over Baghdad, and have strong stylistic affiliations with Seljuk ornament. The Friday Mosque at Barsian (1105) is remarkable in a number of respects. The dome and supporting *muqarnas* squinches in this Seljuk building are remarkably similar to that of the southwest dome chamber of the Friday Mosque in Isfahan (1086). These two buildings were constructed within 20 years of one another and are in relatively close proximity. Like the Seljuk ornament in Isfahan, the Friday Mosque in Barsian also contains several interesting nonsystematic geometric designs. Two of the arched *muqarnas* faces in the *mihrab* of this mosque are decorated with nonsystematic orthogonal patterns with eight-pointed stars. One of these places eight-pointed stars on the vertices of the square repeat, and octagons at the center of the repeat [Fig. 331a]. This is a standard feature of patterns created from the *four-fold system B*. However, the three varieties of irregular pentagon and the irregular triangles are nonsystematic, and the *acute* pattern that the underlying polygonal tessellation produces is unusual. Another one of the *muqarnas* arch faces in the *mihrab* of the Friday Mosque at Barsian is decorated with a nonsystematic orthogonal design comprised of five-, six-, seven-, and eight-pointed stars [Fig. 332a]. Of these, only the six- and eight-pointed stars have regular rotational symmetry. This same design was used in several other locations that were strongly influenced by the Seljuks, including the Danishmend portal of the Great Mosque of



**Photograph 28** A late Abbasid nonsystematic threefold *acute* pattern with 12-pointed stars from the Abbasid Palace of the Qal'a in Baghdad (photograph by K. A. C. Creswell; © Ashmolean Museum, University of Oxford)

Niksar, Turkey[59] (1145); the Ildegizid façade of the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan (1186); and the Qara Qoyunlu portal of the Great Mosque in Van, Turkey (1389-1400). These later examples only differ in their widened interweaving line treatment [Fig. 332b, c]. The same underlying generative tessellation that produced both of these examples was used by Mamluk artists some 200 years later to produce a more complex design at the Amir Sarghitmish *madrasa* in Cairo (1356) [Fig. 332e]. In addition to the examples from Barsian, Nakhichevan, and Niksar, Seljuk artists produced several additional patterns with five-, six-, seven-, and eight-pointed stars; and this seems to have been a somewhat popular geometric theme. The design in the *mihrab* arch spandrel at the Gar mosque (1121-22) in the outskirts of Isfahan employs such a design, although the amount of geometric information contained within each triangular panel is insufficient to definitively determine either the repeat pattern or the underlying



**Photograph 29** A Seljuk nonsystematic fourfold *acute* pattern with 8- and 12-pointed stars from the Friday mosque at Isfahan (© David Wade)

polygonal structure. Nonetheless, the five-, six-, seven-, and eight-pointed star structure is apparent within the limited context. Similarly, among the ornamented arched *muqarnas* faces in the exterior façade of the Gunbad-i Qabud in Maragha (1196-97) is the repetitive use of a design with this same combination of star forms. And once again, such a design was used at the Izzeddin Kaykavus hospital in Sivas, Turkey (1217-18).[60]

The southern interior corner of the southeastern *iwan* of the Friday Mosque at Isfahan includes a small blind arch decorated with a Seljuk example of a carved stucco nonsystematic compound pattern comprised of 8- and 12-pointed stars [Photograph 29]. This mosque went through multiple restorations and additions by subsequent dynasties, and the dating of specific unattributed features is frequently problematic.[61] That said, this example is stylistically similar to the Seljuk geometric ornament within the nearby northeast dome chamber. This design can be constructed from either of the two underlying polygonal tessellations: one with edge-to-edge dodecagons and octagons with concave hexagonal interstice regions [Fig. 379d], and the other with dodecagons and octagons separated by a matrix of irregular pentagons and barrel hexagons [Fig. 379f]. A later Seljuk example of this same pattern was used in an exterior border that runs vertically along the sides of the north *iwan* of the Friday Mosque at Gonabad (1212) [Fig. 379e] [Photograph 23]. Indeed, multiple examples of this same *acute* design

[59] Schneider (1980), pattern no. 352.

[60] Schneider (1980), pattern no. 351.

[61] Ettinghausen, Grabar and Jenkins-Madina (2001), 140–143.

were used widely by succeeding Muslim cultures [Photograph 46]. A slightly later example of this same *acute* pattern was used in the carved stucco ornament at the Abbasid Palace of the Qal'a in Baghdad (c. 1220). A remarkable example of an orthogonal pattern with compound local symmetries is found on one of the small arched surfaces of the *muqarnas* hood in the *mihrab* of the Friday Mosque at Barsian (1105). This unglazed ceramic mosaic ornament is an *acute* design that combines 12- and 16-pointed stars [Fig. 392b]. This compound orthogonal design has considerably greater complexity than other contemporaneous orthogonal designs.

The Seljuks excelled in creating complex nonsystematic geometric patterns from a relatively early date. At its most sophisticated, this variety of geometric pattern will frequently include seemingly irreconcilable combinations of star forms, and will frequently require repeat units other than the standard triangle, square, or regular hexagons. The tradition of especially complex patterns with compound local symmetries reached full maturity under the auspices of the Seljuk Sultanate of Rum in Anatolia and the Mamluks in Egypt during the thirteenth and fourteenth centuries; but the antecedents and earliest examples of this variety of design were established during the twelfth century by the Great Seljuks and their *atabeg* subordinates. In seeking an understanding of the historical development of particularly complex geometric patterns with multiple regions of differentiated local symmetry, it is important to take into account the tremendous loss of early monuments in Transoxiana, Khurasan, Persia, and Iraq through natural disasters, neglect, and especially the Mongol destruction during the thirteenth century. As per the previous cited example, the Friday Mosque at Barsian (1105) is of particular significance to the early history of this variety of geometric pattern. The *mihrab* of this mosque is framed by a very interesting nonsystematic design comprised of seven- and nine-pointed stars that repeats upon an elongated hexagonal grid [Fig. 429]. This is an early example of a geometric design that fills the two-dimensional plane by virtue of a geometric ploy whereby the numeric quality of the alternating star forms is one numerical step above and below the number of stellate points of a more common and convenient design with singular repeating stellations, such as 6-, 8-, 10-, or 12-pointed stars. For example, the fact that six-pointed stars will conveniently repeat is an indication that a compound pattern can be created that employs both five- and seven-pointed stars. The pattern from the *mihrab* of the Friday Mosque at Barsian applies this *principle of adjacent numbers* to the repetitive convenience of the octagon, indicating the potential for a successful, if considerably less geometrically convenient, repetitive pattern with seven- and nine-pointed stars. It is impossible to know whether Muslim artists of the past were aware of this as a design principle per se, or whether their creation of such patterns comprised of 5- and 7-, 7- and 9-, 9- and 11-, or 11- and 13-pointed stars was purely serendipitous. In addition to the orthogonal design with 12- and 16-pointed stars cited above, other examples of particularly complex geometric patterns are included among the arches of the *muqarnas* hood in the *mihrab* from the Friday Mosque at Barsian. These include a pattern comprised of 13-pointed stars, and another comprised of 11- and 12-pointed stars. The limited amount of geometric information contained in each of these two examples is insufficient to conclusively determine either the repetitive structure or the complete underlying generative tessellation, and it is possible that the artist distributed 11-, 12-, and 13-pointed stars into these two small arched regions without their being part of of an actual repetitive structure.

Other Seljuk examples of nonsystematic compound patterns include a design with five-, six-, and seven-pointed stars in the arch spandrels at the top of each exterior wall of the decagonal façade on the Gunbad-i Qubad in Maragha, Iran (1196-97), and an adjacent pattern with eight- and nine-pointed stars that frames the *muqarnas* arch at the top of each exterior wall of the façade of the same building. It is interesting to note what would appear to be the deliberate decision by the artist to juxtapose the pattern with five-, six-, and seven-pointed stars with a pattern comprised of sequenced eight- and nine-pointed stars. As with the exceptionally large repeat unit of the fivefold *obtuse* design that surrounds this tomb tower, the use of two adjacent complex designs that have continuous sequenced numeric qualities is very unusual, and emphasizes the unique character of this building.

Among the greatest achievements of Seljuk geometric artists is the pioneering application of geometric patterns onto the surfaces of domes. Subsequent Muslim dynasties followed in this design convention, and exceptional examples with greater complexity, were produced by the Zangids and Ayyubids in Syria, the Nasrid and Christian *Mudéjar* artists in Spain, the Mamluks in Egypt, the Muzaffarids and Timurids in Persia and Central Asia, and the Mughals in India. By comparison, the early work of the Seljuks appears simplistic. Indeed, the elaborate ribbed vault of the Sultan Sanjar mausoleum in Merv, Turkmenistan (1157), for all its boldness and beauty, does not exhibit particular geometric complexity. This design employs an eight-pointed star at the apex, and the design unfolds upon an eightfold radial division of the domical surface. The earlier Seljuk dome of the Friday Mosque at Golpayegan (1105-18) similarly places an eight-pointed star at the apex, and employs eightfold radial segmentation of the surface. Surrounding the raised brick central eight-pointed star are 8 seven-pointed stars, followed downward by 8 five-pointed stars, and culminating at the periphery with a ring of 8 eight-pointed stars divided in half [Fig. 491]. This stellar matrix,

**Photograph 30** A Seljuk domical geometric design governed by dodecahedral symmetry in the northeast dome chamber of the Friday Mosque at Isfahan (© Tom Goris)

while still rather simple when compared to the non-Euclidean work of subsequent generations of Muslim artists, has all the visual characteristics of a pattern that was produced using the methodology of the polygonal technique. The dome at Golpayegan is the earliest extant example in Islamic architecture of the application of a geometric design to the surface of a dome using radial gore segments as the repetitive device. However, Seljuk artists were also pioneers of the other principal method of applying geometric patterns onto domical surfaces: the use of polyhedral geometry as the repetitive strategy for controlled domical surface coverage. The earliest use of polyhedra for creating a non-Euclidean geometric design is from the northeast dome of the Friday Mosque at Isfahan (1088-89) [Photograph 30].[62] This dome is remarkable on several counts, not the least of which is the fact that the magnificent dome is decorated with a *two-point* pattern derived from the underlying geometry of the dodecahedron [Fig. 496]. The dodecahedron is comprised of 12 pentagonal faces, and the application of *two-point* pattern lines onto each underlying domical pentagon creates the distinctive and unusual fivefold symmetry of this dome. The pentagonal faces of the dodecahedron are spherically projected onto the curved surface of the dome, and it is important to point out that the use of the dodecahedron would ordinarily produce a hemispherical dome. However, the curvature of the northeast dome rises to an apex. While the applied *two-point* pattern is unquestionably derived from the dodecahedron, this otherwise spherical surface has been modified to emphasize the characteristic ascendancy of the traditional Persian pointed dome. The use of underlying generative pentagons projected to a domical surface aligns this example with patterns produced from the *fivefold system*: the difference being that the two-dimensional plane requires at least one other module from the *fivefold system* to accompany the generative pentagons, while the dodecahedron is a spherical tessellation of regular pentagons alone. Along with the two previously discussed fivefold patterns within the small blind arches immediately beneath this dome, the ornament of the northeast dome chamber in the Friday Mosque at Isfahan has the distinction of having the earliest extant examples of Islamic geometric designs with fivefold symmetry: predating both the Ghaznavid nonsystematic fivefold patterns on the minaret of Mas'ud III

[62] Bonner (2016).

[Fig. 206] and the Seljuk example from the interior wall of the Friday Mosque at Golpayegan [Fig. 229b] by 10–20 years. It has been postulated that the great Persian mathematician and poet, ʻUmar Khayyam, may have designed the northeast dome of the Friday Mosque at Isfahan.[63] Certainly he was living in Isfahan at the time of this dome's construction, and enjoyed the scientific patronage of Taj al-Mulk who commissioned the dome. As a prominent mathematician of his time, ʻUmar Khayyam would have been very familiar with polyhedral geometry and spherical projection: a requisite of the designer of this important monument.[64] If true, and especially in light of the relationship between the *two-point* geometric pattern on the dome and those employed within the eight recessed arches of the domed chamber, ʻUmar Khayyam may have been highly significant not just as a mathematician and poet, but also to the historical development of the polygonal technique: the design methodology most responsible for the mature style of Islamic geometric design. Such a confluence of mathematics, poetry, and geometric art is a delight to the imagination.

The fivefold domical geometric design in the northeast dome chamber of the Friday Mosque at Isfahan, together with the series of geometric patterns placed within the eight recessed arches, represents a remarkable advance in the historical development of Islamic geometric design. The many "first occurrences" present in this chamber opened the door to the fully mature geometric design practices that soon followed. As such, the importance of these patterns to the history of Islamic geometric art is paramount, and firmly establishes Seljuk artists as fundamental innovators in the furtherance of this tradition. The design innovations that were first introduced during the construction of this building include the first use of underlying ditrigonal modules within the *system of regular polygons* [Fig. 118a] [Photograph 19]; the earliest occurrence of the classic fivefold *obtuse* design [Fig. 229a] [Photograph 21]; the earliest fivefold *acute* design, in this case a hybrid design with multiple repetitive cells [Fig. 261] [Photograph 25]; the earliest fivefold *two-point* design, in this case on the dome [Photograph 30] [Fig. 496]; the first pattern with sevenfold symmetry created from the *sevenfold system* [Fig. 279] [Photograph 26]; the earliest example of a nonsystematic design [Fig. 309a] [Photograph 27]; and the first occurrence of a domical geometric pattern that uses a polyhedron as its repetitive schema [Fig. 496] [Photograph 30]. What is more, the 3 fivefold designs and the 1 sevenfold pattern are the earliest sophisticated examples of these two types of symmetry known to have been produced by humankind the world over. It is doubtful that any other single room, or even individual building within the totality of Islamic architecture, had such a profound significance to the historical development of Islamic geometric art.[65]

## 1.13 Ghurids (1148-1215)

Following their defeat by the Seljuks at the Battle of Dandanaqan (1040) and the loss of their vast western territories, the Ghaznavids were forced to negotiate a peace treaty with the Seljuks that brought relative stability to Khurasan for approximately a hundred years. In 1150 Ghazna fell to the Ghurid forces of Ala'uddin Hussain. Within two decades the Ghaznavids were driven from their homelands in Khurasan to their eastern territories in Sindh, and in 1187 the Ghurids further defeated the Ghaznavids in Lahore, bringing an end to the Ghaznavid Empire. The Ghurids are thought to have been Tadjiks of eastern Iranian origin that migrated to their homeland of Ghur, in central Afghanistan, at an undetermined time. Ghur is mountainous and provided an ideal defensive location against the largely unsuccessful attempts to conquer this region by the Ghaznavids and Seljuks. Following the final defeat of the Ghaznavids at Lahore, the Ghurids expanded their empire to include most of modern-day Afghanistan and Pakistan, as well as much of northern India. Their first capital was Firuzkuh (present-day Jam) but as they spread eastward they also established capitals in Ghazna, Lahore, and eventually Delhi. The Ghurids were avowed Sunnis and recognized the religious authority of the Abbasid Caliph in Baghdad. As with the Ghaznavids, Persian cultural affinities flourished under Ghurid rule, and great emphasis was placed upon poetry, literature, and arts. Ghurid control over their Afghan territories came to an end in 1215 following their defeat by the Khwarizmshahs, but control over their eastern territories in the Indian subcontinent was maintained through the assumption to power of the Mamluk Sultanate of Delhi.

Like the Seljuks, the Ghurids fully embraced the dynamic architectural practices of the Ghaznavids. Like other Muslim dynasties, the Ghurids approached the design of their architectural monuments, in part, as a way of commemorating their ascendancy as a sovereign force, as well as glorifying Islam within their territories with large non-Muslim populations, such as the Indus Valley and northern India. Significant Ghurid architectural monuments in Khurasan

[63] –Grabar (1990), 85, note 5.
–Özdural (1998), 699–715.
–Hogendijk (2012), 37–43.

[64] The works of the mathematician and astronomer, Abu al-Wafa Buzjani (940–998), would have been familiar to ʻUmar Khayyam, and of especial relevance to this discussion would have been his work on right-angled spherical triangles and spherical trigonometry.

[65] Bonner (2016).

**Photograph 31** A Ghurid *two-point* pattern originally located in Lashkar-i Bazar, Afghanistan, that is easily created from the *system of regular polygons* (© Thalia Kennedy)

include portions of Lashkar-i Bazar, the arch at Bust, the two mausolea at Chisht, the Shah-i Mashhad in Gargistan, the Friday Mosque at Herat, and the minaret of Jam. This latter building is located deep in the Ghur Mountains of central Afghanistan at the confluence of the Hari and Jam rivers. It stands 65 m in height and is the second tallest historical minaret in the Islamic world.

The surviving Ghurid monuments in Khurasan are relatively few, but the sophistication of the ornamental design and the quality of execution are equal to the finest work of the Ghaznavids. As with the Seljuks, the Ghurids added to the established ornamental practices of the Ghaznavids by introducing turquoise glazed faience into their exterior façades: enlivening key ornamental components, such as calligraphy, with vivid color in an otherwise monochrome aesthetic. This innovative approach to architectural ornament was no less focused upon the further development of geometric design, and the use of the *system of regular polygons* continued as a primary methodology employed in the creation of geometric patterns. The carved stucco ornament from a Ghurid *mihrab* at Lashkar-i Bazar[66] (after 1149) employs a threefold geometric design that is easily constructed from an underlying 3.6.3.6 tessellation of triangles and hexagons [Fig. 99d] [Photograph 31]. This is a *two-point* pattern that locates the pattern lines upon two points on each underlying polygonal edge rather than the more common single midpoint location. As demonstrated in this example, the occurrence of multiple closed-loop elements within the pattern matrix is a typical and distinctive feature of the *two-point* family of geometric patterns. This same pattern was used in several locations historically, including an earlier panel in a wooden *maqsura* from the mausoleum of the Seljuk *atabeg* Sultan Duqaq in Damascus (1095-1104). The Ghurid arch soffit of an *iwan* at the Friday

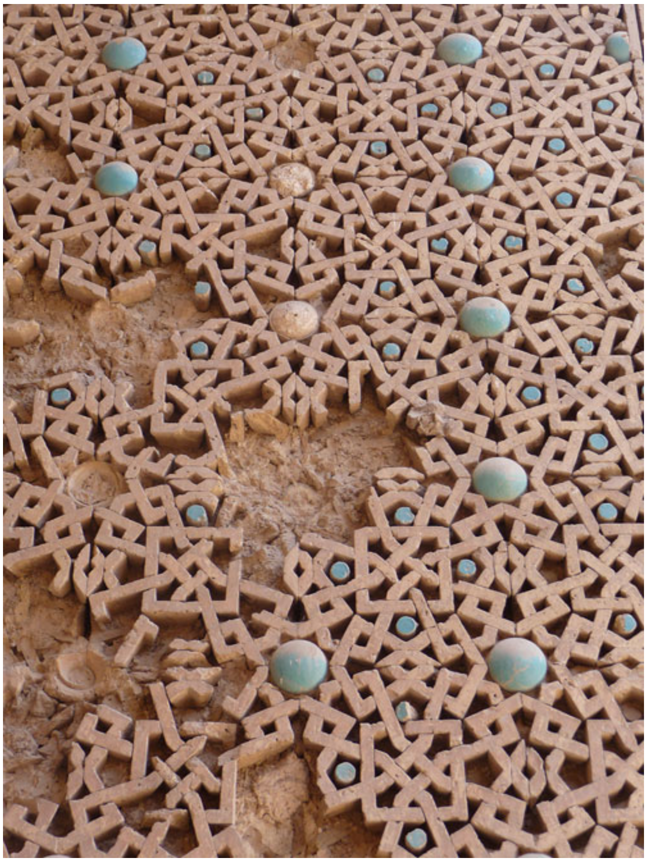
[66] In the collection of the National Museum of Afghanistan in Kabul, Afghanistan.

Mosque in Herat, Afghanistan (1200), employs a raised brick *two-point* pattern of superimposed hexagons that is easily constructed from the $6^3$ tessellation of regular hexagons [Fig. 96d]. The earliest known use of this design was by the Umayyads of Spain in a tenth-century marble window grille. Like much of the initial Ghurid ornament of this monument, this pattern was covered with later Timurid cut-tile mosaic, but eventually revealed through degradation of the Timurid work. The arch spandrel immediately above this *iwan* employs another threefold geometric pattern that can likewise be constructed from an underlying $6^3$ tessellation of regular hexagons [Fig. 98a]. As with several earlier Ghaznavid and Seljuk examples, this pattern applies two varieties of pattern line into adjacent underlying hexagons. The primary underlying hexagons dictate the design by centrally placing six-pointed stars with 60° crossing pattern lines upon the midpoints of each primary hexagon's edges, while four of the edges of each secondary underlying hexagon also place 60° crossing pattern lines upon the midpoints, and a pattern line is drawn between two opposite vertices. This pattern closely resembles a later Ottoman design from the Great Mosque of Bursa (1396-1400) that also is created from alternating active and passive underlying hexagons [Fig. 98b]. The greater regularity in the size of the background elements of the Ottoman design produces a more successful design. The remote ruins of the Shah-i Mashhad in Gargistan, Afghanistan (1176), contain a profuse assortment of ornamental motifs, including knotted *Kufi* calligraphy, highly elaborate braided borders, simple floral designs, and an assortment of geometric patterns. These include several threefold patterns that can be constructed with the *system of regular polygons*. The ornament on this remote Ghurid *madrasa* is mostly of exquisitely executed raised brickwork and molded terra-cotta tiles and inserts. A threefold pattern on one of the remaining *iwans* can be easily produced from the 3.4.6.4 arrangement of triangles, squares, and hexagons as the underlying generative tessellation [Fig. 104c]. This design is reminiscent of the window grille from Córdoba in that both are made up solely of superimposed hexagons in rotation around a hexagonal nodal center. However, in the case of this pattern from Shah-i Mashhad, each of the superimposed hexagons is elongated rather than regular. The façade of the western mausoleum at Chisht, Afghanistan (1167), employs a threefold pattern that can also be made from the 3.4.6.4 underlying tessellation [Fig. 105a]. This raised brick pattern was used inside one of the large exterior blind arches at the side of the main portal. The parallel pattern lines that characterize this design emphasize both the hexagonal grid and its isometric dual. This pattern is simply derived by initially placing a hexagon within each underlying triangle. An identical Seljuk example of this same design was used in the Friday Mosque at Gonabad, Iran, some 50 years later (1212). The minaret of Jam (1174-75 or 1194-95) includes a variation of the 3.4.6.4 design from Chisht. The difference between these two Ghurid examples is that the pattern from Jam has an added hexagonal element at each of the vertices of the triangular repeat [Fig. 105c]. The addition of this hexagon is independent of the underlying tessellation and was a purely arbitrary decision on the part of the artist. This minaret also employs a threefold pattern with 12-pointed stars within one of the panels at the base of the structure that can be easily constructed with the 4.6.12 underlying tessellation of squares, hexagons, and dodecagons [Fig. 109f]. This example is largely characterized by large dodecagons within the pattern matrix. Another Ghurid decagonal pattern that was used more or less contemporaneously at both the minaret of Jam and the Shah-i Mashhad is a fourfold design that repeats on the orthogonal grid, with the underlying dodecagons placed at the vertices of the square repeat units [Fig. 120]. An unusual feature of the underlying tessellation that produces this design is the further infill of each dodecagon with four ditrigons and four triangles. This infill allows for the rather ingenious transformation of the finished pattern from what would have been 12-pointed stars at the vertices of the square repeat unit to regular octagons. This same distinctive design was used on the *minbar* of the Alaeddin mosque in Konya (1219-21) some 50 years later.

One of the Ghurid carved stucco panels from Lashkar-i Bazar[67] (after 1149) is comprised of a network of superimposed four-pointed stars. This is easily created from the $4.8^2$ underlying tessellation of squares and octagons [Fig. 128c]. Artists used this rather simple orthogonal design approximately 100 years later during the Seljuk Sultanate of Rum at the Karatay *madrasa* (1251-55). A raised brick design with eight-pointed stars from the façade of the western mausoleum at Chisht can be produced with several methodologies, including the polygonal technique, whereby the design is easily generated from the underlying $4.8^2$ tessellation of squares and octagons [Fig. 129c]. The Ghurids used this same design on the minaret of Jam (1174-75 or 1194-95), and the wide-ranging popularity of this design is attested to by its use in one of the marble window grilles at the Great Mosque of Córdoba (980-90), and the *minbar* of the al-Aqsa mosque in Jerusalem (1168). The minaret of Jam also employs an example of the classic star-and-cross design that is likewise easily created from the $4.8^2$ tessellation [Fig. 124b].

Multiple Ghurid examples of raised brick patterns that are easily created from the *fourfold system A* include a simple *median* border device on the façade of the western mausoleum at Chisht (1167), as well as at the Shah-i Mashhad in

[67] In the collection of the National Museum of Afghanistan in Kabul, Afghanistan. See Crane and Trousdale (1972), 215–226.



**Photograph 32** A Ghurid *two-point* pattern at the Friday Mosque at Herat, Afghanistan, created from the *fourfold system B* (© Thalia Kennedy)

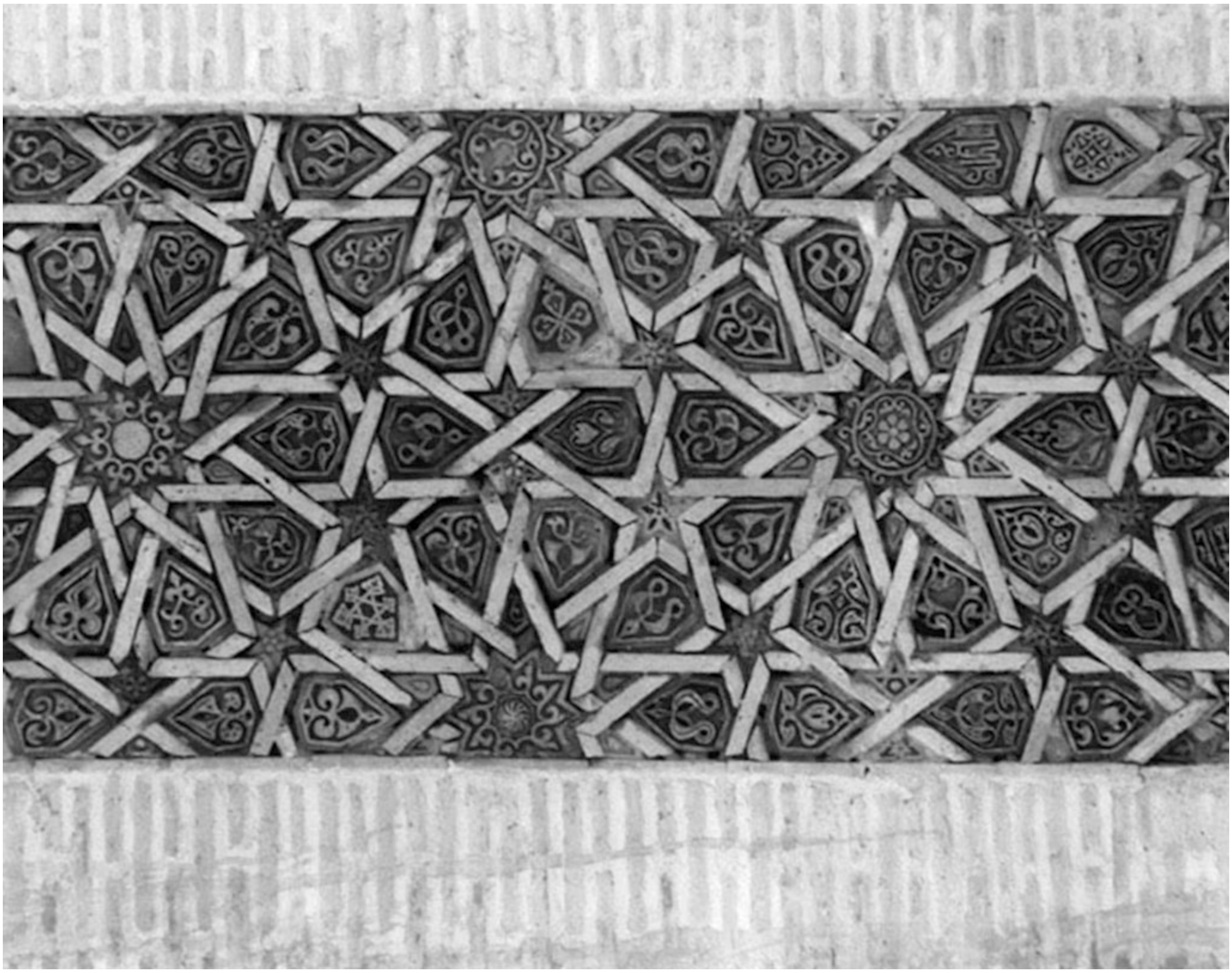
Gargistan, Afghanistan (1176) [Fig. 138c]. This is predated by an identical Seljuk example at the eastern tomb tower in Kharraqan by 100 years (1067). The minaret of Jam in central Afghanistan (1174-75 or 1194-95) has several panels with a *median* pattern that enjoyed ongoing use by many Muslim cultures [Fig. 145]. This design was also used previously by Seljuk artists at both the minaret of the Friday Mosque at Damghan, Iran (1080), and the minaret at Daulalabad, near Balkh, Afghanistan (1108-09) [Photograph 20]. This popular design is closely related to another Ghurid raised brick example from the eastern tomb tower at Chisht, Afghanistan (1197) [Fig. 143a]. This design from Chisht was used 18 years earlier by Saltukid artists at the Great Mosque of Erzurum, Turkey (1179).

An outstanding Ghurid example of a pattern created from the *fourfold system B* is found in the Friday Mosque at Herat, Afghanistan (1200) [Fig. 174b] [Photograph 32]. The specially cut raised brickwork is augmented with circular turquoise glazed plugs set within the backgrounds of the eight-pointed stars and pentagons. This is a *two-point* pattern that includes an arbitrary treatment of the central region of the repeat unit. This design can also be created from the simple $4.8^2$ underlying tessellation of squares and octagons [Fig. 128h]. However, the use of the underlying tessellation from the *fourfold system B* more specifically relates to the geometric composition of the design. The introduction of the arbitrary square element within the pattern matrix was a popular device in *two-point* patterns created from this system. This is associated with the cluster of five pentagons within the underlying generative tessellation. Examples of other patterns that exhibit this distinctive feature include the Qarakhanid southern anonymous mausoleum in Uzgen, Kyrgyzstan (1186) [Fig. 176c]; the Mamluk painted ceiling of the Sultan al-Mu'ayyad Shaykh complex in Cairo (1415-22) [Fig. 176b]; the Sidi Madyan mosque in Cairo (1465) [Fig. 176a]; Bimarhane hospital in Amasya, Turkey (1308-09) [Fig. 174c]; and the Aqbughawiyya *madrasa* in the al-Azhar mosque in Cairo (1340) [Fig. 174a].

The soffit of the Ghurid Arch at Bust, Afghanistan (1149), is beautifully decorated with a pattern that is easily made from the *fivefold system* [Fig. 226c] [Photograph 33]. This is a masterpiece of monochrome architectural ornament: both for its early innovative use of fivefold geometric design and for the precision of the specially cut raised brickwork and the refinement of the vegetal insert plugs that rest below the surface of the geometric pattern. The repeat unit for this geometric design is a rhombus with 72° and 108° angles. This remarkable design was produced at a time when patterns created from the *fivefold system* were just beginning to enter the lexicon of Islamic ornamental motifs, and is the earliest known example of the classic *acute* design that, over time, became ubiquitous to this tradition. The underlying tessellation for this *acute* pattern is comprised of just three polygonal modules: the decagon, pentagon, and barrel hexagon. This same underlying generative tessellation was responsible for the Seljuk *obtuse* pattern used some 60 years earlier in one of the blind arches in the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Fig. 229b] [Photograph 21], and the Seljuk *two-point* pattern in the tympanum over the entry of the Gunbad-i 'Alaviyan in Hamadan, Iran (late twelfth century) [Fig. 231d] [Photograph 22]. When considering the history of this classic *acute* pattern, it is highly significant that the rhombic elements with 72° and 108° included angles that are included in the repetitive make up the fivefold hybrid design in the northeast dome chamber in Isfahan are ornamented with the same *acute* pattern, albeit in association with the pattern lines placed within the other repetitive hybrid components [Fig. 261].

## 1.14 Ildegizids (1136-1225)

The Ildegizids of Azerbaijan came to power as Seljuk *atabegs* in 1136. They gained independence from the Seljuks in 1194, and at the height of their power the Ildegizids



**Photograph 33** A Ghurid example of the classic *acute* pattern created with the *fivefold system* from the soffit of the Ghurid Arch at Bust, Afghanistan (© Bernard O'Kane)

controlled the region stretching from Isfahan in the southeast to the borders of the Kingdom of Georgia to the northwest. The Khwarizmshahs overthrew them in 1225. As with the Seljuks, they were of Turkic origin with strong affiliations for Persian culture and language, and like their Seljuk suzerains, the Ildegizids were patrons of the geometric arts. Indeed, several outstanding examples of Islamic geometric art, created from the polygonal technique, were produced by this culture.

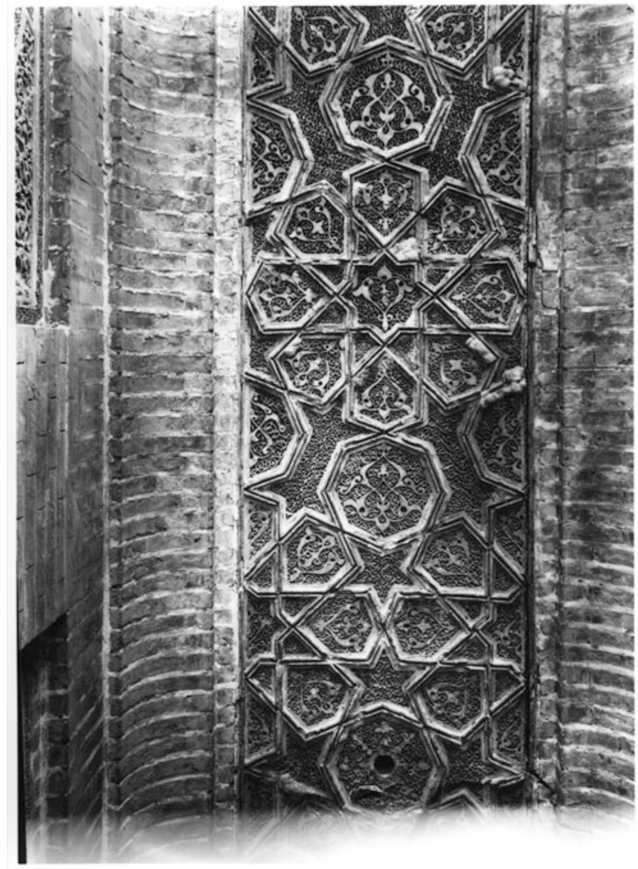
The use of the *system of regular polygons* figured into their geometric aesthetic. The façade of the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan (1186), includes two patterns made from this system. One of these is the frequently used pattern of superimposed interweaving hexagons easily derived from the 3.6.3.6 underlying tessellation, the earliest known example of which is one of the marble window grilles at the Great Mosque Córdoba [Fig. 96e]. This is located in the upper spandrels of the three-tiered *muqarnas* arch at the top of the Mu'mine Khatun mausoleum. The second example is also associated with the 3.6.3.6 tessellation, but with different pattern lines applied to alternating underlying hexagonal and triangular modules [Fig. 101d]. This is located above the portal, and the approximate proportions of this design can also be produced from the isometric grid [Fig. 73c]. Interestingly, this design from the Mu'mine Khatun mausoleum is very similar to the pattern over the portal at the nearby mausoleum of Yusuf ibn Kathir in Nakhichevan (1161-62) [Fig. 101c]. This variation was also used on a carved stone lintel above the Zangid portal of the Bimaristan Arghun at the citadel of Aleppo[68] [Photograph 36], and a Seljuk Sultanate of Rum stone relief at the Hatun Han near Pazar, Turkey (1238-39). The

[68] Terry Allen identifies the origin of the portal as predating the rest of the Bimaristan Arghun. See Allen (1999).

mausoleum of Yusuf ibn Kathir is a small Ildegizid tomb tower with an octagonal plan. The design in one of the exterior façades exhibits the generative 3.6.3.6 tessellation along with included octagons placed upon the vertices of the hexagons and triangles [Fig. 51c]. The harmonious placement of octagons within a threefold design is an example of an *imposed symmetry* design, and works by virtue of the two perpendicular lines of reflective symmetry at each 3.6.3.6 vertex. This design can also be created from an underlying 3.4.6.4 generative tessellation [Fig. 107a], in which case the 3.6.3.6 motif is an arbitrary inclusion. It is interesting to compare this with the geometric structure of the pattern in the adjacent façade. In contrast to the 3.6.3.6 design that places arbitrary fourfold elements (octagons) into the otherwise threefold structure of the design, the pattern inside the adjacent panel arbitrarily places a threefold motif of six-pointed stars and a surrounding hexagon into a fourfold orthogonal pattern matrix [Fig. 52b]. This is accomplished by placing the threefold elements upon the midpoints of each edge of the square repeat unit. This juxtaposition of imposed symmetry designs with converse symmetrical characteristics and arbitrary inclusions indicates an admirable and adroit playfulness on the part of the artist.

Ildegizid artists did not make frequent use of the *fourfold system A*. One fine example is a *median* field design created from an underlying tessellation of large and small hexagons, pentagons, and interstice rhombi [Fig. 141]. As with three of the above-cited Ildegizid examples, this is also from one of the façades of the mausoleum of Yusuf ibn Kathir. This design was subsequently used during the Seljuk Sultanate of Rum at the Haunt Hatun in Kayseri (1238), and the Haci Kiliç mosque and *madrasa* also in Kayseri (1249).[69]

The Mu'mine Khatun mausoleum makes use of several designs created from the *fourfold system B*. The more basic of these is the classic *acute* pattern that was used with great frequency throughout the Islamic world, and had already been featured some 50 years previous at the Friday Mosque at Sin (1134) [Fig. 173a]. The most remarkable of the *fourfold system B* designs at the Mu'mine Khatun mausoleum is an *acute* pattern located in one of the long vertical panels decorating the exterior façade of the tomb [Fig. 182]. This is a hybrid design characterized by the employment of two separate repetitive cells within the overall schema: the square and rhomb. There is a long history of Islamic geometric patterns that fill the two-dimensional plane with more than a single variety of repetitive cell: the earliest extant example being the above-cited fivefold hybrid design from the northeast dome chamber in Isfahan [Photograph 25]. There are two ways of tessellating the plane using just squares and rhombi. One of these places the rhombi in a



**Photograph 34** A late Abbasid hybrid *acute* pattern from the Abbasid Palace of the Qal'a in Baghdad that employs both square and rhombic repetitive elements and is created with the *fourfold system A* (photograph by K. A. C. Creswell; © Ashmolean Museum, University of Oxford)

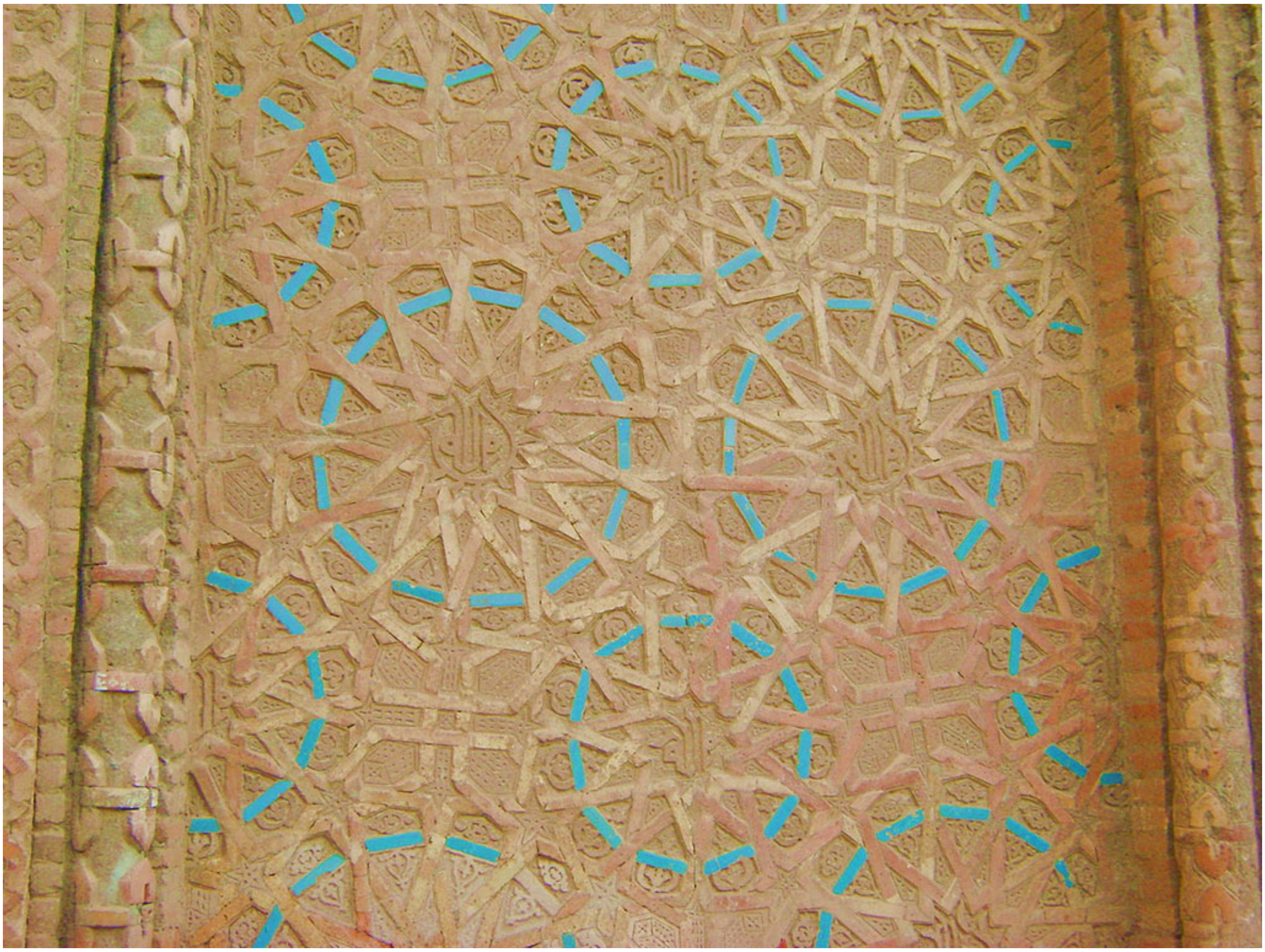
pinwheel-like rotation around each square, and the other places the rhombi and squares in a tessellation of alternating linear bands. The repetitive structure of the example from the Mu'mine Khatun mausoleum is the latter type, with rhombi that have 45° and 135° included angles. Any variety of rhomb will tessellate with a square in such a linear arrangement: the requirement being that the length between the two obtuse angles of the rhomb be equal to the edge length of the square, thus defining the coincident edges of the square and triangle produced from the half rhombi. In order for the pattern to flow seamlessly across these two repeat units it is necessary for the underlying polygonal tessellation to share the same distribution of polygonal modules along the coincident edges of each repeat unit. This hybrid design from the Mu'mine Khatun mausoleum was also used at the Abbasid Palace of the Qal'a in Baghdad (c. 1230) [Photograph 34]. It is worth noting that each of these repetitive cells will work independently to fill the two-dimensional plane. In the case of the example from the Mu'mine Khatun mausoleum, the square repeat unit is the classic *acute* pattern that, on its own, is found on the

[69] Schneider (1980), pattern no. 281.

same building high above the entry door [Fig. 173a]. The rhombic repetitive element at the Mu'mine Khatun mausoleum was likewise used independently within this tradition: for example, at the Izzeddin Kaykavus hospital and mausoleum in Sivas, Turkey (1217) [Fig. 181]. However, this example from the Mu'mine Khatun mausoleum is the first to use this *fourfold system B* repetitive rhombic element, and the first design from the *fourfold system B* to employ a hybrid approach to filling the two-dimensional plane with two distinct repetitive cells.

Among the diversity of geometric patterns that adorn the Mu'mine Khatun is an outstanding panel created easily from the *fivefold system*. This example employs a rhombic repeat unit with 72° and 108° included angles, and, like other designs produced with this system, places the primary underlying decagons upon the vertices of the rhombic grid [Fig. 232g]. This is an *acute* pattern derived from an underlying tessellation of decagons, elongated hexagons, trapezoids, and large concave hexagons with edges that are equal to the long edge of the trapezoid [Fig. 232h]. It is worth noting that this design can also be created from an underlying tessellation of just decagons and concave hexagons [Fig. 232f], as well as an arbitrary modification of the classic *median* pattern [Fig. 227e]. Generative ambiguity of this nature is not uncommon with patterns created from the *fivefold system*; and while this often makes it impossible to know categorically which of two, or even several, underlying polygonal tessellations the original designer employed, it by no means undermines the legitimacy of this methodological practice. Rather, this exemplifies the inherent flexibility of the *fivefold system*. The rhombic underlying tessellation of decagons, barrel hexagons, trapezoids, and large concave hexagons that produces this design is a modification of a rhombic underlying tessellation of decagons, barrel hexagons, and six contiguous pentagons that surround a small rhombi [Fig. 223]. This configuration of six pentagons surrounding a small rhombus was used frequently by geometric artists working with the *fivefold system*. However, it is only well suited to the production of *obtuse* and *two-point* patterns, rather than *acute* and *median* patterns. For acceptable patterns in these latter two families the underlying pentagons are truncated into trapezoids that lend themselves to the pattern characteristics of the *acute* and *median* families. There are two conventions for this truncation [Figs. 198 and 199]: one that truncates just four of the pentagons (leaving a large rhombic interstice region), and the other that truncates all six pentagons (leaving a large concave hexagonal interstice region). With *median* patterns, it is also possible to adjust the pattern lines themselves rather than the underlying tessellation [Fig. 199]. The design from the Mu'mine Khatun is the earliest known example of a fivefold *acute* pattern that utilizes this very effective adjustment to the six clustered pentagons. This design was widely used, and later examples include a Zangid entry door at the Awn al-Din Mashhad in Mosul, Iraq (1248); part of the Mamluk exterior carved stucco ornament on the drum of the dome at the Hasan Sadaqah mausoleum in Cairo (1315-21); a Mamluk wooden door on the *minbar* of the Sultan Qaytbay funerary complex in Cairo (1472-74); a Mamluk cupboard door at the Qadi Abu Bakr Muzhir complex in Cairo (1479-80); and a Mamluk wooden panel along the stair railing of the *minbar* at the Amir Azbek al-Yusufi complex in Cairo (1494-95).

The range of remarkable geometric patterns employed in the exterior ornament of the Mu'mine Khatun mausoleum includes several designs that are created from nonsystematic underlying polygonal tessellations. The most basic of these is the well-used *acute* pattern of 12-pointed stars placed upon the vertices of the orthogonal grid with octagons at the centers of each square repeat [Fig. 336a]. The many other locations of this pattern include the Seljuk minaret of the Great Mosque of Siirt, Turkey (1129); the Artuqid *mihrab* niche at the Great Mosque of Silvan, Turkey (1152-57); and the Seljuk Sultanate of Rum cenotaph at the Izzeddin Kaykavus hospital and mausoleum in Sivas, Turkey (1217). A far less common nonsystematic design from the Mu'mine Khatun mausoleum is a fourfold *acute* pattern that places eight-pointed stars at the vertices of the square repeat unit, six-pointed stars at the midpoints of the repeat unit, and an irregular octagon at the center. The underlying tessellation for this pattern is made up of just three elements: the regular octagon, regular hexagon, and irregular pentagon [Fig. 178c]. This same underlying tessellation was used to create a very similar ceramic panel at the Altinbugha mosque in Aleppo (1318), as well as the design on the exterior of the Mamluk door (1303) at the Vizier al-Salih Tala'i mosque. Another fourfold nonsystematic *acute* pattern from the Mu'mine Khatun mausoleum uses underlying octagons at the vertices of the orthogonal grid that are separated along the edge of the square repeat unit by two regular hexagons rather than just the one from the previous example [Fig. 332a, c]. It must be assumed that the use of these two patterns on the Mu'mine Khatun mausoleum—one with a single underlying hexagon and the other with two underlying hexagons separating the underlying octagons—was a willful and subtle artistic act on the part of the artist. The polygonal matrix within the central region of the repeat employs irregular heptagons, irregular pentagons, and a square at the center of the repeat unit. This same underlying tessellation was first used by Seljik artists at the Friday Mosque at Barsian (1105), and at a later date by Mamluk artists to create a significantly more complex design for a window grille at the Amir Sarghitmish *madrasa* in Cairo (1356) [Fig. 332e]. Of particular interest in the history of this ornamental tradition is a compound pattern from the



**Photograph 35** An Ildegizid nonsystematic *acute* pattern from the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan, with 11- and 13-pointed stars (photograph by Самый древний (Own work) [Public domain], via Wikimedia Commons)

Mu'mine Khutun mausoleum with 11- and 13-pointed stars that is among the most complex nonsystematic geometric designs produced in the long history of this tradition [Fig. 434b] [Photograph 35]. The eccentricity of a design comprised of 11- and 13-pointed stars might appear to challenge the limits of two-dimensional space filling. However, as with the Seljuk pattern with 7- and 9-pointed stars from the *mihrab* of the Friday Mosque at Barsian [Fig. 429], the *principle of adjacent numbers* indicates that the practicality of making designs with 12-pointed stars allows for the likelihood of a successful pattern being created with 11- and 13-pointed stars. And indeed, this pattern from the Mu'mine Khatun is exceptionally successful. This pattern has the unusual characteristic of repeating with either an elongated hexagonal grid with the 13-pointed stars at each vertex or an alternative elongated hexagonal grid with 11-pointed stars at each vertex [Fig. 434a]. What is more, these two repetitive grids are perpendicularly orientated duals of one another. This pattern is beautifully balanced, pleasing to the eye, and a masterpiece of geometric art. While other highly complex geometric patterns with perpendicularly arranged hexagonal dual repeat units were produced subsequently, this particular example is unique to the Mu'mine Khatun mausoleum, and does not appear to have been used by succeeding artists.

## 1.15 Artuqids (1102-1409)

The Artuqids began as military commanders of the Seljuks in Damascus, and rose to power as the Seljuk governors of Jerusalem. Their rule over eastern Anatolia, northern Syria, and al-Jazirah (northern Iraq) vacillated between independence and as vassals to their Seljuk, Zangid, Ayyubid rivals, and eventually the more powerful dynasties of the Sultanate of Rum, Ilkhanids, and Timurids. Their principle capital was Diyarbakir in southeastern Anatolia, which benefited from considerable architectural and artistic patronage. A surviving working drawing for the construction of the bronze doors for the Diyarbakir Palace were produced by a known individual and are adorned with a geometric pattern. This design, along with the accompanying instructions for casting, is the work of Ismail ibn al-Razzaz al-Jazari, and is part of his celebrated *Book on the Knowledge of Ingenious Mechanical Devices*[70] (1206). This geometric pattern is easily created from the 3.4.6.4 underlying tessellation of triangles, squares, and hexagons [Fig. 105e]. It is interesting to note the contrast between the proportional imprecision within his illustration—especially noticeable among the eight-pointed stars—and the geometric accuracy that characterizes the innumerable historic examples of Islamic geometric art. As made clear by the text that accompanies this illustration, this is a working drawing intended to be merely indicative of the final palace door. The text that accompanies this drawing explains, "*In the drawing I have not aimed for completeness. My purpose was to present a general arrangement so that it can be understood in the whole and in detail*."[71] Very few Artuqid buildings have survived intact to the present day, and our knowledge of their use of geometric design is slight at best. A carved stone relief panel at the Great Mosque of Dunaysir in Kiziltepe, Turkey (1204), employs a pattern that directly relates to the $4.8^2$ tessellation of underlying octagons and squares [Fig. 129b]. This is a subtractive variation of the well-known design that locates the pattern lines upon the vertices of the generative tessellation [Fig. 129c]. The Artuqid niche of the *mihrab* at the Maqam Ibrahim at Salihin in Aleppo[72] (1112) is decorated with an *acute* field pattern constructed from just the large hexagonal and square modules of the *fourfold system A* [Fig. 138a]. This simple design places octagons on the vertices of the orthogonal grid, and was occasionally used by subsequent dynasties. The Artuqid use of patterns derived from the *fivefold system* includes a classic *acute* pattern that repeats on the rhombic grid of 72° and 108° angles from the surrounding border of the *mihrab* at the Great Mosque of Dunaysir in Kiziltepe, Turkey (1204) [Fig. 226c]. The wall of the *mihrab* niche at the Great Mosque of Silvan in southeastern Turkey (1152-57) employs a nonsystematic fourfold *median* pattern that places 12-pointed stars on the vertices of the orthogonal grid and octagons at the centers of each square repeat unit [Fig. 336a]. The underlying generative tessellation for this pattern is comprised of dodecagons and two varieties of irregular pentagons. This *median* example from the Great Mosque of Silvan is the earliest known pattern made from this tessellation.

[70] Istanbul, Topkapi Sarayi Müzesi Kütüphanesi, MS A. 3472, fols. 165v–166r.

[71] Necipoğlu (1995), 150–152.

[72] Allen (1999), Chap. 2.

## 1.16 Zangids (1127-1250)

During the twelfth century the Zangids became one of the primary powers in the region of al-Jazirah and Syria. Their founder, Imad al-Din Zengi, was Kurdish, and their rise to power began as the Seljuk *atabegs* of Mosul. Their greatest leader was Nur al-Din whose successes against the crusader kingdoms substantially increased their territorial dominion and aided their standing among Sunni Muslims throughout this region. Nur al-Din's forces were successful in overthrowing the Fatimids in Egypt in 1169, although he died before consolidating Egypt into the Zangid Empire. His death facilitated the rise of Ṣalāḥ ad-Dīn Yūsuf ibn Ayyūb and the founding of the Ayyubid successors to Zangid and Fatimid rule. Following the loss of Syria to the Ayyubids in 1183, the Zangids held on to their northern Iraqi territories until their demise in the mid-thirteenth century. The Zangids were great patrons of architecture, and many outstanding Zangid architectural monuments have survived into the modern era. Zangid architecture is best represented in Aleppo and Damascus in Syria, and to a lesser extent Mosul in Iraq. The refinement of the geometric ornament, including their magnificent use of *muqarnas*, attests to the level of interest that Zangid patrons had for this idiom. As demonstrated by the many examples of geometric pattern, their Seljuk origins allowed for the direct assimilation of the precise knowledge of geometric design methodology.

Like other Muslim cultures of the same period, the Zangids made frequent use of patterns constructed from the *system of regular polygons*. A pattern inside the niche of the wooden *mihrab* at the Lower Maqam Ibrahim at the citadel of Aleppo (1168) is easily created from an underlying $6^3$ tessellation of regular hexagons [Fig. 96h]. The first known use of this design was approximately a hundred years earlier on the Seljuk eastern tomb tower at Kharraqan, and subsequent generations of Muslim artists made regular use of this geometric pattern. A Zangid use of this same design is found on the outer panels of the *minbar* doors at the al-Aqsa mosque in Jerusalem[73] (1168-74). The pattern which adorns the inside surfaces of these same *minbar* doors is created from the underlying 3.6.3.6 tessellation [Fig. 102a3]. This pattern employs two varieties of applied crossing pattern lines to the midpoints of the underlying polygonal edges. As mentioned above, the first known use of this design is from the Ghaznavid South Palace at Lashkar-i Bazar in Afghanistan (before 1036) [Fig. 102a1] [Photograph 13]. The Zangid portal of the otherwise Mamluk Bimaristan

[73] The al-Aqsa Mosque in Jerusalem is primarily a Fatimid building. However, the *minbar* was commissioned by the Zangid ruler Nur al-Din in 1168, placing it within the sphere of Seljuk influence. See Tabbaa (2001), 86–88.

**Photograph 36** An incised stone panel in the Zangid portal of the Bimaristan Arghun in Aleppo with a threefold *median* pattern easily created from the *system of regular polygons* (photo by Nasser Rabbat, courtesy of the Aga Khan Documentation Center at MIT)

Arghun in Aleppo[74] contains a large bold rectangular panel above the entry door that is decorated with a design also created from the underlying 3.6.3.6 tessellation of triangles and hexagons [Fig. 101c] [Photograph 36]. Like the pattern from the interior surface of the doors of the al-Aqsa *minbar*, the construction of this design uses two types of crossing pattern line applied to the alternating underlying hexagons and triangles. This pattern is incised into the alternating colors of *ablaq* stonework in a manner wherein the zigzag divisions between the alternating dark and light stones are determined by the lines of the geometric pattern. This is a highly sophisticated, and virtually unique, ornamental device that indicates the origin of this portal to the high period of Zangid architectural ornament during the second half of the twelfth century. An incised stone panel placed over the door in the portal at the Adilliyya *madrasa* in Damascus (1172-74[75]) is a very fine example of Zangid geometric ornament. This is an isometric design that is made from the $3.12^2$ underlying tessellation of dodecagons and triangles. This pattern is created by placing 60° crossing pattern lines upon the centers of each coinciding polygonal edge [Fig. 108a]. The earliest known use of this design is from the raised brick ornament of the eastern tomb tower at Kharraqan (1067) [Photograph 17], and over time it became widely circulated throughout Muslim cultures. The use of incised lines within stonework ornament was characteristic of Zangid and later Ayyubid architectural ornament. This masonry technique requires considerably less time than carving stone in high relief. The loss of clarity associated with the highlights and shadows of high-relief carved stone was compensated by the introduction of pigments into the incised lines. This has its own bold aesthetic appeal that, to the modern eye, appears overtly graphical. This technique was used primarily on exterior façades, and over time had the disadvantage of loosing its boldness in color contrast if the paint was not refreshed periodically. Without the paint, the incised lines loose their boldness and are experienced more as a subtle presence, with just a vestige of its former ornamental impact. An outstanding example of a Zangid orthogonal pattern created from the *system of regular*

[74] Allen (2003).

[75] Different dates for this portal have been posited. See Allen (1999).

*polygons* was used on the side panel of a wooden *minbar* commissioned by Nur al-Din in 1186, and likely made in Aleppo[76] [Fig. 113c]. This pattern is created from the 3.4.3.12-3.12$^2$ *two-uniform* tessellation with 120° crossing pattern lines placed upon the midpoints of the underlying polygonal edges. This Zangid pattern may have served as an inspiration to later Mamluk artists whose widespread use of this design includes at least two examples of *minbar* side panels: at the Sultan al-Nasir Muhammad ibn Qala'un at the citadel of Cairo (1295-1303), and the Amir Altinbugha al-Maridani mosque in Cairo (1337-39).

The railing of the celebrated al-Aqsa *minbar* (1187) contains an outstanding Zangid geometric pattern that is generated from the 4.8$^2$ underlying tessellation of squares and octagons. This is an early example of a subtractive variation of the classic star-and-cross pattern [Fig. 126f] that was used in a number of locations historically, including an almost contemporaneous design from the façade of the southern portal at the Qarakhanid Maghak-i Attari mosque in Bukhara (1178-79), and one of the panels on the interior of the Mamluk door (1303) of the Vizier al-Salih Tala'i mosque in Cairo. One of the side panels of this same Zangid *minbar* at the al-Aqsa mosque in Jerusalem uses a *two-point* pattern from the same 4.8$^2$ underlying tessellation [Fig. 128g]. This *two-point* pattern has essentially the same geometry as a design that employs the vertices of this same generative tessellation [Fig. 129c]. A Ghurid example of this alternative derivation can be found at the roughly contemporaneous western mausoleum at Chisht, Afghanistan (1167).

There are few Zangid examples of geometric patterns created from the *fourfold system A*. One notable example is a design used for several window grilles at the Nur al-Din Bimaristan in Damascus (1154) [Fig. 161] [Photograph 37]. This design is created from an underlying tessellation of octagons, truncated octagons, and eight-pointed stars. The use of the truncated octagons as underlying generative modules is an unusual feature more typically associated with later *fourfold system A* patterns from the Maghreb, and indicates a significant innovative imagination on the part of the artist who created this design. The widened interweaving lines are unusual in that some of the pattern lines are widened through their being offset on both sides equally, and others are widened through being offset in one direction only. This produces an irregularity to the otherwise predictable design, and is responsible for the distinctive visual character of these windows.

Zangid artists were among the earliest to make use of the *fourfold system B*. The first known use of this

**Photograph 37** A Zangid window grille at the Nur al-Din Bimaristan in Damascus with a fourfold *median* pattern created with the *fourfold system A* (Photo by Nasser Rabbat, courtesy of the Aga Khan Documentation Center at MIT)

methodological system is from the above-mentioned *mihrab* in the Friday Mosque at Sin (1134). By the second half of the twelfth century the *fourfold system B* had been readily adopted by artists working in the western regions of Seljuk influence. The earliest Zangid examples include the same *acute* design that had been used at Sin, and was to become the most widely used pattern created from this design system [Fig. 173a]. This is located on the base of the minaret of the Great Mosque of Nur al-Din at Mosul (1170-72). Its utilization throughout Muslim cultures and artistic media, coupled with its easily recognizable distribution of eight-pointed stars and octagons, qualifies this as the classic *fourfold system B* pattern. Zangid artists were also among the first to use the innovative additive device whereby octagons can be incorporated into the pattern matrix through adjusting the *acute* pattern lines within the underlying large hexagonal modules [Fig. 172b]. Over time, among succeeding Muslim cultures, this became a standard feature of *acute* patterns

[76] This *minbar* is in the collection of the Hama National Museum in Syria.

created from this system. Both the framing border of the wooden *mihrab* in the Lower Maqam Ibrahim in the citadel of Aleppo (1168) and the large triangular side panel of the wooden *minbar* at the al-Aqsa mosque in Jerusalem (1187) employ a particularly beautiful pattern with this octagonal characteristic [Fig. 177b]. The Zangid ruler Nur al-Din commissioned both of these superlative examples of Muslim woodworking.

The al-Aqsa *minbar* also employs one of the earliest Zangid examples of a pattern constructed from the *fivefold system*. This is a classic *acute* pattern that repeats upon a rhombic grid of 72° and 108° angles [Fig. 226c]. As discussed above, this design can be traced back to one of the repetitive cells in the hybrid design at the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Fig. 261] [Photograph 25]; and other early examples include the Ghurid Arch at Bust, Afghanistan (1149) [Photograph 33], and the Seljuk façade of the Friday Mosque at Gonabad, Iran (1212) [Photograph 33]. This predates the panel from the al-Aqsa *minbar* by only 38 years. The interior of the main entry door of the Imam Awn al-Din Mashhad in Mosul (1248) is decorated in raised copper with a very-well-conceived *acute* pattern made from the *fivefold system* [Fig. 232g]. The earliest example of this popular design is from the Ildegizid tomb of Mu'mine Khatun in Nakhichvan, Azerbaijan (1186). As discussed previously, the underlying tessellation of this pattern employs a modification to the cluster of six pentagons at the center of each rhombic repeat unit. This configuration of six pentagons makes very satisfactory *obtuse* and *two-point* patterns, but is not suited to creating acceptable *acute* and *median* designs. In order to overcome this design limitation, the cluster of six pentagons is modified in either of the two ways [Figs. 198 and 199]. As with the Azerbaijani design, the fivefold pattern from the Awn al-Din Mashhad truncates each of the six clustered pentagons into six adjacent trapezoids, creating the distinctive concave hexagonal feature at the center of each rhombic repeat unit.

The Zangids made occasional use of geometric patterns that were created from nonsystematic underlying polygonal tessellations. The exterior of the bronze door at the Nur al-Din Bimaristan in Damascus (1154) is decorated with a threefold pattern comprised of five- and six-pointed stars [Fig. 309c]. A very similar Seljuk design was used earlier in the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Photograph 27]. As explained, the underlying tessellation that produces this pattern places pairs of coinciding pentagons upon each edge of the triangular repeat unit in such manner that they create irregular hexagonal ditrigons within the centers of the triangular repeat units. This pentagonal configuration also produces a six-pointed star at each vertex of the isometric grid. The pattern lines are placed upon the midpoints of each underlying pentagonal edge, and the 36° angular opening of the crossing pattern lines creates a very acceptable *acute* pattern. An incised pattern on the archivolt surrounding the *muqarnas* hood of the Zangid portal at the Bimaristan Arghun in Aleppo makes use of a nonsystematic border pattern of alternating nine- and six-pointed stars. This is a poorly conceived design that has the further problem of being in very poor repair: the linear repetitive structure being obscured beyond interpretation. A far more successful, if common, Zangid nonsystematic geometric pattern was used on the double-entry doors of this same portal. These are decorated with interweaving bronze straps that create an *acute* pattern with 12-pointed stars set on an orthogonal grid [Fig. 337]. The underlying polygonal tessellation for this design is a modification of the tessellation used to generate the *median* pattern in the contemporaneous Artuqid *mihrab* niche of the Great Mosque of Silvan, Turkey (1152-57) [Fig. 336a]. Just as the cluster of six regular pentagons within the *fivefold system* can be modified through truncating the pentagons into trapezoids, so too is this possible with the cluster of four irregular pentagons with coinciding edges that are found in this Artuqid example. These trapezoids are responsible for generating the four dart motifs that are a distinctive feature of this design. Other locations that employed this design include the Ildegizid exterior ornament of the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan (1186); the Zangid entry doors and incised stone ornament at the Zahiriyya *madrasa* in Aleppo; the Mamluk mausoleum of Sultan al-Zahir Baybars in Damascus (1277-81); a pair of wooden doors from the *atabeg* of Cizre in southeastern Turkey (thirteenth century); the *minbar* of the Mamluk funerary complex of Sultan al-Zahir Barquq in Cairo (1384-86); and the *minbar* doors at Amir Taghribardi funerary complex in Cairo (1440). Another example of a nonsystematic Zangid pattern with 12-pointed stars is from a *mihrab* archivolt at the Upper Maqam Ibrahim at the citadel of Aleppo (c.1214). The original mosque at this site was rebuilt by Nur al-Din following a fire in 1212.[77] An interesting feature of the underlying tessellation for this pattern is the truncation of three irregular pentagons into three trapezoids that surround a central triangle [Fig. 320b]. The geometric pattern employed over this *mihrab*, when considered in the abstract, is both beautiful and conceptually satisfying. However, the execution of this pattern on the *mihrab* archivolt is imprecise and clumsy. The incised carving is poorly executed and the geometric distortion that is required when applying a linear motif onto a curve is, in this example, inelegant, and stands in marked contrast to the vastly superior execution in the Zangid archivolt pattern at the Zahiriyya *madrasa* produced just 3 years later. Far more

[77] Allen (1999), Chap. 8.

pleasing subsequent examples of this lovely threefold pattern include a thirteenth-century Christian *khachkar* stele from Dsegh, Armenia,[78] and two vertical panels from a fourteenth-century wooden door from Fez, Morocco.[79]

The most remarkable Zangid geometric pattern created from a nonsystematic underlying tessellation was reported by Ernst Herzfeld to have been used on a pair of doors at the Lower Maqam Ibrahim at the Aleppo citadel[80] (1168). This *acute* design is comprised of an ingenious combination of 12-, 11-, and 10-pointed stars arranged in linear vertical bands. The repeat unit is a long rectangle with 12-pointed stars at the vertices, 10-pointed stars at the midpoints of the long sides of the rectangular repeat, and two 11-pointed stars centered between the 10- and 12-pointed stars within the rectangular repeat [Fig. 427]. This design is equal to the most complex designs from the Anatolian Sultanate of Rum and Mamluks of Egypt. A pattern on the portal of the kiosk at Erkilet near Kayseri, Turkey (1241), employs the same exact sequence of linear star forms [Fig. 425]. Despite accolades to the contrary,[81] the pattern that was reported to have been in the doors of the Lower Maqam Ibrahim contains areas where the pattern matrix is problematic: specifically, the crowding of the pattern lines at the centers of the triangular regions defined by the 11- and 12-pointed stars. The overall balance of the conceptually similar Sultanate of Rum design from Erkilet is far superior to the example recorded by Herzfeld at the Lower Maqam Ibrahim.

During the twelfth century, the Zangids experimented with the Great Seljuk innovations in applying geometric patterns onto domical surfaces. The Zangid artists working in this specialized field of ornament were woodworkers, and were arguably the most skilled craftsmen working within the Zangid artistic milieu. This form of design and fabricating technology is highly specialized and required a practical knowledge of spherical geometry. The exceptional woodwork from this period includes a quarter dome hood in the *mihrab* niche of the Lower Maqam Ibrahim in the citadel of Aleppo (1168): signed by Ma'ali ibn Salam: clearly a master of this art. This quarter dome is ornamented with a beautiful geometric pattern derived from a spherical projection of underlying polyhedral geometry. As with the Seljuk ornament of the northeast dome of the Friday Mosque at Isfahan, the use of polyhedral geometry as an organizing principle for domical ornament is unusual, but very effective. The great majority of domes with geometric decoration are based upon segmented radial gores, and only a relatively small number of domes utilize the geometry of Platonic or Archimedean polyhedra. The quarter dome hood of the *mihrab* niche in the Lower Maqam Ibrahim employs a pattern with five- and six-pointed stars that is based upon a spherical projection of the truncated icosahedron as the formative underlying structure [Fig. 498]. The truncated icosahedron is comprised of 12 pentagons and 20 hexagons in a $5.6^2$ configuration at each vertex. As a spherical projection, this polyhedron is often associated with the soccer ball. The geometric pattern in this quarter dome places 60° crossing pattern lines at the midpoints of the coinciding pentagonal and hexagonal edges. This creates a *median* pattern that flows across the spherical surface in a very cohesive and pleasing fashion, placing five-pointed stars within the projected pentagons, and six-pointed stars inside each projected hexagon.

## 1.17 Fatimids (909-1171)

The Fatimid dynasty was founded by Kutama Berbers from the northeastern coastal region of Algeria in the early tenth century. They rose to prominence rapidly, and by the second half of the tenth century they conquered Egypt, founding Cairo as their capital in 969. By the close of the tenth century they controlled an empire stretching from the Maghreb in the west, across all of North Africa and Egypt, Sicily, and portions of southern Italy, Malta, the northern Red Sea coastal region of the Hijaz, and the Levant. The Fatimid Caliphate was founded in 909 and rivaled the Abbasid Caliphate until the Zangids defeated the substantially weakened Fatimids in 1169. The Fatimids were Shi'a Muslims, and were ruled by members of the Ismaili sect. In contradistinction to rival empires, their governance recognized merit over heredity in rewarding advancements. With aptitude as the primary qualifier, Sunni Muslims, Christians, and Jews were entrusted with high levels of responsibility and authority. Under Fatimid patronage, Cairo became one of the great cities of the world. The great wealth amassed through sea fairing trade in the Mediterranean and Indian Ocean, as well as with China enabling extravagant building projects and widespread support for the arts. With their Mediterranean origins and influential communities of Byzantine and Coptic Christians, the aesthetic sensibilities of the Fatimids were distinct from the prevalent artistic trends occurring in the eastern regions of the Islamic world. Within the minor arts, representational motifs with animal and human figures were used more widely than similar work from eastern Muslim

[78] Azarian (1973), pl. 58.

[79] Dar al-Athar al-Islamiyya Kuwait National Museum.

[80] This pair of doors is no longer present at the Lower Maqam Ibrahim in Aleppo. Ernst Herzfeld published a drawing and description of this pattern, and this is the only record of its existence. See Herzfeld (1954–56), Fig. 56.

[81] "It is the most complicated design ever produced by that branch of art. The almost unsolvable problem of a design based on horizontal groups of 11-pointed stars is solved by alternative intercalation of a parallel group of 12-pointed and one of 10-pointed stars between them": Herzfeld (1943), 65.

cultures, often exhibiting a level of facility and playfulness that contradicts the commonly held view that the depiction of human and animal forms is anathema to all Muslim cultures. While still placing great emphasis upon calligraphy, Fatimid artists generally favored the floral idiom over the geometric in the ornamentation of their architectural monuments. The extent to which Fatimid floral preferences originated from an antipathy toward the cultural mores of their neighboring Sunni rivals or simply a genuine partiality toward this more naturalistic idiom is uncertain.[82] The ornament of the earlier Fatimid period embraced the Samarra floral styles exemplified at the ibn Tulun mosque, as well as the aesthetics of their North African ancestral homelands.[83] The Fatimid eschewal of geometric designs and *muqarnas* vaulting began to change during the twelfth century. The gradual incorporation of the advances to the geometric arts that were carried out under the auspices of the Ghaznavids, Ghurids, Qarakhanids, and Seljuks appears to have been initially advanced by non-imperial patronage,[84] and only later fully adopted into the fabric of Egyptian aesthetics. What is certain from the historic record is that the Fatimids incorporated geometric patterns and *muqarnas* vaulting with less frequency than the contemporaneous Sunni cultures to the north and east. Despite this floral predilection, the twelfth-century geometric ornamental advances nonetheless made their way into the fabric of Fatimid culture—however tenuously—and these examples include a number of patterns of great beauty.

The majority of Fatimid geometric designs were produced from the *system of regular polygons*. One of the earliest is a pattern made up of superimposed interweaving dodecagons from a window grille in the chamber of the Dome of al-Hafiz at the al-Azhar mosque in Cairo (970-72) that is easily created from the $6^3$ hexagonal grid [Fig. 97c]. This same design was used some 10–20 years later in one of the window grilles at the Great Mosque of Córdoba, and indeed was widely used throughout Muslim cultures. A window grille from the al-Aqmar mosque in Cairo (1125) uses a design that can also be derived from the $6^3$ hexagonal grid. This is comprised of superimposed hexagons [Fig. 96e], and also shares provenance with one of the window grilles in the Great Mosque of Córdoba (980-90). The carved stucco *mihrab* of the al-Amri mosque in Qus, Egypt (1156), appears to employ a 3.4.6.4 design that extends the coincident triangular and square edges of the generative tessellation to create a very successful design with 12-pointed stars within each hexagon [Fig. 106b]. This derivation is unusual in that the 12-pointed stars are not created from an underlying dodecagon. This design is very similar to an example from the Seljuk Sultanate of Rum that employs a 4.6.12 underlying generative tessellation [Fig. 109a], and indeed the Fatimid design can also be made from this tessellation. The rear portion of the portable Fatimid *mihrab* at the Sayyid Ruqayya Mashhad in Cairo (1133) includes a threefold pattern with 12-pointed stars that is generated from the $3.12^2$ tessellation of triangles and dodecagons [Fig. 108a]. This is the same pattern that was used for the first time on the eastern tomb tower at Kharraqan some 66 years earlier [Photograph 17]. In this Fatimid example, an arbitrary 6-pointed star motif has been added into the center of each 12-pointed star. This is a sixfold example of a form of additive pattern modification that was more commonly applied to *obtuse* patterns created from the *fivefold system* [Fig. 224b]. Later examples of this design—without the modification—include a Zangid incised stone panel at the Adilliyya *madrasa* in Damascus (c. 1172), and the top and bottom panels from the Mamluk double doors of the Vizier al-Salih Tala'i mosque in Cairo (1303). The same portable wood *mihrab* of the Sayyid Ruqayya Mashhad in Cairo employs a lovely geometric design that dominates the front surface framing the niche. This design can be created from either of the two underlying polygonal tessellations: the 3.6.3.6 semi-regular tessellation with alternating active and passive hexagonal and triangular cells [Fig. 101b], and a *three-uniform* tessellation of triangles, squares, and hexagons in a $3^4.6\text{-}3^3.4^2\text{-}3^2.4.3.4$ arrangement, wherein the 60° crossing pattern lines are applied to the midpoints of just selected polygonal edges [Fig. 114c]. This is an unusual and highly inventive use of the *system of regular polygons*. A Fatimid pattern from the triangular side panel of the wooden *minbar* (1091-92) of the Haram al-Ibrahimi in Hebron, Palestine, can be created from at least three different underlying tessellations: the simple $6^3$ tessellation of regular hexagons [Fig. 98d]; the 3.4.6.4 tessellation of triangles, squares, and hexagons [Fig. 106c]; and the $3^4.6\text{-}3^3.4^2\text{-}3^2.4.3.4$ *three-uniform* tessellation of triangles, squares, and hexagons [Fig. 114a]. While it is impossible to know which method the artist used in any given example, the high degree of available methods to create just a single design speaks to the flexibility of this design methodology. Another historical occurrence of this design is from the *minbar* of the al-Amri mosque in Qus, Egypt (1156). The niche of the above-cited portable wooden *mihrab* at the Sayyid Ruqayya Mashhad in Cairo also employs a very simple *acute* design created from the $6^3$ hexagonal grid as an underlying generative structure [Fig. 95a]. This pattern places 30° crossing pattern lines at the midpoints of each hexagonal edge, resulting in a design comprised of six-pointed stars and distinctive ditrigonal shield shapes. This very becoming, if rather simple, design never generated the level of pan-Islamic interest as its close relatives that can be created from the same regular

[82] Tabbaa (2001), 80–84.

[83] Ettinghausen, Grabar and Jenkins-Madina (2001), 195.

[84] Bloom (1988), 27–28.

hexagonal grid with 60°, 90°, and 120° crossing pattern lines. Earlier Seljuk examples of this simple design are found at the northeast domed chamber of the Friday Mosque at Isfahan (1089-90) [Photograph 18], and the Friday Mosque at Sin, Iran (1133).

Fatimid geometric patterns created from the underlying $4.8^2$ tessellation of squares and octagons include a fine example of the classic star-and-cross design from the stucco *mihrab* of the Umm Kulthum and al-Qasim Abu Tayyib mausoleum in Cairo (1122) [Fig. 124b]. This bears an unmistakable resemblance to the aesthetic treatment of the much earlier example of this pattern on the arch soffits at the No Gunbad mosque in Balkh, Afghanistan (800-50) [Photograph 10]. The multiple small circles applied linearly within the interweaving straps of the design, as well as the circularity of the floral motifs within each eight-pointed star, are so similar that it is possible that the artist who designed this Fatimid *mihrab* may have been familiar with the earlier Afghan example. A more complex geometric design created from the same underlying $4.8^2$ tessellation is found in the Fatimid *mihrab* from the mausoleum of Sayyidah Nafisah in Cairo (1138-46) [Fig. 127d]. This same design was used by Mengujekid artists at the Great Mosque of Divrigi, Turkey (1228-29), and later still by Mamluk artists on the minaret of the Sultan Qaytbay funerary complex in Cairo (1472-74).

Fatimid artists appear to have limited their use of the polygonal technique to the *system of regular polygons*. They did not make use of either the *fourfold system A* or *B* for creating geometric designs, and their use of the *fivefold system* was rare at best.[85] Similarly, the Fatimids did not incorporate nonsystematic patterns within their architectural ornament. These omissions are surprising considering the widespread adoption of all of these design methodologies by their Muslim neighbors to the north and east during the eleventh and twelfth centuries. This may have been a deliberate rejection based upon the association of such designs with their Seljuk and Abbasid Sunni rivals, or it may simply have been due to the absence of knowledge of these more advanced methodologies within the community of artists working under Fatimid patronage. Whichever the case, the fact of the virtually exclusive use of the *system of regular polygons* to create their geometric ornament can be regarded, at least in part, as more a willful continuation of the earlier methodological practices and geometric aesthetic of their Tulunid predecessors, and less an influence by the art of their Sunni rivals.

## 1.18 Ayyubids (1171-1260)

In 1169, the Zangid ruler Nur ad-Din Zangi sent his general Asad ad-Din Shirkuh on a campaign to overthrow the Fatimids in Egypt. Shirkuh died soon after his successful defeat of the Fatimids, and was succeeded by his nephew Ṣalāḥ ad-Dīn Yūsuf ibn Ayyūb. While establishing relative autonomy in Egypt, Ṣalāḥ ad-Dīn remained faithful to Nur ad-Din. Upon the death of the Fatimid Caliph, at Nur ad-Din's request, Ṣalāḥ ad-Dīn reestablished the authority of the Abbasid Caliphate in Baghdad, returning Egyptian rule to Sunni Islam. Ṣalāḥ ad-Dīn's military successes against the Crusader Kingdoms won him the lasting respect of Muslims throughout this region. Following the death of Nur ad-Din in 1174, Ṣalāḥ ad-Dīn's rise to power as the first Ayyubid Sultan was more the result of the high esteem to which he was held throughout the Zangid territories than through military conquest. Like his Zangid predecessors, Ṣalāḥ ad-Dīn was Kurdish, and his superior military tactics and honorable conduct of warfare won him the lasting respect of his Christian Crusader adversaries, many of whom honored him as a paragon of knightly virtue. At the height of their power, the Ayyubids controlled a region stretching from Tripoli in the west, across the North African coastal zone, all of greater Egypt and Nubia, large portions of the Arabian Peninsula including Yemen, the Levant, Syria, al-Jazirah, and much of southeastern Turkey.

Considering the close historical connection between the Zangid and Ayyubid dynasties, it is not surprising that Ayyubid architecture and ornament was, in essence, a furtherance of the Zangid aesthetic practices and preferences.[86] The architectural attention paid to Aleppo and Damascus by the Zangids continued under the Ayyubids, and with Egypt now integrated into the sphere of Sunni influence, the new construction commissioned by Ayyubid patrons spread this distinctive style to Cairo. It was during this period that several ornamental devices became prevalent in the architecture of Cairo, including *muqarnas* vaulting and *ablaq* masonry: the bold use of alternating light and dark stone that originated in Syria in the early twelfth century became an important ornamental feature of Ayyubid, Mamluk, the Sultanate of Rum, and Ottoman architecture. And under Ayyubid patronage in Cairo, the fledgling attention paid to

[85] One possible example of the Fatimid use of the *fivefold system* is a window grille in the northeast wall at the al-Hakim Mosque in Cairo. This is an *acute* dart motif generated from just the barrel hexagon, and one of the simplest fivefold field patterns. However, it is very likely that this window grille dates to the post-earthquake restoration by Amir Baybars al-Jashankir in 1303, or the restorations by Sultan al-Nasir Hasan in 1360.

[86] For detailed accounts of Zangid and Ayyubid architecture and architectural ornament in Aleppo and Damascus, see
–Allen (1999).
–Tabbaa (2001).

geometric design during the Fatimid period received far greater prominence and influence.

The Ayyubids continued and refined the established conventions of geometric pattern making, and in this way, the *system of regular polygons*, both of the fourfold systems, as well as the *fivefold system* were all used in their architectural ornament. As well as being innovators, many examples of patterns that were used by previous Muslim dynasties were likewise incorporated into the Ayyubid ornamental milieu. These include a star-and-cross pattern on a barrel vault in the Burg al-Zafar in Cairo (1176-79) [Fig. 124b]. An Ayyubid example of the well-used pattern made from the $6^3$ underlying tessellation of hexagons that was first used at the eastern tomb tower in Kharraqan, as well as previously by both Fatimid and Zangid artists, was used on an arched portal of the Zahiriyya *madrasa* in Aleppo (1217) [Fig. 96h]. The Ayyubid use of this design is noteworthy for the highly unusual manner in which the pattern continues across the 90° change in angle between the archivolt and intrados of the arch. As per the convention established by their Zangid predecessors, this pattern is expressed in an incised line technique, and the absence of contrasting pigment within the incised lines makes this pattern relatively difficult to discern from a distance. The successful application of what would otherwise be a linear band of geometric pattern onto the curve of the archivolt requires considerable skill. This example from the Zahiriyya *madrasa* accomplishes the requisite distortion so successfully that the finished design appears completely natural, as if the pattern should always appear in this fashion. What makes this curvilinear distortion all the more remarkable is the fact that the matching pattern on the surface of the intrados is purely linear in its layout. While the geometric pattern itself is not particularly complex, and was certainly well known by the time this example was produced, the artist responsible for this archway was clearly endowed with considerable geometric skill and ingenuity.

The teakwood cenotaph at the Imam al-Shafi'i mausoleum in Cairo (1211) is decorated with two designs with 12-pointed stars made from the *system of regular polygons*. One of these is the same threefold design that was first used at the eastern tomb tower at Kharraqan (1067) [Fig. 108a] [Photograph 17]. This is a *median* pattern that is derived from the $3.12^2$ semi-regular underlying tessellation of triangles and dodecagons. This Ayyubid example from the Imam al-Shafi'i mausoleum uses the same 6-pointed star additive motif within the center of the 12-pointed stars as the Fatimid example from the Sayyid Ruqayya Mashhad in Cairo (1133). Despite the 78 years separating their production, the close physical proximity of these two Cairene examples may explain their similarity. It is the juxtaposed presence of the second pattern from the cenotaph at the Imam al-Shafi'i mausoleum in Cairo that makes these two patterns exceptional. The second pattern has fourfold symmetry, and is created from a *two-uniform* underlying tessellation in a $3.4.3.12\text{-}3.12^2$ configuration [Fig. 113a]. The edges of both the square repeat unit of the fourfold pattern and the triangle repeat of the threefold pattern have the same arrangement of edge-to-edge dodecagons, and hence the pattern lines that are generated from these tessellating dodecagons are likewise identical upon their respective repetitive edges. While it is certainly possible that the artist responsible for the cenotaph at the Imam al-Shafi'i mausoleum merely replicated these two patterns from two earlier local buildings, the remarkable concordance in the edge configuration of the respective repeat units indicates the artist's knowledge of the special geometric relationship between these two patterns, and that their selection was not coincidental. Had the artist wanted, these two repeat units could have been used together to create a single hybrid composition, and indeed several historical examples of hybrid designs made up of both square and triangular repeat units are known, and invariably, the edge configurations are, per force, identical [Fig. 23].

As stated, Ayyubid artists made use of the $4.8^2$ underlying tessellation of squares and octagons to create a particularly bold example of the classic star-and-cross *median* design that covers the surface of a barrel vault at the Burg al-Zafar in Cairo (1176-93) [Fig. 124b]. A later example of their use of this well-known pattern was used in the inlaid stone ornaments of the Sharafiyya *madrasa* in Aleppo (1242). The Firdaws *madrasa* in Aleppo (1235-36) employs an unusual variant of the classic *median* design created from this tessellation that uses 60° crossing pattern lines at the midpoints of each underlying polygonal edge [Fig. 126a]. This 60° angular opening is more commonly associated with isometric patterns that have triangles and hexagons within their underlying polygonal matrix, and the use of this angular opening within this orthogonal design produces a pleasing alternative to the standard star-and-cross design. An Ayyubid example of an *acute* pattern created from the $4.8^2$ underlying tessellation is found at the Sahiba *madrasa* in Damascus (1233-45). This differs from the standard *acute* design [Fig. 124a] through the incorporation of small eight-pointed stars within the underlying square modules [Fig. 125b].

An interesting orthogonal design with 12-pointed stars was used in the Ayyubid wooden *mihrab* (1245-46) from the Halawiyya mosque and *madrasa* in Aleppo. The underlying tessellation for this pattern places a dodecagon upon each of the four vertices of the square repeat unit. These are edge to edge with an octagon located at the center of the repeat. This arrangement of dodecagons and octagons produces concave hexagonal interstice regions [Fig. 333a]. This design is unusual in that the octagon does not play a direct formative role in deriving the pattern. Rather, the pattern lines within these polygons and the concave interstice regions are continuations of the 60° crossing pattern lines created from the dodecagons. This same underlying tessellation of edge-to-edge dodecagons and octagons will produce many very acceptable geometric designs [Figs. 379–382].

A small stone lintel over a door at the Sahiba *madrasa* in Damascus is decorated with a simple pattern derived from the *fourfold system A*. This is an *acute* field pattern that makes use of only the square and large hexagon in its underlying generative tessellation [Fig. 138a]. As with other Zangid and Ayyubid examples, the ornamental carving in this panel is incised into the stone, requiring far less time and cost than carved high relief. This same field pattern was used to decorate the *mihrab* niche of the Sharafiyya *madrasa* in Aleppo (1242). Both of these Ayyubid examples may have been inspired by the identical *acute* field pattern that was used over a century earlier in the Artuqid *mihrab* niche at the Maqam Ibrahim at Salihin in Aleppo (1112), and subsequent examples include a Seljuk Sultanate of Rum exterior faience border design at the Sirçali *madrasa* in Konya (1242).

The Ayyubids used the *fourfold system B* more widely than the *fourfold system A*. The Farafra *khanqah* in Aleppo (1237-38) employs the well-known *acute* pattern created from the *fourfold system B* that makes use of just the underlying octagons and small pentagons from this system [Fig. 173a]. This classic example from the Farafra *khanqah* is from a wooden soffit over one of the door openings. The vertical flanking panels on the *mihrab* at the Zahiriyya *madrasa* in Aleppo (1242) are decorated with the same design, as is the niche of the wooden *mihrab* (1245-46) of the Halawiyya mosque and *madrasa* in Aleppo. These three identical examples were produced in Aleppo within 10 years of one another, and it is certainly possible that a single artist was responsible for each. Over time, the popularity of this very-well-balanced pattern spread widely throughout the Islamic world. A particularly fine example of an Ayyubid *obtuse* design made from the *fourfold system B* is a very bold *ablaq* geometric pattern used at the top of the portal façade at the Palace of Malik al-Zahir at the citadel of Aleppo (before 1193[87]) [Fig. 175d]. This same pattern is found at the Taybarsiyya *madrasa* (1309) in the al-Azhar mosque in Cairo, and in the Great Mosque at Bursa in Turkey (1396-1400). The polygonal modules that make up the underlying tessellation that creates this pattern are the octagon, small hexagon, and pentagon. It is perhaps significant that this underlying tessellation was used only 7 years previously to produce a Qarakhanid *two-point* pattern at the southern anonymous mausoleum at Uzgen (1186) [Fig. 176c]. An Ayyubid example of another design created from a similar underlying tessellation created from the *fourfold system B* is from the pierced marble balustrades on the minaret of the Aqsab mosque in Damascus (1234) [Fig. 177a]. This is an *acute* pattern created from an underlying tessellation that replaces the small hexagons with the large hexagons from this system. The proportions of the pentagons in this example are specific to the arrangement of octagons and hexagons, and are unique to this single tessellation. As such, this new pentagonal element is not typical to the set of standard polygonal modules that comprise the *fourfold system B*. The most complex Ayyubid design created with this system is from the niche wall of the *mihrab* at the Imam al-Shafi'i mausoleum in Cairo (1211). This is a *two-point* pattern that is as outstanding for its beauty and ingenuity, and likely influenced the complex *two-point* aesthetic of the succeeding Mamluks. This pattern places 16-pointed stars at the vertices of the orthogonal grid, a cluster of four pentagons at the center of each square repeat unit, and an 8-pointed star within each quadrant of the repeat unit [Fig. 185b]. The eight-pointed stars are located at the vertices of the $4.8^2$ tessellation of octagons and squares, and indeed this is a governing structural feature of this design. This same underlying tessellation was used by Nasrid artists to create an equally fine *acute* pattern that was used in a cut-tile mosaic panel in the Sala de las Aleyas at the Alhambra[88] (fourteenth century).

The portal of the Malik al-Zahir in Aleppo contains a fourfold pattern created out of the arbitrary placement of six-pointed stars upon the midpoints of each edge of the square repeat units [Fig. 52a]. This is an example of a class of geometric design that imposes radial symmetry—in this case sixfold—into a repetitive structure that is generally incompatible—in this case orthogonal. The extension of the lines of the six-pointed stars creates the overall pattern matrix, and includes the four-pointed star at the center of each repeat unit, the square at the vertices of the repeat unit, and the four irregular octagons that surround the four-pointed stars. A remarkable and highly distinctive feature of this *imposed symmetry* pattern is the continuous application of the incised pattern across the dark and light colored alternating *ablaq* masonry voussoirs that surround the doorway. It is interesting to note the geometric similarity between this imposed symmetry design and the fourfold pattern from the mausoleum of Yusuf ibn Kathir in Nakhichevan, Azerbaijan (1161-62) [Fig. 52b]. The earlier

[87] For details on the dating of this portal, see Allen (1999), Chap. 5.

[88] This mosaic panel was moved to the Christian chapel of the Mexuar by Morisco artists during the sixteenth century, and now resides at the Museo Nacional de Arte Hispanomusulmán in Granada. See Dodds [ed.] (1992), 374–375.

Ildegizid design also arbitrarily places six-pointed stars in the same location of the square repeat. However, the smaller size of these stars provides for the inclusion of bounding hexagons at these same locations, which in turn creates an irregular eight-pointed star at the center of each repeat.

The use of geometric patterns in Ayyubid architecture predominantly relied upon designs created from the *system of regular polygons* and the *fourfold system B*. The extent to which this was due to aesthetic preferences or some other more prosaic reason remains unclear. It may simply be that the Ayyubid architectural designers who were most successful in receiving patronage had a comparatively limited knowledge of the broad range of available design methodologies and consequent pattern types. Whatever the reason, there are relatively few Ayyubid examples of patterns produced from the *fivefold system*, or more complex patterns created from nonsystematic underlying polygonal tessellations. One Ayyubid example of a fivefold *acute* pattern is an incised stone surround of a domical hood from a courtyard portal at the Palace of Malik al-Zahir at the citadel of Aleppo. This design has many of the characteristics of the classic fivefold *acute* pattern that repeats upon a rhombic grid [Fig. 226c]. Regrettably, very little of the original geometric panel has survived, and it is impossible to determine the full systematic character of the design, or even whether the repeat unit is rhombic or rectangular.

Among the few examples of nonsystematic Ayyubid geometric pattern is a carved stone design found in the city walls of the Bab Antakeya in Aleppo (1245-47). This is a very common *median* pattern with 8- and 12-pointed stars that was used subsequently by other Muslim cultures [Fig. 380b], but deviates from the standard design by introducing curvilinear lines within the central region of each 12-pointed star. This design was subsequently used by Ilkhanid and Timurid artists and a particularly fine later example is a Muzaffarid cut-tile mosaic panel in the lower section of the southern *iwan* at the Friday Mosque of Yezd (c. 1365).

The Ayyubids inherited the traditions of domical geometric ornament from their Zangid predecessors. One of the earliest Ayyubid geometric domes is located within the hood of the *mihrab* (before 1205) at the al-Sharafiyah *madrasa* in Aleppo. As with both the Seljuk northeast dome of the Friday Mosque at Isfahan, and the quarter dome in the *mihrab* of the Lower Maqam Ibrahim, this example from the al-Sharafiyah *madrasa* uses polyhedral geometry as the fundamental structure upon which the geometric pattern rests [Fig. 500]. Specifically, the domical portion of the *mihrab* niche is based upon a spherical projection of the octahedron. This Platonic solid is comprised of eight triangular faces, with four triangles at each of the six vertices. This quarter dome employs just two of the triangular projections of the octahedron, equaling just 1/4 of the spherical surface that results after both horizontal and vertical divisions. An inscription on this *mihrab* credits it to ʻAbd al-Salâh Abû Bakr,[89] and the concept for his design is both simple and elegant. Being that the 1/4 sphere is comprised of two projected equilateral triangles, the artist employed a two-dimensional design that uses a triangle as its repetitive unit, and applied this to the three-dimensional triangular surface of the octahedron. The two-dimensional progenitor is a well-known nonsystematic pattern that places a dodecagon on each vertex of the isometric grid, and a ring of 12 pentagons around each dodecagon, three of which are clustered at the centers of the triangular repeats [Fig. 300a *acute*]. Locations of earlier examples of this two-dimensional design include the *mihrab* of the Great Mosque at Niksar, Turkey (1145). The quarter dome in the *mihrab* of the al-Sharafiyah *madrasa* replaces the 12-pointed stars that rest upon the vertices of the two-dimensional originator with 8-pointed stars. This is due to there being just four triangles at each vertex, and each triangular corner having just two points for the star. In two dimensions, these two points are arrayed around each vertex 6 times, making 12 points, whereas on the octahedron they are only repeated 4 times, resulting in 8 points. It is interesting to note that when the same triangular repeat is applied to the 20 triangular faces that make up the icosahedron; each of the 12 vertices becomes the host for 10-pointed stars.

The geometric design in the highly refined quarter dome of the Zangid wooden *mihrab* at the Lower Maqam Ibrahim in the citadel of Aleppo was undoubtedly the inspiration for the quarter dome of the magnificent Ayyubid wooden *mihrab* (1245-46) of the Halawiyya mosque and *madrasa* in Aleppo. This later *mihrab* is the work of Abu al-Husayn bin Muhammad al-Harrani ʻAbd Allah bin Ahmed al-Najjar: a master of his art. Like the previous example from the al-Sharafiyah madrasa, the geometric design of this dome is derived from the octahedron, and likewise places eight-pointed stars at the vertices of the four repetitive triangular faces. The central region of each spherically projected triangular face is populated with nine-pointed stars, and the eight- and nine-pointed stars are separated by a network of five-pointed stars and darts [Fig. 501]. The earlier Seljuk and Zangid polyhedral designs were derived from the actual faces of their respective polyhedra—in effect, using the polyhedral faces as the underlying generative tessellation. By contrast, both of these Ayyubid domical hoods employ their projected repetitive faces as a substructure for their applied underlying generative tessellation. This is a spherical analogue to the very common application of underlying generative tessellations to triangular repetitive cells on the two-dimensional plane. Indeed, when used two-dimensionally, the pattern contained within each

[89] Allen (1999), Chap. 8.

triangular repeat from the domical hood in the mihrab of the Halawiyya mosque and madrasa produces a very successful design with 9- and 12-pointed stars that was used by Mengujekid artists at the Great Mosque and hospital of Divrigi (1228-29) less than twenty years previous [Fig. 346a]. As with the example from the al-Sharafiyah madrasa the 12-pointed stars of the two-dimensional analogue are replaced with eight-pointed stars by virtue of there being four triangles at each vertex rather than six.

An Ayyubid quarter dome hood that rests upon two tiers of *muqarnas* in the stone portal of the Farafra *khanqah* in Aleppo (1238) is decorated with an incised geometric design that is based upon the cubeoctahedron. The vertices of the cubeoctahedron have bilateral symmetry, and are composed of two opposing squares that alternate with two opposing triangles. This pattern places a six-pointed star at each vertex, and orientates the star to coincide with the bilateral symmetry of the vertex. The placement of the six-pointed stars onto the vertices creates a subtle incompatibility wherein the symmetry of the sixfold division does not precisely reconcile with the rigid symmetry of the cubeoctahedron. The pattern lines that stem from, and interact with, these six-pointed stars are consequently ill suited to comfortably fill the regions defined by the projected squares and triangles of the cubeoctahedron. This creates noticeable irregularities that result in a design that, however ambitious, is considerably less elegant than that of the quarter dome of the wooden *mihrab* (1245-46) at the Halawiyya mosque and *madrasa*.

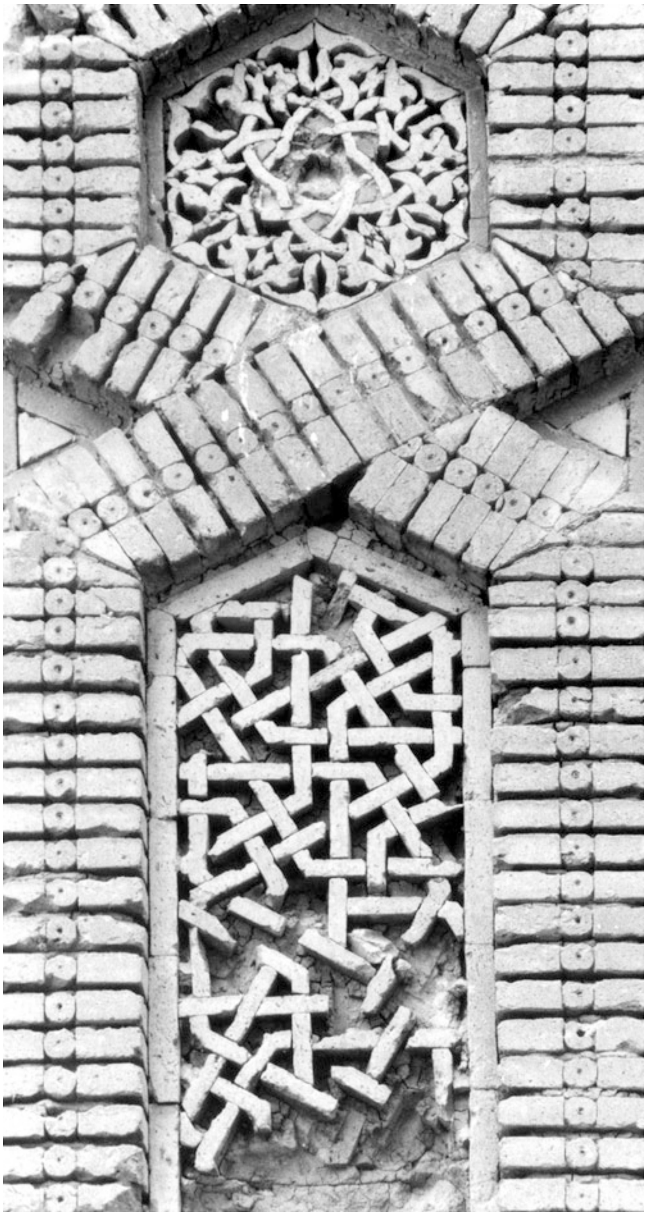
## 1.19 Khwarizmshahs (1077-1231)

The Khwarizmshah lineage is traced back to pre-Islamic times. Their homeland is the vast Khwarizm oasis formed by the Amu Darya delta immediately south of the Aral Sea in present-day Turkmenistan and Uzbekistan. Throughout the reign of the Samanids, Qarakhanids, Ghaznavids, Seljuks, and Qara Khitai they maintained their rule over Khwarizm through military strength and by accepting the suzerainty of sequential empires. As vassals to the Seljuks and Qara Khitai they became immensely powerful. During the last decade of the twelfth century the Khwarizmshahs defeated the Qara Khitai forces in Transoxiania, and within decades they came to rule over an immense territory that stretched from Azerbaijan and portions of Iraq in the west, across all of Persia, through all of Afghanistan in the east, and Transoxiana in the north. Within just 5 years of their victory over the Ghurids in Khurasan, in 1220 the Mongol onslaught brought the Khwarizmshah Empire to a crushing defeat.

Few extant examples of Khwarizmshahid architecture remain, but those that have survived clearly indicate the influence of Seljuk aesthetics. Khwarizmshahid architectural decoration continued the Seljuk practice of raised brick and faience ceramics. Indeed, both the mausoleum of Sultan Tekesh in Konye-Urgench, Turkmenistan (c. 1200), and the Zuzan *madrasa* in northeastern Iran (1219) are notable for their exuberant use of these ornamental media. The Zuzan *madrasa* is one of the earliest extant Muslim buildings to have expanded the use of ceramic faience ornament beyond a single color. The color palette of this building includes turquoise, dark blue, and white combined with unglazed terra-cotta. This building is also remarkable for the monumental scale of the *iwan*: prefiguring the architectural predilections of later Muslim cultures of this region. The exterior façade of the Zuzan *madrasa* has two interesting patterns made from the *system of regular polygons* and executed in raised brick. One of these is a *two-point* pattern with threefold symmetry that is created from the 4.6.12 semi-regular underlying tessellation of squares, hexagons, and dodecagons. This pattern is a remarkable concatenation of interweaving dodecagons, octagons, and arbitrary six-pointed stars [Fig. 109e]. The second is also a *two-point* pattern comprised from a *two-uniform* underlying tessellation made up of hexagons, squares, and triangles in an orthogonal $3^2$.4.3.4-3.4.6.4 configuration [Fig. 90]. This design from the Zuzan *madrasa* repeats upon a root-4 (double-square) rectangular grid [Fig. 112b] [Photograph 38]. Were it not for the 90° rotation of the underlying hexagonal modules, and the alternating orientation of the six-pointed stars that results from this rotation, this noteworthy pattern would repeat on a standard square grid. A very similar design, albeit with a significant difference in the line treatment, was used by post-Ilkhanid artists in the mausoleum of Muhammad Basharo in the remote village of Mazar-i Sharif in Tajikistan[90] (1342-43) [Fig. 111a]. The similarity in aesthetic character between these two *two-point* designs results from their identical pattern line application to the square modules of the underlying tessellation. Both utilize *two-uniform* tessellations with $3^2$.4.3.4-3.4.6.4 vertices, the difference being that the *two-uniform* tessellation from the Zuzan *madrasa* is orthogonal, while that of the mausoleum of Muhammad Basharo is isometric.

The ornament of the Zuzan *madrasa* includes a round faïence panel with a highly original geometric design that places octagons upon the vertices of the $3^2$.4.3.4 tessellation of triangles and squares [Fig. 103] [Photograph 39]. The two edge-to-edge triangles produce an underlying rhombic component, and their rotational orientation around each square element produces the repetitive structure common to all

[90] Not to be confused with the city of Mazar-i Sharif in Afghanistan. The village of Mazar-i Sharif, where the Mausoleum of Muhammad Basharo is sited, is approximately 25 km east of Penjikent, Tajikistan, and located in the Zarafshan River valley.



**Photograph 38** A Khwarizmshahid *two-point* pattern created from the *system of regular polygons* located at the Zuzan *madrasa* in Iran (© Sheila Blair and Jonathan Bloom)

**Photograph 39** A Khwarizmshahid oscillating square pattern created from the *system of regular polygons* located at the Zuzan *madrasa* in Iran (© Caroline Mawer)

oscillating square designs [Figs. 23–26]. Patterns based upon the $3^2.4.3.4$ tessellation are unusual, and more readily place 12-pointed stars at the vertices; such as an example from the Topkapi Scroll[91] [Fig. 23d]. The use of octagons at the vertices of this tessellation from the Zuzan *madrasa* does not readily conform to the angles of the underlying vertex configuration, and the success of their use is reliant upon the bilateral symmetry at each vertex.

Among the multiple roundel motifs that make up the highly elaborate interior frieze at the Zuzan *madrasa* are two identical *obtuse* patterns, with radial symmetry, created from the *fivefold system* [Fig. 260b] [Photograph 40]. As is often the case with fivefold *obtuse* patterns, this design can be created from two distinct underlying tessellations: one that places pattern lines with 108° angular openings into underlying network of a central decagon surrounding by pentagons, barrel hexagons, and thin rhombi, and the other with 72° angular openings applied to an underlying tessellation comprised of a central decagon surrounded by long hexagons and concave hexagons from the *fivefold system*. *Obtuse* patterns from this system invariably have 108° angular openings, while those with 72° are typically associated with the *median* family. However, in tessellations comprised of just decagons, long hexagons, and concave hexagons, and devoid of pentagons and barrel hexagons in particular, the 72° crossing pattern lines replicate the distinctive character of the *obtuse* family. For this reason, regardless of the specific underlying generative tessellation that the artist employed to create this example, it is rightfully identified as an *obtuse* design.

[91] Necipoğlu (1995), diagram no. 35.

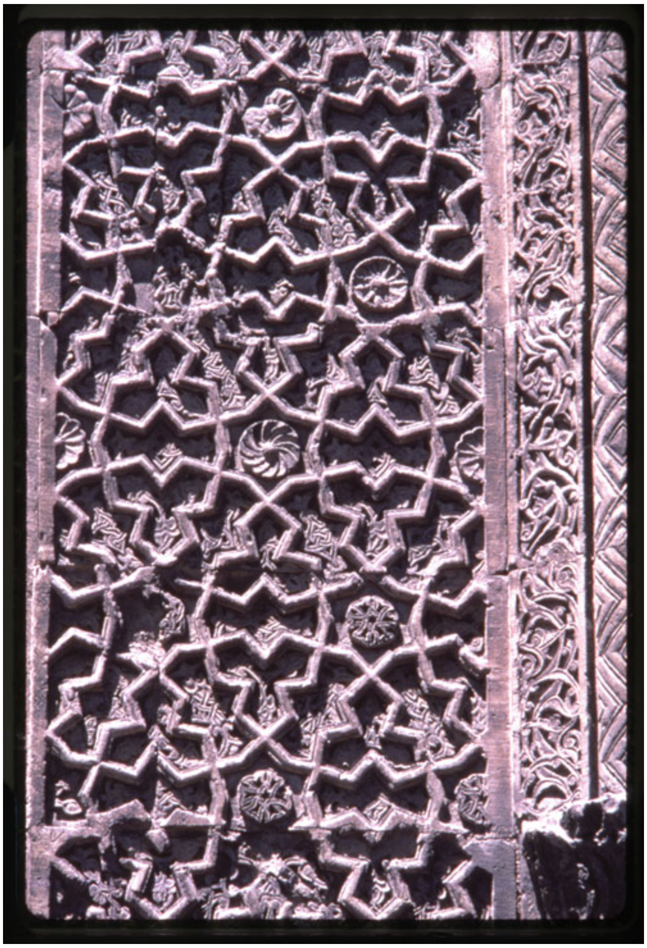
**Photograph 40** A Khwarizmshahid *obtuse* pattern created from the *fivefold system* located at the Zuzan *madrasa* in Iran (© Caroline Mawer)

## 1.20 Seljuk Sultanate of Rum (1077-1307)

In 1071 the Seljuks defeated the Byzantine forces at the battle of Manzikert, near Lake Van in eastern Anatolia. Within a decade, the Seljuk military commander Suleyman bin Kutalmish (d. 1086) declared himself Sultan over the conquered Byzantine territories in Anatolia, and established his capital in Isnik. By the middle of the twelfth century, from their capital in Konya, the Seljuk Sultanate of Rum controlled all of central Anatolia and portions of both the Mediterranean and Black Sea coastlines. In the thirteenth century, at the height of their power, the Seljuk Sultanate of Rum averted the loss of their empire by becoming vassals to the conquering Mongols. As such, they prevented the high level of destruction that befell so many of the great cities and centers of culture across Iraq, Persia, Khurasan, and Transoxiana. In marked contrast to the Mongol destruction in the eastern regions, the Seljuk architectural heritage in Anatolia remained relatively intact, and many of the most significant extant examples of Seljuk geometric ornament are from this region. Anatolia fell from Seljuk rule through internal strife and the rising power of rival dynasties: the last Seljuk Sultan of Rum being murdered in 1307.

The architecture of the Sultanate of Rum is predominantly stone. This was the material of choice for the Christian population of Anatolia and Armenia. The Seljuk conquerors of these regions readily adopted the highly evolved stone masonry practices of their new subjects. Over time, the ornamental use of stone in the architecture of the Sultanate of Rum developed a level of sophistication that is unsurpassed in the historical use of this material. Like the brick architecture of the eastern regions, the carved stone ornamental façades built by Seljuk artists in Anatolia were primarily monochrome with high relief; creating an aesthetic effect of stunning boldness that was largely reliant upon texture, light, and shadow. Carved stone provided an ideal medium for monumental epigraphy, floral design, and complex geometric patterns. The Seljuk architecture of Anatolia continued the practice of augmenting important parts of a building with colorful faience mosaics. The faience *mihrab* and dome of the Alaeddin mosque in Konya (c. 1219-21) is lavishly decorated with floral and geometric designs, as well as both cursive and knotted Kufi script. The density of the faience, nearing total coverage, is a forerunner of the *muarak* cut-tile mosaic aesthetic that came to prominence under the auspices of the Muzaffarids and Kartids approximately a century later in Persia and Khurasan respectively, from whom it was readily adopted by the Timurids. Other superb examples of faience mosaic from Seljuk Anatolia are found at the Izzeddin Kaykavus hospital and mausoleum in Sivas (1217) and the Sirçali *madrasa* in Konya (1242). Rather than being used as an accent for especially important parts of a building, such as a *mihrab*, the faience mosaics in both of these latter examples is applied throughout the main façade of the building. These are examples of transformative architectural ornament, creating a polychromatic aesthetic from what had been primarily monochromatic.

The architectural ornament of the Seljuk Sultanate of Rum exhibits multiple examples of geometric patterns that were used earlier in the eastern regions, including patterns made from the *system of regular polygons*. Several of these are sufficiently unusual as to strongly suggest direct inspiration rather than their independent re-creation, thereby substantiating the westward transmission of methodological knowledge. A possible example of such influence is an ornamental panel from the Izzeddin Kaykavus hospital and mausoleum in Sivas, Turkey (1217) [Fig. 106a], that closely resembles one of the patterns created from the 3.4.6.4 underlying tessellation at the Qarakhanid anonymous tomb at Uzgen, Kyrgyzstan (1186) [Fig. 104d]. The example from Uzgen was produced some 30 years earlier. Another unusual pattern made from the *system of regular polygons* that was used in both Khurasan and Anatolia is derived from the unusual pattern line extraction process wherein alternating underlying hexagons from the 3.6.3.6 tessellation are treated differently. The narrow border pattern that surrounds the door at the Seh Gunbad tomb tower in Orumiyeh, Iran (1180) [Fig. 101a] is also found at the Great Mosque of Niksar, Turkey (1145), and an outstanding variation is



**Photograph 41** A Seljuk Sultanate of Rum threefold pattern that is easily created from the *system of regular polygons* located at the Çifte Minare *madrasa* in Sivas, Turkey (courtesy of the Yasser Tabba Archive, Aga Khan Documentation Center at MIT)

from the portal of the Çifte Minare *madrasa* in Sivas, Turkey (1271) [Photograph 41]. The only differences between these examples are slight variations in the angles of the pattern lines, and the absence of the small triangle centered within the trefoil elements. This difference is purely stylistic, and results from an arbitrary aesthetic determination on the part of the artist rather than directly determined by the design methodology. Further evidence of the westward migration of artistic practices comes from an inscription on a faience panel at the Sirçali *madrasa* in Konya, Turkey (1242), wherein it is stated "made by Muhammad, son of Muhammad, son of Othman, architect of Tus." This artist traveled from Khurasan to Anatolia during the period of instability following the Mongol invasion.[92] During this tumultuous period, the exodus of those artists who were privy to the methodology used to create complex geometric designs shifted the nexus of innovation firmly westward. As discussed above, during the twelfth century knowledge of the polygonal technique for designing geometric patterns had spread from Khurasan and Transoxiana westward into Ildegizid Azerbaijan, Seljuk and Artuqid Anatolia, Seljuk Iraq and al-Jazirah, as well as Egypt, Syria, and North Africa under successive Fatimid, Zangid, and Ayyubid rule. During the period of destruction, and the destabilized aftermath wrought by the Mongols throughout Transoxiana, Khurasan, Persia, and Iraq during the greater part of the thirteenth century, the western regions that either accepted the suzerainty of their Mongol overlords or successfully repelled the Mongols militarily became the primary benefactors of this ongoing design tradition. The Seljuk Sultanate of Rum and their rivals, the Mamluk Sultanate in Egypt were of particular importance in this ongoing westward growth in interest in the further development of Islamic geometric art.

Along with more complex varieties of geometric pattern, the *system of regular polygons* received considerable innovative attention from artists working in Anatolia during the Sultanate of Rum. The considerable quantities of monuments that have survived from Seljuk Anatolia provide a remarkable diversity of Islamic geometric patterns. Within just the *system of regular polygon*s there are dozens of beautiful patterns originating from this cultural milieu. A distinctive design that can be conveniently created from the underlying $6^3$ hexagonal grid was used at the Great Mosque at Divrigi (1228-29) [Fig. 96i]. This design was used with some frequency by artists in Anatolia, including in the carved stone façade of the Hatuniye *madrasa* in Karaman (1382). A more complex design also created from the simple $6^3$ grid is found in the south portal of the Great Mosque at Bayburt[93] (1220-35) [Fig. 97b]. This pattern is characterized by superimposed interweaving nonagons, set in rotation around the vertices of the visually exposed hexagonal grid, producing six-pointed stars at each vertex of the isometric dual grid. The same design was used as a border design at the Çifte Minare *madrasa* in Erzurum (late thirteenth century), and in the faience mosaic *mihrab* of the Ahi Serafettin mosque in Ankara (1289-90). A faience mosaic panel from the Ali Tusin tomb tower in Tokat, Turkey (1233-34), employs an isometric design that is easily created from the underlying 3.6.3.6 tessellation of hexagons and triangles[94] [Fig. 99f]. This is a very simple and well-balanced design comprised of meandering lines that do not lead back to themselves to make closed circuits. It is surprising that Muslim geometric artists did not use this very becoming pattern more frequently. The *mihrab* of the Great Mosque of Siirt (1129) uses an isometric design produced from the

[92] Wilber (1939), 40.

[93] Schneider (1980), pattern no. 198.

[94] Schneider (1980), pattern no. 250.

3.6.3.6 tessellation that is made up of two interlocking elements: the six-pointed star and a motif with threefold rotation symmetry [Fig. 100a]. This distinctive pattern is part of a group of similar interlocking isometric designs found within the historical record. A hallmark of these designs, including this example, is their ability to easily be created from the isometric grid [Fig. 73a]. As is often the case, it is impossible to know for certain which methodology the artist working in Siirt used to create this fine pattern. A much later example of this interlocking design is found at the tomb of I'timad ad-Dawla in Agra, India (c. 1628-30). Another example of an Anatolian pattern made from the 3.6.3.6 underlying semi-regular tessellation was employed on the northern portal at the Great Mosque at Divrigi[95] (1228-29) [Fig. 100c]. This is a Mengujekid monument that is noteworthy for the unique and highly elaborate baroque quality of the floral ornament. Yet the many examples of geometric design within this building conform to the aesthetic practices of their neighboring artists working under the patronage of the Seljuk Sultanate of Rum. A later Seljuk example of this pattern was used at the Muzaffer Barucirdi *madrasa* in Sivas (1271-72). A design that uses alternating active and passive hexagons and triangles from the 3.6.3.6 underlying tessellation was widely used by artists working in the Seljuk Sultanate of Rum [Fig. 101c]. Examples include the portal of the Great Mosque of Niksar (1145); the portal of the Alaeddin mosque in Konya (1219-21); and the Huand Hatun complex in Kayseri (1237). This design was also used at the Zangid portal of the Bimaristan Arghun at the citadel of Aleppo (twelfth century) [Photograph 36].

A well conceived 3.4.6.4 pattern from the north portal of the Sungur Bey mosque in Nigde (1335) places nonagons upon the vertices of the hexagonal grid[96] [Fig. 107c]. These nonagons are located on the centers of the triangles of the underlying 3.4.6.4 tessellation, with superimposed hexagons added into the pattern matrix. It is interesting to note that the perpendicular arrangement of the *two-point* pattern lines that cross the shared edges of the triangular and square modules produces a similar aesthetic to the design created from the same 3.4.6.4 underlying tessellation at the Chaghatayid mausoleum of Tughluq Temür in Almaliq, western China (1363) [Fig. 105d] [Photograph 70]. A border pattern in the east *iwan* of the Çifte Minare *madrasa* in Erzurum (late thirteenth century) employs a very successful isometric pattern made from the 3.4.6.4 underlying tessellation of triangles, squares and hexagons.[97] This design is conceptually similar to the above referenced pattern at the Great Mosque at Bayburt that places six nonagons in rotation around each hexagonal repeat [Fig. 97b]: the difference being that the design from the Çifte Minare *madrasa* places six *octagons* around each hexagon. This is achieved by locating the center of each octagon upon the center of each underlying square of the 3.4.6.4 tessellation [Fig. 107a]. The application of octagons into an isometric repetitive structure qualifies this as an *imposed symmetry* design wherein a star or polygon with an *n*-fold rotational symmetry—in this case eightfold—is placed within a seemingly incompatible repetitive structure—in this case isometric. A 3.4.6.4 design from the Cincikh mosque in Aksaray (1220-30) [Fig. 107b] is conceptually similar to the above-cited example from the Çifte Minare *madrasa*: a similar distribution of superimposed octagons has hexagons added into the pattern matrix.[98] The simple addition of the hexagons augments the complexity considerably, and radically changes the overall appearance of the design. Another innovative Anatolian geometric pattern created from the 3.4.6.4 underlying tessellation is from the portal of the Gök *madrasa* and mosque in Amasya, Turkey (1266-67).[99] The angular openings of the applied pattern lines of this design are determined by the placement of regular octagons within the square modules of the underlying tessellation [Fig. 104b]. This lovely design was used nearly 240 years later in the stone mosaic ornament in the *mihrab* niche of the Sultan Qansuh al-Ghuri complex in Cairo (1503-05).

Patterns with 12-pointed stars created from underlying dodecagons were also widely used by the Seljuk Sultanate of Rum. Two of the earliest Anatolian examples of the well-known threefold pattern created from the underlying semi-regular $3.12^2$ tessellation are found at the Great Mosque of Kayseri (1205) and the *mihrab* of the Great Mosque of Akşehir near Konya (1213) [Fig. 108a]. These were preceded by the example of this pattern at the eastern tomb tower at Kharraqan by approximately 150 years [Photograph 17]. Several nearly identical examples of an isometric pattern with 12-pointed stars that are easily created from the 3.4.6.4 underlying tessellation are found in the city of Ahlat on Lake Van in eastern Turkey. These include examples from the Usta Sagirt tomb[100] (1273) [Fig. 105g], the Huseyin Timur tomb[101] (1279) [Fig. 105i], and a number of the highly ornamented gravestones for which this town is renowned. A variation of this pattern from the Seljuk Sultanate of Rum was widely used in the Maghreb [Fig. 105h].

[95] Schneider (1980), pattern no. 206.

[96] Schneider (1980), pattern no. 226.

[97] Schneider (1980), pattern no. 227.

[98] Gerd Schneider illustrates the close relationship between these examples in his patterns 227 and 228. However, in keeping with the totality of his illustrations, he does not provide generative methodologies or underlying formative structures. Schneider (1980).

[99] Schneider (1980), pattern no. 215.

[100] Schneider (1980), pattern no. 397.

[101] Schneider (1980), pattern no. 403.

This places an arbitrary eight-pointed star within the underlying square elements. Among the many examples of this variation are a *zillij* mosaic panel at the Alcazar in Seville (fourteenth century), and the carved stucco ornament from the Córdoba Synagogue (1315). Each of these *two-point* patterns is constructed in an identical manner, and only differs in the arbitrary stylistic treatment of the pattern elements within the underlying square and triangular modules. The construction of this particular group of 3.4.6.4 patterns is not limited to the polygonal technique, and can also be produced using the methodology of *extended parallel radii*. Artists working under patronage of the Seljuk Sultanate of Rum were pioneers of this alternative technique for generating geometric patterns; and though this methodology was never as widely utilized as the polygonal technique, it is certainly possible that the multiple examples of these designs from Ahlat were generated using this alternative technique [Figs. 80 and 81]. Patterns created from the 4.6.12 tessellation are less common, and two fine examples include a pattern from the drum of the dome of the Izzeddin Keykavus hospital and mausoleum in Sivas[102] (1217-18) [Fig. 109d] and an Ottoman carved stone relief panel from the Hasbey Darül Huffazi *madrasa* in Konya[103] (1421) [Fig. 109a]. A very fine orthogonal pattern created from the 3.4.3.12-3.12$^2$ *two-uniform* tessellation was used at the Hasbey Darül Huffazi Han near Kayseri (1235-41) [Fig. 113c]. Other examples of this pattern from the Seljuk Sultanate of Rum are found at the Sultan Han in Kayseri (1232-36), and the tomb of Sultan Mesud in Amasya (fourteenth century). This pattern was used at an earlier date on the side panels for a Zangid *minbar* commissioned by Nur al-Din in 1186, as well as many subsequent examples produced by Mamluk artists.

The high degree of artistic innovation during the Sultanate of Rum is reflected in a group of patterns created from the *system of regular polygons* that employ non-regular modules within their otherwise fully regular underlying tessellations. An orthogonal design from the *mihrab* of the Alaeddin mosque in Konya (1219-21) is the earliest of many Sultanate of Rum examples of an unusual fourfold pattern made up of dodecagonal underlying polygonal elements that are filled in with four ditrigonal shield shapes and four triangles[104] [Fig. 120]. Without the ditrigons and triangular inclusions, this would be the 3.4.3.12-3.12$^2$ *two-uniform* generative tessellation that was commonly used for creating Islamic geometric patterns [Fig. 113]. This same design was used some 40 years earlier at the Shah-i Mashhad and the minaret of Jam in Afghanistan: providing further evidence for the westward dissemination of specific designs into the eastern areas of Seljuk influence. A fine example of a threefold pattern that incorporates this same ditrigonal module within the underlying generative tessellation is located in the *mihrab* of the Yelmaniya mosque in Cemiskezck, Turkey[105] (1274) [Fig. 117]. The underlying generative tessellation for this pattern is comprised of dodecagons, squares, and triangles, with irregular ditrigons at the center of each triangular repeat unit. This same design was subsequently used in a number of historical locations, including the *mihrab* of the Aqburghawiyya *madrasa* (1340) at the al-Azhar mosque in Cairo and the Ottoman ornament of the Dome of the Rock in Jerusalem. Another threefold pattern that uses this ditrigonal module in its underlying generative tessellation is from the portal of the Huand Hatun complex in Kayseri (1237), and was also used in the *mihrab* of the Ahi Serafettin mosque in Ankara (1289-90) [Fig. 118c]. This design can also be constructed using the $6^3$ tessellation of hexagons [Fig. 97a]. The ditrigons in this generative tessellation are identically proportioned to those from the Yelmaniya mosque in Cemiskezck. However, in this design the angles of the applied crossing pattern lines are determined by the placement of octagons upon the underlying polygonal vertices with 90° angles. The underlying tessellation that produces this design is actually a well-known *median* pattern in its own right: created from the simple hexagonal grid [Fig. 95c]. This use of an existing geometric pattern as an underlying generative structure is unusual, but not unique. An Ottoman design from the *mihrab* of the Yesil mosque in Bursa (1419-21) is identical to the example from the Huand Hatun complex except that six-pointed stars arbitrarily replace the hexagonal elements within the pattern matrix. A very similar pattern created from the same underlying tessellation to the example from the Huand Hatun complex is from one of the Mamluk window grilles at the al-Anzar mosque in Cairo [Fig. 118b]. A further example from the Karatay *madrasa* in Antalya, Turkey (1250) employs a variation within the underlying six-pointed star module that is derived from the inclusion of pentagons and a central hexagon into the underlying tessellation [Fig. 118d]. Another pattern from the Seljuk Sultanate of Rum that adds non-regular modules into the underlying tessellation of regular polygons is from the tomb of Seyit Mahmut Hayrani in Aksehir near Konya[106] (1275) [Fig. 122]. This elegant orthogonal pattern is created from an arrangement of underlying squares and triangles such that the interstice regions provide for the creation of four coinciding irregular pentagons. The cluster of four pentagons has shared characteristics with some *fourfold system B* designs, except

[102] Schneider (1980), pattern no. 256.
[103] Schneider (1980), pattern no. 413.
[104] Schneider (1980), pattern no. 298.
[105] Schneider (1980), pattern no. 441.
[106] Schneider (1980), pattern no. 325.

that the proportions of the pentagons are slightly different, and do not tessellate systematically.

As with other Muslim cultures, patterns that relate to the $4.8^2$ underlying tessellation were commonly used by the Seljuk Sultanate of Rum. An otherwise standard *acute* pattern [Fig. 124a] from the minaret of Hotem Dede in Malatya (twelfth century) radically alters the appearance of this standard design through a subtractive variation [Fig. 125a], while the additive variation of this same standard design from the Sirçali *madrasa* in Konya (1242-45) likewise alters the visual quality of the original design significantly [Fig. 125d]. Several unique, if very simple, *two-point* designs were also likely produced from this tessellation, including a pattern comprised of superimposed four-pointed stars from the Karatay *madrasa* in Konya (1251-55) [Fig. 128c]; a pattern made up of two sizes of octagons from the Divrigi hospital (1228-29) [Fig. 128e]; and a pattern composed of superimposed concave hexagons from the Great Mosque of Divrigi (1228-29) [Fig. 128f]. Another example from the Great Mosque of Divrigi is an *obtuse* pattern that employs arbitrarily placed eight-pointed stars into the underlying octagonal regions [Fig. 127d]. Fatimid artists used this pattern over half a century earlier in the wooden *mihrab* of the mausoleum of Sayyidah Nafisah in Cairo (1138-46), and a later Mamluk example was used on the upper shaft of a minaret at the Sultan Qaytbay funerary complex in Cairo (1472-74).

The architectural ornament of the Seljuk Sultanate of Rum made wide use of the *fourfold system A*, including examples that were already known in the eastern regions, and many that the historical record suggests were original to artists working in Anatolia. The least complex designs created from this system are field patterns. These eschew the use of the underlying large octagons, and hence have no eight-pointed stars within their pattern matrix. As discussed, the earliest examples of field patterns created from the *fourfold system A* were produced by Ghaznavid, Qarakhanid, Ghurid, and Seljuk artists working in the eastern regions, and artists working under the patronage of the Seljuk Sultanate of Rum readily adopted this category of geometric design. Examples of such field patterns include multiple examples of a very basic pattern comprised of interlocking concave octagons, the earliest example of which is found in the *mihrab* of the Great Mosque of Niksar (1145) [Fig. 136b]. This pattern is generated from a tessellation of just the small hexagons from the *fourfold system A*. At least two field patterns were used during the Seljuk Sultanate of Rum that are easily produced from an underlying tessellation of large hexagons and squares. Each was used in multiple locations over a wide span of time. The first of these is an *acute* pattern that was used at the Tepsi minaret in Erzurum (1124-32). This was built by the Saltukids within decades of its first use by the Artuqids at the Maqam Ibrahim at Salihin in Aleppo (1112) [Fig. 138a]. The Saltukids were an Anatolian *beylik* that were allied with the Great Seljuks in Persia, and who were overthrown by the rise of the Seljuk Sultanate of Rum. The second pattern created from this underlying tessellation is a well-known *median* design that was used throughout the Islamic world. Among its earliest locations in Anatolia is in the portal of the Alay Han near Aksaray (1155-92) [Fig. 138c]. A variation of this *median* pattern was used in the *mihrab* of the Great Mosque of Erzurum (1179) [Fig. 139]. A *median* field pattern that can be created from an underlying tessellation of large hexagons and pentagons was also used widely by artists in Anatolia [Fig. 140]. The earliest known example is from the Sultan Han in Kayseri (1232-36). Generally, the greater the number of underlying polygonal modules used to create a design the more complex the resulting pattern. A very successful *median* field pattern from the Huand Hatun complex in Kayseri (1237) makes use of four different underlying modules, and is one of the more complex field patterns created from the *fourfold system A* [Fig. 141].[107] This design was used soon after at the Haci Kiliç mosque and *madrasa* also in Kayseri (1249), but its earliest use appears to have been by Ildegizid artists at the mausoleum of Yusuf ibn Kathir in Nakhichevan (1161-62).

Original Seljuk Sultanate of Rum patterns created from the *fourfold system A* include a large number of *median* designs with 90° crossing pattern lines placed at the midpoints of each underlying polygonal edge. This provides these patterns with similar features that are easily recognized as a family. Examples of this family include a design from the Great Mosque at Erzurum (1179) that was used subsequently in many locations in Anatolia[108] [Fig. 143a]; a border design on the exterior façade of the Alay Han, 35 km northeast of Aksaray, Turkey[109] (1155-92) [Fig. 148]; a border design placed around the drum of the dome at the Gök *madrasa* in Amasya (1266-67) [Fig. 143b]; and a wooden screen railing in the Esrefoglu Süleyman Bey in Beysehir, Turkey (1296-97) [Fig. 142]. An original *acute* carved stone border from the portal façade of the hospital at the Çifte *madrasa* in Kayseri[110] (1205) is unusual in that it uses two types of crossing pattern line. This design can be created from either of two underlying tessellations [Figs. 146 and 147a]. Interestingly, these are not duals of one another. A variation of this design was used at the Friday Mosque in Gonabad, Iran (1212), some 7 years after the example from Kayseri. The distinctive quality of this design, and their close dates of production, implies a direct causal relationship between these two examples of this pattern. A fine example

[107] Schneider (1980), pattern no. 281.
[108] Schneider (1980), pattern no. 302.
[109] Schneider (1980), pattern no. 303.
[110] Schneider (1980), pattern no. 306.

of a unique Sultanate of Rum *two-point* pattern was used on the *iwan* of the Sirçali *madrasa* in Konya[111] (1242) [Fig. 152]. This example is unusual in that the edge length of the underlying octagons matches the longer rather than the smaller edge of the triangular module from this system, resulting in a proportionally larger underlying octagonal module and consequent eight-pointed star motif. This same underlying tessellation was used 63 years earlier by Qarakhanid artists to create a pattern comprised of two sizes of superimposed octagons at the Maghak-i Attari mosque in Bukhara, Uzbekistan (1178-79) [Fig. 151].

All of these examples repeat upon the standard orthogonal grid with eight-pointed stars located at the vertices of the repeat. By contrast, a remarkable orthogonal *median* pattern created from the *fourfold system A* located in the *mihrab* in the mosque of the Huand Hatun complex in Kayseri[112] (1237) places the primary eight-pointed star motifs at the vertices of a square and rhombus symmetrical structure rather than at the vertices of the more broad scaled square repeat [Fig. 156]. The 64.4712. . .° and 115.5288. . .° included angles of the rhombi are eccentric, and do not readily conform to eightfold geometry, yet in this instance they combine with the square repetitive cells to produce a design that is as visually successful as it is unusual. These rhombi are placed in rotation around each square in the same manner as the $3^2$.4.3.4 semi-regular tessellation of regular triangles and squares [Fig. 89]. The application of the underlying polygonal modules from the *fourfold system A* to this repetitive structure is unconventional in two respects: (1) the eight-pointed stars within the pattern matrix are generated with two alternating underlying polygonal arrangements, and (2) the layout of the underlying tessellation does not have coinciding edges that symmetrically align with the square and rhombic coinciding edges. In contradistinction to other patterns that have square and rhombic hybrid structures—such as the aforementioned design from the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijanin [Fig. 182]—the layout of the underlying polygonal modules of this design from the Huand Hatun mosque prevents either the squares or rhombic elements from functioning as repeat units on their own. Another unusual feature of this design is the discrepancy between the *plane symmetry group* of the underlying polygonal tessellation and that of the pattern itself. Ordinarily, both the underlying tessellation and its generated design will share an identical plane symmetry group (unless and until a design's crossing pattern lines acquire chirality through their being provided with an interweaving treatment). The underlying tessellation that creates this design from the Huand Hatun falls into the *cmm* plane symmetry group, while the pattern itself is in the *p4g* group. These unique geometric characteristics qualify this example as perhaps the most symmetrically complex pattern created from the *fourfold system A* throughout the length and breadth of the Islamic ornamental tradition. However, despite this extremely eccentric geometric character, it is nonetheless very balanced and pleasing to the eye.

Artists working under the auspices of the Sultanate of Rum experimented with geometric patterns that employ an additive swastika device within the square components of particular designs. Most of these are based upon patterns that were created from the *fourfold system A*, although this same additive device was also applied to other varieties of design: the operative qualifier being the presence of squares within the pattern matrix. Most of the Anatolian designs with this variety of additive treatment are rather simplistic,[113] but a particularly sophisticated example was used in the faïence ceramic ornament of the Karatay *madrasa* in Konya[114] (1251-55) [Fig. 150a]. This variety of additive motif became especially popular under Timurid patronage.

In keeping with Zangid and Ayyubid practices to the south, and unlike the aesthetic predilections of the Great Seljuks to the east, the *fourfold system B* was more widely used than the *fourfold system A* under the patronage of the Sultanate of Rum. The classic *acute* pattern from this system [Fig. 173a] was used ubiquitously[115], with the earliest known Anatolian example located on the *minbar* of the Great Mosque of Aksaray (1150-53). This was produced within two decades of its first apparent use in the arch spandrels of the *mihrab* at the Friday Mosque at Sin, Iran (1134). This design uses only the underlying octagons and pentagons from this set of modules. An *acute* design that is similar in appearance, but slightly more complex, was used at the Sultan Han in Aksaray[116] (1229) [Fig. 177a]. The underlying tessellation that creates this pattern incorporates the same octagons and pentagons, but with elongated hexagons separating the octagons. Tessellations with this configuration of polygons, albeit with variable proportions to the hexagonal module, were used to create a wide variety of patterns in each of the four pattern families [Figs. 175–178]. The Zangid artists who produced the *minbar* (1187) for the al-Aqsa mosque used this same underlying tessellation for the notable *acute* pattern that adorn the triangular side panels [Fig. 177b]. The difference between the visual characteristics of the earlier Zangid *acute* pattern and that

[111] Schneider (1980), pattern no. 236.

[112] Schneider (1980), pattern no. 330.

[113] Schneider (1980), pattern numbers 91–98.

[114] Schneider (1980), pattern no. 91.

[115] Schneider (1980), pattern no. 321. Schneider identifies no less than 38 examples of this pattern scattered throughout the many monuments built by the Sultanate of Rum: pp. 183–4.

[116] Schneider (1980), pattern no. 217.

of the Sultan Han results from the treatment of the applied pattern lines to the underlying elongated hexagonal modules [Fig. 172], and the treatment of the pattern lines within the cluster of four pentagons at the center of the square repeat unit. Another popular Anatolian Seljuk *acute* pattern created by the *fourfold system B* employs rhombic repeat units with 45° and 135° included angles, and eight-pointed stars at the rhombic vertices [Fig. 181]. The earliest known use in Anatolia is in the portal of the Izzeddin Keykavus hospital and mausoleum in Sivas[117] (1217-18). However, this repetitive element was used in conjunction with a square repetitive element to create the distinctive hybrid design some 30 years earlier at the Ildegizid tomb of Mu'mine Khatun in Nakhichevan, Azerbaijan (1186) [Fig. 182]. This rhombic design was widely used by artists during the Sultanate of Rum. An aesthetically similar *acute* design, also from the Izzeddin Keykavus hospital and mausoleum in Sivas, incorporates far more geometric information within each square repetitive unit, and like its rhombic neighbor, employ the same pattern line variation within the underlying elongated hexagons that create the distinctive octagons within the pattern matrix[118] [Fig. 179a]. This orthogonal design places eight-pointed stars upon the vertices of the $4.8^2$ semi-regular grid, and as with the rhombic design from this same location, many subsequent examples of this design were used by Anatolian artists during this period. Another Anatolian design created from the same underlying tessellation as the orthogonal design from the Izzeddin Keykavus hospital and mausoleum in Sivas is an *obtuse* pattern from the Hudavent tomb in Nidge (1312) [Fig. 179b]. A *two-point* pattern from a portal at the Bimarhane hospital in Amasya (1308-09) [Fig. 174c] is nearly identical to the notable Ghurid *fourfold system B* raised brick panel in the Friday Mosque at Herat [Photograph 32] from just over a hundred years earlier [Fig. 174b]. The unusual quality of this design argues against independent development, and for the possibility that knowledge of this design was imported from Khurasan to Anatolia.

The quantity of geometric patterns created from the *fivefold system* by artists working in Anatolia under the Sultanate of Rum is vast, and far exceeds the confines of this study. As one would expect, the classic *acute* pattern from this system [Fig. 226c] was used multiple times in the architecture of the Sultanate of Rum[119], with the earliest known Anatolian example located in the *minbar* (1155) of the Alaeddin mosque in Konya. The underlying polygonal tessellation that produces the classic *acute* pattern is also responsible for equally classic patterns in each of the other pattern families [Figs. 85–88]. The earliest Anatolian *obtuse* design created from this tessellation is from the Great Mosque of Siirt (1129): just decades later than the earliest known occurrence at the Friday Mosque at Isfahan in Iran [Fig. 229b] [Photograph 21]. And the earliest Anatolian use of the *two-point* pattern created from this tessellation is from the Huand Hatun complex in Kayseri (1237): only decades later than its earliest known use at the Gunbad-i 'Alaviyan in Hamadan, Iran (late twelfth century) [Fig. 231d] [Photograph 22]. As with other varieties of geometric pattern, the spirit of experimentation among artists in the Sultanate of Rum was widely applied to the *fivefold system*, with the result of there being a greater concentration of diverse fivefold patterns in Anatolia than found in any other extant Islamic architectural tradition. This fivefold diversity included designs with very broad repetitive structures, multiple examples of field patterns, many repetitive strategies, and various additive treatments to the ten-pointed stars that are inherent to this system.

Artists working under the patronage of the Seljuk Sultanate of Rum produced many original patterns from the *fivefold system* that repeat upon the rhombic grid of 72° and 108° angles. Many of these were used multiple times throughout Anatolia; some were adopted by succeeding Muslim cultures; and others are only known to exist in a single location. Examples of this variety of original fivefold pattern include: the Huand Hatun complex in Kayseri[120] (1237) [Fig. 235d]; the Agzikara Han near Aksaray (1231-40) [Fig. 233b]; and a pattern from the portal of the Gök *madrasa* in Tokat[121] (1275-80) [Fig. 234b]. The design from the Gök *madrasa* is unusual in that the underlying polygons applied to the central region of the rhombic repeat create an interstice region that is filled by simply extending the pattern lines from the adjacent underlying polygons into the open region. More complex fivefold patterns that repeat upon the rhombic grid of 72° and 108° included angles include an outstanding *median* pattern from the Sultan Han in Kayseri[122] (1232-36) [Fig. 237] [Photograph 42]. The ten-pointed stars located at the vertices of the repetitive grid are not usual to the *median* family. Typically, these will have 72° crossing pattern lines placed at the midpoints of each decagonal edge. However, rather than decagons, the underlying tessellation of this example from Kayseri has large ten-pointed star interstice regions at each vertex of the repetitive grid. The 72° crossing pattern lines that are placed upon

[117] Schneider (1980), pattern no. 322.

[118] Schneider (1980), pattern no. 320.

[119] Schneider (1980), pattern no. 219. Schneider identifies 45 examples of the fivefold classic *acute* pattern in the many Anatolian Seljuk buildings he studied.

[120] Schneider (1980), pattern no. 279.

[121] Schneider (1980), pattern no. 377.

[122] Schneider (1980), pattern no. 392.

**Photograph 42** A Seljuk Sultanate of Rum *median* pattern created from the *fivefold system* located at the Sultan Han in Kayseri, Turkey (© Serap Ekizler Sönmez)

the edges of this interstice region are extended to create the ten-pointed stars that are atypical to this pattern family.

Seljuk Sultanate of Rum patterns that repeat upon the more acute fivefold rhombus with 36° and 144° included angles include a very pleasing *acute* pattern from the Huand Hatun complex (1237) that uses irregular pentagons within the underlying generative tessellation that are not a part of the standard modules used in this design system [Fig. 242]. Later examples of this design include a panel from a Mamluk *minbar* in the collection of the Victoria and Albert Museum[123] (c. 1300), and a Kartid pair of wooden doors of the mausoleum of Shaykh Ahmed-i Jam at Torbat-i Jam in northeastern Iran (1442-45). A very successful *obtuse* pattern that repeats upon this more acute rhombus is from the portal of the Muzaffar Barucirdi *madrasa* in Sivas[124] (1271-72) [Fig. 241b]. This same design was employed by Mamluk artists at the *mihrab* of the Amir Altinbugha al-Maridani mosque in Cairo (1337-39).

Artists working in the Seljuk Sultanate of Rum frequently used rectangular repeat units of various proportions when making patterns from the *fivefold system*. One of the most basic fivefold rectangular designs, with the least amount of geometric information contained within the repeat unit, is found at the Sultan Han in Aksaray (1229) [Fig. 245a]. The underlying polygonal tessellation for this *obtuse* design places decagons at the vertices of the rectangular repeat units, with two edge-to-edge pentagons separating the decagons along the short edge of the repeat, and barrel hexagons in the long dimension [Fig. 203]. This arrangement creates the cluster of six pentagons at the center of the repeat unit: a configuration that produces very acceptable *obtuse* and *two-point* patterns, but requires adjustment for acceptable *acute* and *median* designs [Figs. 197 and 198]. It should be noted that this design can also be created with equal ease by using an underlying tessellation of contiguous decagons in the short dimension of the repeat, and the concave hexagon separating the decagons in the long dimension. The earliest known use of this very popular rectilinear pattern was at the Maghak-i Attari mosque in Bukhara, Uzbekistan (1179-79), and the variety of later locations include the Amir Zadeh mausoleum in the Shah-i Zinda funerary complex in Samarkand, Uzbekistan (1386); the Abdullah Ansari complex in Gazargah near Herat, Afghanistan (1425-27); the Gur-i Amir complex in Samarkand (1403-04); and the tomb of Akbar in Sikandra, India (1613). Rectangular fivefold designs that are original to the Sultanate of Rum include an *obtuse* design from the *iwan* of the Sirçali *madrasa* in Konya[125] (1242-45) [Fig. 247]; an *obtuse* design from the *iwan* of the Yusuf ben Yakub *madrasa* in Cay[126] (1278) [Fig. 249]; and a *median* pattern from the *mihrab* of the Külük mosque in Kayseri[127] (1280-90) [Fig. 251]. A design from a door panel of the Hekim Bey mosque in Konya[128] (1270-80) has the unusual feature of transitioning from the *acute* family at the ends of the very elongated rectangular repeat unit to the *median* family throughout the rest of the rectangular repeat [Fig. 269]. This is achieved through the use of two scales of underlying polygonal modules. Patterns with variable scaled underlying polygonal modules are extremely rare, and

[123] A nineteenth century reproduction of the original panel is in the collection of the Victoria and Albert Museum, London; museum number 887–1184.

[124] Schneider (1980), pattern no. 374.

[125] Schneider (1980), pattern no. 376.

[126] Schneider (1980), pattern no. 380.

[127] Schneider (1980), pattern no. 370.

[128] Schneider (1980), pattern no. 388. This door panel currently resides in the Ince Minare *madrasa* History Museum in Konya, Turkey.

appear to be exclusive to Turkey. A later example is from an Ottoman wooden door at the Sultan Bayezid II Kulliyesi in Istanbul (1501-06) [Fig. 270].

Field patterns made from the *fivefold system* have a distinct quality that sets them apart from non-fivefold varieties of field pattern, and indeed, other fivefold patterns with their characteristic ten-pointed stars. These will often employ rectangular or hexagonal repeat units with a minimum of geometric information. Examples of the former include a *median* pattern on an arch at the Kayseri hospital[129] (1205-06) [Fig. 209]; a *two-point* design from the *iwan* of the Great Mosque at Malatya[130] (1237-38) [Fig. 207]; and an *obtuse* pattern that is used as a linear band at the Haci Kiliç *madrasa* in Kayseri (1275) [Fig. 208]. Rectangular field patterns with greater complexity include a *median* pattern from the west portal of the Huand Hatun complex in Kayseri[131] (1237) [Fig. 210]. Field patterns with diversely proportioned small hexagonal repeat units were also well known to this tradition, and examples include the Sitte Melik tomb in Divrigi[132] (1196) [Fig. 213]; a *median* design from a courtyard portal at the Sultan Han in Kayseri[133] (1232-36) [Fig. 216]; the Huand Hatun complex in Kayseri[134] [Fig. 218]; the Great Mosque in Malatya[135] (1237-38) [Fig. 220]; the Hekim Bey mosque in Konya[136] (1270-80) [Fig. 219]; and the Çifte Minare *madrasa* in Erzurum[137] (late thirteenth century) [Fig. 215]. Two very nice linear border designs with hexagonal repeat units were created from the same underlying tessellation: one of these is a *median* pattern from the Alaeddin mosque in Konya [Fig. 214a], and the other is an *obtuse* pattern from the Çifte Minare *madrasa* in Erzurum [Fig. 214c]. Yet another *fivefold system* field pattern with a hexagonal repeat from the Çifte Minare *madrasa* in Erzurum has characteristics that are equally *obtuse* (pentagons), and *median* (kite shapes) [Fig. 215]. An interesting field pattern with minimal geometric information within each repetitive element is found at the Sahib Ata mosque in Konya[138] (1258) [Fig. 211]. Like the above-cited hexagonally repeating designs from the Huand Hatun and Sultan Han in Kayseri, the underlying tessellation of this *median* design employs a distinctive kite-shaped module that is 2/5 of the decagon [Fig. 188]. The difference with this design from Konya is that the pattern is created exclusively from the kite-shaped underlying polygonal module. This is achieved by setting the kite-shaped polygons into alternating linear bands with coincident long edges. The earliest example and likely progenitor of this design was produced by Seljuk artists at the Khwaja Atabek mausoleum in Kerman (1100-1150). This group of field patterns demonstrates the diversity of methods the *fivefold system* offers for filling the two-dimensional plane with a single atypical repeat unit.

Artists working in the Seljuk Sultanate of Rum occasionally introduced either of two additive motifs into the ten-pointed stars of fivefold *median* Patterns [Fig. 224]. This type of pattern variation was developed by the Seljuks in Persia and the earliest known use is found at the Gunbad-i Qabud in Maragha (1196-97) [Fig. 240] [Photograph 24]. This additive technique has the affect of transforming the overall design into a field pattern. A very successful Anatolian Seljuk example of this additive technique is from the Huand Hatun complex in Kayseri (1237) [Fig. 257c]. The modification to the standard *median* ten-pointed stars in this pattern replaces them with a five-pointed star motif [Fig. 224b]. This results in a highly cohesive design that is unsurpassed in beauty by the many outstanding field patterns produced in Anatolia during this period. It is interesting that this simulated field pattern from Kayseri is one of the only Anatolian Seljuk fivefold designs with underlying decagons in the generative tessellation that repeats upon a hexagonal grid. Another remarkable example of a fivefold *median* pattern that arbitrarily fills the ten-pointed stars in a similar fashion is from the Karatay *madrasa* in Konya (1251-55) [Fig. 238]. This example repeats upon the fivefold rhombus with 72° and 108° included angles, and the number of underlying polygonal modules within this repeat is significantly greater than usual. This results in a rather complex design whose initial complexity is augmented through the additive treatment of the decagonal regions. A fundamental feature of the governing structure used in the creation of this design is the placement of a ring of ten edge-to-edge decagons at each vertex of the rhombic repeat [Fig. 238a]. These decagons are either filled with further underlying polygonal modules [Fig. 238b], or with the arbitrary modification to the generated ten-pointed stars that introduces a pentagon at the center of each decagon [Figs. 224a and 238c]. An interesting feature of this pattern is the ten-pointed star rosette within each ring of ten decagons. These introduce the characteristics of the *acute* family into what is otherwise a *median* pattern, and the overall affect is highly successful. It is interesting to note that the initial layout of the ring of ten decagons placed at each vertex of the rhombic repeat was also used some 50 years previously on the celebrated decagonal tomb tower of the Gunbad-i Qabud in Maragha (1196-97) [Figs. 239 and 240] [Photograph 24]. While the initial

[129] Schneider (1980), pattern no. 363.
[130] Schneider (1980), pattern no. 362.
[131] Schneider (1980), pattern no. 367.
[132] Schneider (1980), pattern no. 367.
[133] Schneider (1980), pattern no. 369.
[134] Schneider (1980), pattern no. 360.
[135] Schneider (1980), pattern no. 365.
[136] Schneider (1980), pattern no. 368.
[137] Schneider (1980), pattern no. 204.
[138] Schneider (1980), pattern no. 361.

decagonal layout is identical, both of these examples incorporate very different secondary polygonal infill into their decagons and interstice regions, as well as very different locations for the arbitrary modifications to their ten-pointed stars. However, the conceptual similarity suggests a direct influence of the earlier upon the latter.[139]

Artists working in the Sultanate of Rum augmented the complexity of patterns created from the *fivefold system* by combining otherwise stand-alone repeat units into single hybrid constructions. In its broad context, the overall repeat for each of these examples is a rectangle; but these broad rectangular repeats are the direct product of, and best understood as, a tessellating conglomerate of smaller repetitive units. For such designs to be successful, the pattern-lines located upon the *n*-length and *x*-length edges of each independent repetitive cell must precisely match: which is to say, the underlying polygonal modules that are placed upon each repetitive edge of equal length must have the same coinciding edge configuration. In this respect, these fivefold hybrid patterns employ the same principle that was used in the above-cited *fourfold system B* Ildegizid hybrid example from the Mu'mine Khatun tomb tower in Nakhichevan, Azerbaijan (1186) [Fig. 182]. In keeping with the rich diversity of innovative fivefold patterns at the Huand Hatun complex in Kayseri, this monument also includes a fivefold hybrid design in one of its portals[140] [Fig. 262d]. This exceptional example is a *median* pattern, and employs two repetitive units: a rhombus with 72° and 108° included angles, and an elongated hexagon. It is worth noting that each of these will tessellate independently. Indeed, the rhombic component is the classic *median* pattern used with great frequency throughout the Islamic world, including at the Huand Hatun complex [Figs. 87 and 227a]. As with other fivefold *median* patterns produced during the Seljuk Sultanate of Rum, this design arbitrarily modifies the standard ten-pointed stars with a central pentagon surrounded by five rhombi and distinctive trefoil elements [Fig. 224a]. This effectively transforms the original design into a field pattern. A hybrid design from the Izzeddin Kaykavus hospital and mausoleum in Sivas[141] (1217) [Fig. 263c] employs four repetitive elements: a small rhombus of 72° and 108° included angles, a larger rhombus of the same proportion, a more acute rhombus with 36° and 144° included angles, and a triangle that is half the acute rhombus, which is to say a 1/10 segment of the decagon [Fig. 263a]. Each of these three rhombi, with their associated pattern lines, was used on its own for surface coverage within this Anatolian design tradition. The pattern within the small rhombus is the above-mentioned classic *median* design; the larger rhombic repeat element was used on its own at the Huand Hatun complex in Kayseri (1237) [Fig. 235d]; and the acute rhombus was used on its own at the Muzaffar Barucirdi *madrasa* in Sivas (1271-72) [Fig. 241b]. This example from the Izzeddin Kaykavus hospital and mausoleum is historically significant in that it is the earliest example of a hybrid design that overtly employs more than two repetitive cells within its overall structure. The most complex fivefold hybrid designs are two examples from the Karatay Han (1235-41), 50 km east of Kayseri [Figs. 264c[142] and 265c[143]]. Both of these are *acute* patterns and share several of the same repetitive units, and their similarity clearly indicates that the same artist produced both. The first of these employs four repetitive elements: the rhombus with 72° and 108° included angles; the rhombus with 36° and 144° included angles; a triangle with the proportions of 1/5 of a pentagon, which is half the more obtuse rhombus; and an elongated hexagon with the same proportion as the barrel shape from the polygonal modules of the *fivefold system* [Fig. 264a]. The pattern within the more obtuse rhombus in this set of repetitive elements is the classic fivefold design that was used ubiquitously during the Seljuk Sultanate of Rum, and harkens back to the outstanding design used on the Ghurid soffit of the Arch at Bust, Afghanistan (1149) [Figs. 85 and 226c], and earlier still to one of the repetitive cells within the hybrid design in the northeast dome chamber at the Friday Mosque at Isfahan (1088-89) [Fig. 261] [Photograph 25]. These same four repetitive units, with the same pattern line application, were also used in the second hybrid design from the Karatay Han in Karadayi, but with the further addition of a rectangular and elongated hexagonal element [Fig. 265a]. Until the development of fivefold dual-level patterns in fourteenth-century Spain and fifteenth-century Persia, these hybrid patterns from the Sultanate of Rum represent the most sophisticated examples of Islamic geometric design created from the *fivefold system*.

Artists working in the Sultanate of Rum either appropriated or rediscovered the simple, but elegant, method of creating an underlying tessellation from six regular heptagons placed together in an edge-to-edge arrangement with bilateral symmetry. By drawing lines that connect the centers of the heptagons, an elongated hexagonal repeat unit is established. The interstice of these six heptagons is comprised of two irregular pentagons that meet edge to edge in the center of the heptagon cluster. As detailed above, this same arrangement of heptagons was first used in the Ghaznavid minaret of Mas'ud III in Ghazni,

[139] The author is indebted to both Emil Makovicky and Jean-Marc Castéra for independently discovering the geometric similarity between these two fivefold patterns. See Castéra (2016).

[140] Schneider (1980), pattern no. 366.

[141] Schneider (1980), pattern no. 382.

[142] Schneider (1980), pattern no. 386.

[143] Schneider (1980), pattern no. 387.

Afghanistan (1099-1115), approximately 100 years earlier. The two Ghaznavid patterns derived from this tessellation are considerably more complex [Figs. 280 and 281]. These two examples also include a large number of pattern lines that are arbitrarily placed within the pattern matrix rather than the strict product of a systematic schema. As such, while they are sevenfold patterns, they do not fall into the category of having been created from the *sevenfold system*. By contrast, the three Anatolian Seljuk designs created from this same underlying tessellation are among the earliest examples of patterns created from underlying polygonal modules that eventually became recognized as components of the *sevenfold system*. The rarity and simplicity of sevenfold patterns created by artists working in the Sultanate of Rum suggests that these artists were not fully aware of the systematic potential of these underlying modules. This is in clear distinction from the artists working under the Mamluks some 150 years later when this system came to full maturity. The three Anatolian examples are an *acute* pattern from the *mihrab* of the Great Mosque of Dunaysir in Kiziltepe[144] (1204) that was also used at the Alaeddin mosque in Nidge (1223) [Fig. 282a]; an *obtuse* design from the Eğirdir Han[145] (1229-36) [282c]; and a *two-point* pattern from the façade of the Great Mosque of Malatya[146] (1237-38) [Fig. 282d]. These were produced within some 30 years of one another, and may well have been the product of the same artist, lineage, or atelier. Although it does not appear to have been used historically, the *median* pattern created from this same underlying tessellation is equally acceptable [Fig. 282b]. However, this *median* design was the foundational basis for one of the highly complex sevenfold designs from the minaret of Mas'ud III [Fig. 281b].

The architectural ornament of the Seljuk Sultanate of Rum includes a large number of very fine geometric patterns that are nonsystematic. These range from more basic designs that were first used by earlier Muslim cultures to highly innovative original constructions that are among of the most complex nonsystematic geometric patterns from the totality of this artistic tradition. Unlike their neighbors to the east, the continuance of the Seljuk Sultanate of Rum in the face of the Mongol onslaught of the thirteenth century insured that there was no consequent interruption in the developmental continuity of the geometric arts in Anatolia. On the contrary, the Sultanate of Rum and the Mamluks directly benefited from the exodus of skilled artists and craftspersons fleeing the Mongol destruction. Along with their Mamluk contemporaries, artists working under the patronage of the Seljuk Sultanate of Rum were responsible for bringing the nonsystematic use of the polygonal technique to its full geometric sophistication; and the nonsystematic geometric ornament of subsequent Muslim cultures, for all its originality and aesthetic distinction, never surpassed the innovative developments of these two important dynasties.

Among the nonsystematic designs that repeat upon the hexagonal grid are a number of interesting patterns with nine-pointed stars. Six nonagons will cluster when placed edge to edge in sixfold radial symmetry. The central region of an underlying tessellation constructed from this configuration is an interstice six-pointed star, and the pattern lines that extend into this interstice region likewise form a six-pointed star [Fig. 310]. This simple tessellation was used to create a very successful *acute* pattern located in the *Turkish triangle* pendentives in the dome of the *mihrab* at the Alaeddin mosque in Konya[147] (completed in 1219-21). The placement of nonagons on the vertices of the hexagonal grid also allows for their being separated by a ring of nine pentagons. As with the previous example, this arrangement creates an underlying six-pointed star interstice region at the center of each hexagonal repeat unit. An *acute* design created from this closely related underlying tessellation was also included in the Turkish triangle pendentives at the Alaeddin mosque.[148] [Fig. 312c]. This same underlying tessellation was used to create an equally successful *median* pattern that was used in several locations, including the Alay Han near Aksaray (1155-92) [Photograph 43]; the Huand Hatun in Kayseri; and the Agzikara Han[149] [Fig. 312b]. These differ from the example in the *Turkish triangles* in the treatment of the pattern lines within the central region, as well as slight variations in the angular opening of the crossing pattern lines. An *acute* design from the Izzeddin Kaykavus hospital and mausoleum in Sivas (1217) employs an underlying tessellation that is essentially the same, with nonagons at the vertices of the hexagonal grid that are separated by mirrored pentagons. However, the central region of this example places six contiguous barrel hexagons around a regular hexagon at the center of each repetitive hexagonal cell [Fig. 313c]. The ornament of the Great Mosque of Malatya includes an unusual design that employs underlying nonagons placed on the vertices of the isometric grid, with equilateral triangles separating each nonagon. Despite this placement, due to the nonagon's odd number

[144] Schneider (1980), pattern no. 216.

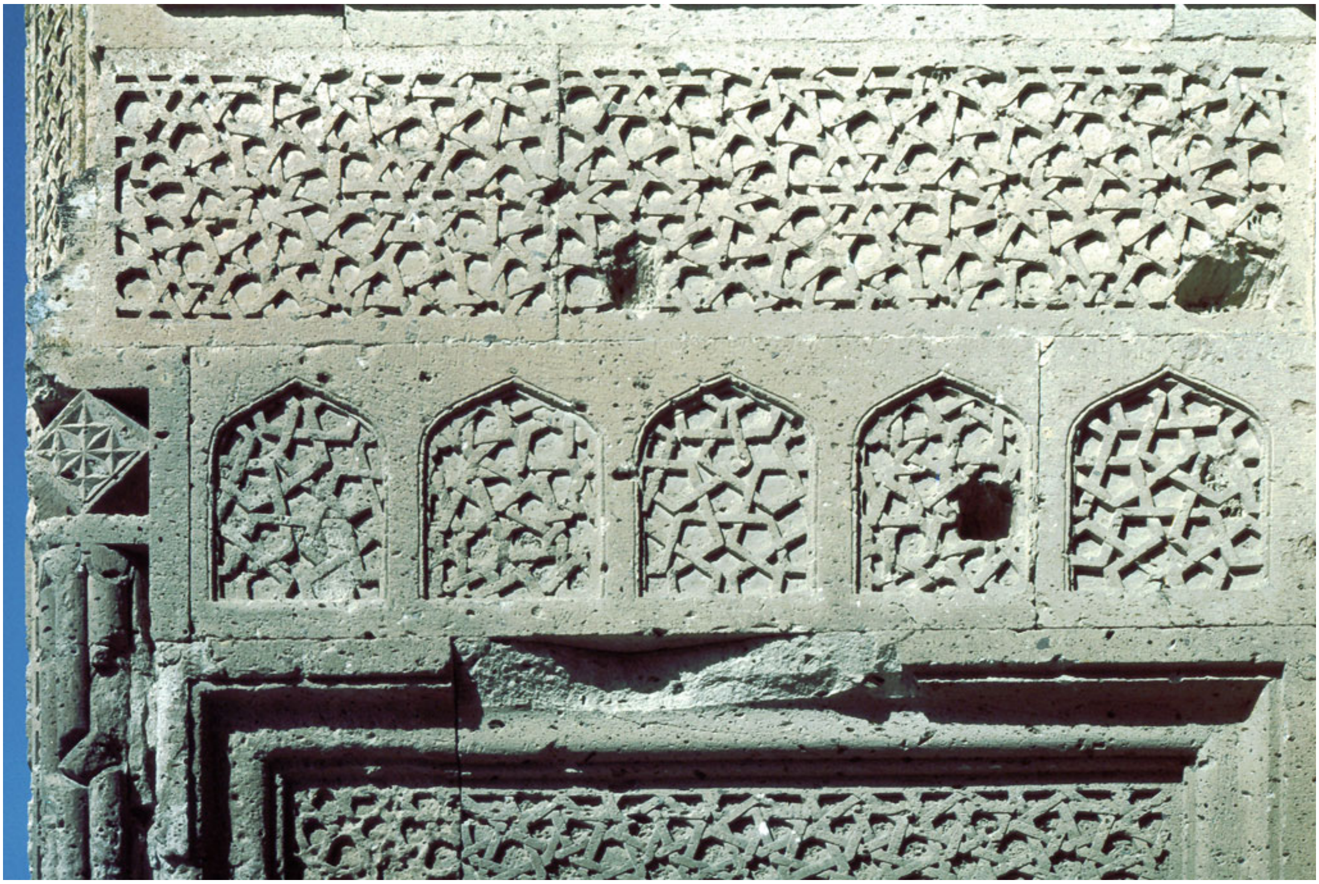
[145] Schneider (1980), pattern no. 205.

[146] Schneider (1980), pattern no. 209.

[147] Schneider (1980), pattern no. 359.

[148] Schneider (1980), pattern no. 218 (pl. 19 and 34).

[149] Schneider (1980), pattern no. 218 (pl. 34). Schneider compares the similarity between the nonagonal pattern from the Alaeddin Mosque in Konya with those from the Huand Hatun and Agzikara Han in this figure. However, he does not identify the reason for their similarity: that being the single underlying polygonal tessellation that is responsible for both these *acute* patterns.



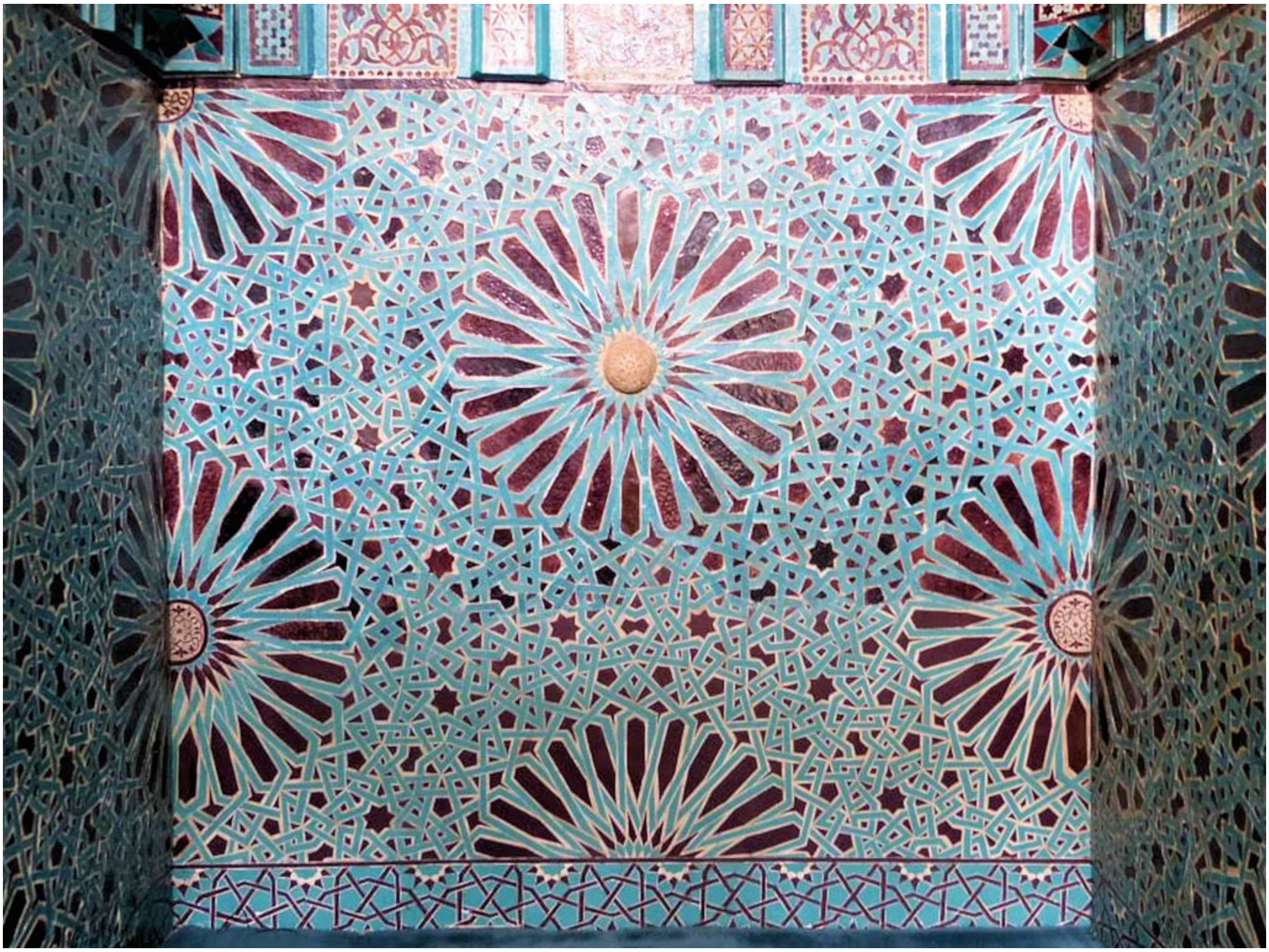
**Photograph 43** Seljuk Sultanate of Rum geometric panels in carved stone relief at the Alay Han near Aksaray, Turkey (© David Wade)

of sides, this design repeats upon a rhombic grid with 60° and 120° angles [Fig. 311]. This example from the Great Mosque of Malatya is a *median* pattern with an unusual threefold rotational devise generated from the ditrigonal hexagons that are edge to edge with the three nonagons and their triangles.[150] Another nonagonal design from the Seljuk Sultanate of Rum is from the Gök *madrasa* and mosque in Amasya, Turkey (1266-67). This separates each underlying nonagon with a barrel hexagon, and places a ring of 12 pentagons that surround a central irregular dodecagonal interstice region. In an innovative tour de force, the *acute* pattern lines in the dodecagonal interstice region place regular octagons into the pattern matrix [Fig. 315]. One of the most interesting patterns created from an underlying tessellation that utilizes nonagons at the vertices of the regular hexagonal grid is a second such example from the Great Mosque of Malatya.[151] This rather exceptional *median* pattern employs both nine- and seven-pointed stars in a fashion that is reminiscent of the aesthetic quality of *median* patterns created from the *fourfold system A* [Fig. 318]. The use of these two star forms is an example of the *principle of adjacent numbers* wherein the ease of generating repetitive patterns with the eight-pointed star indicates that successful patterns can also be made with nine- and seven-pointed stars. As with other nonagonal designs, the nine-pointed stars are located at the vertices of the hexagonal grid, while a ring of 6 seven-pointed stars rests within the field of the hexagonal repeat unit, and a six-pointed star is located at the center of each repeat unit. The underlying polygonal matrix that connects the nonagons and heptagons is comprised of irregular pentagons and hexagons that cleverly imitate those of the *fourfold system A*.

Like other preceding and neighboring Muslim cultures, the Sultanate of Rum also employed nonsystematic patterns that place 12-pointed stars on the vertices of the isometric grid. The underlying tessellation in one of the most basic patterns of this type places a ring of pentagons around each dodecagon, with three pentagons meeting at the center of each triangular repeat. One of the earliest examples of an *acute* pattern made from this underlying tessellation is from

[150] Schneider (1980), pattern no. 211.

[151] Schneider (1980), pattern no. 356.



**Photograph 44** A threefold Seljuk Sultanate of Rum nonsystematic *two-point* pattern with 7- and 24-pointed stars from the Esrefoglu Süleyman Bey mosque in Beysehir, Turkey (© Mirek Majewski)

the *mihrab* of the Great Mosque of Niksar in north central Turkey[152] (1145) [Fig. 300a *acute*]. This was produced under the patronage of the Danishmend Dynasty: early rivals of the Seljuks in Anatolia. Later Anatolian examples of this design include the cenotaph of the Izzeddin Kaykavus mausoleum in Sivas (1217-18), and the side panel of the *minbar* at the Great Mosque of Divrigi (1228-29). A variation of this *acute* pattern is created from truncating the three coinciding pentagons in the underlying tessellation such that they become trapezoids with coincident edges with the central equilateral triangle[153] [Fig. 320]. Anatolian examples with this design variation include the vertical side panel of the Mengujekid *minbar* in the Great Mosque of Divrigi (1228-29), and in a portal niche at Çifte Minare *madrasa* in Erzurum (late thirteenth century). Another underlying tessellation with dodecagons placed upon the vertices of the isometric grid is from the kiosk of the Keybudadiya at Kayseri (1224-26). The *acute* pattern generated from this underlying tessellation was widely used throughout Muslim cultures[154] [Fig. 321b *acute*]. The underlying tessellation of this example also places a cluster of three coincident pentagons at the center of each triangular repeat unit, and introduces an elongated hexagon that separates the dodecagons. Another early use of this *acute* pattern that dates to the same approximate period is a carved stucco panel at the Abbasid Palace of the Qal'a in Baghdad (c. 1220) [Photograph 28]. Artists working for the Sultanate of Rum also created patterns that place 24-pointed stars onto the vertices of the isometric grid. The portal of the Nalinci Baba tomb and *madrasa* in Konya (1255-65) is decorated with a very beautiful *two-point* pattern that incorporates seven-pointed stars into the pattern matrix that surround 24-pointed stars [Fig. 327]. This exceptional design was also used in the *mihrab* niche at the Esrefoglu Süleyman Bey mosque in Beysehir, Turkey (1296-97) [Photograph 44].

[152] Schneider (1980), pattern no. 402.

[153] Schneider (1980), pattern no. 398.

[154] Schneider (1980), pattern no. 401.

Isometric patterns from the Seljuk Sultanate of Rum frequently employed more than a single region of local symmetry. Compound patterns with 12-pointed stars at the vertices of the isometric grid and 9-pointed stars within the centers of each triangular repeat were especially popular. The most commonly used underlying tessellation with this form of compound symmetry separates the dodecagons from the nonagons with a ring of irregular pentagons, and places an elongated hexagon between the nonagons. Patterns from the Seljuk Sultanate of Rum that are created from this underlying tessellation include an *obtuse* design from the *mihrab* of the Great Mosque of Aksehir[155] (1213) [Fig. 347a] and an *acute* design from a faience ceramic panel on the façade of the Cincikli mosque in Aksaray[156] (1220-30) [Fig. 346a]. The design from Aksaray is the earliest known use of this *acute* pattern, and over time this was used throughout the Islamic world. A significantly more complex *median* pattern with the same combination and location of 12- and 9-pointed stars was used in a portal at the Susuz Han in the village of Susuzköy[157] (1246) [Fig. 354d].

Artists working under the Seljuk Sultanate of Rum created a number of isometric designs with significantly greater complexity in their diversity of local symmetries. In addition to the vertices and centers of the isometric grid, such designs will place additional regions of local symmetry upon the midpoints of the triangular repeat, and occasionally into the field of the repeat. Multiple examples of a particularly ambitious *acute* design with 9-, 10-, 11- and 12-pointed stars placed onto these locations include a stone relief panel from the Egridir Han[158] (c. 1229-36); the courtyard portal at the Seri Han near Avanos (1230-35); and a framing border in the entry to the mosque at the Karatay Han (1235-41). This design places the 12-pointed stars at the vertices of the isometric grid, 9-pointed stars at the center of each triangular repeat, 10-pointed stars upon the midpoints of each triangular edge, and 11-pointed stars within the triangular field[159] [Fig. 367]. The technically demanding construction of this complex design, coupled with the closeness in age and proximity of these three examples argues for each to have been the product of a single workshop. Another example with a similar geometric arrangement of star forms is from a gravestone in Ahlat[160] (thirteenth–fifteenth centuries). This is an *acute* pattern that also places 12-pointed stars at the vertices of the isometric grid, but nonagons rather than 9-pointed stars at the centers of each triangular repeat, and 8-pointed stars rather than 10-pointed stars at the midpoints of each repetitive edge, and serendipitous heptagons into the field of the design [Fig. 361]. This is the only known historical example of this exceptionally well-balanced design. An outstanding isometric pattern with multiple centers of local symmetry was used in the *mihrab* niche of the Great Mosque of Ermenek (1302). This is an *acute* design that places 24-pointed stars upon the vertices of the grid, 12-pointed stars at the centers of each triangular repeat unit, and 8-pointed stars upon the midpoints of each edge of the repeat unit[161] [Fig. 365].

Artist working in the Seljuk Sultanate of Rum were equally innovative in their focus upon nonsystematic patterns based upon the orthogonal grid. As with nonsystematic isometric patterns, these most commonly place 12-pointed stars upon the vertices of each repeat unit—in this case squares. The underlying polygonal tessellation for the most basic of such patterns places a ring of pentagons around each dodecagon, with four of these pentagons meeting at the centers of the square repeat. An early Anatolian example is found on the minaret of the Great Mosque of Siirt[162] (1129) [Fig. 335b], dating from less than 10 years after the earliest known use of this design in the northeastern *iwan* of the Friday Mosque at Isfahan [Fig. 335a]. Multiple instances of this ever-popular design were used subsequently by artist working for the Seljuk Sultanate of Rum. A variation of this pattern truncates the cluster of four underlying pentagons at the centers of the square repeat units [Fig. 337]. An example of an *acute* pattern created from this variation is found on the pair of bronze doors from the Anatolian Seljuk *atabeg* of Cizre, Turkey (thirteenth century), and the earliest known use is from a Zangid bronze door at the portal of the Bimaristan Arghun in Aleppo (twelfth century).

Orthogonal patterns with multiple centers of local symmetry were widely employed by the Sultanate of Rum. The most common nonsystematic pattern of this type places 12-pointed stars on the vertices of square repeat units, and 8-pointed stars at the center. The earliest known example of this variety of pattern was created by Mengujekid artists in the portal of the Kale mosque in Divrigi[163] (1180-81) [Fig. 379b]. Multiple later examples of orthogonal designs with 8- and 12-pointed starts were used both in Anatolia and throughout Muslim cultures. A very pleasing *acute* pattern with 16-pointed stars on the vertices of the square repeat unit and 8-pointed stars at the center of each repeat was used in

155 Schneider (1980), pattern no. 412.

156 Schneider (1980), pattern no. 358.

157 Schneider (1980), pattern no. 414.

158 Now spolia in the city walls of Egridir.

159 Schneider (1980), pattern no. 418.

160 Schneider (1980), pattern no. 407.

161 Schneider (1980), pattern no. 435.

162 Schneider (1980), pattern no. 408.

163 Schneider (1980), pattern no. 406.

the *mihrab* of the Keykavus hospital in Sivas[164] (1217-18) [Fig. 389a]. This same design was used by ʻAbd Allah ibn Muhammad al-Hamadani in the illumination of the 30 volume Quran of Uljaytu (1313), and by *Mudéjar* artist in a window grille at the ibn Shushen Synagogue of Toledo (1180), referred to today as the Santa Maria la Blanca.

As with isometric designs, more complex patterns made from nonsystematic orthogonal underlying polygonal tessellations will frequently incorporate additional areas of local symmetry at the midpoints of the square repeat units, and within the field of the repeat. Geometric artists from the Seljuk Sultanate of Rum were particularly resourceful in producing designs of this type. An outstanding case in point is a pattern from the Kayseri hospital (1205-06) that places 12-pointed stars on the vertices of the square repeats, octagons at the center of the repeat, 10-pointed stars at the midpoint of each edge of the repeat, and 9-pointed stars within the field[165] [Fig. 400]. This same design was used in several later Anatolian locations, including the Agzikara Han near Aksaray (1231-40), and the Ince Minareli *madrasa* in Konya[166] (1264-65). Several examples of orthogonal compound patterns were created that have 16-pointed stars at the vertices of the square repeat, 8-pointed stars at the centers of the repeat, 12-pointed stars at the midpoints on each edge of the repeat, and 10-pointed stars within the field of the repeat. Notable among these is from the *iwan* of the Kemaliya *madrasa* in Konya[167] (1249) [Fig. 404].

Artists working for the Sultanate of Rum also created nonsystematic patterns with compound symmetry that employed repetitive schema other than the isometric and orthogonal grids. This variety of pattern is especially complex, and is generally comprised of three types: those that have rectangular repeat units, those with elongated hexagonal repeat units, and those that are characterized by linear bands of primary star forms. Technically, this last category repeats with an especially broad rectangle, but the visual quality is sufficiently distinct from other rectangular designs as to warrant its own separate consideration. An early Anatolian example of a nonsystematic rectangular design with 10- and 12-pointed stars was used on the wooden *minbar* of the Great Mosque at Aksaray[168] (1150-53) [Fig. 414]. The dual of a rectangle is an identical rectangle; and the repeat unit for this design can be regarded equally as having either the 10- or 12-pointed stars placed upon the vertices, with the other star form located at the center of the repeat unit. This example is an *acute* pattern, and the underlying generative tessellation makes equally successful designs with each of the other three pattern families, although none are known within the historical record [Fig. 415]. This design with 10- and 12-pointed stars is the only known architectural example, although it is interesting that the same design is illustrated in the anonymous Persian treatise *On Similar and Complementary Interlocking Figures*,[169] as well as in the Topkapi Scroll.[170] A very pleasing Mengujekid *acute* pattern that borders the interior of a window at the Great Mosque of Divrigi (1228-29) places 12-pointed stars on the vertices of the rectangular repeat units, 8-pointed stars at the midpoints of the long edges of each repeat, and two 9-pointed stars within the field of each repeat[171] [Fig. 421]. Another design from this general region that repeats upon a rectangular grid is a highly complex *acute* pattern with 10- and 11-pointed stars that was used on a stone *khatchkar* in Noravank, Armenia, created by Momik, a monk and artist who worked between the years 1282 and 1321 [Fig. 423]. This is not strictly speaking the product of the Sultanate of Rum. However, this tradition of Armenian Christian commemorative stone crosses was greatly influenced by the carved stone ornament of the Anatolian Seljuks. Their incorporation of Islamic geometric and floral design motifs is in aesthetic conformity with the contemporaneous work of their Anatolian neighbors. Among the many geometric patterns that were used on Armenian *khachkars* are several with complex geometry. This example by Momik is particularly complex, and one of the earliest signed examples of such a pattern. It is also one of the most sophisticated examples of the non-Muslim adoption of Islamic geometric art, even if rather disproportionate in the relative sizes of the five-pointed stars and the shape of the 11-fold rosettes. An example of an Anatolian compound pattern that employs an elongated hexagon as the repeat unit is found in the courtyard portal of the Karatay Han[172] (1235-41). This pattern employs 9-pointed stars at the vertices of the hexagonal repeat unit, with 12-pointed stars at the midpoints of the two opposite parallel edges of the repeat, an 8-pointed star at the center of the repeat, and four 10-pointed stars within the repetitive field [Fig. 439]. An outstanding design that is characterized by linear bands of star-forms arranged in an alternating sequence of 12-, 11-, 10-, 11-, and 12-pointed stars was used on the portal of the Kiosk at Erkilet near Kayseri[173] (1241) [Fig. 425]. This highly complex pattern was used a second time in Kayseri: in the *mihrab* of the Çifte

[164] Schneider (1980), pattern no. 423.
[165] Schneider (1980), pattern no. 429.
[166] This stone panel currently resides in the Museum of Wooden Artifacts and Stone Carving in Konya: collection number 157092.
[167] Schneider (1980), pattern no. 427.
[168] Schneider (1980), pattern no. 416.
[169] Bibliothèque Nationale de France, Paris, MS Persan 169, fol. 195b.
[170] Necipoğlu (1995), diagram no. 44.
[171] Schneider (1980), pattern no. 421.
[172] Schneider (1980), pattern no. 420.
[173] Schneider (1980), pattern no. 417.

Kümbet (1247): the work almost certainly of the same artist. This is an identical numeric sequence to the earlier, and inferior, Zangid design that was reported by Ernst Herzfeld to have been used on a pair of doors from the Lower Maqam Ibrahim in the citadel of Aleppo[174] (1168) [Fig. 427].

Artists working in the Seljuk Sultanate of Rum also applied their knowledge of geometric design to the decoration of domes and semi-spheres. During the same general period that Ayyubid artists were working in this same specialized discipline, artists in Anatolia created several fine examples that utilized both radial and polyhedral geometry. The renowned faience mosaic dome of the Karatay *madrasa* in Konya (1251-52) is an overt homage to the number 24: with a complex geometric matrix of multiple 24-pointed stars applied within the 24 gore segments that provide the repetitive schema for this domical ornament. There are a number of examples of polyhedral ornament from this dynasty that apply geometric designs onto the surfaces of domical hemispheres that protrude from the ornamental design of their otherwise two-dimensional backgrounds. Most of these historical examples are carved stone and are based upon the geometry of the dodecahedron and include: a pattern that places five-pointed stars associated with the *acute* family into each projected pentagonal face at the Huand Hatun complex in Kayseri (1237); a second example from the Huand Hatun complex in Kayseri that places five-pointed stars from the *median* family onto each pentagonal face [Fig. 497]; and a *two-point* pattern from the Sahib Ata mosque in Konya (1258) that is identical in geometric concept to the ornament in the northeast dome of the Friday Mosque at Isfahan[175] [Fig. 496] [Photograph 30]. A more complex example of one of these projecting hemispherical stone ornaments is constructed from the spherical projection of an underlying truncated cube in a portal at the Susuz Han in the village of Susuzköy (1246) [Fig. 499].

## 1.21 Mamluks of Egypt (1250-1517)

The Mamluk dynasty of Egypt was founded by former Turkic slaves who gained positions of military and political power during Ayyubid rule. Their loyalty to the Ayyubid Sultans, and their military prowess, made the Mamluk martial guard a crucial aspect of the Ayyubid governance. Many Mamluk members of the military were awarded freedom from slavery, and appointed to high-ranking positions within government. With the collapse of the Ayyubids, these highly placed political and military professionals assumed governance. The Mamluk Empire lasted for over 250 years. At its peak, this great empire included all of Egypt, part of Libya to the west, Nubia to the south, the Hijaz to the east, and Palestine, Syria, and part of southern Anatolia to the north. Evidence of their military strength was the defeat of the invading Mongol forces of Hulagu Khan at the battle of Ain Jalut, near Nazareth, in 1260, bringing an end to the Mongol's westward expansion in the Levant.

The Mamluk tactics at the battle of Ain Jalut were devised by the military commander Baybars al-Bunduqdari, who also led the vanguard of the Mamluk forces. Following this victory, he succeeded to the position of Sultan. Baybars proved to be as adept in diplomacy as he was in battle. When the Mongols conquered Baghdad in 1258, they executed the Abbasid Caliph al-Musta'sim, along with most of his family. In 1261, Baybars offered a surviving descendant of al-Musta'sim refuge in Cairo. This invitation led to the reestablishment of the Abbasid Caliphate in the new location of Cairo. Baybars extended his dominion to include the Hijaz region of the Arabian Peninsula. As the protector of the holy cities of Mecca and Medina, victor over the invading Mongols, and benefactor to the transplanted caliphate, Baybars became one of the most greatly respected Muslim leaders of his time. Equally important in spreading his reputation among Sunni Muslims were his many victories over the Christian crusader kingdoms. During Mamluk reign, Cairo maintained its exalted reputation and position of importance throughout the Islamic world.

The Mamluk dynasty was responsible for some of the most beautiful art and architecture of the Islamic world. Their artists worked in all media, and at a level of skill that was unsurpassed. Mamluk patronage gave particular attention to the book arts, and the Quranic calligraphy, illumination, and bookbinding of this period represent one of the high points of this most important Islamic art. Great emphasis was also given to calligraphic epigraphy, and, as with other Muslim cultures, such inscriptions were often elaborated with highly refined floral backgrounds. A supremely beautiful example of this form of ornament is found in the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63), where a continuous running band of calligraphy and floral ornament surrounds the interior in an embrace of Quranic revelation. This is one of the most beautiful examples of *Kufi* script with floral ornament from anywhere in the Islamic world.

The Mamluk metal work of Cairo and Damascus rivaled the best of Mosul, Tabriz, Shiraz, or Herat. Under the Mamluks, the Mosul style of inlaying bronze vessels with silver and gold achieved further refinements. To this end, many of the finest metalworkers from Mosul are known to have relocated to Damascus and Cairo during the Mamluk period. All manner of vessels were produced under Mamluk

[174] Herzfeld (1954–6), Fig. 56.

[175] Schneider (1980), pattern numbers 437, 438, and 439.

patronage, including vases, basins, lamps, candle holders, incense burners, pen and ink holders, ewers, as well as weapons and scientific instruments. Mamluk metalwork was held in the very highest regard throughout the Islamic world, as well as in Europe: as exemplified by the use of an especially fine Mamluk basin, called the Baptistere de St Louis, as a baptismal bowl for the kings of France. As with metal work, Mamluk knotted carpets were the equal of the finest carpets from al-Andalus and Persia.

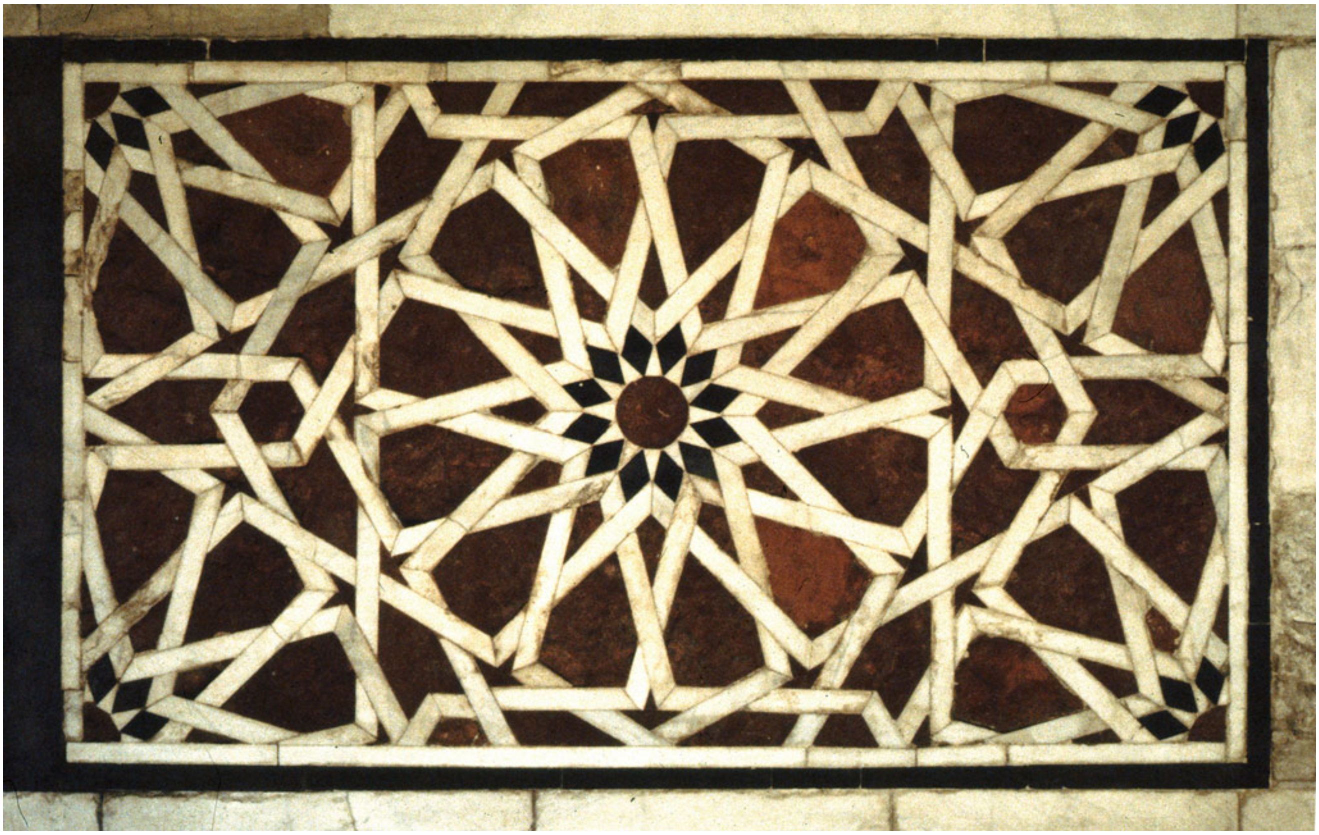
Mamluk architecture is one of the great Islamic classical styles, and the exceptional beauty of historical Cairo is primarily due to its Mamluk heritage. The Mamluk architectural style was a direct beneficiary of Ayyubid and Zangid architectural traditions, with stone remaining the primary material for both construction and ornamentation. The earlier conventions of *ablaq* polychrome stone ornament was fully embraced by Mamluk artists, and the exuberant *ablaq* vegetal designs that were created during this period represent one of the pinnacles of floral ornamental expression throughout the Islamic world. The Mamluks also expanded upon the Ayyubid and Zangid practice of applying geometric patterns to the carved stone the quarter domes of entry portals and *mihrab* niches to include the application of geometric patterns onto the entire exterior surfaces of domes. The Mamluks rose to power during the period of upheaval in Transoxiana, Khurasan, and Persia brought on by the invading Mongols. Like the Seljuks of Rum, and as stated, the Mamluks benefited from the influx of artists fleeing the Mongol onslaught. Several eastern architectural features were introduced into Egypt during this period. The grand entry portal of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63) has several characteristics that are more common to Persia: including its monumental size and height, and the use of recessed spiral columns at its corners. Originally this great entry portal had twin minarets on each side: another distinctive Persian feature. These were discarded following the collapse of one of the minarets soon after completion, killing many orphaned children in an adjacent school. Rather than rebuilding the fallen minaret it was considered more prudent to remove the remaining minaret. Eastern influences on Mamluk Quranic illumination include the occasional incorporation of distinctive Mongol floral devices such as stylized lotus and peony flowers. It is an interesting fact that these Mongol influenced floral motifs rarely found expression in Mamluk architectural ornament.

As with Quranic illuminations, Mamluk architecture made full use of the fully mature tradition of Islamic geometric design. The exterior of Mamluk monumental architecture was frequently ornamented with very bold geometric patterns. These geometric patterns were simple, and their very large scale gives emphasis to the monumentality of the building itself. This was a Fatimid ornamental devise that the Mamluks further refined, and provides an architectural façade with an ornamental boldness that can be appreciated from a considerable distance. Within the interior of Mamluk buildings, geometric patterns were also used widely. The Egyptian tradition of pierced geometric window grilles was continued, but the geometric patterns used by the Mamluks were more complex than those used in earlier times. Complex geometric patterns were also regularly used in the inlaid marble ornament of *mihrab* and fountains. Among the most noteworthy incorporation of geometric patterns are panels from the exceptionally beautiful wooden *minbars* for which the Mamluks are renowned. As with other aspects of their architectural ornament, this focus upon wooden *minbars* was inherited from their Zangid and Ayyubid predecessors. Rather like the contemporaneous carved stone ornament of the Seljuk Sultanate of Rum, this rich tradition is characterized by the use of a wide variety of very complex geometric patterns. These *minbars* are masterpieces of design and craftsmanship, and rank among the finest examples of Islamic art,[176] and the application of geometric patterns within Mamluk *minbars* represent one of the most sophisticated expressions of the geometric idiom from the whole of the Islamic world.

Mamluk geometric artists built upon the practices inherited from their predecessors, and applied the polygonal technique to new heights of sophistication and complexity. Their work with two-dimensional systematic pattern making continued with the widespread use of the *system of regular polygons*, both fourfold systems and the *fivefold system*, as well as the use of diverse nonsystematic designs already known to this ornamental tradition. What is more, Mamluk artists were responsible for bringing the *sevenfold system* of pattern generation to full maturity; and the relatively small number of patterns that were created from this system are remarkable for their beauty and ingenuity. To a very limited extant, this system was adopted by Ottoman and Timurid arts. Like their contemporaries in the Seljuk Sultanate of Rum, Mamluk geometric artists also produced many outstanding examples of highly complex non-systematic designs with multiple centers of differentiated symmetry. These compound patterns represent the full maturity of this nonsystematic ornamental tradition. The innovation of Mamluk artists is also exemplified in the many stone domes and quarter domes ornamented with geometric designs. Their work with highly complex nonsystematic patterns, applied geometric patterns onto domical surfaces, together with their development of the *sevenfold system*, is evidence of the important contributions by Mamluk artists to the diversity, maturity and richness of Islamic geometric ornament.

[176] Atil (1982), 195–196.

The Mamluk use of patterns created from the *system of regular polygons* was widespread and diverse. Many of these designs were already well known throughout the Islamic world. In addition to the beauty that less complex, easily comprehended, and immediately recognizable designs contribute to an overall ornamental schema, such geometric patterns can be regarded as a unifying device that helped establish an aesthetic continuity and cultural affiliation among preceding Muslim cultures. For example, numerous previously established patterns that are easily created from the $6^3$ tessellation of regular hexagons were used as architectural ornament during the Mamluk period, including the relatively uncommon *acute* design with 30° crossing pattern lines used in the exterior carved stone ornament at the Imam al-Shafi'i mausoleum in Cairo (1211) [Fig. 95a], and the simple *median* design with 60° angular openings used in the stone window grilles at the Sultan Qala'un funerary complex in Cairo (1284-85) [Fig. 95b] [Photograph 55]. This latter example is a classic threefold *median* pattern that was used universally by Muslim cultures. The design of the window grilles immediately adjacent to this example from the Sultan Qala'un funerary complex is far less common [Fig. 99f] [Photograph 55]. This adjacent *two-point* pattern is directly associated with the 3.6.3.6 underlying tessellation of triangles and hexagons, and its use at the Sultan Qula'un funerary complex was some 50 years after its use by artist during the Seljuk Sultanate of Rum at the Ali Tusin tomb tower in Tokat, Turkey (1233-34). Complementing the wide use of well-known designs made from the *system of regular polygons*, Mamluk artists also used this system to create new and original geometric patterns. A very successful *two-point* pattern that appears to be derived from the 3.6.3.6 underlying tessellation was employed in the mosaic spandrel above the *mihrab* niche of the Aqbughawiyya *madrasa* (1340) at the al-Azhar mosque in Cairo [Fig. 100d]. This unusual design utilizes the hexagons within the generative tessellation as part of the completed pattern. A more complex pattern produced from this same 3.6.3.6 underlying tessellation was used as a border design that surrounds a door at the *manzil* (house) of Zaynab Khatun in Cairo (1468) [Fig. 102b]. This rather unusual pattern is comprised of superimposed dodecagons and ditrigonal shield shapes: the latter being generated by applying the 90° crossing pattern lines at the midpoints of alternating underlying hexagons, and allowing these crossing pattern lines to extend into the adjacent triangles and hexagons until they meet with other extended pattern lines, and the former being the product of simply applying dodecagons so that they cross the underlying triangles in an aesthetically acceptable fashion. A considerably more complex Mamluk design created from the 3.6.3.6 underlying tessellation is from the central panels of the double doors at the Vizier al-Salih-i Tala'i mosque in Cairo [Fig. 100c]. These Mamluk doors were added to this Fatimid mosque during its restoration following an earthquake in 1303. This design is unusual in that it incorporates nonagons centered upon each underlying triangular module. The Mengujekids of Anatolia used this same pattern many decades earlier at the Great Mosque and hospital of Divrigi in Turkey (1228-29), as did Seljuk artists in a narrow border at the Gunbad-i 'Alaviyan in Hamadan, Iran (late twelfth century) [Photograph 22]. Particularly successful examples of Mamluk 3.4.6.4 designs include a *median* pattern from the *mihrab* spandrel of the Aydumur al-Bahlawan funerary complex in Cairo (1364) [Fig. 104a], and a stone mosaic *obtuse* pattern from the *mihrab* niche at the Sultan Qansuh al-Ghuri complex in Cairo (1503-05) [Fig. 104b]. The earliest known use of this latter pattern is from the Gök *madrasa* and mosque in Amasya, Turkey (1266-67). A strong characteristic of this pattern is the application of octagons within the square modules of the underlying tessellation. Many examples of patterns created from the $3.12^2$ underlying tessellation were employed by Mamluk artists, including a *median* pattern from the *mihrab* arch spandrel of the Amir Salar and Amir Sanjar al-Jawli funerary complex in Cairo (1303-04) [Fig. 108a]. The quarter dome hood of the *mihrab* niche also employs this design. However, the artist naively forced this two-dimensional design onto the spherical surface, thereby causing significant distortion. This forced fit is surprising in that this pattern could have uniformly fit the domical surface had the artist employed either an octahedral or icosahedral layout of the multiple triangular repetitive units. Two exquisite stone mosaic panels at the Amir Aq Sunqar funerary complex in Cairo (1346-47) were created from this same underlying tessellation. One of these is a *two-point* pattern [Fig. 108f], and the other is an *obtuse* pattern [Fig. 108d] [Photograph 45]. The occurrence of these two mosaic panels with their shared generative origin would appear to be a deliberate, if subtle, feature of the ornamental schema, and provides peripheral evidence for the use of the polygonal technique within this tradition. This *two-point* pattern was also used by Mamluk artists at the Amir Aq Sunqar funerary complex in Cairo (1346-47), as well as during the Ilkhanid period on an illuminated frontispiece of a Baghdadi Quran illuminated by Muhammad ibn Aybak ibn 'Abdullah (1306-07). The same *obtuse* pattern that was used at the Amir Aq Sunqar funerary complex was later used on a pair of wooden cupboard doors at the Sultan Qansuh al-Ghuri complex in Cairo (1503-05). Perhaps the most renowned Mamluk geometric pattern easily created from the $3.12^2$ tessellation is a frontispiece from the 30-volume Quran written and illuminated by 'Abd Allah ibn Muhammad al-Hamadani in 1313. The visual appeal of this outstanding illumination is augmented by the curvilinear



**Photograph 45** A threefold Mamluk *obtuse* pattern with 12-pointed stars that can be created from the *system of regular polygons* located at the Amir Aq Sunqar funerary complex in Cairo (© David Wade)

treatment of the pattern lines[177] [Fig. 108c]. An *obtuse* pattern derived from the 4.6.12 underlying tessellation was particularly popular among Mamluk artists. This pattern places octagons within the square modules of the generative tessellation [Fig. 109b]. The many Mamluk buildings that employed this pattern include: one of the exterior carved stucco roundels at the base of the dome at the Amir Sanqur al-Sa'di funerary complex in Cairo (1315); the entry door of the Amir Ulmas al-Nasiri mosque and mausoleum in Cairo (1329-30); the entry door of the Sultan Qansuh al-Ghuri *madrasa* (1501-03); and the entry door of the Sultan Qansuh al-Ghuri Sabil-Kuttab in Cairo (1503-04). The shared patron and time period of the latter two examples indicates the likelihood of their being produced by the same artist.

In addition to the use of regular and semi-regular tessellations, Mamluk artists also made frequent use of *two-uniform* and *three-uniform* tessellations when using the *system of regular polygons*. The 3.4.3.12-3.12$^2$ tessellation was especially relevant to the Mamluk geometric idiom. A particular *obtuse* pattern created from this tessellation was used with great frequency by Mamluk artists [Fig. 113c]. The 120° crossing pattern lines are easily determined by the application of regular hexagons placed within each underlying triangles, and by applying lines that skip one polygonal edge within the dodecagon—as per a 12-*s*2 pattern line application. Mamluk examples of this pattern include the side panel of the *minbar* at the mosque of Sultan al-Nasir Muhammad ibn Qala'un at the citadel of Cairo (1295-1303); a window grille at the Amir Sanqur al-Sa'di funerary complex in Cairo (1315); the side panels of the *minbar* at the Amir Altinbugha al-Maridani mosque in Cairo (1337-39); the arch spandrel over the *mihrab* at the Araq al-Silahdar mausoleum in Damascus (1349-50); a frontispiece in an illuminated Quran[178] written by Ya'qub ibn Khalil al-Hanafi in 1356; and a stone mosaic floor at Fort Qaytbey in Alexandria (c. 1480). A very beautiful *two-point* pattern [Fig. 113e] made from this same 3.4.3.12-3.12$^2$ *two-uniform* tessellation was used on the side panels of the *minbar* at the Amir Azbak al-Yusufi complex in Cairo (1494-95) [Photograph 46], as well as in the *minbar* railing at the mosque of Amir Qijmas al-Ishaqi in Cairo (1479-81). And an eccentric *median* pattern created from the 3.4.3.12-3.12$^2$ was used in

[177] Cairo, National Library, 72, pt. 19.

[178] Cairo, National Library, 8, ff. IV-2r.

**Photograph 46** Mamluk *minbar* at the Amir Azbak al-Yusufi complex in Cairo (© John A. and Caroline Williams)

the stone *minbar* of the Zawiya wa-Sabil Faraj ibn Barquq in Cairo (1400-11) [Fig. 113f]. This design employs two distinct pattern line treatments within the alternating dodecagons: a feature quite common in the Maghreb, but very unusual in Mamluk ornament. A Mamluk pattern that can be created from a *three-uniform* tessellation was used in the window grilles of the main façade at the Sultan Qala'un funerary complex in Cairo (1284-85). The application of 60° crossing pattern lines into the $3^4.6$-$3^3.4^2$-$3^2.4.3.4$ tessellation of triangles, squares, and hexagons produces this outstanding *median* design [Fig. 114b]. It is worth noting that this pattern can also be created from the 4.6.12 tessellation of squares, hexagons, and dodecagons [Fig. 109c]. When using this underlying tessellation to generate the design, the central six-pointed stars inside each of the underlying dodecagons are an arbitrary modification of what would otherwise be 12-pointed stars. The fact that this pattern can be created from more than just one underlying tessellation demonstrates the inherent methodological flexibility of the polygonal technique. These two derivations have slightly different proportions within the extracted pattern lines, but those created from the *three-uniform* tessellation precisely match the proportions and pattern density of the window grille in the Sultan Qala'un funerary complex. This design shares characteristics with the pattern from the earlier Zangid portal of the Bimaristan Arghun at the citadel of Aleppo (twelfth century) [Fig. 101c] [Photograph 36].

Mamluk artist occasionally employed the previously mentioned ditrigonal module that is part of the *system of regular polygons*. This hexagonal module has three 90° included angles that alternate with three 150° included angles. A threefold *median* pattern that incorporates this module into its underlying generative tessellation was used in the *mihrab* of the Amir Altinbugha al-Maridani mosque in Cairo (1337-39), as well as in one of the small blind arches within the Mamluk *mihrab* niche of the Aqbughawiyya *madrasa* (1340) at the al-Azhar mosque in Cairo [Fig. 117]. The closeness in time and location suggests that these two designs may have been the work of the same artist or atelier. The underlying tessellation is comprised of dodecagons located at the vertices of the isometric grid, separated by a vertex-to-vertex square surrounded by four coincident triangles. The ditrigon is located at the center of each triangular repeat unit, and can be regarded as the interstice of the regular polygonal modules. A fourfold pattern from the side panels of the *minbar* (c.1300) at the Vizier al-Salih Tala'i mosque in Cairo uses four radially arrayed underlying ditrigonal modules within alternating dodecagons of the otherwise $3.4.3.12$-$3.12^2$ generative tessellation [Fig. 119]. The pattern lines that are generated from the cluster of four ditrigons create an octagon at the center of the repeat unit. This same pattern was used as a border in the mosaic *mihrab* of the Mamluk Tabarsiyya *madrasa* (1309) at the al-Azhar mosque in Cairo. This same location has a second pattern that also employs the ditrigon within its underlying generative tessellation. A window grille from this *madrasa* employs a design that is created from an underlying tessellation that places six ditrigons around and interstice six-pointed star [Fig. 118b]. This underlying tessellation is, itself, the classic *median* pattern created from the $6^3$ tessellations of hexagons [Fig. 95c]. The earliest known pattern created from this underlying tessellation of ditrigons and six-pointed stars was produced by Seljuk artists working on the northeast dome chamber of the Friday Mosque at Isfahan (1088-89) [Fig. 118a] [Photograph 19].

One of the most elegant examples of the standard *acute* pattern created from the $4.8^2$ tessellation of squares and octagons is from the stucco window grille on the façade of the entry court at the Sultan Qala'un funerary complex in Cairo (1284-85) [Fig. 124a]. Mamluk artist used the well-known subtractive version of the standard *median* pattern created from this tessellation on the door of the Vizier al-Salih Tala'i mosque in Cairo (1303) [Fig. 126f], and an example of an exceptional variation to the standard *obtuse* pattern created from this tessellation surrounds the upper shaft of a minaret at the Sultan Qaytbay funerary complex in Cairo (1472-74) [Fig. 127d].

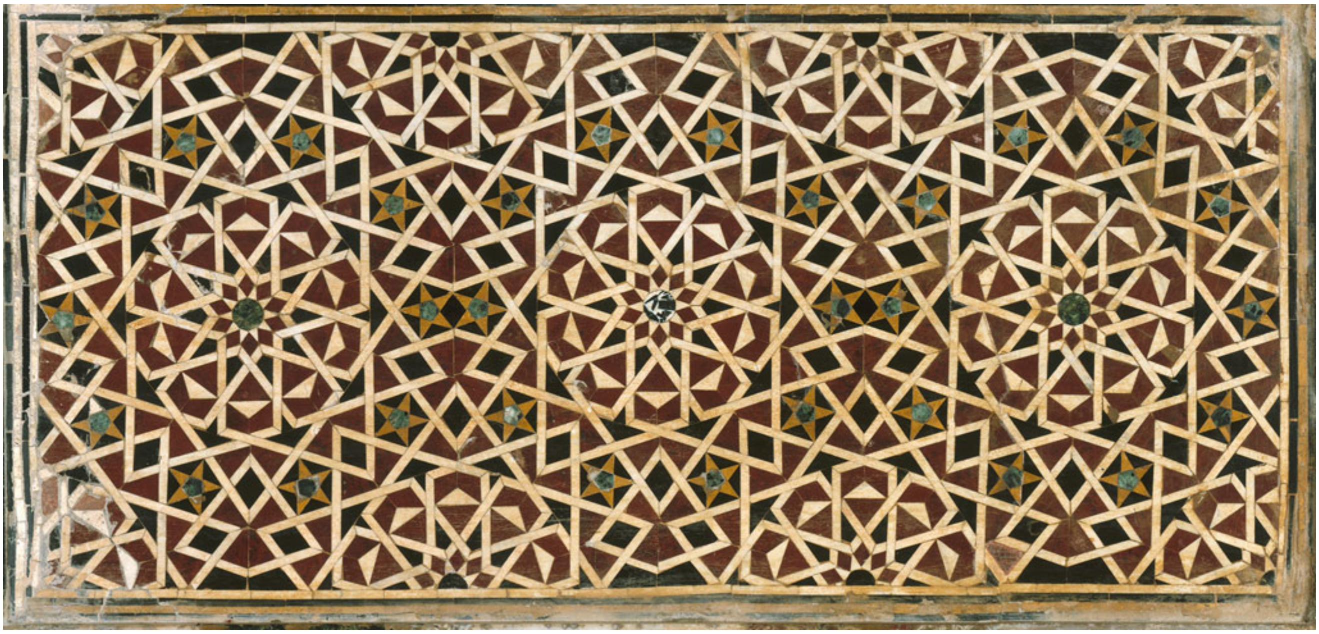
The Mamluks were less disposed toward patterns created from the *fourfold system A*. Of the relatively few patterns from this system, most were recreations of existing patterns that had been used by prior Muslim cultures. Such examples include a *median* pattern surrounding the circular shaft in the upper portion of the minaret at the Amir Taghribardi funerary complex in Cairo (1440) that was used by Qarakhanid artists nearly 300 years previously at the Maghak-i Attari mosque in Bukhara (1178-79) [Fig. 151] [Photograph 16]. An example of a Mamluk *median* field pattern created from this system frames an entrance to the Khan al-Sabun in Aleppo (1492) [Fig. 138c]. This is the ubiquitous design first found at the Seljuk east tomb tower at Kharraqan (1067-68). A *median* pattern on the minaret of the Attar mosque in Tripoli, Lebanon (1350), appears to be an original construction, although it is not overly complex and may well have been used previously. While this design repeats upon an orthogonal grid, the center points of the eight-pointed stars are placed upon the vertices of the $4.8^2$ tessellation of squares and octagons [Fig. 154]. Patterns that use this repetitive schema are most frequently produced from the *fourfold system B* [Figs. 179 and 180], and one of the relatively few additional occurrences of the Mamluk design in the Attar mosque is at the Mughal tomb of I'timad al-Daula in Agra (1622-28) [Photograph 73].

The Mamluk use of the *fourfold system B* was far more pervasive than that of the *fourfold system A*. It would appear significant that the ornament of the Fatimids, Zangids, and Ayyubids all shared in the relative absence of geometric patterns constructed from the *fourfold system A*. Reasons for this bias are lost to history, but one can surmise that the small body of artists working for successive dynasties in this geographical region, and who were the inheritors of the polygonal technique as a principle design methodology, were substantially less familiar with this particular system than their eastern counterparts.

Among the many Mamluk patterns created from the *fourfold system B* are multiple examples of the classic *acute* design derived from the underlying tessellation of just octagons and irregular pentagons [Fig. 173a]. An early Mamluk example of this *acute* design was used in the pierced stone window grilles of the Sultan Qala'un funerary complex in Cairo (1284-85) [Photograph 55]. Later Mamluk examples of this well-known pattern include the lower mosaic panels of the *mihrab* niche of the Mamluk Taybarsiyya *madrasa* (1309) at the al-Azhar mosque in Cairo; the minaret of the Aydumur al-Bahlawan funerary complex in Cairo (1364); and a curvilinear variation from a carved stone relief panel at the entry of the Qadi Nur al-Din mosque in Cairo (1466). A *two-point* pattern made from the same underlying tessellation [Fig. 173d] was used in the *mihrab* niche of the Mamluk Aqbughawiyya *madrasa* (1340) in the al-Azhar in Cairo, as well as the entry portal of the Ashrafiyya *madrasa* in Jerusalem (1482). A variation to this *two-point* pattern was used in the magnificent painted ceiling of the Sultan al-Mu'ayyad Shaykh complex in Cairo (1415-22) [Fig. 174a]. A stylistically similar *two-point* pattern created from an underlying tessellation of octagons, pentagons, and elongated hexagons was used in the entry portal of the Sidi Madyan mosque in Cairo (1465) [Fig. 176a]; and the same underlying tessellation was used to derive a very pleasing *obtuse* pattern in two adjacent upper panels of the Mamluk mosaic *mihrab* at the Taybarsiyya *madrasa* (1309) in the al-Azhar mosque in Cairo [Fig. 175d]. The earliest known use of the *obtuse* design from the Taybarsiyya *madrasa* is from the Ayyubid portal façade at the Palace of Malik al-Zahir at the citadel of Aleppo (before 1193). Like their counterparts in other Muslim cultures, in using the *fourfold system B* Mamluk artists employed the attractive variation to the *acute* pattern line application into the underlying elongated hexagonal module that provides for the creation of regular octagons within the pattern matrix [Fig. 172b]. One of the most outstanding Mamluk examples of this type of *fourfold system B* design is from the wooden *minbar* (1296) at the mosque of ibn Tulun in Cairo[179] [Fig. 179a]. This *minbar* was part of the restoration of the mosque by Sultan Lajan (r. 1296-1299) stemming from his gratitude at having successfully escaped his enemies by hiding in the derelict mosque. This pattern places eight-pointed stars upon the vertices of the $4.8^2$ tessellation. This same *acute* pattern was used during the Mamluk period in several instances; including a very beautiful illuminated frontispiece of a Mamluk Quran[180] (before 1369), and an exterior stone panel at the Cathedral of St. James in Jerusalem that was likely produced by local Armenian stone carvers during the Mamluk period. A carved stone lintel above a recessed bay at the Sultan Qaytbey Sabil in Jerusalem (1482) is interesting in that the lines of the *obtuse* pattern are irregularly placed within portions of the underlying polygonal tessellation [Fig. 184]. This irregularity results from the application of 90° crossing patterns lines at select locations within the otherwise pattern matrix of 118° crossing pattern lines. This variation in pattern angles creates an unusual dynamic that is very successful, and has analogous aesthetic characteristics with *obtuse* patterns created from the *fivefold system*. All of these cited examples employ the orthogonal grid in their repetition. However, Mamluk artists working with this system occasionally created patterns that repeat on a rhombic grid. A panel above the *mihrab* at the al-Mar'a mosque in Cairo (1468-69) is a

179 The panel from this *minbar* is in the collection of the Victoria and Albert Museum, London: museum number 1051–1869.

180 Mamluk *mashaf*: Quranic manuscript No. 16; Islamic Museum, al-Aqsa Mosque, al-Haram al-Sharif, Jerusalem.



**Photograph 47** Mamluk inlaid stone panel with a *two-point* pattern created from the *fivefold system* (The Metropolitan Museum of Art: Gift of the Hagop Kevorkian Fund, 1970: www.metmuseum.org)

case in point [Fig. 183]. This rather clever *two-point* pattern utilizes the octagon, pentagon, hexagon, and rhombus as underlying modules in the pattern construction.

The Mamluk use of geometric patterns created from the *fivefold system* was pervasive and incorporated the full range of diversity in repetitive schema. As with other Muslim cultures, the most commonly used fivefold repeat unit was the more obtuse rhombus with 72° and 108° angles. Among these are multiple examples of patterns created from the most commonly employed underlying tessellation of decagons, pentagons, and barrel-shaped elongated hexagons [Fig. 226a]. The Mamluk use of the classic *acute* pattern that is created from this underlying tessellation was less frequent than that of other Muslim cultures. Two examples of this design are found on Mamluk doors: one at the al-Azhar mosque, and another that is currently in the courtyard of the French Embassy in Giza [Fig. 226c]. A Mamluk carved stone relief panel on the main façade of the Sultan Qansuh al-Ghuri complex in Cairo (1503-05) makes use of a widened line version of the *median* design created from this underlying tessellation [Fig. 87]. Multiple examples of the *obtuse* design produced by this tessellation were used during the Mamluk period, and examples include a window grille within the dome of the Sultan Qala'un funerary complex in Cairo (1284-85), and several blind arches surrounding the drum of the dome at the Amir Sanqur al-Sa'di funerary complex in Cairo (1315) [Fig. 229b]. Similarly, the Mamluks were particularly disposed toward the *two-point* pattern made from this same tessellation, and examples include a panel in the entry portal of the Qadi Abu Bakr Muzhir complex in Cairo (1479-80); an inlaid stone panel from the Sultan Qaytbay Sabil-Kuttab in Cairo (1479); and a contemporaneous Mamluk polychrome stone mosaic panel in the collection of the Metropolitan Museum of Art, New York City[181] [Fig. 231d] [Photograph 47]. Mamluk geometric artist frequently used a pattern that was first used on the Ildegizid mausoleum of Mu'mine Khatun in Nakhichevan, Azerbaijan (1186), although they were more likely influenced by less distant Ayyubid or Zangid examples such as that found at the Imam Awn al-Din Meshhad in Mosul (1248) [Fig. 232g]. This pattern also repeats with the 72° and 108° rhombus, and can be produced from either of two separate underlying tessellations: from the tessellation of just decagons and concave hexagons [Fig. 232f], and from a tessellation of decagons, barrel hexagons, and trapezoids that surround a large concave hexagon [Fig. 232h]. As mentioned previously, this design can also be produced in yet a third manner: through manipulating the *median* pattern lines from the standard design created from the most basic rhombic underlying tessellation of decagons, pentagons, and barrel hexagons [Fig. 227e]. Among the many Mamluk locations of this design are the Sultan Qala'un funerary complex in Cairo (1284-85); Amir Sanqur al-Sa'di funerary complex in Cairo (1315); the Hasan Sadaqah mausoleum in Cairo (1315-21); the Sultan Qaytbay funerary complex in Cairo (1472-74);

[181] Metropolitan Museum of Art: gift of the Hagop Kevorkian Fund; 1970.327.8.

**Photograph 48** Mamluk illuminated frontispiece from a Quran commissioned by Sultan Faraj ibn Barquq with a design created from the *fivefold system* (British Library Board: BL Or. MS 848, ff. 1v-2)

the Qadi Abu Bakr Muzhir complex in Cairo (1479-80); and the Amir Azbak al-Yusufi complex in Cairo (1492-95) [Photograph 46]. An underlying tessellation of decagons, pentagons, barrel hexagons, and small rhombi was used to create a fine *obtuse* pattern that was used in the *mihrab* niche of the Sultan Qala'un funerary complex in Cairo (1284-85), as well as an illuminated frontispiece in the 30-volume Mamluk Quran commissioned by Sultan Faraj ibn Barquq (1399-1411) [Fig. 233b] [Photograph 48]. As with other examples, this design can alternatively be created from the dual of this tessellation: in which case the generative tessellation is comprised of decagons, elongated hexagons, and concave hexagons. Either of these same dualing tessellations will produce a very satisfactory *two-point* pattern that was used in the *mihrab* at the Sultan Qansuh al-Ghuri complex in Cairo (1503-05) [Fig. 233e]. Another very successful pattern that repeats on this rhombus was used on a pair of matched frontispieces from a Quran (1313) originally owned by Sultan Nasir al-Din Muhammad, and illuminated by Aydoğdu bin Abdullah al-Badri and Ali bin Muhammad al-Rassam[182] [Fig. 235c]. The pattern from this Quran can also be made from two distinct underlying polygonal tessellations made from the components of the *fivefold system*. Both derivations are equally valid, and the original artist is as likely to have used one as the other. This same design was used during the Seljuk Sultanate of Rum at the Huand Hatun complex in Kayseri (1237) [Fig. 235d]. A highly complex fivefold pattern that uses the same obtuse rhombic repeat with 72° and 108° included angles is found on one of the metal doors of the *madrasa* of Qadi Abu Bakr ibn Muzhir[183] (1479-80) [Fig. 267]. This pattern is distinctive for its use of 20-pointed stars: each placed upon a vertex of the rhombic repeat. Each of the 20-pointed stars is surrounded by ten 10-pointed stars. Interestingly, these 10-pointed stars are located upon the vertices of a secondary tessellation of decagons and concave hexagons. These two distinct repetitive cells identify this as the only known Mamluk example of a hybrid design.

Mamluk artists also produced a variety of geometric patterns that employ the more acute fivefold rhombus comprised of 36° and 144° angles. The historical occurrence of fivefold patterns that repeat with this rhombus are significantly less common, and the Mamluk examples are a testament to the exploratory approach to geometric design among these artists. A fine example of a *two-point* pattern that repeats on this more acute rhombus was used on the bi-fold doors of a *minbar* that was commissioned by Sultan Qaytbey[184] (r. 1468-96) [Fig. 244]. Another example of this variety of fivefold repeat was used in at least two Mamluk locations: a *minbar* door panel of uncertain provenance in the Victoria and Albert Museum in London[185] and the *minbar* railing at the *khanqah* and mosque of Sultan al-Ashraf Barsbay funerary complex in Cairo (1432-33) [Fig. 242b]. The earliest known examples of this pattern are two contemporaneous locations: a pair of Kartid wooden doors at the Turbat-i Shaykh Ahmad-i Jam in Torbat-i Jam in northwestern Iran (1236); and in the carved stone portal of the Huand Hatun *madrasa* in Kayseri in central Anatolia (1237). This *acute* pattern is unusual in that the underlying generative tessellation includes irregular pentagons and associated rhombi that give the pattern lines associated with these modules qualities that are characteristic of the

[182] This Quran is in the collection of the Museum of Turkish and Islamic Arts; Sultanahmet, Istanbul, Turkey: Museum Inventory Number 450.

[183] Mols (2006), cat. no. 46/1, pl. 191–194.

[184] This *minbar* is in the collection of the Victoria and Albert Museum in South Kensington, London: 1050: 1 to 2–1869.

[185] This fivefold pattern is from a nineteenth-century copy of the original Mamluk *minbar* door panel, and is part of the collection of the Victoria and Albert Museum in South Kensington, London: 887–1884.

*median* pattern family. The application of this pattern to *minbar* doors is particularly appropriate in that the typical proportions of a door necessitate long and narrow panels, and the more acute proportions of this rhombus fit nicely within these design constraints. An *obtuse* pattern from a Mamluk mosaic panel that repeats with this same acute repeat unit was used in the *mihrab* niche of the Amir Altinbugha al-Maridani mosque in Cairo (1337-39) [Fig. 241]. Once again, this pattern can be made from either of two dualing tessellations with equal facility. Two matching stone relief panels above the door at the southeast entrance of the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) employ an interesting *two-point* pattern that repeats upon the more acute rhombi [Fig. 243d]. The floating rhombic elements that separate the ten-pointed stars give this design a somewhat non-cohesive quality. This example is not known to have been used elsewhere.

Like the artists in preceding dynasties, Mamluk artists also applied the *fivefold system* to designs that repeat upon a rectangular grid. The wall of the *mihrab* niche at the Qadi Abu Bakr Muzhir complex in Cairo (1479-80) includes a relatively simple *two-point* pattern executed in polychrome stone mosaic [Fig. 245c]. A considerably more complex *two-point* pattern, with much broader rectangular repeats, was use to decorate a side panel of the *minbar* commissioned by Sultan Qaytbay[186] (r. 1468-96) [Fig. 248]. This same design was also used on the *minbar* of the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81), as well as the *minbar* at the Amir Azbak al-Yusufi complex in Cairo (1494-95) [Photograph 46]. Both these *minbars* utilize this design on the side panels adjacent to the platform, and the fact that Sultan Qaytbay and Amir Qijmas were contemporaries indicates that the same geometric artists likely worked on both. This rectangular *two-point* pattern has the interesting feature of a ten-pointed star being placed at the center of the repeat unit with radii that are not aligned with the radii of the ten-pointed stars at the corners of each repeat unit. This skewed orientation provides an unusual and dynamic quality. Other Mamluk *two-point* patterns with non-aligned radii between the ten-pointed stars at the vertices of those at the centers of the rectangular grid include a second design from the Amir Qijmas al-Ishaqi mosque (1479-81) [Fig. 252] and a carved stone lintel at the Sultan Qaytbay Sabil in Jerusalem (1482) [Fig. 250d]. The underlying generative tessellation that produces this latter design was also used by the Mamluks at two other locations: an *obtuse* design on the stone *minbar* of the Sultan Barquq mausoleum in Cairo (1384-86) [Fig. 250f] [Photograph 57], and an *acute* pattern in a bronze window grille at the al-Azhar mosque in Cairo [Fig. 250b]. The artist who created this *acute* design recognized the inherent problem when applying 36° crossing pattern lines to the long hexagon within this system [Fig. 187]. Rather than adjusting the underling tessellation itself, this artist arbitrarily changed the angles of the lines within this module. At first glance, this appears to be an acceptable solution. However, upon closer inspection, the break in the angles of the pattern lines is awkward and poorly resolved, and does not follow the well-established conventions for fivefold patterns. A more acceptable Mamluk *acute* pattern that repeats upon a rectangular grid was used on the *minbar* door (thirteenth century) of the otherwise Fatimid mosque of Vizier al-Salih Tala'i [Fig. 255]. Indeed, this is an exceptional example of a complex fivefold *acute* pattern, and was used many hundreds of years later by Mughal artists at the tomb of Akbar in Sikandra, India (1612).

The Mamluks rarely used hexagonal repeat units with the *fivefold system*. Of the few examples are two identical raised stone panels in the entry portal of the Ashrafiyya *madrasa* in Jerusalem (1482) [Fig. 258]. The *two-point* design of these two panels follows the occasional Mamluk convention of representing only a minimum portion of the design. In the case of this design from Jerusalem, the limited view of the pattern obscures the clarity of the total repetitive unit. If the featured image is reflected and repeated with translation symmetry, as per convention, the repeat unit for the total design is hexagonal. The interstice region at the center of the repeat unit creates pattern elements that are atypical of the *fivefold system*, as are the very close parallel lines placed within the underlying rhombic modules. The rectangular cropping of this example cleverly divides the underlying rhombi in half, thereby eliminating the problem of the two overly close parallel lines [Fig. 258b].

Mamluk artists were less disposed toward fivefold field patterns than their contemporaries from Anatolia. Among the relatively rare Mamluk examples of this variety of five-fold design is a simple, but pleasing, carved stone relief above the door at the entry portal of the Amir Ghanim al-Bahlawan funerary complex in Cairo (1478) [Fig. 212]. This is a *median* pattern that repeats upon a hexagonal grid with *two-point* characteristics that result from the pattern line application to the long edges of the underlying triangles. This form of *median* pattern line treatment can be traced back to the Khwaja Atabek mausoleum in Kerman (1100-1150) [Fig. 211], and was a particularly popular ornamental devise among artists working in the Seljuk Sultanate of Rum.

Mamluk geometric patterns created from the *system of regular polygons*, both fourfold systems and the *fivefold system*, for the most part continued the working practices and aesthetic predilections of their Zangid and Ayyubid predecessors. Yet Mamluk artists were among the most innovative in the long history of this design tradition. This

[186] This *minbar* is in the collection of the Victoria and Albert Museum in South Kensington, London: 1050: 1 to 2–1869.

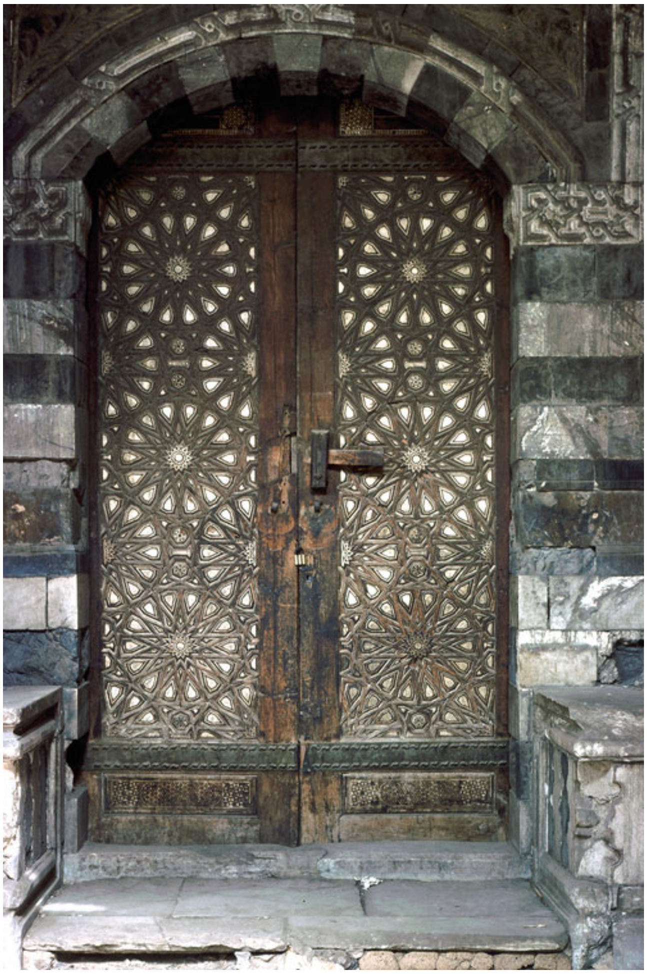
innovation is particularly evident in three areas of geometric design: the application of geometric patterns to the surfaces of domes; the further development of complex non-systematic geometric patterns with multiple regions of differentiated local symmetry; and the bringing to maturity the class of geometric patterns that are created from the *sevenfold system* of pattern generation. Earlier examples of sevenfold patterns were used in Seljuk Persia and Turkey, but these are so few in number that it is impossible to determine the extent to which the responsible artists knew that the underlying polygonal elements with their associated pattern lines formed part of a comprehensive system, much like the *fivefold system*. As cited above, these earlier Seljuk examples employed only four polygonal modules: two varieties of irregular hexagon in the Persian example, and the heptagon and irregular pentagon in the three examples from the Sultanate of Rum. This paucity of underlying modules argues against the artist's knowledge of this as a distinct *system* per se, as does the fact that there are so few examples of sevenfold designs from this earlier period. Had there been knowledge of the *sevenfold system* at this earlier time, one would assume that there would be many more examples of such patterns in the historical record. Rather, it appears that these Seljuk artists, in their quest to apply the polygonal technique to the creation of sevenfold patterns, happened upon several of the underlying polygonal tessellations that were, in time, discovered to be part of a comprehensive *sevenfold system*.

The Mamluk development of sevenfold patterns began in the early fourteenth century, and from their earliest examples, the underlying generative polygons included a greater diversity than previous sevenfold designs. In particular, the underlying generative tessellations of these Mamluk examples included the 14-sided tetradecagon at the vertices of the repetitive grid. This module is responsible for the 14-pointed stars that characterize patterns created from this system in its fully mature expression. The earliest Mamluk example of a sevenfold *median* pattern is from a carved stone lintel on the south elevation of the Qawtawiyya *madrasa* in Tripoli, Lebanon[187] (1316-26) [Fig. 286a]. Just as there are two rhombi associated with fivefold symmetry [Fig. 5], there are three rhombi associated with sevenfold symmetry [Fig. 10]. However, only the two more obtuse sevenfold rhombi were used historically as repeat units. The example from Tripoli repeats on the more acute of these two rhombi: comprised of 2/14 and 5/14 included angles. A variation of the same underlying generative tessellation is associated with a *median* pattern on the carved stone exterior façade of the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) [Fig. 286b]. In keeping with a common Mamluk decorative convention pertaining to geometric designs, only a portion of the overall design is shown. This same design, in its full reveal, was used at a somewhat earlier date by Timurid artists to create a carved stucco wall panel at the Amir Burunduq mausoleum at the Shah-i Zinda complex in Samarkand (1390-1420). A side panel from the *minbar* at the Sultan Barsbay complex at the northern cemetery in Cairo (1432) employs a fine sevenfold *obtuse* pattern that repeats upon this same rhombus[188] [Fig. 287b]. This was copied for the entry door of the Hanging Church in Cairo (al-Mu'allaqa), a Coptic church dedicated to St. Mary. This door is stylistically Mamluk in both the sophistication of the design and woodwork. A subtractive variation of this pattern was used on an earlier Ottoman door panel at the Bayezid Pasa mosque in Amasya, Turkey (1414-19) [Fig. 287c].[189] An example of a sevenfold *obtuse* pattern that repeats upon the more obtuse rhombus comprised of 3/14 and 4/14 included angles was used on the double doors of the *minbar* at the Haram al-Ibrahimi in Hebron, Palestine [Fig. 290]. While this *minbar* is Fatimid (1092), and brought to Hebron by the great Ayyubid leader Ṣalāḥ ad-Dīn, some components are clearly later Mamluk additions. In particular, the style of the patterns used in the *minbar* doors and back panel of the platform are of Mamluk origin. A similar sevenfold pattern was used during the Ottoman period on the incised marble ceiling of the small rectangular water feature within the courtyard of the Suleymaniya mosque in Istanbul (1550-58) [Fig. 289] [Photograph 81]. This later Ottoman example can be created from a very similar underlying polygonal tessellation wherein the concave decagonal interstice regions remains free of additional polygonal modules, and the kite motifs within this region have a distinctive *two-point* quality. The same more obtuse rhombic repeat unit was used for two outstanding Mamluk sevenfold designs that are created from the same underlying polygonal tessellation: an *obtuse* pattern from the *minbar* door of the 'Abd al-Ghani al-Fakri mosque in Cairo (1418) [Fig. 292a], and a *two-point* pattern from the large congregational Quran Stand at the Sultan Qahsuh al-Ghuri

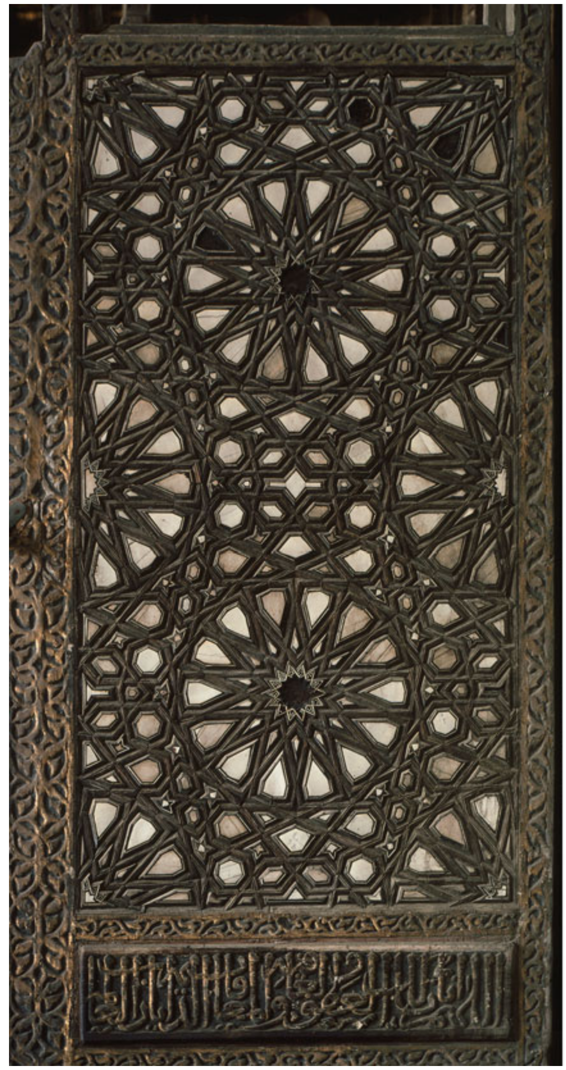
[187] This design was illustrated in the Monument Survey of Tripoli, Lebanon by Hala Bou Habib, Karl Sharro, and Hind Abu Ibrahim for the American University of Beirut, Department of Architecture, 1991 and 1992.

[188] Bourgoin (1879), pl. 166. As with all Bourgoin's illustrations, this pattern is not shown with its formative structure.

[189] This pattern is also identical to a pattern in raised brick in one of the ground-level blind arches in the courtyard of the Mustansiriyah in Baghdad (1227–34). This building stems from the late Abbasid period just decades prior to the Mongol conquest. However, the incorporation of this sevenfold design appears to date from the nineteenth-century Ottoman restoration of this building. The earlier Ottoman provenance of this sevenfold pattern appears to have been the source of influence for the example at the Mustansiriya in Baghdad.



**Photograph 49** A Mamluk entry door at the Sultan Qansuh al-Ghuri complex in Cairo with a pattern created from the *sevenfold system* (© David Wade)



**Photograph 50** A side panel from the Mamluk *minbar* at the Sultan al-Mu'ayyad Shaykh complex in Cairo with a pattern created from the *sevenfold system* (© David Wade)

complex in Cairo[190] (1503-05) [Fig. 292b]. The underlying generative tessellation for the obtuse example from the 'Abd al-Ghani al-Fakri mosque was first attributed to this design by Ernest Hanbury Hankin.[191] One of the entry doors at the Sultan Qansuh al-Ghuri complex in Cairo, Egypt (1503-05) [Photograph 49] is also decorated with a fine sevenfold pattern that can be produced from either of two underlying tessellations [Fig. 288]. As with designs produced from the *fivefold system*, patterns made from the *sevenfold system* can also repeat upon a rectangular grid, although only one such pattern is known from the historical record: a fine Mamluk example from one of the side panels of the *minbar* at the Sultan al-Mu'ayyad Shaykh complex in Cairo[192] (1415-22) [Fig. 294] [Photograph 50]. This design places 14-pointed stars at the vertices of the rectangular grid, the proportions of which are nearly a square. A 14-pointed star is also placed at the center of each rectangular repeat unit. The two underlying mirrored contiguous triangles that separate the underlying tetradecagon at the center of the repeat from those at the corners create a skewed orientation between the 14-pointed stars. This snub-like quality contributes to the powerful dynamic of this exceptional geometric pattern.

[190] Bourgoin (1879), pl. 168.

[191] In *The Drawing of Geometric Patterns in Saracenic Art* Hankin illustrates this underlying tessellation along with its associated *obtuse* pattern lines, but does not attribute the historical location of this design. As with his other published pattern analyses, he does not represent the polygonal elements used in creating his design examples as being part of a systematic methodology for pattern generation. In analyzing this design, it is likely that Hankin worked from the pattern collection of Joules Bourgoin (1879), plate 167. Hankin (1925a).

[192] Bourgoin (1879), pl. 169.

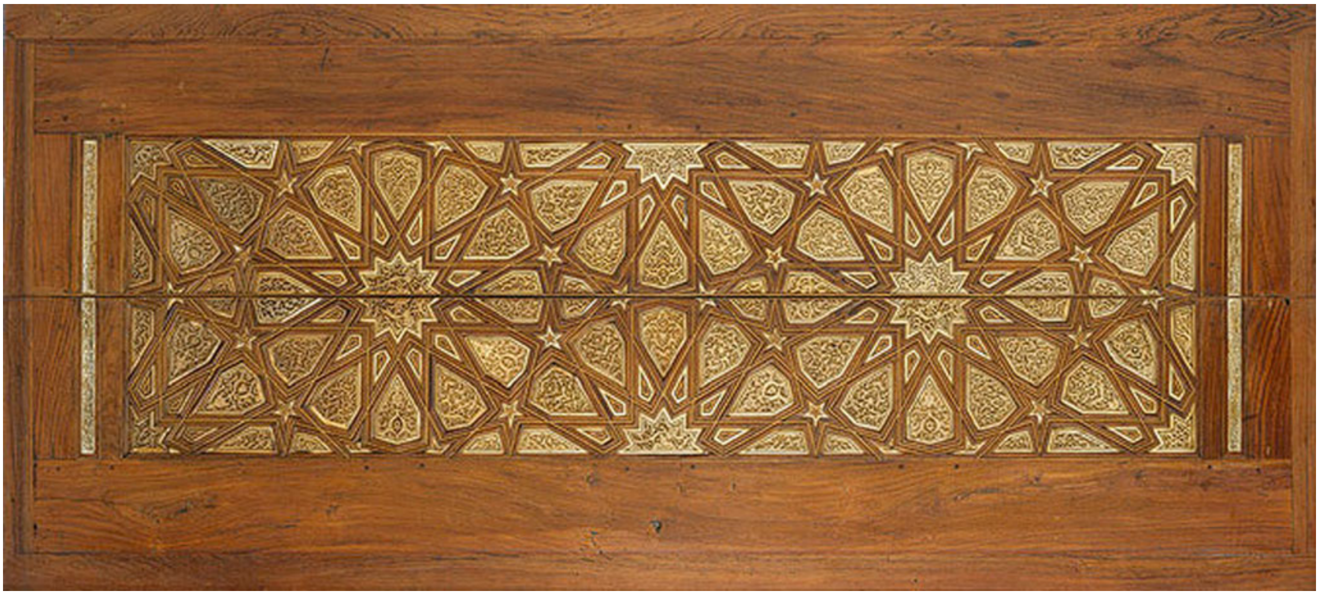
The significance of the *sevenfold system* is belied by the paucity of examples from the historical record. The development of this system represents a landmark achievement both for the beauty of the resulting designs and their geometric ingenuity. The small number of known examples warrants the inclusion of a further design created from this system that was recorded by Jules Bourgoin in his nineteenth-century collection of Islamic geometric patterns: *Les Eléments de l'art arabe: le trait des enterlacs*.[193] Unfortunately, Bourgoin did not provide the provenance of the designs he recorded. However, he worked principally in Egypt, and one must assume that the stylistic similarities to known Mamluk designs created from this system indicate a Mamluk provenance. The unidentified sevenfold pattern in Bourgoin's collection repeats upon the more obtuse rhombus, and is a very successful and well-balanced design. This is an *acute* pattern that uses an underlying tessellation that is closely related to those employed in the examples from the Suleymaniya mosque in Istanbul [Fig. 289] [Photograph 81], and the Haram al-Ibrahimi in Hebron [Fig. 290]. The difference between the underlying tessellations of these three designs is in the infill treatment of the identical concave decagonal regions.

Like their contemporaries in the Seljuk Sultanate of Rum, Mamluk artists applied the polygonal technique to a vast number of highly diverse nonsystematic geometric patterns. As inheritors of the artistic traditions of their Zangid and Ayyubid predecessors, Mamluk artists made frequent use of geometric designs that were already well known within the Islamic world. Yet the Mamluks contributed greatly to the full maturity of this design idiom by also developing original compound patterns with multiple centers of local symmetry, often with diverse repetitive schema. As in the Seljuk Sultanate of Rum, the creative attention of Mamluk artists was responsible for one of the last great innovative periods for nonsystematic geometric pattern making. Both of these cultures survived the onslaught of the Mongols and were able to continue the process of design innovation uninterrupted by political and cultural chaos. What is more, and as stated previously, the arts of both cultures benefited from the influx of artists fleeing the destruction in the east while seeking patrons in more stable Muslim lands.

Two identical adjacent Mamluk panels with a particularly successful nonsystematic design comprised of nine-pointed stars placed at the vertices of the hexagonal grid were used above the door in the entry portal at the Ashrafiyya *madrasa* in Jerusalem (1482). This is a *two-point* pattern created from an underlying tessellation of nonagons surrounded by a ring of nine irregular pentagons. This arrangement of pentagons creates an interstice six-pointed star at the center of each hexagonal repeat unit [Fig. 313b]. This *two-point* pattern appears to be unique to this location. However, an *acute* pattern made from this same underlying tessellation was used on the side panel of the *minbar* at the Sultan Qaytbay funerary complex in Cairo (1472-74) [Fig. 312a] [Photograph 51]. The close proximity in time and location between these two Mamluk examples raises the possibility of their being created by the same artist. This same underlying generative tessellation was used by earlier artists from the Seljuk Sultanate of Rum to create a *median* pattern that was first used at the Alay Han near Aksaray (1155-92) [Fig. 312b] [Photograph 43], as well as by later Shaybanid artists in the creation of an *obtuse* pattern that was used in several locations, including the Kukeltash *madrasa* in Bukhara, (1568-69) [Photograph 83]; the Nadir Diwan Begi *madrasa* and *khanqah* in Bukhara; and the Tilla Kari *madrasa* in Samarkand (1646-60) [Fig. 313a]. Separated over distance and time, this underlying tessellation was used to create designs in each of the four pattern families.

**Photograph 51** A side panel from the Mamluk *minbar* at the Sultan Qaytbay funerary complex in Cairo with a nonsystematic threefold *acute* pattern with six- and nine-pointed stars (© David Wade)

[193] Bourgoin (1879), pl. 165.



**Photograph 52** A door panel from the *minbar* of the Amir Qawsun mosque in Cairo with a nonsystematic threefold *acute* pattern with 12-pointed stars (The Metropolitan Museum of Art: Edward C. Moore Collection, Bequest of Edward C. Moore, 1891: www.metmuseum.org)

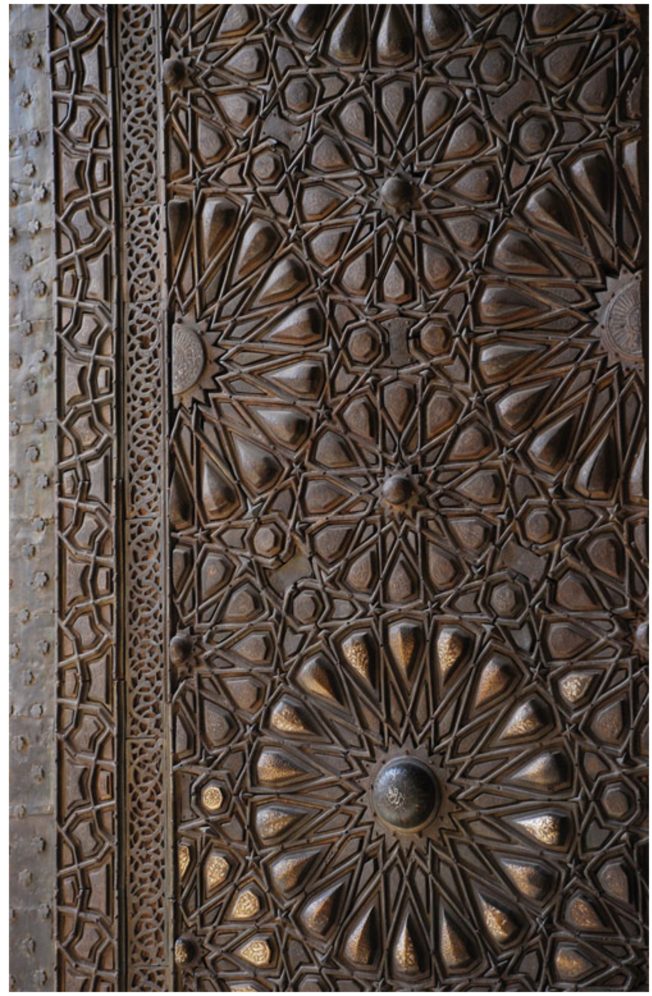
As with other Muslim cultures, the isometric grid was widely used by Mamluk artists for the repetition of nonsystematic geometric patterns; especially for designs with 12-pointed stars placed at the vertices of triangular repeat units. An underlying polygonal tessellation with dodecagons surrounded by a ring of 12 irregular pentagons was as widely employed by Mamluk artists as by the artists of other Muslim cultures. A fine *acute* example was used to decorate a pair of bifold doors thought to be from the *minbar* of the Amir Qawsun mosque in Cairo[194] (1329-1330) [Fig. 300a *acute*] [Photograph 52]. Another underlying generative tessellation that places dodecagons at the vertices of the isometric grid connects each dodecagon with elongated hexagons, and places three edge-to-edge irregular pentagons at the center of each triangular repeat unit [Fig. 321a]. This underlying tessellation was used by Mamluk artists to create a variety of designs, including an *acute* pattern from a door at the Al-Azhar mosque in Cairo [Fig. 300b *acute*]; an *obtuse* design from the stone window grilles of the Sultan Qala'un funerary complex in Cairo (1284-85) [Fig. 321f]; a *two-point* pattern that was used on both the *minbar* rail and an interior wooden door at the Sultan al-Mu'ayyad Shaykh complex in Cairo (1415-22); the portal of the Ribat Khawand Zaynab in Cairo (1456); as well as a carved stone lintel at the Ashrafiyya *madrasa* in Jerusalem (1482) [Fig. 300b *2-point*]. A variation of this tessellation, with pentagons that are truncated into trapezoids also makes fine designs [Fig. 321i]. The *acute* pattern that is created from this modified tessellation [Fig. 321j] can also be produced from the $3.12^2$ tessellation from the *system of regular polygons* [Fig. 108d]. This design was used by Mamluk artists in a mosaic panel from the Amir Aq Sunqar funerary complex in Cairo (1346-47), as well as in the doors of a cupboard at the Sultan Qansuh al-Ghuri complex in Cairo (1503-05).

The Mamluk use of nonsystematic patterns that repeat upon the isometric grid frequently included more complex geometric representations with multiple regions of diverse local symmetry. By far the most common Mamluk example of such a design was the already well established *acute* pattern that places 12-pointed stars at the vertices of the isometric grid, and 9-pointed stars at the centers of each triangular repeat unit [Fig. 346a]. Two Mamluk examples of this *acute* pattern include: a particularly fine example from a bronze door at the Sultan al-Zahir Baybars *madrasa* in Cairo[195] (1262-63), and a very becoming curvilinear example from one of the bronze doors of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63). The *two-point* pattern created from the same underlying tessellation was also used by Mamluk artists, and examples include one of the side panels of the *minbar* at the Qadi Abu Bakr Muzhir complex in Cairo (1479-80), as well as a side panel from the *minbar* at the Sultan Qansuh al-Ghuri complex (1503-05) [Fig. 347b] [Photograph 53]. The geometric logic of these patterns calls for the vertices of the isometric

[194] This pair of *minbar* doors is in the collection of the Metropolitan Museum of Art in New York City: accession number 91.1.2064.

[195] This Mamluk bi-fold door presently serves as the entry door of the French Embassy in Egypt.

**Photograph 53** A side panel from the Mamluk *minbar* at the Sultan Qansuh al-Ghuri complex in Cairo with a nonsystematic threefold *two-point* pattern with 9- and 12-pointed stars (© David Wade)



**Photograph 54** A Mamluk bronze entry door at the Sultan al-Zahir Barquq *madrasa* and *khanqah* in Cairo with a nonsystematic threefold *acute* pattern with 12- and 18-pointed stars (© David Lewis)

grid to be populated by *n*-pointed stars whose points are multiples of 6, and the *n*-pointed stars at the vertices of the hexagonal dual of the isometric grid (i.e., the centers of each triangular repeat) to be multiples of 3. Mamluk artists created a number of more complex isometric compound patterns that follow this design stratagem. An example with 12-pointed stars at the vertices of the isometric grid (2 stellar points × 6), and 15-pointed stars at the vertices of the dual hexagonal grid (5 stellar points × 3) was used on a carved stone lintel at the Qartawiyya *madrasa* in Tripoli, Lebanon (1316-26) [Fig. 355d]. This is a *median* pattern that employs a variation to the 15-pointed stars that are typical of Mamluk *median* designs.[196] An *acute* design from the *minbar* railing in the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) places 18-pointed stars at the vertices of the isometric grid and 9-pointed stars at the vertices of the hexagonal dual grid [Fig. 357]. This also introduces octagons at the midpoint intersections of these dual grids that function similarly to the roughly contemporaneous isometric design from the Sultan Qaytbay funerary complex in Cairo (1472-74) [Fig. 109b]. Another outstanding Mamluk *acute* design with 18-pointed stars located at the vertices of the isometric grid was used in the bronze entry door of the Sultan al-Zahir Barquq *madrasa* and *khanqah* in Cairo (1384-1386) [Photograph 54]. As distinct from the previous

[196] The author has extrapolated the reconstruction of this pattern from the Qartawiyya Madrasa in Tripoli, Lebanon, from an indistinct photograph taken by Hana Alamuddin [Aga Khan Visual Archive, Massachusetts Institute of Technology; catalogue number IHT0078]. This is the only image of this compound isometric pattern that the author has been able to find. The analysis represented in Fig. 355d is based upon the inherent logic of the 15- and 12-fold regions of local symmetry as exemplified in the indistinct proportions indicated within this photograph. A closer examination of this example may reveal slightly different angles in the crossing pattern lines, and pattern line relationships.

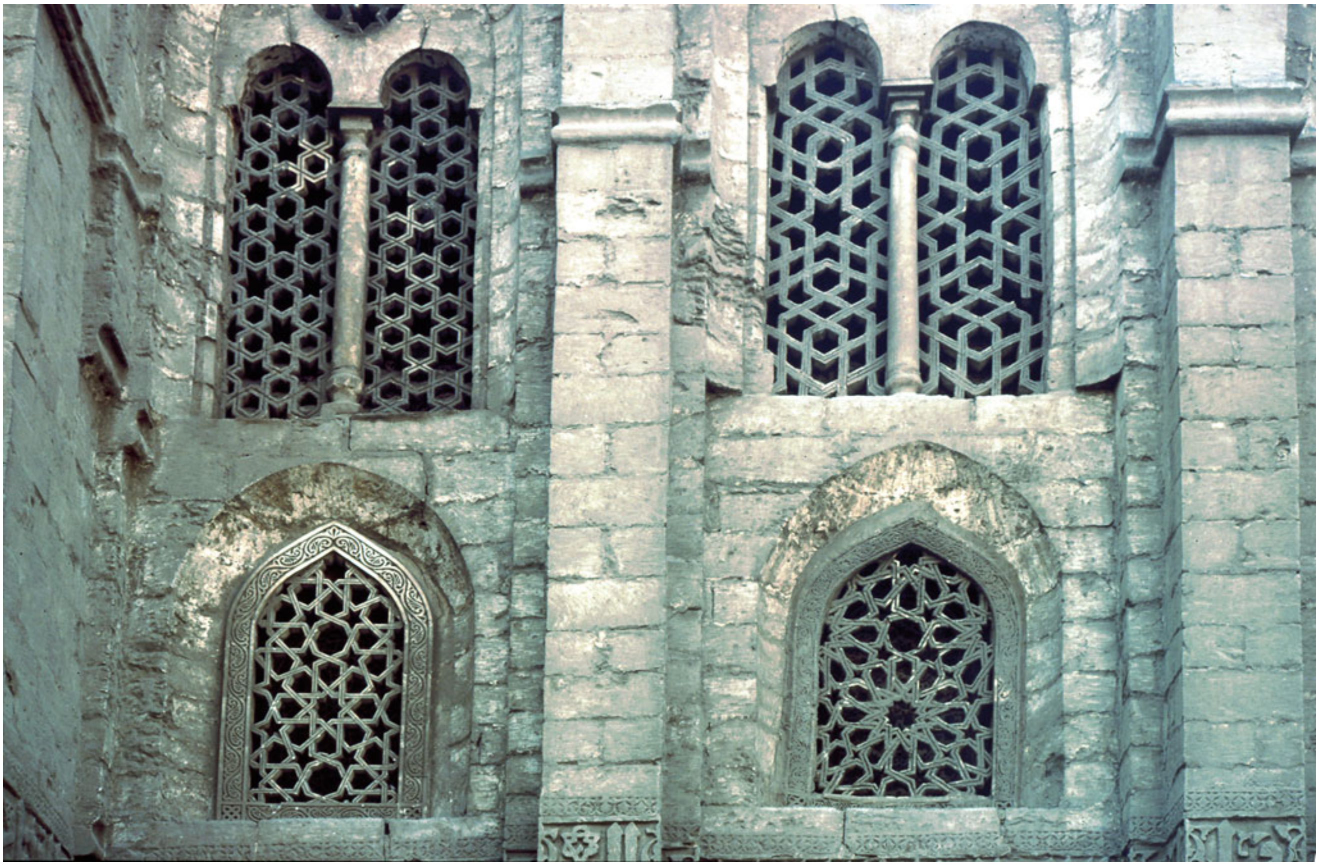
example, this places 12-pointed stars on the vertices of the dual-hexagonal grid [Fig. 359].

The Mamluks also showed great versatility in creating nonsystematic geometric patterns that repeat upon the orthogonal grid. At the most basic level, these will place primary star forms solely upon the vertices of this grid, and these stars will have *n*-fold rotational symmetry that is a multiple of 4. Most Islamic geometric patterns with 8-pointed stars are created from either the *fourfold systems A* or the *fourfold system B*, and the primary stars of most nonsystematic designs that repeat on the square grid are 12-pointed. An example of a Mamluk nonsystematic design with primary eight-pointed stars located at the vertices of the orthogonal grid is from a window grille at the Amir al-Sayfi Sarghitmish *madrasa* in Cairo (1356). The primary underlying octagons are separated along the edges of the repeat unit by two regular hexagons with coincident edges bisecting the midpoint of each edge of the square repeat unit. The underlying polygonal infill of the repeat unit is comprised of four irregular pentagons, and four irregular heptagons that are clustered around a single square at the center of the repeat unit. This creates a very-well-balanced pattern composed of five-, six-, seven-, and eight-pointed stars [Fig. 332e]. The earliest known pattern created from this same underlying tessellation is from the Seljuk ornament in the *mihrab* of the Friday Mosque at Barsian (1105) [Fig. 332a]. The most common Mamluk nonsystematic design with only a single primary region of local symmetry places 12-pointed stars at the vertices of the orthogonal grid. Each underlying generative dodecagon is surrounded by a ring of 12 edge-to-edge pentagons. This tessellation creates an *acute* pattern that was well known throughout the Islamic world [Fig. 335b], and three representative Mamluk examples include one of the carved stone lintels at the Khatuniyya *madrasa* in Tripoli, Lebanon (1373-74); the *minbar* door at the Amir Taghribardi *madrasa* complex in Cairo (1440); and a side panel from the *minbar* at the Sultan Barsbay complex at the northern cemetery in Cairo (1432). This latter example is noteworthy for the curvilinear treatment of the pattern lines. A *two-point* pattern produced from the same underlying tessellation was used in the *minbar* of the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) [Fig. 336d]. An *acute* pattern created from a simple variation to this underlying tessellation also enjoyed popularity among Mamluk artists. This variation calls for the truncation of the four clustered pentagons at the center of the square repeat: transforming the four adjacent five-pointed stars into four dart motifs [Fig. 337]. The many Mamluk examples of this design include a bronze entry door, and incised stone border around one of the interior doorways at the Zahiriyya *madrasa* and mausoleum of Sultan al-Zahir Baybars in Damascus (1279); and a fourteenth- or fifteenth-century Egyptian *minbar* panel of very high quality construction.[197] Another underlying tessellation with dodecagons at the vertices of the orthogonal grid separates these dodecagons with squares, thereby creating large interstice regions in the centers of the square repeats. These are divided into a cluster of four irregular pentagons [Fig. 334]. This tessellation was used to create an *acute* pattern in one of the window grilles at the Sultan Qala'un funerary complex in Cairo (1284-85) [Photograph 55]. As with other patterns that incorporate the cluster of four underlying pentagons, this configuration generates an octagon within the pattern matrix that is a primary feature of this design. It is interesting to note that the artist working on adjacent windows of the Sultan Qala'un funerary complex juxtaposed this design with another pattern that also places an octagon within the center of the square repeat that is created from four coincident pentagons. The pattern in this adjacent window is the classic *fourfold system B acute* pattern [Fig. 173a] [Photograph 55]; and whereas the octagons are located at the same respective positions within these two window grilles, the vertices of the former have 12-pointed stars, while those of the latter have eight-pointed stars. Two very becoming examples of Mamluk nonsystematic orthogonal designs with singular primary star forms were employed as the frontispieces in a Quran (1369) commissioned by Sultan Sha'ban for the *madrasa* founded by his mother in Cairo.[198] [Fig. 344d]. The first of these is a *median* pattern created from an underlying tessellation that separates the 16-gons with barrel hexagons, and an atypical infill comprised of further barrel hexagons, pentagons, quadrilateral kites, and a central square. The treatment of the 16-pointed stars in this design was modified in a manner common among Mamluk artists working with *median* patterns [Fig. 344c]. The second such design from this Quran also places 16-gons at the vertices of the orthogonal grid. Each 16-gon is surrounded by a ring of 16 edge-to-edge pentagons, which in turn are surrounded by eight barrel hexagons. At the center of each repeat unit is a cluster of four contiguous pentagons. The arrangement of these underlying polygons produces four irregular octagons within each square repeat. The large degree of distortion in these octagons would ordinarily produce unsatisfactory pattern conditions. However, the artist who created this pattern devised a very clever, and visually acceptable solution that is unique to this ornamental tradition [Fig. 345].

As with isometric patterns, the Mamluks made wide use of nonsystematic orthogonal patterns with differentiated regions of local symmetry. The most basic of these are derived from an underlying tessellation that places octagons at the vertices of the square repeat, separated by regular

[197] In the collection of the Royal Museum of Scotland: museum inventory number A.1884.2.1.

[198] Cairo National Library; 7, ff. IV-2r.



**Photograph 55** Mamluk pierced window grilles from the Sultan Qala'un funerary complex in Cairo (© David Wade)

hexagons on each midpoint of the repetitive edge [Fig. 178]. Once again, there is a cluster of four pentagons at the center of each repeat. This tessellation is similar in concept to those from the *fourfold system B* [Figs. 175–177] except that the elongated hexagons are replace with regular hexagons. The proportions of the pentagons are, by necessity, changed to suit these new circumstances, and as a consequence become nonsystematic. An outstanding *acute* pattern created from this underlying tessellation was used on the exterior of the Mamluk door (1303) at the Vizier al-Salih Tala'i mosque in Cairo [Fig. 178a]. Depending on the angular openings of the crossing pattern lines, this underlying tessellation will create designs with quite different visual qualities, as seen in another Mamluk example from a glazed ceramic panel at the Altinbugha mosque in Aleppo (1318) [Fig. 178b]. The earliest known pattern created from this tessellation is from the façade of the Ildegizid mausoleum of Mu'mine Khatun in Nakhichevan, Azerbaijan (1186) [Fig. 178c]. The example from the Vizier al-Salih Tala'i mosque employs more acute crossing pattern lines that are 32.2042°: an angle that is determined by drawing lines that connect two adjacent vertices of the underlying hexagon with the midpoint of the opposite edge of the same hexagon. As applied to the cluster of four pentagons, this produces the distinctive flattened octagon at the center of each square repeat unit. The application of the crossing patterns lines to the examples from the Mu'mine Khatun and Altinbugha mosque predominantly employ 60° angular openings, which qualifies them as *median* patterns. As with multiple other examples, the crossing pattern lines at the cluster of four pentagons of the design from the Altinbugha mosque replaces the 60° angular openings with 45° angular openings, thus creating regular octagons at these central locations.

Perhaps more than in any other Muslim culture, Mamluk artists applied the greatest design diversity to orthogonal patterns with multiple centers of local symmetry. Of particular significance was their use of designs with 8- and 12-pointed stars in each of the four pattern families that are derived from either of two underlying tessellations: that of dodecagons placed at the vertices of the square grid, with edge-to-edge octagons located at the centers of each square repeat, and concave hexagonal interstice regions, and that of proportionally smaller dodecagons and octagons placed at the same locations, but separated by pentagons and barrel hexagons. These two underlying tessellations have a dual relationship with one another, and the wide diversity of resulting patterns can be created from either with equal facility [Figs. 379–382]. The Mamluk use of patterns that can be created from either of these underlying tessellations exceeded that of other Muslim cultures, with designs in each of the four pattern families represented within their ornamental canon. Mamluk examples of *acute* patterns generated from either of these underlying tessellations include a curvilinear *acute* pattern from a bronze entry door at the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63), and a square panel in the railing of the *minbar* at the Amir Azbek al-Yufusi complex in Cairo (1494-95) [Fig. 379e] [Photograph 46]. The triangular side panels from the *minbar* of the Sultan Mu'ayyad mosque in Cairo (1415-21) employ a typical Mamluk variation to the standard *median* pattern that changes the character of the 12-pointed stars by replacing the crossing pattern lines that are ordinarily located at the midpoints of the dodecagonal edges with an arbitrary star rosette surrounded by darts. In accordance with Mamluk geometric aesthetics, this also introduces heptagonal elements into the pattern matrix [Fig. 380e]. Examples of Mamluk *obtuse* patterns created from either of these two underlying tessellations include the blind arches surrounding the exterior drum of the dome at the Hasan Sadaqah mausoleum in Cairo (1315-21), and a window grill in the drum of the dome at the contemporaneous Amir Sanqur al-Sa'di funerary complex in Cairo (1315) [Fig. 381b]. Their proximity in location and date, and similarity in media and design expression, suggests the likelihood of their being produced by the same artist or atelier. A *two-point* pattern that can be derived from either of these tessellations was used in the side panels of the *minbar* at the Princess Asal Bay mosque in Fayyum, Egypt (1498) [Fig. 382b]. A considerably more complex Mamluk orthogonal *two-point* pattern with 8- and 12-pointed stars is derived from an underlying tessellation that is also comprised of dodecagons, octagons, elongated hexagons, and pentagons; although in this case the pentagons are clustered in an edge-to-edge arrangement around a thin rhombus. This configuration of pentagons is a corollary with the *fivefold system*, and similarly only works (without adjustment) with *obtuse* and *two-point* patterns [Fig. 197]. This was used in the triangular side panel of the *minbar* at the Sultan Qaytbay funerary complex in Cairo (1472-74), as well as in a carved stone panel of Mamluk origin at the al-Azhar mosque in Cairo [Fig. 383].

Mamluk artists also made common use of several compound patterns that place 16-pointed stars at the vertices of the orthogonal grid. The least complex of these incorporate eight-pointed stars at the centers of the square repeat unit. An *acute* pattern on the small side door into the interior of the *minbar* at the Sultan Mu'ayyad mosque in Cairo is just such a design [Fig. 388] [Photograph 60]. As with the *median* design on the triangular side panels of the same *minbar* with 8- and 12-pointed stars [Fig. 380e], this design can be generated from two distinct underlying tessellations. The first of these uses 16-gons placed at the vertices of the repetitive grid and coinciding octagons located at the center of each square repeat. The alternative underlying tessellation is comprised of 16-gons and octagons that are separated by a network of pentagons. This underlying tessellation can be modified so that the eight pentagons that surround each octagon are truncated into trapezoids, creating a square with triangles placed on each edge that is located at the center of the square repeat unit. This variation replaces the eight-pointed stars with octagons. A Mamluk pattern that employs this modified underlying tessellation was used in the ornament of the Sultan al-Nasir Hasan funerary complex in Cairo [Fig. 391]. Several varieties of orthogonal compound design exhibiting 12- and 16-pointed stars were also produced by Mamluk artists. One of the most interesting is created from an underlying tessellation comprised of dodecagons and 16-gons separated by rings of pentagons that was used on the original bronze entry doors of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63) [Fig. 392d].[199] This example employs a variation of the otherwise standard *median* pattern that has the added feature of placing heptagons within the pattern matrix. The 12- and 16-pointed star rosettes of this design are modified in the common Mamluk fashion whereby the ring of five-pointed stars is transformed into darts that make up an alternative ten-pointed star [Fig. 223]. Truncating the six pentagons that surround the small rhombi can modify the underlying tessellation that otherwise creates this example from the Sultan al-Nasir Hasan funerary complex; thereby producing an altogether new pattern [Fig. 393]. This alteration of the underlying generative tessellation follows the convention established within the *fivefold system* by changing the pattern lines to conform with the cluster of six truncated pentagons. [Fig. 198]. Mamluk designs that employ this modified underlying tessellation include a window grill at the mosque of Altinbugha al-Maridani in Cairo (1337-39) [Photograph 56]; a curvilinear variation in one of the bronze entry doors of the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63); and the triangular side panel of a wooden *minbar* (1468-96) commissioned by Sultan Qaytbay and currently on display at the Victoria and Albert Museum in London. It is interesting to note that the earliest use of this design appears to be Seljuk: in one of the large *muqarnas* panels of the *mihrab* hood in the Friday Mosque at Barsian, Iran (1105). Further complexity is provided to patterns with 12- and 16-pointed stars by the incorporation of two mirrored 7-pointed stars into the pattern matrix. These are located at the midpoints of each edge of the square repeat unit. The underlying tessellation that creates this design

[199] Moved in 1416–17 to the Sultan Mu'ayyad Mosque in Cairo where it functions as the main entry door to this day. See Mols (2006), 214.

**Photograph 56** A Mamluk window grille at the mosque of Altinbugha al-Maridani in Cairo with an *acute* pattern comprised of 12- and 18-pointed stars (© David Wade)

separates the dodecagons and 16-gons with barrel hexagons, with a cluster of ten pentagons that surround two edge-to-edge irregular heptagons [Fig. 395]. This underlying tessellation was used by Mamluk artists to create several *two-point* designs, including the triangular side panel of the stone *minbar* of Sultan al-Zahir Barquq complex in Cairo (1384-86) [Photograph 57]; the triangular side panel of the wooden *minbar* at the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81); and one of the side panels of the wooden *minbar* at the Sultan Qansuh al-Ghuri complex in Cairo (1503-05). This underlying tessellation was also used to create the carved stone *median* pattern at the base of the minaret at the Mughulbay Taz mosque in Cairo (1466) [Fig. 396b].

The Mamluks did not produce as many geometric patterns with more than two primary regions of differentiated symmetry as their contemporaries in the Seljuk Sultanate of Rum. A stunning exception is an *acute* pattern from one of the side panels of the *minbar* at the Sultan Qaytbay funerary complex at the northern cemetery in Cairo (1472-74). This exceptionally well-balanced design places 16-pointed stars upon the vertices of the orthogonal grid, 12-pointed stars at the center of each repeat unit, and 10-pointed stars on the midpoints of each edge of the repeat unit [Fig. 402].

Examples of nonsystematic compound patterns with repetitive grids that are neither isometric nor orthogonal are less common among the Mamluks than their contemporaries in the Seljuk Sultanate of Rum. Among the few examples are two designs from the Sultan al-Nasir Hasan funerary complex in Cairo (1356-63). The first of these is a very nice *acute* field pattern from an incised stone border that places octagons at the vertices of a rectangular repeat unit, as well as at the center of the repeat [Fig. 412] [Photograph 58]. This border pattern is located in the entry portal of this mosque. The second example from the Sultan al-Nasir Hasan funerary complex is immediately adjacent to the first, adorning the back wall of a niche on the sidewalls of the main entry portal, and is fashioned in inlaid polychrome stone [Fig. 413] [Photograph 58]. This design is noteworthy in that it is one of the few historical examples of a geometric pattern that expressly shows the underlying generative tessellation as part of the ornament. As such, this panel provides important historical evidence for the use of the polygonal technique of geometric pattern construction. Mamluk examples of more complex nonsystematic compound patterns that do not repeat with either the isometric or orthogonal grids are unusual. A rhombic *acute* example is found on the wooden entry door at the Zaynab Khatun Manzil (house) in Cairo (1468). This places 24-pointed stars on the vertices of the rhombus, with 12-pointed stars at the midpoints of each edge of the rhombic repeat, and 8-pointed stars within the field. The proportions of this rhombic repeat are governed by $2 \times 3/24$ and $2 \times 9/24$ included angles. While the complexity of this Mamluk pattern is the equal of earlier examples created under the auspices of Seljuk patronage, this design from the Zaynab Khutun Manzil entry door is poorly proportioned and ill conceived: in large part due to the position of the underlying octagon in relation to the underlying 24-gons and dodecagons. Their respective radii are not congruent, or near enough to appear as such. A far more successful Mamluk nonsystematic compound *acute* pattern was used on the entry door of the ‘Abd al-Ghani al-Fakhri mosque in Cairo (1418). This pattern repeats upon a rectangular grid, with 10-pointed stars located at the vertices of the rectangle, 10-pointed stars at the midpoints of the long edge of the repeat, and two 11-pointed stars within the field of the repeat [Fig. 417]. This is perhaps the most successful Mamluk geometric pattern with complex local symmetries, and is superior in overall balance to the only other known geometric pattern comprised of 10- and 11-pointed stars: the *acute* pattern produced by the Armenian Christian monk Momik (between 1282-1321) for one of his stone *khachkar* crosses in Noravank [Fig. 423].

During the later Mamluk period, many stone domes with highly ornate exterior surfaces were constructed in Cairo. Noteworthy among these are a small number of domes that

**Photograph 57** A side panel of the Mamluk *minbar* at the Sultan al-Zahir Barquq complex in Cairo with a *two-point* pattern comprised of 7-, 12-, and 18 pointed stars (© David Wade)

are ornamented with geometric designs in high relief. This was a new and distinctly Mamluk development. Other subsequent Muslim cultures, such as the Safavids, also decorated the exterior of their domes with geometric designs. However, what makes the Mamluk domes so distinctive is their monochrome aesthetic: emphasizing the design with high relief and the consequent play of light and shadow. These Mamluk geometric domes invariably have radial symmetry wherein the gore segments are provided with an underlying generative tessellation upon which the geometric design is constructed. This is a highly specialized application of the polygonal technique requiring an additional facility with three-dimensional geometry and especially close collaboration with the architectural designer.

Mamluk examples of domical geometric ornament include three domes at the Sultan al-Ashraf Barsbay funerary complex at the northern cemetery in Cairo (1432-33). The geometric design on the dome over the Sultan Barsbay mausoleum is a dense interweave that is reminiscent of a basket weave [Photograph 61]. This pattern places a ring of half eight-pointed stars around the periphery of the dome, ascending to second ring of eight-pointed stars, followed by sequential rings of seven-, six-, and five-pointed stars [Fig. 493a]. The initial double course of eight-pointed stars is the well-known classic *acute* pattern created from the tessellation of octagons and squares [Fig. 124a]. The transmission through the eight-, seven-, six-, and five-pointed stars results from the diminishing width of the gore segment, and the narrow proportions of this gore segment result from the 20-fold radial segmentation of the dome's surface. In this way, the geometric design culminates at the apex in a 20-pointed star. Immediately adjacent to this dome is the much smaller dome of the Amir Gani Bak al-Ashrafi mausoleum. This hosts a curvilinear design comprised of six half 12-pointed stars around the periphery, and a ring of six


**Photograph 58** A Mamluk niche in the entry portal of the Sultan al-Nasir Hasan funerary complex in Cairo with a nonsystematic pattern that includes its underlying generative tessellation, as well as a nonsystematic border pattern that surrounds the niche (© Scott Haddow)

10-pointed stars in the mid-section of the pattern matrix. These primary regions of local symmetry are connected with 6-pointed stars and octagonal regions, culminating in a 12-pointed star at the apex. The third geometric dome at this complex covers an anonymous tomb of a Barsbay family member. The geometric design on this dome is the most successful in its overall balance, and is comprised of a ring of six half 12-pointed stars surrounding the periphery of the dome, followed by a ring of six 8-pointed stars, and a second ring of six 12-pointed stars, culminating in a 12-pointed star at the apex [Fig. 493b]. Perhaps the most notable Mamluk geometric dome is at the Sultan Qaytbay funerary complex in the northern cemetery in Cairo (1472-1474) [Photograph 2]. The powerful visual appeal of this dome is the result of the augmentation of the geometric design with a meandering floral devise that fills the background. The geometric design itself is less complex than the three examples from the Sultan Barsbay funerary complex, but the addition of the floral element provides a richer overall ornamental affect. This geometric pattern places a ring of eight half 10-pointed stars at the base of the dome, followed by a ring of eight 9-pointed stars, a further ring of sixteen 5-pointed stars, and culminating in a 16-pointed star at the apex of the dome [Fig. 493c]. The exterior treatment of this dome is an outstanding example of Mamluk monochrome geometric and floral ornament.

Mamluk artists also continued the Zangid and Ayyubid tradition of decorating the interior quarter domes above entry portals and *mihrab* niches with geometric patterns. Several of the earlier Mamluk examples were clumsy in their use of geometry: for example the *mihrab* hood at the Haram al-Ibrahimi in Hebron, Palestine (fourteenth century), is decorated with a geometric design that forces hexagons onto an orthogonal repeat. The further incompatibility of the orthogonal grid with the surface of a sphere results in a pattern that is poorly conceived and rife with geometric inconsistencies. The forced application of otherwise two-dimensional patterns with triangular repeat units onto quarter dome surfaces was also occasionally practiced, and examples include the *mihrab* hood at the Amir Salar and Amir Sanjar al-Jawli funerary complex in Cairo (1303-04) and the *mihrab* hood at the Faraj ibn Barquq Zawiya and Sabil in Cairo (1408). However, none of these examples comes close to equaling the sophistication in design and construction of the earlier geometric quarter domes of the Zangids and Ayyubids.

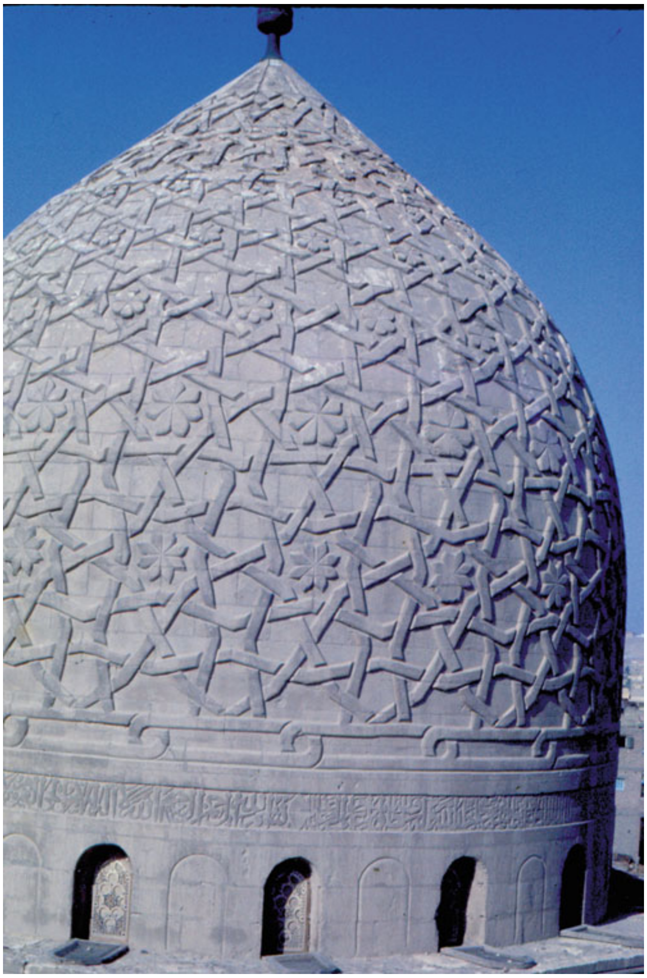
Two relatively simple Mamluk geometric semidomes are found in Cairo that were constructed within a decade of one another: a *two-point* pattern in the *mihrab* niche at the al-Mar'a mosque in Cairo (1468-69) and a curvilinear *acute* pattern comprised of six-pointed stars in the hood of the entry portal at the Timraz al-Ahmadi mosque in Cairo (1472). Each of these exhibits a bold simplicity that is sufficiently distinct from other examples to suggest the possibility of their being designed by the same individual. In contrast to the above examples, the interior geometric ornament of several Mamluk niches rivals the complexity and sophistication of the finest Zangid and Ayyubid work. The stone mosaic *mihrab* hood from the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) places half 12-pointed stars at the base of the quarter dome, 10-pointed stars at the middle of the radial gore, and an 8-pointed star at the apex [Fig. 493d]. The stone quarter dome in the entry portal of the Amir Ahmad al-Mihmandar funerary complex in Cairo (1324-25) is one of the most spectacular examples of Egyptian three-dimensional geometric art [Photograph 59]. The archivolt that surrounds the quarter dome is decorated with a *median* pattern comprised of 10- and 11-pointed stars that are centered on the edge of the arched opening to the hood. The pattern turns the corner of the archivolt to continue directly onto the surface of the quarter dome and the pattern on the domical surface has nine- and ten-pointed stars. This is an immensely complex geometric schema that is unique to this location, but was likely inspired by the archivolt of the

**Photograph 59** A Mamluk quarter dome in the entry of the Amir Ahmad al-Mihmandar funerary complex in Cairo with a surrounding nonsystematic *median* pattern comprised of 10- and 11-pointed stars, and a contiguous pattern on the domical surface with 9- and 10-pointed stars (© David Wade)

**Photograph 60** A Mamluk side door to the *minbar* of the Sultan Mu'ayyad mosque in Cairo with a nonsystematic *median* pattern comprised of 8- and 16-pointed stars (© David Wade)

Ayyubid entry portal at the Zahiriyya *madrasa* in Aleppo (1217). While this earlier example also turns the corner at the arch opening, the pattern only continues onto the ascending plane of the intrados of the interior side of the arch rather than onto an actual domical surface.

It is noteworthy that almost all of the many Mamluk examples of geometric domical ornament, be they the exceptional monochrome exterior domes or the quarter domes in *mihrab* and entry portals, are based upon decorated gore segments and consequent radial symmetry. Mamluk artists were either ignorant of the design potential of the Platonic and Archimedean polyhedra or they preferred the aesthetic qualities of decorated gore segments. For the most part, these artists did not follow the earlier polyhedral precedents of their Zangid and Ayyubid predecessors. A very beautiful Mamluk exception to this deficiency is from the domical hood of the entry portal at an anonymous mausoleum in the Nouri district of Tripoli, Lebanon. This spherical design is based on the geometry of the cubeoctahedron. Unlike its Levantine wood predecessors, this example of polyhedral-geometric design was produced in polychromatic inlayed stone as per the gore segmented *mihrab* hood at the Amir Qijmas al-Ishaqi mosque. This example from Tripoli is a *two-point* pattern that places ten-pointed stars at the vertices of the cubeoctahedron, and eight-pointed stars at the centers of each projected square face of the polyhedra [Fig. 502]. Each polyhedral vertex is made up of two squares and two triangles in a 3.4.3.4 configuration, and the ten-pointed stars are suited to this location by virtue of three points being allocated to each square corner, and two points to each triangular corner. The underlying generative tessellation that is applied to both varieties of polyhedral face is comprised of decagons and octagons that are separated by a connective polygonal network of pentagons and barrel hexagons. This is one of the most complex and beautiful polyhedral domes known to the historical record.



**Photograph 61** The dome over the Mamluk mausoleum of Sultan al-Ashraf Barsbay with a geometric pattern comprised of ascending eight-, seven-, six-, and five-pointed stars (© John A. and Caroline Williams)

## 1.22 Design Developments in the Western Regions

The history of the art and architectural ornament of al-Andalus is a complex interplay of cross-cultural influences. From as early as the eighth century, following the Muslim conquest of the Iberian Peninsula, the aesthetic predilections of the Umayyads helped to create an ornamental style that was distinct from other Muslim cultures. The successive Almoravid and Almohad Berber invasions from North Africa introduced a more austere approach to architectural ornament that was in part informed by their Abbasid affiliations. Throughout this history was the ongoing interaction with indigenous Christian artists, and the *Mudéjar* ornamental style that grew out of this cultural overlap continued long after the forced departure of Muslims and Jews from Spain. From the thirteenth to fifteenth centuries the inherently Muslim ornamental style of al-Andalus was equally embraced by Christian and Jewish patronage. The community of artists and master craftsmen engaged in this work included both Muslims and Christians, and the ecumenism between these three faiths allowed for artists to work for patrons who adhered to a different faith.

The rise of the Almoravids in the mid-eleventh century brought about cultural and artistic change in al-Andalus. This Berber tribe had come to dominate much of North Africa, and in 1090 they defeated the Umayyad Caliphate in Spain. The Almoravids were Sunnis, and accepted the authority of the Abbasid Caliphate in Baghdad. The coming to power of a dynasty with strong religious and political affiliations with the Abbasids introduced North Africa and Spain to a wealth of new ornamental design traditions. This was particularly the case with an increased emphasis on geometric ornament. Prior to the Almoravids, the use of geometric patterns in Spain primarily followed the earlier example set by the Umayyads in Damascus: the application of interlaced geometric patterns into window grilles. A number of very beautifully carved marble window grilles dating from the late tenth century demonstrate the continuity of this use of geometric patterns. However, these later examples differ in that they are not the products of compass work, but are significantly more sophisticated in the methods used in construction. The mature nature of these window grille designs raises an interesting question: If artists working during the late tenth century Umayyad Caliphate of al-Andalus were able to create complex and beautiful geometric patterns, why was it that these patterns were only applied to window grilles, and not more generally to areas of architectural mass? Nor was there any appreciable utilization of geometric patterns to the minor arts during the Umayyad period. It would appear that this conscious limitation to the application of geometric patterns was, on the one hand, born out of a wish to remain true to their earlier Syrian roots, and on the other hand, a possible desire to assert their cultural distinction from the might of the Abbasid empire to the east. By contrast, the Almoravids appear to have been both familiar with, and sympathetic to the ornamental developments that were taking place in Baghdad and the Islamic east, for it was during this period that *muqarnas* vaulting was first introduced to the Maghreb and al-Andalus; and it was also during this period of Sunni ascendancy that the tradition of complex geometric pattern making finally became a central aspect of the ornamental arts of the western regions.[200]

The Tiafa kingdoms in al-Andalus continued the aesthetic approach to architectural ornament of their Umayyad

[200] Necipoğlu (1995), 101.

predecessors. Many fine examples of geometric design were incorporated into the fabric of their building, and patterns created from the *system of regular polygons* were especially common. A fine, if geometrically simple, example is from the entry door of the Aljafería Palace in Zaragoza that is created from the $6^3$ hexagonal grid, and comprised of six-pointed stars and hexagons [Fig. 95b]. The exterior façade of the nearby Cathedral of San Salvador includes a *Mudéjar* brickwork pattern created from the 3.6.3.6 underlying tessellation of triangles and hexagons. This design is comprised of superimposed hexagons [Fig. 99e]. This pattern adheres to the specific proportions that result from the 3.6.3.6 generative schema rather than the alternative $6^3$ derivation that produces an almost identical design [Fig. 96e]. The architectural ornament of the succeeding Almoravids continued the stylistic developments of their Umayyad and Tiafa predecessors: the abundant use of the floral idiom with rudimentary geometric patterns created mostly from the *system of regular polygons*. The Almoravid recognition of Abbasid religious authority and cultural affinity may have facilitated the introduction of more overtly geometric forms of ornament, such as *muqarnas*. Unfortunately, most Almoravid architecture was destroyed by their Almohad conquerors. Of that which remains, the carved stucco ornament in the Great Mosque of Tlemcen (1136) stands out as especially beautiful. The dome in front of the *mihrab* is made up of 12 rib-arches that form a 12-pointed star at the center of the dome. Within the center of the 12-pointed star is a fine example of early Maghrebi *muqarnas* vaulting. To add to the stunning visual effect of this dome, the floral ornament inside the 12 arches is pierced so that sunlight can filter through to the interior space. This is one of the most outstanding domes produced in the western Islamic regions.

The Almohads were also a Berber tribe who originated from the Atlas Mountains of North Africa. Like the Almoravids, they were Sunni Muslims. They nevertheless had strong religious differences with Almoravids, as well as with the Abbasid Caliphate in Baghdad. Led by their religious zealotry, they won victories against the Almoravids in both North Africa and Spain during the first quarter of the twelfth century. The Almohads ruled over the Maghreb from their capitol in Marrakesh, and in al-Andalus from their capital in Seville. The rise of the Almohads had a strong influence on the art and architecture of al-Andalus and the Maghreb. The Almohads regarded the opulence of Almoravid ornament as decadent, and an indication of their moral and religious inferiority. Their puritanical zeal led to the destruction of almost all Almoravid architecture. However, their dislike for architectural splendor notwithstanding, they constructed many beautiful examples of Islamic architecture, albeit in a more austere and restrained style. While the Almohads employed both geometric patterns and *muqarnas* vaulting, the level of sophistication was still far short of the contemporaneous work of the Zangids and Ayyubids to the east.

The Nasrids rose to power following the defeat of the Almohads in al-Andalus by the Christian forces from the northern regions of the Spanish peninsula. The Nasrids founded their capitol in Granada in 1238. The Nasrid dynasty lasted nearly 250 years, until it was finally defeated by the Christian *reconquista* in 1492. It was at this time that Muslim rule over the last stronghold of Iberian territory ended, drawing to a close almost 800 years of Muslim dominance in al-Andalus. The western style of Islamic ornament reached its full maturity during the Nasrid period. Perhaps as a reaction to the austerity of the Almohad ornamental style, the ornament of the Nasrids was especially ornate, and replete with highly refined detail. The Nasrids rejoiced in creating an architectural feast for the eyes. Virtually no space was left unadorned, and beautiful calligraphic inscriptions were accented through the use of complex geometric patterns, *muqarnas* vaulting, and beautifully stylized floral designs. The greatest Nasrid architectural monument to survive to the present day is the Alhambra in Granada. This is both a fortress and a palace, and was the seat of Nasrid power. It stands out as one of the most remarkable and beautiful architectural complexes in the world. A wide variety of ornamental materials were used throughout this complex, including carved and painted stucco, ceramic tile and cut-tile mosaic, carved and painted wood, pierced stone, carved stone, and cast bronze. At the hands of less skilled artists, such an exuberant use of so wide a range of materials might be expected to create a cacophony of ornamental overload. It is a remarkable attribute of the Nasrid ornamental tradition that these diverse materials and abundant design elements were harmoniously brought together to create an architectural environment that is as tranquil as it is luxuriant.

The Nasrids of al-Andalus were closely allied with the Marinid dynasty in North Africa. The Marinids were a Berber tribe from the Sahara desert who entered Morocco in 1216. By 1250 they had taken control of Fez, where they founded their capital. The exceptional beauty of this city is a legacy of Marinid rule. In 1269 they brought an end to the Almohad dynasty by conquering their capital of Marrakesh. During the Marinid period, the city of Fez became a great center of Islamic culture. By the middle of the fourteenth century, the architecture of the Marinids had reached a level of beauty and refinement that was equal to that of the Nasrids in al-Andalus. Of particular note are the al-'Attarin *madrasa* in Fez (1323), and the Bu 'Inaniyya *madrasa* in Fez (1350-55). The architectural ornament in both these buildings is fully mature, and along with that of the Alhambra, represents the fulfillment of the western style of Islamic design. The Marinids of the Maghreb and the Nasrids of al-Andalus were close allies and trading partners. The close political and cultural relations between these neighboring dynasties included the ability for skilled artists to receive patronage on both sides of the Straits of Gibraltar, and resulted in the aesthetic synthesis of style and quality in the ornament in these two regions during this period.

The Nasrids and Marinids were prodigious patrons of the geometric arts. With the exception of the *sevenfold system*, Nasrid and Marinid artists created many fine examples of geometric patterns that are associated with each of the polygonal systems. What is more, in addition to Muslim patronage, Christian and Jewish patrons also commissioned geometric designs of the highest quality. Of the countless number of patterns created from the *system of regular polygons* one example was particularly popular: a design created from the 3.4.6.4 underlying tessellation that is characterized by 6- and 12-pointed stars, with arbitrary 8-pointed stars introduced into the underlying square modules [Fig. 105h]. This pattern was employed widely in the western regions during this period, and examples include a cut-tile mosaic panels at the Alcazar in Seville (fourteenth century), and a carved stucco panel in the Córdoba Synagogue (1315). Several examples of essentially the same design produced under the patronage of the Sultanate of Rum are found in Ahlat, eastern Turkey, from approximately 40 years earlier [Fig. 105gi]. A later Alawid design from the Moulay Ismail Palace in Meknès, Morocco (seventeenth century) employs a becoming additive variations within the 12-pointed stars of an *obtuse* pattern created from the $3.12^2$ generative tessellation [Fig. 108e]. A fine Nasrid *obtuse* pattern created from the $3.4.3.12\text{-}3.12^2$ *two-uniform* tessellation was used in the *zillij* cut-tile mosaic of the Alhambra [Fig. 113d]. This design is identical to contemporaneous Mamluk examples from Egypt [Fig. 113c] except for the arbitrary incorporation of an eight-pointed star within the underlying square modules.

Nasrid and Marinid examples of patterns created from the $4.8^2$ underlying tessellation of squares and octagons are plentiful. Indeed, each of the four pattern families, along with countless stylistic variations, is represented in the derivation of patterns from this single tessellation. One of the more basic *zillij* mosaic panels at the Alhambra includes the standard *acute* pattern created from this tessellation [Fig. 124a]; and one of the wooden ceilings from the Alhambra is ornamented with an elaborated version of the classic star-and-cross *median* pattern [Fig. 126b]. Examples of the standard *obtuse* pattern include a wooden screen from the Sultan's Palace in Tangier [Fig. 124c]; and an exceptional Nasrid example of the standard *two-point* design created from the $4.8^2$ underlying tessellation was used on a silk brocade weaving dating from the fourteenth century[201] [Fig. 124d]. The Bu 'Inaniyya *madrasa* in Fez (1350-55) includes several patterns that can be easily produced from the $4.8^2$ tessellation. These are provided with greater complexity through arbitrary additions to the pattern matrix [Fig. 126e, g, h], while a more simplified variation of the above-referenced *two-point* design used on the Nasrid silk brocade enjoyed great popularity among artists of both cultures [Fig. 128a]. Many of the patterns associated with the $4.8^2$ underlying tessellation can be created with alternative methodologies, and it is often impossible to know for certain how a given example was produced. A pattern from the Alhambra with *two-point* characteristics may well have been created using the orthogonal graph paper technique instead [Fig. 128b], and a very simple design with octagonal characteristics from a tile mosaic panel in Fez may, or may not, have employed this tessellation in its creation [Fig. 127f]. A design that places the pattern lines upon the vertices of the underlying $4.8^2$ tessellation was used at both the Alhambra and the Alcazar in Seville (1364-66) [Fig. 129d]. This design is identical to a much earlier Ghurid pattern [Fig. 129c] except that it has arbitrary small eight-pointed stars placed at the underlying square modules.

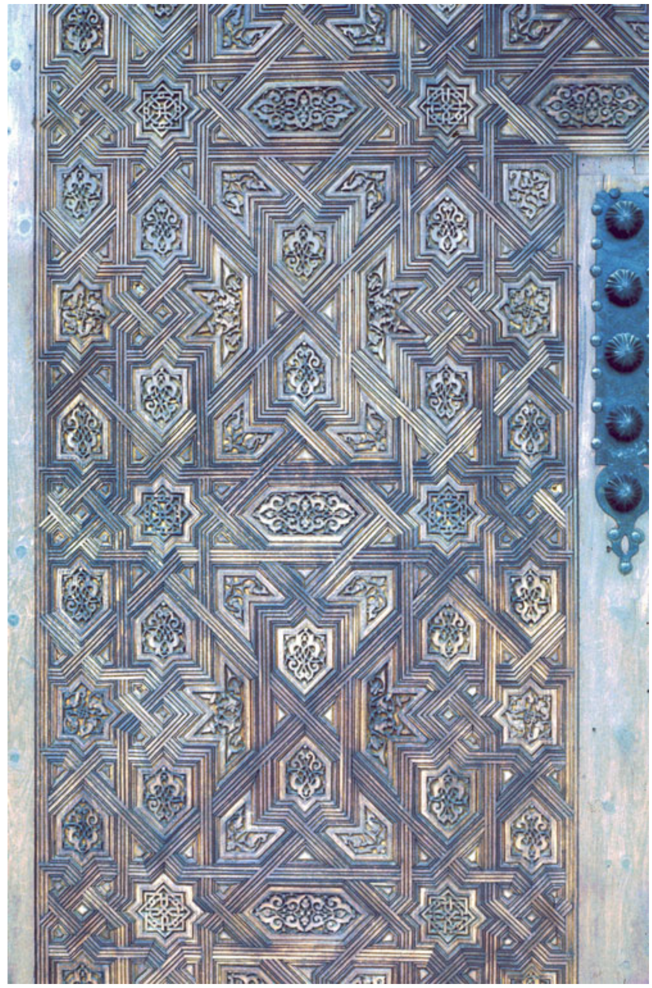
A cut-tile mosaic panel from the Alhambra utilizes an unusual underlying generative tessellation comprised of dodecagons, equilateral triangles and rhombic interstice modules [Fig. 339]. This places the dodecagons onto the vertices of the orthogonal grid in a vertex-to-vertex rather than the more typical edge-to-edge orientation. Each square repeat unit has four equilateral triangles that are coincident with the dodecagons and meet at the center of the repeat. This underlying generative tessellation is atypical of the *system of regular polygons*, and has characteristics that are akin to nonsystematic design methodology. The pattern generated from this tessellation is in the *acute* family. The designer of this cut-tile mosaic panel incorporated multiple variations to the treatment of the pattern lines within the dodecagons [Figs. 340 and 341]. These variations are independent of the underlying tessellation, and are arrived at via arbitrary design considerations that adhere to the geometric aesthetics of the Nasrids and Marinids. While such variations to the primary star form were a common feature of geometric patterns in the Maghreb, this particular example is worthy of note for its use of multiple variations within a single design. As such, this mosaic panel from the Alhambra exemplifies the diversity and playfulness of the western geometric design tradition.

Patterns associated with the *fourfold system A* were especially popular among Nasrid and Marinid artists. Indeed, their use of this system exceeded other Muslim cultures in the creation of especially complex and sophisticated patterns. This increased complexity is due to two primary factors: very broad repeat units comprised of large numbers of underlying polygonal modules, and an innovative approach to varying the applied pattern lines through

[201] In the collection of the Metropolitan Museum of Art, New York, Fletcher Fund, (1929), 29.22.

arbitrary design decisions. These western practices created patterns with significantly greater visual diversity and design variation. This is in marked distinction to the more uniform aesthetics of eastern Muslim cultures wherein the repetitive structure was more readily evident. Multiple examples of this more complex use of the *fourfold system A* were seen at the Alhambra in Granada. A fine example of the incorporation of arbitrary variations in the application of the pattern lines to diverse modules in the underlying tessellation is demonstrated in one of the wooden ceilings at the Palace of the Myrtles (1370) at the Alhambra. This is an *acute* design that places 45° crossing pattern lines placed at the midpoints of each short edge of the underlying polygonal modules, and 90° crossing pattern lines at the midpoints of each longer polygonal edge [Fig. 149b]. The standard *acute* pattern created from this underlying tessellation is certainly satisfactory [Fig. 149a], but the visual character of this design from the Palace of the Myrtles is pleasingly altered by adjusting the application of the pattern lines within the underlying square modules.

Artists in the Maghreb built upon the standard set of underlying polygonal modules that comprise the *fourfold system A*, thereby increasing the design potential of this system. These additional design modules are easily derived through either truncating the large and small octagons, or by identifying interstice regions when tessellating with the standard modules [Fig. 158]. An outstanding wooden door from the Alhambra demonstrates the efficacy of these additional polygonal modules as used by Nasrid and Marinid master artists [Fig. 162] [Photograph 62]. This example also demonstrates a less dogmatic approach to the application of the pattern lines to the underlying tessellations by artists of this region. This is similar to the pattern lines associated with the underlying square regions of the above referenced design from the Palace of Myrtles [Fig. 149b], although in the example from the wooden door from the Alhambra, the arbitrary pattern line adjustments apply to the truncated octagons, trapezoids, and triangles. In each case, the angles and placement of the applied *obtuse* patterns lines are determined more by their associated neighbors than by the pedantic iterative placement of the same applied pattern lines onto a given polygonal module in all circumstances and locations. This is a more creative process wherein the artist is actively making arbitrary decisions that nonetheless conform to the geometry of the underlying tessellation. The design of the door at the Alhambra utilizes two distinct rectangular repetitive elements; and either of these will work on their own as perfectly acceptable patterns. Many of the *zillij* mosaic panels at the Alhambra have increased complexity resulting from the incorporation of added polygonal modules and similar variations within the applied pattern lines. An especially beautiful example from the Hall of Ambassadors places pattern variations into selected octagons within the



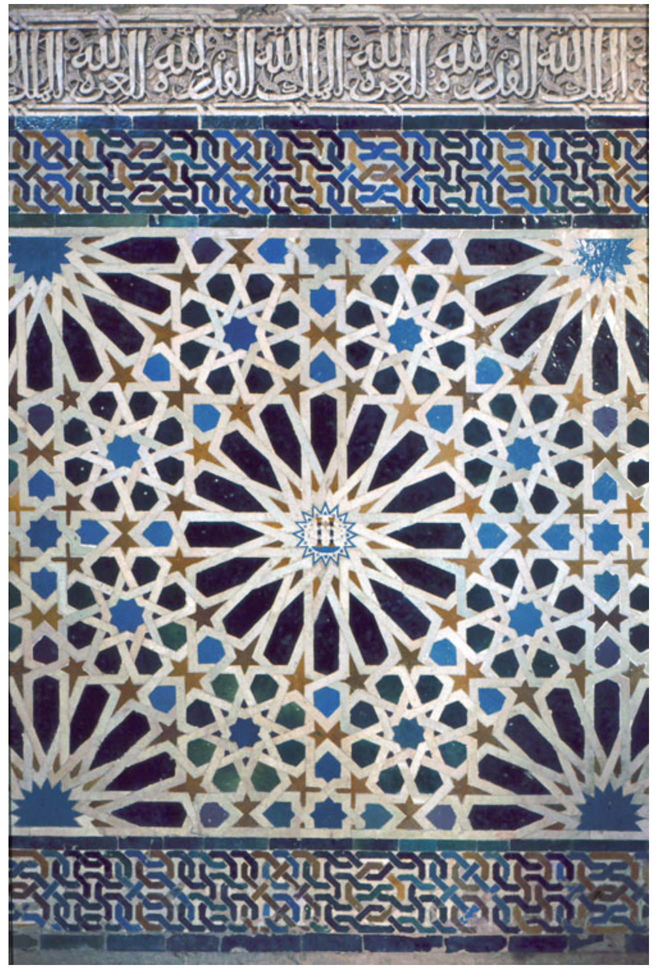
**Photograph 62** Nasrid wood joinery from a door at the Alhambra with an *obtuse* pattern created from the *fourfold system A* (© David Wade)

underlying tessellation [Fig. 163]. This panel also arbitrarily rotates the central eight-pointed star by 22.5°, causing the set of parallel lines within this star to be out of sync with the rest of the overall design. This dynamic central feature is illustrative of the flexible approach to geometric design as practiced in the Maghreb.

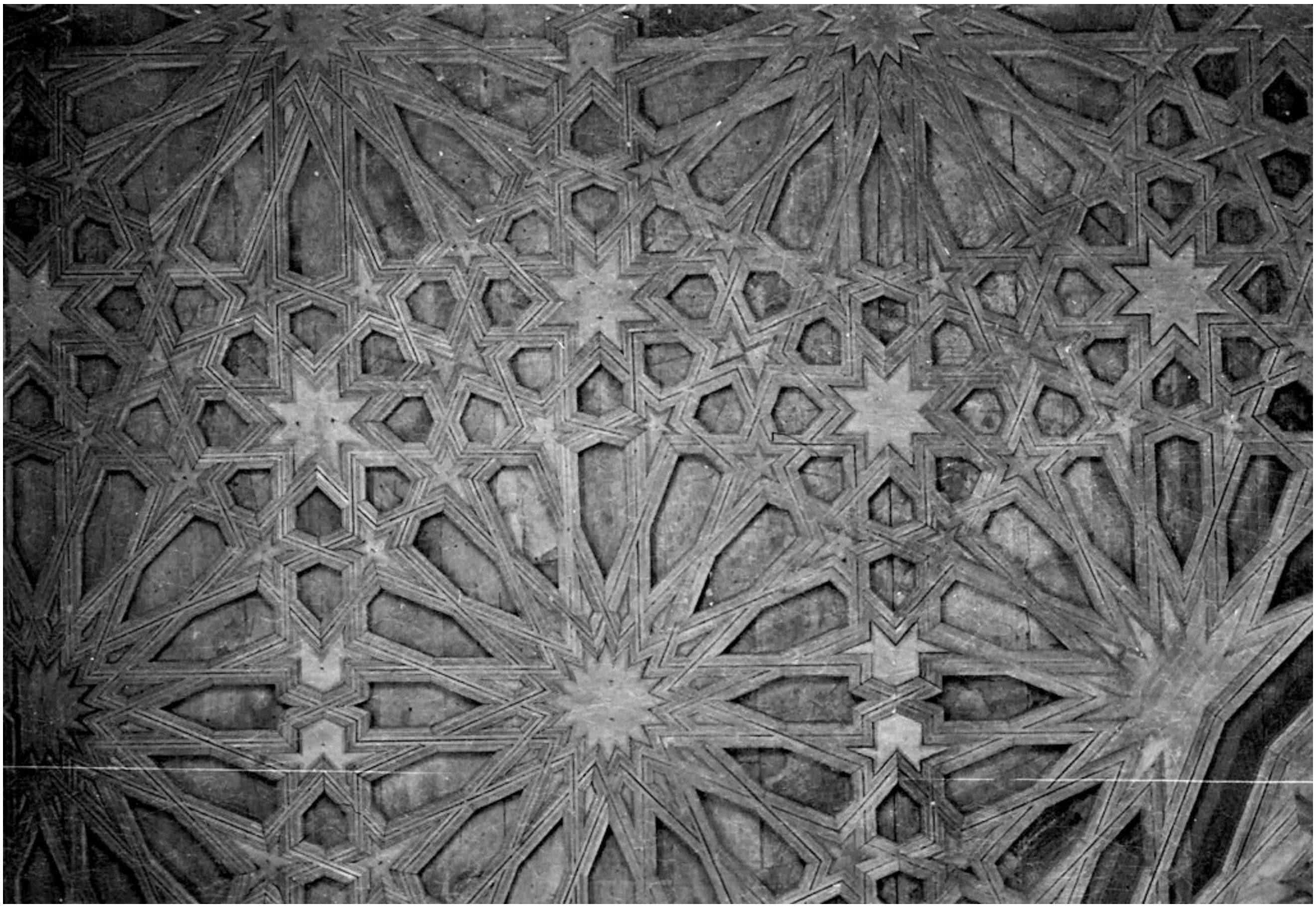
Nasrid and Marinid artists developed a variation to patterns created from both the fourfold systems that incorporate 16-pointed stars into the pattern matrix. This was achieved through the discovery that the modules of these systems can be circularly arranged to have 16-fold symmetry, allowing for further infill of the 16-pointed star motif. This polygonal arrangement is very elegant, and it is surprising that artists from other Muslim cultures did not discover this inherent, if obscure, 16-fold capacity within the fourfold systems. As pertains to the *fourfold system A*, this variety of pattern invariably employs the expanded set of generative polygonal modules. A *Mudéjar* stucco window grille from the Sinagoga del Tránsito in Toledo (1360) has a very fine example of such a pattern among its many window grilles [Fig. 165]. A similar design was used in the upper portion of three identical adjacent window grilles at the Alhambra [Fig. 166]. The lower portion of these windows adds a rectangular repetitive element beneath the upper square that is created from the expanded modules of the *fourfold system A*. Both the square and rectangular repetitive elements from this window can be used independently as repeat units. Two additional examples of this variety of design were found within the *zillij* mosaic wainscotings at the Alhambra, and include an example with moderate complexity (as measured by the amount of geometric information within the square repeat unit) [Fig. 167], and a design composed of considerably more underlying polygonal components within the square repeat unit, with consequent increased overall complexity [Fig. 168].

Patterns created from the *fourfold system B* were also well known in the Maghreb, including multiple examples of designs that had been used previously by Muslim cultures in the east. Yet even the use of established designs was often imbued with innovative flourish. A case in point is a wooden ceiling at the Alhambra that incorporates curvilinear elements into an otherwise well-known *acute* pattern that had been widely used by earlier Muslim cultures [Fig. 177d]. Moreover, and in keeping with Maghrebi practices, this design is provided greater complexity through the application of two distinct varieties of pattern line application to the underlying polygonal edges to create the overall design. A Marinid *acute* design from the portal of the Sidi Abu Madyan mosque in Tlemcen, Algeria (1346), is produced from the same underlying tessellation, except with the small hexagons rather than the large hexagons. This design arbitrarily extends the lines within the pentagons to create an eight-pointed star at the center of each repeat unit surrounded by four dart motifs [Fig. 175b].

Perhaps the most distinctive augmentation of the *fourfold system B* by Nasrid and Marinid artists are a category of *acute* designs that add 16-pointed stars into the pattern matrix. These are identical in methodological concept to designs with 16-fold regions of local symmetry that are created with from the *fourfold system A* [Figs. 165–168]. In fact, both rely on a circular arrangement of alternating octagons (or elements that fill the octagon) and hexagons within their set to create the regions with 16-fold symmetry. There are more modules within the expanded *fourfold system A*, and consequently far greater tessellating potential for creating patterns with 16-pointed stars. By contrast, the more limited set of modules within the *fourfold system B* provides less design potential, and explains why there are fewer designs created from this system than from the *fourfold system A*. Nonetheless, the designs with 16-pointed stars produced from the *fourfold system B* are exceptionally elegant, albeit generally less complex. A *zillij* panel from the Alhambra is a representative case in point [Fig. 185a] [Photograph 63]. This *acute* design can be differentiated from similar examples created from the *fourfold system A* by virtue of the pentagons along the edges of the repeat unit. The proportions of these are specific to the *fourfold system B* [Fig. 170]. As with other examples, the underlying tessellation places octagons upon the vertices of the $4.8^2$ grid of squares and octagons. This same design, with or without variations, was used in many locations in Spain and Morocco, including a *zillij* panel from the Sa'dian tombs in Marrakesh, Morocco (sixteenth century), and an Alawid stucco ceiling at the Moulay Ishmail mausoleum in Meknès (seventeenth century). A very becoming *acute* pattern created from the *fourfold system B* was used in a wooden ceiling at the Bu 'Inaniyya madrasa in Fez (1350-55) [Fig. 186] [Photograph 64]. This juxtaposes the 16-pointed stars in a very similar fashion as the above-cited *zillij* panel from the Alhambra, except that it cleverly repeats upon a rhombic grid with 16-pointed stars at the vertices, and equally upon



**Photograph 63** A Nasrid *zillij* mosaic panel from the Alhambra with an *acute* pattern made up of 8- and 16-pointed stars created from the *fourfold system B* (© David Wade)



**Photograph 64** A Marinid wooden ceiling at the Bu ʻInaniyya madrasa in Fez, Morocco, comprised of an *acute* pattern with 8- and 16-pointed stars that is created from the *fourfold system B* (© David Wade)

the hexagonal dual grid with the 8-pointed stars located at the vertices.

The Nasrid and Marinid use of the *fivefold system* was less pervasive than the two fourfold systems. Nonetheless, many examples were used in the architectural ornament of these allied cultures. However, Maghrebi artists working with this system did not apply the same level of innovation and variation that was a hallmark of their work with the *system of regular polygons* and both fourfold systems. Specifically, the many western examples of patterns created from the *fivefold system* do not employ analogous variations to the primary ten-pointed stars. What is more, the variety of fivefold design was invariably limited to the *acute* family with 36° crossing pattern lines. In these regards, their use of this system was generally less innovative than the fivefold work of the eastern regions. A typical example of a Nasrid fivefold design is from one of the stucco window grilles at the Alhambra. Like so many of the fivefold examples from this region, this is a classic *acute* pattern [Fig. 226c], but has the added feature of the design being modified at the periphery to create a framing parallel line motif, referred to as the river, that is typical of the geometric ornament of the Maghreb.[202] This feature strays noticeably from the pattern lines of the actual *acute* design at the lower portions of the panel where the lines of the framing motif do not conform with the five directions of the pattern lines in the design itself, as well as in the upper arched portion of the window grill were the parallel framing lines within the river are somewhat at odds with the pattern itself. Despite the small sacrifices to the integrity of the *acute* pattern lines that comprise the standard design, this framing device is visually attractive. Two carved stucco arched panels in the courtyard façade of the Bu ʻInaniyya *madrasa* in Fez (1350-55) are decorated with a fivefold pattern that is an exception to the relative lack of Maghrebi innovation with this system. This design places 20-pointed stars at the vertices of the rectangular repeat, as well as at the center of the repeat unit [Fig. 268]. The incorporation of 20-pointed stars into a system that is ordinarily limited to 10-pointed stars is analogous to the Maghrebi introduction of 16-pointed stars into both the fourfold systems. This is a rare design phenomenon,

[202] Castéra (1996).

and one of the only other instances of such a pattern is a Mamluk example from the Qadi Abu Bakr ibn Muzhir in Cairo (1479-80) [Fig. 267]. Indeed, this Marinid fivefold pattern is a masterpiece of geometric art. While the overall repeat is a rectangle, the internal repetitive structure is comprised of rhombi and decagons. The pattern within the rhombic elements is the classic *acute* pattern constructed from underlying decagons, pentagons, and barrel hexagons [Fig. 226b]. The use of these two repetitive elements within a broader repeat unit conforms with the relatively small group of fivefold hybrid patterns created within this tradition [Figs. 261–268].

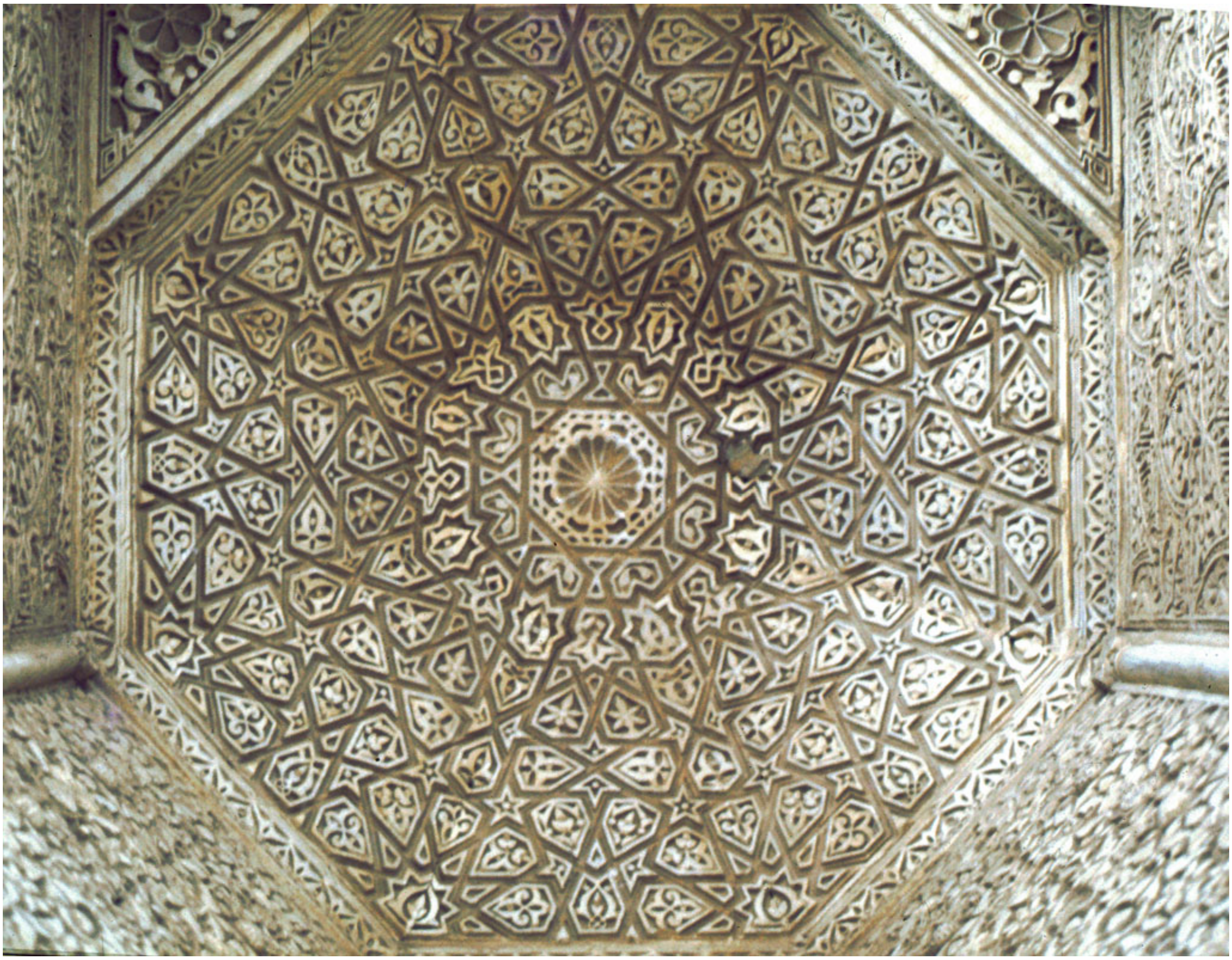
Another notable exception to the relative lack of fivefold innovation in the western region is found in several *zillij* dual-level panels at both the al-'Attarin *madrasa* (1323) [Fig. 476] and Bu 'Inaniyya *madrasa* (1350-55) in Fez [Fig. 474]. While still of the *acute* family, these reach high degrees of added complexity through very broad repeat units comprised of large numbers of underlying polygonal modules. This variety of pattern is characterized by two levels of design that are differentiated through the application of color. Most Maghrebi dual-level designs were created from the *fourfold system A*, but a very few were created from the *fivefold system*. While the visual character is distinct, the methodology of the dual-level tradition in the Maghreb is essentially the same as that of the eastern regions that followed some hundred years hence. The history of this variety of geometric design, in both the western and eastern regions, is examined later in this chapter.

The Marinid and Nasrid use of nonsystematic patterns was almost exclusively limited to designs that repeat upon either the orthogonal or isometric grids. As such, previously originated compound patterns with 8- and 12-pointed stars, and 9- and 12-pointed stars, were commonly used by the local Muslim, Christian and Jewish communities alike. Maghrebi examples of orthogonal patterns with 8- and 12-pointed stars are mostly of the *acute* family [Fig. 379h] and include a series of stucco window grilles from the Alhambra; a *Mudéjar* painted wood panel from the Casa de Pilatos in Seville (sixteenth century); and a series of stucco window grilles at the Sinagoga del Tránsito in Toledo (1360). Similarly, the most common nonsystematic isometric examples are *acute* patterns with 9- and 12-pointed stars, including several cut-tile mosaic panels at the Alhambra, and an illuminated frontispiece from a Moroccan Quran written for the Sharifi Sultan 'Abd Allah ibn Muhammad[203] (1568) [Fig. 346a] [Photograph 65]. The most outstanding Maghrebi example of a nonsystematic geometric design is from two identical adjacent window grilles also found at the Sinagoga del Tránsito. This design is comprised of 8-, 14-, and 18-pointed stars that repeat upon a rectangular grid [Fig. 419]. Although it may not appear so at first glance, this is a hybrid design that utilizes two distinct rectangular repeat units, either of which will cover the plane on its own. The complexity of this remarkable design rivals that of the most complex compound patterns from the Seljuk Sultanate of Rum.

**Photograph 65** A Sharifi illuminated frontispiece from a Quran produced for Sultan 'Abd Allah ibn Muhammad in Morocco comprised of a nonsystematic *acute* pattern with 9- and 12-pointed stars (British Library Board: BL Or. MS 1405, ff. 370v-371r)

In al-Andalus the tradition of using geometric patterns as ornament for domes was roughly contemporaneous with the Mamluk practice: the principle difference being the partiality toward wood rather than either carved stone or stone mosaic, and the application to interior surfaces exclusively. The Islamic geometric pattern in the wooden *artesonado* cupola over the Capilla de Santiago at the Convent of Las Huelgas near Burgos, Spain (late thirteenth century), is an early Andalusian example of a form of ceiling vault that, while not domical, is closely related in aesthetic character and fabricating technology to the true wooden domes that soon followed. This variety of pseudo-dome is comprised of a series of flat trapezoidal panels that connect along their nonparallel edges, and is generally surmounted by a flat

[203] London; British Library, Or. 1405, ff. 370v–371r.



**Photograph 66** The Nasrid octagonal cupola of the *madrasa* of Yusuf I at the Alhambra with an *acute* pattern produced from the *fivefold system* (© David Wade)

ceiling panel at the apex to enclose the cupola. Multiple examples of this form of truncated pyramidical ceiling are found at the Alhambra, including the square-based cupola from the Hall of the Ambassadors (c. 1354-91) with its highly ornate matrix of 16- and 8-pointed stars; the octagonal cupola of the *madrasa* of Yusuf I (c. 1333-54) with 12- and 8-pointed stars; a small stucco cupola with an *acute* design produced from the *fivefold system* [Photograph 66]; and the 16-sided cupola of the Torre de las Damas at the Palacio del Partal (c. 1302-09) upon which the 16 trapezoids are decorated with 8-pointed stars joined together in a appealing geometric matrix that continues onto the 16-sided flat panel at the apex with a bold 16-pointed star. The multitude of sides in this cupola gives it the feel of a true dome with both vertical and horizontal curvilinearity. The most visually arresting domical geometric ornament in al-Andalus is in the Hall of Ambassadors in the Alcázar of Seville (1364) [Photograph 67]. This palace was built by Pedro I (Pedro the Cruel) on the site of an earlier Almohad palace, and the geometric dome in the Hall of the Ambassadors is the work of Diego Roiz. This dome is a significant example of the Islamic geometric idiom, even if created by a Christian artist. However, the geometric pattern appears more complex than it actually is, and for all its beauty, it is not without problems. The basic iterative unit is the same rhombic repeat as found in the classic fivefold *acute* pattern [Fig. 226c]. This rhomb has been slightly distorted to fit the curvature of the dome, and is repetitively placed upon the surface of each 1/12 gore segment of the dome. There is a basic problem with this approach: the rhombic repeat units do not fit accurately into the precise curvature of the 1/12 segment, requiring the geometric pattern lines to be inelegantly truncated where they cross the edges of each gore segment. The design only repeats radially

**Photograph 67** The *Mudéjar* dome over the Hall of Ambassadors in the Alcázar of Seville with an *acute* pattern produced from the *fivefold system* (© Jean-Guillaume Dumont)

by virtue of each truncated line meeting an identical reflected line along the joint of each gore segment; thereby establishing reflected symmetry and a semblance of order. The one area that falls far short of appearing purposeful is in the midsection of the overall geometric matrix wherein the centers of the 12 horizontally arrayed 10-pointed stars fall just inside the edges of each segment such that, when reflected, an elongated 11-pointed star is thereby created. The frustration caused by this ring of 12 irregular stars is ameliorated by the beauty of the 12-pointed star at the apex of the dome, and the overall beauty of the complete dome. A *Mudéjar* wooden dome at the Casa de Pilatos in Seville (sixteenth century) has a similar ceiling that was doubtless inspired by the example from the Hall of Ambassadors. It is interesting that this later and smaller example suffers from the exact same problem, complete with the central ring of 12 distorted 11-pointed stars. The two Nasrid wooden domes in the projecting porticos of the Court of Lions (c. 1354-91) at the Alhambra are of comparable beauty to the geometric dome in the Hall of Ambassadors in Seville, but far exceed this dome in geometric ingenuity. The geometry of one of these hemispherical domes is particularly interesting in that its repetitive structure is polyhedral rather than based on radial gore segments. The artist who devised this dome worked with a polyhedral subdivision of the surface of the sphere into 6 squares, 40 isosceles triangles, and 8 interstice triangles distributed in rhombic pairs around the hemispherical base whose proportions are determined by the distribution of the squares and isosceles triangles. As detailed by Emil Makovicky,[204] this polyhedral face configuration has two vertex conditions, and is similar to the octacapped truncated octahedron[205] except that it utilizes two types of triangle (one of which is equilateral) rather than a single non-equilateral triangle. The genius of the geometric pattern in this dome is the placement of regular 11-pointed stars at the vertices of the square and triangular spherical faces. When used as a repeat unit on the two-dimensional plane, the angles of both the triangle and square allow for the placement of 12-pointed stars at their corners: each 90° corner of the square having 3 of the 12 points, and each 60° corner of the triangle having just 2 of the 12 points. Similarly, a rhombus with two 30° acute angles and two 150° obtuse angles will receive a single point of a 12-pointed star at the acute angles, and five points at the obtuse angles. In short, in two dimensions, each of these polygonal faces is compatible with 12-fold symmetry at the vertices. As applied to the sphere, these three projected polygons tessellate together in a manner that creates 11-fold symmetry at both vertex conditions: square with four isosceles triangles $[3 + (2 \times 4) = 11]$, and five isosceles triangles with the acute angle of the rhomb $[1 + (2 \times 5) = 11]$. This is an ingenious solution to the covering of a dome with a geometric design. A similar approach was employed in the geometric ornament of the long wooden barrel vault in the Sala de la Barca (c. 1354-91) that is adjacent to the Hall of the Ambassadors at the Alhambra. The geometric schema of this vault is unique in Islamic architectural history. This barrel vault is capped on each end with half of a hemispherical dome, and the geometric pattern that runs the length of the barrel vault is adapted to seamlessly continue onto the surface of the domical regions at each end of the vault. If one were to remove the barrel vault and bring the two half domes together, a single hemispherical dome would be created with pattern qualities that closely resemble those of the dome in the Court of Lions. In fact, the repetitive square motif in the Sala de la Barca is identical to the central square motif in the dome at the Court of Lions. That said, the polyhedral structure of these two domical designs from the Alhambra are geometrically distinct from one another: the example from the Court of Lions being a distorted octacapped truncated octahedron, and the quarter domes at the Sala de la Barca being based upon the geometry of the rhombicuboctahedron. While the spherical projection of the square repeat units in the dome at the Court of Lions places the 11-pointed stars at the vertices, the example from the Sala de la Barca places the 12-pointed stars at the center of each repetitive square. The placement of the primary stars at the centers of each square repeat unit dispenses with the

[204] Makovicky (2000), 37–41.

[205] O'Keeffe and Hyde (1996).

need for the vertices to have 11-fold symmetry, and provides for a simple continuation of the design into the projected equilateral triangles. The similarity in polyhedral concept, and the proximate time and place of these two examples of Nasrid domical ornament is strong evidence of their having been designed and produced by the same individuals.

Another example of Nasrid polyhedral geometric ornament is from the Jineta sword of Muhammad XII, the last Nasrid ruler also known as Boabdil.[206] The hilt of this sword is elaborately ornamented with carved ivory and cloisonné enamel, much of which includes the common star-and-cross pattern that is easily created from the $4.8^2$ tessellation of squares and octagons [Fig. 124b]. The spherical pommel at the end of the hilt is ornamented with a three-dimensional corollary of the star-and-cross pattern that is created from a spherical projection of the truncated cube. Each vertex of this polyhedron is comprised of an equalateral triangle and two octagons in a $3.8^2$ configuration that is analogous to the two-dimensional $4.8^2$ tessellation of squares and octagons. Whereas the underlying square of this two-dimensional tessellation is responsible for the fourfold cross element within the standard star-and-cross design, the underlying triangular component of the spherical projection produces the threefold analogue found in this sword pommel. This is the only known historical example of a spherical geometric design used in immediate proximity to its two-dimensional analogue.

A distinctive feature of the later style of geometric ornament in the western regions was the development and utilization of geometric patterns with star forms placed at key nodal points that have an unusually large number of points.[207] Most of the designs of this variety have fourfold symmetry and repeat upon the square grid. As such, the stellar rosettes will commonly have 16, 20, or 24-points; and stars with higher numbers, such as 40- and 64-points, were also used. Less common are patterns of this type that employ the hexagon as a repeat unit; with large numbered star forms comprised of multiple of 3 (e.g., 18-, 21-, and 24-pointed stars). This variety of geometric design developed after the expulsion of Muslims from Spain following the final *reconquista*, and is mostly associated with the work of Moroccan artists during and after the Alawid dynasty. A typical example of this regional style is a wooden ceiling at the Bahia Palace in Marrakech (nineteenth century) that employs an orthogonal design with 8- and 16-pointed stars, and a large 32-pointed star at the center of the design. Generally, the tradition of Islamic geometric patterns is remarkably cohesive throughout the Muslim cultures of the Turks, Persians and Arabs.[208] The aesthetic differentiation in the use of geometric patterns between Muslim cultures is primarily determined by four interdependent factors: (1) available or preferred media; (2) specific fabrication technologies; (3) cultural predilections favoring different varieties of geometric pattern; and (4) different conventions for expressing and embellishing the design (e.g., thin or thick line weights, interweaving vs. interlocking characteristics, arbitrary additive elements). The primarily Alawid geometric style that uses higher numbered star forms is one of the few design traditions that is uniquely distinctive in geometric structure and resulting aesthetic character, and is exclusive to a single region and period of time.

## 1.23 Further Design Developments in the East After the Mongol Destruction

The societal upheaval that followed the Mongol overthrow of the Khwarizmshahs and the Abbasid Caliphate in the first half of the thirteenth century greatly impacted the arts and architecture throughout the regions of Transoxiana, Khurasan, Persia, and much of Iraq. Many great centers of Muslim culture, such as Merv, Samarkand, Bukhara, Herat, Balkh, Tus, and Nishapur experienced extensive destruction. In 1258, Baghdad was sacked by Hulagu Khan (d. 1265), the grandson of Chingiz Khan and founder of the Ilkhanid dynasty. The caliph al-Musta'sim was executed: ending the 500 years of religious authority and political hegemony of the Abbasid Caliphate of Baghdad. The westward expansion of Hulagu Khan's army was eventually checked in 1260 at the battle of Ain Jalut in eastern Galilee by the Mamluk forces of Sultan Baybar I (d. 1277). The number of buildings and works of art destroyed during this period of Mongol upheaval is incalculable, and a detailed and accurate knowledge of the early developmental history of Islamic art and architecture, including geometric patterns, is consequentially forever compromised. The Mongols generally

[206] In the collection of the Museo del Ejército, Madrid: no. 24.902.

[207] For more information on this distinctive regional style see:
–Piccard (1983).
–Castéra (1996).

[208] The cultural adoption of Islamic geometric patterns as a primary ornamental devise was primarily promulgated under the auspices of Turkic, Persian, and Arab patronage. The ornamental traditions of the Muslim populations in the more peripheral regions of sub-Saharan Africa, southeastern Europe, central Russia, the southern portion of the Indian subcontinent, southeast Asia, and China utilized this design aesthetic to a far less degree. When geometric patterns were employed, they were, more often than not, simplistic and derivative. All of these more peripheral cultures had their own distinctive and rich ornamental traditions that would have satisfied the aesthetic expectations of their artists and patrons. However, it is possible that the wider incorporation of more sophisticated Islamic geometric patterns would have likely appealed to these Muslim cultures had their artists been privy to the very specific design methodology required for their production.

spared those cities that did not resist them and accepted their authority; and even in cities such as Merv in Turkmenistan, where wholesale genocide was tragically employed as a military tactic, there is historical evidence of the lives of some artists having been spared.[209] Those artists who survived the Mongol destruction found themselves living in a world where the established system of patronal support was broken. Many artisans fled their homelands to settle in more stable regions,[210] and it can be assumed that these refugees would have included specialists in the geometric arts. The Seljuk Sultanate of Rum and the Mamluks of Egypt were among the direct beneficiaries of this artistic exodus: the former by virtue of their acceptance of Mongol suzerainty, and the latter by having conclusively repelled the Mongol advance on the Levant and Egypt at Ain Jalut.

As a direct result of the societal chaos that followed the Mongol invasion, the developmental momentum of Islamic geometric design that continued under Mamluks and Seljuk Sultanate of Rum patronage was arrested throughout Transoxiana, Khurasan, and Persia. The early Mongol rulers of the Ilkhanid Dynasty favored Christianity and Buddhism over Islam. It was not until Ghazan Khan succeeded the Ilkhanid throne in 1293 that Islam once again became the state religion of the vast region under Ilkhanid control, and the quintessentially Islamic artistic conventions that preceded the Mongol invasion began to reassert themselves into the new cultural paradigm. The reign of Ghazan Khan eventually reestablished many of the vanquished cities as important centers of Islamic culture, attracting tremendous wealth through public works and active trade, particularly with the Yuan Dynasty of China. Ghazan Khan was an avid and enthusiastic builder, purportedly ordering the building of a mosque and bathhouse in every town: with the proceeds from the bathhouses used to support the mosques. He moved the capitol of his empire to Tabriz, which became an influential center of Islamic arts and culture. His greatest architectural undertaking was the Ghazaniyya (1297-1305): his palace complex in Sham outside of Tabriz. Very little has survived to the present, but in its day it was vast on an unprecedented scale, with "monasteries, *madrasas*, a hospital, library, philosophical academy, administrative palace, observatory and palatial summer residences, as well as arcades and gardens of exceptional charm."[211] The mausoleum of Ghazan Khan in the Ghazaniyya is reported to have had a richly ornamented dome some 45 m in height, which 14,000 people worked on over a period of 4 years to complete.[212] Ghazan Khan was succeeded by his younger brother, Sultan Uljaytu Khudabanda, who was also a dedicated patron of the arts. The mausoleum of Uljaytu in Sultaniya, Iran (1307-13), was similarly lavish. Much of this building is still standing, and is regarded as the most significant extant Ilkhanid building, and one of the most important examples of Islamic funerary architecture in Iran.

Despite the tremendous Ilkhanid emphasis on architectural projects, the level of geometric sophistication of their architectural ornament did not generally parallel the work of their Mamluk and Seljuk neighbors to the west. Without doubt, the geometric ornament at such buildings as the Sultaniya is of great beauty, and exhibits a distinctive Ilkhanid aesthetic. However, the ornamental originality is primarily in the use of materials and color rather than a pioneering approach to geometric design. As such, their use of geometric patterns was, for the most part, informed by the geometric ornament of their pre-Mongol predecessors, and included patterns made from the *system of regular polygons*, both fourfold systems, and the *fivefold system*. Nonsystematic patterns were also widely utilized, including examples with compound symmetry. Most of these were patterns that had already been used previously by Muslim artists, and tend to be less complex designs such as orthogonal patterns with 8- and 12-pointed stars, and isometric patterns with 9- and 12-pointed stars.

The later Ilkhanids and their Muzaffarid successors in central and southern Persia had an aesthetic predilection for additive geometric patterns. This type of pattern is created by applying additional pattern elements into an existing design, resulting in a heightened level of geometric complexity. This additive practice is relatively simple and does not require particular skill or specialized knowledge, and was used to a limited extent by earlier Muslim cultures: for example, the Seljuk arched panel over the entry door at the Gunbad-i Surkh in Maragha (1147-48). As typical of later Ilkhanid and Muzaffarid additive geometric designs, the additive elements of this Seljuk example are differentiated with color. Among the more significant Ilkhanid additive patterns are several examples from the mausoleum of Uljaytu at Sultaniya (1313-14). These include two designs created from the simple hexagonal grid: a *median* pattern with 90° crossing pattern lines that was also used by Ilkhanid artists at the Khanqah-i Shaykh ʻAbd al-Samad in Natanz,

[209] The Persian historian Ata al-Mulk Jujayni wrote in his account of the Mongols, *Ta'rikh-i jahan-gusha* (History of the World Conqueror) that the order was given for the whole population of Merv, including women and children, to be put to death except for 400 artisans.

[210] An inscription on a panel of faience mosaic at the Sirçali Madrasa in Konya, Turkey (1242) states that the work was carried out by "Muhammad, son of Muhammad, son of Othman, architect of Tus." See Wilber (1939), 40.

[211] Pope (1965), 171.

[212] Wilber (1955), 124–126.

Iran (1304-25), into which octagons are added at the centers of each intersection of the primary pattern [Fig. 64], and a very basic *median* design with 60° crossing pattern lines that places additional 6-pointed stars at the same centers as the original 6-pointed stars, but rotated 30°, thereby creating 12-pointed stars in an isometric arrangement [Fig. 65]. One of the most outstanding additive designs at the Uljaytu mausoleum is a *median* pattern made from the *fourfold system A* [Fig. 66]. This design repeats on a rhombic grid, and the primary motif on its own was used subsequently by Timurid artists in the Bibi Khanum in Samarkand, Uzbekistan (1398-1404) [Fig. 157b]. An arch spandrel from the Uljaytu mausoleum contains an additive *two-point* design created from a nonsystematic underlying polygonal tessellation of octagons surrounded by coinciding triangles, pentagons and squares [Fig. 331b] [Photograph 68]. This is a rare example of an Ilkhanid design created from an underlying tessellation that appears to have no prior use. Furthermore, this is unusual in that additive patterns are almost always elaborations of patterns that were created from one or another of the generative systems, whereas this example is nonsystematic. The resulting design is arguably the most elaborate Ilkhanid additive pattern.

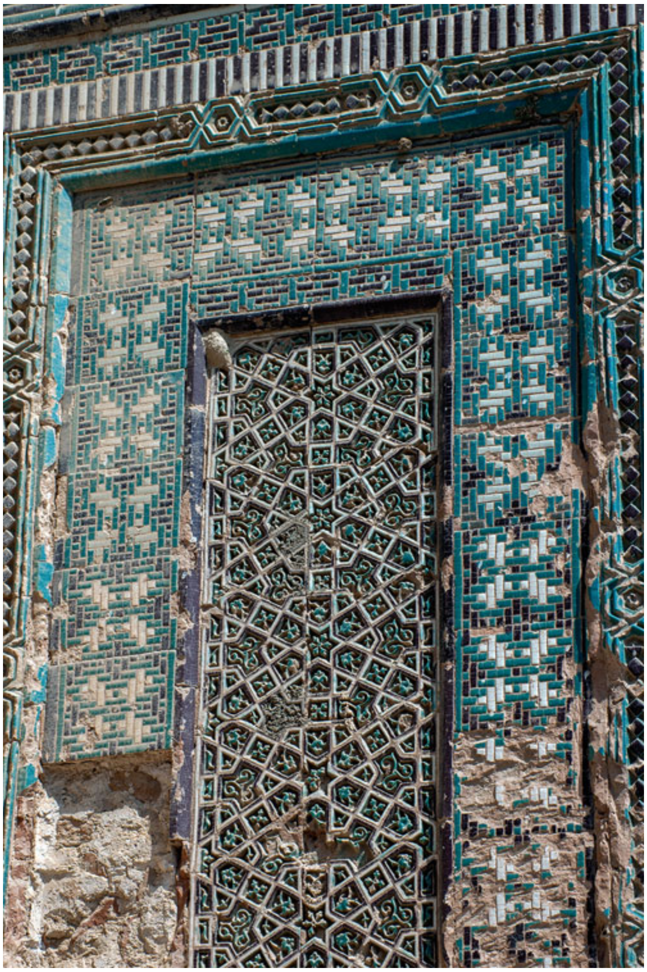
The use of geometric patterns by Muslim cultures in the regions affected by Mongol conquest, albeit largely derivative of earlier work, is nonetheless refined and beautiful. While lacking the creative vitality and methodological innovation of the contemporaneous work of Egyptian and Anatolian artists, the quality of execution was outstanding. This is especially the case with the increased application of geometric patterns to the burgeoning tradition of cut-tile mosaic that took place among the Muzaffarid, Kartid, Qara Qoyunlu, Aq Qoyunlu, and Timurid successors to the Ilkhanids. These cultures continued the prolific use of systematic geometric methodologies, and innumerable examples are found in diverse media. Designs created from the *system of regular polygons* were especially popular and many fine examples were employed throughout this vast region by succeeding dynasties. The original creation of most of these designs took place during the period of high innovative development in the twelfth and thirteenth centuries, and predates the period of the Mongol destruction. Notable examples of earlier designs created from the *system of regular polygons* that were incorporated into the post-Mongol work in the eastern regions include an Ilkhanid isometric design from the 30-volume Quran (1310)

**Photograph 68** An Ilkhanid cut-tile mosaic and stucco arch spandrel from the mausoleum of Uljaytu at Sultaniya, Iran, with a nonsystematic additive *two-point* pattern (© David Wade)

commissioned by Sultan Uljaytu that has the precise proportions of the pattern derivation associated with the 3.6.3.6 underlying tessellation[213] [Fig. 99c], as distinct from the proportions created from the use of the $6^3$ hexagonal grid [Fig. 96d]; and a Tughluqid raised relief ceramic panel from the tomb of Shah Rukn-i ‘Alam in Multan, Pakistan (1320-24), that has the proportions of the 3.4.6.4 derivation of this otherwise similar design [Fig. 107d] [Photograph 69], as distinct from the proportions produced from other underlying polygonal tessellations [Figs. 97c and 99b]. A *two-point* pattern from a Chaghatayid ceramic relief panel at the mausoleum of Tughluq Temür in the ancient city of Almaliq (present-day Huocheng) in western China (1363). This is created from the 3.4.6.4 underlying tessellation [Fig. 105d] [Photograph 70]. This is very similar in structure to the somewhat more complex Qarakhanid *two-point* pattern at the southern anonymous tomb at Uzgen dating from approximately 350 years earlier [Fig. 105b], as well as a Ghurid *two-point* pattern from the minaret of Jam (1174-75 or 1194-95) [Fig. 105c]. A Khoja Khanate *two-point* design from the Apak Khoja mausoleum in Kashi, China (c. seventeenth century), employs a wooden window grille created from the 3.6.3.6 tessellation that is identical to a carved stone relief pattern from an Armenian *khatchkar* dating to the fourteenth century [Fig. 100b]. The design of the window grille from Kashi is very similar to a fine pattern created from the $3^2$.4.3.4-3.4.6.4 *two-uniform* underlying tessellation that was used in the entry portal of the post-Ilkhanid mausoleum of Muhammad Basharo in the village of Mazar-i Sharif, Tajikistan (1342-43) [Fig. 111a]. This example is a *two-point* pattern that is similar in concept to the above-referenced Chaghatayid design from the mausoleum of Tughluq Temür in western China (Fig. 105d). The mausoleum of Muhammad Basharo has an immediately adjacent second design that is a variation to the $3^2$.4.3.4-3.4.6.4 *two-uniform* design [Fig. 111a], except that it replaces the central

**Photograph 69** A Tughluqid ceramic panel from the tomb of Shah Rukn-i ‘Alam in Multan, Pakistan, with a pattern comprised of superimposed dodecagons that is easily created from the *system of regular polygons* (© Aga Khan Trust for Culture-Aga Khan Award for Architecture/Jacques Betant [Photographer])

[213] Calligraphed by ‘Ali ibn Muhammad al-Husayni in Mosul (1310). British Library, Or. 4945, ff. IV-2r.



**Photograph 70** A Chaghatayid ceramic panel from the mausoleum of Tughluq Temür in Huocheng, western China, with a *two-point* pattern that is easily constructed with the *system of regular polygons* (© Daniel C. Waugh)

cluster of underlying triangles, squares, and central hexagon with a single dodecagon [Fig. 111b]. This modification adroitly provides for the attractive 12-pointed star located at the center of the ornamental panel.

Among the Ilkhanid examples of design created from the *system of regular polygons* are two noteworthy Quranic frontispieces that utilize the $3.12^2$ polygonal tessellation. The first of these is a *two-point* pattern from a Quran illuminated in Baghdad by Muhammad ibn Aybak ibn ʻAbdullah[214] (1306-07) [Fig. 108f]; and the second is a curvilinear design from the 30 volume Quran of Uljaytu,[215] illuminated by ʻAbdallah ibn Muhammad al-Hamadani (1313) [Fig. 108c]. Other post-Mongol patterns that feature 12-pointed stars and are created from the *system of regular polygons* include an Ilkhanid triangular pendentive for one of the vaults at the mausoleum of Uljaytu in Sultaniya that is decorated with the well-known isometric *median* pattern created from the $3.12^2$ underlying tessellation [Fig. 108a]; an *obtuse* design in one of the ceiling vaults at this same mausoleum [Fig. 108d]; and a Qara Qoyunlu arched entry portal of the Great Mosque at Van in eastern Turkey (1389-1400) that is decorated with a very nice representation of the equally well-known orthogonal *median* pattern created from the $3.4.3.12\text{-}3.12^2$ underlying *two-uniform* tessellation [Fig. 113a]. A Timurid cut-tile mosaic design from the exterior of the Abu'l Qasim shrine in Herat, Afghanistan (1492) employs a *median* pattern created from the $3.4.3.12\text{-}3.12^2$ that is very similar to the far more widely used design from the Great Mosque of Van [Fig. 113b]: the difference being in the pattern line treatment within the underlying triangular and square elements. A rare example of a *three-uniform* pattern created from the *system of regular polygons* was used by Qara Qoyunlu artists at the Great Mosque of Van in eastern Turkey (1389-1400) [Fig. 115]. This is a particularly complex *median* pattern created from the $3^6\text{-}3^3.4^2\text{-}3^2.4.12$ tessellation.

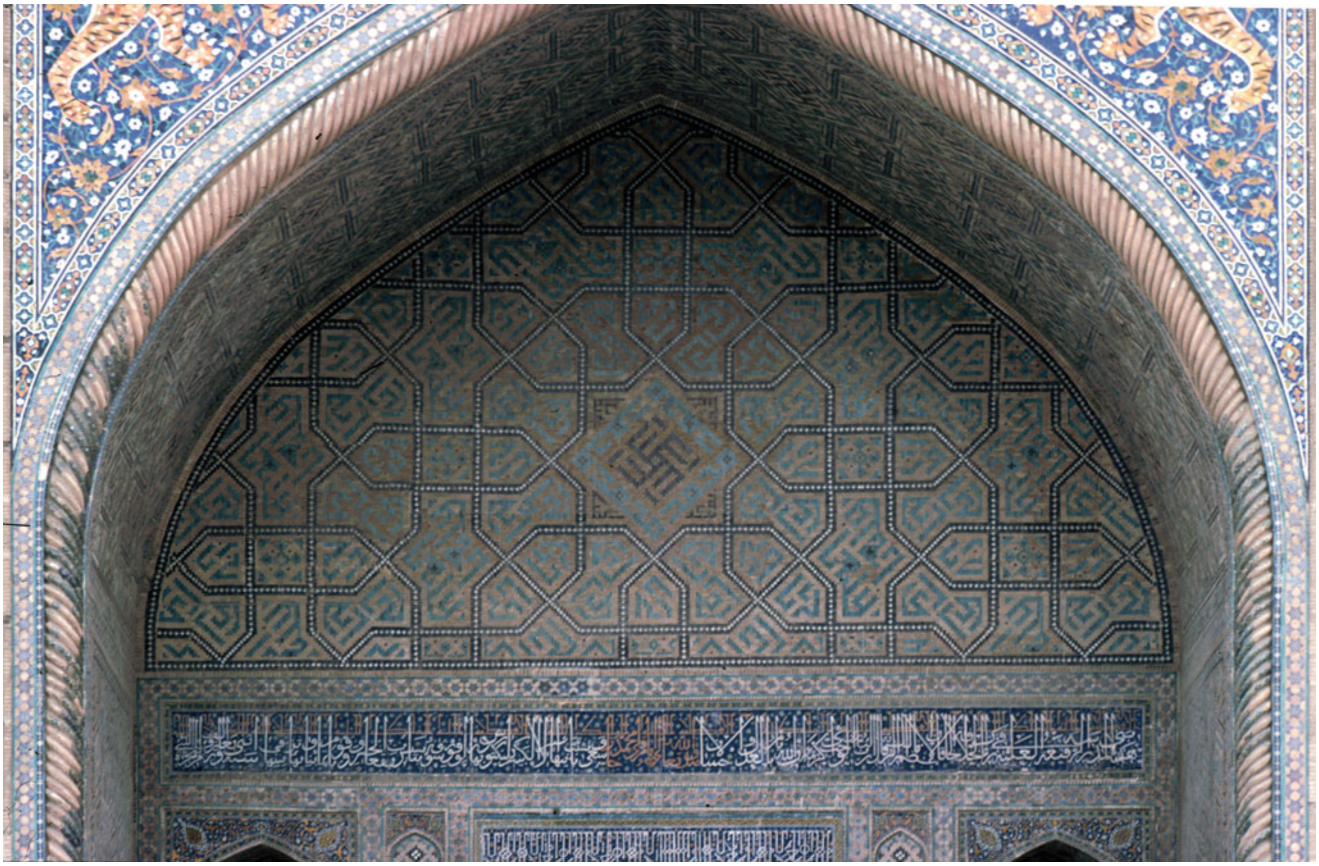
There were many designs created in the eastern regions after the Mongol destruction that are easily created from the $4.8^2$ underlying tessellation. A beautifully executed Chaghatayid ceramic border in the tomb of Tughluq Temür in Almaliq (Huocheng), China (1363), is an example of the classic star-and-cross pattern [Fig. 124b]; and a Timurid example of this same pattern from the Ghiyathiyya *madrasa* in Khargird, Iran (1438-44), is provided with greater complexity by emphasizing the generative tessellation equally with the final pattern [Fig. 126d]. This design also has a secondary eight-pointed star with 45° included angles arbitrarily added inside each eight-pointed star of the *median* pattern. An Ilkhanid illuminated frontispiece to a Quran (1304) employs a version of the standard *obtuse* design with pattern lines that extend into the underlying squares: creating two sizes of octagon within the pattern matrix [Fig. 127a]. Muzaffarid examples of the star-and-cross design include a very fine cut-tile mosaic border at the Friday Mosque at Yazd (1324) that is further developed with additive pattern lines that interweave with the standard design. The Tughluqid use of the star-and-cross pattern at the Adina mosque in Pandua, West Bengal, India (1375) serves as an overall textural background to the exterior façade. The combination of the small scale of design and low-level relief provides the carved stone with a subtle aesthetic unlike that of other Muslim cultures. Mughal artists also made wide use of patterns created from this underlying tessellation, including: a painted mural with the standard *acute* pattern from the tomb of Jahangir in Lahore (1637) [Fig. 124a]; and the simple but elegant red and white octagonal paving at the Taj Mahal in Agra (1632-53) [Fig. 127f].

[214] Chester Beatty Library Ms. 1614 (Arberry No. 92).

[215] Cairo National Library; 72, pt. 19.

Both the fourfold systems were less widely used by Ilkhanid and Muzaffarid artists, but regained popularity under the patronage of succeeding dynasties of the Kartids and the Timurids. The many exquisite designs in the ceiling vaults at the Ilkhanid mausoleum of Uljaytu in Sultaniya include the ever-popular *median* pattern created from the *fourfold system A* that was used extensively by Seljuk and Ghurid artists in Khurasan as early as the late eleventh century [Fig. 145]. Ilkhanid artists used this same design in two additional locations: at the Mashhad-i Bayizid Bastami in Bastam, Iran (1300-13), and in the carved stucco *mihrab* of the Imamzada Rabi'a Khatun shrine in Ashtarjan, Iran (1308). Later examples of this design include a very beautiful Mughal inlaid stone panel at the mausoleum of Akbar in Sikandra, India (1613), and a contemporaneous carved stone border that surrounds a window in the Bayt Ghazalah private residence in Aleppo created during the Ottoman period. A Muzaffarid *median* pattern from the Friday Mosque at Kerman (1349) is unusual in that it uses 60° crossing pattern lines [Fig. 144a]. This arrangement is more typical of designs with 6- and 12-pointed stars. An interesting *median* field pattern was created by Qara Qoyunlu artists for the *mihrab* arch spandrels at the Great Mosque at Van [Fig. 138d]. This shares the same variation to the standard pattern line application within the large hexagonal modules as the design in the wooden railing at the Esrefoglu Süleyman Bey in Beysehir, Turkey (1296-97) [Fig. 142]. In fact, these two designs are identical except that the example from the Esrefoglu Süleyman Bey has elegantly incorporated eight-pointed stars within the pattern matrix. Under the Timurids and Shaybanids, as well as the later Safavids and Qarjars, the *fourfold system A* was applied widely to the glazed *banna'i* brickwork façades that feature prominently in the architecture of these cultures. Part of the aesthetic of this brickwork tradition is the emphasis on designs that are comprised of just vertical, horizontal and 45° diagonal lines expressed via the orthogonal layout of the brick modules. This layout provides for smooth edges for the pattern lines that run vertically and horizontally, but stepped edges for those lines that run diagonally. This creates a very distinct visual quality that softens the rigidity of the geometric design. What is more, the limitation to just four directions of pattern line is ideally compatible with the constraints of *median* patterns created from the *fourfold system A*. Innumerable examples of this variety of ceramic ornament were used since the fourteenth century, and typical examples include a Timurid arched panel in the exterior façade of the Bibi Khanum mosque in Samarkand (1398-1405) [Fig. 157b] and part of the Shaybanid exterior façade of the Tilla Kari *madrasa* in Registan Square, Samarkand (1646-60) [Fig. 138c] [Photograph 71]. An especially common practice was the application of this form of *fourfold system A* design to arched tympanums in the back walls of entry *iwans*. Timurid examples of this form of architectural ornament are found at the Gawhar Shad mosque in Mashhad, Iran (1416-18); the Ulugh Beg *madrasa* in the Registan Square in Samarkand (1417-20); and the Khwaja Akhrar funerary complex in Samarkand (1490). Later Shaybanid examples are found at the Shir Dar *madrasa* in Registan Square, Samarkand (1619-36) [Photograph 72], and the Tilla Kari *madrasa* (1646-60) in the same square in Samarkand. Both of these introduce a central eight-pointed star within a *median* field pattern [Fig. 145]. Among the many Mughal examples of patterns created from the *fourfold system A* is a fine stone mosaic panel from the tomb of I'timad al-Daula in Agra (1622-28) [Fig. 154] [Photograph 73]. The underlying generative tessellation for this *median* design is easily created by placing coinciding octagons at the vertices of the $4.8^2$ tessellation of octagons and squares, and infilling the central region with an underlying octagon surrounded by eight pentagons. A particularly eccentric pattern that can be created from an underlying tessellation of

**Photograph 71** Shaybanid polychrome brickwork from the Tilla Kari *madrasa* in Samarkand with a *median* pattern created from the *fourfold system A* (© David Wade)



**Photograph 72** Shaybanid polychrome brickwork from the Shir Dar *madrasa* in Samarkand with a *median* pattern created from the *fourfold system A* (© David Wade)

eight squares in eightfold rotation around eight-pointed star interstice regions was used in the entry portal of the Task-Kala caravanserai in Konye-Urgench, Turkmenistan (fourteenth century) [Fig. 147b]. This building was built under either Chaghatayid or Sufid patronage, but is in the emerging Timurid style. The arrangement of squares in the underlying tessellation is identical to that of a pattern used more than a hundred years previously at both the Çifte *madrasa* in Kayseri, Turkey (1205), and the Friday Mosque in Gonabad, Iran (1212) [Fig. 147a]. The application of the pattern lines to the underlying tessellation in the example from Konye-Urgench is highly unusual, and not in keeping with the standard methodological practices associated with the *fourfold system A*. The pattern is initiated by first placing regular hexagons at key locations within the eightfold geometric structure, and extending the lines of these hexagons until they meet with other extended lines. While aesthetically pleasing, the resulting pattern is visually distinct from more overtly systematic orthogonal designs. An example of a Timurid design created from the *fourfold system A* that repeats on a rectangular grid is found in a border design at the Shah-i Zinda complex in Samarkand [Fig. 164] [Photograph 74]. This *median* pattern places eight-pointed stars at the vertices of the rectangular grid, as well as at the vertices of the rectangular dual grid. In fact, the geometric information contained within the repeat unit and the dual repeat unit are identical.

Designs created from the *fourfold system A* were occasionally given an additive treatment that incorporates a swastika device within square components of the pattern matrix. This variety of additive variation was used to a limited extend by Seljuk artists working in the Sultanate of Rum, but greater variation and ingenuity was employed by post-Mongol artists, particularly during the Timurid period. A fine, if rather predicable, example encompasses the marble shaft of a column found at the Gawhar Shad *madrasa* and mausoleum in Herat (1417-38) [Fig. 150b]. The Topkapi Scroll illustrates several examples of this variety of additive feature. One is particularly interesting in that it repeats upon a rhombic grid[216] [Fig. 157a]. The designer of this pattern used the implicit squares contained within the elongated hexagonal modules in the pattern to incorporate the swastika motif.

[216] Necipoğlu (1995), diagram number 67.

**Photograph 73** A Mughal inlaid stone panel at the tomb of I'timad al-Daula in Agra, India, with a *median* pattern created from the *fourfold system A* (© David Wade)

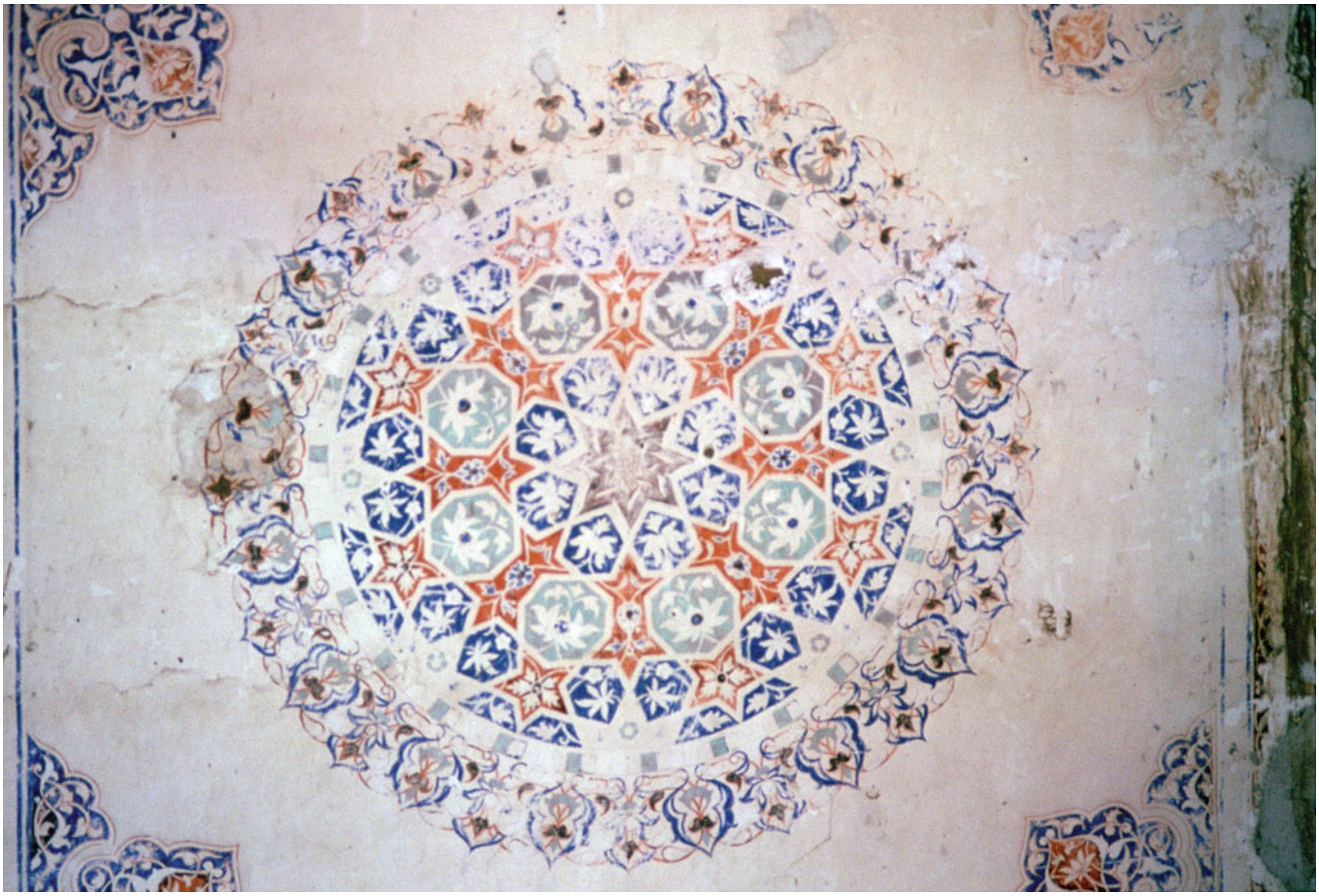
Examples of the Ilkhanid use of the *fourfold system B* include the classic *acute* pattern created from underlying octagons and pentagons located in another of the ceiling vaults at the mausoleum of Uljaytu in Sultaniya (1307-13) that was used some 175 years previously at the Friday Mosque at Sin, Iran (1134) [Fig. 173a], and an *acute* pattern created from the underlying tessellation of octagons, pentagons, and elongated hexagons in the arched tympanum over the entry door of the round tomb tower of Hulagu Khan's sister in Maragha, northeastern Iran (thirteenth century) [Fig. 177c]. A Muzaffarid design from the exterior façade of the Friday Mosque at Kerman (1349) has a *median* pattern created from an underlying tessellation that is essentially the same accept that it uses the small hexagons rather than the large hexagons from this system [Fig. 175c]. A painted fresco in the mausoleum of Shaykh Ahmed-i Jam at Torbat-i Jam in northeastern Iran (1442-45) is a fine Kartid design that utilizes the variation to the pattern lines within the hexagon that allow for regular octagons within the design [Fig. 179a] [Photograph 75]. Timurid examples created from this system are mostly derivative of earlier work, and include: several mosaic panels with the classic *acute* pattern at the Abdulla Ansari complex in Gazargah near Herat, Afghanistan (1425-27) [Fig. 173a] [Photograph 76]; and a rhombic *acute* pattern produced in carved stucco from the mausoleum of Amir Burunduq in the Shah-i Zinda in Samarkand (1390) [Fig. 181]. This was used earlier by artists in the Seljuk Sultanate of Rum at the Izzeddin Keykavus hospital and mausoleum in Sivas (1217-18). The Mughals made occasional use of the *fourfold system B*, and most of these examples are the classic *acute* design produced from just the underlying tessellation of octagons and pentagons [Fig. 173a]. A distinctive example in high-relief carved stone is found in the Agra Fort (1565-73). However, the most remarkable Mughal use of this classic *acute* design is a marble *jali* screen from the tomb of Salim Chishti at Fatehpur Sikri (1605-07) [Photograph 77]. This is one of several Mughal *jali* screens that prominently portray the generative tessellation as part of the completed design. In addition to being stunningly beautiful, these are important examples of historical evidence for the use of the polygonal technique as a traditional design methodology.

The use of the *fivefold system* enjoyed continued popularity throughout the eastern regions during this period, and

**Photograph 74** Timurid cut-tile mosaic ornament from the Shah-i Zinda complex in Samarkand with a *median* pattern created from the *fourfold system A* (© David Wade)

included examples of established designs in all four of the pattern families, as well as original patterns with diverse repeat units comprised of a relatively large number of underlying polygonal modules. Artists working during the period that followed the Mongol destruction were evidently very familiar with the subtleties of the *fivefold system*; as evidenced by the large number of original patterns created from this system, and the high level of innovation that was applied to these designs. As pertains to the tradition of Islamic geometric patterns, the mausoleum of Uljaytu in Sultaniya, Iran (1307-13), is the most important surviving building from the period of Ilkhanid reconstruction, and several very successful fivefold designs were produced for this monumental tomb. A particularly beautiful example is an unglazed raised brick geometric design with unglazed ceramic cast relief inserts in the background [Fig. 246]. The raised relief geometric design, and the cast inserts are elegantly separated by a thin outline of *lājvard* (dark blue) glazed ceramic. This Ilkhanid example is an *obtuse* pattern that can be created with equal ease from either of two dualing underlying tessellations. The design methodology of an Ilkhanid faïence mosaic panel at the Gunbad-i Gaffariyya in Maragha (1328) is unusual in that the pattern lines are placed upon the vertices of the underlying tessellation rather than their standard placement upon the midpoints [Fig. 259]. The resulting design is equally unusual, and while the pentagrams are akin to those of the standard *acute* pattern, their vertex-to-vertex orientation within each underlying pentagon is virtually unique,[217] and does not conform to any of the four pattern families. Another atypical aspect of this design is the convergence of multiple non-coincident pattern lines upon a single point rather than the standard crossing of two pattern lines. Except for the pattern lines within the underlying decagonal modules, this feature disallows the pattern lines from interweaving with one another. A Muzaffarid *acute* design that surrounds the north portal of the courtyard of the Friday Mosque at Kerman (1349) has a particularly large rectangular repeat unit with considerably more underlying polygonal modules than was typically employed within this tradition [Fig. 254c]. This is a supremely successful fivefold Islamic geometric pattern. A Kartid cut-tile mosaic panel at the Shamsiya *madrasa* in Yazd (1329-30) employs a *median* pattern created from an unusual underlying tessellation of modules in the *fivefold system* [Fig. 236]. Rather than the standard underlying decagon, the decagons in this example are proportioned to the width of the short half of the underlying wide rhombus from the *fivefold system*. The matrix of the resulting *median* pattern has distinctive large decagons located at each vertex of the repeat unit, and the ten-pointed stars within these decagons are as becoming as they are atypical. This same distinctive *median* design was used by Timurid artists on an arch soffit at the Ulugh Beg *madrasa* in Samarkand (1417-20) approximately 100 years later. The unusual qualities of this design are unlikely to have been independently derived, and it would appear likely that this design variation was directly influenced by an Anatolian design from the Sultan Han in Kayseri (1232-36) that applies the same treatment to the ten-pointed stars [Fig. 237] [Photograph 42]. The identical underlying decagonal condition, with its distinctive ten-pointed star, was also used in a Timurid *median* border pattern in the cut-tile mosaic ornament at the Imam Reza shrine complex in Mashhad, Iran (fourteenth century) [Fig. 253]. This design is made more dynamic by alternating the unconventional

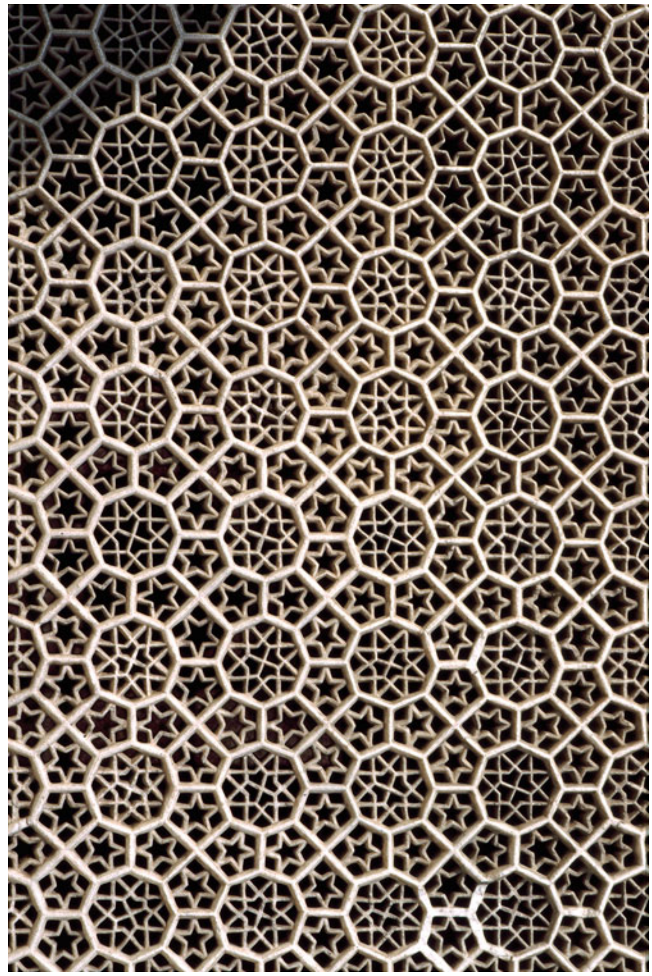
[217] Another example of this unusual arrangement of acute five-pointed stars is found in the pattern that fills the tympanum of the arched entry portal at the hospital of the Great Mosque of Darussifa in Divrigi, Turkey (1228–29): although this Mengujekid pattern is simplistic by comparison.



**Photograph 75** A Kartid fresco roundel at the mausoleum of Shaykh Ahmed-i Jam at Torbat-i Jam in northeastern Iran that employs a *median* pattern created from the *fourfold system B* (© Sheila Blair and Jonathan Bloom)

ten-pointed stars with those created from the standard decagon from this system. The Anatolian example from the Sultan Han, together with the above cited Kartid example from the Shamsiya *madrasa* in Yazd and this design from Mashhad are the only known fivefold patterns that utilize this unusual decagonal feature within the underlying generative tessellation, and this rarity suggests they may share a common origin, perhaps through association with the same *tumar*. A comparatively simple Timurid *acute* pattern that repeats upon a rectangular grid was used in a cut-tile mosaic panel at the Shah-i Zinda complex in Samarkand (fourteenth century) [Fig. 254]. While far less complex, this is nonetheless an elegant design that, surprisingly, was not more widely used. A panel from a Shaybanid wooden door at the Kukeldash *madrasa* (1568-69) in the Lab-i Hauz complex in Bukhara employs a very successful *acute* pattern with a comparatively large number of polygonal modules used in its underlying generative tessellation [Fig. 256] [Photograph 78]. This design includes the distinctive partial ten-pointed star motifs, in this case 2/10 and 3/10 [Fig. 196], that are a frequent feature of fivefold *acute* patterns that originated during the later period in the eastern regions. This same design was also used on a wooden door panel produced during the Janid Khanate at the Bala Hauz mosque in Bukhara (1712). A similarly proportioned rectangular repeat is found on a Mughal inlaid stone *acute* border design from the mausoleum of Akbar in Sikandra, India (c. 1612) [Fig. 255]. This design places ten-pointed stars at the vertices of the rectangular repeat, as well as at the center of the repeat unit. As is often the case with such patterns, at first glance, this arrangement of primary star forms gives the impression of repeating on a rhombic grid. However, the tenfold radii at these locations do not align with one another; and their skewed orientation causes the repetitive structure to be more accurately defined as rectangular. This design has the distinctive and unusual feature wherein the geometric information contained within the rectangular repeat unit is identical to that of the dual repeat. A very beautiful inlaid stone panel from the mausoleum of Humayun in Delhi (1562-72) is a relatively rare example of the use of *two-point* methodology among Mughal artists [Fig. 245b] [Photograph 79]. One of the pierced marble *jali* screens at the I'timad al-Daula in Agra (1622-28) makes use of a fascinating rendering of the classic *acute* pattern

**Photograph 76** Timurid cut-tile mosaic ornament from the Abdulla Ansari complex in Gazargah, Afghanistan, with an *acute* pattern created from the *fourfold system B* (© Thalia Kennedy)



**Photograph 77** A Mughal pierced marble *jali* screen at the tomb of Salim Chishti at Fatehpur Sikri, India, that employs an *acute* pattern created from the *fourfold system B* along with its underlying generative tessellation (© David Wade)

[Fig. 226c] that is one of the most widely used patterns created from the *fivefold system*. This example is unusual in that it prominently incorporates the generative tessellation with the standard *acute* pattern into the finished screen. This Mughal exposure of the underlying generative tessellation is also found in the above-cited *fourfold system B* design from the mausoleum of Salam Chishti in Fatehpur Sikri (1605-07) [Photograph 77], and like the earlier example from Fatehpur Sikri, this is an important piece of historical evidence for the use of the polygonal technique in generating Islamic geometric patterns.

In addition to patterns that repeat upon both rhombic and rectangular grids, artists working in the eastern regions following the Mongol destruction occasionally applied the *fivefold system* to patterns with radial symmetry. Among the most common are the secondary patterns of dual-level designs that are incorporated into the primary pattern elements with five or tenfold rotational symmetry; for example pentagons, decagons, five-pointed stars, and ten-pointed stars. Many examples of this variety of fivefold radial design were used in the dual-level designs produced by Timurid and Qara Qoyunlu artists [e.g. Fig. 453]. The Topkapi Scroll is replete with examples of this form of radial design application [Fig. 22]. Another type of radial design places two-dimensional patterns created from the *fivefold system* onto domical surfaces. This method of domical geometric ornament makes use of eight 1/10 segments of a tenfold pattern for application to the eight gore segments of a dome. The resultant distortion is minimal and undetectable to the eye. Ernest Hanbury Hankin first identified this form of domical ornament when writing about the Samosa Mahal at Fatehpur Sikri, India[218] (sixteenth century) [Fig. 21]. The Safavid exterior ceramic ornament of the large dome at the Mashhad-i Fatima in Qum (c. 1519) employs the same

[218] Hankin (1925a), Figs. 45–50.

**Photograph 78** Shaybanid wood joinery from a door panel at the Kukeldash *madrasa* in Bukhara, Uzbekistan, that employs an *acute* pattern created from the *fivefold system* (© Thalia Kennedy)

decorative methodology in its lower portion, but breaks from the *fivefold system* in the upper quarter as it approaches the apex. The Topkapi Scroll appears to have another example of this form of fivefold domical ornament, although only the 1/10 decagonal triangle is represented. Without any associated text, it is impossible to know for certain whether this was intended for use on a dome[219] [Fig. 260e].

The method of employing more than one repetitive cell, each with its own geometric pattern, into a single larger hybrid fivefold construction was a practice first developed by Seljuk artists in Persia, and later in Anatolia [Figs. 261–265]. Still later, Mamluk and Marinid artists engaged in this practice to a lesser extent, but with exceptional results [Figs. 267 and 268]. Mughal artists also experimented with fivefold hybrid designs, although such work is comparatively rare. A fine example was used in the stone mosaic façade of the I'timad al-Daula in Agra (1622-28) [Fig. 266]. This exceptional *acute* design has the unusual characteristic of having regions within the design that have rotational point symmetry. The hybrid repetitive cells that comprise this design are of three types: a rectangle that includes the point symmetry, a rhombus with 72° and 108° angles, and a half rhombus.

Ilkhanid artists devised an additive treatment to fivefold patterns that was popularly adopted by artists in several subsequent eastern dynasties. This variety of additive pattern places arbitrary pattern lines into the standard design in such

[219] Necipoğlu (1995), diagram number 90a.

**Photograph 79** A Mughal inlaid stone panel at the mausoleum of Humayun in Delhi with a *two-point* pattern created from the *fivefold system* (© David Wade)

**Photograph 80** Muzaffarid cut-tile mosaic ornament at the Friday Mosque at Yazd, Iran, that employs an additive variation of an *obtuse* pattern created from the *fivefold system* (© Jean-Marc Castéra)

manner as to fill the background regions with a meandering mazelike device. An early example of this variety of additive design that employs the classic fivefold *acute* pattern as its starting motif was used in a mosaic panel at the mausoleum of Uljaytu in Sultaniya [Fig. 226d]. This fivefold additive design is similar in concept to the fourfold patterns with additive swastikas that were also popular among the later eastern cultures in Persia, Khurasan, and Transoxiana [Figs. 150a, b and 157a]. A Muzaffarid cut-tile mosaic panel from the Friday Mosque at Yazd (1324) [Photograph 80] is an outstanding example of an additive *obtuse* design with the swastika aesthetic [Fig. 230]. This same design was used many centuries later by Safavid artists at the Shah mosque in Isfahan (1611-38), and an example is included in the repertoire of designs illustrated in the Topkapi scroll.[220] And just as Seljuk artists in Anatolia were among the first to develop this variety of fourfold additive design, so also were they early developers of the use of swastika additive elements within the *fivefold system*.[221]

An artist working during the Ottoman period employed the *fivefold system* to create a rather remarkable design wherein the individual polygonal modules that comprise the underlying generative tessellation transition between two distinct scales. This unusual design technique was used in a door panel from the Sultan Bayezid II Kulliyesi in Istanbul (1501-06) [Fig. 270]. The use of differently scaled polygonal modules within a single generative tessellation is conceptually the same as the earlier Seljuk example from the Hekim Bey mosque in Konya (1270-80) [Fig. 269], although the scaling factor of the Ottoman example is considerably larger. While the earlier Seljuk design transitions between *acute* pattern lines in the smaller modules and *median* pattern lines within the larger polygonal modules, the Ottoman example employs *acute* pattern lines within the smaller underlying polygons and *two-point* pattern lines within the larger underlying polygons. Due to the use of the 72° angular openings in the *two-point* pattern line application, the Ottoman example includes five-pointed stars typically associated with the *median* family. In this way, this design contains pattern characteristics of the *acute*, *two-point*, and *median* families within a single construction.

By far the most sophisticated designs to be created from the *fivefold system* in the post Mongol eastern regions are a series of highly complex dual-level patterns produced under Qara Qoyunlu and Timurid patronage. These designs also employ two scales of generative polygons; although rather than the modules transitioning between scales within a single tessellation, these dual-level designs apply a smaller secondary tessellation to an already created primary pattern—thereby creating the secondary pattern within the overall design. The history of this class of Islamic geometric design is examined later in this chapter, and the methodology is detailed in Chap. 3.

Examples of geometric patterns created from the *sevenfold system* that originate from the eastern regions following the Mongol destruction are quite rare. Among the relatively few is a carved stucco relief panel from the Timurid mausoleum of Amir Burunduq at the Shah-i Zinda complex in Samarkand (1390-1420) [Fig. 286b, c]. This *median* pattern can be produced from either of two distinct underlying tessellations. Mamluk artists utilized a very similar generative schema in at least two locations: a 14-*s*2 *obtuse* design

[220] Necipoğlu (1995), diagram no. 8.

[221] Schneider (1980), pattern no. 73.

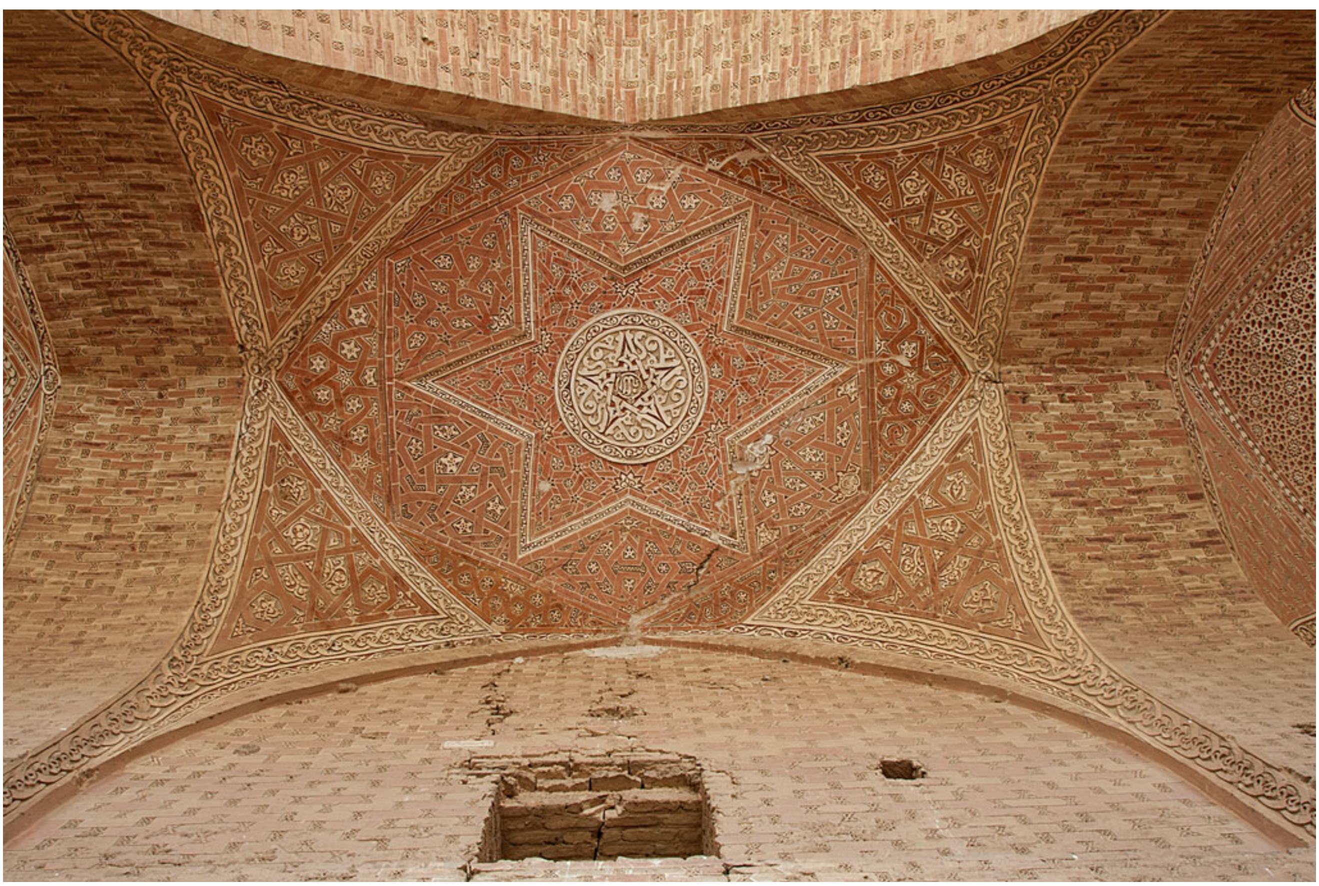
from the Qawtawiyya *madrasa* in Tripoli, Lebanon (1316-26) [Fig. 286a], and a design from the Amir Qijmas al-Ishaqi mosque in Cairo (1479-81) that is identical to the earlier Timurid example. A fine example of 14-*s*3 pattern created from this system was used in several of the deeply recessed blind arches in the courtyard of the Timurid shrine complex of Imam Reza in Mashhad, Iran (1405-18) [Fig. 293c]. The pattern line application to the pentagons and barrel hexagons is analogous to the *median* design within the *fivefold system*. Ottoman artists also produced fine patterns from the *sevenfold system*. These include a door panel from the Bayezid Pasa mosque in Amasya, Turkey (1414-19) [Fig. 287c]. This example uses a subtractive variation that produces a distinctive trefoil motif. This design, without the subtractive treatment, was also used by Mamluk artists in one of the side panels of the *minbar* at the Sultan Barsbay complex at the northern cemetery in Cairo (1432), as well as in the entry door of the Hanging Church (al-Mu'allaqa) in Cairo [Fig. 287b]. Another fine Ottoman example is a 14-*s*6 *acute* design from the incised marble ceiling in the small rectangular water feature within the courtyard of the Suleymaniya mosque in Istanbul (1550-55) [Fig. 289] [Photograph 81].

The significant innovations in creating evermore complex and varied nonsystematic geometric patterns among artists working under Mamluk and Sultanate of Rum patronage was not, for the most part, equaled by artists in the eastern regions following the Mongol destruction. Even with the reestablishment of societal stability during the fourteenth century, the post-Ilkhanid cultures of Transoxiana, Khurasan, Persia, and Iraq, never placed the degree of emphasis upon highly complex geometric design as practiced contemporaneously by their fellow artists in Egypt and Anatolia. It can be assumed that in the wake of the loss of methodological knowledge following the Mongol destruction, the necessary skills for creating highly complex nonsystematic designs were slow to return to these eastern regions. The Ilkhanids and their successors relied heavily upon systematic design methodologies, and the post-Mongol eastern examples of nonsystematic patterns are mostly recreations of existing designs rather than expressive of an innovative spirit. This general de-emphasis toward complex nonsystematic geometric design continued into the

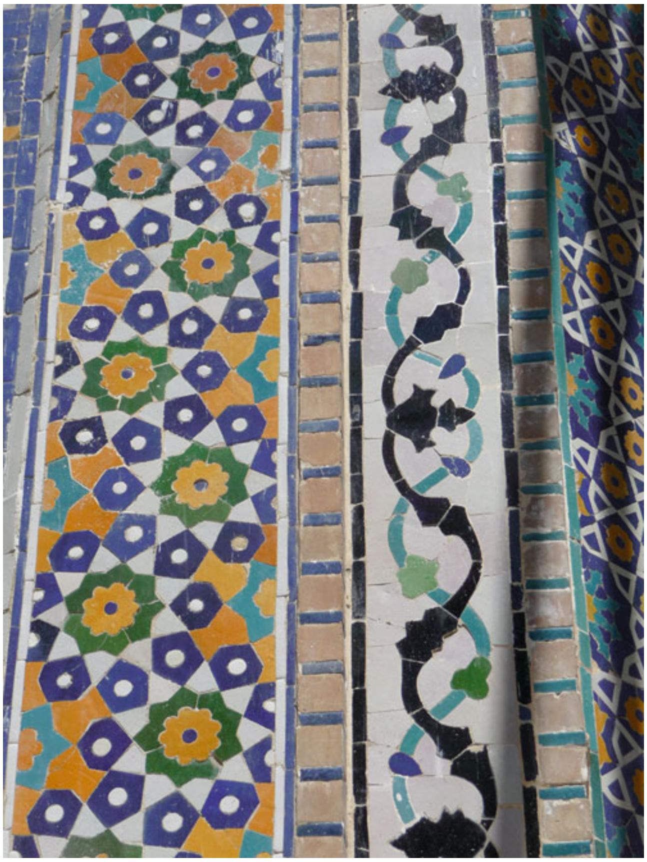
**Photograph 81** An Ottoman incised stone ceiling in a water feature of the courtyard at the Suleymaniya mosque in Istanbul that employs an *acute* pattern created from the *sevenfold system* (© Serap Ekizler Sönmez)

gunpowder dynasties of the Ottomans, Safavids, and Mughals, well after cultural stability and trade throughout these regions had allowed for the aesthetic predilections and artistic practices of neighboring cultures to be more widely known. It is reasonable to speculate that the relative paucity of especially complex nonsystematic patterns in the eastern regions following the Mongol destruction was as much to do with an aesthetic preference for more easily ascertained geometric constructions as with a loss in methodological knowledge. Indeed, these two conditions would appear to be intimately entwined.

Among the previously originated post-Mongol nonsystematic patterns with isometric symmetry are many with a single primary star form, and many with multiple primary star forms. Examples of the former include a *median* pattern with 12-pointed stars in one of the ornate Ilkhanid vaults at the mausoleum of Uljaytu in Sultaniya, Iran (1307-13) [Photograph 82], created from an underlying tessellation of just dodecagons and pentagons [Fig. 300a *acute*]. One of the most remarkable post-Mongol examples of this same design is from a pierced *jali* screen in one of the marble brackets at the tomb of Salim Chishti at Fatehpur Sikri (1605-07). This example is significant in that it overtly includes the generative tessellation within the finished screen. As such, this is an important source of historical evidence for the nonsystematic use of the polygonal technique as the preeminent geometric design methodology. Another isometric example with 12-pointed stars in one of the ceilings at the mausoleum of Uljaytu can be created from either of two underlying tessellations. The first separates the underlying dodecagons with barrel hexagons and places three contiguous pentagons at the center of each triangular cell that are truncated into trapezoids [Fig. 321j]. This design can also be created from the $3.12^2$ tessellation of triangles and dodecagons [Fig. 108d]. A Qara Qoyunlu *obtuse* design created from the same nonsystematic underlying tessellation was used in a cut-tile mosaic wainscoting within the *iwan* of the Imamzada Darb-i Imam in Isfahan (1453). The standard design was modified such that the 12-pointed stars within each underlying dodecagon become 6-pointed stars, providing the visual quality of a field pattern [Fig. 321f]. An almost identical example of this modified design is found at the Sultan Qala'un funerary complex in Cairo (1284-85). Multiple examples of a design the employs nine-pointed stars at the vertices of the hexagonal grid, and six-pointed stars at the center of each repeat unit were used in the eastern

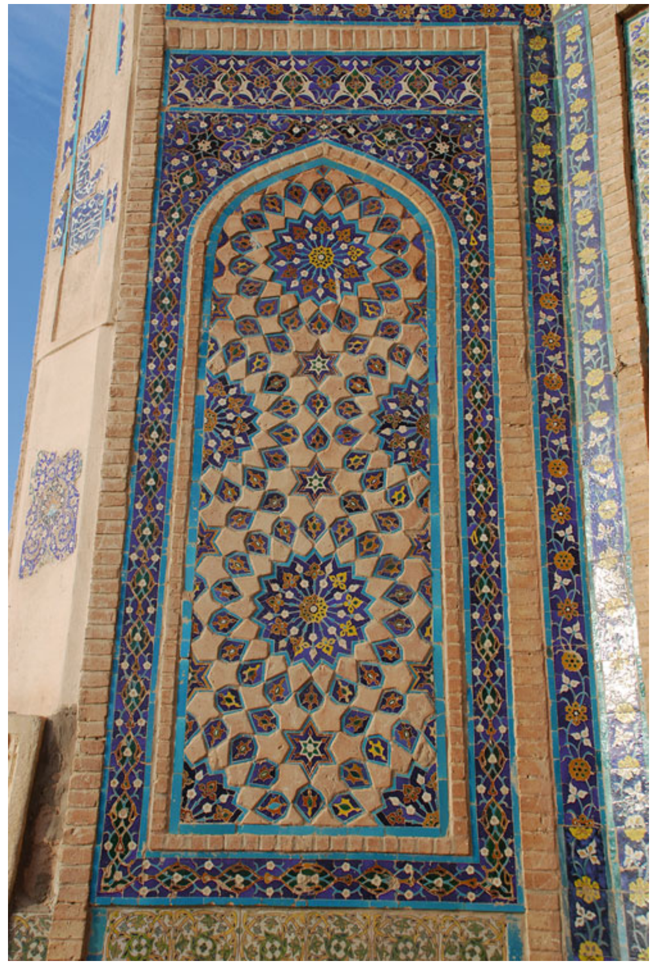


**Photograph 82** An Ilkhanid vault in the mausoleum of Uljaytu in Sultaniya, Iran, with multiple geometric designs (© Daniel C. Waugh)



**Photograph 83** Shaybanid cut-tile mosaic ornament from the Kukeltash *madrasa* in Bukhara, Uzbekistan, with a nonsystematic *obtuse* pattern comprised of six- and nine-pointed stars (© Thalia Kennedy)

regions during this later period. A nonagonal design that places nine-pointed stars upon the vertices of the hexagonal grid was used by Shaybanid artists at both the Kukeltash *madrasa* in Bukhara (1568-69) [Photograph 83], and the Tilla Kari *madrasa* in Samarkand (1646-60) [Fig. 313a]. The underlying generative tessellation for this *obtuse* pattern is the same used by Mamluk artists for creating a *two-point* pattern at the Ashrafiyya *madrasa* in Jerusalem (1482) [Fig. 313b], as well as an *acute* design produced during the Seljuk Sultanate of Rum and located at the Izzeddin Kaykavus hospital and mausoleum in Sivas (1217) [Fig. 313c]. One of the most remarkable isometric designs with only a single primary star form is from a Timurid cut-tile mosaic panel at the Abdulla Ansari complex in Gazargah near Herat, Afghanistan (1425-27) [Photograph 84]. This pattern places 18-pointed stars upon the vertices of each triangular repetitive cell, and is an original design not known to have been used elsewhere [Fig. 322]. The distinctive visual character of this design appears more complex than the rather simple underlying tessellation might suggest.



**Photograph 84** A Timurid cut-tile mosaic panel at the Abdulla Ansari complex in Gazargah, Afghanistan, that employs a nonsystematic threefold *median* pattern with 6- and 18-pointed stars (© Thalia Kennedy)

Inherited nonsystematic isometric designs with more than one region of local symmetry include multiple examples of the most commonly used design comprised of 9- and 12-pointed stars. One of the ceiling vaults at the mausoleum of Uljaytu includes an *acute* pattern of this variety [Fig. 346a]. The *median* pattern created from this underlying tessellation was also used by Mughal artists in an inlaid stone panel from the Friday Mosque at Fatehpur Sikri (1566) [Fig. 346b], and the *obtuse* pattern created from this underlying tessellation was used by Qara Qoyunlu artists at the Great Mosque at Van in eastern Turkey (1389-1400), by Mughal artists at the tomb of I'timad al-Daula in Agra (1622-28), and by Timurid artists at the Abdulla Ansari complex in Gazargah near Herat, Afghanistan (1425-27) [Fig. 347a] [Photograph 85]. A particularly beautiful Qara Qoyunlu *median* design from the stucco ornament of the Great Mosque of Van in eastern Turkey (1389-1400) places 12-pointed stars at the vertices of the isometric grid, and 10-pointed stars upon the midpoints of each triangular edge

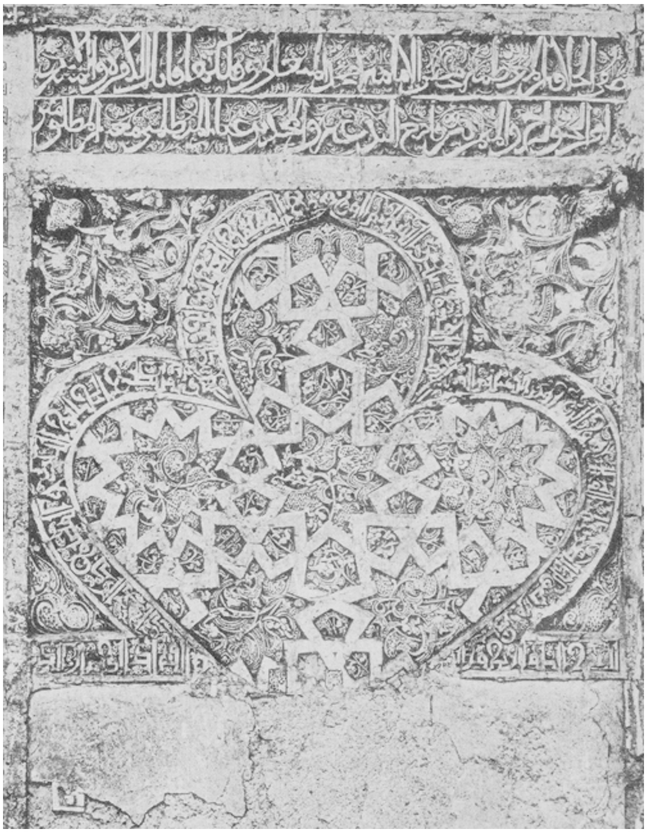
**Photograph 85** A Timurid cut-tile mosaic border at the Abdulla Ansari complex in Gazargah, Afghanistan, that employs a nonsystematic threefold *obtuse* pattern with 9- and 12-pointed stars (© Thalia Kennedy)

[Fig. 363] [Photograph 86]. This example is unique to this location. For the points of the 10- and 12-pointed stars to meet, they must be distorted slightly, causing these primary stars to be non-regular. This is an atypical feature of this design tradition, and could be considered a flaw were it not for the strong visual appeal of this design.

Among the many examples of orthogonal nonsystematic designs with a single primary region of local symmetry used by later Muslim cultures in the eastern regions is an outstanding illumination from the celebrated Ilkhanid 30-volume Quran written and illuminated by ʻAbd Allah ibn Muhammad al-Hamadani in 1313.[222] This *acute* design places 12-pointed stars upon the vertices of the orthogonal grid and octagons at the centers of each square repeat unit, and is created from an underlying tessellation of dodecagons and two varieties of non-regular pentagons [Fig. 335d]. Like the majority of nonsystematic designs of this post-Mongol period, this design had been used earlier: two examples being from the Great Mosque of Siirt in Turkey (1129), and the Mu'mine Khatun in Nakhichevan, Azerbaijan (1186). The same underlying tessellation that created this illuminated example was used to produce a *median* pattern at the Friday Mosque in Kerman, Iran, during the Qarjar period [Fig. 336a]. One of the earliest examples of this popular design is from the Artuqid *mihrab* in the Great Mosque of Silvan in Turkey (1152-57). Another especially beautiful Ilkhanid orthogonal design incorporates tweleve 7-pointed stars that surround the 12-pointed stars placed upon each vertex of the square grid [Fig. 342]. This is a *median* pattern that was used in one of the ceiling vaults at the mausoleum of Uljaytu in Sultaniya (1313-14) [Photograph 87] and does not appear to have been used elsewhere. The underlying tessellation is comprised of 12 non-regular edge-to-edge heptagons that surround each vertex of the orthogonal grid. These successive rings of heptagons create a cluster of four heptagons surrounding a square at the centers of each repeat unit. The application of the pattern lines to this underlying star produces a 12-pointed star that shares an aesthetic treatment with many patterns in the eastern regions.

Preexisting nonsystematic patterns with multiple regions of local symmetry were also frequently employed in the

[222] This Quran is often given the appellation of the Uljaytu Quran. National Museum, Cairo; 72, part. 22.



**Photograph 86** Qara Qoyunlu carved stucco ornament at the Great Mosque at Van in eastern Turkey that employs a nonsystematic threefold *median* pattern comprised of 10- and 12-pointed stars (photo by Walter Bachmann, courtesy of the Aga Khan Documentation Center at MIT)

eastern regions following the Mongol invasion. Among the most common are designs with 8- and 12-pointed stars, and noteworthy examples include two Ilkhanid *obtuse* patterns [Fig. 381b]: one from a ceiling vault at the mausoleum of Uljaytu at Sultaniya (1307-13), and the other from a cut-tile mosaic border in the entry portal of the Gunbad-i Gaffariyya in Maragha (1328). An Ilkhanid variation of this *obtuse* design, also at the mausoleum of Uljaytu, modifies the 12-pointed stars so that they become 6-pointed [Fig. 381e]. This sixfold modification is analogous to the more common convention established within the *fivefold system* [Fig. 224a]. The standard *obtuse* design was also used in the Qarjar compound entry portal of the Aramgah-i Ni'mat Allah Vali shrine in Mahan, Iran, and by Mughal artists at the tomb of Akbar in Sikandra, India (1612). *Acute* examples [Fig. 379] created from this same underlying tessellation include a Muzaffarid cut-tile mosaic panel from the Friday Mosque at Yazd (1324), and a Kartid painted ceiling at the Shamsiya *madrasa* in Yazd (1329-30) that employs an atypical curvilinear treatment within the 12-pointed stars. The Friday Mosque in Yazd also includes a *median* pattern created from this underlying tessellation [Fig. 380b].

Despite the preponderance of nonsystematic designs with earlier origins, the later orthogonal patterns from the eastern regions that have more than one variety of local symmetry include a number of very beautiful examples that appear to be original rather than recreations of earlier work. The In'juid tympanum in the east portal of the Friday Mosque at Shiraz (1351) is decorated with an unusual *median* design comprised of 8- and 12-pointed stars that is created from an underlying tessellation of dodecagons separated by elongated hexagons along the edges of the square repeat unit, and a central array of eight rhombi that collectively create an 8-pointed star at the center of the underlying tessellation [Fig. 384a]. The neighboring Muzaffarids used a variation of this same unusual pattern some 15 years later in a cut-tile mosaic border at the Friday Mosque at Yazd (1365), and there is also a representation of this pattern included in the Topkapi scroll[223] [Fig. 384b]. Considering the relative proximity in time and place, it is likely that there was a direct causal influence of the earlier upon the latter. Another fine example with 8- and 12-pointed stars is a *median* pattern from the Ulugh Beg *madrasa* in Samarkand (1417-20) [Fig. 386] [Photograph 88]. This Timurid cut-tile mosaic panel has several similarities with the previous example from Shiraz: specifically the underlying polygonal origin of the eight-pointed stars within the central regions. However, the underlying tessellation for this design contains many more polygonal elements, and the resulting pattern has significantly greater geometric information within the square repeat unit. This lovely orthogonal pattern shares distinctive characteristics with a fivefold design that was also used at the Ulugh Beg *madrasa* [Fig. 236]. Despite the differences in their respective symmetry, both of these *median* patterns employ regions in their underlying tessellations comprised of four coincident rhombi; both have principle regions of local symmetry that are separated in the same fashion by two underlying pentagons that are rotated so that their vertices are orientated toward the centers of the neighboring primary polygons rather than their edges; and the applied pattern lines within these underlying star forms in both designs have the same atypical visual character. There can be no doubt that the artist responsible for these two exceptional designs employed them within the same building with full knowledge of the geometric concordance between these otherwise disparate varieties of geometric design. A Timurid cut-tile mosaic panel from the main entry *iwan* at the Ulugh Beg *madrasa* in Samarkand (1417-20) uses an orthogonal *acute* design that places 16-pointed stars at the vertices of the square grid and 8-pointed stars at the centers of each repeat unit [Photograph 89]. The underlying tessellation for this design places a ring of pentagons around both the octagon

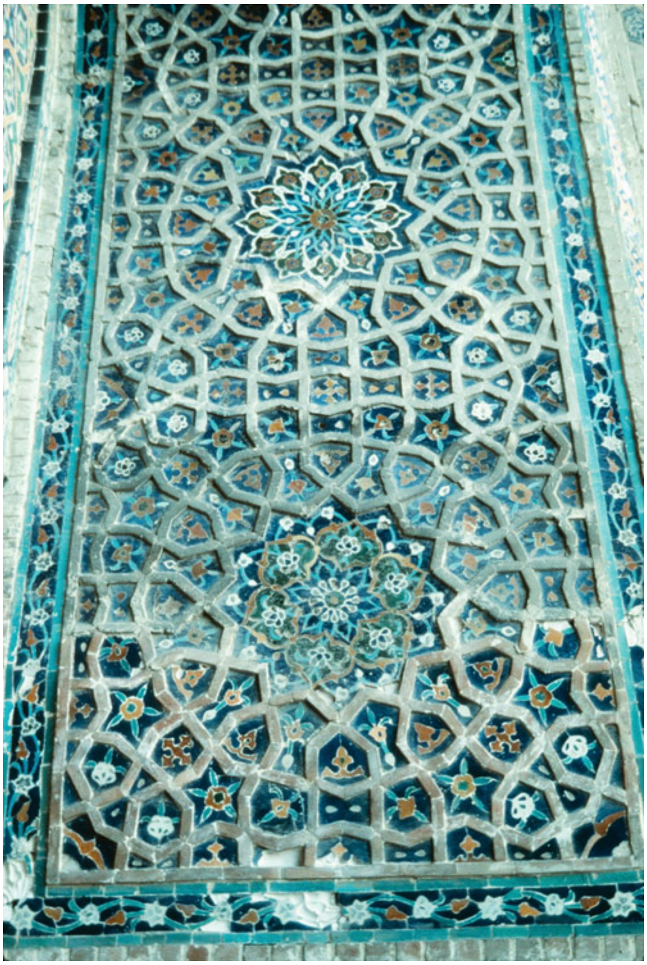
[223] Necipoğlu (1995), diagram no. 72d.

**Photograph 87** An Ilkhanid vault in the mausoleum of Uljaytu in Sultaniya, Iran, with a fourfold nonsystematic *median* pattern comprised of 7- and 12-pointed stars (© Daniel C. Waugh)

and 16-gon [Fig. 389b]. A more complex orthogonal design with 16-pointed stars placed at the vertices of the square grid is illustrated in the Topkapi Scroll.[224] This remarkable *median* pattern incorporates four 13-pointed stars within the field of each square repeat unit [Fig. 398]. The matrix of edge-to-edge polygons that connect the tridecagons (13-gons) and hexadecagons (16-gons) is comprised of barrel hexagons and pentagons.

At least three examples of nonsystematic hybrid designs were produced in the eastern regions following the Mongol invasion. A Jalayirid border design that surrounds the arch and arch spandrel at the Mirjaniyya *madrasa* in Baghdad (1357) is cleverly comprised of both square and triangular repeat units. The geometric patterns within each of these repeat units were well known at the time that this was constructed, but their combined use within a single design was unusual. The triangular repetitive element contains an *acute* pattern created from the underlying tessellation of dodecagons surrounded by a ring of 12 pentagons, three of which are clustered at the center of the repeat [Fig. 300a *acute*], and the square elements contain an *acute* pattern with the same edge configuration of dodecagons and pentagons within its underlying tessellation, and a cluster of four coincident pentagons at the center [Fig. 335b]. This Jalayirid hybrid design was likely inspired by the late Abbasid hybrid pattern in the carved stucco ornament in the entry portal of the Mustansiriyah *madrasa* in Baghdad (1227-34). This earlier Baghdadi example is conceptually similar in its combined use of regular triangular and square repetitive elements to populate the border that surrounds the arched entry. However, the design of the earlier Abbasid example uses different patterns within the two repetitive elements, both of which can be used independently to cover the plane through translation symmetry. The triangular elements conform with the well known *acute* design that is created from and underlying tessellation of dodecagons separated by barrel hexagons, with three pentagons clustered at the center of the triangle [Fig. 321b], while the pattern within the square element is derived from an underlying tessellation that shares the same edge configuration, but includes an octagon at the center of the square that is surrounded by eight pentagons [Fig. 379f]. It is interesting to note that the *acute* hybrid design from the earlier Abbasid example is

[224] Necipoğlu (1995), diagram no. 30.



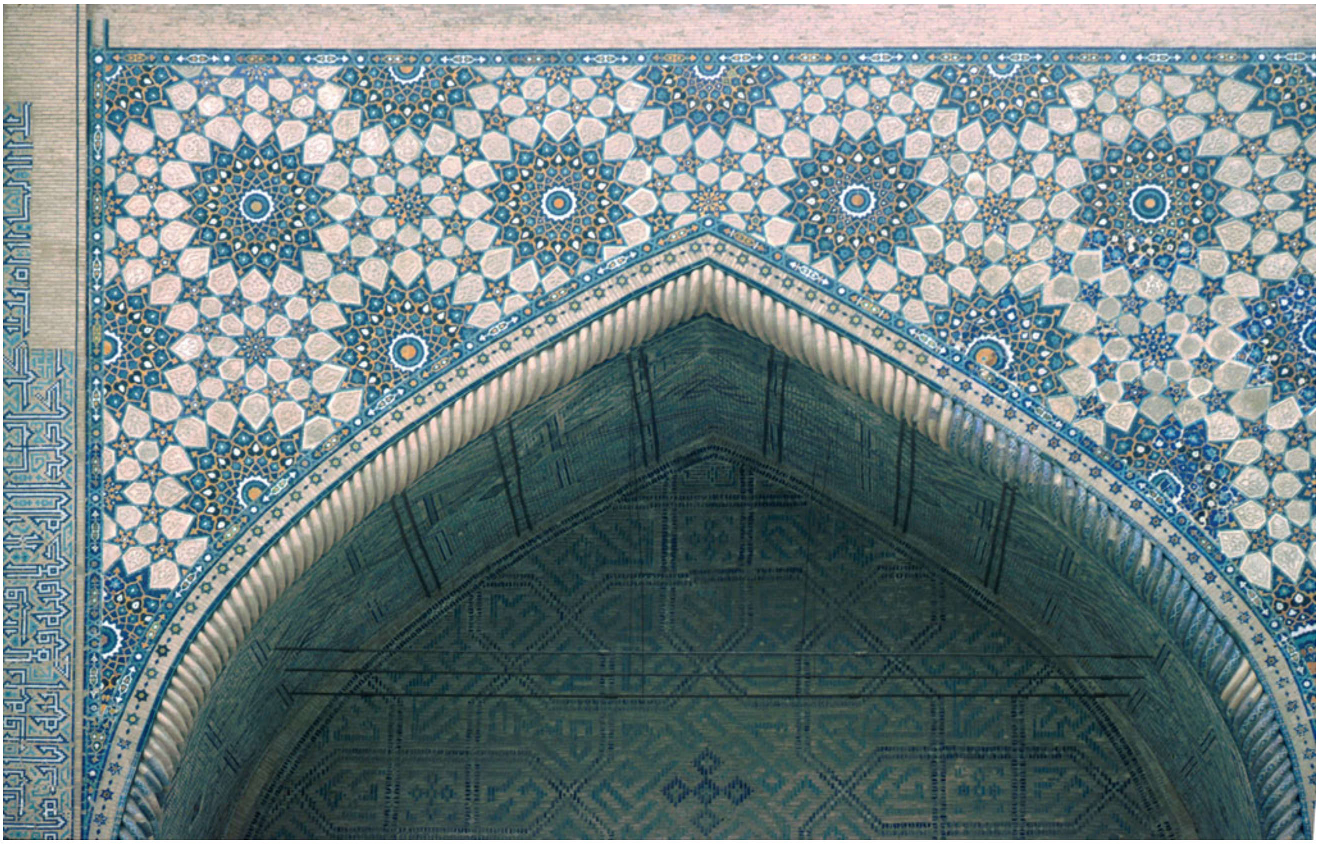
**Photograph 88** A Timurid cut-tile mosaic border at the Ulugh Beg *madrasa* in Samarkand that employs a nonsystematic *median* pattern made up of 8- and 12-pointed stars (photo by Hatice Yazar, courtesy of the Aga Khan Documentation Center at MIT)

essentially identical to one of the other post-Mongol hybrid designs from the eastern regions: an example from the Topkapi Scroll[225] [Fig. 23d]. The only difference between these historical examples is in the angle of the applied pattern lines within the pentagonal elements of the underlying tessellation: the version from the Topkapi Scroll having angles that are more readily associated with the *median* pattern family. The prominent arc that runs through the illustrated example in the Topkapi Scroll suggests the intended use within an arched tympanum. The layout of the triangles and squares follows the $3^2$.4.3.4 semi-regular tessellation wherein mirrored triangles are placed in rotation around each square [Fig. 89]. The third hybrid design from the post-Mongol period in the eastern regions is also found in the Topkapi Scroll.[226] This uses the same arrangement of triangles and squares, but with a much simpler application of pattern lines into these repetitive elements [Fig. 23g]. The pattern contained within the triangle is identical to the most basic isometric *median* design governed by 90° crossing pattern lines [Fig. 95c], while the pattern within the square elements is identical to the classic star-and-cross *median* pattern that is ubiquitous to this tradition. These two elements work together by virtue of their both placing 90° crossing pattern lines at the midpoints of each repetitive module, and when placed together in this fashion, non-regular seven-pointed stars are produced at each vertex of the repetitive grid. The earliest known use of this simple hybrid design is from the Malik mosque in Kerman, Iran (eleventh century).

Although less common than isometric and orthogonal nonsystematic patterns, artists working in the post-Mongol eastern regions produced a variety of noteworthy nonsystematic designs with less typical repeat units. While fewer in number than found in the work of their Seljuk neighbors in Anatolia, the level of beauty and sophistication occasionally rivaled those from the Sultanate of Rum. Unlike the especially complex nonsystematic designs created by Seljuk artist in Anatolia, patterns with greater complexity from the eastern regions rarely have more than two primary varieties of local symmetry. As in earlier Muslim cultures, this variety of design utilizes a diverse range of repetitive schema that includes rectangles, rhombi, and radial symmetries. The use of non-regular hexagons as repeat units does not appear to have been practiced in the post-Mongol eastern regions. The Mughal inlaid stone ornament in the Friday Mosque at Fatehpur Sikri (1566) includes a very beautiful border design comprised of 14-pointed stars that repeats on a rhombic grid.[227] Ordinarily, patterns with these features are created from the *sevenfold system*. However, this design separates the underlying tetradecagons located on each vertex with a square. This arrangement dictates the proportions of the underlying elongated hexagonal and pentagonal elements that complete the generative tessellation; and while these elements work well together to create this lovely *median* pattern, they do not reassemble into addition tessellations, and are not, therefore, part of a systematic methodology.

Some of the most complex post-Mongol eastern nonsystematic geometric patterns are found in the Topkapi scroll. This anonymous scroll, or *tumar*, dated to the fifteenth or sixteenth century, is thought to have originated in central or western Iran, and reflects the ongoing influence of Timurid aesthetics within this region.[228] It is of added

[225] Necipoğlu (1995), diagram no. 35.
[226] Necipoğlu (1995), diagram no. 81a.
[227] Hankin (1925a), Fig. 34, pl. VII.
[228] Necipoğlu (1995), 37–38.



**Photograph 89** A Timurid cut-tile mosaic arch spandrel at the Ulugh Beg *madrasa* in Samarkand that employs a nonsystematic *acute* pattern made up of 8- and 16-pointed stars (© David Wade)

significance in that the artist or artists who produced this scroll frequently illustrated the underlying generative tessellation in addition to the geometric patterns themselves. In many cases the underlying tessellations are overtly illustrated as dotted red lines, and in other cases more subtly indicated with non-inked "dead" lines scribed with a steel point. An *acute* design that repeats upon a rectangular grid places 12-pointed stars at the vertices of the grid and 10-pointed stars at the center of each rectangular repeat.[229] Conversely, the dual of this repetitive grid places the 10-pointed stars at each rectangular vertex, and the 12-pointed stars at the centers [Fig. 414]. This *acute* pattern also appears in the anonymous Persian treatise *On Similar and Complementary Interlocking Figures,*[230] and the earliest known architectural example is the product of Anatolian Seljuk artists working at the Great Mosque at Aksaray (1150-53). One of the Topkapi Scroll designs indicated with bold arcs for use within an arched tympanum employs 8-, 10-, and 12-pointed stars.[231] However, this *median* pattern is poorly conceived, with strained symmetrical relationships between the primary underlying polygons. This creates multiple distortions throughout the underlying polygonal network and, consequently, the resulting geometric pattern. A far more successful complex design—in fact, one of the most remarkable patterns with just two regions of local symmetry in the history of this tradition—has the distinction of being the only known historical example of a design comprised of 9- and 11-pointed stars[232] [Fig. 431]. The repeat for this *acute* pattern is an elongated hexagon that places the 11-pointed stars on each vertex of the repetitive grid. Remarkably, the dual of this grid is also an elongated hexagon, but of differing proportions and orientated perpendicularly. This dual grid has the nine-pointed stars located upon its vertices. Either of these hexagons can equally be regarded as the repeat unit. It is interesting to note that this pattern shares a remarkable correspondence with two other examples from the historical record: the Seljuk border design from the *mihrab* of the Friday Mosque at Barsian, Iran (c. 1100) that employs 7- and 9-pointed stars [Fig. 429d]; and one of the patterns on

[229] Necipoğlu (1995), diagram no. 44.

[230] Bibliothèque Nationale de France, Paris, MS Persan 169, fol. 195a.

[231] Necipoğlu (1995), diagram no. 39.

[232] Necipoğlu (1995), diagram no. 42.

the exterior of the Mu'mine Khatun mausoleum in Nakhichevan, Azerbaijan (1186), is made up of 11- and 13-pointed stars [Fig. 434] [Photograph 35]. Each of these repeats with dual-elongated hexagons with one primary star form placed on the vertices of one hexagonal grid, and the other star form placed upon the vertices of the perpendicular dual-hexagonal grid. What is more, each of these three designs exhibit the *principle of adjacent numbers* wherein the convenience of 8-pointed stars anticipates the example with 7- and 9-pointed stars; the ease of making patterns with 10-pointed stars paves the way for the example with 9- and 11-pointed stars; and the flexibility of designing with 12-pointed stars allows for the example with 11- and 13-pointed stars.

A number of very fine nonsystematic geometric designs with radial symmetry were produced during this period in the eastern regions. Of particular note are a series of designs that fill the flat horizontal star shaped soffits within the outstanding Safavid *muqarnas* vault in the southeast *iwan* of the Friday Mosque at Isfahan. These soffit elements include four-, five-, seven-, eight-, and ten-pointed stars: each decorated with radial geometric patterns that are appropriate to the symmetry of the given star. The geometric design inside the bounding 7-pointed star soffit is an *obtuse* design that places a 14-pointed star at the center of the design, with seven 11-pointed stars placed at the acute included angles of the bounding 7-pointed star. There are partial nine-pointed stars at each of the seven reflex angles of the bounding seven-pointed star. As with the *fivefold system*, this design can be created from either of two underlying tessellations [Fig. 440]. Another soffit in this *muqarnas* ceiling is a bounding ten-pointed star containing a *median* pattern that places a ten-pointed star at the center, surrounded by a ring of 10 seven-pointed stars, with partial ten-pointed stars at vertices of the obtuse angles of the star panel, and partial seven-pointed stars at the reflex angles of the ten-pointed star panel [Fig. 441] [Photograph 90]. The use of 90° crossing pattern lines in association with the radial configuration of seven-, nine-, and ten-pointed stars provides this design with the visual character of a *median* pattern created from the *fourfold system A* [Figs. 154 and 159].

Like their Mamluk contemporaries, post-Mongol artists in Persia, Khurasan, and Transoxiana produced many outstanding examples of domical geometric ornament. And like many of the examples produced by the Mamluks, these utilize radial gore segments as the repetitive units upon

**Photograph 90** Detail of a ten-pointed star soffit from the Safavid *muqarnas* in the southeast *iwan* of the Friday Mosque at Isfahan that employs a nonsystematic radial *median* pattern with seven- and ten-pointed stars (© David Wade)

**Photograph 91** Muzaffarid dome at the Friday Mosque at Yazd, Iran, with a geometric design comprised of ascending 6-, 7-, 6-, 5-, and 4-pointed stars, and culminating in a 16-pointed star at the apex (© Muhammad Reza Domiri Ganji)

which the nonsystematic underlying polygonal tessellations are applied. Among the earlier examples produced after the Mongol devastation is a shallow dome from the Ilkhanid tomb of Uljaytu in Sultaniya, Iran (1313-14), that is an 8-pointed star in plan with a 16-pointed star at the apex surrounded by sixteen 7-pointed stars in the field. This is a very shallow dome and the geometric design merely projects the otherwise two-dimensional pattern onto the slight curvature of the vault. Muzaffarid geometric domes were produced in cut-tile mosaic (*muarak*), and excellent examples include: the main interior dome at the Friday Mosque at Yazd (1324) [Photograph 91] with sixteen half 6-pointed stars at the base, ascending to a ring of sixteen 7-pointed stars, followed by sixteen 6-, sixteen 5-, and finally sixteen 4-pointed stars, with a 16-pointed star at the apex [Fig. 495a]; and a niche hood from this same building in Yazd that transitions the classic fivefold *acute* pattern on the walls of the niche onto a domical surface with a ring of 9-pointed stars, followed by two rings of 7-pointed stars, with an 8-pointed star at the apex. A magnificent cut-tile mosaic geometric dome with gore segmentation was produced by artists during the short lived Sufid Dynasty at the mausoleum of Turabek-Khanym in Konye-Urgench, Turkmenistan (1370). This places twelve half 10-pointed stars around the periphery, ascending to another ring of tweleve 10-pointed stars, followed by a ring of tweleve 9-pointed stars, and surmounted by a 24-pointed star at the apex [Fig. 495b] [Photograph 92]. A relatively simple Muzaffarid geometric design in a quarter dome hood of a niche at the Friday Mosque at Kerman, Iran (1349), places half 8-pointed stars at the base, 5-pointed stars in the field, and a partial 12-pointed star at the apex, and a second quarter dome example at the same building in Kerman places 6-pointed stars at the base and an 8-pointed star at the apex. A rare Qara Qoyunlu geometric quarter dome is found in the hood of an arched niche at the Muzaffariyya mosque in Tabriz (1465). This example utilizes ten- and nine-pointed stars in fine quality cut-tile mosaic. A relatively simple, but powerful Timurid example is found on the interior of the dome at one of the anonymous mausolea at the Shah-i Zinda funerary complex in Samarkand (1385). This has an eight-pointed star at the apex whose lines descend

**Photograph 92** Sufid cut-tile mosaic dome at the mausoleum of Turabek-Khanym in Konye-Urgench, Turkmenistan, with a geometric design comprised of ascending 10-, 10-, and 9-pointed stars, with a 24-pointed star at the apex (© Pete Martin)

into a simple geometric matrix. Later Safavid examples include the late-sixteenth-century decoration on the interior of a dome at the Friday Mosque at Saveh, Iran. This is comprised of a ring of 8 ten-pointed stars at the base, ascending to a ring of nine-pointed stars, followed by more ten-pointed stars, then seven-pointed stars, with an eight-pointed star at the apex. The exterior of this dome is also ornamented with a geometric design: with a ring of half 12-pointed stars at the base, ascending to a ring of 8-pointed stars, 11-pointed stars, 9-pointed star, and a 12-pointed stars at the apex [Fig. 495c]. The renowned geometric design of the exterior dome at the Aramgah-i Ni'mat Allah Vali in Mahan, Iran (1601) [Photograph 93] places half 8-pointed stars at the base, ascending to a ring of 10-pointed stars, followed by 9-pointed stars, 11-pointed stars, 12-pointed stars, and 9-, 7-, and 5-pointed stars; with a 12-pointed star at the apex [Fig. 495d]. The significant distortion in the *n*-fold symmetry of the stars in this design is only a minimal distraction from its great beauty. The Ottoman aesthetic did not generally include the application of geometric patterns onto the surfaces of domes. A rare exception is the exterior dome of the Haydar Khanah in Baghdad (1819-27) that is simply made up of several bands of six-pointed stars. Stylistically, this has more in common with Safavid than Ottoman aesthetics.

The Mughals in the Indian subcontinent also used radial gore segments for decorating a number of their geometric domes. In his praiseworthy early twentieth century article *The Drawing of Geometric Patterns in Saracenic Art,*[233] E. H. Hankin describes the interior geometric decoration of several Mughal domes from Fatehpur Sikri in India (1570-80). Hankin's work is of primary historical interest to the study of Islamic geometric star patterns in that it represents the first European discovery of the polygonal technique as a generative methodology. Hankin concludes his paper with an analysis of several designs that were applied to domes at Fatehpur Sikri,[234] and demonstrates the ingenious traditional technique used by Mughal artists for applying the patterns to domical surfaces. Each of these makes use of the *fivefold system*, and utilizes just eight segments of a tenfold radial geometric design [Fig. 21]. Applying a two-dimensional 1/10 radial segment to a three-dimensional 1/8 domical

[233] Hankin (1925a).

[234] Hankin (1925a), pl. XIII, Figs 45–50.

**Photograph 93** The Safavid dome at the Aramgah-i Ni'mat Allah Vali in Mahan, Iran, with a geometric design comprised of ascending 8-, 10-, 9-, 11-, 12-, 9-, 7-, and 5-pointed stars, with a 12-pointed star at the apex (© Aga Khan Trust for Culture-Aga Khan Award for Architecture/Khosrow Bozorgi [Photographer])

gore segment is an effective means of introducing the beauty of fivefold geometric patterns onto domical surfaces with minimal distortion. Examples of this form of domical design methodology are exclusive to Mughal India, with one notable exception being the large Safavid polychromatic dome at the Mashhad-i Fatima in Qum, Iran (seventeenth century). The exterior cut-tile mosaic decoration of this dome uses the latter truncated technique described by Hankin; although this dome breaks with the regularity of the ten-pointed stars in the uppermost portion of the dome.

## 1.24 Dual-Level Designs

The last great innovations in the tradition of Islamic geometric design were advances in the development of dual-level designs during the fourteenth and fifteenth centuries. This form of design further elaborates a primary geometric pattern with the inclusion of a smaller scaled secondary pattern of the same or similar variety. The Muslim proclivity for dual-level ornament precedes the development of mature dual-level geometric design by many hundreds of years. In addition to geometric design, the dual-level aesthetic found expression in both the floral and calligraphic idioms. Within the floral tradition, dual-level designs reached a high level of maturity during the fourteenth century under Timurid patronage wherein the primary and secondary motifs are differentiated by both scale and contrasting depths of relief. Timurid dual-level floral designs are typically monochromatic by virtue of their being carved from a single material such as wood or marble. Exceptional marble examples were occasionally used on sarcophagi during the fifteenth century, including that of Ghiyathuddin Mansur at the *madrasa* of Sultan Husain Mirza Baiqara in Herat (1492-93), and several from the Abdullah Ansari funerary complex in Gazargah near Herat, Afghanistan (1425-27) [Photograph 94]. Safavid dual-level floral designs are also of particular significance, especially the style that places a secondary floral scrollwork motif at the center of the background of the primary scrollwork design. This form of Safavid floral ornament was commonly carried out in cut-tile mosaic, such as that of the exterior dome of the Mardar-i Shah *madrasa* in Isfahan (1706-14) [Photograph 95]. Calligraphic dual-level examples are primarily architectural, where greater design flexibility and stylistic variation was accepted over the more constrained requirements of Quranic calligraphy. Dual-level calligraphy often places smaller scale *Kufi* script in the upper area of a calligraphic composition so that it runs through the ascending letters, such as the *alif*,[235] of a cursive primary text such as *Thuluth*. A fine Ilkhanid example of this variety of dual-level ornament is found in the carved stucco calligraphic band at the Friday Mosque at Varamin in Iran (1322). Muslim artists began experimenting with dual-level geometric designs as early as the ninth century, and two early examples include a window grille in an arch soffit [Photograph 8] at the mosque of ibn Tulun in Cairo (876-79). Within the geometric idiom, prior to the mature dual-level styles, many of the dual-level designs achieve their secondary component via additive processes. The most sophisticated example among these earlier patterns is the aforementioned fivefold field pattern that surrounds the exterior of the Gunbad-i Qabud tomb tower in Maragha, Iran (1196-97) [Fig. 67] [Photograph 24]. The aesthetics of this dual-level additive pattern anticipates the fully mature style developed approximately 250 years later in the same general region. Other early dual-level geometric patterns that can be

[235] The *alif* is the first letter of the Arabic alphabet. It is an ascender that is made from a single vertical stroke.

**Photograph 94** A Timurid dual-level floral design from a sarcophagus at the Abdullah Ansari funerary complex in Gazargah, Afghanistan (© Thalia Kennedy)

regarded as formative to this tradition include: the ornamental exterior of the minaret of the Yakutiye *madrasa* in Erzurum, Turkey (1310); and an exterior panel from the Ilkhanid minaret of the Qabr Dhu'l Kifl shrine near Hillah, Iraq (1316), wherein a simple threefold geometric design is placed within the triangulated *Kufi* script. Each of these examples is visible from far and near: "allowing for the dynamics of scale to provide travelers with a progressive appreciation of the primary design from a relatively great distance, and the secondary elements upon closer proximity."[236] While calligraphic and floral expressions of dual-level ornament are exceptionally beautiful, they did not significantly enhance the aesthetic importance of these two ornamental modalities within the overall history of Islamic art and architecture. By contrast, the dual-level innovations that were applied to the geometric arts eventually led to an altogether new form of geometric design that is a significant historical addition to the breadth of this ornamental tradition.

The mature style of Islamic dual-level geometric design developed along two distinct historical paths. The earliest occurrence of such patterns was during the fourteenth century in the western regions of Morocco and al-Andalus under patronage from the Marinid and Nasrid dynasties. A century later, fully mature dual-level geometric designs were introduced to the architectural ornament of Transoxiana, Khurasan, and Persia under rival Timurid, Qara Qoyunlu, and Aq Qoyunlu patronage. It is unknown whether these two design traditions developed independently of one another or whether the preceding design methodologies from the Maghreb had a causative influence upon the development of these design conventions in the eastern regions. While the systematic methodology in the creation of dual-level designs from both regions is essentially the same, their respective aesthetic characteristics are very different. When considered from the perspective of Islamic art history, the tradition of

[236] Bonner (2003), 3.

**Photograph 95** A Safavid dual-level floral design from a dome at the Mardar-i Shah *madrasa* in Isfahan (© David Wade)

dual-level design represents the pinnacle of systematic geometric pattern making, and was the last great innovation in the illustrious tradition of Islamic geometric star patterns. As pertains to the history of mathematics, many of the fourteenth- and fifteenth-century Islamic dual-level designs are consistent with the modern geometric criteria for self-similarity: the property of an object or overall structure to have an identical or analogous scaled-down substructure that, in the abstract, is or can be recursively scaled down ad infinitum. These dual-level designs are especially significant in that they appear to be the earliest anthropogenic examples of sophisticated self-similar geometry.[237]

Dual-level patterns invariably employ one of the established generative systems for creating both the primary and secondary designs. As such, historical dual-level patterns always have threefold, fourfold or fivefold symmetry. The artists working with this methodology never applied the *sevenfold system* in creating dual-level patterns, although this generative system is also well suited for such use.[238] The self-similarity within the fully mature tradition of dual-level patterns is of growing interest to contemporary artists, art historians, and mathematicians alike. This remarkable artistic tradition is the direct result of the recursive manipulation of the generative polygonal modules that comprise these modular systems whereby proportionally scaled-down polygonal modules are applied into the structure of the primary design. While self-similar recursive processes are theoretically infinite—be they cosmological, geographical, biological, or anthropogenic—the practical manifestation of self-similar recursion within the arts is constrained by the medium in which it occurs. The historical examples of this variety of Islamic geometric design never exceed a single recursion; with both design levels employing constituent modules from the same set of pre-decorated underlying polygons. In this way, the scaled-down recursive use of the same set of generative polygonal modules, with the same family of pre-applied pattern lines, is responsible for the self-similarity. Can an object be self-similar if it has only a single recursion? The answer is yes, provided that the relationship between both levels satisfies the criteria for

[237] Bonner (2003).

[238] One has to assume the likelihood that the artists who developed the systematic dual-level methodology were unfamiliar with the *sevenfold system* of pattern generation.

–Bonner and Pelletier (2012), 141–148.

–Pelletier and Bonner (2012), 149–156.

self-similarity, and the recursion has the theoretical capacity for infinite scaled-down iteration. The recursive character of Islamic self-similar geometric designs can be identified as substitution tilings that are based upon *n*-inflation symmetry being applied within the primary underlying polygonal tessellating modules. Among the historical examples of Islamic dual-level geometric design, this inflationary process invariably takes place within a repetitive unit cell, and the resulting self-similarity is, therefore, not quasiperiodic,[239] nor is it the product of Penrose matching rules. Rather, the historical examples of self-similarity within this design tradition are comprised of "motifs of different scales [that] resemble each other in style or composition but are not replicas."[240] It is important to note that despite the high level of sophistication, there is no evidence to suggest that the artists responsible for this design tradition had any prescient knowledge or concept of self-similar geometry per se, just as there is no evidence that they were familiar with modern concepts of aperiodicity or quasicrystallinity. That said, the generative and recursive capabilities of the various polygonal systems have tremendous potential for contemporary designers who are interested in producing true aperiodic, quasicrystalline and self-similar designs with multiple levels of recursion.

Not all mature dual-level geometric designs satisfy the criteria for self-similarity. Many examples will use a different family of pattern in the primary and secondary levels. Strictly speaking, this difference in pattern families precludes such examples from qualifying as self-similar. However, the iterative use of the same polygonal tessellating modules at both levels allows for the design methodology to be regarded as self-similar, if not the design itself. Most of the examples of mature dual-level geometric ornament in both the east and the west are architectural, and were fabricated in cut-tile mosaic. A number of examples in the east were also produced in wood and on paper. The examples from the Topkapi Scroll are particularly significant in that they reveal the systematic polygonal methodology behind their construction. In all cases, the fact that the Muslim artists responsible for these masterpieces of geometric art limited themselves to just two levels of design is more to do with the material constraints of their chosen medium than any lack of geometric ingenuity.

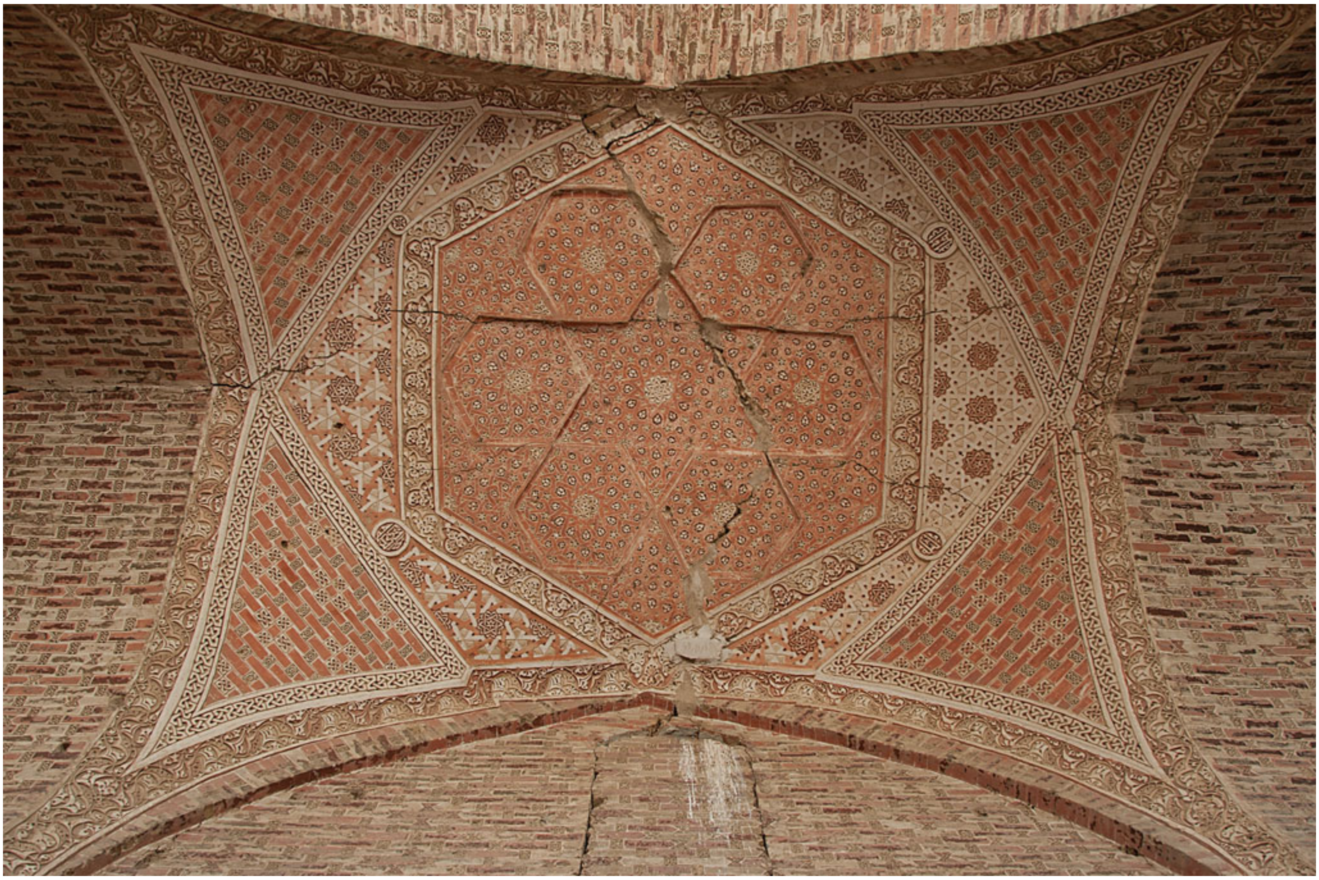
Muslim artists developed four distinct varieties of self-similar design. For purposes of clarification, these are being identified as *types A, B, C*, and *D*.[241] Each of the first three is from the eastern regions, and the fourth is from the Maghreb. *Type A* designs are characterized by the primary design expressed as a bold single line of contrasting color, with the reduced scale secondary pattern filling the *entire background* of the primary design. This variety of dual-level design typically locates scaled-down stars upon the vertices of the primary design. The earliest example of a *type A* design is from one of the Ilkhanid ornamental vaults at the mausoleum of Uljaytu in Sultaniya (1307-13) [Photograph 96]. Indeed, this is one of the earliest examples of a true dual-level geometric design from the eastern regions, and represents the transition toward the fully mature style. The secondary level is a *median* pattern with 10- and 12-pointed stars upon the isometric grid, and the primary design is created by emphasizing through relief selected lines of this grid such that the classic threefold *median* pattern with 60° crossing pattern lines is produced [Fig. 95b]. The use of the 10-pointed stars at given vertices creates problems in the pattern alignment between both levels of the design, and could have been avoided through the use of a more compatible isometric design with, for example, just 12-pointed stars. Outstanding examples of fully mature *type A* designs are found in a wide variety of architectural locations, and significant examples include an Qara Qoyunlu cut-tile mosaic arched panel at the Imamzada Darb-i Imam in Isfahan[242] (1453-54) [Photograph 97] wherein both the primary and

[239] It has been suggested that the Persian artists responsible for a dual-level pattern within an arch spandrel at the Imamzada Darb-i Imam in Isfahan applied quasiperiodic substitution rules while designing this example of dual-level geometric design; and that these artists may have had specific knowledge of the science of quasiperiodicity some 500 years before the discoveries of Sir Roger Penrose in the 1970s. However, the fact that the recursive use of the *fivefold system* of pattern generation *can* be used to create true quasiperiodic designs does not mean that the dual-level use of this system at the Imamzada Darb-i Imam *is* actually quasiperiodic. A rudimentary examination of the cited example reveals that both levels of the overall design repeat within the same rhombic unit cell: the very definition of periodic tiling. The claim to have found quasicrystallinity in the design from the Imamzada Darb-i Imam is based upon overlooking the unit cell in favor of arbitrarily isolating and analyzing limited portions of the overall structure. See Lu and Steinhardt (2007a). See also:
–Makovicky and Hach-Ali (1996), 1–26.
–Saltzman (2008), 153–168.
–Cromwell (2009), 36–56 and (2015).

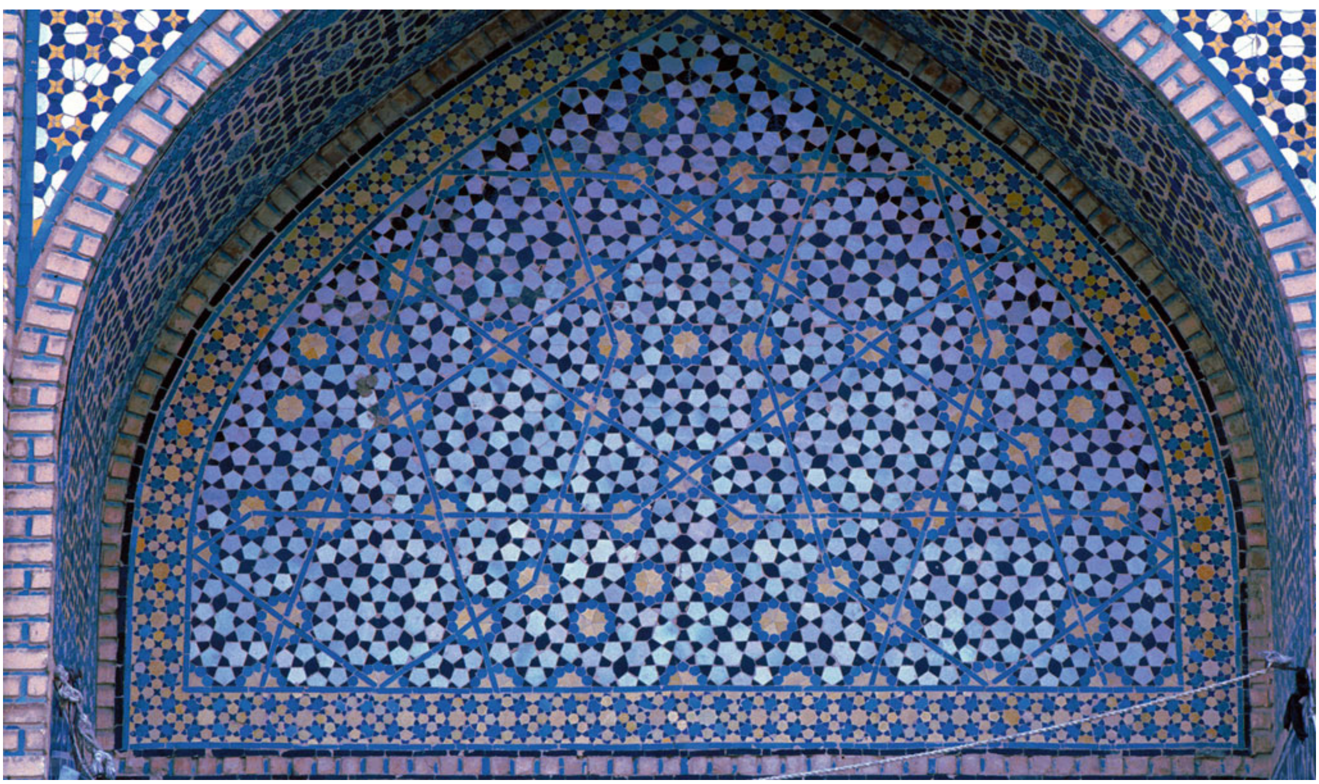
[240] Cromwell (2009), 47.

[241] In an earlier publication the author identified just three varieties of Islamic geometric self-similar design, but has since identified a fourth historical variety as a hybrid of his original *type A* and *type B*. As such, in this work the hybrid form is designated as *type C*, and the former *type C* is now renamed as *type D*. See Bonner (2003).

[242] The Imamzada Darb-i Imam employs a second example of this particular fivefold *type A* dual-level design in a pair of arch spandrels. This is a vastly inferior representation of this fine design, with multiple mistakes in the application of the secondary design. It is also poorly constructed with grossly disproportional polygonal figures, such as the pentagons, in the primary design. Its poor construction and myriad mistakes in the layout of the secondary elements lead one to assume that this was produced by a separate set of artists possibly working at a later date.



**Photograph 96** An Ilkhanid vault in the mausoleum of Uljaytu in Sultaniya, Iran, with a central *type A* dual-level geometric design (© Daniel C. Waugh)



**Photograph 97** A Qara Qoyunlu cut-tile mosaic arch at the Imamzada Darb-i Imam in Isfahan that employs a *type A* self-similar dual-level design that is constructed from the *fivefold system* (© David Wade)

**Photograph 98** A Qara Qoyunlu cut-tile mosaic panel at the Imamzada Darb-i Imam in Isfahan that employs a *type B* self-similar dual-level design that is constructed from the *fivefold system* (© David Wade)

secondary levels are fivefold *obtuse* patterns [Fig. 451]; a Safavid cut-tile mosaic panel from the Madar-i Shah *madrasa* in Isfahan (1706-14) wherein the primary level is the classic fivefold *acute* pattern, and the secondary level is an *obtuse* pattern [Fig. 453]; and an Aq Qoyunlu arch spandrel (c. 1475) at the Friday Mosque at Isfahan. *Type B* designs are characterized by widened primary pattern lines, with an analogous scaled-down secondary pattern placed within the widened primary design. The specific proportion of the widened line is geometrically determined to allow for the application of the secondary polygonal modules, with the primary polygons generally placed at the vertices of the widened lines. The polygonal background regions of *type B* designs are typically filled with either floral or calligraphic motifs. Exceptional architectural examples of this variety of dual-level design include a Timurid cut-tile mosaic border in the southern *iwan* of the courtyard at the Gawhar Shad mosque in Mashhad (1416-18) that is constructed from the *fourfold system A* [Fig. 460]; a Qara Qoyunlu cut-tile mosaic panel from the Imamzada Darb-i Imam in Isfahan that is created from the *fivefold system* wherein the widened primary design is an *acute* pattern, with the secondary infill design from the *obtuse* family [Fig. 463] [Photograph 98]; and an Aq Qoyunlu cut-tile mosaic panel created from the *fourfold system A* at the Friday Mosque of Isfahan wherein the widened primary design of octagons, concave octagons, and four-pointed stars is filled with a secondary design of eight-pointed stars[243] [Fig. 462] [Photograph 99]. A simplified form of *Type B* dual-level design was used frequently during the fifteenth and sixteenth centuries. These utilize either the isometric or the orthogonal grid as the basis for the primary design, and the widened line effect is achieved by isolating selected cells within the grid, and placing predesigned geometric patterns with either triangular or square repeat units into the selected cells. Despite the simplicity of this technique, these dual-level designs can be very beautiful, and especially fine examples include: a threefold Timurid cut-tile mosaic panel from the Friday Mosque at Varzaneh in Iran (1442-44) that places 12-pointed stars upon the vertices of the isometric grid of the primary design [Fig. 457]; a threefold Janid cut-tile mosaic border from the

[243] This may have been produced during the sixteenth century during Safavid rule.
–Necipoğlu (1995), 37.

**Photograph 99** An Aq Qoyunlu cut-tile mosaic panel from the Friday Mosque at Isfahan that employs a *type B* dual-level design that is constructed from the *fourfold system A* (© David Wade)

Nadir Divan Beg in Bukhara (1622-23) [Photograph 100] that places secondary 6-pointed stars at each prescribed vertex of the primary isometric grid and 9-pointed stars within the secondary pattern matrix [Fig. 455]; and a fourfold Aq Qoyunlu[244] cut-tile mosaic panel in the southwest *iwan* of the Friday Mosque at Isfahan (c. 1475) that places 12-pointed stars on the vertices of the rotating kite primary grid. As with some hybrid designs (e.g. Fig. 23), this example has the further quality of combining triangular and square repetitive cells in creating the widened line effect of the rotating kite primary design. *Type C* dual-level designs are essentially a fusion of *types A* and *B* in which the primary design is widened and filled with a secondary design in exactly the same fashion as *type B*, but the secondary design continues to flow into the background regions of the widened primary design, thus filling the entirety of the overall design with secondary patterning much like *type A* designs. Differentiation between the two levels of design is achieved in two ways: through emphasizing the widened lines of the primary design, and through coloring the secondary pattern within the widened lines differently from the secondary pattern inside the background regions. The comparatively few examples of *type C* designs from the historical record include: an outstanding fourfold Muzaffarid cut-tile mosaic panel (1470) over the eastern entry portal of the Friday Mosque at Yazd wherein both the widened primary design and secondary design are created from the *fourfold system A*[245]; a fivefold Safavid cut-tile mosaic arched panel from the Mardar-i Shah in Isfahan wherein the widened primary design is the classic fivefold *obtuse* pattern, and the secondary design is also an *obtuse* pattern [Fig. 468]; and a Shaybanid wooden ceiling at the Khwaja Zayn al-Din mosque and *khanqah* in Bukhara (c. 1500-50) wherein the primary design is a standard threefold *median* pattern with 60° crossing pattern lines that is created simply from the regular hexagonal grid [Fig. 95b], and the secondary design is a simple device that places six-pointed stars at each vertex of the widened primary design. The wooden ceiling's bold

[244] This may date from the Safavid period.

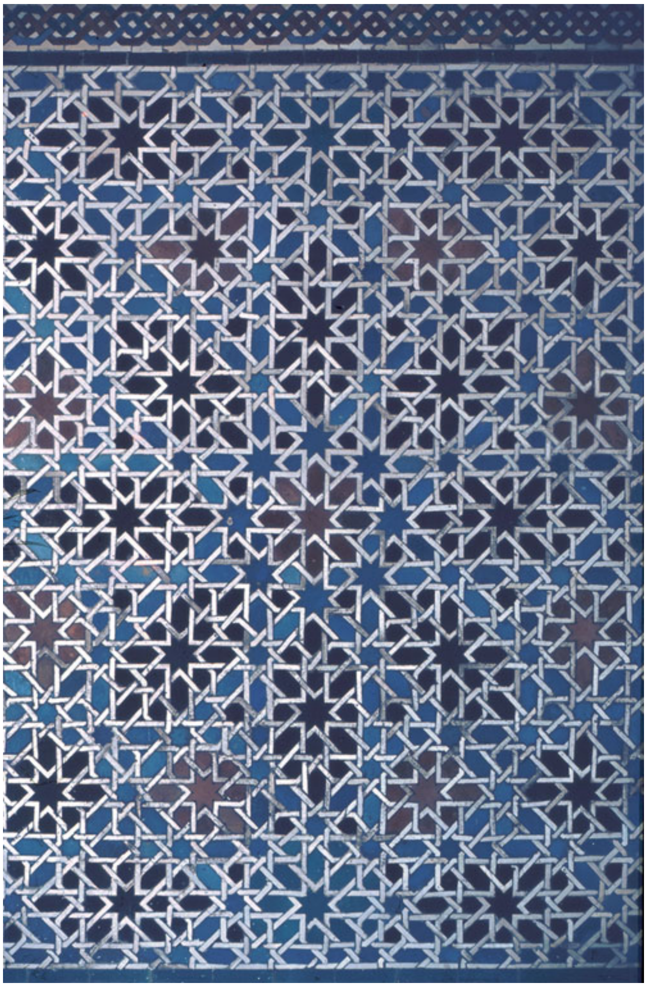
[245] Peter Cromwell's detailed methodological analysis of the *type C* dual-level design in the entry portal of the Friday Mosque at Yazd demonstrates the use of the *fourfold system A* in its creation. See Cromwell (2012a), 159–168.

**Photograph 100** A Janid cut-tile mosaic border at the Nadir Divan Beg in Bukhara, Uzbekistan, that employs a *type B* dual-level design (© David Wade)

relief provides the differentiation between the secondary design in the widened lines and those of the background regions. This is a noteworthy feature of many dual-level designs with widened lines *(types A* and *B*), and was used very successfully at the mausoleum of Uljaytu in Sultaniya, as well as the dual-level designs at both the Friday Mosque at Isfahan and the nearby Imamzada Darb-i Imam. *Type D* designs were developed in Morocco and al-Andalus in the fourteenth century: preceding the mature dual-level traditions in the eastern regions by approximately a century.[246] The designs from the Maghreb are invariably expressed in *zillij*—the Moroccan tradition of cut-tile mosaic. While the basic methodology in creating the two levels through the application of scaled-down polygonal modules to strategic locations of the primary design is the same, this western dual-level tradition differs in the manner of emphasizing the two levels. In the Maghreb, the primary design is expressed exclusively through the contrasting color of the background areas of the secondary design. The secondary pattern in *type D* designs is an interweaving widened line that is given its own distinct mosaic color, typically white, within the overall color scheme. The primary design is differentiated from the secondary design by providing the requisite secondary background elements their own color. Depending on the color of the mosaic pieces that emphasize the primary design, the dual-level quality can be either bold or subtle. The color distribution of the remaining secondary background elements is determined according to the aesthetic predilections of the artist. Almost all of the dual-level designs in the Maghreb are created from the *fourfold system A*, but the *fivefold system* was used in at least two locations. Especially fine examples include a fourfold *Mudéjar* wall panel in the Patio de las Doncellas at the Alcazar in Seville[247] (1364-66) [Fig. 470] [Photograph 101]; a fourfold Nasrid wall panel from the Alhambra in Granada [Fig. 472] [Photograph 102]; and a fivefold Marinid wall panel from the Bu 'Inaniyya *madrasa* in Fez (1350-55) [Fig. 474]. The primary design in this last example is actually an arrangement of decagons that touch corner to corner, separated by interstice regions in the shape of non-regular four-pointed stars. Although the primary design is not a traditional fivefold geometric pattern, the method of highlighting the decagonal design through the background coloring of the secondary design is the same as used in the *type D* fourfold designs of the Maghreb. The use of tessellating decagons as a primary design was also used by the Marinids in a more complex dual-level *zillij* panel from the al-'Attarin *madrasa* in Fez (1323) [Fig. 476] [Photograph 103]. The Marinids and Nasrids were closely allied and artists were known to have traveled across the Straights of Gibraltar to work in both al-Andalus and Morocco. This explains the remarkable unanimity in the architectural ornament of these two cultures generally, as well as the exactitude in stylistic interpretation of dual-level designs more specifically. While this form of architectural ornament survived among succeeding dynasties in Morocco, with few exceptions the dual-level design methodology of the Nasrids did not survive the final *reconquista*, and the post-1492 art of the *Mudéjar* Christians in al-Andalus never reached the level of geometric sophistication as experienced under the courtly patronage of the Nasrids.

[246] The large number of examples of *type D* dual-level patterns at the Alhambra has led Jean-Marc Castéra, a renowned specialist in Islamic geometric art, to refer to this variety of design as the *Alhambra Technique*. See Castéra (1996), 276–277.

[247] –Makovicky and Hach-Ali (1996), 1–26.
–Bonner (2003), 10–11.



**Photograph 101** A *Mudéjar zillij* mosaic panel at the Patio de las Doncellas at the Alcazar in Seville, Spain, that employs a *type D* self-similar dual-level design constructed from the *fourfold system A* (© David Wade)

The mature tradition of dual-level design developed in early fifteenth century Persia under the auspices of the Timurids, and, by mid-century, additional patronage of both the Qara Qoyunlu and Aq Qoyunlu. Stylistically, the ornament of this period falls within the prevailing Timurid aesthetic, and despite political tension between these powerful rival dynasties, at least one artist is known to have received commissions from all three: Sayyid Mahmud-i Naqash.[248] The vast majority of Islamic geometric patterns used as architectural ornament are unsigned. To a large extent this anonymity remains true of the dual-level designs in the eastern regions. However, the significance of Sayyid Mahmud-i Naqash is not just that he is one of the few individuals known by name to have worked within the geometric design tradition generally. His association with the development of the mature style of dual-level designs in the eastern regions is of particular art historical relevance. Relatively few architectural monuments were decorated with this methodologically complex art form, and it appears likely that only a select corps of elite artists possessed the requisite skills to create these dual-level designs. The earliest example of a fully mature dual-level geometric design from the eastern regions appears to be the fourfold *type B* border design in the *iwan* at the Gawhar Shad mosque in Mashhad[249] (1416-18) [Fig. 460]. The earliest known piece to have been signed by Sayyid Mahmud-i Naqash is the threefold *type B* panel from the Friday Mosque at Varzaneh[250] (1442-44) [Fig. 457]. The rarity of these dual-level designs, together with the timeframe of these two Timurid monuments, suggests that Sayyid Mahmud-i Naqash was likely affiliated with the master artist responsible for the work in Mashhad: perhaps as an apprentice working in the atelier that produced the earlier work in Mashhad. His name also appears in the outstanding cut-tile mosaics (c. 1475) of the Friday Mosque at Isfahan. This work was created under the patronage of the Aq Qoyunlu ruler Uzun Hasan. A *type A* dual-level border design in the northwest *iwan* of the Friday Mosque at Isfahan is identical to the earlier unsigned Qara Qoyunlu fivefold design in the arch at the Imamzada Darb-i Imam (1453-54) [Photograph 97]. These buildings are only some 300 m apart and were built within 20 years of one another, and it appears likely that the multiple examples of dual-level design at both these monuments were created by Sayyid Mahmud-i Naqash, or at least artists working within the same atelier or guild. This is supported by the fact that the diversity of dual-level work in both these buildings is of the highest caliber of design and execution, and appears to be the work of a single individual or guild. What is more, the dual-level work of Sayyid Mahmud-i Naqash "deserves recognition not just as a great artist and designer, but also as a pioneer of self-similar geometry some 500 years ahead of his time."[251]

The methodological practices responsible for the remarkable rise in dual-level maturity and sophistication that occurred in the fifteenth century under the guidance of a relatively small number of artists working in Mashhad and Isfahan continued for some hundreds of years in the eastern regions. Yet this distinctive form of design was not widely distributed throughout the monuments of successive dynasties. Rather, additional locations of dual-level panels is limited to only a handful of buildings, including the Darb-i Kushk in Isfahan (1496-97); Khwaja Zayn al-Din mosque and *khanqah* in Bukhara (c. 1500-50); the Nadir Divan Beg in Bukhara (1622-23); and the Madar-i Shah in Isfahan

248 Hutt and Harrow (1979), 61–65.

249 O'Kane (1987), 70.

250 Hutt and Harrow (1979), 61.

251 Bonner (2003), 5.

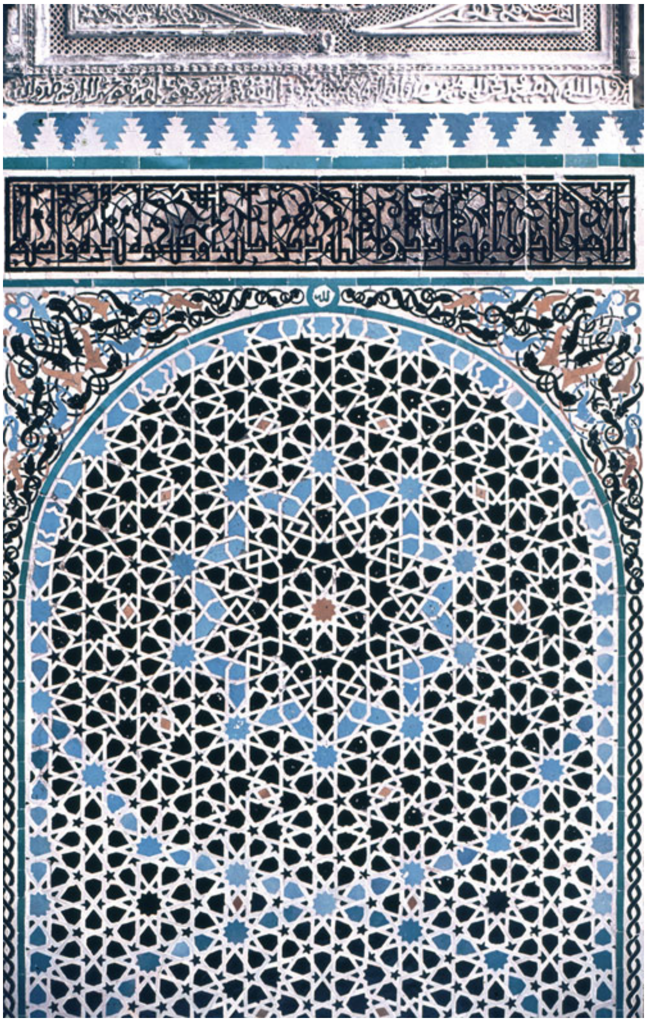
**Photograph 102** A Nasrid *zillij* mosaic panel at the Alhambra in Granada, Spain, that employs a *type D* self-similar dual-level design constructed from the *fourfold system A* (© David Wade)

(1706-14). This suggests that the requisite methodology for creating these dual-level designs was not widely known amongst artists of the period, but was preserved over time through a more restricted master-to-student inherited methodological lineage. It is likely that *tumar* design scrolls contributed to this transference of knowledge. The main purpose of these scrolls appears to have primarily been as an *aide memoire* for master artists. However, it is likely that in addition to serving as a reference manual, these scrolls may also have been used for teaching. As such, they would have been an important facet in the preservation, dissemination and transference of specific patterns, as well as design methodology more generally.[252] These scrolls were made by gluing new sheets of paper onto the end of an already existing scroll, effectively lengthening the scroll with added designs. This additive process allows for the strong possibility that these scrolls were added to over time by successive owners, rather than the product of only a single individual.

Very few *tumar* design scrolls, such as the Topkapi Scroll, are known publicly and available for study by contemporary historians. Of those that are known, and at the time of writing, only the Topkapi Scroll illustrates dual-level geometric designs. Outside the architectural record, the Topkapi Scroll is the largest and most important repository of dual-level design from the eastern regions. Because the dual-level designs in the Topkapi Scroll also show the underlying generative tessellations (as overt solid or dotted lines differentiated by color, or by more subtle non-inked "dead" lines scribed into the surface of the paper with a steel graver) this document is exceptionally important as the only historical evidence of the polygonal design methodology behind the creation of these designs. The Topkapi Scroll illustrates seven dual-level geometric designs. Five of these are *type A* designs, and two are *type B*. Several of these exhibit the qualities of self-similarity and are the equal in design ingenuity to the architectural examples produced by Sayyid Mahmud-i Naqash. The *type A* designs are all fivefold (nos. 28, 29, 31, 32, and 34)[253] and follow the same formula of utilizing *median* patterns at both the primary and secondary levels that was used by Sayyid Mahmud-i Naqash

[252] Necipoğlu (1995), Chap. 1.

[253] The author is using the diagram numbers for each separate design as attributed by Gülru Necipoğlu: See Necipoğlu (1995).



**Photograph 103** A Marinid *zillij* mosaic panel at the al-'Attarin *madrasa* in Fez, Morocco, that employs a *type D* dual-level design constructed from the *fivefold system* (© David Wade)

at the Friday Mosque at Isfahan, and later at the Imamzada Darb-i Imam [Photograph 97]. This recursive similitude qualifies these five designs as self-similar. Only design no. 28 expressly represents—in dotted red lines—the underlying generative tessellation. The underlying generative tessellations for designs 29, 31, 32, and 34 are indicated by un-inked scribed lines that are only visible through close inspection. Design nos. 38 and 49 are *type B* designs: the former comprised of threefold symmetry, and the latter of fivefold symmetry. With the added element of color as part of the composition, design no. 38 is the most visually arresting. This colorization may indicate an intended application to cut-tile mosaic. This example is the more simple variety of *type B* design wherein the widened pattern line is produced from a tessellation of repetitive polygonal cells: in this case triangles, squares and hexagons [Fig. 458]. The secondary infill pattern is applied to just the triangles and squares, with the hexagons being open background elements. The geometric designs that are applied to both these repeat units were well known throughout Muslim cultures. The nonsystematic threefold *median* pattern made from an underlying tessellation comprised of a ring of 12 pentagons surrounding the dodecagons [Fig. 300b *median*], and the nonsystematic fourfold *median* pattern created from an underlying tessellation of dodecagons and octagons separated by pentagons and hexagons [Fig. 379f]. The use of these two repeat units is also represented in the single-level hybrid *median* design no. 35 from the Topkapi Scroll [Fig. 23d–f]. The method of transitioning from the interior pattern of decorated triangles and squares with open hexagons to the surrounding rectangular border is both clever and beautiful [Fig. 459a]. Similar formulae for bordering *type B* and *type C* designs were commonly employed within the architectural record. The no. 49 *type B* fivefold design in the Topkapi Scroll is a magnificent example of Islamic dual-level design, and, indeed, the most complex *type B* design from the historical record [Fig. 465]. The surrounding rectangular border is resolved in the same fashion as the previous threefold *type B* design from this scroll, albeit with rectangular repeat units rather than squares and triangles [Fig. 466d].

## 1.25 The Adoption of Islamic Geometric Patterns by Non-Muslim Cultures

Throughout its long and illustrious history, the evolution of Islamic ornament into its many and varied branches has been greatly influenced by the artistic conventions of non-Islamic cultures. The genius of Muslim artists to assimilate and reorient foreign design elements into their own distinctive ornamental tradition can be traced back to the earliest Islamic period. For example, the application of the Hellenistic geometric compass-work technique to the pierced stone window grilles of the Great Mosque of Damascus was an early Islamic innovation of great visual impact, as well as of continuing influence to subsequent Islamic cultures. Similarly, the highly stylized carved stucco vegetal ornament of Samarra appears to have been influenced by earlier Hellenistic and Sassanian vegetal motifs[254]: and as with pierced window grilles, the stucco design innovations of Samarra were to have a lasting influence on Islamic ornament well into the fifteenth century. Many centuries later, the Mongols introduced Chinese and Indian design motifs into the ornamental vocabulary of Islamic artists and designers.

Just as Islamic cultures were able to assimilate many of the artistic and architectural conventions of non-Muslim

[254] Allen (1988), 1–15.

peoples with whom they had close contact; it was inevitable that the rich and varied beauty of Islamic ornament should, in turn, become an influence to the art and architecture of many non-Muslim cultures. Non-geometric examples of such influence include the Tulunid form of the Samarra beveled style being used in the architectural ornament of the Egyptian Coptic community. Most notably, this style was employed in the Coptic monastery of Dayr as-Suryani (914) in the Wadi Natrun.[255] A strong late Fatimid influence is evident in the *muqarnas* vault in the Cappella Palatina in Palermo, Sicily (c. 1140). This church was built for Roger II, the Norman king of Sicily, and is also remarkable for the use of *Kufi* calligraphy, and distinctively Fatimid stylization of the painted human figures that adorn the *muqarnas*. But for the use of human figures, this Sicilian *muqarnas* parallels contemporaneous examples from Egypt.

In Spain, where Muslim, Christian, and Jewish communities lived side by side for many centuries, the degree of cultural interaction was to have a profound effect upon both the Christian and Jewish artistic practices of the region. Islamic geometric and floral patterns were freely used by Christians and Jews alike. Except in the use of Hebrew rather than Arabic, the Jewish synagogues in al-Andalus were stylistically Moorish in every respect. Only a few have survived relatively untouched by later Christian acquisition. Both the Santa Maria la Blanca (1180) and Nuestra Señora del Transito (1360) in Toledo were originally synagogues. The Christian use of Hispano-Moresque ornamental devices was mostly the work of *Mudé jar* artists and craftsmen. These were Muslims who lived in areas under Christian control. *Mudéjar* art also refers to the continuation of Islamic ornament after the final surrender of the Nasrids in Granada in 1492, and the expulsion of virtually all remaining Muslims and Jews from Spain. At its best, this highly influenced form of Christian architecture is often indistinguishable from the work from contemporaneous Arab patrons. Many magnificent buildings were built by *Mudéjar* craftsmen; two very noteworthy examples being the Palace of the Alcazar in Seville (1364-66), and the Convento de la Concepcion Francisca in Toledo (1311). Of particular note is the wooden geometric dome in the Hall of the Ambassadors at the Palace of the Alcazar in Seville [Photograph 67]. This distinctly Islamic styled geometric dome is the work of Diego Roiz, a Christian *Mudéjar* artist who was clearly well versed in the Islamic geometric idiom. To a limited extent, the *Mudéjar* use of Islamic geometric patterns even made its ways to the New World. An outstanding example is from the entry door of the Cathedral of Santa Domingo in Cusco, Peru (c. 1560-1654). This employs the classic nonsystematic pattern that places 12-pointed stars upon the vertices of the isometric grid, and 9-pointed stars at the center of each triangular repeat. The Christian tradition of early Spanish manuscript illumination was particularly influenced by Islamic work. Rather than being carried out by *Mudéjar* craftsmen, the work of the Leonese School of manuscript illumination was created by Christian monks who, through prolonged close cultural contacts, were greatly influenced by Islamic artists. This style of highly influenced Christian art is referred to as *Mozarab*; from the Arabic word *mustarib*, which translates as Arabized. The general layout, and especially the interweaving geometric border designs of the Moralia in Iob, written in the monastery of Valeranica in 945, is a good example of the Islamic influence upon Mozarab art.

Just as the Seljuks in Anatolia were influenced by the stone masonry traditions of Armenian Christians, the Islamic geometric and floral ornament of the Seljuks had a very distinctive reciprocal impact on the carved stone ornament of the Armenians. This is especially apparent in the remarkable tradition of *khachkar* stela. These are large rectangular stone obelisks, at least twice their height as width, that invariably employ a central cross in deep relief as a primary motif. The cross is often winged and resting upon a circular rosette, and framed in a border of geometric and floral designs. (The reverse sides are provided with inscriptions.) *Khachkars* were presented to the church by patrons and benefactors in commemoration of a person or event, and as a means of securing religious favor. In such circumstances, these monuments were often set into the walls of churches. *Khachkars* also served as grave markers, and were set upon tombs in churchyards or exposed to the elements in open fields. It is noteworthy that the large number of Ahlatshah Muslim tombstones in Ahlat, Turkey are of the same approximate size and shape as the nearby Armenian *khachkars*, and the ornamental treatment of these tombstones is remarkably similar—except for the absence of any figurative elements, and Christian symbols. This similarity suggests the possibility of a reciprocal influence between these cultures. The Ahlatshahs were an Anatolian Turkish *beylik* closely allied with the Great Seljuks of Persia who ruled the region northwest of Lake Van that bordered on Armenia during the twelfth century. The tradition of *khachkars* developed during the second half of the ninth century, at a time when the Armenians had won back their independence from the Abbasid Caliphate. However, during the period of Seljuk dominion over Anatolia, the Armenian tradition of ornamental stone carving took on distinctively Seljuk characteristics. The nature of the geometric and floral ornament that was used on the *khachkars* of this period is, in many respects, identical to that of their neighboring Seljuk rivals to the south. The interweaving knotted borders, simple geometric field patterns, and meandering floral designs found on these monuments could easily be mistaken for the work of Muslim

[255] Kuhnel (1962), 58.

artists. For example, the floral and geometric patterns on the outer frame of the thirteenth century Siroun *Khachkar* from Toumanian are classically Seljuk in stylization, as is the floral rosette beneath the cross, and the *bandi-rumi*, or Anatolian knot-work, in the crown. One of the few Armenian ornamental devices that is stylistically distinct from Seljuk work is the occasional transformation of the interweaving lines of the geometric pattern into the floral design. The *khachkar* tradition is regarded as having reached its stylistic perfection in the thirteenth and fourteenth centuries in the Sunik and Azizbekov regions of Armenia. The work of one man in particular is of especial importance. Momik (1282-1321) was a renowned scribe, painter, architect, and sculptor, and was responsible for some of the finest *khachkars* found within this tradition. The church in Noravank has an especially refined example of his work that prominently features a very complex nonsystematic geometric pattern made up of 8-, 10-, and 11-pointed stars [Fig. 423]. This pattern approaches the sophistication of the geometric work produced during the same period by neighboring artists working for the Seljuk Sultanate of Rum. The geometric sophistication of this pattern indicates that Momik likely received training from Muslim artists in the traditional methods of constructing complex geometric patterns, despite his being a Christian. It is worth noting however that this design is not without problems. Generally, the variable size and distribution of the background elements are unbalanced, causing regions of greater and lesser pattern density. Of particular concern are the distorted geometric rosettes that surround each 11-pointed star.

Throughout the Islamic world, many non-Muslim minority communities adopted the ornamental conventions of the Muslim culture they lived in. The Coptic churches of Cairo frequently employ Islamic geometric patterns in their architectural ornament. For example, the Hanging Church (al-Mu'allaqa) dedicated to St. Mary is replete with a diversity of geometric designs, as well as floral motifs and *muqarnas* that is stylistically identical to the contemporaneous work in the mosques of Cairo. The architectural ornament of the Armenian Christian community in the New Julfa district of Isfahan fully embraces the practices and aesthetics of the Safavids, including the wide use of geometric patterns. As in Spain, synagogues in cities such as Fez, Tunis, Cairo, Baghdad, Istanbul, Isfahan, Kabul, and Samarkand frequently employ Islamic geometric patterns in their ornamentation.

In India, the architectural style of the Mughals had a tremendous influence on Hindu and Sikh architecture. In particular, the Hindu Rajput princes in Rajasthan were greatly influenced by Mughal courtly life, and freely adopted Mughal customs and practices. The art and architecture of the Rajputs was virtually a complete abandonment of earlier Hindu forms in favor of the Mughal style. Except for the fact that they were built by Hindu rulers, cities such as Udaipur, Jodhpur, Jaipur, and Jaisalmer are essentially Mughal in conception. And much like the later ornament of the Mughals themselves, the eighteenth and nineteenth century Rajput architecture and ornament tended toward an overabundant decadence. The Sri Harmandir Sahib in Amritsar is the principal temple of the Sikh religion. This three-story building sits upon an island in the center of a reservoir, and the architectural style is strongly derivative of late Mughal work. The Hindu and Jain adoption of Mughal architectural standards included geometric design, most notably in their use of pierced stone *jali* screens.

A number of European artists have shown an interest in Islamic design. Both Leonardo di Vinci (1452-1519) and Albrecht Durer (1471-1528) produced remarkable meandering concatenations: pen and ink rosettes of complex interweaving lines that are highly reminiscent of Islamic floral designs.[256] Hans Holbein (1497-1543) incorporated his studies of arabesque ornament into his paintings. In the nineteenth and early twentieth centuries numerous studies and collections of Islamic geometric and floral ornament were assembled by European scholars for the purposes of inspiring the ornamental arts of Europe. These include: Jules Bourgoin, Prisse d'Avennes, Owen Jones, E. Hanbury Hankin, and Archibald H. Christy. These collections were an integral aspect of the nineteenth-century Orientalist movement. More recently, the twentieth-century Dutch artist M.C. Escher (1898-1972) was greatly influence by the geometric ornament he encountered while visiting the Alhambra in Spain. Truly, the geometric ornament of Islamic cultures served as a source of inspiration for countless generations of artists, designers, and craftsmen throughout the Islamic world and beyond.

## 1.26 The Decline of Islamic Geometric Patterns

The gradual decline in the use of Islamic geometric pattern began with the three modern era Muslim empires: the Ottoman Turks, Persian Safavids, and Indian Mughals. Each of these cultures continued to employ geometric designs in their art and architecture; yet the spirit of innovation was substantially lost. With several notable exceptions, such as the geometric domes of the Safavids and Mughals, geometric pattern construction became highly derivative of previous work. During the sixteenth and seventeenth centuries, floral ornamentation was progressively given far greater emphasis over geometric design in each of these three empires. The reason for this aesthetic shift is unclear. What

[256] Coomaraswamy (1944), 109–28.

is certain is that creative vitality within the geometric idiom was highly reliant upon the very specific methodological knowledge of the polygonal technique. As patrons increasingly favored floral ornament, those artists knowledgeable of the polygonal technique would have found less work, and fewer apprentices to carry this tradition forward. Over time, this break with the past regrettably led to the inability of Muslim artists to create new and original geometric patterns, eventually relegating geometric patterns to the mere making of copies.

The early Ottoman use of geometric patterns continued the tradition inherited from the Seljuks, although in a less grand scale and with less geometric sophistication. By the end of the fifteenth century the Grand Ottoman style, exemplified in the works of Sinan, utilized the ornamental quality of geometric patterns to a far lesser extent. The emphasis of the floral idiom by the Ottoman Turks seemed to know no bounds: with textiles, metalwork, leatherwork, bookbinding, stained glass, painting and illumination, carved ivory, jewelry, and inlaid woodwork all receiving the prodigious floral talents of the Ottoman artists. Nevertheless, many fine examples of geometric design were created during the sixteenth century, albeit mostly derivative of earlier work. Inlaid and joined wood were especially popular media for their applied geometric designs, as seen in many of the finest examples of Ottoman *minbars*, doors, and furniture of this period.

Being from the same region as the Seljuk Sultanate of Rum, it is surprising that Ottoman artists were not more inclined to adopt the sophisticated nonsystematic design methodologies used in Anatolia by their predecessors. Yet the Ottoman use of geometric patterns tended toward more conventional designs with less symmetrical ambiguity. This would appear to be due to an aesthetic predilection toward more easily ascertained geometric structures that are less visually demanding. And as stated, it is also possible that knowledge of the more complex methodologies was not transferred to subsequent generations of artists. Of the relatively few examples of more complex Ottoman geometric design, the side panels beneath the platform of the wooden *minbar* at the Great Mosque of Bursa (1396-1400) is particularly significant. The long vertical panel adjacent to the wall is made up of an *acute* pattern with five vertically arranged stars from top to bottom in the following order: a 10-pointed star at the top, followed by another 10-pointed star, followed by two 9-pointed stars, and an 11-pointed star at the base. This design does not have a repeat unit per se. Rather, the underlying generative tessellation is arranged top to bottom without a satisfactory resolution at the edges, thus causing the pattern to be cut off in a fashion that would not repeat nicely were the panel repeated horizontally or vertically. This is an atypical feature that suggests the artist was less skilled than previous artists who produced patterns with multiple regions of differentiated symmetry. The triangular side panel of the *minbar* at the Great Mosque of Bursa employs an orthogonal *acute* pattern that places 12-pointed stars at each corner of the square repeat unit, an octagon at the center of the repeat unit, a 10-pointed stars at the midpoint of each edge of the repeat unit, and 9-pointed stars within the field of the repeat unit [Fig. 400]. This same orthogonal design was used at the Kayseri hospital in Kayseri, Turkey (1205-06), the Agzikara Han near Aksaray, Turkey (1231-40), and the Çifte Kumbet in Kaysari (1247). However, in the later Ottoman example of this design the artist choose to replace the original octagons at the center of each square repeat unit with eight-pointed stars. This design variation is not the product of an adjustment to the underlying generative tessellation, and the arbitrary replacement of the eight-pointed stars into this central location is consequentially forced and clumsy in appearance. These two examples from this minbar indicate that Ottoman artists, for all their brilliance in other artistic disciplines, were not equally skilled in the methodology of more complex geometric patterns with multiple regions of differentiated symmetry.

The earliest significant Mughal building is the tomb of Humayun in Delhi. This beautiful building was constructed in 1560, and combines Persian elements with distinctive Indian influences. This combination of influences created a bold new architectural style. In time, Mughal architecture developed into one of the most refined and beautiful traditions within the Islamic world. Mughal architecture reached its full maturity during the reign of Jalal al-Din Akbar (1556-1605). His fortress/palace complex at Fatehpur Sikri, built between 1570 and 1580, is one of the great Islamic monuments in all of India, and is of equal importance and beauty to the other two great Islamic fortress/palaces complexes which have survived to the present day: the Alhambra in Spain, and the Topkapi in Istanbul. The most celebrated Mughal building is the Taj Mahal in Agra. This building was built by Shah Jahan between 1632 and 1647, and is the tomb of his wife Mumtaz. Like that of the earlier Indian Sultanate period, the architectural decoration of the Mughals was mostly stone. The Mughals introduced several important ornamental elements from Persia; including more complex geometric patterns, star vaulting, and distinctive floral styles. One of the primary Mughal uses of geometric patterns was in their pierced stone *jali* screens. These were made from either marble or red sandstone. Pierced stone screens have always been a popular form of geometric ornament within Islamic cultures; and, as mentioned earlier, many examples exist from the earliest Islamic period. The Mughals refined this tradition to a remarkable degree. Of particular interest to the question of design methodology are the aforementioned *jali* screens that employ the underlying generative tessellation along with the geometric pattern that the tessellation creates. As discussed previously, the most innovative Mughal application of geometric patterns was to the interior surfaces of domes. Yet the variety of geometric patterns that were most commonly used by Mughal artists were not particularly complex, and with rare exceptions were already well known in Persia and Transoxiana.

The last great Persian architectural and ornamental traditions were those of the Safavid dynasty. This was an important time in the history of Persian culture. It was during this period that Shia Islam became the official religion of Persia. The Safavids are descendants of Shaikh Safi al-Din (d. 1334), the founder of the Safawiyya Sufi order in Azerbaijan. The Safavid dynasty was founded by Shah Ismail who lived between 1501 and 1524. He was a popular leader who united Persia under a single leadership. Shah Ismail claimed to be a direct descendent from Ali, the son-in-law of the Prophet Mohammad, providing Shah Ismail and all subsequent Safavid rulers a religious authority that was strongly embraced by his Persian subjects. The architectural ornament of the Safavids is primarily characterized by the abundant use of floral designs and calligraphy. The preferences for floral and calligraphic ornament notwithstanding, many exceptional examples of geometric domes and panels were produced by the Safavids.

Following the fall of the Safavids in the first half of the eighteenth century, Persia, Afghanistan, and Transoxiana came under the rule of several rival dynasties, including the Qarjars, Durannis, and Uzbeks. The architectural ornament of the Qarjars and the Shaybanid Uzbeks in particular was strongly influenced by the work of their Timurid predecessors, and the use of geometric patterns took on greater emphasis than during the Safavid period. However, knowledge of the methodology for the polygonal technique appears to have been lost, as the geometric work of these cultures is, at best, derivative of earlier geometric design. Most of this ornament is undertaken in ceramic tile, and as with the late Ottoman use of ceramics, the color palette was radically altered by the introduction of European ceramic colors that were unknown in this region previously. Adding to this change was an emphasis on more naturalistic floral designs such as found at the Vakil mosque in Shiraz (1766). The architectural decoration of the Qarjars is, generally, of much poorer quality in design and technique than the work of earlier Persian traditions. The baroque-inspired floral designs, as well as the new color palette, give many Qarjar buildings an overworked decadent quality. The Uzbek architectural focus upon continuing the Timurid aesthetic was more successful than the parallel attempts by the Qarjars. This was dealt a crippling blow with their defeat and occupation by the Russians in the nineteenth century.

By the nineteenth century, the unfortunate decline of Islamic geometric patterns had become irreversible throughout the Islamic world. In addition to the huge areas under Turkish, Persian, and Indian influence, the Islamic regions of Central Asia and North Africa also lost the vitality of this tradition. The history of the decline of geometric patterns in Morocco is similar to that of Uzbekistan. Although the architectural decoration of Morocco has continued to use geometric designs up to the present day, Moroccan artists and designers also lost their skills in creating new and original geometric patterns from the polygonal technique. The architectural record indicates that the significant decline of this methodological tradition in Morocco was well advanced by the eighteenth century. As in other Muslim cultures, most Moroccan geometric art of the eighteenth and nineteenth centuries was derivative of earlier work, and appears to have been created from less innovative design methodologies such as the grid paper technique, and assemblages of *zillij* tesserae into different known arrangements. With the loss of knowledge for constructing original complex geometric patterns from the polygonal technique, artists throughout the Islamic world ended up with little alternative but to *copy* existing patterns from the past. Of course the copying of geometric patterns is a perfectly acceptable traditional practice, and can be traced back to the earliest Islamic period. Patterns found in the compass-work window grilles of the Great Mosque of Damascus were also used in the carved stucco arch soffits in the mosque of Ibn Tulin: and without doubt, specific geometric patterns were used repeatedly throughout Muslim cultures. However, a vital artistic tradition cannot be sustained and advanced by mere copying. Without the methodological knowledge required for the creation of new and original geometric patterns being handed-down to successive generations of artists, this tradition sadly slid from decline to inexorable demise. Yet through reawakening the traditional design methodologies that engendered the creative vitality that sustained generations of Muslim geometric artists, this remarkable artistic discipline can once again provide inspiration to new generations of artists and designers.