Chapter 5
Building European Collaboration in Technology-Enhanced Learning in Mathematics

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Abstract This chapter is concerned with the work that Kaleidoscope Network of Excellence made possible on technology-enhanced learning in mathematics. It presents some findings from two complementary initiatives that were carried out in this field: TELMA European Research Team and the Special Interest Group on Learning and Technology at Work. TELMA initiative, starting from the acknowledgement of the difficulties generated in mathematics education by the diversity and fragmentation of existing theoretical frameworks and methodological approaches, worked towards the collaboration and integration of European research teams involved in the use of digital technologies in mathematics education. Some common concepts and a methodology based on the cross-experimentation of ICT-based tools for school mathematics were elaborated and tested in real classroom settings, with the aim of analysing the intertwined influence played, both implicitly and explicitly, by the different contextual characteristics and theoretical frames assumed as reference by the diverse teams participating in TELMA. The work developed by the Learning and Technology at Work group gave the possibility to enlarge the usual perspective on mathematics learning since it allowed considering not only indications coming from school education, but also needs coming from the world outside the school and, in particular, from the workspace, where novel kinds of mathematical knowledge, techno-mathematical literacies, have become of critical importance.

Keywords Technology-enhanced mathematics education · Learning environments · Theoretical frameworks · Cross-experiments · Techno-mathematical literacy

5.1 Introduction

The advent of the microcomputer in the early 1980s brought with it high expectations regarding the potential of technology to drive change and innovation in schools. Notwithstanding the positive results produced in experimental settings

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by a number of research projects and the considerable budget invested by many governments for equipping schools with hardware and software tools, it is nevertheless true that these expectations appear to have remained unfulfilled at the level of wide school practice (Pelgrum, 1996; Sutherland, 2004; Venezky & Davis, 2002). This is true in particular for mathematics, even if, from the beginning, a wide number of researchers have been concerned with the study of the opportunities brought about by new technologies to the teaching and learning of this discipline (Cornu & Ralston, 1992; Lagrange, Artigue, Laborde, & Trouche, 2003).

Many reasons can be considered for this outcome, from those related to the traditional resistance of both the school system and teachers themselves to change to reasons more deeply related to the fact that technology has often been introduced as an addition to an existing, unchanged classroom setting (de Corte, 1996; Grasha & Yangarber-Hicks, 2000).

If one considers the character of the recommendations frequently adopted at the beginning to promote the integration of ICT in school practice, many of them seem to assume (often implicitly) that the character of ICT "use" in teaching and learning is relatively independent from the specific context of application and unproblematic (Jones, 2005). The problem was that software tools for education were often evaluated on the basis of very general, ill-defined expectations, resulting in a lack of understanding about the theoretical frameworks and the conditions under which the educational use of such tools could have been meaningful and productive (Noss, 1995).

A more critical perspective was adopted at the research level, where digital technologies have been seen as vehicles to promote change in education and to implement didactical strategies in line with the different theoretical frameworks and principles that, in the course of time, have typified the evolution of didactical research.

The tension between theory and practice has deeply characterized the educational use of digital technology and, in particular, the use of technology in mathematics education.

Moreover, mathematics education in the last decades had to confront not only the problem of how ICT might be used to improve teaching and learning processes to achieve existing curricular goals, but also the problem of the changing nature of the knowledge required in workplaces or in everyday life: what Papert calls the "what" as opposed to the "how" of learning (Papert, 2006).

One of the most acute issues in this regard, arising from recent research in workplaces (Kent, Hoyles, Noss, Guile, & Bakker, 2007), is the finding that, over the last two decades, the nature of mathematical knowledge required in workplaces has been influenced by two significant changes. The first change has been a dramatic increase in the deployment of information technologies within workplace practices. The second change is the shift to customer focus and greater transparency of processes. Taken together, these two changes have impacted on the nature of the skills (and particularly, the mathematical skills) required in modern workplaces.

New work practices increasingly involve quantitative or symbolic data processed by information technology, as part of the interactions between employees, and between employees and customers. "Techno-mathematical literacies" are needed to reason with this kind of information and integrate it into decision-making and communication (see, for example, Noss, Bakker, Hoyles, & Kent, 2007). This change in what is required in the world beyond school is a critical issue for the "what" of school and college curricula and presents a significant challenge for those who are concerned with the analysis of how the use of ICT in classroom activities can produce significant changes both in the nature of the knowledge imparted and in the processes involved in acquiring it.

Within the frame outlined above, in this chapter two complementary perspectives, coming from the work that Kaleidoscope Network of Excellence made possible on mathematical learning with digital technologies, are considered. Both perspectives address crucial issues and needs that, up to now, have been underestimated in the research field of mathematics education. The first perspective, which is examined more in detail, is concerned with the work performed by the Kaleidoscope European Research Team in the area of technology-enhanced learning in mathematics (TELMA). The second perspective has to do with the work performed by the Kaleidoscope Special Interest Group on Learning and Technology at Work.

On the one hand, the TELMA initiative, starting from the acknowledgement of the difficulties generated in mathematics education by the diversity and fragmentation of existing theoretical frameworks and methodological approaches, worked towards the collaboration and integration of European research teams that, within different contexts and cultures, are all involved in the use of digital technologies for mathematics education in school.

On the other hand, the work developed by the Learning and Technology at Work group gave the possibility to enlarge the usual perspective on mathematics learning since it allowed considering not only indications coming from school education, but also needs coming from the world outside the school and, in particular, from the workspace, where novel kinds of mathematical knowledge, techno-mathematical literacies, have become of critical importance.

5.2 Technology-Enhanced Learning in Mathematics: The TELMA Joint Research Activity

Among the different joint research activities in Kaleidoscope, TELMA initiative has been established to focus on the improvements and changes that technology can bring to teaching and learning processes in mathematics. It includes six European teams with a strong tradition in the field.[1] TELMA's main aim is to promote

[1] TELMA teams (whose acronyms are indicated in brackets) belong to the following institutions: Consiglio Nazionale delle Ricerche, Istituto Tecnologie Didattiche, Italy (CNR-ITD); Università di Siena, Dipartimento di Scienze Matematiche ed Informatiche, Italy (UNISI); University of Paris 7 Denis Diderot, France (DIDIREM); Grenoble University and CNRS, Leibniz Laboratory, Metah, France (LIG); University of London, Institute of Education, United Kingdom (UNILON); National Kapodistrian University of Athens, Educational Technology Laboratory, Greece (ETL-NKUA).

networking and integration among such teams and to favour the development of shared projects, common methodologies and research priorities.[2]

Each team has brought to the project particular focuses and theoretical frameworks, adopted and developed over a period of time. Most of these teams have also designed, implemented and experimented, in different classroom settings, computer-based systems for supporting teaching and learning processes in mathematics. Since it was clear from the beginning that, to connect the work of groups that have different traditions and frameworks, it was necessary to find some common perspectives from which to look at the different approaches adopted by each team, to find similarities and to clarify differences, it was decided to concentrate the analysis on three interrelated topics: the theoretical frameworks within which the different research teams face research in mathematics education with technology, the role assigned to representations provided by technological tools and the way in which each team plans and analyses the educational context in which the technology is employed.

As a first step towards this analysis, an investigation on current technological tools in mathematics education with a specific attention on those designed and/or used by each TELMA team was made together with the definition of a common notion able to facilitate the comparison and the interpretation of the different research projects. Then a more operative phase followed aimed at designing and testing a new methodological approach for networking research teams: the cross-experiments methodology.

In the following we briefly examine these two phases and provide some findings and observations that we have derived from such work.

5.2.1 Evolution of Perspectives in ICT-Based Systems for Mathematics Education

Research on technology-enhanced teaching and learning has undergone a deep transformation in the course of time, due to the opportunities offered by the extraordinarily rapid progress of technology and by the evolution of educational, pedagogical and cognitive science theories (Bottino, 2004; European Commission, 2004). TELMA teams have a long tradition in working in this field and, even if in the course of time their work evolved in different directions and along with different theoretical references, it is possible to single out some common perspectives and considerations.

A first consideration regards the theoretical frameworks that TELMA teams refer to. They reflect the general trends and major evolutions of the field. Even with different interpretations and focuses, the prevailing orientation is on socioconstructivist and sociocultural perspectives with an interest for tools such as microworlds [see Hoyles (1993) and Balacheff & Sutherland (1994) for an historical overview of the

[2] TELMA web site: http://telma.noe-kaleidoscope.org/

term and meaningful examples]. Microworlds are environments characterized by some primitives (objects and functions) that can be combined in order to produce a desired effect (computational, graphical, etc.). Examples of microworlds developed and used by TELMA teams are the Fraction Slider microworld developed by the ETL-NKUA team or the microworlds incorporated in the ARI-LAB-2 system developed by CNR-ITD team to support the development of arithmetic problem-solving abilities. Microworlds are built up around a given knowledge domain which has to be explored by the students interacting with the program (often in a direct manipulation modality). The Fraction Slider, for instance, provides immediate visual feedback following student manipulation of either symbolic (Logo) or dynamic (slider) representations, indicating the relative sizes of fractions by the relative positions of slider cursors. ARI-LAB-2 microworlds have been designed to model common situations in everyday life such as “purchase and sales” or “time measure” problems or to model different arithmetic fields and tools for solving problems (graphs, spreadsheets, etc.). For example, to solve a problem involving a money transaction the student can enter the “Euro” microworld where s/he can generate Euros, move them on the screen to represent a given amount, change them with other Euro coins or banknotes of an equivalent value, and so on.

In a socioconstructivist/constructionist framework, students interact with and manipulate the representations provided by the microworlds, making sense of their behaviours taking into account both the interaction and the feedback provided by the tool and the social context of the classroom.

TELMA researchers share a common sensitiveness on the fact that learning processes cannot be understood just by looking at the learners and their inner cognitive processes in interaction with the tool, but that this understanding requires to take into consideration the context in its situational, institutional and cultural dimension. The underestimation of the role played by these contextual characteristics has certainly contributed to the difficulties met in fulfilling the expectations of ICT in education. Consequently, one of the crucial areas to be investigated by TELMA teams was that related to the study of the role played by contextual issues with the aim of understanding how different backgrounds, technologies and content-related educational objectives and cultures can shape different learning environments.

The concept itself of learning environment is understood in a broader perspective, considering not only the relationship of the learner with a digital technology but the teaching and learning situation as a whole (that is, considering not only the tool but also the tasks proposed, the settings, the role played by the different actors). This is in line with current research approaches in educational computing where progressive consideration has been given to the definition of meaningful practices through which technology can be used effectively. Focus has moved to the teachers and to their needs, to the social context in which technology is used, to the ways in which teaching and learning activities integrating technology are organized, etc. (Griffiths & Blat, 2005; Monaghan, 2004).

The analysis of the social dimension of the learning process has been faced in a variety of ways that depend on the different theories assumed as frameworks. Such frameworks answer to different needs even if they share a common sensitivity to the social and cultural dimensions of the teaching and learning processes. Some of these frameworks are strictly related to the mathematics education area as the Theory of Didactic Situations (Brousseau, 1997), deeply used by the TELMA French teams, while others, as activity theory (Cole & Engeström, 1991) referred to by CNR-ITD team or the theory of semiotic mediation (Vygotsky, 1978) referred to, for example, by the UNISI team, are more general and not specifically developed for educational purposes.

Moreover, French researchers pay a specific attention to the instrumental dimension of teaching and learning processes mediated by technology, considering, from one side, Chevallard's anthropological approach (Chevallard, 1992) and, on the other side, the views developed by Rabardel in cognitive ergonomy (Rabardel, 1995). A specific attention is thus paid to institutional values and norms and to the development of instrumented techniques, avoiding reducing them to mere skills. A fundamental role is attributed to instrumental genesis (Artigue, 2002; Lagrange et al., 2003), that is, to the process that produces in the learner the transformation of artefacts into instruments. As an example of tools produced under such framework, Aplusix, an algebra-learning assistant, developed by the LIG team can be mentioned. Aplusix has been implemented with the aim to be fully integrated into the regular work of secondary school classes and it is centred on the feedback provided by the system to students' calculations, thus helping them to verify step by step the acquisition of algebraic rules.

The brief excursus made above suggests that to understand, at a not superficial way, how the different frameworks adopted as reference by the various TELMA teams have been concretely applied to the design, practical implementation and analysis of learning environments integrating technology, it was necessary to go beyond the simple reading of papers and reports made by each team and to move towards a more concrete phase where comparison and integration among teams could be promoted in an operative way. As the matter of fact, in the research papers provided by each team, theoretical references were explicitly mentioned but it was very difficult to infer from what was written the exact role these had played in the design and management of the research projects, and thus in the analysis of the data collected and in the identification of the results obtained. The same was true for the impact of contextual characteristics, making it difficult to figure out up to what point the experience and knowledge gained in one team could be useful for the others and on what basis collaboration and integration could be undertaken.

A first level of integration was then pursued through the elaboration of the notion of *didactical functionalities of an ICT-based tool* (Cerulli, Pedemonte, & Robotti, 2007). This notion was developed as a means to link theoretical reflections to the concrete pedagogical plans that one has to face when designing or analysing effective uses of digital technologies. It individuates three main dimensions to be analysed when considering a learning environment where an ICT-based tool is integrated:

1. A set of features/characteristics of the considered ICT-based tool.
2. An educational goal.

3. The modalities of use of the tool in the teaching and learning activity enacted to reach such goal.

These three dimensions are interrelated: although characteristics and features of an ICT-based tool can be identified through an a priori inspection, these features only become functionally meaningful when understood in relation to the educational goal for which the tool is used and to the modalities of its use. Moreover, it is worthwhile to point out that when designing an educational ICT-based tool, designers necessarily have in mind some specific didactical functionalities, but these are not necessarily those which emerge when the tool is used, especially when it is used outside the control of its designers or in contexts different from those initially envisaged.

A new approach was then implemented by the TELMA group: the cross-experiments methodology (Artigue et al., 2007) where the notion of didactical functionalities has been operatively used to implement guidelines of experiments and to analyse the results obtained.

5.2.2 The Cross-Experiments Methodology

The key idea around which this methodology was built was the design and the implementation by each TELMA team of an experiment in a real classroom setting making use of an ICT-based tool developed by another team. Such experiments were constructed in order to provide a systematic way of gaining insight into theoretical and methodological similarities and differences in the work of the various TELMA teams. This is a new approach to collaboration that seeks to facilitate common understanding across teams with diverse practices and cultures and to elaborate integrated views that transcend individual team cultures. There are two principal characteristics of the cross-experiments methodology elaborated by TELMA that distinguish it from other forms of collaborative research:

1. The design and implementation by each research team of a field experiment making use of an ICT-based tool developed by another team.
2. The joint construction of a common set of questions to be answered by each team in order to frame the process of cross-team communication.

In the development of cross-experiments, an important role was given to TELMA young researchers and doctoral students. This choice was coherent with the general philosophy of Kaleidoscope and was suggested also by the wish to have “fresh” eyes looking at teams’ approaches, theoretical frameworks and consolidated practice, in order to better make explicit those factors that often remain implicit in the choices made by more experienced researchers.

Each team was asked to select an ICT tool among those developed by the other teams, as shown in Table 5.1. This decision was expected to induce exchanges between the teams and to make more visible the influence of theoretical frames through comparison of the didactical functionalities developed by the designers of

Table 5.1 The ICT-based tools employed by TELMA teams in the cross-experiments

ICT tool	Developed by	Experimented by
Aplusix[1]	LIG (France)	CNR-ITD (Italy), UNISI (Italy)
E-Slate[2]	ETL-NKUA (Greece)	UNILON (UK)
ARI-LAB-2[3]	ITD (Italy)	LIG (France), DIDIREM (France), ETL-NKUA (Greece)

[1]http://aplusix.imag.fr
[2]http://etl.ppp.uoa.gr
[3]http://www.itd.ge.cnr.it/arilab_english/index.html

the tool and those implemented by the team developing a field experiment using such tool. Moreover, in order to facilitate the comparison between the different experimental settings, it was also agreed to address common mathematical knowledge domains (arithmetic and introduction to algebra), to carry out the experiments with students between the fifth and eighth grades and to perform them for about the same amount of time (1 month).

Cross-experiments were developed with the aim of acquiring a better understanding of what happens when an ICT-based learning environment is implemented using a tool that has been designed under theoretical frameworks and in a context different from that of the experimenting team. This approach allowed making some step further in the analysis of the complexities involved in designing and implementing learning environments integrating technology. Each experiment had its specific goals but was also an object of collective research for TELMA, and the following issues have been particularly considered:

- What does it mean to "tune" the use of a tool to the specific pedagogical aims and research objectives of a team that has not developed it?
- What are the similarities and differences in the educational settings set up by each team to develop a teaching experiment involving the use of an ICT-based tool?
- Is it possible to unpack some of the implicit aspects embedded in tools?
- Is it possible to understand implicit theoretical assumptions that characterize the design and the development of a learning environment involving the use of an ICT-based tool?

Experiments' guidelines were collectively built for monitoring the whole process: from the design and the a priori analysis of the experiments to their implementation, the collection of data and the a posteriori analysis. Guidelines contained all the research questions to be addressed and the experimental plans developed by each team. These plans included information on the experimental settings, on the modalities of employment of the tool and on the methods used to collect and analyse data. The research questions included in the guidelines were both questions to be addressed before the experiments and questions to be addressed after them.

At the end of the experiments, reflective interviews based on stimulated recall were organized in order to make clear the exact role theoretical frames and contextual characteristics had played in the different phases of experimental work, explicitly or in a more naturalized and implicit way.

It was hypothesized that introducing an "alien" technology would be problematic, and thus can better contribute to make visible design decisions and practices that generally remain implicit when one uses tools developed within his/her research and educational culture, and that this visibility would be increased by making explicit the requirements of the guidelines. Cross-experiments made also possible the comparison of the designs and analyses produced for the experiments with those already produced by the teams having developed the tools. Moreover, since most tools were experimented with by two different teams, it was also possible to compare their designs, implementations and analysis. All these comparisons were expected to contribute to the visibility of the role played by theoretical frames and contexts and help understand their respective influence.

For supporting such understanding, TELMA teams introduced a second basic notion: that of "key concerns" (Artigue, 2005). Concerns are issues considered functionally important as far as a specific aspect or characteristic. Behind this choice lies the hypothesis that the level of concerns is a good level for establishing useful connections between theoretical frameworks, as concerns approach these in terms of functionality, focusing on the needs they respond to. A set of key concerns was thus a priori attached to each of the dimensions of the didactical functionality construct. For instance, as regards the characteristics of a given tool, key concerns considered are related to the mathematical objects implemented and their relationships, to the actions available on these objects, to the possible interaction with other agents, to the support provided to the professional work of the teacher and to the distance with institutional and/or cultural habits and values. Similarly, as regards educational goals, it seemed interesting to investigate key concerns of epistemological nature referring to mathematics as a domain of knowledge or as a field of practice, to concerns of a cognitive nature focusing on the student in his/her relationship with mathematical knowledge, to concerns focusing on the social dimension of learning processes, and so on.

5.2.3 Some Findings from the Cross-Experiments

The first evidence provided by the cross-experiments project was that theoretical frameworks, while influencing design and analysis, were far from playing the role they are usually given in the literature. They mainly acted in the design as implicit and naturalized frames, and more in terms of general principles than of operational constructs. Even if some interesting variations can be noticed, all the teams pointed out the gap they experienced between the support offered by theoretical frames and the decisions to be taken in the design process. Theoretical frames were in general much more explicitly active in the analysis and interpretation of collected data.

This does not mean that theoretical frames did not have a serious influence on the identification of didactical functionalities and thus on the design. For instance, the influence of the theory of didactical situations (Brousseau, 1997) and of the anthropological theory of didactics (Chevallard, 1992) was evident in the choices made by the French teams. It was clear that they were expecting the tools to provide a “milieu” for the students’ work with a strong potential in terms of a-didactic[3] adaptation. This led them to pay particular attention to the feedback that tools offer to students’ actions. They were also very sensitive to the necessity of maintaining a reasonable distance between the mathematics implemented in the tool and the French institutional one, and to limit the instrumental needs. This sensitivity was increased in that specific case by the limited duration of the experiment. Such factors influenced the selection of the tools to be used, the specific educational goals attached to them and the pedagogical plans built. The other teams did not impose to their constructions the same constraints and were more open to exploratory activities. They did not feel so obliged to anticipate the possible mathematical outcomes of the student’s interaction with the tool and were less concerned with the way in which responsibilities were shared between the students and the teacher and to what could be institutionalized and how from the students’ activity.

Conversely, they were more sensitive to other key concerns. For instance, the Italian teams, relying on theories of activity, were especially concerned by the way the representations provided by the tools could act as semiotic mediators of mathematical knowledge. Their scenarios tried to maximize the learning effect of such semiotic mediations to be orchestrated by the teacher (Bartolini Bussi & Mariotti, 2008).

The cross-experiments also confirmed that the differences observed were not just resulting from differences in theoretical approaches. What was at stake was more an intertwined influence of theoretical and contextual characteristics. Some of these contextual characteristics are situated at a rather global level. For instance, the institutional pressure was stronger in France than in Italy and Greece, reducing the space of freedom of the researchers and teachers involved in the experiments. Some are more local. They contribute to explain why teams sharing the same culture (as was the case for the two Italian teams), and using the same tool (Aplusix), developed quite different pedagogical plans.

Another point that is worth mentioning is that it was useful to compare not only the experimental designs but also the way the different teams analysed the data they had collected, and how they invested in this analysis their theoretical constructs. This comparison showed the TELMA teams how their respective tools for design analysis could complement each other to provide a better understanding of the learning phenomena at stake and, in some cases, challenge the interpretations made by one team providing it with alternative ways of thinking, or make unexpected events highly predictable. From this point of view, the results of the a posteriori interviews (Artigue, 2006) were especially valuable.

Finally, thanks to cross-experiments and to the constructs developed for planning and evaluating them, the assumptions lying behind the design of the tools considered

[3] The notion of a-didactic adaptation is attached to the notion of a-didactic situation, a core concept in the theory of didactical situations. This notion denotes a situation where students behave “mathematically”, forgetting for a while that the situation has been built with a precise educational goal, freeing themselves from the pressure of the didactic contract. For an elementary introduction to the theory of didactical situations, see Warfield (2006).

were made clearer and teams got a clearer vision of the kind of theoretical integration they could achieve. Moreover, developers were provided with new ways of employing their tool and, thus, new perspectives to the design process itself were offered. Teams also gained the conviction that theoretical networking or integration cannot be achieved just by reading and discussing. Knowledge in this domain, as in any other, can only result from collective practice, organizing the communication between different cultures in appropriate ways. In TELMA this is done with the cross-experiments methodology and with the didactical functionality construct and the meta-language of key concerns.

As shown by the research on communities of practices, communication can be also supported by the identification of some boundary objects (Lee, 2007). In the TELMA cross-experiments, two different notions have apparently played such a role: the notion of instrument and that of a priori analysis, which as expressed in Artigue (2007)

> has become progressively shared, not, of course, for each of us with the meaning given to it in the theory of didactical situations, where it originated, but filled with what our different approaches found reasonable to try to anticipate and control (p. 79).

Such notions are to be more widely tested to investigate their potential for supporting comparison as well as the development of connections and complementarities among teams.

5.3 Technology-Enhanced Learning in Mathematics: Considering Techno-mathematical Literacies Outside School

The analysis of ICT evolution in education indicates that there is a widely assumed appreciation that in the design of ICT-based learning environments the whole learning situation should be considered, that is, not only the tool, but the teachers who will be using the software, the ways in which it will be used, the curriculum objectives, the social context and way in which learning is organized. TELMA work shows up to what point such systemic views are also necessary following collaboration and integration between research teams working in different contexts and cultures about the educational use of digital technology.

However, at this point, we reinstate our earlier remarks concerning the novel kinds of mathematical knowledge – techno-mathematical literacies – that have become necessary as a result of the ubiquitous but largely invisible mathematical relationships built into ICT systems in workplaces, and elsewhere. In recent workplace-based studies[4] focusing on mathematical knowledge (see, for example,

[4] The work referred to was performed both within the Learning and Technology at Work Kaleidoscope Special Interest Group and in the Techno-mathematics in the Workplace project (funded by the Teaching and Learning Research Programme of the Economic and Social Research Council – 2004–2007) by the group composed of Richard Noss, Celia Hoyles, Phillip Kent, Arthur Bakker, Chand Bhinder and David Guile.

Noss et al., 2007), the techno-mathematical literacies (TmL) needed by a wide variety of employees in four manufacturing and service sectors in the UK were investigated. From the point of view of this chapter, the relevant findings emerged from a series of iterative design-based experiments undertaken with employer-partners, to design learning opportunities to develop the TmL identified in the first phase. Learning opportunities incorporated technologically enhanced "boundary objects" that modelled elements of the work process or were reconstructions of symbolic artefacts from workplace practice (Lee, 2007). The learning opportunities were embedded in activity sequences largely derived from authentic episodes recorded in the ethnographic studies or reported by employer-partners and aimed to allow exploration and discussion of the interconnections between the different inputs and outputs within the (normally invisible) models.

The researchers isolated three aspects of workplace learning that were consistently successful across the workplace sectors, namely

- Authenticity, in which situations derived from actual workplace events can be the subject of discussion and reflection.
- Visibility, in which hitherto invisible relationships become visible and manipulable.
- Complexity, in which relationships are represented in non-trivial ways that reflect real situations, but alternative representations are used which avoid conventional and usually problematic algebraic symbolism.

While these principles concern workplace learning, they do, we think, have lessons for broader learning contexts (including schools); moreover, they illustrate the reciprocal relationship between knowledge and pedagogy – how, for example, an engagement with new kinds of knowledge can catalyse new approaches to learning (and vice versa). The promise of digital technologies, particularly, in allowing authentic and complex models to be probed, manipulated and modified, offers genuinely novel epistemological as well as didactical opportunities to introduce modelling as mathematical knowledge in new and hardly tested ways (see, for example, Wilensky, 2003).

Moreover, the increasing necessity to pay attention to a knowledge characterized by significant new attributes such as accelerated production, continuous change, distribution in terms of geography and community through a variety of media and tools brings with it that an increasing importance has to be given to contextual aspects and to skills such as, for example, logical and strategic reasoning. Since problems posed in social and work settings are currently subject to constant change and do not lend themselves to pre-determined solution schemes, critical thinking, under an increasing mass of stimuli, is to be systematically cultivated as a key factor for growth. Further, the increasing need to wade through vast amounts of distributed information emphasizes the importance of capacities related to information problem-solving (Vakkari, 1999), especially the capacity to select, re-organize and integrate information and to be able, as mentioned before, to deal with quantitative information presented in different visual and iconic representations.

The workplace and societal perspectives thus add new epistemological and concrete indications to school mathematics, especially for the provision of a basic knowledge that takes into account the new needs posed by the digital revolution. Techno-mathematical literacies are required to be developed in order to provide all students with skills and abilities that can support them in becoming effective members of a flexible, adaptable and competitive workforce and to engage in lifelong learning.

5.4 Conclusion

In conclusion, we are acutely aware of the importance of coordinating different perspectives and methodologies for throwing light on problems of technology-enhanced learning. This is a general issue, transcending the school disciplines, but each of these disciplines raises specific problems. Regarding the particular case of mathematics, the Kaleidoscope Network of Excellence allowed European researchers involved in mathematics education to approach this issue from a diversity of facets, both in its transversal and specific dimensions. We have tried to reflect these characteristics in this chapter by presenting the complementary advances made possible through the ERT TELMA and the SIG Learning and Technology at Work. The interrelationships between the various teams participating in TELMA allowed a productive investigation of contexts, settings and methodologies, which would have been difficult – if not impossible – without the involvement of a network like Kaleidoscope, allowing us to use differences between groups to assess what really might be invariant among them. TELMA allowed the joint development of a methodology, the cross-experimentation methodology, and of specific constructs, such as didactical functionality and key concerns. The initiative showed the effectiveness of these developments for promoting communication and coordination among different theoretical perspectives and contexts in research studies concerning technology-enhanced learning in mathematics. Even if nested in a specific discipline, mathematics, these results have certainly a more general value.

In a complementary way, the advances of the Learning and Technology at Work group (and its national "TmL" project) allowed to expand the approach on technology-enhanced learning in mathematics beyond the sole school perspective so common in research studies in this area. The work performed opened up the possibility of bringing perspectives from the workplace where, thanks to a reflection carried out in a different and more global context, novel kinds of mathematical knowledge, techno-mathematical literacies, have assumed a critical importance.

This cross-fertilization – of school and workplace settings – is a pointer, perhaps, to a more interesting issue which merits further investigation. Mathematics in school is a rather special kind of entity, an (almost) arbitrary "sliver" (as Papert has called it) of mathematical thought, and one which is most often divorced from any contextual reality (except, of course, the artificial reality of mathematical "problems"). Workplaces are, on the contrary, rich in contextual knowledge, and in so far as they deal with abstractions at all, these are always embedded within situations and – most crucially – technologies. By bringing these two settings together, we hardly solve the problem of making mathematics more meaningful for learners, but we can, at least, delineate some of the roles for technology in both contexts.

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