Chapter 22
Connectivity and Virtual Networks for Learning

Celia Hoyles, Ivan Kalas, Luc Trouche, Laurent Hivon, Richard Noss and Uri Wilensky

Abstract We present papers that indicate the potential and challenge of connectivity within or between mathematics classroms.

Keywords Collaboration • Virtual networks

22.1 Introduction

Celia Hoyles Digital technologies are already changing the ways we think about interacting with mathematical objects, especially in terms of dynamic visualizations and the multiple connections that can be made between different kinds of symbolic representation. At the same time, we are seeing rapid developments in the ways that it is possible for students to share resources and ideas and to collaborate through technological devices both in the same physical space and at a distance. Given that these developments are becoming more and more available to all students as the Web becomes increasingly accessible across the world, ICMI Study 17 was keen to explore the potential and challenges for mathematics education of these new levels of connectivity, both within and between classrooms. It was envisaged that there would be considerable impact on teaching and learning in the short,

---

C. Hoyles (✉) and R. Noss
London Knowledge Lab, Institute of Education, University of London, UK

I. Kalas
Comenius University, Bratislava, Slovakia

L. Trouche
INRP: National Institute for Pedagogical Research and LEPS, University of Lyon, Lyon Cedex, France

L. Hivon
IREM: Institute for Research on Mathematics Teaching, Orléans, France

U. Wilensky
Northwestern University, Evanston, IL, USA

C. Hoyles and J.-B. Lagrange (eds.), *Mathematics Education and Technology-Rethinking the Terrain,* DOI 10.1007/978-1-4419-0146-0_22,

medium, and long term. A theme of connectivity and virtual networks was therefore set out in the original plans for the Study and the following questions presented as guides to the submission of proposals within this theme:

- How can theoretical frameworks and methodologies developed for interpreting activity, learning, and teaching in technology-integrated classrooms be extended to assist in understanding the distance-learning context? What kinds of changes and refinements are needed?
- What is the potential contribution to mathematics learning of different levels of interactivity and different modalities of interaction, and how might this potential be realized?
- What is special about the potential of physically separated collaborative study of mathematics, and how might this potential be harnessed so as to support mathematics learning?
- What is the potential for creating virtual communities for mathematics learning and permitting communication between individuals from different educational settings?

In the event, there were rather few papers submitted to this theme, no doubt due to the fact that rather little research had been completed at that time around the impact of connectivity on mathematics teaching and learning. But rather than drop the theme, a group of researchers in this field was invited to participate in a plenary panel at the Study Conference – either at the meeting itself or through a video link: as mentioned in the introduction to this volume this latter mode was considered to be particularly appropriate for this panel as an illustration of the potential of this form of communication.

This chapter comprises the papers written by the four panelists following from their contributions to the plenary panel. There are common threads running through the papers. All point to the importance of design: of the technical aspects that shape what students can do with the technology, what they can share and how they can interact; and of the activities themselves, how they exploit connectivity and stimulate student participation. Some contributions describe experiments that take advantage of connectivity within one classroom while others focus on between-classroom interactivity. In both scenarios, teachers' actions in supporting new communities of practice are recognized as crucial, and new roles for the teacher are noted while acknowledging that these roles had as yet been under theorized.

To complete this summary, we note that other ongoing research in this area (see for example Hegedus and Penuel 2008) supports some of the ideas presented in this chapter, most notably in suggesting how "networks can link private cognitive efforts to public social displays thus – potentially at least – enhancing students' metacognitive ability to reflect upon their own work in reference to others" (Moreno-Armella et al. 2008). More radically, these authors argue in a similar way to panel members that this type of connectivity means that the introduction of technology will lead – at last – to a real transformation of practice in classrooms. This remains to be seen. There is no doubt that connectivity will transform how students interact with each other – simply consider the widespread ownership and use of the mobile phone – a technology that is truly personal for a rapidly increasing number of students. Yet if and how connectivity, in whatever form, transforms mathematical practices in school is a matter of future investigation. It is clear from the papers in this chapter that design will continue to be a crucial research theme in the future, as it will be design decisions that will shape what can be shared in terms of resources, information, student solutions, or part-solutions. But an even more fundamental theme emerges that concerns how the technology, activities, and teacher strategies together can motivate students to engage in and take responsibility for mathematical discussion of the process by which they construct their own knowledge and the justifications they propose for solutions to mathematical conjectures.

22.2 Developing Microworlds for On-Line Collaborative Learning

Ivan Kalas

22.2.1 Background Issues

In our department we have considerable experience in developing flexible software platforms for learning, such as Super Logo, Thomas the Clown, Imagine Logo and others, and in the process of their development we have tried to create effective opportunities for communication including being able to work in a common learning space. Such spaces have different forms: a physically common learning space in one place like an interactive smart board; a virtual common learning space within one classroom, such as several computers within one classroom with groups of learners collaborating with and between groups, and a virtual common learning space shared over the network. In our research projects we are trying to address questions like:

- What are the properties of a flexible software platform that support the development of microworlds for effective collaboration?
- What are the important criteria for developing collaborative microworlds?[1]

The aim of our CoLabs project (see http://matchsz.inf.elte.hu/Colabs/) was to examine obstacles that obstruct collaborative learning, see for example Turcsanyi-Szabo and Kalas (2005). In CoLabs, we used Imagine Logo as a platform for developing and exploring collaborative microworlds – called *collaboratories* – which would allow children to communicate and cooperate – either locally in one place or through the network among different schools, towns, or even among different countries in spite of the many technical, linguistic, and cultural obstacles.

As one such *collaboratory* we created Visual Fractions – a complex dynamic interactive computer environment, which allowed groups of children to explore and discover fractions and fractional relations, see Fig. 22.1. Visual Fractions provides dynamic jigsaw puzzle pieces for children to build their own understanding of the topic, see Lehotska and Kalas, 2005. The evaluation of the Visual Fractions environment by a group of future teachers suggested that building and exploring these dynamic playgrounds of dependent

---

[1] In Kalas and Winczer 2006, we presented our attempt to summarize all known aspects in a framework for the development of collaborative microworlds. We do however accept the argument presented in the panel by Hivon and Trouche that a complete list is probably impossible to generate.

Fig. 22.1 Visual fractions: dynamic environment for discovering fractions and fractional relations

visual and interactive representations of fraction objects and relations required (and further developed) the same competencies as programming, see Lehotska (2006).[2]

22.2.2 A Further Example

Since the CoLabs project, we have concentrated mainly on how to provide support for collaboration, again within an Imagine Logo environment. Our ambition is to build an environment that could be offered to teachers, researchers, and enthusiastic amateur developers who want small, immersive, open, interactive, flexible, and collaborative microworlds developed for everyday learning situations.

During this process, we distinguished four dimensions that each needed to be addressed: the technicalities, the connection interface, the aspects to be shared and the features of the activities. We were particularly interested in building proper metaphors that would mediate computational support for collaboration among teachers and learners in the most intuitive and inspiring ways.

Figure 22.2 illustrates an experimental microworld in which several connected users (here represented by letters A, B, C…) own their personal technical panels in which they are given several visually represented parts of a whole, that is a selection of fractions. All users additionally share a common workspace in which they are expected to piece together a given quantity, expressed in shaded circles as an improper fraction. In the example shown in Fig. 22.2, the goal is produce 2 and ¼. The users (in this case four users A, B, C, and D) can bring their own pieces into the common workspace by dragging them into their local representation of that space, or by manipulating the pieces of other children (comparing them, rearranging, or rotating them etc.). However, only a user's own pieces can be dragged back to his/her personal technical panel or individual workspace, to be "weighted" there[3] or divided into smaller pieces and then reused in the common space.

---

[2] Other researchers (Pratt and his colleagues) used Visual Fractions for other kind of observations more closely related to mathematics education (see, for example, Jones and Pratt 2006).

[3] When a child drags a piece or several pieces one by one (i.e. the visual representations of fractions), into the dark area of his/her technical panel, the environment "weighs" or "measures" them all together and shows the total sum (value) for example 5/12 or 1/6 or 1/6.

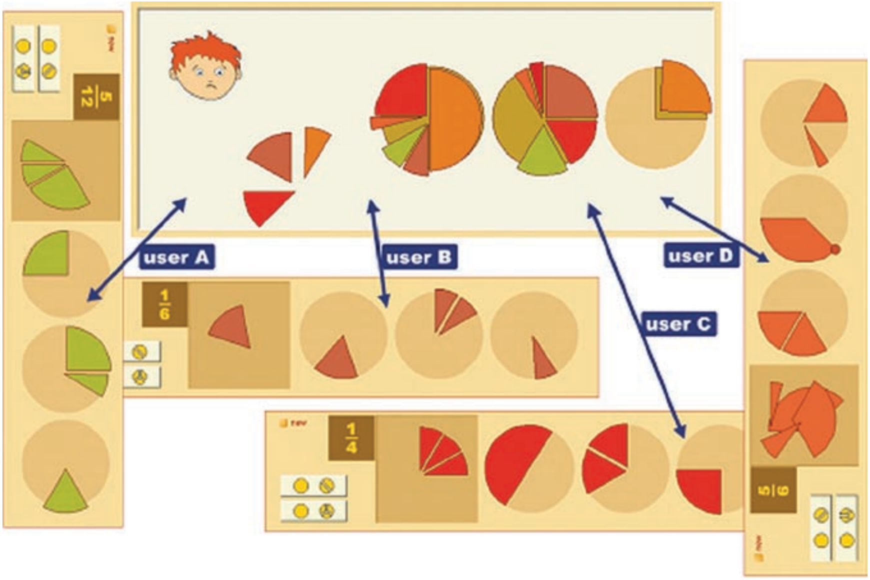


Fig. 22.2 A common virtual learning space (the rectangle in the top middle) shared by four collaborating distant learners, A, B, C, and D

Based on this work, we conjecture that an approach to collaboration that involves constructing common spaces in which children can compose together, explore, construct, communicate, pose, or solve problems can be employed successfully with children from preschool to upper secondary stages; and not only in mathematics learning but also in the development for example of language skills, in art and design, science, and citizenship.

22.2.3 Some Reflections and Observations

In our research on developing microworlds for on-line collaborative learning we have also examined whether digital technologies can motivate children to collaborate and communicate and how specifically designed microworlds support these phenomena. Most of all, we want to identify the critical factors for efficient collaboration, motivation, and engagement in the learning process. Below is a summary of our observations distinguished by some widely held claims (for more detail see Kalas and Winczer 2006):

Claim 1 "Our interfaces for collaborative learning always have an amateur look and are therefore far less attractive than the professional activities and games that many children know and use elsewhere". Although this claim is probably more true for boys than for girls, I wanted to say that we conducted a survey and we found following that children were asked to rate the importance of several aspects of collaborative environments that a *clear and intuitive interface scored more highly* than the professional look of the environment. We thus concluded that an intuitive interface was the key factor in the children's motivation and engagement in the cooperation.

Claim 2 "*The intensity of communication increases during an activity as does its efficiency*". We found on the contrary that the number of brief interchanges of information communicated between the children was high at the beginning of an activity but decreases considerably during the activity, and finally reaches nearly minimal, yet optimal, flow. This can be explained since initially children always explored all the possible communication channels of the environment and exploited them heavily – even without any obvious reason. For example when they discovered that it was possible "to chat" in the environment, they immediately paused their main activity to exchange messages with nearly no content with each participant. Only after these phases did they resume the primary tasks.

Claim 3 "*Competition is important for motivation*". Perhaps surprisingly, we found that competitiveness was not in conflict with collaboration. Rather both phenomena could be stimulated in parallel in activities with two or more competing teams.

22.2.4 Some Concluding Remarks

Although we are rather successful in overcoming a range of technical, linguistic, and cultural obstacles in our experimental collaborative microworlds, we have to admit we still need to find ways to face the hardest obstacle of all, namely the educational obstacles to implementation. It seems to us that our formal educational systems are not yet quite prepared to assimilate computational support for effective on-line collaboration.

22.3 Connectivity: New Challenges for the Ideas of Webbing and Orchestrations

Luc Trouche and Laurent Hivon

22.3.1 Introduction

It is not easy to speak about the implications of connectivity since the word itself calls up a set of connected questions for research:

- What is possible to do for mathematics learning with ICT either face-to-face or through distance learning that transcends just the ability to communicate?
- What are the implications for each learner of the potential of "cognitive connectivity," that is being able to establish links between a situation and an idea and being able to move more or less easily from one mathematical frame to another?

And more generally:

- What are the relationships between what we call *orchestrations* (Trouche 2004; Drijvers and Trouche 2008) – the intentional organization by the teacher of the

Fig. 22.3 TI Navigator system

various tools available in a learning environment, and creativity of the learners who form part in this situation?

In this short contribution, we focus on an environment dedicated to a particular type of connectivity, namely the TI Navigator, providing wireless communication between students' TI graphing calculators and the teacher's personal computer (Fig. 22.3), with activities designed following the collaborative work of a team of teachers and tried out in ordinary classrooms.

We introduce the following questions that guided the investigation:

- How should orchestrations be conceived in order to optimize the chances that the tools serve as efficient instruments for mathematics learning?
- What new difficulties and opportunities become evident for students, using the technology to interact each other, and with the teacher?
- What new difficulties and opportunities become evident for teachers, and what new professional practices are necessary?

Finally, from a theoretical point of view:

- What are the challenges that need to be addressed in new formulations of the two theoretical concepts: of *webbing* – "a structure that learners can draw upon and reconstruct for support – in ways that they choose as appropriate for their struggle to construct meaning for some mathematics" (Noss and Hoyles 1996; p. 108) – and of orchestrations?

22.3.2 Some Elements on the Experiment

Working with INRP and IREM,[4] a team of six high-school teachers near Orleans, France, studied how to introduce and work with the TI Navigator System in their classrooms. The team had two main hypotheses; namely that the integration of this new device into classrooms:

- Would lead to new orientation to mathematics teaching, particularly from the point of view of orchestrations
- Would foster interactions between students, and motivate peer debate

The research began with studying the device and its integration into the French school system (10th grade), sorting out issues of installation and familiarization with the device, and then the design of some specific activities. The research focused also on the development of collaborative work that integrated the new technological tools. The device incorporates many new technical developments, allowing three main configurations:

- Displaying all (or some) of the pupils' calculator screens in quasi-real time (*screen mosaic* configuration)
- Displaying all of the pupils' data, for example, points or curves, in a single coordinate system (*common coordinate system* configuration)
- Displaying immediately the results of a class vote between two (or more) contradictory proposals (*consultation* configuration)

These three configurations have the common property of establishing a *common workspace* on the class screen. The teacher can choose between several ways of using these configurations such as:

- For the screen mosaic configuration, he/she can choose whether or not to display the name of the corresponding pupil on each screen (Fig. 22.4).
- For the common coordinate system configuration the teacher can decide whether or not to give the pupils the option to change their answer, make one or more uploads and whether or not to perform these uploads simultaneously (Fig. 22.5).

To test the two hypotheses of the research, some specific activities (mathematical problems and orchestrations) were designed and tested in five classrooms (Hivon et al. 2008).

---

[4] INRP: National Institute for Pedagogical Research; IREM: Research Institutes on Mathematics Teaching.

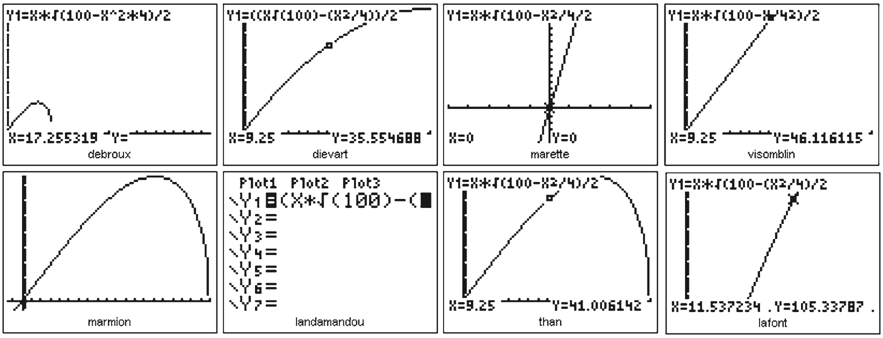
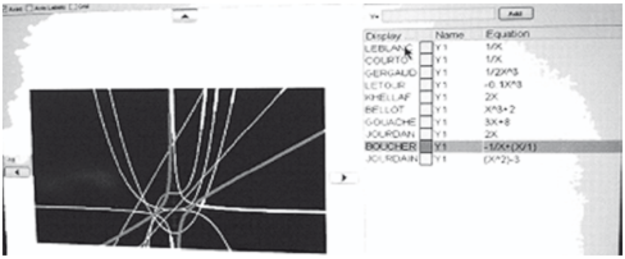


Fig. 22.4 Screen mosaic sent to the common workspace by different students



Display		Name	Equation
LEBLANC	☐	Y1	1/X
COURTO	☐	Y1	1/X
GERGAUD	☐	Y1	1/2X^3
LETOUR	☐	Y1	-0.1X^3
KHELLAF	☐	Y1	2X
BELLOT	☐	Y1	X^3+2
GOUACHE	☐	Y1	3X+8
JOURDAN	☐	Y1	2X
BOUCHER	■	Y1	-1/X+(X/1)
JOURDAIN	☐	Y1	(X^2)-3

Fig. 22.5 Example of common screen configuration

22.3.3 Some Results

The work with the TI Navigator was found to foster an emergent real community of practice (Wenger 1998) in the classroom in which we could distinguish three fundamental aspects, *participation*, *reification*, and the existence of shared resources, whose major elements are summarized below:

- Participation with the engagement of students in the mathematical activity and debate
- Reification with the collaborative creation of mathematical objects (a good example being the collective creation of the graph of a function that gradually becomes an easily identifiable object, cf. Figure 22.6, see also Wilensky in this chapter)
- Shared resources most notably the public shared board, which is a place where every student can show her/his mathematical creation. Each student is confronted with her/his production and those of other pupils

In traditional classrooms, speech or writing on the board are the ways students can express themselves and share with others, at the request of the teacher. With TI Navigator, the situation is very different, for two main reasons:

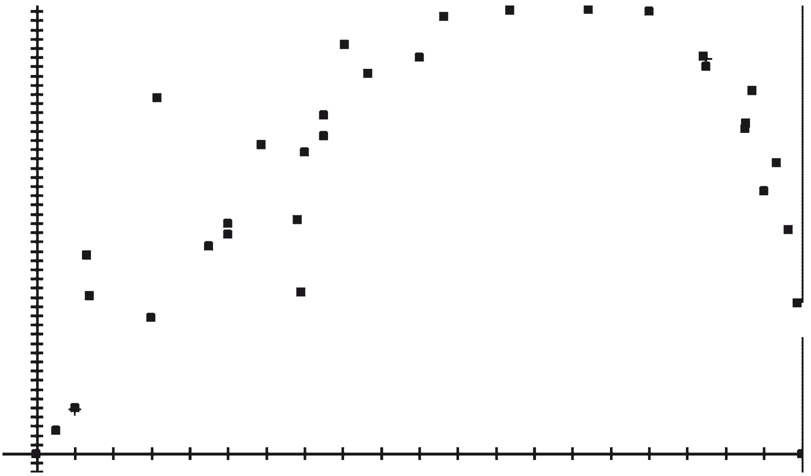


Fig. 22.6 Points sent by every student as a result of a modeling problem, collected in one space

- A new interactivity is fostered between the artefact and the student, and between students themselves: students convey their messages through the artefact, the artefact acts on the students enabling them to distance themselves from their productions thus freeing them to become more easily involved in peer exchanges. Thus the common space becomes a space of debate and exchange that aims to elaborate a “social mathematical truth”.
- Each student becomes detached from his/her production as a distance is created between student and the expression of her/his creation and this distance seemed to improve collective reflection on practice. The student becomes involved in the class activity in a different way as the tool maintains this distance between a student and the results proposed to the class and to the teacher.

Thus our first conclusions point to the renewal of relationships and exchanges inside the class. However, other elements must be borne in mind:

(a) Daily use of the device is difficult due to the complex equipment. Thus the device was not often used which has two contradictory consequences. On the one hand each new usage of TI Navigator needs time for re-appropriation, and on the other, these rare moments of use tend to be remembered by all students.

(b) As the responses proposed by the students were often very different and there were many solution processes opened up for discussion, not just the one used in a traditional course, the students often tried to produce the most sophisticated solution they could possibly find.

(c) The use of the devices deeply changed the way of the class had to be managed, that is the way the class mathematical activity was orchestrated. This added a

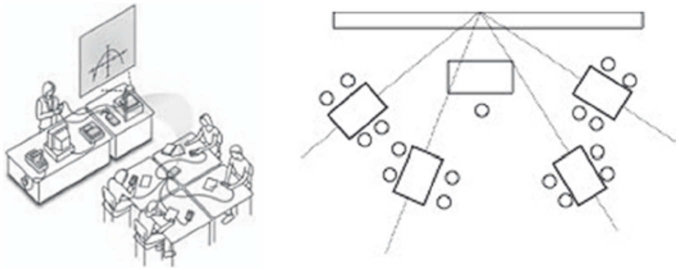


Fig. 22.7 From the intended configuration by the manufacturer to the configuration chosen by the teachers

new complexity to the teacher's work. The complexity of facing the integration of ICT in mathematics education is well known (see Guin et al. 2004), but the necessity to manage both students' tools and the collective tool (the calculator network) makes this integration much more complex. For example, students' activity is deeply sensitive to the organization of the classroom space, for example where teachers change the orientation of the students' desks (Fig. 22.7).

(d) The collaborative work of the teachers involved in this experiment seemed absolutely necessary to face all this complexity and to produce the design of appropriate mathematical situations.

(e) The cross-observations of teachers in their own classes helped them to create a distance from their own practice and to develop a reflexive attitude to the orchestration of students' activity.

22.3.4 Questions to Be Considered in More Depth

In the future, when students will be used to working with the system, we intend to undertake a deeper analysis of students' learning processes in this environment although we recognize the difficulties in doing this for three major reasons:

- *The complexity of the orchestrations:* as a lesson is made up of many stages (personal work, interactions within each group, interactions in the whole classroom and debates), it is not easy to observe the way a single student changes her/his mind.
- *The multiple instruments used in the students' work*: the students use paper, screen calculator, public screens, so it is difficult to know what they do and in which order they do it.
- *The interaction between the phases of classroom work*: we could classify the students' work in the classroom into three stages: first, they expressed their

personal point of view (especially at the beginning of the lesson); second, they expressed themselves as members of their group (they worked in groups of four); and third, they expressed themselves as members of the class. Of course, the three ways are mixed, which adds to the complexity of analysis.

Other questions arose for future investigation are:

(a) *The teacher's behavior and professional development*. S/he created the conditions for students to build a mathematical object, but this object would, partly, be built by the community of students. Thus a student does no longer only plays the music written by the conductor, rather s/he is writing part of the music. The question then has to be faced as to how the teacher can create conditions to make the music not too different from what s/he wanted it to be, or to enrich his/her own partition with the – sometimes – unexpected students' improvizations.
(b) *The teachers' use of the computer*. A recent report of the European Commission[5] showed that French teachers "do not use computers and the internet very frequently and intensively in schools." Could connectivity tools like the Navigator change this situation?
(c) *The sharing of students' conceptions*. In a situation of connectivity, a student constructs her/his knowledge in collaboration with other students. As everyone takes part in this construction, will the others' conceptions help her/him to build her/his own knowledge? How will the students learn to manage this new situation? What influence will it have on the way they build their conceptions of an object?
(d) *The influence of the private practice of connectivity* (blogs, chat, MSN) on how connectivity is used inside the classroom? For example, will this private practice of connectivity make the instrumentalization processes (the way a user appropriates, modifies, a given artefact) more important than in a nonconnectivity activity in the school?

All these questions need to be addressed in new experiments.

22.3.5 Some More General Considerations

This first experiment was derived from a particular context (a classroom in a high school, in a given technological environment), but our conviction, based on other experimentation (Guin and Trouche 2005) is that several elements of this context are more generally relevant, such as to distance learning. These elements are:

(a) The idea of a *common workspace*, for the pupils as well as for the teacher, in which each learner has to orchestrate the part of the game over which s/he is in charge (see also Kalas this chapter). This part is much more important than in an ordinary classroom given there are many results, many mathematical objects and semiotic registers all appearing at the same time on her/his own machine and in the common work place. As we know from students, such an approach appears to motivate,

---

[5] http://ec.europa.eu/information_society/newsroom/cf/itemlongdetail.cfm?item_id =2888

with mathematics appearing like a game. But of course it must be recognized that this is not the case for all the pupils, as some will remain passive. Also the teacher's role is complex as s/he has to manage the different instruments used in the classroom, as well as the collective instruments (in our experiment the network of calculators). The question of time is crucial: many results appear very quickly and the teacher has to make didactical choices swiftly and all the time. We have thus observed, as also noted by Noss and Hoyles, this chapter, in most of our experiments, that *successful knowledge construction is critically dependent on teacher intervention directly to facilitate, encourage, and foster interactions*.

(b) The idea of *collaboratories*. Connectivity and collaborative work are strongly connected. Connectivity enriches and is enriched by collaborative work, both among learners and among teachers: that is to say, the emergence of a community of practice is a condition for connectivity to work, while connectivity in turn facilitates such communities to emerge.

(c) The possibility of *building a detailed map of all aspects important for developing collaborative microworlds* (as suggested by Kalas in this chapter). It is certainly possible to suggest some features for developing collaborative microworlds, but agreeing on all aspects is certainly impossible. Rather it must be recognized that some aspects are necessarily dependent on the community using them and many are simply not predictable. Therefore, there is a necessity for flexible adaptive environments. Behind this, there are ideas of distributed design, between designers themselves and users, what Rabardel (1995), French ergonomist, calls *conceptions in use*; the need to rethink the notion of orchestration and the notion of webbing (see Noss and Hoyles 1996). As Hoyles et al. (2004) point out, these two metaphors are not referring to the same thing and are not exactly at the same level. On the one hand, it is important to have in mind a necessary assistance (the notion of orchestration) of students' mathematical activity, and on the other, it is crucial to let the students free to think and establish connections (the idea of webbing): Hadamard (1954), a French mathematician put in evidence some extraordinary moments of *illumination*, based on very quick internal and external connections. For example, the image of the concept of function, as a teacher said, appears, for each student, at once, as the result of the sum of the contributions of the whole class. It is a sort of depersonalization: the object is no more on my screen; it is in the common work place, enriched by all the community. But it is certainly the result of a given mathematical situation and of a particular orchestration by the teacher, which makes necessary new processes of *documentation* (Gueudet and Trouche, online) for teachers.

In this sense and as a summary of our work, connectivity raises many new didactical challenges for the teaching of mathematics.

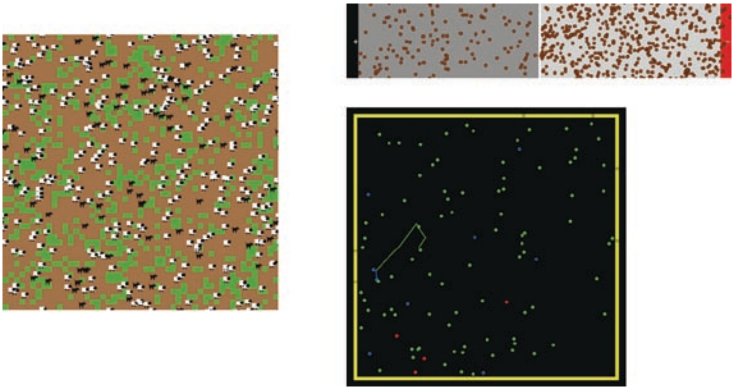


Fig. 22.8 NetLogo models of predator and prey, electricity, and ideal gases

22.4 Concurrent Connectivity: Using Netlogo's Hubnet Module to Enact Classroom Participatory Simulations

Uri Wilensky

In the panel, I presented an outline of our 20 years of work with agent-based modeling and NetLogo (Fig. 22.8) (Wilensky 1999) and described how this work can be enhanced though classroom connectivity.

Much of the discussion of connectivity in education has focused on the potential of asynchronous collaboration and distance learning. Moreover, the vision for connectivity is usually about connecting people who are geographically separated and need such connectivity in order to work together. But, there is another, more neglected affordance of connectivity: the ability to give people a shared interactive experience in a classroom context. This use builds on gamelike scenarios wherein players interact with each other in a simulated world. Such games have great holding power for children and that same holding power can be leveraged for educational benefits in the classroom.

In our many years of working with NetLogo in middle and secondary classrooms, we have endeavored to bring to students descriptions of complex systems at a micro-level and connect those micro-level descriptions to macro-level and observable phenomena. Typically when we have taught students about systems that can be construed as complex, we have concentrated on aggregate equations that summarize system behavior. For example, to describe the behavior of ideal gases, we rely on equations such as $PV = nRT$. But agent-based modeling enables students to more directly control and examine the behavior of elements of the system and connect this behavior to the system emergent behavior. Thus in NetLogo's GasLab model suite, students come to understand the ideal gas as composed of myriad interacting gas molecules and see that $PV = nRT$ is an emergent result of these interactions.

There are hundreds of NetLogo models we have used in classrooms. Students examine a wide range of phenomena such as the spread of a disease through a population, or the interactions of predator and prey in an ecosystem or the flow of electricity through a circuit or traffic on a highway, etc. There is considerable research that shows that it is hard for students (and people in general) to reason about such systems (Centola et al. 2000; Penner 2000; Wilensky and Reisman 2006; Wilensky and Resnick 1999). We have argued that this is largely because the aggregate descriptions do not shed light on the mechanisms of action and, conversely, it has been impractical to have students do the extensive computations required for the micro-level approach.

The use of agent-based modeling (ABM) has changed the terms of use – both in scientific practice and for classrooms. ABM languages and environments enable students to focus on the systems parts and their interactions and to rapidly compute the emergent results and experiment with a host of alternative scenarios. In recent years, a number of ABM-based curricula have been developed that have been quite successful in classrooms, especially at the secondary school and university levels (Abrahamson and Wilensky 2002; Blikstein and Wilensky 2005; Levy et al. in press; Sengupta and Wilensky 2005; Stieff and Wilensky 2003; Wilensky et al. 2006).

However, despite considerable efforts to "lower the threshold" of entry into agent-based modeling, it remains difficult for elementary students to master both the programming and modeling skills needed. A remedy for this that we and others have tried is to have the teacher present and explore a model with the entire class. This approach has considerable merits, but it leaves the student somewhat passive as only a few can be engaged at any one time and they are limited to discussion of model behaviour.

One possible solution to this dilemma is to enable students to collectively participate within the simulation, controlling elements of it and collectively observing and discussing the results of their actions. This approach enables all classroom students to be simultaneously active while giving them an experience of a complex system that they all share. It also empowers them to try to change the system by their actions and to see how much they can affect the system and how much they are constrained by it.

To accomplish this aim, we added a networked architecture to the NetLogo software. This added module, HubNet (Fig. 22.9) (Wilensky and Stroup 1999a), enables a host of devices to connect to a NetLogo simulation and control agents within that simulation. We designed HubNet to be able to accept a range of client devices, including computers, graphing calculators, handheld devices and phones. All of these devices have bee implemented with HubNet, but the two most robust client devices are full computers, which use the computer-HubNet interface and TI graphing calculators, which use the Calc-HubNet interface. We worked with Texas Instruments for many years on networked calculator products which has led to the current TI-Navigator interface which includes HubNet activities.

By adding synchronous connectivity to NetLogo, the modeling activity is transformed into a participatory simulation (Wilensky and Stroup 1999b). This

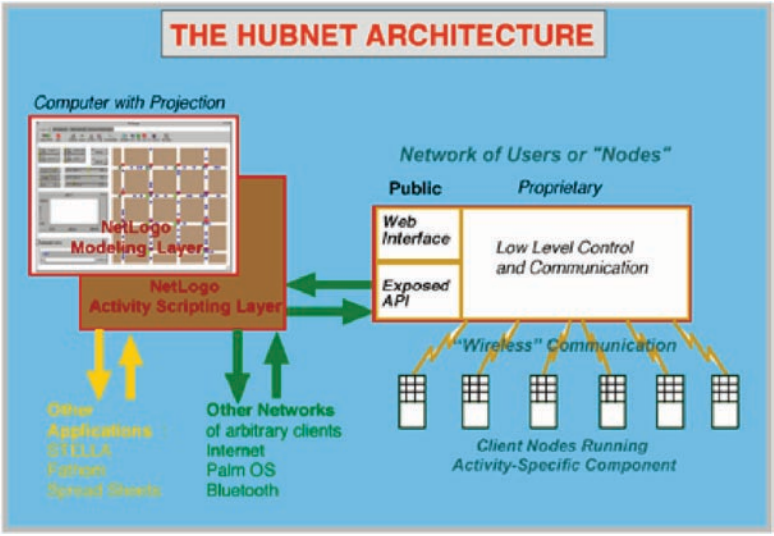


Fig. 22.9 The HubNet architecture

transformation has several important benefits for learning. For example the modeling activity:

- Becomes more engaging – especially for younger learners. It becomes a social activity and captures much of the same draw as online games.
- Promotes greater student participation. Every student can be actively involved at the same time. Because they often require continuous action on the part of the students, they are "in-the moment" motivated to participate. Such universal participation is very hard to achieve in a traditional classroom.
- Enables a shared experience of a complex system. There are very few opportunities, in the classroom or in life, for students to collectively witness the same complex system unfolding. Focal attention to such a system is hard to achieve outside of the virtual and, even when achieved, if the viewing does not connect the micro-level behavior to the macro-level outcomes, then only the appearance is shared, not the mechanisms of action.
- Facilitates classroom discussion of the system and examination of "what-ifs." Student can suggest experiments with varying critical system parameters and/or agent-rules, hypothesize the observed behavioral change, run the simulation and refine the experiment.
- Scaffolds individual modeling and analysis. Once students have experienced several opportunities to collectively model and analyze complex systems, they become much better prepared (and motivated) to conduct such inquiry on their own. Often students have suggestions for model experiments that are not

> explored in class. These questions are potent seeds of further student inquiry, experimentation, and model revision.

NetLogo comes with a bundle of HubNet activities. At Northwestern's Center for Connected Learning and Computer-Based Modeling (CCL), we have authored many of these and tested them in classrooms. We have explored a wide range of content domains and simulation forms including simulations of ecologies, economies, disease transmission, traffic patterns, and many more. Some of these activities can be freely downloaded with NetLogo from ccl.northwestern.edu/netlogo. Many more are in classroom tests and in development.

22.5 Designing for Exploiting Connectivity Across Classrooms

Richard Noss and Celia Hoyles

This section is based around two large-scale projects that have occupied us for much of the last decade: the Playground project and the WebLabs project, both funded by the European Union, and co-directed by ourselves. Both projects set out to investigate ways that students could be motivated to collaborate while physically separated.

22.5.1 The Playground Project

In the Playground project (Noss et al. 2002), we attempted to tap in to children's games culture by adding a new dimension whereby they built their *own* games. The central idea was to design and try out computational worlds – playgrounds – in which the objects in a game *and* the means for expressing them are engaging; where the programming of a game is itself a game (we used *ToonTalk*[6] as the major programming environment, and we also created an icon-based language of our own, called *Pathways*[7]).

Fig. 22.10 The stones combined into rules for a monster

---

[6] See http://www.toontalk.com

[7] This prototype system was subsequently published as *Magic Forest*. See http"//www.logo.com/cat/view/magicforest.html

We set ourselves the task of working with young children (aged as young as 4 and at most 8) where it was obvious that we could not rely on the written word as a means of communication. This challenged us considerably and forced us to take seriously other modalities of interaction, such as speech as well as direct manipulation.

Children populated their games with objects which had "behaviors" – sets of rules that determined their actions. Behaviors were defined using collections of iconic rules, which could be viewed by opening a "scroll of paper" attached to the object. Each rule was expressed as a visible "sentence" or string of graphic icons which combined a condition and a series of actions to be executed whenever the condition was true. The icons representing the conditions and actions were represented as "stones," small concrete manifestations of the concepts that could be strung together to constitute a rule (see Fig. 22.10). Action stones had a convex left side so that conditions with their concave right sides could naturally fit to their left. Any object could accept any number of these iconic rules, all of which would be executed in parallel whenever the conditions for their execution were satisfied. Figure 22.1, for example, illustrates three rules for a "monster." Pathways provided 13 conditions and 25 actions, together with a wide range of object parameters (such as speed and heading) that could be set by using sliders and other manipulable tools. Pathways also included predrawn objects, backgrounds and – in the final version – a mobile phone icon that allowed players to send messages to each other. Objects could be edited (e.g. size and color changed), copied, deleted, and pasted. For examples of children's activities with these rules, see Hoyles et al. (2002).

We gave the children the opportunity to construct creative and fun games (see Fig. 22.11), and at the same time, offered them an appreciation of – and a language for – the rules which underpin them, and the mathematical structures that they had to engage with in order to make their games function. The motor for this latter stage was that one group of children would share their games either face-to-face in their own classroom, or with another group in a remote classroom, either synchronously or asynchronously using the Web. In this latter scenario, the remote group could comment on the game, and amend and extend it as they saw fit by changing the

Fig. 22.11 A space game built by children, involving the monsters from Fig. 22.10, together with new elements (spaceship, scoring – see *top left* and *right* – and a space background)

rules, introducing new ones, and typically, merging existing objects (and their corresponding behaviors) into the games to add complexity and interest.

Our findings confirmed that while working both face-to-face and remotely on their games, children could collaboratively explain phenomena arising from rules we characterized as either *player rules* (an agreed regulation), or *system rules* (a formal condition and action for the behavior of the game). We found that in face-to-face collaboration, the children centered their attention on narrative, and addressed the problem of translating the narrative into system rules which can be programmed into the computer. This allowed the children to debug any conflicts between system rules in order to maintain the flow of the game narrative.

When we added remote communication to the system by enabling the sending and receiving of games from within the *Playground* system – we found that children were encouraged to add complexity and innovative elements to their games, not by the addition of socially-constructed or "player" rules but rather through additional system rules which elaborate the formalism (games were created using two different kinds of programming system, neither of which employed textual modality. This shift of attention to system rules occurs at the same time, and perhaps as a result of, a loosening of the game narrative that is a consequence of the remoteness of the interaction.

This phenomenon was particularly evident in the case of asynchronous interaction where, stripped of even the semantics of gestures, our extremely young students found it increasingly natural to try to communicate meaning via the various formalisms we provided. Thus a key historical claim for programming, that it offers a key motivation and model for immersion in a formal system, came to life as children struggled to modify and add rules of their programs that achieved the effects they desired. And it is worth stressing that asynchronous communication, while somewhat less attractive to the students at the time (we should not underestimate the impact of online synchronous video communication, in 2000, with children in other countries), allowed students to reflect on, and therefore use more effectively, the formal rules of their games.

The Playground project left us with a modest set of corroborative data that leads to the general conclusion that online collaboration catalyzed some interesting outcomes. The shift from narrative to system/formal rules does, in fact, seem to be a direct result of the necessity to formalize in the absence of all the normal richness of interaction that characterizes face-to-face collaboration. Moreover, the contrast was all the more vivid when we compared the children's later work with their initial constructions, in which the narrative was clearly foregrounded, and the focus of attention was necessarily the translation of the narrative into a form that the computer could accept. This initial form of engagement made it possible to debug the system rule conflict that occurred. There were, inevitably, some difficulties. First, we noted that harder games did not necessarily mean harder mathematics – sometimes the games simply became more complicated rather than more complex. Second, peer-to-peer connectivity was severely limited in scope for knowledge building and sharing (the project began in the previous century!).

Rita's guess my robot

Created by Rita - Created: 27-01-04 - Modified: 25-04-04

My Sequence:

I created this sequences of numbers. My five firsts terms of the sequence are:

2 16 72 296 1192

○ Can you explain?
○ What if...
○ I have a conjecture...
○ This doesn't work because...
○ Try this instead...
● Other:

Nasko's response

Posted by: yish at 05-02-04

Hi Rita,

your sequece is very nice, but I have maneged to solve it.

See the solution below.

Best regards NASKO!!!

Here is also a sequence generated by the same robot. Two questions:

1. What was the input of my robot?

2. Can your robct generate it?

9.5 14 16.25 17.375 17.9375

Fig. 22.12 A WebReport. Rita (in Portugal) has challenged other students to find the mathematical function that generates her sequence. Nasko (in Bulgaria) answers in an interesting – and surprising – way

22.5.2 *The Weblabs Project*

In a second project, we decided to address these issues directly. The *WebLabs* Project (http://www.weblabs.eu.com, European Union, Grant # IST-2001–32200), aimed to explore new ways of constructing, expressing, and sharing mathematical and scientific knowledge in communities of young learners. Some of the ideas we asked students to engage with were indeed sophisticated (for example, the convergence of infinite sequences and the properties of infinite decimals). Once again, we used ToonTalk as our primary platform for construction, building open toolsets for students to construct models, and supplementing these with other appropriate tools as necessary (for example, *Excel*). From the point of view of the panel and this chapter, a key focus was our ambition to design and build a web-based collaboration system for sharing and discussing student constructions. This was considerably more sophisticated than the primitive system of sending files back and forth as in *Playground*, and consisted of a set of functionalities – named *Web Reports* (http://www.weblabs.org.uk/wlplone) – that allowed models to be shared and included prompts for students to add comments, conjecture about the best approaches, and most efficient models (see Fig. 22.12 for an example). It is worth noting that this project was ahead of its time: had we engaged in this a few years later, we would have been able to tap into web 2.0 applications that are now taken for granted: essentially we had to invent a genre of mathematical social software! (While we were constructing WebReports, we became aware of Knowledge Forum, and drew inspiration from this work: for recent work, see, for example, Scardamalia and Bereiter 2006).

WebReports allowed students to embed seamlessly their models in free-form text documents and publish them on the web. Thus the central tenet of the approach was that students simultaneously *build* and *share* models of their emerging mathematical knowledge.

Our pedagogical approach was based on encouraging students to propose conjectures or derive concrete questions to explore (real, and complex ones: e.g. are there more integers than even integers?), which were then formulated by us into modeling/programming tasks.

Students completed these tasks individually or in pairs and published their individual models (*ToonTalk* programs) along with their observations about them, in their personal webreports, commenting on each other's models, which were then used as input to an instructor-led group discussion. The product of this discussion was a *group* webreport which represented the shared understandings of the group, a process that we intended would encourage students to reflect on their work, to acknowledge the need to construct rigorous arguments for their claims, and to negotiate socio-mathematical and socio-technical norms within the (international) community (in the sense of Cobb et al. 2002). As an (ideal) final step, *Web report* would be reviewed by another group, perhaps in another country, and an inter-group online discussion would ensue: (we would now probably call this a math-blog and the students would need little, if any, tutoring on how to use it!).

Once again, we found that collaboration and discussion played a central role in the construction of individual and group knowledge. The need to publish their thoughts in writing, and in a public medium, provoked students to reflect on their experiences and intuitions. The process of writing a joint report required that they find a shared mathematical language, and revisit their arguments. Reading others' reports critically, encouraged attention to detail. Yet all these results were contingent on two major facets: that the students had *something engaging to talk about*, and that they had a *reason to talk about it*. In our case, the former consisted of their models and conjectures, and the latter was built into the activity structure.

In fact, we rather seldom succeeded in orchestrating lively discussion, largely due to pragmatic limitations but also because of the difficulty in establishing a distributed community of practice. The modal thread length of interaction when building a webreport was 1, and the average only slightly greater than one. However, we had some outstanding successes – for example, "Guess my Robot" (in which the challenge is to write the program/robot/function that generates a given sequence of numbers) had a modal thread length of more than 20. In considering why we achieved this kind of success, we identified the teacher as critical: as a facilitator who maintained and supported the interaction, and as a mechanism for validating what did and did not make sense in terms of knowledge building.

22.5.3 Concluding Remarks

Alongside overcoming not inconsiderable technical challenges, establishing an appropriate set of socio-technical/mathematical norms that prioritized collaboration was crucial in exploiting connectivity. We found that the school culture – with its relative lack of expectation to reuse knowledge, and the difficulty, in assessment-heavy systems, of simply “being wrong” – was a formidable challenge. Here again, the teacher could play a crucial role. And finally, the role of the teacher was essential in finding ways to exploit connectivity to encourage students to permeate the layers of our system: to move, for example, from running models to writing or modifying the programs that generated them.

We knew already, of course, that the teacher is crucial. But here we are delineating new, even more demanding roles for the teacher, to be aware – across not only her own classroom but those in remote locations – of the evolution of discussion, the mathematical substance of what is and what is not discussed, and the need all the while to find ways to keep students on task without removing the exploratory and fun elements of the work. This is, surely, a demanding set of roles for the teacher! And it is with this in mind that we have begun new research, in which we are exploring the extent to which the technical system may be able to assist in helping teachers in these roles, by working on building intelligence into the system to achieve this. See www.migen.org and for some early results, see Pearce et al. (2008a, b), Noss (2008).

References

Abrahamson, D., & Wilensky, U. (2002). *Statistics as situated probability: The design and implementation of S.A.M.P.L.E.R.*

Blikstein, P., & Wilensky, U. (2005). Less is more: Agent-based simulation as a powerful learning tool in materials science. *Proceedings of the 4th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2005)*, Utrecht, The Netherlands.

Centola, D., Wilensky, U., & McKenzie, E. (2000). A hands-on modeling approach to evolution: Learning about the evolution of cooperation and altruism through multi-agent modeling – The EACH Project. *Proceedings of the fourth Annual International Conference of the Learning Sciences(ICLS)*, Ann Arbor, MI, June 14–17, 2000.

Cobb, P., Yackel, E., & McClain, K. (Eds.) (2002). *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse Tools and Instructional Design* (pp. 361–383). Mahwah, NJ: Lawrence Erlbaum Associates.

Drijvers, P., & Trouche, L. (2008). ‘From artifacts to instruments: A theoretical framework behind the orchestra metaphor’. In G. W. Blume & M. K. Heid (Eds.), *Research on Technology and the Teaching and Learning of Mathematics: Vol. 2. Cases and Perspectives*, (pp. 363–392). Charlotte, NC: Information Age.

Gueudet, G., & Trouche, L. (online). *‘Towards New Documentation Systems for Mathematics Teachers?’*. Educational Studies in Mathematics. DOI 10.1007/s10649-008-9159

Guin, D., & Trouche, L. (2005). ‘Distance training, a key mode to support teachers in the integration of ICT? Towards collaborative conception of living pedagogical resources’. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education, CERME 4*, Sant Feliu de Guíxols, Spain.

Guin, D., Ruthven, K., & Trouche, L. (Eds.) (2004). *The Didactical Challenge of Symbolic Calculators: Turning a Computational Device into a Mathematical Instrument*. Berlin: Springer.

Hadamard, J. (1954). The Psychology of Invention in the Mathematical Field. Dover Gueudet, G., & Trouche, L. (online). ‘Towards New Documentation Systems for Mathematics Teachers?’. Educational Studies in Mathematics. DOI 10.1007/s10649-008-9159

Hegedus, S., & Penuel, W. (2008). Studying new forms of participation and classroom identity in mathematics classrooms with integrated communication and representational infrastructures. *Special Issue of Educational Studies in Mathematics: Democratizing Access to Mathematics through Technology – Issues of design and Implementation*, 68(2), 171–184.

Hivon, L., Péan, M., & Trouche, L. (2008). *‘D’un réseau de calculatrices à la construction collaborative du savoir dans la classe’*. Repères-IREM, 72, 79–102.

Hoyles, C., Noss, R., & Adamson, R. (2002). Rethinking the microworld idea. *Journal of Educational Computing Research*, 27(1&2), 29–53

Hoyles, C., Noss, R., & Kent, P. (2004). ‘On the integration of digital technologies into mathematics classrooms’. *International Journal for Computers in Mathematical Learning*, 9(3), 309–326.

Jones, I., & Pratt, D. (2006). Connecting the equals sign. *International Journal of Computers for Mathematical Learning*, 11(3), 301–325

Kalas, I., & Winczer, M. (2006). Building interfaces for on-line collaborative learning. *Education and Information Technologies*, 11(3/4), 371–384

Lehotska, D. (2006). Visual fractions in teacher training. In *Imagining the Future for ICT and Education: Abstracts and Proceedings from the WG 3.1 & 3.5 Joint Conference*. Norway: Alesund University College.

Lehotska, D., & Kalas, I. (2005). *LVF – Interface for Dynamic Fractions. Proceedings of the 7th International Conference on Technology in Mathematics Teaching, Vol. 1*, (pp. 108–116). Bristol: University of Bristol.

Levy, S., Novak, M., & Wilensky, U. (in press). Students’ Learning with the Connected Chemistry (CC1) Curriculum: Navigating the Complexities of the Particulate World. *Journal of Science Education and Technology*.

Moreno-Armella, L., Hegedus, S., & Kaput, J. (2008). Static to dynamic mathematics: Historical and representational perspectives. *Special Issue of Educational Studies in Mathematics: Democratizing Access to Mathematics through Technology – Issues of design and Implementation*, 68(2), 99–111.

Noss, R. (2008). *Expressing generality: A computational approach. Proceedings of the Asian Technology Conference in Mathematics*.

Noss, R., & Hoyles, C. (1996). *Windows on Mathematical Meanings. Learning Cultures and Computers*. Berlin: Springer.

Noss, R., Hoyles, C., Gurtner, J-L., Adamson, R., & Lowe, S. (2002). Face-to-face and online collaboration: appreciating rules and adding complexity. *International Journal of Continuing Engineering Education and Lifelong Learning*, 12(5/6), 521–540.

Pearce, D., Geraniou, E., Mavrikis, M. & Gutierrez, S. (2008a). “Issues in the Design of an Environment to Support the Learning of Mathematical Generalisation” (accepted). *European Conference on Technology Enhanced Learning (EC-TEL’08)*. Maastricht, The Netherlands.

Pearce, D., Mavrikis, M., Geraniou, E., Gutiérrez, S., & Kahn, K. (2008b). An Environment for Expressing Mathematical Generalisation using Pattern Construction and Analysis, *Workshop for HCI for Technology Enhanced Learning, at HCI2008 Culture, Creativity, Interaction*.

Penner, D. E. (2000). Explaining systems: Investigating middle school students' understanding of emergent phenomena. *Journal of Research in Science Teaching*, 37, 784–806.

Rabardel, P. (1995). Les hommes et les technologies contemporaines, Paris: Armand Colin, English version People and technology, a cognitive approach to contemporary instruments, available at http://ergoserv.psy.univ-paris8.fr/

Scardamalia, M., & Bereiter, C. (2006). Knowledge Building: Theory, Pedagogy and Technology. In Sawyer, K. (Ed.), *Cambridge Handbook of the Learning Sciences*, (pp. 97–118). Cambridge, U.K.: C.U.P

Sengupta, P., & Wilensky, U. (2005). *N.I.E.L.S:* An Emergent Multi-Agent Based Modeling Environment for learning Physics. In *Agent-Based Systems for Human Learning, workshop of 4th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2005)*. Utrecht, The Netherlands.

Stieff, M., & Wilensky, U. (2003). Connected Chemistry – Incorporating interactive simulations into the chemistry classroom. *Journal of Science Education and Technology*, 12(3), 285–302.

Trouche, L. (2004). 'Managing Complexity of Human/Machine Interactions in Computerized Learning Environments: Guiding Student's Command Process Through Instrumental Orchestrations'. *International Journal of Computers for Mathematical Learning*, 9, 281–307

Turcsanyi-Szabo, M., & Kalas, I. (2005). *Collaboration – a tool for learning*. *Proceedings of 10th European Logo Conference EuroLogo 2005,* Warsaw, (pp. 54–65).

Wenger, E. (1998). *Communities of Practice. Learning, Meaning, Identity*. New York: Cambridge University Press.

Wilensky, U. (1999). *NetLogo [Computer Software].* Evanston, IL: Center for Connected Learning and Computer-Based Modeling, Northwestern University, http://ccl.northwestern.edu/netlogo.

Wilensky, U., & Reisman, K. (2006). Thinking like a wolf, a sheep or a firefly: Learning biology through constructing and testing computational theories. *Cognition and Instruction*, 24(2), 171–209.

Wilensky, U., & Resnick, M. (1999). Thinking in levels: A dynamic systems approach to making sense of the world. *Journal of Science Education and Technology*, 8(1), 3–19.

Wilensky, U., & Stroup, W. (1999a). *HubNet*. Evanston, IL: Center for Connected Learning and Computer-Based Modeling, Northwestern University.

Wilensky, U., & Stroup, W. (1999b). Learning through participatory simulations: Network-based design for systems learning in classrooms. *Proceedings of the Computer Supported Collaborative Learning Conference (CSCL '99)*, Stanford, CA.

Wilensky, U., Novak, M., & Rand, W. (2006). *BEAGLE Evolution Curriculum*. Evanston, IL: Center for Connected Learning and Computer Based Modeling, Northwestern University.