Chapter 16
Power Laws in Urban Supply Networks, Social Systems, and Dense Pedestrian Crowds

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16.1 Scaling Laws in Urban Supply Networks

The classical view of the spatio-temporal evolution of cities in developed countries is that urban spaces are the result of (centralized) urban planning. After the advent of complex systems' theory, however, people have started to interpret city structures as a result of self-organization processes. In fact, although the dynamics of urban agglomerations is a consequence of many human decisions, these are often guided by optimization goals, requirements, constraints, or boundary conditions (such as topographic ones). Therefore, it appears promising to view urban planning decisions as results of the existing structures and upcoming ones (e.g. when a new freeway will lead close by in the near future). Within such an approach, it would not be surprising anymore if urban evolution could be understood as a result of self-organization (Batty & Longley, 1994; Frankhauser, 1994; Schweitzer, 1997).

Comparison with biological systems promises further insight. Quantities like metabolic rates, population growth, life-span, etc. have been discovered to scale with the average body mass of biological species over about 20 orders of magnitude (West, Brown, & Enquist, 1997; Enquist, Brown, & West, 1998). The corresponding power laws reflect the underlying function, structure, and organization of biological species and even extend to the realm of ecological systems such as natural forests with different sized trees. For example, it turns out that all trees of one size class consume the same amount of solar energy as trees of a different size class (Enquist et al., 1998).

It would be interesting to find out, whether a system of cities could be viewed as an ecological system with similar relationships. In this connection, it is useful to remember Zipf's (1949) law, according to which the population sizes of cities are inversely proportional to their rank. This implies the relationship $n_k \propto 1/N_k$ for the number n_k of cities of size class k (e.g. with more than 5×10^k but less than $5 \times 10^{k+1}$ inhabitants). Therefore, as the energy usage E_i by the population of a city i (when

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Table 16.1 Scaling exponents and their 95% confidence intervals for different variables of electric energy supply in Germany as function of population size. For details, (see Kühnert et al., 2006)

Variable	Exponent	95% Confidence interval
Usable electric energy	1.1	[1.04, 1.13]
Electric energy delivery to households	1.0	[0.96, 1.06]
Length of low-voltage cables	0.9	[0.82, 0.92]

neglecting the energy consumption by industrial production) grows linearly with the population N_i (see the entry "electric delivery to households" in Table 16.1), the number of cities of size k times their energy usage is constant. In other words, the inhabitants of all cities of one size class k consume the same energy as the inhabitants of all cities of any other size class, similar to the ecological example of trees in a forest.

Among the many different approaches trying to explain Zipf's law (e.g., Simon, 1955; Steindl, 1965; Schweitzer, 2003), the one by Gabaix (1999) is surprising because of its simplicity. According to Gabaix, the simplest stochastic model with multiplicative noise $\xi_i(t)$, namely

$$\frac{dN_i}{dt} = [A + \xi_i(t)]\, N_i(t), \tag{16.1}$$

is able to generate Zipf's distribution. In agreement with "Gibrat's law" (Gibrat, 1931; Sutton, 1997), it assumes that the growth rates $A_i(t) = A + \xi_i(t)$ are stochastically distributed and varying around a characteristic value A independent of the (population) size $N_i(t)$ of a city i. Note, however, that the exponent of Zipf's law seems to be different from 1 in some countries (Pumain, Paulus, Vacchiani, & Lobo, 2006).

Therefore, let us discuss the consequences if the deterministic part of the growth law would be slightly different from Equation (16.1), namely of the form

$$\underbrace{dN_i/dt}_{Growth} = \underbrace{BN_i(t)^\beta}_{\substack{Resource\\Generation}} - \underbrace{CN_i(t)^\gamma}_{Maintenance} \tag{16.2}$$

This equation reflects that the difference between the generation of resources *Ni* of system i and its maintenance determines its growth *dNi/dt* in time. B and C are treated as constants. The powers β and γ allow one to take into account scaling exponents different from 1. While the case $\beta = \gamma = 1$ corresponds to Equation (16.1), any difference of one of the exponents β or γ from 1 would have dramatic consequences.

Equation (16.2) has a surprisingly rich variety of solutions (see Fig. 16.1). If $BN_i(0)^\beta - CN_i(0)^\gamma < 0$, we have either a decay to a finite value, a decay to zero, or an unexpected, delayed decay to zero, depending on whether β is smaller than, equal to, or greater than γ. In the case $BN_i(0)^\beta - CN_i(0)^\gamma > 0$, we find a limited growth for $\beta < \gamma$, and an exponential growth for $\beta = \gamma$, as for the deterministic version

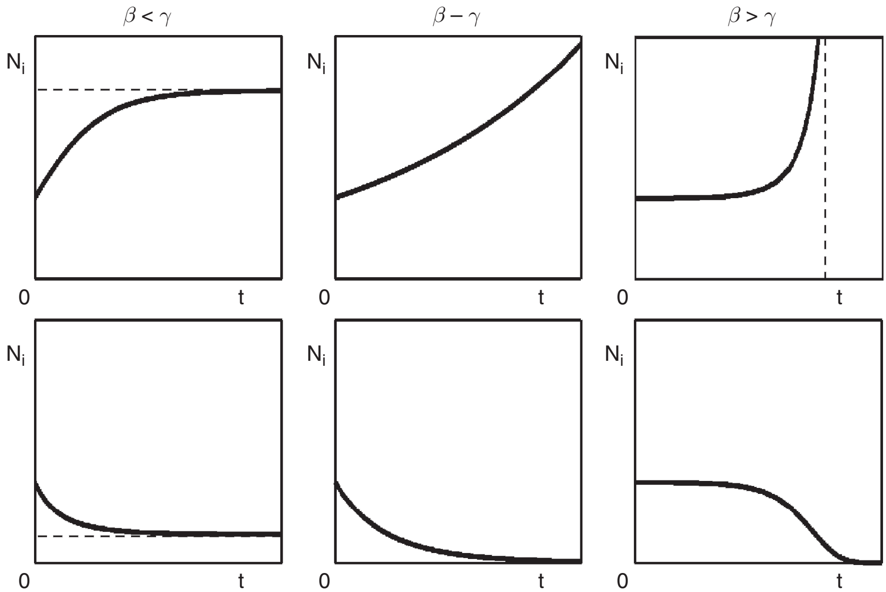

Fig. 16.1 Schematic illustration of the different possible solutions of the growth Equation (16.2). The course of the growth behavior depends on the relations between the parameters β, γ, and on the initial value $N_i(0)$. The *top row* is for $BN_i(0)^{\beta} > CN_i(0)^{\gamma}$, while we have $BN_i(0)^{\beta} < CN_i(0)^{\gamma}$ in the *bottom row*

of Equation 16.1. However, if β were greater than γ, the growth curve would have a singularity, i.e. it would increase without limits within finite time. This possibility would have dramatic implications for urban systems, as the system would sooner or later go out of control. It would also be a distinguishing feature from biological systems, as these are usually characterized by scaling exponents β smaller than 1 and $\gamma = 1$. Moreover, growth processes of biological species sooner or later saturate similar to the curve displayed in Fig. 16.1a.

To determine the nature of urban growth processes, we have analyzed data of European cities to reveal some of the fundamental forces at play in the formation and development of urban organization. Our empirical results show that, in spite of the enormous variation of particular features (climate, economic specialization, age), cities are unified by mechanisms that are on average simple scaling functions of their population size. Our data sets of urban supply systems for European cities i larger than 50,000 inhabitants contained information about the local energy consumption in German cities and Western European points of interest collected by TeleAtlas© for route guidance and geo-information systems. We have evaluated variables X_i and countries for which the data sets were either close to complete or a good statistical representation. The underlying rationale for our empirical investigation was to collect measures of resource production and consumption as a function of urban size, measured in terms of population N_i.

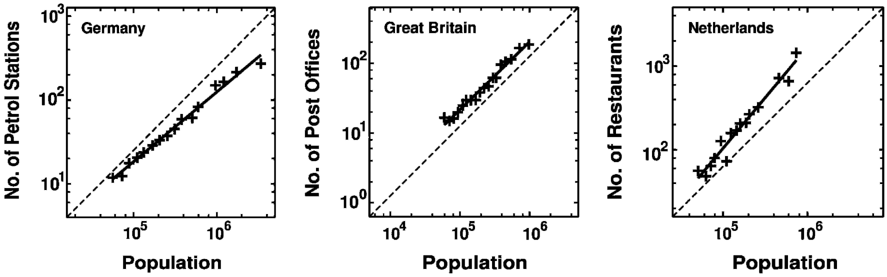

Fig. 16.2 Examples of supply systems with (**a**) sublinear, (**b**) linear, and (**c**) superlinear scaling. The figures show the number of supply stations as a function of the respective population sizes of cities in double-logarithmic representation, using the logarithmic binning method. For details, see Kühnert et al. (2006)

The first empirical fact to emphasize is that scaling is a wide-spread property of urban organization. For most countries, we found power-law scaling relations over two orders of magnitude in population size N^i (see, for example, Fig. 16.2). The scaling relations have the simple form

$$X_i = X_0 N_i^{\beta} \tag{16.3}$$

where N_i is the population size, X_0 a normalization constant independent of N_i, and β the scaling exponent. Our results are summarized in Table 16.1 and Fig. 16.3. For details, see Kühnert, Helbing, D., and West (2006).

Despite the width of the confidence intervals, one can draw several interesting conclusions:

1. The scaling exponents of different countries are consistent, i.e. of the same order. In fact, the 95% confidence intervals tend to have a common subset, which may be used for a more precise determination of the respective scaling exponent, if universality (i.e. country-independence) is assumed. Statistical analysis of variance tests support this picture.
2. A proportionality of the number of "supply stations" to the population size corresponding to a scaling exponent of 1 is only found for some supply systems. This includes hospitals and hospital beds, post offices, and pharmacies.
3. There are also cases of sub-or superlinear relationships. For example, the scaling exponents for the number of car dealers and petrol stations are smaller than 1 (sublinear case), while the scaling exponent for restaurants is larger than 1 (superlinear case).

What are the reasons for observed differences in the scaling exponents? The proportionality of post offices, pharmacies, and doctors to the population size is probably dictated by comparable individual demands, combined with the requirement of a certain level of reachability (by foot). Moreover, it is often regulated by government. As a consequence, each size class of cities offers approximately the same number of these "supply stations."

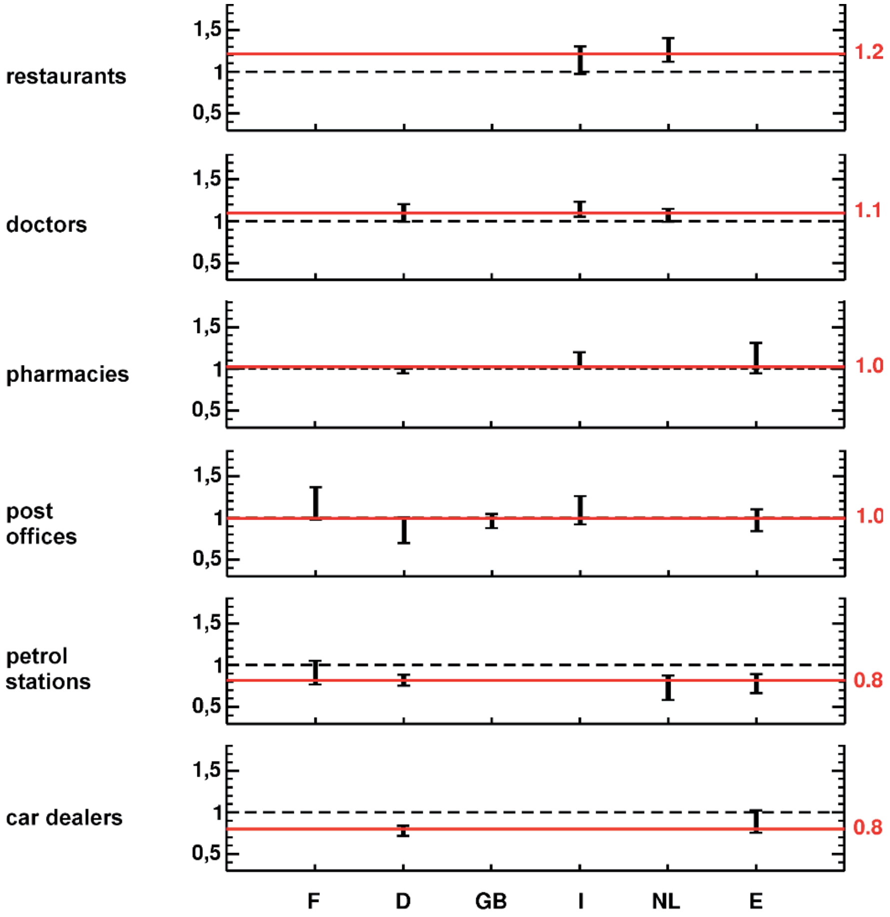

Fig. 16.3 Scaling exponents and confidence intervals for different supply systems and countries: France (F), Germany (D), Great Britain (GB), Italy (I), The Netherlands (NL), and Spain (E) (after Kühnert et al. 2006)

Sublinearly scaling quantities, such as the number of petrol stations or car dealers, indicate an "economy of scales," i.e. efficiency gains by serving larger agglomerations. In other words, one such "supply station" serves more people in a larger town and distributes larger quantities (e.g., sells more fuel per month). This is certainly reasonable and typical for material supply systems, which are more profitable for larger population sizes or governed by a free market. Therefore, sublinear scaling supply systems profit from higher population densities and a more efficient usage of capacities in larger service units (e.g., by better utilization or reduction of the relative statistical variation etc.). Sublinear scaling is also expected for the number of shopping centers or for polyclinics.

But why do some supply systems scale superlinearly? This question concerns, for example, the number of restaurants, but a similar relations seem to hold for museums, theaters, colleges, etc. We recognize that these supply systems satisfy social and communicative needs. That is, information exchange seems to increase overproportional with the number of inhabitants in a town. The number of patents, as a function of the population size (Strumsky, Lobo, & Fleming, 2005), and other variables (Pumain et al., 2006) confirm this conclusion. The same applies to other non-conserved variables such as money or wealth (Bettencourt, Lobo, Helbing, Kühnert, & West, 2007). If these variable would determine city growth, a finite time singularity would be expected (which would have to be avoided by increasing innovation or friction).

16.2 Scaling Laws in Urban Road Networks

Let us now turn to the questions of how the scaling laws we identified relate to spatial structures of urban organization and to fractal features of supply and transportation systems. In biological systems, the power laws mentioned in Section 16.1 can be explained by a minimization of energy losses in the respective biological supply system with the constraint that the supply system is space-filling, as all elements (e.g., all cells in the body) must be reached (West et al., 1997). This organization principle implies hierarchical and self-similar structures such as the system of blood vessels.

Therefore, are urban transportation networks also organized in a hierarchical, self-similar way? Self-similar, fractal features have, in fact, been found in the organization of cities, according to Christaller's (1980) theory of central places, and in the structure of public transportation systems (Frankhauser, 1994; Hołyst, Sienkiewicz, Fronczak, & Suchecki, 2005). The same applies to urban boundaries and urban sprawl (Batty & Longley, 1994; Frankhauser, 1994; Makse, Havlin, & Stanley, 1995; Schweitzer, 1997, 2003). But what about urban road networks?

Despite distances being very crucial for logistic, geographical, and transportation networks, surprisingly little attention has been paid to the spatial structure of urban networks in the past. Urban road networks with links and nodes representing road segments and junctions, respectively, exhibit unique features different from other classes of networks (Newman, 2002; Jiang & Claramunt, 2004; Buhl et al., 2006; Crucitti, Latora, & Porta, 2006; Gastner & Newman, 2006; Porta et al., 2006). As they are almost planar, they show a very limited range of node degrees. Thus, they can never be scale-free like airline networks or the internet (Gastner & Newman, 2006) Nevertheless, road and airline networks can both be viewed as solutions of an optimization process minimizing average travel costs, if the travel costs for one airline connection are approximately equal, but the travel costs of road traffic are proportional to the length of links. Hence, a small-world network with a hub-and-spoke structure results for air traffic, while a Poisson node distribution is typical for road networks (Gastner & Newman, 2006).

For an empirical analysis, we have extracted road network data of the administrative areas of the 20 largest German cities from the geographical database Tele Atlas

MultiNet™, which is typically used for real-time navigation systems or urban planning and management. The data provide a geo-coded polygon for each road segment as well as a series of properties, e.g., the length, average expected travel-time, speed limit, driving direction, etc. Since the road network of Hanover, which ranked 11th, could not be extracted unambiguously, it was excluded from our analysis.

Note that, according to human perception, the effort of traveling is not measured in distances, but in terms of the energy consumption by the body required to perform the travel activity (Kölbl & Helbing, 2003). This means that travel times are the relevant quantities for the destination and route choice of car drivers. This implies that routes along faster roads appear "shorter" than along slower ones. A distant, but well accessible, destination is virtually closer than a near one with a longer access time. The heterogeneity of road speeds also has an impact on the distribution of vehicular traffic over the road network. Faster roads are more attractive for human drivers, resulting in a concentration of traffic along these "arterial" roads, see Fig. 16.4a.

The importance of a road or a junction can be characterized by the number of cars passing through it within some time interval. This can roughly be approximated with the measure of link betweenness centrality, b_e, and node betweenness centrality, b_v. It is given by the number of shortest paths, with respect to travel-time, between all pairs of nodes in the corresponding graph, of which, the particular link e or node v is a part (Albert & Barabási, 2002; Newman, 2002; Brandes & Erlebach, 2005; Costa & da Rocha, 2006; Porta et al., 2006). The road networks of Germany's largest cities show an extremely high node betweenness centrality b_v at only a small number of nodes, while its values are very low at the majority of nodes. As a consequence, the distribution of its frequency density distribution $p(b_v)$ follows a power law $p(b_v) \sim \mathrm{b_v}^{-\delta}$ with exponent $\delta \approx 1.4$ (see Fig. 16.4b and Table 16.2). Note that values of $\delta > 1$ indicate a high concentration of traffic over a few important intersections. In Dresden, for example, 50% of all road meters carry as little as 0.2% of the total traffic volume only, while almost 80% of the total traffic volume are concentrated on no more than 10% of the roads. Most interestingly, half of the total traffic volume is handled by only 3.2% of the roads in the network.

The bundling of traffic streams on a few arterial roads (see Fig. 16.4a) reflects the clear hierarchical structure of the roads (Levinson & Yerra, 2006). However, the usage pattern does not display the regularity of hierarchical networks such as Cayley trees, in contrast to many supply networks in biology, such as vascular systems (Brown & West, 2000).

Is this result just an effect of the respective urban topography? Or is it a result of the fact that the time span of urban evolution is short compared to biological evolution, so that deviations from optimal (resource-efficient) structures occur? Or do the boundary conditions of urban growth change so fast that urban systems are always in a transient state?

The apparent universality of the scaling exponent, $\delta \approx 1.4$, suggests that there must be other reasons for the irregular and not strictly hierarchical structure of urban road networks. Universal scaling laws are, in fact, a very surprising feature of urban road networks in view of all the particularities of cities regarding their history, climate, economic specialization, etc. At least in Germany, universal power laws are

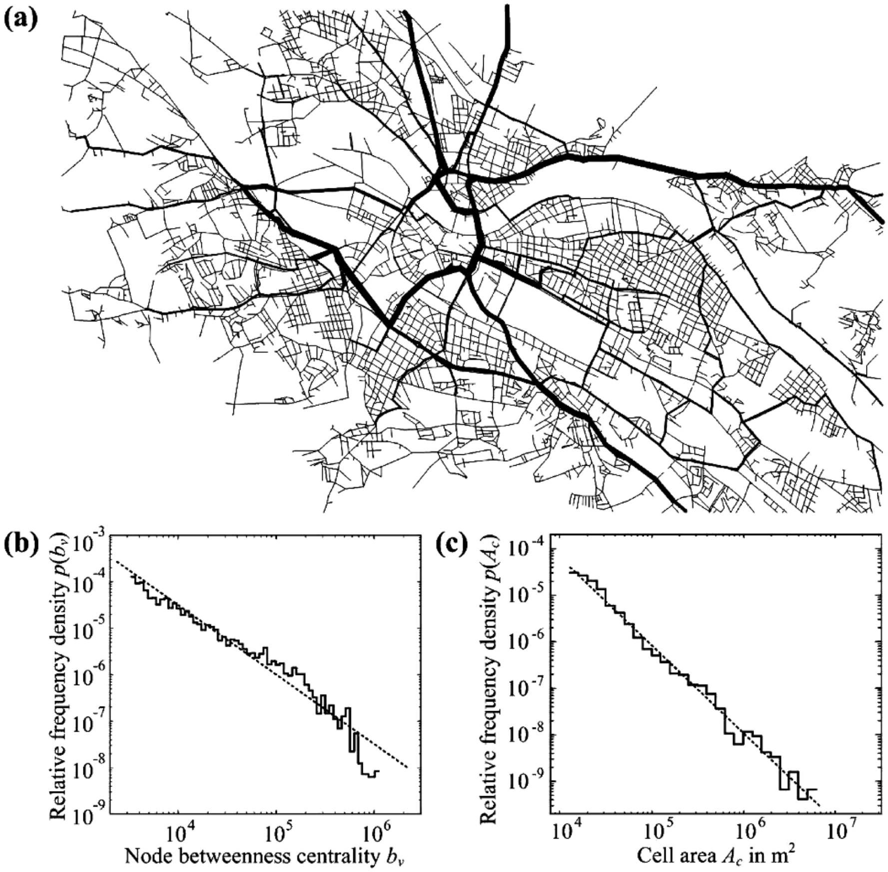

Fig. 16.4 (**a**) Street network of Dresden, Germany. The width of the links represents the respective betweenness centrality be, which is a simple measure of the estimated amount of traffic on the roads. (**b**) The corresponding distribution of the node betweenness centrality bv obeys the power-law $p(b_v) \sim \mathrm{b_v}^{-\delta}$ with exponent $\delta = 1.36$ (*dotted line*). (**c**) The distribution of surface areas enclosed by roads is also power-law distributed with an exponent of 1.89. (After Lämmer et al., 2006)

also found for the size distribution of the areas enclosed by roads (see Fig. 16.4c and the cell size exponent in Table 16.2) and other quantities like the effective dimension or the Gini index (see Lämmer, Gehlsen, & Helbing, 2006 for details).

16.3 Deficiencies of Strictly Hierarchical Organizations

Nodes (intersections) and links (roads) of urban networks are often blocked by building sites, accidents, or congestion. This restricts the reliability of nodes and links considerably. We believe that this is a strong reason for network structures

Table 16.2 The 20 largest cities of Germany and their characteristic coefficients; see main text (after Lammer et al., 2006)

City rank	City name	Population in 1,000	Area in km^2	No. of nodes	No. of links	Betweenness exponent δ	Gini index g	Cell size exponent β
1	Berlin	3, 392	891	37, 020	87, 795	1.481	0.871	2.158
2	Hamburg	1, 729	753	19, 717	43, 819	1.469	0.869	1.890
3	Munich	1, 235	311	21, 393	49, 521	1.486	0.869	2.114
4	Cologne	969	405	14, 553	29, 359	1.384	0.875	1.922
5	Frankfurt	644	249	9, 728	18, 104	1.406	0.873	2.009
6	Dortmund	591	281	10, 326	22, 579	1.340	0.875	1.809
7	Stuttgart	588	208	10, 302	21, 934	1.377	0.894	1.901
8	Essen	585	210	11, 387	24, 537	1.368	0.892	1.932
9	Düsseldorf	572	218	8, 237	16, 773	1.380	0.849	1.964
10	Bremen	543	318	10, 227	21, 702	1.351	0.909	1.931
11	Hanover	517	204	1, 589	3, 463	–	–	–
12	Duisburg	509	233	6, 300	14, 333	1.480	0.900	1.924
13	Leipzig	495	293	9, 071	21, 199	1.320	0.880	1.926
14	Nuremberg	493	187	8, 768	18, 639	1.420	0.854	1.831
15	Dresden	480	328	9, 643	22, 307	1.355	0.870	1.892
16	Bochum	389	146	6, 970	15, 091	1.337	0.847	1.829
17	Wuppertal	364	168	5, 681	11, 847	1.279	0.881	1.883
18	Bielefeld	325	259	8, 259	18, 280	1.337	0.872	1.735
19	Bonn	309	141	6, 365	13, 746	1.374	0.889	2.018
20	Mannheim	309	145	5, 819	12, 581	1.455	0.897	1.959

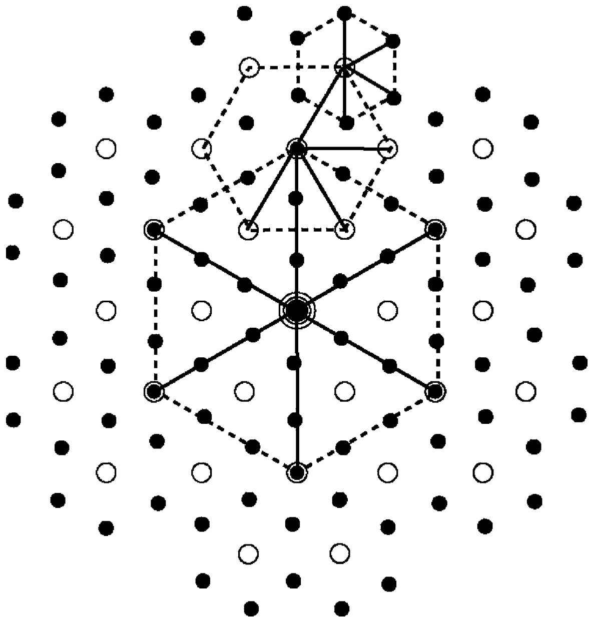

Fig. 16.5 Illustration of a strictly hierarchical "arterial" road network capable of connecting all cities, assuming a spatial organization according to Christaller's theory of central places. Note that not all links are shown here in order to avoid an overloaded picture

that are not organized in a strictly hierarchical manner (in contrast to Fig. 16.5). As will be illustrated for the example of information flows in organizations, strict hierarchies are only optimal under certain conditions, particularly a high reliability of nodes and links.

Experimental results on the problem solving performance of groups (Ulschak, 1981; Tubbs, 2003) show that small groups can find solutions to difficult problems faster than any of their constituting individuals, because groups profit from complementary knowledge and ideas. Small groups also have a potential to assess situations and future developments better than their single members (Chen, Fine, & Huberman, 2003). The actual performance, however, sensitively depends on the organization of information flows, i.e., on who can communicate with whom. If communication is unidirectional, for example, this can reduce performance. However, it may also be inefficient if everybody can talk to everyone else. This is, because the number of potential (bidirectional) communicative links grows like $N(N-1)/2$, where N denotes the number of group members. As a consequence, the number of information flows explodes with the group size, which may easily overwhelm the communication and information processing capacity of individuals. This explains the slow speed of group decision making, i.e. the inefficiency of committees. It is also responsible for the fact that, after some transient time, (communication) activities in large (discussion) groups often concentrate on a few members only, which reminds of the bundling of traffic flows discussed in the last section. A similar effect is observed in insect societies such as bee hives. When a critical colony size is exceeded, a few members develop hyperactivity, while most colony members become lazy (Gautrais, Theraulaz, Deneubourg, & Anderson, 2002).

These findings indicate that there may be an optimal size of companies and organizations (Huberman & Loch, 1996). Considering the limited communication and information processing capacities of individuals, the optimal number of group members seems to be seven (or less) (Miller, 1956; Baddeley, 1994). This implies the need for bundling and compressing information flows, which is, for example, satisfied by hierarchical organizations. But are there better forms of organization than strictly hierarchical ones? Some of the relevant questions are:

- How robust is the communication or organization network with respect to failure of nodes (due to illness, holidays, quitting the job) or links (due to difficulty personal relationships)?
- How suitable is the organization for crisis management?
- How well does an organization interconnect interrelated activities?
- What is the degree of information loss when communication within an organization network is imperfect?

Similar to road networks and biological supply networks (such as the respiratory system), organizations must be organized space-filling in their covered competence field with staff members playing the role of terminal units. For matters of illustration, we will focus on regular, two-dimensional space-filling kinds of subdivision, as they are particularly suited for a modular organization structure. They share some properties with urban road networks, while the tree-like organization of arterial, water or respiratory supply systems in biological species is three-dimensional (West et al., 1997).

Regular area-filling kinds of subdivision can be triangular, quadratic, or hexagonal. These subdivisions are all compatible with a strictly hierarchical organization, see Fig. 16.6. If the top level consists of one individual (the CoE) and each member of a certain level, except for the lowest one, has N_D subordinates, the number of staff members in a system with L hierarchies is given by

$$N = \sum_{l=1}^{L} N_D^{l-1} = \frac{N_D^L - 1}{N_D - 1} \tag{16.4}$$

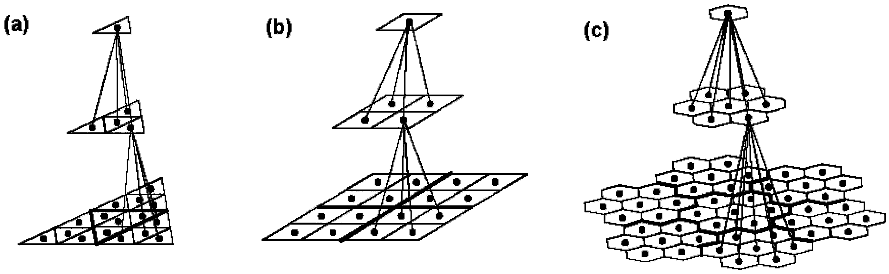

Fig. 16.6 Examples of strict hierarchies based on (**a**) a triangular, (**b**) a quadratic, or (**c**) a hexagonal area-covering organization. Dots represent staff members, while the links indicate the communication pathways (after Helbing, Johansson, et al., 2006)

Table 16.3 Number of members in a hierarchical organization as a function of the hierarchy levels, when everyone (apart from the lowest organizational level) has four or seven subordinates, respectively (after Helbing, Johansson, et al., 2006)

	Four subordinates		Seven subordinates	
Levels	No. of members	Cumulative no.	No. of members	Cumulative no.
1	1	1	1	1
2	4	5	7	8
3	16	21	9	57
4	64	85	343	400
5	256	341	2, 401	2, 801
6	1, 024	1, 365	16, 807	19, 608
7	4, 048	5, 413	117, 649	137, 257
8	16, 192	21, 605	823, 543	960, 800

(see Table 16.3). While triangular and quadratic structures correspond to $N_D = 4$, subordinates, hexagonal structures are compatible with $N_D = 4, 5, 6$, or 7 (Helbing, Johansson, Mathiesen, Jensen, & Hansen, 2006). As a consequence, for a given number N of individuals, the number of hierarchical levels can be reduced by a hexagonal kind of organization (see Table 16.3). Note, however, that a strictly hierarchical organization of the road system for Christaller's (1980) hexagonal system of central places corresponds to $N_D = 4$ (see Fig. 16.5); otherwise some cities would have multiple access routes.

As the number of hierarchical levels reflects the number of intermediate steps from the bottom level to the top (and vice versa), on the one hand, it is desirable to have a small number of hierarchical levels ("flat hierarchies") to minimize information delays. On the other hand, assuming that the information compression is roughly proportional to the inverse $1/N_D$ of the number N_D of subordinates, flat hierarchies have a higher degree of information loss from one hierarchical level to the next higher one. (This assumes that a fixed amount of communication and information processing capacity is basically divided among the number of subordinate. Given a fixed number N of members of an organization, let us calculate the probability P that certain information from the basis (i.e., the lowest hierarchical level) reaches the top level (or vice versa). Considering that information is compressed by a factor of $1/N_D$ from one level to the other and lost with some probability $p > 0$, we get

$$P = (1-p)^{L-1}\left(\frac{1}{N_D}\right)^{L-1} = (1-p)^{L-1}\left(\frac{1}{N_D^{L-1}}\right) \approx (1-p)^{L-1}\frac{1}{N} \qquad (16.5)$$

because of $N \approx {N_D}^{L-1}$. That is, the larger the number of hierarchy levels, the greater the chance that some potentially relevant information from the bottom level will never reach the top level.

Let us now discuss how the value of P can be improved by redundant information flows. In disaster response management, strictly hierarchical organizations tend to show certain weaknesses with potentially serious consequences:

- Important information is lost due to information compression.
- Information takes too much time to get from the sender to the intended receiver because of too many hierarchical levels to be crossed.
- Information never reaches its destination, because some information node or link does not function.

These weaknesses can be mitigated by additional side links (information flows within the same hierarchy level) and shortcuts between different hierarchy levels (see Fig. 16.7). Since the information flow, over an existing communication link, is basically proportional to $(1 - p)$, redundant links will always reduce the probability that information is lost, as additional information channels are available. However, establishing and maintaining additional information channels is costly, at least it

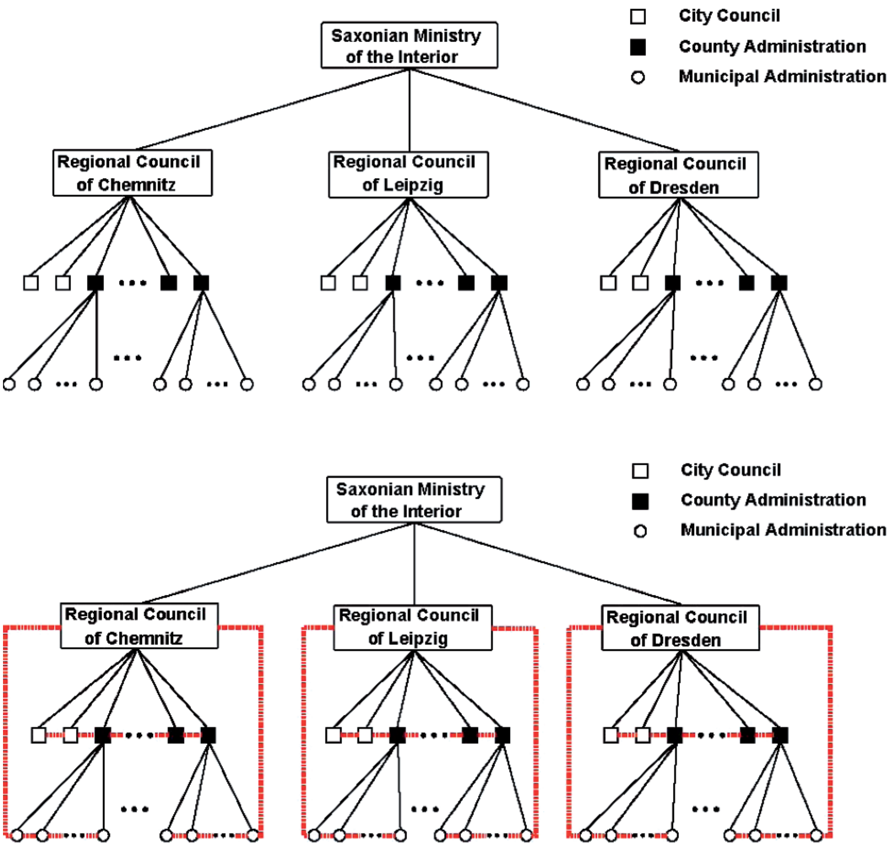

Fig. 16.7 *Top*: Illustration of the hierarchical information flows in the disaster response management during the floodings in Saxony, Germany, in August 2002 (after Helbing, Johansson, et al., 2006). *Bottom*: Improved information flows can be reached by additional side links and shortcuts (*dashed thick lines*)

requires time. Therefore, their optimum number depends on the reliability of nodes and links. Generally speaking, it increases with the failure rate.

One interesting question is how to establish the most urgently needed links. If some information link breaks down or does not function properly, an alternative information link should be used or established. The same applies, if the capacity of some information channel is not high enough (e.g., due to a lack of communication time or communication ability). That is, information channels should be adaptively strengthened, when needed. This can either be done by extending or redistributing communication times or by establishing additional links.

Regarding the identification of missing links, it is interesting to see how Amazon (www.amazon.com) recommends books to customers, based on their previous purchase decisions and those of other customers. This method is based on an evaluation of correlations among different purchasing activities. A similar method has been recently suggested by Adamic and Adar (2003), who have identified missing links by analysis of e-mail communication. In some sense, it is recommended to establish a new link (a "shortcut"), if it would reduce information flows via many nodes, i.e. if it would reduce detours.

16.4 Spontaneous Self-Organization of Hierarchies

Note that it can be difficult to establish a hierarchical organization. Social systems are complex systems in which the non-linear interactions between its individuals can dominate efforts to control their behavior. However, a hierarchical organization can often emerge by self-organization of its elements. One example is the phenomenon of "crowd turbulence." Fruin, 1993) reports:

> At occupancies of about seven persons per square meter the crowd becomes almost a fluid mass. Shock waves can be propagated through the mass, sufficient to ... propel them distances of three meters or more. ... People may be literally lifted out of their shoes, and have clothing torn off. Intense crowd pressures, exacerbated by anxiety, make it difficult to breathe, which may finally cause compressive asphyxia. The heat and the thermal insulation of surrounding bodies cause some to be weakened and faint. Access to those who fall is impossible. Removal of those in distress can only be accomplished by lifting them up and passing them overhead to the exterior of the crowd.

This drastic picture visualizes the conditions in extremely dense crowds quite well, but it does not provide a scientific analysis and interpretation.

Our detailed analysis of video recordings of the pilgrimage in Mecca has now revealed how extremely dense crowds, after a previous transition from laminar flows to stop-and-go waves (Helbing, Ammoser, & Kühnert, 2006), develop a turbulent dynamics characterized by random displacements of pedestrians into all possible directions (see Fig. 16.8a). These displacements are measured as the distance moved between two successive stops of a pedestrian and can reach magnitudes up to 12 m or more (see Fig. 16.8b).

We suggest comparing extreme crowding with driven granular media of high density. Under quasi-static conditions (Radjai & Roux, 2002), these are building up

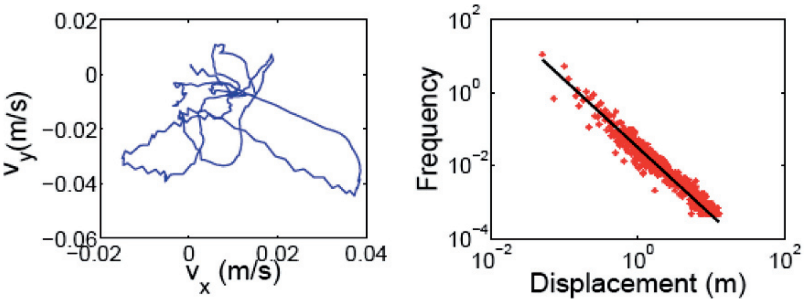

Fig. 16.8 *Left*: Typical time-dependence of both components of velocity in the course of time during turbulent crowd motion. One can clearly see the motion into all possible directions, including the change from forward to backward motion. *Right*: The double-logarithmic representation of the frequency of differently sized displacements between stopping events reveals a power law. (After Helbing, Johansson, & Al-Abideen, (2007)

force chains (Cates, Wittmer, Bouchaud, & Claudin, 1998), which are characterized by strong fluctuations in the strengths and directions of the local forces. As in earthquakes (Bak, Christenson, Danon, & Scanlon, 2002; Johnson & Jiz, 2005), this can lead to events of sudden, uncontrollable stress release with power-law distributed displacements. Such a power-law has, in fact, been discovered by our video-based crowd analysis (Fig. 16.8b). It indicates a self-similar behavior. However, instead of the vortex cascades in turbulent fluids, one observes another kind of hierarchical organization: at extreme densities, individual motion is replaced by collective motion. That is, there are groups of people moving at the same speed. These groups form clusters moving at similar speeds, which again form larger clusters, etc.

Note that the spontaneous formation of hierarchies is quite typical in social systems: individuals form groups, which form companies, organizations, and parties, which make up a society or nation. A similar situation can be found in biology, where organelles form cells, cells form tissues, tissues form organs, and organs form bodies. Another example is well-known from physics, where elementary particles form nuclei, which combine to atoms with electrons. The atoms form chemical molecules, which organize themselves as solids. These make up celestial bodies, which form solar systems, which again establish galaxies.

Obviously, the non-linear interactions between the different elements of the system give rise to a formation of different levels, which are hierarchically ordered one below another. While changes on the lowest hierarchical level are fastest, changes on the highest level are slow.

On the lowest level, we find the strongest interactions among elements. This is obviously the reason for the fast changes on the lowest hierarchical level. If the interactions are attractive, bonds will arise. These cause the elements to behave no longer completely individually, but to form units representing the elements of the next level. Since the attractive interactions are more or less "saturated" by the bonds, the interactions within these units are stronger than the interactions between them. The relatively weak residual interactions between the formed units induce their relatively slow dynamics. Consequently, a general interdependence between the interaction strength, the changing rate, and the formation of hierarchical levels can be found.

16.5 Summary and Conclusions

In this chapter, we have started with an empirical study of urban supply networks. We have found various power laws: While a linear scaling with the population size was found for the number of doctors or pharmacies in a city, quantities like petrol stations, supermarkets or hospitals scale sublinearly, indicating an economy of scales. Non-material quantities such as information, money, or social interactions, however, scale superlinearly. If these factors determine the speed of urban growth, this implies a finite-time singularity which can only be avoided by friction or innovation (Bettencourt et al., 2007).

Furthermore, we have compared urban systems with biological and ecological systems. Despite some interesting analogies, the differences are significant. For example, there is no strict hierarchical organization of urban transport networks. Nevertheless, we find power-laws for the distribution of traffic flows and the distribution of areas enclosed between urban roads. The power-law exponents are universal, at least for Germany's 20 largest cities.

A deviation from a strictly hierarchical organization is reasonable when the functioning of the nodes or links of a network is not reliable (e.g., due to failures). In such cases, redundant links (side links and shortcuts) increase the robustness of the system. However, the possibilities to design an urban or social system are limited, as their organization and dynamics is, to a large extent, the result of self-organization. Nevertheless, hierarchies result quite naturally based on non-linear interactions among the system elements. As an example, we have discussed the case of "turbulence" in extremely dense pedestrian crowds.

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