1 Introduction

This book represents Part 2 of a larger work on the structural synthesis of parallel robots. The originality of this work resides in combining new formulae for the structural parameters and the evolutionary morphology in a unified approach of structural synthesis giving interesting innovative solutions for parallel robots. Part 1 (Gogu 2008a) presented the methodology of structural synthesis and the systematisation of structural solutions of simple and complex limbs with two to six degrees of connectivity systematically generated by the structural synthesis approach. Part 2 of this work focuses on the structural solutions of *translational parallel robotic manipulators* (TPMs) with two and three degrees of mobility.

This section recalls the terminology, the new formulae for the main structural parameters of parallel robots (mobility, connectivity, redundancy and overconstraint) and the main features of the methodology of structural synthesis based on the evolutionary morphology presented in Part 1.

1.1 Terminology

Robots can be found today in the manufacturing industry, agricultural, military and domestic applications, space exploration, medicine, education, information and communication technologies, entertainment, etc.

We have presented in Part 1 various definitions of the word *robot* and we have seen that it is mainly used to refer to a wide range of mechanical devices or mechanisms, the common feature of which is that they are all capable of movement and can be used to perform physical tasks. Robots take on many different forms, ranging from humanoid, which mimic the human form and mode of movement, to industrial, whose appearance is dictated by the function they are to perform. Robots can be categorized as robotic manipulators, wheeled robots, legged robots swimming robots, flying robots, androids and self reconfigurable robots which can apply themselves to a given task. This book focuses on parallel robotic manipulators which are the counterparts to the serial robots. The various definitions of *robotics* converge towards the integration of the design and the end use in the

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studies related to robotics. This book focuses on the conceptual design of parallel robots.

Although the appearance and capabilities of robots vary greatly, all robots share the features of a mechanical, movable structure under some form of control. The structure of a robot is usually mostly mechanical and takes the form of a mechanism having as constituent elements the links connected by joints.

Serial or parallel kinematic chains are concatenated in the robot mechanism. The *serial kinematic chain* is formed by links connected sequentially by joints. Links are connected in series as well as in parallel making one or more closed-loops in a *parallel mechanism*. The mechanical architecture of *parallel robots* is based on parallel mechanisms in which a member called a *moving platform* is connected to a reference member by at least two *limbs* that can be simple or complex. The robot *actuators* are integrated in the limbs (also called legs) usually closed to the fixed member, also called the *base* or the *fixed platform*. The moving platform positions the robot end-effector in space and may have anything between two and six degrees of freedom. Usually, the number of actuators coincides with the degrees of freedom of the mobile platform, exceeding them only in the case of redundantly-actuated parallel robots.

The paradigm of parallel robots is the *hexapod*-type robot, which has six degrees of freedom, but recently, the machine industry has discovered the potential applications of lower-mobility parallel robots with just two, three, four or five degrees of freedom. Indeed, the study of this type of parallel manipulator is very important. They exhibit interesting features when compared to hexapods, such as a simpler architecture, a simpler control system, high-speed performance, low manufacturing and operating costs. Furthermore, for several parallel manipulators with decoupled or uncoupled motions, the kinematic model can be easily solved to obtain algebraic expressions, which are well suited for implementation in optimum design problems. Parallel mechanisms can be considered a well-established solution for many different applications of manipulation, machining, guiding, testing, control, etc.

The terminology used in this book is mainly established in accordance with the terminology adopted by the International Federation for the Promotion of Mechanism and Machine Science (IFToMM) and published in (Ionescu 2003). The main terms used in this book concerning kinematic pairs (joints), kinematic chains and robot kinematics are defined in Tables 1.1–1.3 in Part 1 of this work. They are completed by some complementary remarks, notations and symbols used in this book.

IFToMM terminology (Ionescu 2003) defines a *link* as a mechanism element (component) carrying kinematic pairing elements and a *joint* is a physical realization of a kinematic pair. The *pairing element* represents the assembly of surfaces, lines or points of a solid body through which it may contact with another solid body. The *kinematic pair* is the mechanical model of the connection of two pairing elements having relative motion of a certain type and degree of freedom.

In the standard terminology, a *kinematic chain* is an assembly of links (mechanism elements) and joints, and a *mechanism* is a kinematic chain in which one of its links is taken as a "frame". In this definition, the "*frame*" is a mechanism element deemed to be fixed. In this book, we use the notion of *reference element* to define the "frame" element. The reference element can be fixed or may merely be deemed to be fixed with respect to other mobile elements. The *fixed base* is denoted in this book by 0. A mobile element in a kinematic chain G is denoted by n_G ($n = 1, 2, \ldots$). Two or more links connected together in the same link such that no relative motion can occur between them are considered as one link. The identity symbol "≡" is used between the links to indicate that they are welded together in the same link. For example, the notation $1_G \equiv 0$ is used to indicate that the first link 1_G is the fixed base.

A kinematic chain G is denoted by the sequence of its links. The notation $G\ (1_G \equiv 0 – 2_G – \ldots – n_G)$ indicates a kinematic chain in which the first link is fixed and the notation $G\ (1_G – 2_G – \ldots n_G)$ a kinematic chain with no fixed link.

We will use the notion of mechanism to qualify the whole mechanical system, and the notion of kinematic chain to qualify the sub-systems of a mechanism. So, in this book, the same assembly of links and joints G will be considered to be a kinematic chain when integrated as a sub-system in another assembly of links and joints and will be considered a mechanism when G represents the whole system. The systematization, the definitions and the formulae presented in this book are valuable for mechanisms and kinematic chains.

We use the term *mechanism element* or *link* to name a component (member) of a mechanism. In this book, unless otherwise stated, we consider all links to be rigid. We distinguish the following types of links:

(a) *Monary link* – a mechanism element connected in the kinematic chain by only one joint (a link which carries only one kinematic pairing element).

(b) *Binary link* – a mechanism element connected in the kinematic chain by two joints (a link connected to two other links).

(c) *Polinary link* – a mechanism element connected in the kinematic chain by more than two joints (ternary link – if the link is connected by three joints, quaternary link if the link is connected by four joints).

The IFToMM terminology defines open/closed kinematic chains and mechanisms, but it does not introduce the notions of simple (elementary) and complex kinematic chains and mechanisms. A *closed kinematic chain* is a kinematic chain in which each link is connected with at least two other links, and an *open kinematic chain* is a kinematic chain in which there is at least one link which is connected in the kinematic chain by just one joint. In a *simple open kinematic chain* (open-loop mechanism) only monary and binary links are connected. In a *complex kinematic chain* at least one ternary link exists. We designate in each mechanism two extreme elements called reference element and final element. They are also called *distal links*. In an open kinematic chain, these elements are situated at the extremities of the chain. In a single-loop kinematic chain, the final element can be any element of the chain except the reference element. In a parallel mechanism, the two extreme elements are the *mobile and the reference platform*. The two platforms are connected by at least two simple or complex kinematic chains called limbs. Each limb contains at least one joint. A *simple limb* is composed of a simple open kinematic chain in which the final element is the mobile platform. A *complex limb* is composed of a complex kinematic chain in which the final element is also the mobile platform.

IFToMM terminology (Ionescu 2003) uses the term *kinematic pair* to define the mechanical model of the connection of links having relative motion of a certain type and degree of freedom. The word *joint* is used as a synonym for the kinematic pair and also to define the physical realization of a kinematic pair, including connection via intermediate mechanism elements. Both synonymous terms are used in this text.

Usually, in parallel robots, *lower pairs* are used: *revolute R*, *prismatic P*, *helical H*, *cylindrical C*, *spherical S* and *planar pair E*. The definitions of these kinematic pairs are presented in Table 1.1 – Part 1. The graphical representations used in this book for the lower pairs are presented in Fig. 1.1a–f. Universal joints and *homokinetic joints* are also currently used in the mechanical structure of the parallel robots to transmit the rotational motion between two shafts with intersecting axes. If the instantaneous velocities of the two shafts are always the same, the kinematic joint is homokinetic (from the Greek "homos" and "kinesis" meaning "same" and "movement"). We know that the universal joint (*Cardan joint* or Hooke's joint) are *heterokinetic joints*. Various types of homokinetic joints are known today: Tracta, Weiss, Bendix, Dunlop, Rzeppa, Birfield, Glaenzer, Thompson, Triplan, Tripode, UF (undercut-free) ball joint, AC (angular contact) ball joint, VL plunge ball joint, DO (double offset) plunge ball joint, AAR (angular adjusted roller), helical flexure U-joints, etc. (Dudiță et al. 2001a, b). The graphical representations used in this book for the universal homokinetic joints are presented in Fig. 1.1g–h.

Joints with idle mobilities are commonly used to reduce the number of overconstraints in a mechanism. The *idle mobility* is a potential mobility that is not used by the mechanism and does not influence a mechanism's mobility in the hypothesis of perfect manufacturing and assembling precisions. In theoretical conditions, when no errors exist with respect to parallel and perpendicular positions of joint axes, motion amplitude in an idle mobility is zero. Real life manufacturing and assembling processes introduce errors in the relative positions of the joint axes and, in this case, the idle mobilities become effective mobilities usually with small amplitudes, depending on the precision of the parallel robot.

A parallel robot can be illustrated by a physical implementation or by an abstract representation. The physical implementation is usually illustrated by robot photography and the abstract representation by a CAD model, structural diagram and structural graph.

Figure 1.2 gives an example of the various representations of a Gough-Stewart type parallel robot largely used today in industrial applications.

The physical implementation in Fig. 1.2a is a photograph of the parallel robot built by Deltalab (http://www.deltalab.fr/). In a CAD model (Fig. 1.2b) the links and the joints are represented as being as close as possible to the physical implementation (Fig. 1.2a). In a *structural diagram* (Fig. 1.2c) they are represented by simplified symbols, such as those introduced in Fig. 1.1, respecting the geometric relations defined by the relative positions of joint axes. A *structural graph* (Fig. 1.2d) is a network of vertices or nodes connected by edges or arcs with no geometric relations. The links are noted in the nodes and the joints on the edges. We can see that the Gough–Stewart type parallel robot has six identical limbs denoted in Fig. 1.2c by A, B, C, D, E and F. The final link is the mobile platform $4 \equiv 4_A \equiv 4_B \equiv 4_C \equiv 4_D \equiv 4_E \equiv 4_F$ and the reference member is the fixed platform $1_A \equiv 1_B \equiv 1_C \equiv 1_D \equiv 1_E \equiv 1_F \equiv 0$. Each limb is connected to both platforms by spherical pairs.

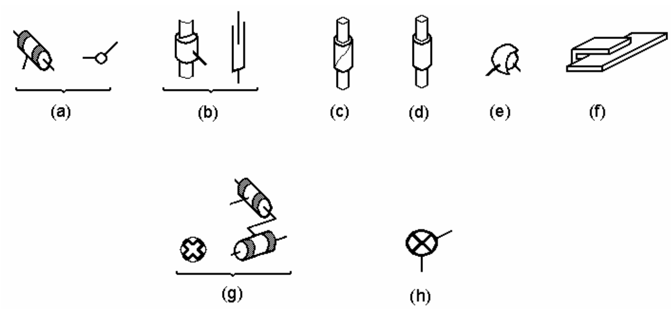

Fig. 1.1. Symbols used to represent the lower kinematic pairs and the kinematic joints: (**a**) revolute pair, (**b**) prismatic pair, (**c**) helical pair, (**d**) cylindrical pair, (**e**) spherical pair, (**f**) planar contact pair, (**g**) universal joint and (**h**) homokinetic joint

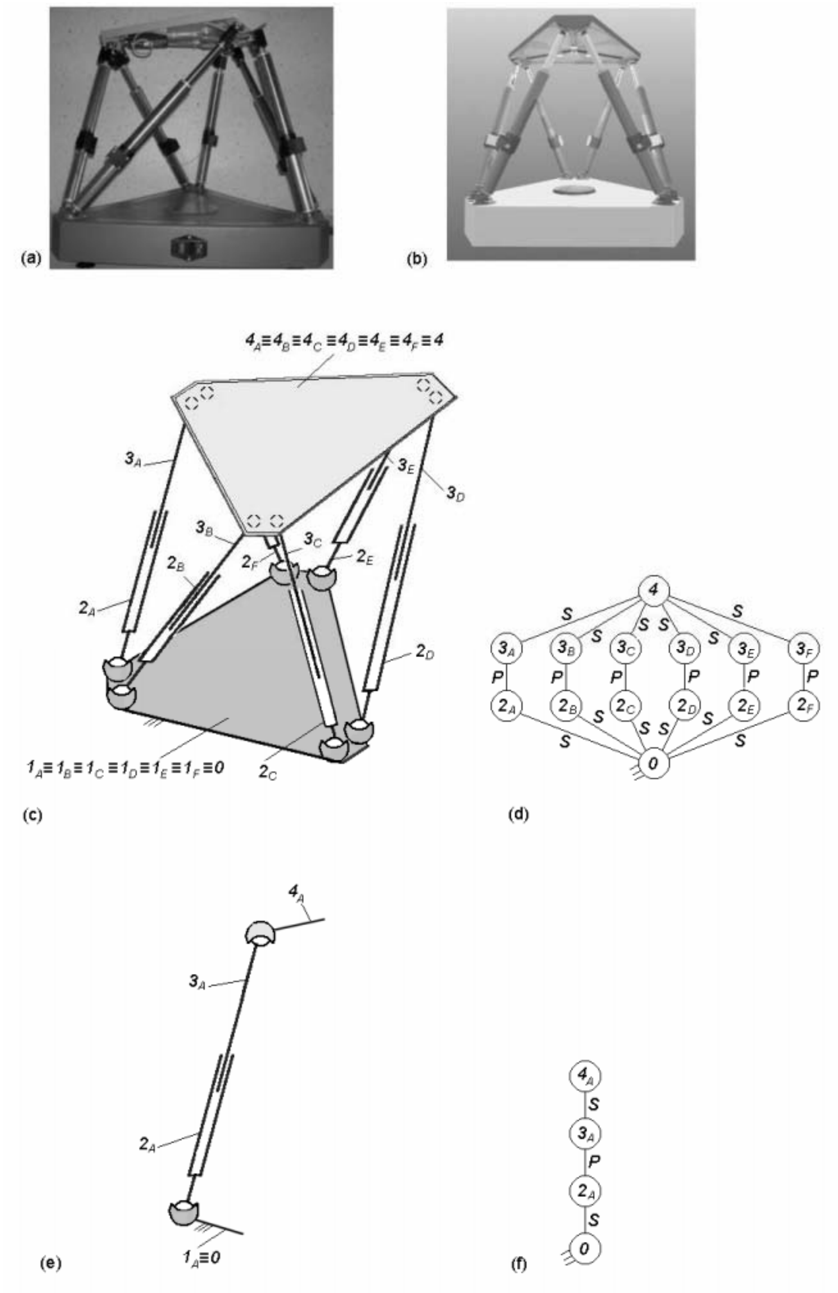

Fig. 1.2. Various representations of a Gough–Stewart type parallel robot: physical implementation (**a**), CAD model (**b**), structural diagram (**c**) and its associated graph (**d**), A-limb (**e**) and its associated graph **(f)**

A prismatic pair is actuated in each limb. The spherical pairs are not actuated and are called *passive pairs*. The two platforms are polinary links, the other two links of each limb are binary links. The mechanism associated with the Gough–Stewart type parallel robot is a complex mechanism with a multi-loop associated graph (Fig. 1.2d).

The simple open kinematic chain associated with *A*-limb is denoted by A ($1_A \equiv 0–2_A–3_A–4_A \equiv 4$) – Fig. 1.2e and its associated graph is tree-type (Fig. 1.2f).

We consider the general case of a robot in which the end-effector is connected to the reference link by $k \geq 1$ kinematic chains. The *end-effector* is a polynary link called a mobile platform in the case of parallel robots, and a monary link for serial robots. The reference link may either be the fixed base or may be deemed to be fixed. The kinematic chains connecting the end-effector to the reference link can be simple or complex. They are called limbs or legs in the case of parallel robots. A serial robot can be considered to be a parallel robot with just one simple limb, and a hybrid robot a parallel robot with just one complex limb.

We denote by $F \leftarrow G_1–G_2–\ldots G_k$ the kinematic chain associated with a general serial, parallel or hybrid robot, and by G_i ($1_{Gi}–2_{Gi}–\ldots–n_{Gi}$) the kinematic chain associated with the *i*th limb ($i = 1,2,\ldots,k$). The end effector is $n \equiv n_{Gi}$ and the reference link $1 \equiv 1_{Gi}$. If the reference link is the fixed base, it is denoted by $1 \equiv 1_{Gi} \equiv 0$. The total number of robot joints is denoted by p.

A *serial robot* $F \leftarrow G_1$ is a robot in which the end-effector $n \equiv n_{G1}$ is connected to the reference link $1 \equiv 1_{G1}$ by just one simple open kinematic chain G_i ($1_{Gi}–2_{Gi}–\ldots n_{Gi}$) called a serial kinematic chain.

A *parallel robot* $F \leftarrow G_1–G_2–\ldots G_k$ is a robot in which the end-effector $n \equiv n_{Gi}$ is connected in parallel to the reference link $1 \equiv 1_{Gi}$ by $k \geq 2$ kinematic chains G_i ($1_{Gi}–2_{Gi}–\ldots n_{Gi}$) called limbs or legs.

A *hybrid serial-parallel robot* $F \leftarrow G_1$ is a robot in which end-effector $n \equiv n_{G1}$ is connected to reference link $1 \equiv 1_{G1}$ by just one complex kinematic chain G_i ($1_{Gi}–2_{Gi}–\ldots n_{Gi}$) called complex limb or complex leg.

A *fully-parallel robot* $F \leftarrow G_1–G_2–\ldots G_k$ is a parallel robot in which the number of limbs is equal to the robot mobility ($k = M \geq 2$), and just one actuator exists in each limb.

1.2 Methodology of structural synthesis

Recent advances in research on parallel robots have contributed mainly to expanding their potential use to both terrestrial and space applications including areas such as high speed manipulation, material handling, motion platforms, machine tools, medical applications, planetary and underwater exploration. Therefore, the need for methodologies devoted to the systematic design of highly performing parallel robots is continually increasing. *Structural synthesis* is directly related to the conceptual phase of robot design, and represents one of the highly challenging subjects in recent robotics research. One of the most important activities in the invention and the design of parallel robots is to propose the most suitable solutions to increase the performance characteristics.

The challenging and difficult objective of structural synthesis is to find a method to set up the mechanical architecture to achieve the required structural parameters. The mechanical architecture or *topology* is defined by number, type and relative position of joint axes in the parallel robot. The structural parameters are mobility, connectivity, redundancy and the number of overconstraints. They define the number of actuators, the degrees of freedom and the motion-type of the moving platform. A *systematic approach* of structural synthesis founded on the theory of linear transformations and an evolutionary morphology have been proposed in Part 1 (Gogu 2008a). The approach integrates the new formulae for mobility, connectivity, redundancy and overconstraint of parallel manipulators (Gogu 2005d, e) and a new method of systematic innovation (Gogu 2005a).

1.2.1 New formulae for mobility, connectivity, redundancy and overconstraint of parallel robots

Mobility is the main structural parameter of a mechanism and also one of the most fundamental concepts in the kinematic and the dynamic modelling of mechanisms. IFToMM terminology defines the *mobility* or the *degree of freedom* as the number of independent coordinates required to define the configuration of a kinematic chain or mechanism.

We note that the mobility of a mechanism can be defined by the number of independent finite and/or infinitesimal displacements in the joints needed to define the configuration of the mechanism (Gogu 2008a).

Mobility M is used to verify the existence of a mechanism ($M > 0$), to indicate the number of independent parameters in robot modelling and to determine the number of inputs needed to drive the mechanism.

Earlier works on the mobility of mechanisms go back to the second half of the nineteenth century. During the twentieth century, sustained efforts were made to find general methods for the determination of the mobility of any rigid body mechanism. Various formulae and approaches were derived and presented in the literature. Contributions have continued to emerge in the last few years. Mobility calculation still remains a central subject in the theory of mechanisms.

In Part 1 (Gogu 2008a) we have shown that the various methods proposed in the literature for mobility calculation of the closed loop mechanisms fall into two basic categories:

(a) Approaches for mobility calculation based on setting up the kinematic *constraint equations* and calculating their *rank* for a given position of the mechanism with specific joint locations.

(b) Formulae for a quick calculation of mobility without the need to develop the set of constraint equations.

The approaches used for mobility calculation based on setting up the kinematic constraint equations and their rank calculation are valid without exception. The major drawback of these approaches is that the mobility cannot be determined quickly without setting up the kinematic model of the mechanism. Usually this model is expressed by the closure equations that must be analyzed for dependency. The information about mechanism mobility is derived by performing position, velocity or static analysis by using analytical tools (screw theory, linear algebra, affine geometry, Lie algebra, etc.). For this reason, the real and practical value of these approaches is very limited in spite of their valuable theoretical foundations. Moreover, the *rank* of the constraint equations is calculated in a given position of the mechanism with specific joint locations. The mobility calculated in relation to a given configuration of the mechanism is an instantaneous mobility which can be different from the general mobility (global mobility, full-cycle mobility). The *general mobility* represents the minimum value of the instantaneous mobility. For a given mechanism, general mobility has a unique value. It is a global parameter characterizing the mechanism in all its configurations except its *singular* ones. Instantaneous mobility is a local parameter characterizing the mechanism in a given configuration including singular ones. In a singular configuration the instantaneous mobility could be different from the general mobility. In this book, unless otherwise stated, general mobility is simply called mobility.

Note 1. In a *kinematotropic mechanism* with branching singularities, full-cycle mobility is associated with each branch. In this case, the *full-cycle mobility* (global mobility) is replaced by the branch mobility which represents the minimum value of the *instantaneous mobility* inside the same branch. As each branch has its own mobility, a single value for global mobility cannot be associated with the kinematotropic mechanisms. The term kinematotropic was coined by K. Wohlhart (1996) to define the linkages that permanently change their global mobility when passing by a singularity in which a certain transitory infinitesimal mobility is attained. Various kinematotropic parallel mechanisms have been recently presented (Fanghela et al. 2006; Gogu 2008b, c).

A formula for quick calculation of mobility is an explicit relationship between the following structural parameters: the number of links and joints, the motion/constraint parameters of joints and of the mechanism. Usually, these structural parameters are easily determined by inspection without any need to develop the set of constraint equations.

In Part 1, we have shown that several dozen approaches proposed in the last 150 years for the calculation of mechanism mobility can be reduced to the same original formula that we have called the Chebychev-Grübler-Kutzbach (CGK) formula in its original or extended forms. These formulae have been critically reviewed (Gogu 2005b) and a criterion governing mechanisms to which this formula can be applied has been set up in (Gogu 2005c). We have explained why this well-known formula does not work for some multi-loop mechanisms. New formulae for quick calculation of mobility have been proposed in (Gogu 2005d) and demonstrated via the *theory of linear transformations*. More details and a development of these contributions have been presented in Part 1.

The *connectivity* between two links of a mechanism represents the number of independent finite and/or infinitesimal displacements allowed by the mechanism between the two links.

The *number of overconstraints* of a mechanism is given by the difference between the maximum number of joint kinematic parameters that could lose their independence in the closed loops, and the number of joint kinematic parameters that actually lose their independence in the closed loops.

The *structural redundancy* of a kinematic chain represents the difference between the mobility of the kinematic chain and connectivity between its distal links.

Let us consider the case of the *parallel mechanism* $F \leftarrow G_1–G_2–\ldots G_k$ in which the mobile platform $n \equiv n_{Gi}$ is connected to the reference platform $1 \equiv 1_{Gi}$ by k simple and/or complex kinematic chains G_i ($1_{Gi}–2_{Gi}–\ldots n_{Gi}$) called limbs.

In Part 1, the following parameters have been associated with the parallel mechanism $F \leftarrow G_1–G_2–\ldots G_k$:

R_{Gi} – the *vector space* of relative velocities between the mobile and the reference platforms, n_{Gi} and 1_{Gi}, in the kinematic chain G_i disconnected from the parallel mechanism F,

R_F – the vector space of relative velocities between the mobile and the reference platforms, $n \equiv n_{Gi}$ and $1 \equiv 1_{Gi}$, in the parallel mechanism $F \leftarrow G_1–G_2–\ldots G_k$, whose basis is

$$(R_F)=(R_{G1} \cap R_{G2} \cap ... \cap R_{Gk}), \tag{1.1}$$

S_G – the connectivity between the mobile and the reference platforms, n_{Gi} and 1_{Gi}, in the kinematic chain G_i disconnected from the parallel mechanism F,

S_F – the connectivity between the mobile and the reference platforms, $n \equiv n_{Gi}$ and $1 \equiv 1_{Gi}$, in the parallel mechanism $F \leftarrow G_1–G_2–...G_k$.

We recall that the connectivity is defined by the number of *independent motions* between the mobile and the reference platforms. The notation $1 \equiv 1_{Gi} \equiv 0$ is used when the reference platform is the fixed base. The vector spaces of relative velocities between the mobile and the reference platforms are also called *operational velocity spaces*.

The following formulae demonstrated in Chapter 2 – Part 1 (Gogu 2008a) for *mobility* M_F, *connectivity* S_F, *number of overconstraints* N_F and *redundancy* T_F of the parallel mechanism $F \leftarrow G_1–G_2–...G_k$ are used in structural synthesis of parallel robotic manipulators:

$$M_F = \sum_{i=1}^{p} f_i - r_F , \tag{1.2}$$

$$N_F=6q-r_F , \tag{1.3}$$

$$T_F=M_F-S_F , \tag{1.4}$$

where

$$S_{Gi} = dim(R_{Gi}) , \tag{1.5}$$

$$S_F = dim(R_F) = dim(R_{G1} \cap R_{G2} \cap ... \cap R_{Gk}) , \tag{1.6}$$

$$r_F = \sum_{i=1}^{k} S_{Gi} - S_F + r_l , \tag{1.7}$$

$$p = \sum_{i=1}^{k} p_{Gi} , \tag{1.8}$$

$$q=p-m+1, \tag{1.9}$$

and

$$r_l = \sum_{i=1}^{k} r_{Gi} . \tag{1.10}$$

We note that p_{Gi} represents the number of joints of G_i-limb, p the total number of joints of parallel mechanism F, m the total number of links in mechanism F including the moving and reference platforms, q the total number of independent closed loops in the sense of graph theory, f_i the mobility of the ith joint, r_F the total number of joint parameters that lose their independence in mechanism F, r_{G_i} the number of joint parameters that lose their independence in the closed loops of limb G_i, r_l the total number of joint parameters that lose their independence in the closed loops that may exist in the limbs of mechanism F. In Eqs. (1.5) and (1.6), *dim* denotes the *dimension of the vector spaces*.

We denote by k_1 the number of simple limbs and by k_2 the number of complex limbs ($k = k_1 + k_2$). Eq. (1.8) indicates that the limbs of the parallel mechanism $F \leftarrow G_1–G_2–…G_k$ must be defined in such a way that a joint must belong to just one limb; that is the same joint cannot be combined in two or more limbs.

In Chapter 5 – Part 1 the following structural conditions have been established:

(a) For the *non redundant parallel robots* ($T_F = 0$)

$$S_F = M_F \le M_{Gi} \quad (i=1,...,k), \tag{1.11}$$

$$M_{Gi} = S_{Gi} \le 6 \quad (i=1,...,k), \tag{1.12}$$

(b) For the *redundant parallel robots* with $T_F > 0$

$$S_F < M_F \le M_{Gi} \quad (i=1,...,k), \tag{1.13}$$

$$M_{Gi} > S_{Gi} \le 6 \quad (i=1,...,k), \tag{1.14}$$

(c) For the *non overconstrained parallel robots* ($N_F = 0$)

$$M_F = \sum_{i=1}^{p} f_i - 6q, \tag{1.15}$$

(d) For the *overconstrained parallel robots* with $N_F > 0$

$$M_F > \sum_{i=1}^{p} f_i - 6q. \tag{1.16}$$

We recall that

$$M_{Gi} = \sum_{i=1}^{p_{Gi}} f_i - r_{Gi}. \tag{1.17}$$

We note that the intersection in Eqs. (1.1) and (1.6) is consistent if the vector spaces R_{Gi} are defined by the velocities of the same point situated on the moving platform with respect to the same reference frame. This point is called the *characteristic point*, and denoted by H. It is the point with the most restrictive motions of the moving platform.

The connectivity S_F of the moving platform $n \equiv n_{Gi}$ in the mechanism $F \leftarrow G_1\text{–}G_2\text{–}\ldots G_k$ is less than or equal to the mobility M_F of mechanism F.

The *basis* of the vector space R_F of relative velocities between the moving and reference platforms in the mechanism $F \leftarrow G_1\text{–}G_2\text{–}\ldots G_k$ must be valid for any point of the moving platform $n \equiv n_{Gi}$.

Note 2. When there are various ways to choose the bases of the vector spaces R_{Gi} in Eqs. (1.1) and (1.6), the bases (R_{Gi}) are selected such that the minimum value of S_F is obtained by Eq. (1.6). By this choice, the result of Eq. (1.2) fits in with the definition of general mobility as the minimum value of the instantaneous mobility.

The parameters used in the new formulae (1.1)–(1.17) can be easily obtained by inspection without calculating the rank of the homogeneous linear set of constraint equations associated with loop closure or without calculating the rank of the complete screw system associated to the joints of the mechanism. An analytical method to compute these parameters has also been developed in Part 1 just for verification and for a better understanding of the meaning of these parameters. These formulae have been successfully applied in Part 1 to structural analysis of various mechanisms including so called "paradoxical" mechanisms. These formulae are useful for the structural synthesis of various types of parallel mechanisms with $2 \leq M_F \leq 6$ and various combinations of independent motions of the moving platform. These solutions are obtained in a systematic approach of structural synthesis by using the limbs generated by the method of evolutionary morphology presented in Part 1.

1.2.2 Evolutionary morphology approach

Evolutionary morphology (EM) is a new method of systematic innovation in engineering design proposed by the author in (Gogu 2005a). EM is formalized by a 6-tuple of design objectives, protoelements (initial components), morphological operators, evolution criteria, morphologies and a termination criterion. The *design objectives* are the structural solutions, also called topologies, defined by the required values of mobility, connectivity overconstrained and redundancy and the level of motion coupling. The *protoelements* are the revolute and prismatic joints. The *morphological* *operators* are: (re)combination, mutation, migration and selection. These operators are deterministic and are applied at each generation of EM.

At least $M_F = S_F$ generations are necessary to evolve by successive combinations from the first generation of protoelements to a first solution satisfying the set of design objectives. Morphological migration could introduce new constituent elements formed by new joints or combinations of joints into the evolutionary process.

Evolutionary morphology is a complementary method with respect to *evolutionary algorithms* that starts from a given initial population to obtain an optimum solution with respect to a fitness function. EM creates this initial population to enhance the chance of obtaining a “more global optimum”. Evolutionary algorithms are optimization oriented methods; EM is a conceptual design oriented method. A detailed presentation of the evolutionary morphology can be found in Chapter 5 – Part 1.

1.2.3 Types of parallel robots with respect to motion coupling

Various levels of *motion coupling* have been introduced in Chapter 4 – Part 1 in relation with the *Jacobian matrix* of the robotic manipulator which is the matrix mapping (i) the actuated joint velocity space and the end-effector velocity space, and (ii) the static load on the end-effector and the actuated joint forces or torques.

Five types of *parallel robotic manipulators* (PMs) are introduced in Part 1: (I) *maximally regular* PMs, if the Jacobian J is an identity matrix throughout the entire workspace, (ii) *fully-isotropic* PMs, if J is a diagonal matrix with identical diagonal elements throughout the entire workspace, (iii) PMs with *uncoupled motions* if J is a diagonal matrix with different diagonal elements, (iv) PMs with *decoupled motions*, if J is a triangular matrix and (v) PMs with *coupled motions* if J is neither a triangular nor a diagonal matrix.

The term maximally regular parallel robot was recently coined by Merlet (2006) to define isotropic robots. We use this term to define just the particular case of fully-isotropic PMs, when the Jacobian matrix is an identity matrix throughout the entire workspace.

Isotropy of a robotic manipulator is related to the *condition number* of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is one. Thus, the condition number of the Jacobian matrix is an interesting *performance index* characterizing the distortion of a unit sphere under this linear mapping. The condition number of the Jacobian matrix was first used by Salisbury and Craig (1982) to design mechanical fingers and developed by Angeles (1997) as a kinetostatic performance index of the robotic mechanical systems. The isotropic design aims at ideal kinematic and dynamic performance of the manipulator (Fattah and Ghasemi 2002). In an isotropic configuration, the sensitivity of a manipulator is minimal with regard to both velocity and force errors and the manipulator can be controlled equally well in all directions. The concept of kinematic isotropy has been used as a criterion in the design of various parallel manipulators (Zanganeh and Angeles 1997; Tsai and Huang 2003).

Fully-isotropic PMs give a one-to-one mapping between the actuated *joint velocity space* and the *operational velocity space*. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission.

1.3 Translational parallel robots

This book focuses on the structural synthesis of *translational parallel robotic manipulators* (TPMs) with two and three degrees of freedom. In such a robot, the moving platform can undergo two or three independent translational motions and its orientation remains unchanged.

The translational parallel robots with two degrees of freedom are also called *T2-type parallel robots* and give two translational velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in the basis of the operational velocity vector space $(R_F) = (\boldsymbol{v}_1, \boldsymbol{v}_2)$ along with a constant orientation of the moving platform. We consider the *xy*-plane as the plane of motion of the moving platform ($\boldsymbol{v}_1 = \boldsymbol{v}_x$ and $\boldsymbol{v}_2 = \boldsymbol{v}_y$). *T2*-type parallel robots have mobility $M_F = 2$ and the connectivity between the moving and fixed platforms is $S_F = 2$.

These kinds of parallel robots are useful in *pick-and-place* operations when the end-effector only needs to undergo purely translational motion in one plane. Pick and place parallel robot mechanisms are typically used in light industries such as the electronics industry and packaging industries. They have to repeat accurately a simple transfer operation many times over with relatively high speed in two degrees of freedom planar motion without altering the orientation of the moving platform (Vainstock 1990; Hesselbach and Frindt 1999; Dagefoerde et al. 2001; Bergmeyer 2002; Huang et al. 2003, 2004, 2005, 2006).

The *direct kinematic model* of the *T2*-type parallel robots becomes

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [J]_{2\times 2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \tag{1.18}$$

where:
$v_1 = v_x = \dot{x}$ and $v_2 = v_y = \dot{y}$ are the translational velocities of the characteristic point *H* of the moving platform, $\dot{q}_1$ and $\dot{q}_2$ are the velocities of the actuated joints, $J_{2\times 2}$ is the Jacobian matrix.

As presented in Chapter 4 – Part 1, for translational parallel robots, the design and conventional Jacobians have the same expression and they are simply called Jacobians or Jacobian matrices.

To obtain a non redundant *T2*-type translational parallel robot, a basic limb presented in Figs. 6.1 and 6.2 – Part 1 is associated with at least one simple or complex limb with $2 \le M_{Gi} = S_{Gi} \le 6$ that integrates velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in the basis of its operational space. In this way, a large set of solutions can be obtained. We recall that the basic legs in Figs. 6.1 and 6.2 – Part 1 give rise to two independent translations along with a constant orientation of the moving platform.

The translational parallel robotic manipulators with three degrees of mobility are also called *T3-type parallel robots* and give three translational velocities $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ in the basis of the operational velocity vector space $(R_F) = (\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3)$ along with a constant orientation of the moving platform. *T3*-type parallel robots have mobility $M_F = 3$ and the connectivity between the moving and fixed platforms is $S_F = 3$.

This kind of parallel robots are useful in pick-and-place operations when the end-effector only needs to undergo purely translational motion in space. Pick and place parallel robots are also typically used in light industries such as the electronics and packaging industries. They have to repeat accurately a simple transfer operation many times over at a relatively high speed in three degrees of freedom spatial motion without altering the orientation of the moving platform.

The *direct kinematic model* of the *T3*-type parallel robots becomes

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = [J]_{3\times 3} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \tag{1.19}$$

where:
$v_1 = v_x = \dot{x}$, $v_2 = v_y = \dot{y}$ and $v_3 = v_z = \dot{z}$ are the translational velocities of the characteristic point H of the moving platform, $\dot{q}_1$, $\dot{q}_2$ and $\dot{q}_3$ are the velocities of the actuated joints, $J_{3\times 3}$ is the Jacobian matrix.

To obtain a non redundant *T3*-type translational parallel robot of type $F \leftarrow G_1$–G_2–G_3, a basic limb presented in Figs. 7.1–7.11 – Part 1 is associated with other two simple or complex limbs with $3 \leq M_{Gi} = S_{Gi} \leq 6$ that integrate velocities $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$ in the basis of their operational velocity spaces. We recall that the basic limbs in Figs. 7.1–7.11 – Part 1 give rise to three independent translations along with a constant orientation of the moving platform. In this way, a large set of solutions with coupled, decoupled, uncoupled motions along with maximally regular solutions can be obtained by using three simple or complex limbs with $3 \leq M_{Gi} = S_{Gi} \leq 6$ that respect the condition $(R_F) = (R_{G1} \cap R_{G2} \cap R_{G3}) = (\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3)$.

Translational parallel robots are largely used in classical manipulating processes, jiggle mechanisms (Li et al. 2005) and micro and nanomanipulation (Jensen et al. 2006; Xu and Li 2006) or MEMS fabrication (Bamberger et al. 2007).

The various architectures of TPM presented in the literature use the following types of kinematic pairs: revolute *R*, prismatic *P*, helical *H*, cylindrical *C*, spherical *S*, planar contact *E*, universal joint *U* as well as the

Table 1.1. Examples of implemented translational parallel robotic manipulators

No.	Robot name	Type	References
1	DELTA	*3-RRPaR*, *3-RUU*, *3-RPa^{ss}*, *3-PRPaR*	Clavel (1988, 1990, 1991)
2	University of Maryland manipulator	*3-RRPaR*	Tsai and Stamper (1996)
3	NUWAR	*3-RPa^{ss}*	Miller (1999, 2001)
4	Urane Sx	*3-PUU*	Company and Pierrot (2002)
5	Orthoglide	*3-PRPaR*	Wenger and Chablat (2000) Chablat and Wenger (2002)
6	Cartesian Parallel Manipulator	*3-PRRR*	Kim and Tsai (2002, 2003)
7	Tripteron	*3-CRR, 3-PRRR*	Gosselin and Kong (2002) Kong and Gosselin (2002b, c) Gosselin et al. (2004)
8	Isoglide*3-T3*	*3-PRRR*	Gogu (2002)

Table 1.2. Literature dedicated to the study of the parallel robots of DELTA topology

No.	Type of study	References
1	Dimensional synthesis and optimization	Company and Pierrot (2002) Bruzzone et al. (2002) Kosinska et al. (2003) Stock and Miller (2003) Chablat et al. (2004a, b) Johannesson et al. (2004) Yoon et al. (2004) Lou and Li (2006) Laribi et al. (2007)
2	Dynamic modelling	Stamper and Tsai (1998)
3	Isotropic conditions	Baron et al. (2002)
4	Kinematic analysis	Company and Pierrot (2002) Bruzzone et al. (2002) Yoon et al. (2004) Lee et al. (2005) Laribi et al. (2007) Lou and Li (2006)
5	Kinematic calibration	Vischer and Clavel (1998)
6	Kinematic and dynamic modelling	Pierrot et al. (1990, 1991)
7	Modelling and control	Pierrot et al. (1991)
8	Singularities	Di Gregorio (2004) Liu et al. (2003)
9	Stiffness	Liu et al. (2003) Yoon et al. (2004)
10	Workspace	Miller (1999, 2002) Di Gregorio and Zanforlin (2003) Liu et al. (2003) Chablat et al. (2004a, b)

parallelogram loop *Pa* which can be considered as a complex pair of circular translation (Huang and Li 2003; Liu and Wang 2003; Hervé 2004; Liu et al. 2004). Examples of implemented translational parallel robotic manipulators are presented Table 1.1.

As a matter of fact, some architectures of TPMs are quite popular already, for instance the *DELTA* robots of types *3-RRPaR*, *3-RUU*, *3-PRPaR* proposed by Clavel (1988, 1990). Much literature is dedicated to the study of the parallel robots of DELTA topologies (Table 1.2). and industrial implementation of these solutions has already been reached by Demareux, ABB, Hitachi, Mikron, Renault-Automation, Comau and other.

Various solutions are derived from this topology by integrating parallelogram loops in different configurations: *3-RHPaR* Y-star, the *3-RPPR* Prism-Robot, the *2-RHPaR + 1PRPaR* H-robot (Hervé and Sparacino 1991, 1992, 1993; Hervé 1995), *3-RRPaR* University of Maryland manipulator (Tsai and Stamper 1996), *3-PUU* Urane Sx (Company and Pierrot 2002).

A special case of *3-PRPaR*-type is *Orthoglide* having the linear actuators on three orthogonal directions. (Wenger and Chablat 2000; Chablat and Wenger 2002). The literature dedicated to the study of this parallel robot is presented in Table 1.3.

Various other architectures have been proposed to achieve three pure translational motions of the platform by using limbs with three, four and five degrees of freedom (Hervé and Sparacino 1991, 1992, 1993; Tsai 1998; Frisoli et al. 2000; Tsai and Joshi 2000; Zhao and Huang 2000; Di Gregorio 2001; Carricato and Parenti-Castelli 2001, 2003a, 2004b; Huang and Li 2002a, b, 2003; Gao et al. 2002; Zlatanov and Gosselin 2004; Kong and Gosselin 2004a, b, 2007; Li et al. 2005; Alizade et al. 2007; Lee and Hervé 2006.). Many studies have been dedicated to these various architectures of translational parallel robots (Table 1.4).

The first solutions of maximally regular and implicitly fully-isotropic *T3*-type translational parallel robots were developed at the same time and independently by Carricato and Parenti-Castelli at University of Genoa, Kim and Tsai at University of California, Gosselin and Kong at University of Laval, and the author at the French Institute of Advanced Mechanics (*IFMA*). In 2002, the four groups published the first results of their works (Carricato and Parenti-Castelli 2002; Kim and Tsai 2002; Gosselin and Kong 2002; Kong and Gosselin 2002a, b, c; Gogu 2002). Each of the last

Table 1.3. Literature dedicated to the study of the Orthoglide parallel robot

No.	Type of study	References
1	Architecture optimization	Chablat and Wenger (2002, 2003)
2	Dextrous workspace and design parameters	Chablat et al. (2004a, b)
3	Design and geometric synthesis	Majou et al. (2002a, b) Pashkevich et al. (2005)
4	Dynamic modelling	Guégan an Khalil (2002)
5	Kinematics and workspace	Pashkevich et al. (2006)
6	Sensitivity analysis	Caro et al. (2006)
7	Stiffness analysis	Majou et al. (2005)

Table 1.4. Literature dedicated to the study of translational parallel manipulators

No.	Type of study	References
1	Calibration	Bleicher and Günther (2004)
2	Dynamic performances	Di Gregorio and Parenti-Castelli (2004)
3	Dynamic balancing	Wu and Gosselin (2005)
4	Dimensional synthesis	Callegari and Marzetti (2003) Wolf and Shoham (2006)
5	Kinematic analysis	Joshi and Tsai (2002) Tsai and Joshi (2002) Carricato and Parenti-Castelli (2003b, c) Ji and Wu (2003) Kim and Chung (2003) Li et al. (2004b, 2005) Shen et al. (2005) Zeng et al. (2006) Zhao et al. (2006)
6	Kinetostatic indices	Gogu (2007)
7	Mobility analysis	Li and Huang (2004) Rico et al. (2005)
8	Position accuracy	Han et al. (2002) Frisoli et al. (2007) Xu and Li (2007)
9	Optimal design and modelling	Miller (2004)
10	Workspace analysis and optimization	Badescu et al. (2002) Zhao et al. (2008) Tsai and Joshi (2002)
11	Singularity analysis	Wolf and Shohan (2003) Liu et al. (2003) Zhao et al. (2005) Di Gregorio and Parenti-Castelli (2002 Li et al. (2004) Callegari and Tarantini (2003)

three groups has built a prototype of this robot in their research laboratories and has called this robot *CPM* (Kim and Tsai 2002), *Orthogonal Tripteron* (Gosselin et al. 2004) or *Isoglide3-T3* (Gogu 2004a). The first practical implementation of this robot was the CPM developed at University of California by Kim and Tsai (2002).

The various methods used in structural synthesis of TPM are systematized in Table 1.5.

Table 1.5. Approaches used in the structural synthesis of translational parallel manipulators

No	Approach	References
1	Additional passive limb	Brogårdh (2002) Hess-Coelho (2007)
2	Algebraic methods	Danescu (1995)
3	CAD functionalities	Lu (2004)
4	Constraint method	Huang and Li (2002a, b, 2003)
5	Evolutionary morphology	Gogu (2002, 2004a, b, 2008a)
6	Group theory	Hervé (1995, 2004) Hervé and Sparacino (1991, 1992, 1993); Lee and Hervé (2006); Rico et al. (2006). Kong and Gosselin (2007)
7	Mobility formulae	Alizade and Bayram (2004); Alizade et al. (2007) Tsai (1998, 1999) Yu et al. (2006)
8	Plücker coordinates	Gao et al. (2002, 2005)
9	Screw theory	Tsai (1999); Frisoli et al. (2000); Kong and Gosselin (2001, 2002a, b, c, 2004a, b), Huang and Li (2002a, b, 2003); Li and Huang (2004) Fang and Tsai (2004)
10	Structural parameters	Gogu (2002, 2004a, b, 2008a)
11	Theory of linear transformations	Gogu (2002, 2004a, b, 2008a)
11	Units for single-opened-chains	Jin and Yang (2004)
12	Velocity-loop equations	Di Gregorio and Parenti-Castelli (1998); Di Gregorio (2002), Carricato and Parenti-Castelli (2001, 2002, 2003a, b, c, 2004b)